

Probing the equation of state of dense matter with neutron stars

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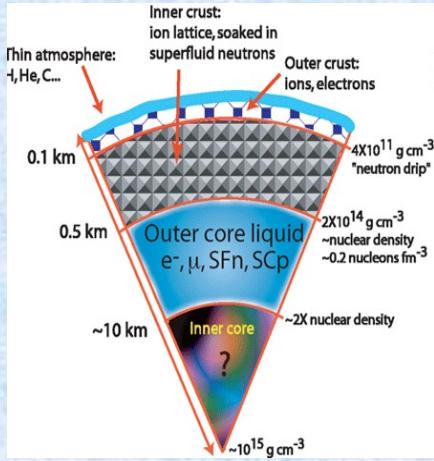
Outline

- ❖ Introduction and motivation
 - Inhomogeneous matter in astrophysical (compact-star) context
 - Theoretical approach (brief introduction)
- ❖ Some (selected) examples : recent results
 - NS crust at $T = 0$: pasta-phase, impact of empirical parameters, global structure
 - NS crust at finite T : impurities in (proto-)NS
 - Accreting NS crust
- ❖ Conclusions & outlooks



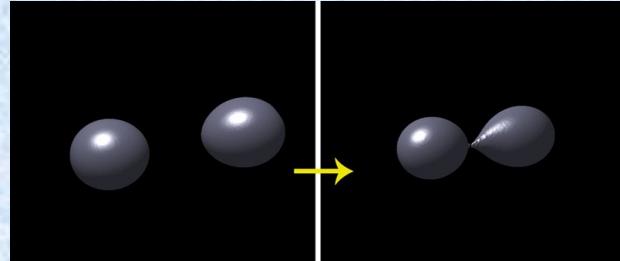
(In)homogeneous matter (at $T = 0$)

Mature neutron stars (NSs)



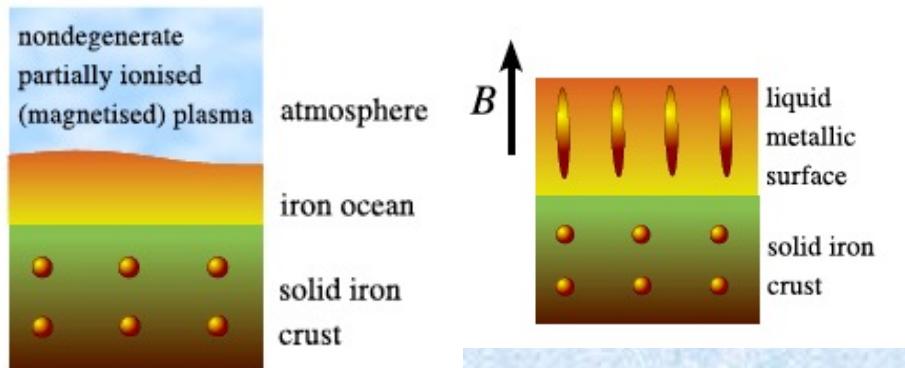
<http://www.physics.montana.edu>

Binary NS mergers : approx. OK for inspiral phase



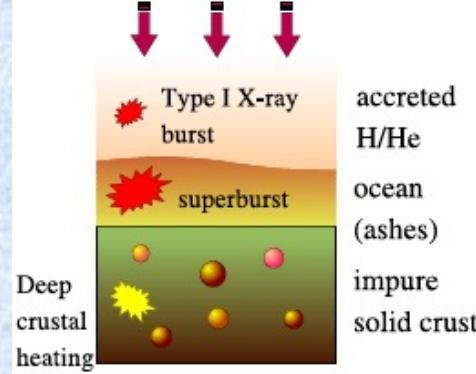
Simulation MPA Garching
(Goriely, Bauswein, Janka, ApJ 738, 2011)

Magnetised NSs



Weakly and strongly magnetised surface

Accreting NSs (if cold enough)



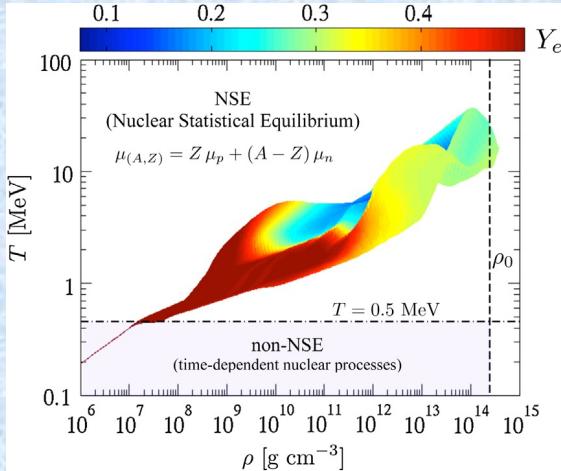
Chamel & Hansel, Liv. Rev. Relat. 11, 10 (2008)

N.B.: apart from accreting NSs, (beta-) equilibrium is assumed!



(In)homogeneous matter (at finite T)

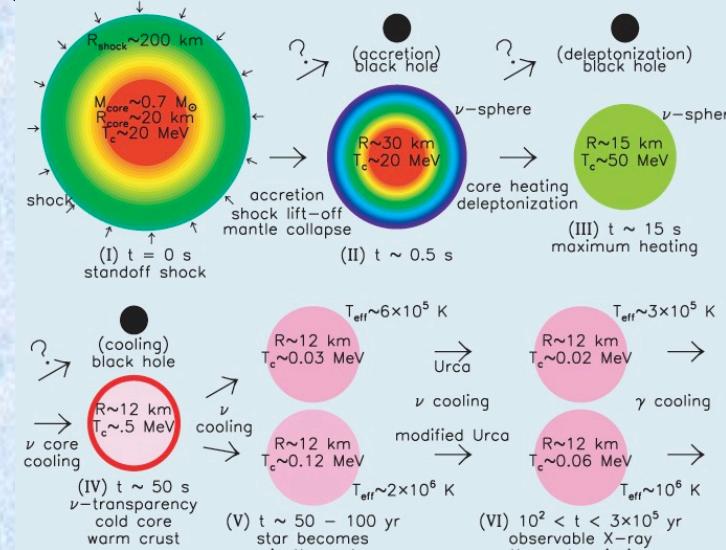
Core-collapse supernovae (SNe)



Simulation T. Fischer

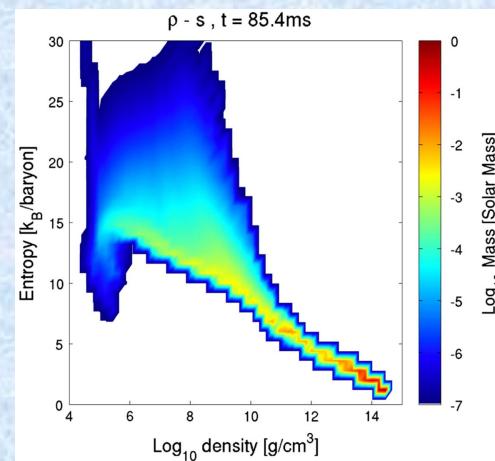
Oertel et al., Rev. Mod. Phys. 89, 015007 (2017)

(Proto-) NS crust & cooling



Lattimer & Prakash, Science 304, 536 (2004)

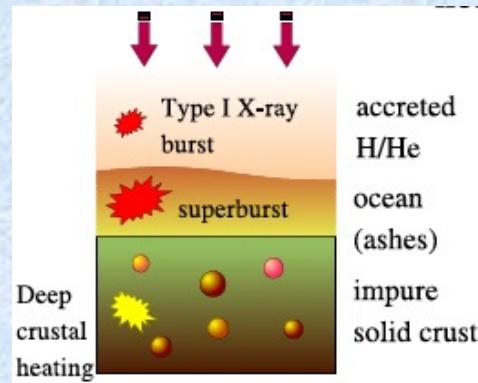
Binary NS mergers



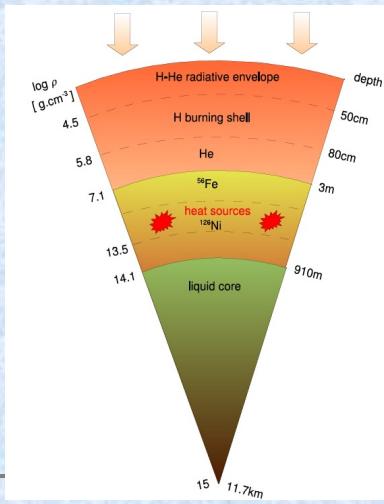
Simulation A. Perego

Oertel et al., Rev. Mod. Phys. 89, 015007 (2017)

Accreting NSs



Chamel & Hansel, Liv. Rev. Relat. 11, 10 (2008)

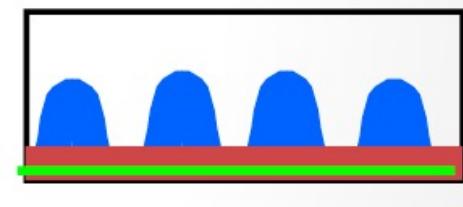


N.B.: also off-(beta-) equilibrium !

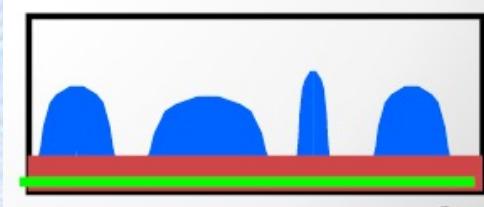


Which approach ?

- Which theoretical framework for clustered matter (at finite T) ?
 - *ab-initio methods*, but: not affordable for inhomogeneous matter in all range
 - *phenomenological methods* → **Nuclear energy-density functional theory**
- One-component (OCP) single-nucleus (SNA) vs Multi-component (MCP)



→ energy minimisation gives favoured *cluster* (A, Z)



→ energy minimisation gives favoured *cluster distribution*

$$p(A, Z) \propto \exp \left[-(k_B T)^{-1} (F_N + \delta\Omega - \mu_n N - \mu_p Z) \right]$$

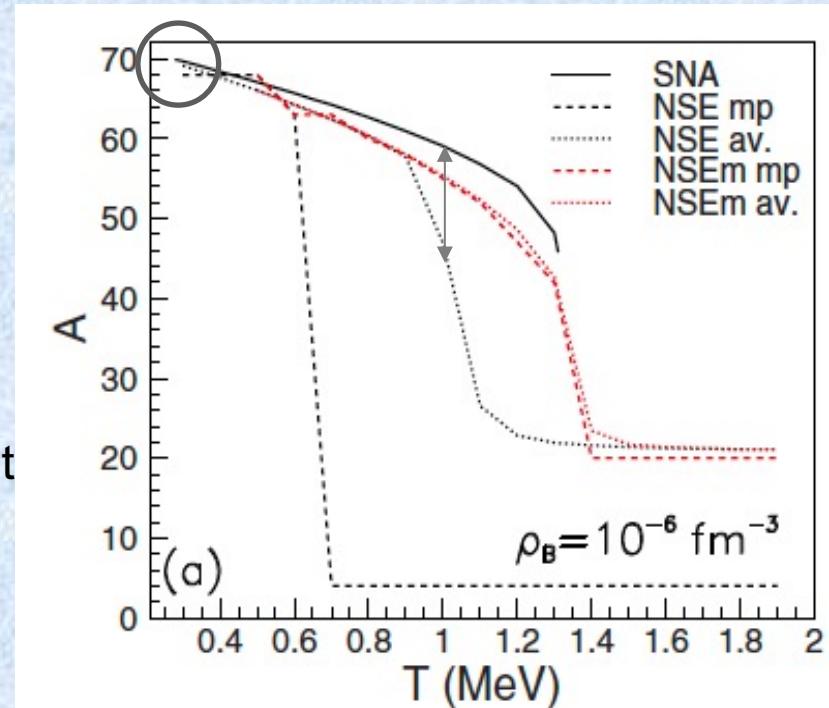
- ✓ OCP OK at $T = 0$ for thermodynamic quantities and faster computationally
- ✗ but: at finite T more microstates populated → nuclear distributions
- ✗ MCP needed to compute reaction (electron-capture) rates (CCSN, cooling, ...)



Treatment of clusters

→ Need to go beyond the standard Wigner-Seitz (one-component) approximation

- Possible strategies:
 1. Micro calculations (MD, TDHF)
→ computationally expensive
(e.g. Sebille et al., 2009, 2011;
Nandi & Schramm 2018, ...)
 2. Statistical (NSE) models
→ cluster degrees of freedom
→ more flexible but more difficult to treat
beyond mean-field effects
(e.g. Gullminelli & Raduta 2015,
Grams et al. 2018, Pais et al. 2020, ...)



Gulminelli & Raduta, PRC 92, 055803 (2015)

for a review: Oertel et al., Rev. Mod. Phys. 89, 015007 (2017);

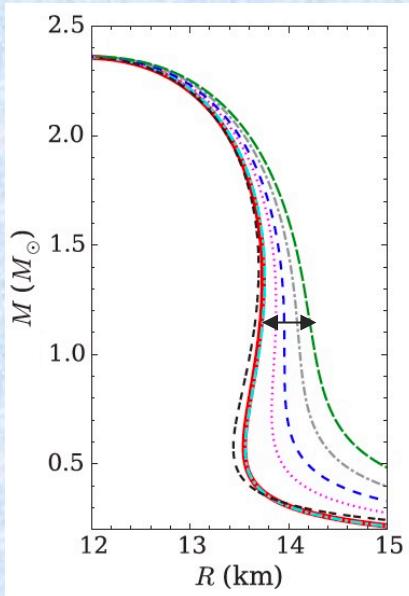
Burgio & Fantina, Astrophys. Space Sci. Lib. 457, 255 (2018)



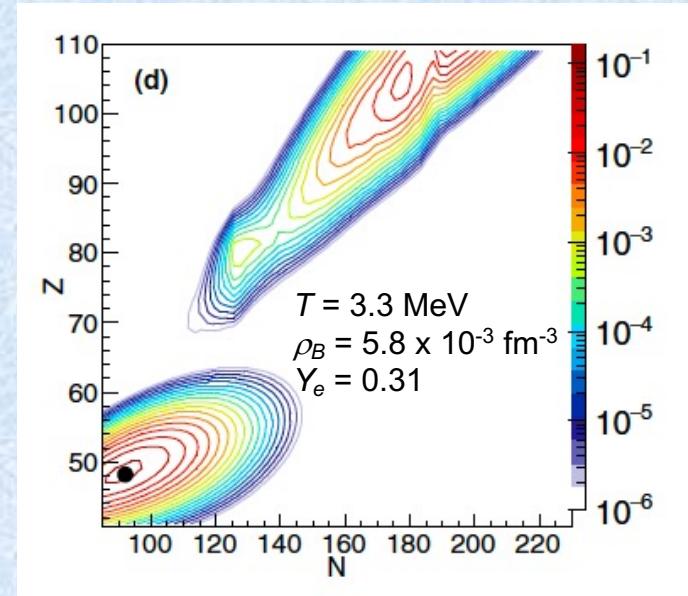
Why a unified treatment ?

Unified treatment of finite nuclei & infinite matter
→ same nuclear model employed in different regions of star

- Challenging because of wide range of density, temperature, isospin asymmetry
- Challenging because different states of matter (cluster, “pasta”, homogeneous matter)
- But: essential to avoid spurious non-physical effects in numerical modelling



Fortin et al., PRC 94, 035804 (2016); $T = 0$

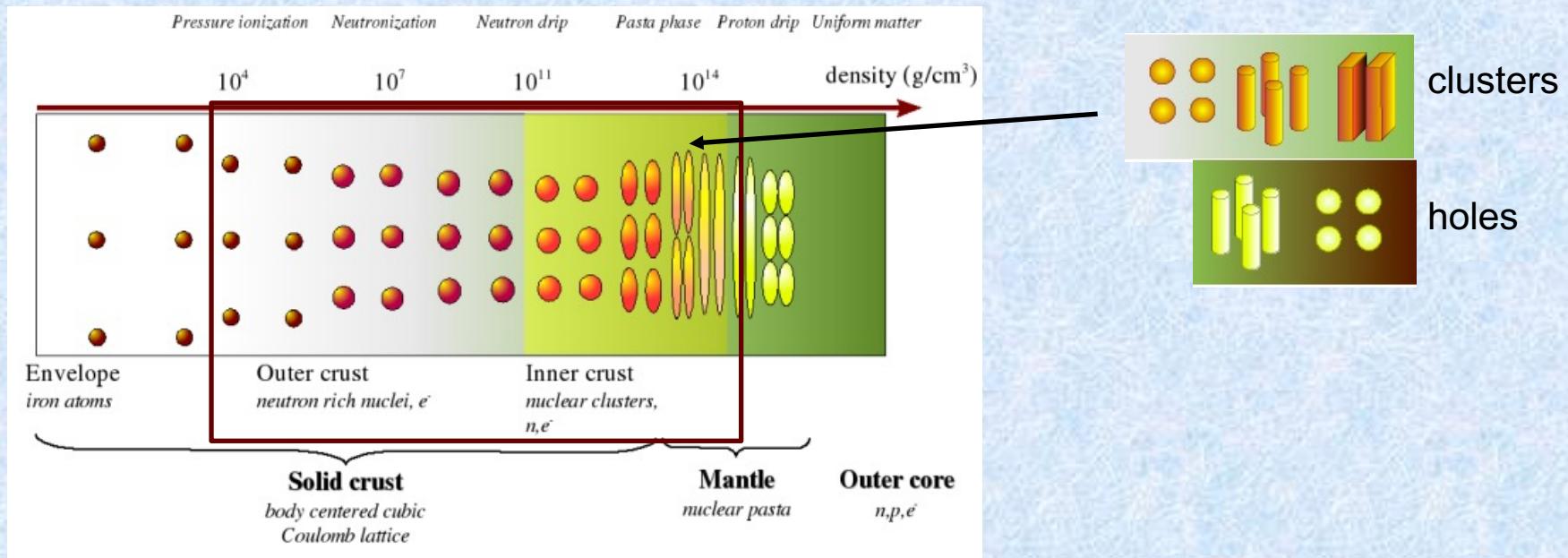


Grams et al., PRC 97, 035807 (2018); CCSN, $T \neq 0$



NS crust ($T = 0$)

“Cold” (catalysed) NS ($T = 0$ approx., full equilibrium)



Chamel & Haensel, Liv. Rev. Relativ. 11, 10 (2008)

see also : Chamel & Blaschke, ASSL 457, 337 (Springer, 2018)

→ possible existence of “pasta” layer
at the bottom of the in the crust



Crust properties: meta-model

CLD approach :

- **Meta-model** approach for nucleons (nucleons only considered): flexible functional (“quasi” agnostic)
→ expansion in density and asymmetry around n_{sat} and $\delta = 0$ (with m_q^* included)

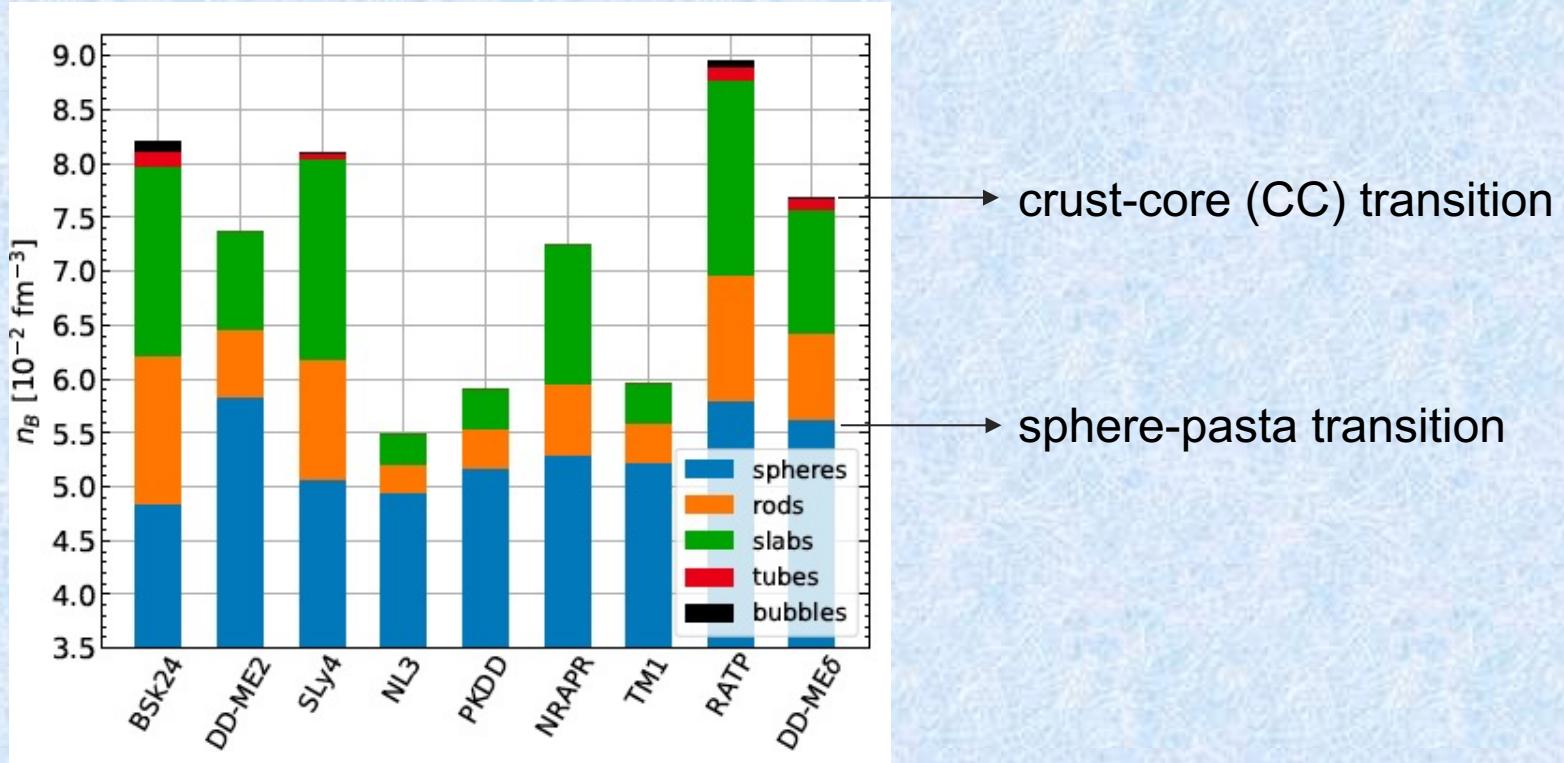
$$\epsilon_B(n, \delta) \approx n \sum_{m=0}^4 \frac{1}{m!} \left(\left. \frac{d^m e_{\text{sat}}}{dx^m} \right|_{x=0} + \left. \frac{d^m e_{\text{sym}}}{dx^m} \right|_{x=0} \delta^2 \right) x^m \quad x = (n - n_{\text{sat}})/3n_{\text{sat}} \\ \delta = (n_n - n_p)/n$$

Empirical parameters (bulk) $\mathbf{X}_{\text{sat,sym}} = E_{\text{sat}}, K_{\text{sat}}, Q_{\text{sat}}, E_{\text{sym}}, L_{\text{sym}}, K_{\text{sym}}, \dots$ } 13 bulk
+
5 surface parameters

- + surface and Coulomb term → surface parameters ($\sigma_0, \sigma_{0,c}, \beta, b_s, p$)
→ dimensionality of pasta only enters here
→ fixed by fit to nuclear masses (AME2016)



Pasta-phase properties



Dinh Thi et al., A&A 654, A114 (2021)

→ Pasta predicted (robustly) but model dependences



Crust properties: Bayesian analysis

$$p_{\text{post}}(\vec{X}) = \mathcal{N} w_{\text{LD}}(\vec{X}) w_{\text{HD}}(\vec{X}) e^{-\chi^2(\vec{X})/2} p_{\text{prior}}(\vec{X})$$

Low-Density filters
→ ab-initio (EFT)

(Drischler et al, PRC 93,
054316 (2016))

High-Density filters
→ causality, stability,
 $M_{\text{NS,max}}, e_{\text{sym}} > 0$

flat non-informative prior
→ span large parameter space

nuclear masses
(AME2016)
→ surf. param. ($\sigma_0, \sigma_{0,c}, \beta, b_s, p$)

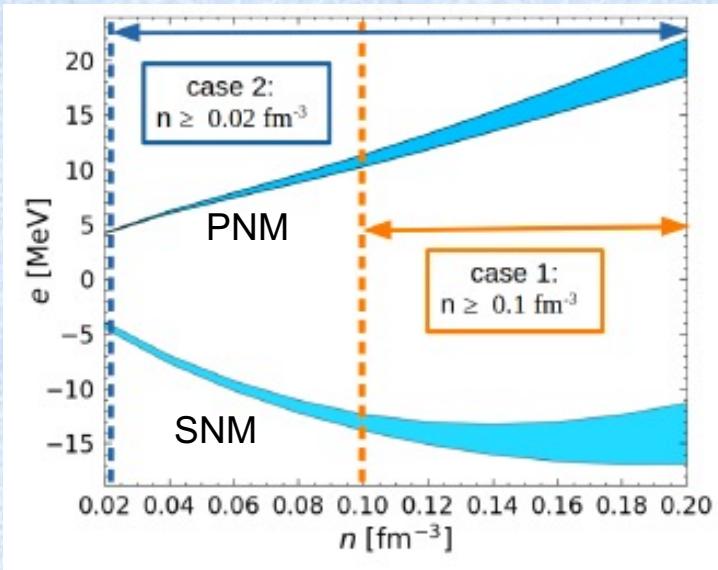
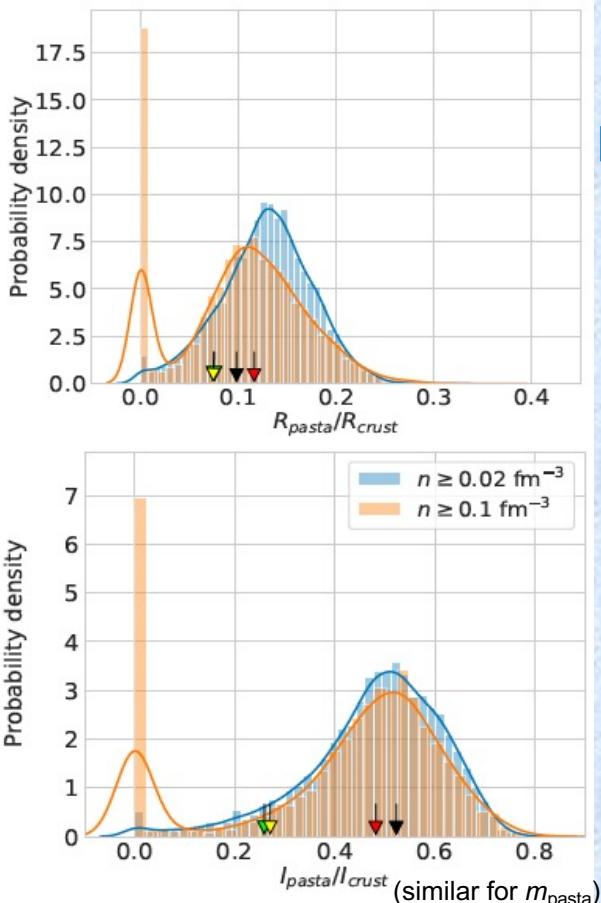


Fig. courtesy of H. Dinh Thi

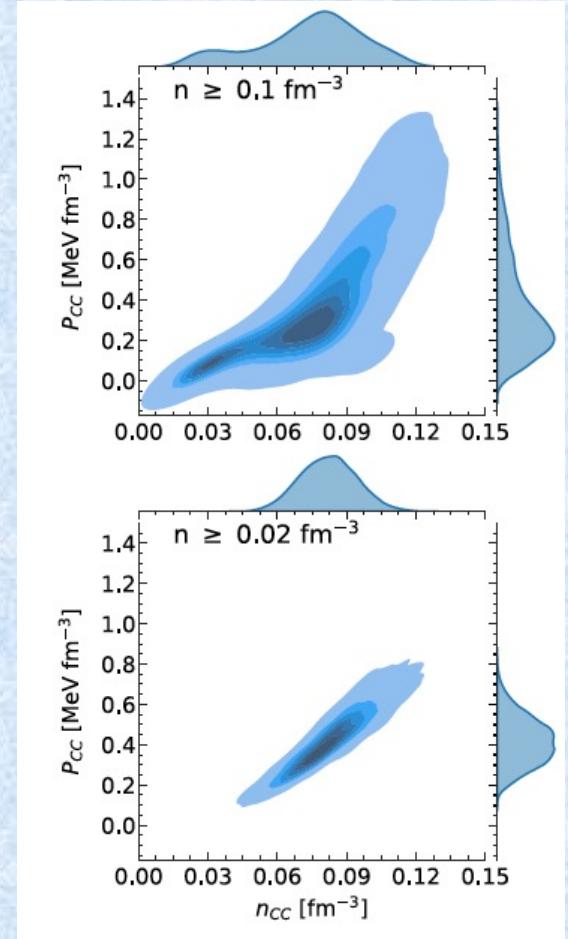


Pasta-phase properties, CC transition



- $l_{\text{pasta}} \sim 0.5 l_{\text{crust}}$;
- $m_{\text{pasta}} \sim 0.5 m_{\text{crust}}$;
- $R_{\text{pasta}} \sim 0.1 R_{\text{crust}}$
- models predicting almost no pasta filtered out

uncertainties reduced when constraining from lower densities



Dinh Thi et al., EPJA 57, 296 (2021)

Dinh Thi et al., A&A 654, A114 (2021)

→ importance of (low-density) nuclear physics constraints



Correlations & empirical parameters

n_{CC}															
LD+HD ($n \geq 0.02 \text{ fm}^{-3}$)	-0.04	-0.07	0.11	-0.05	-0.02	-0.30	-0.57	-0.15	0.45	-0.15	0.05	0.52	-0.15	-0.04	0.51
LD+HD ($n \geq 0.1 \text{ fm}^{-3}$)	-0.06	-0.06	0.33	-0.46	0.17	-0.15	-0.29	-0.10	0.39	-0.16	0.06	0.34	-0.11	-0.08	0.33
Prior	0.14	0.09	0.13	-0.18	0.02	0.08	-0.56	0.11	0.20	-0.05	-0.17	0.07	0.29	0.18	0.18
	E_{sat}	n_{sat}	K_{sat}	Q_{sat}	Z_{sat}	E_{sym}	L_{sym}	K_{sym}	Q_{sym}	Z_{sym}	σ_0	b_s	σ_{0c}	β	p
n_p															
LD+HD ($n \geq 0.02 \text{ fm}^{-3}$)	-0.86	0.20	0.32	-0.27	0.02	0.32	-0.20	-0.25	0.22	-0.00	0.87	0.08	-0.73	-0.84	-0.09
LD+HD ($n \geq 0.1 \text{ fm}^{-3}$)	-0.44	0.03	0.39	-0.42	0.14	0.18	-0.19	-0.40	0.45	-0.14	0.44	0.02	-0.36	-0.47	-0.04
Prior	-0.28	0.01	0.09	-0.11	0.03	-0.04	0.06	-0.49	0.44	-0.06	0.29	0.06	-0.26	-0.22	-0.04
	E_{sat}	n_{sat}	K_{sat}	Q_{sat}	Z_{sat}	E_{sym}	L_{sym}	K_{sym}	Q_{sym}	Z_{sym}	σ_0	b_s	σ_{0c}	β	p
$I_{\text{pasta}}/I_{\text{crust}}$															
LD+HD ($n \geq 0.02 \text{ fm}^{-3}$)	0.59	-0.21	-0.17	0.12	-0.02	-0.43	-0.22	0.09	0.15	-0.09	-0.59	0.48	0.42	0.59	0.66
LD+HD ($n \geq 0.1 \text{ fm}^{-3}$)	0.27	-0.10	0.09	-0.30	0.15	-0.26	-0.05	0.19	0.07	-0.06	-0.28	0.42	0.15	0.26	0.51
Prior	0.37	0.01	0.04	0.06	0.07	0.20	-0.05	0.35	0.29	0.08	-0.40	-0.01	0.45	0.36	0.24
	E_{sat}	n_{sat}	K_{sat}	Q_{sat}	Z_{sat}	E_{sym}	L_{sym}	K_{sym}	Q_{sym}	Z_{sym}	σ_0	b_s	σ_{0c}	β	p

Dinh Thi et al., EPJA 57, 296 (2021)

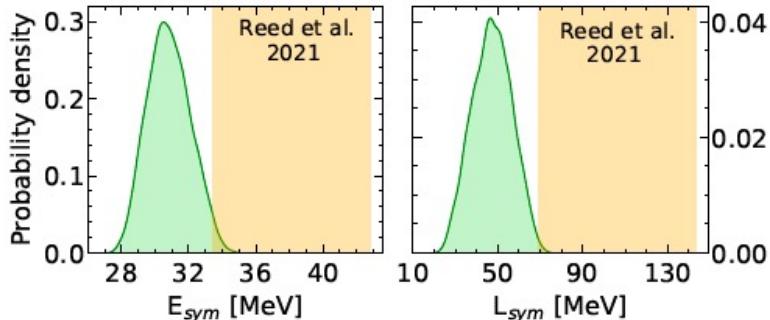
bulk

surface

→ Importance of parameters (both bulk and surface) on observables

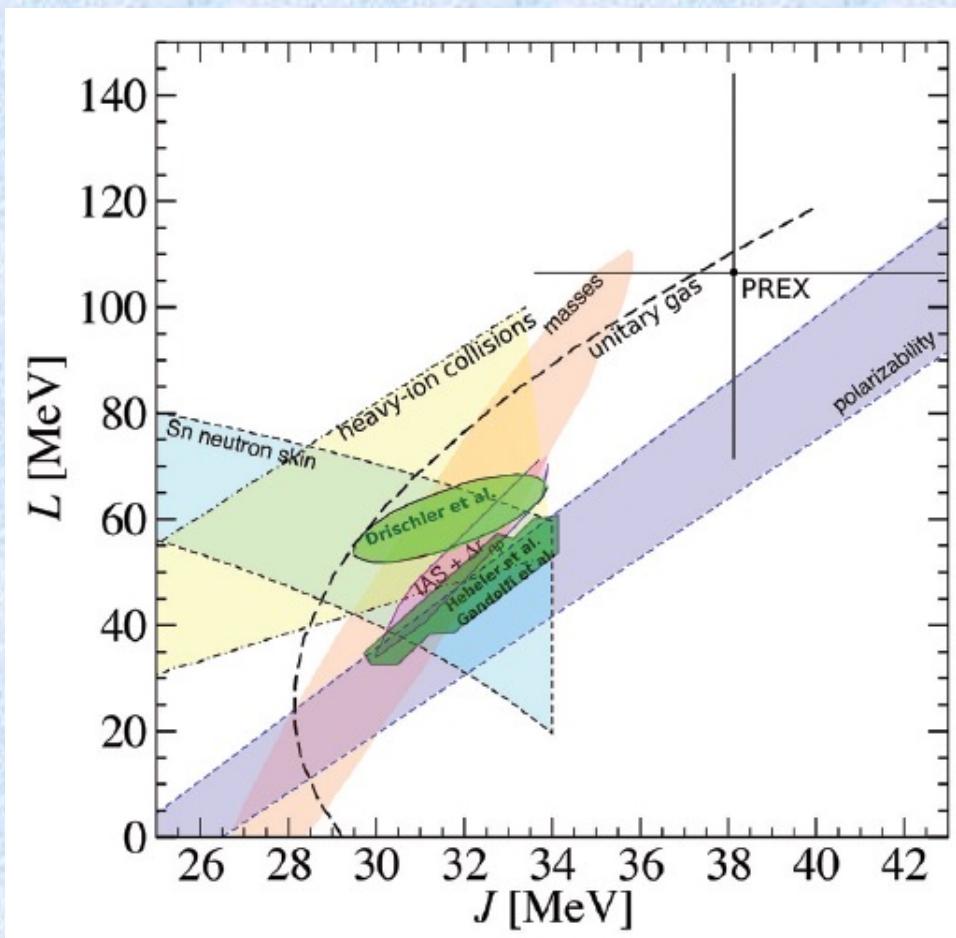


Symmetry energy parameters



Dinh Thi et al., A&A 654, A114 (2021)

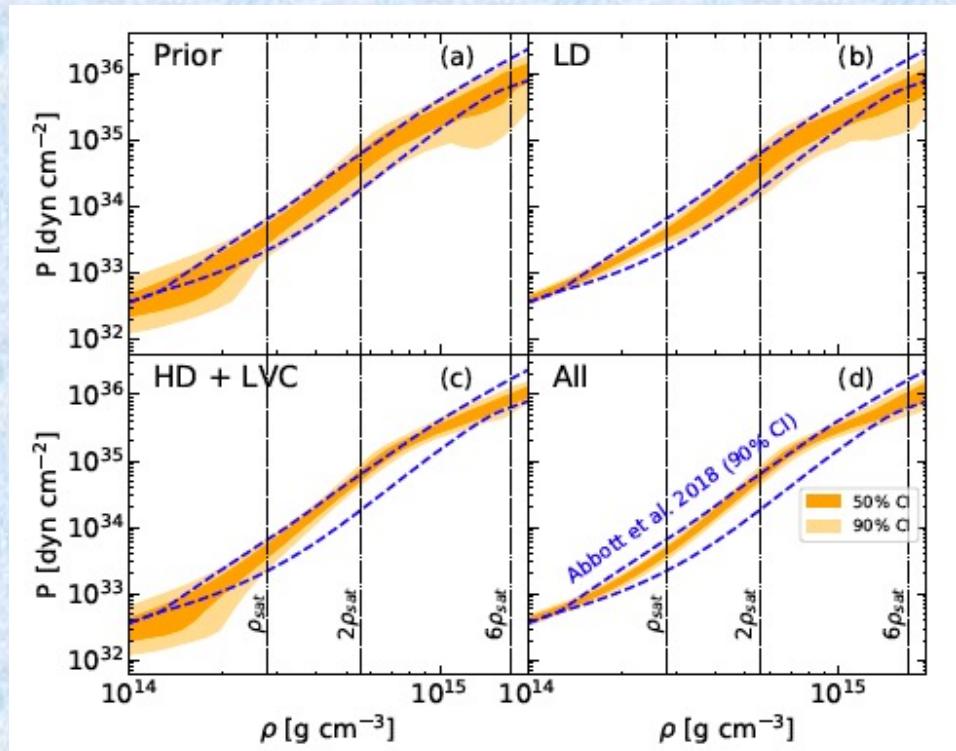
→ “tension” in parameter estimation



Gulminelli & Fantina, Nucl. Phys. News 31, 2 (2021)
(see also Burgio & Fantina, ASSL 457, 255 (2018);
Reed et al., PRL 126, 172503 (2021) and refs. therein)

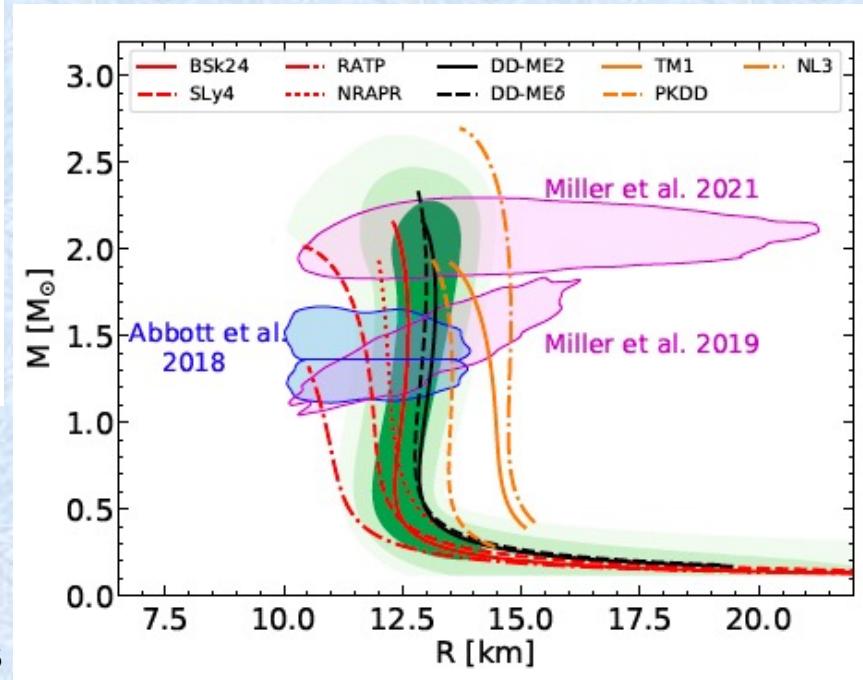


EoS : effect of LD/HD constraints



Dinh Thi et al., Universe 7, 373 (2021)

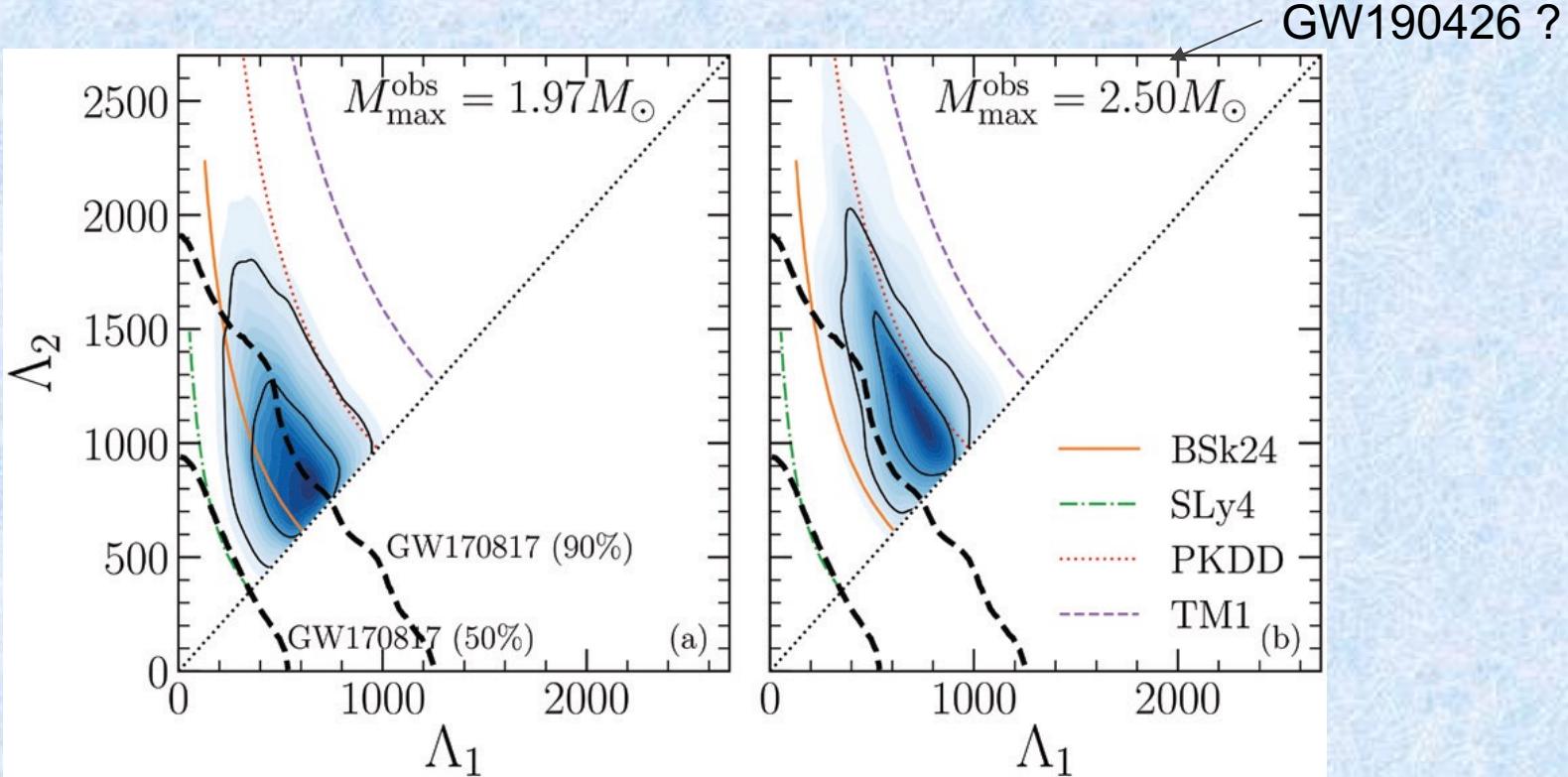
- posterior compatible with observations
but: some popular models are not !
- nucleonic hp compatible with observations



Dinh Thi et al., A&A 654, A114 (2021)



Challenge for nucleonic hp?



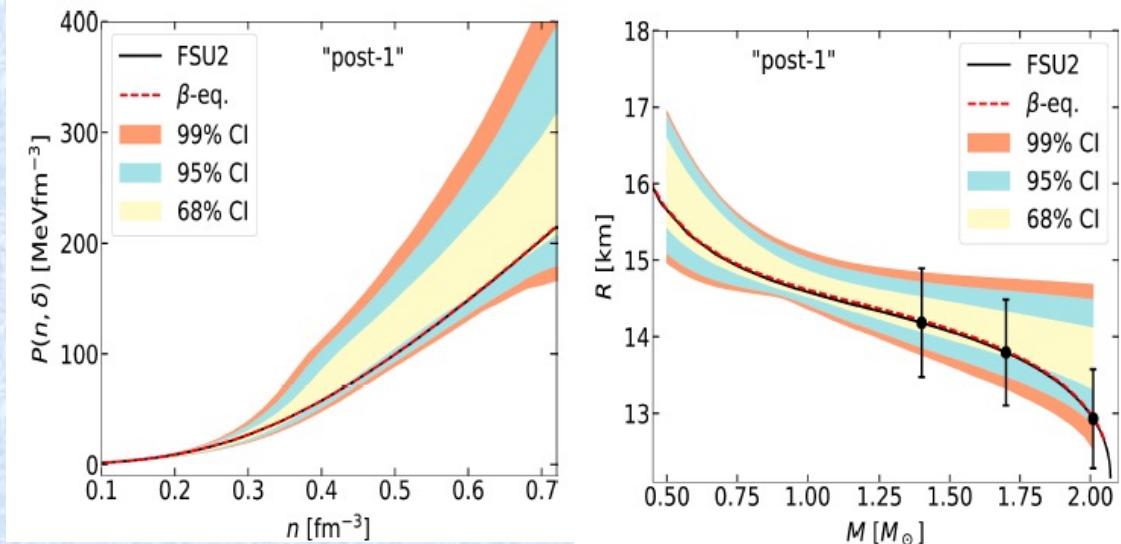
Gulminelli & Fantina, Nucl. Phys. News 31, 2 (2021)

T. Carreau, PhD Thesis (2020)

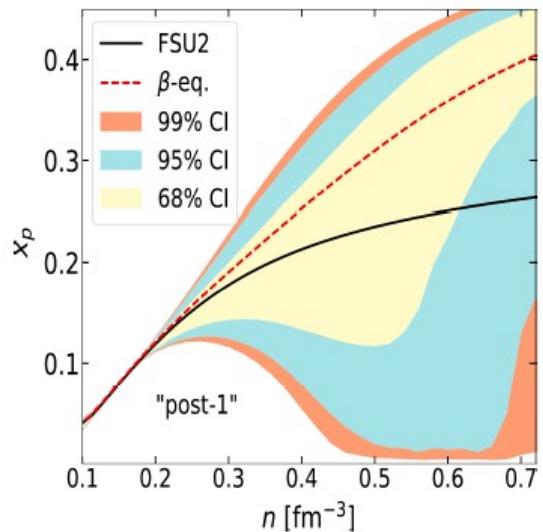
- posterior (nucleonic matter) compatible with observations
- but: if $M_{\text{max}} \sim 2.5 M_{\odot}$ → challenge for nucleonic hypothesis !
- meta-model can be used as null hp



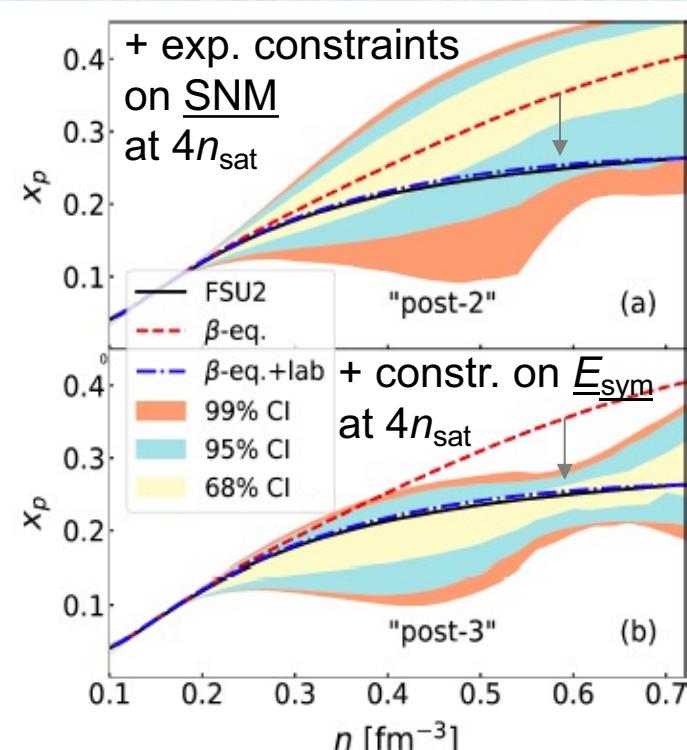
Uniqueness of EoS \longleftrightarrow composition ?



- non-uniqueness of composition!
- better estimation if (experimental) data above n_{sat} available



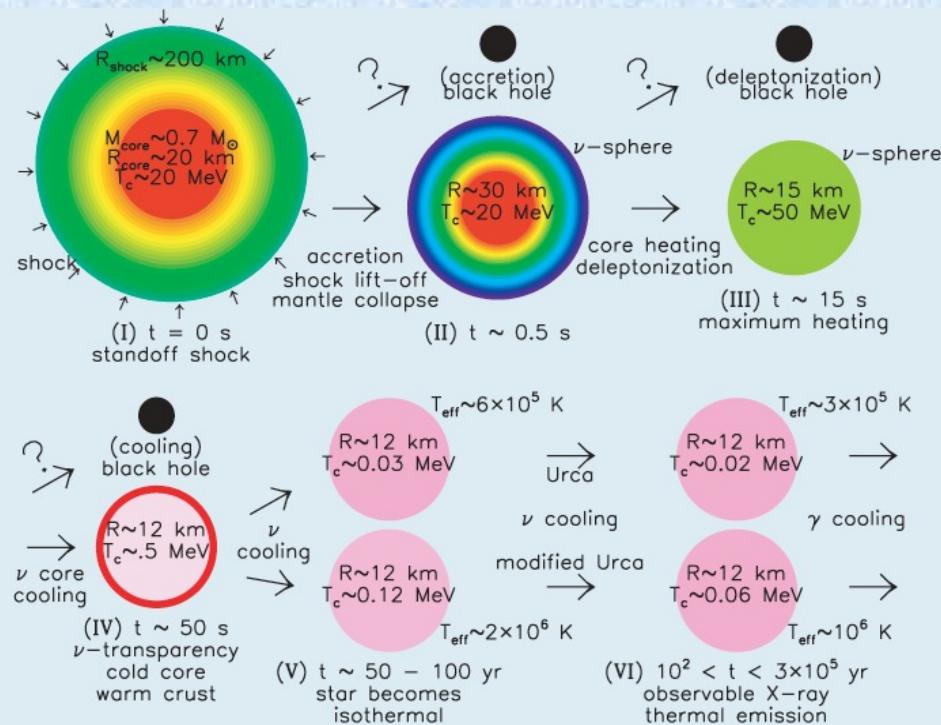
→ one-to-one correspondence
EoS & $M(R)$



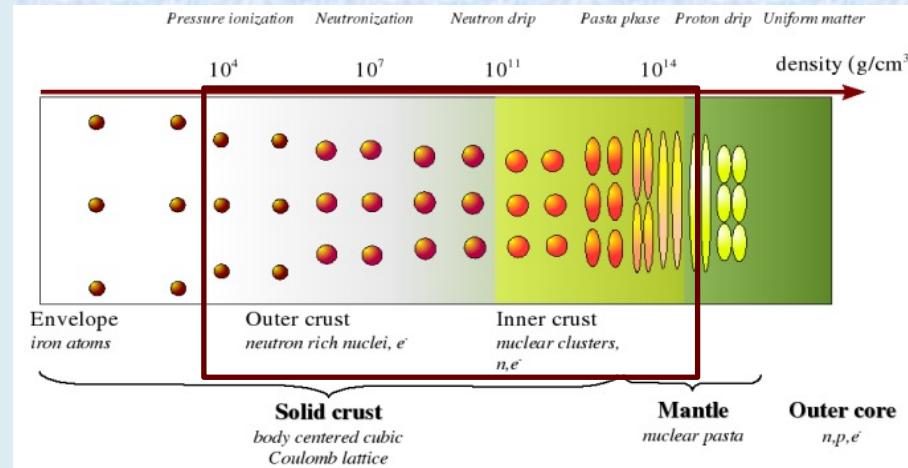


(Proto-)NS crust (finite T)

NS formation from CCSN (here from shock stall)



**“Cold” (catalysed) NS
($T = 0$ approx. → OCP)**



Chamel & Haensel, Liv. Rev. Relativ. 11, 10 (2008)

Lattimer & Prakash, Science 304, 536 (2004)

→ NS are born hot from core-collapse SN ($T > 1$ MeV)
→ ensemble of nuclei expected at formation



Impurities in the (P)NS crust

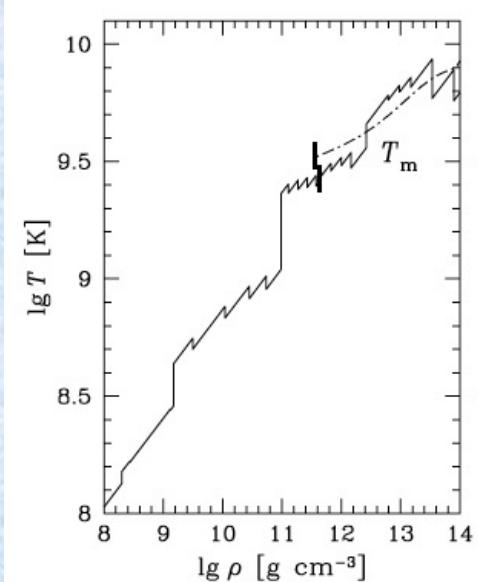
- Crystallization temperature in crust :
 $10^9 < T_m < 10^{10}$ K (\sim few 100 keV)
- Static properties (e.g. EoS)
similar to ground-state cold catalysed matter ($T=0$)
- Dynamic, magneto-rotational, and transport
properties affected by impurities



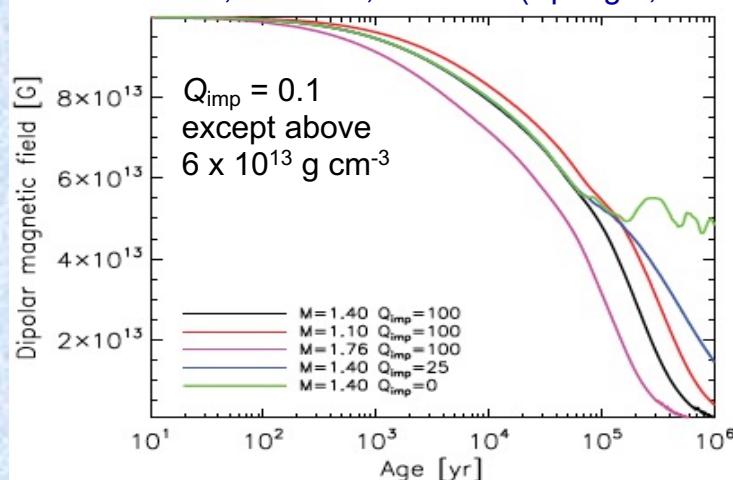
- ❖ Cooling simulations use ground-state EoS and composition
- ❖ “**Impurity factor**” (free parameter adjusted on observational data)

$$Q_{\text{imp}} = \sum_j p(Z^{(j)})(Z^{(j)} - \langle Z \rangle)^2$$

- **consistent calculation of nuclear distributions, Q_{imp} (and transport properties)**



Haensel, Potekhin, Yakovlev (Springer, 2007)

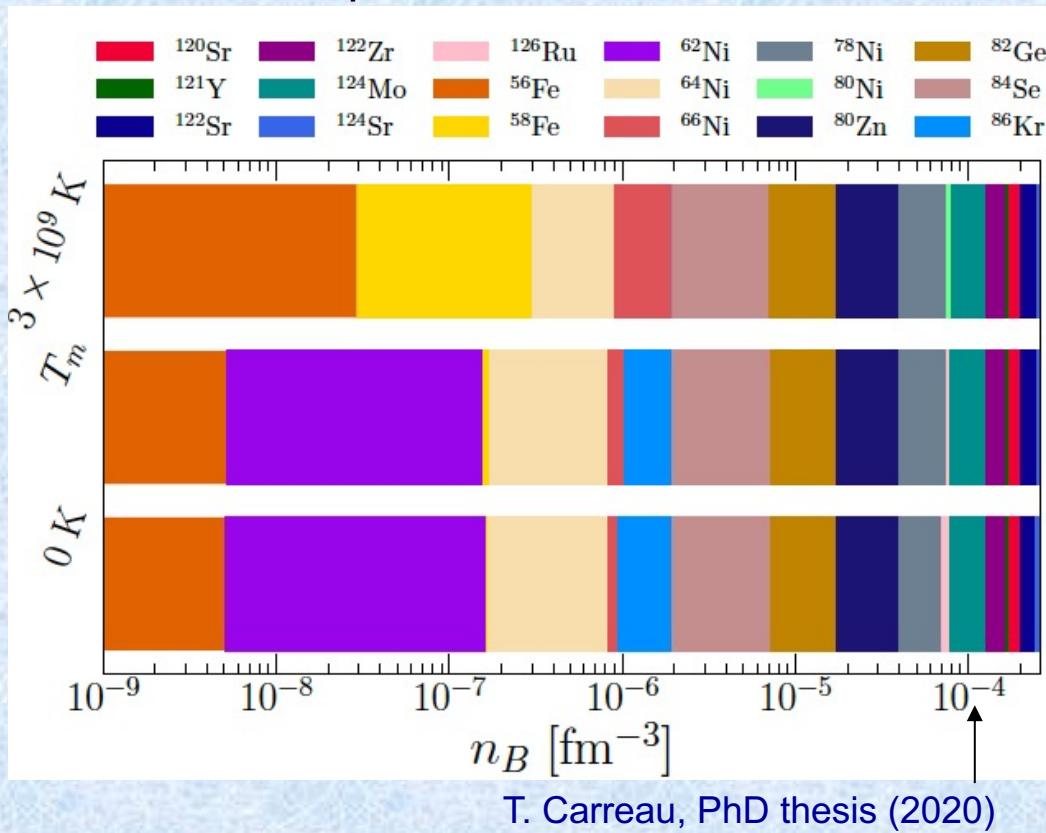


Viganò et al., MNRAS 434, 123 (2013)

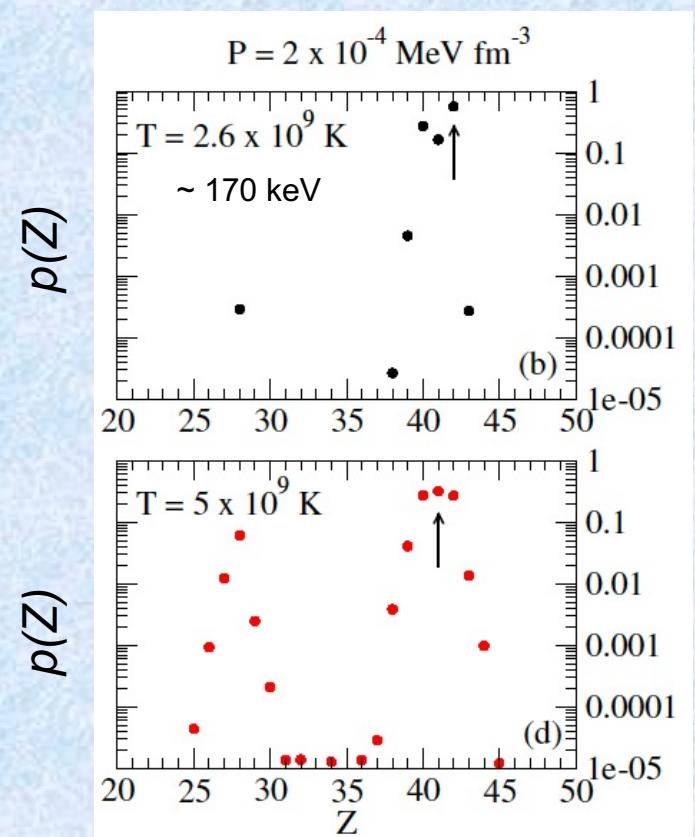


Impurities in the (P)NS crust

Composition of outer crust



- if composition frozen at $T > T_m$
- different from ground state !
- OCP approx. very good at lower P (or n_B), T
- larger distribution at higher $T \rightarrow$ higher Q_{imp}

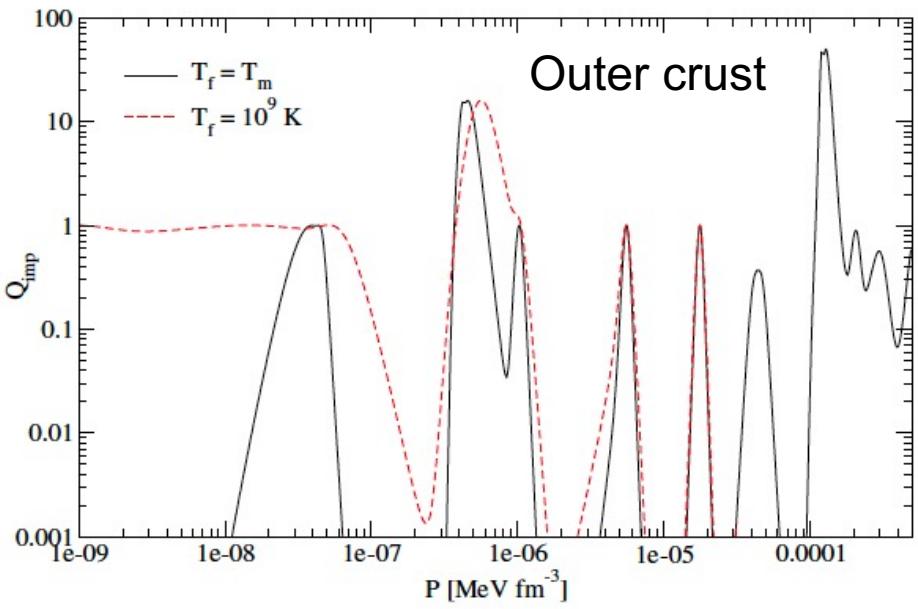


Fantina et al., A&A 633, A149 (2020)
Exp. masses (AME2016) + HFB-24

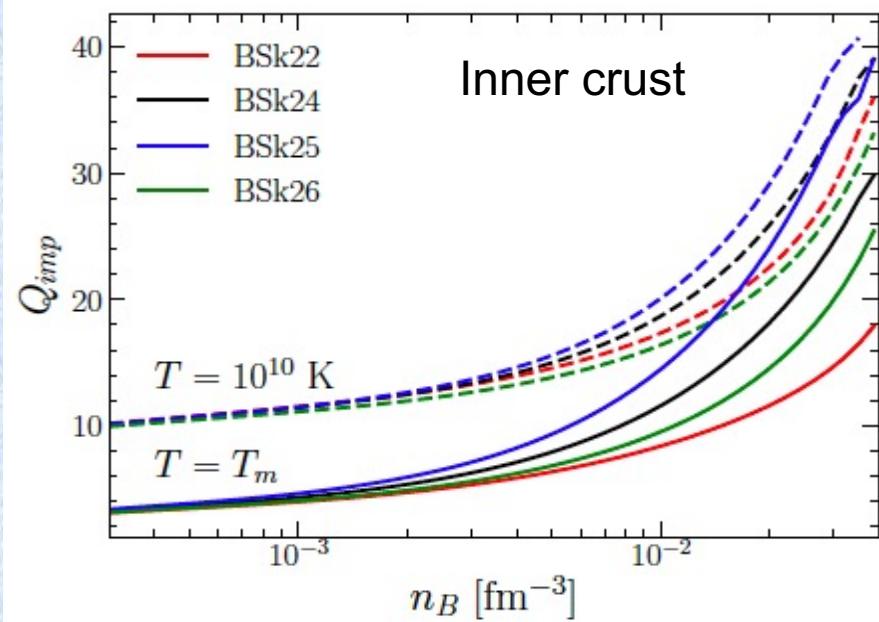


Impurities in the (P)NS crust

- ✓ First self-consistent (microscopic) calculation of $Q_{\text{imp}} = \sum_j p(Z^{(j)})(Z^{(j)} - \langle Z \rangle)^2$



Fantina et al., A&A 633, A149 (2020)



Carreau et al., A&A 640, A77 (2020)

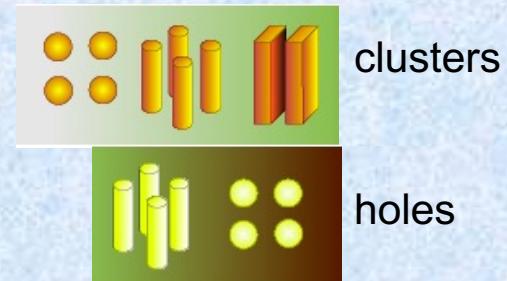
- variation of Q_{imp} → alternation of pure (conductive) and impure (resistive) layers
- larger Q_{imp} at higher pressure/density and T
- consistent calculations of transport coefficients/properties needed
- implementation in numerical simulations of (P)NS cooling



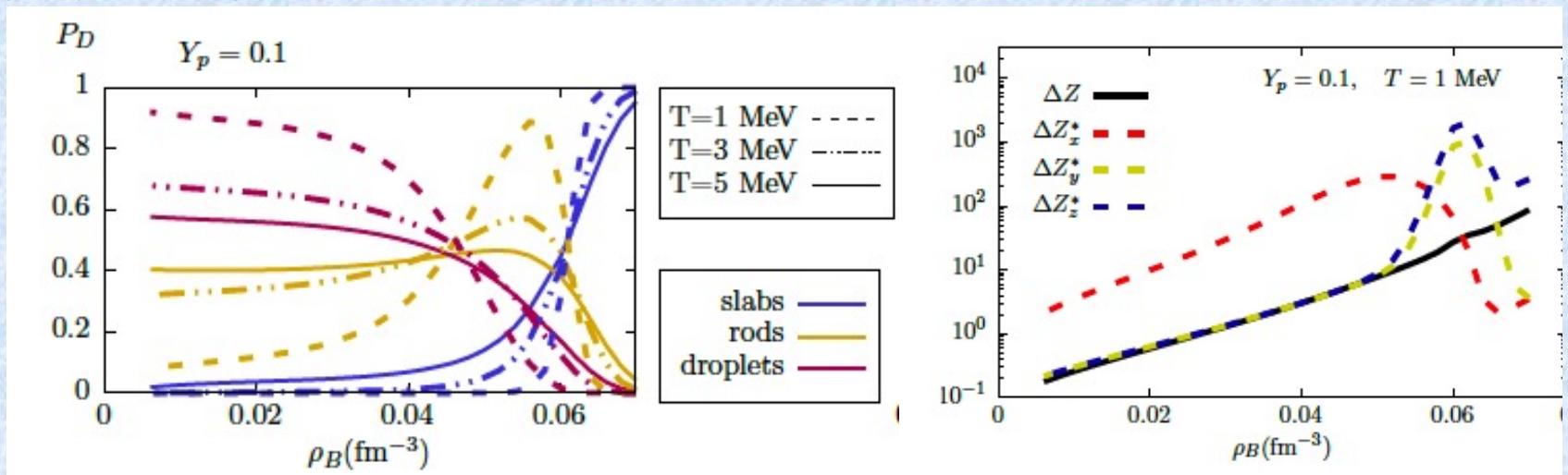
Impurities in the (P)NS crust & pasta

What about non-spherical nuclei/clusters (“**pasta**” phases) ?

$$p(A, Z) \propto \exp \left[-(k_B T)^{-1} (F_{N,d} + \delta\Omega_d - \mu_n N - \mu_p Z) \right]$$



✓ (very recent) calculations at finite T



Pelicer et al., PRC 104, L022801 (2021) - RMF model

- different geometries can co-exist in large fraction of pasta phase
- dependence on spatial direction
- can affect transport properties



Conclusions

- ❖ Crust at $T = 0$ and at finite T in a *unified* approach
- ❖ OCP vs MCP → impurity parameter and transport coefficients (in progress)
- ❖ From observational data to energy functional :
analytic inversion + Bayesian approach
- ❖ Out-of-equilibrium crust → accreting crust, heat released
- ❖ In-medium effect and comparison with data



Thank you