

Topological Properties at Fractal Dimensions

Cristiane Morais Smith¹

¹Utrecht University, Utrecht, The Netherlands

c.demoraissmith@uu.nl

Abstract

Fractals have been used since long as decorative art, but only in the last century they have been classified mathematically. In the 80's and 90's, the foundational work of Mandelbrot triggered an enormous activity in the field. The focus was on understanding **classical fractals**. This century, the task is to understand **quantum fractals**. In 2019, we realized a Sierpinski gasket using a scanning tunneling microscope to pattern adsorbates on top of Cu(111) and showed that the wavefunction describing electrons in this fractal has the Hausdorff dimension $d = 1.58$ [1]. However, STM techniques can only describe **equilibrium** properties.

Last year, we unveiled the **quantum dynamics** in fractals using photonics experiments. By injecting photons in waveguide arrays arranged in a fractal shape, we were able to follow their motion and understand their transport properties with unprecedented detail. We built 3 types of fractal structures to reveal the influence of different Hausdorff dimensions and geometry [2]. Finally, we will discuss topological properties of self-formed fractals of Bi on InSb. In these systems, the spin-orbit coupling is very strong, thus potentially leading to a quantum spin Hall effect. Muffin-tin calculations indeed reveal corner states and edge modes in these fractal structures.

References

- [1] S.N. Kempkes, M.R. Slot, S.E. Freoney, S.J.M. Zevenhuizen, D. Vanmaekelbergh, I. Swart, and C. Morais Smith, "Design and characterization of electronic fractals", *Nature Physics* 15, 127 (2019) [see also 15 years of Nature Physics, *Nature Physics* 16, 999 (2020)].
 - [2] X.-Y. Xu, X.-W. Wang, D.-Y. Chen, C. Morais Smith, and X.-M. Jin, "Quantum transport in fractal networks," *Nature Photonics* 15, 703 (2021).
-

Figures

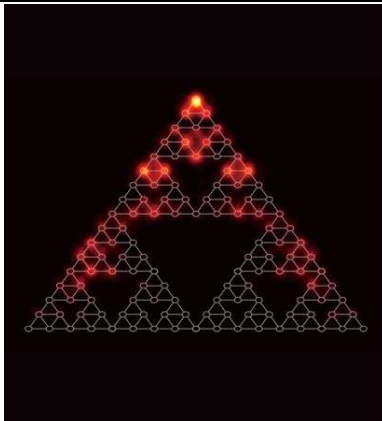


Figure 1: Photon dynamics in a Sierpinski triangle [2].