Muon g-2
A Brief Overview

Muon magnetic moment

Muons (even resting ones) possess a magnetic moment sourced by their spin angular momentum

electric charge

$$\vec{\mu}_\mu = g_\mu \left(\frac{Q_\mu}{2m_\mu} \right) \vec{S}_\mu$$

Landé g-factor

mass

For elementary particles
Dirac equation predicts

$$g = 2$$

Yet vacuum fluctuations induce a (small) correction

$$g_\mu = 2(1 + a_\mu)$$

magnetic moment 'anomaly'

Measuring anomalous magnetic moments

Polarized muon from P-violating weak pion decay

$$\pi^- \to \mu^- + \bar{\nu}_\mu$$

Spin precession around momentum in B field [Thomas 1927]

$$\vec{\omega}_a = \frac{Q_\mu}{m_\mu}\left[a_\mu \vec{B} - \left(a_\mu - \frac{1}{\gamma_m^2 - 1}\right)\vec{\beta}\times\vec{E}\right]$$

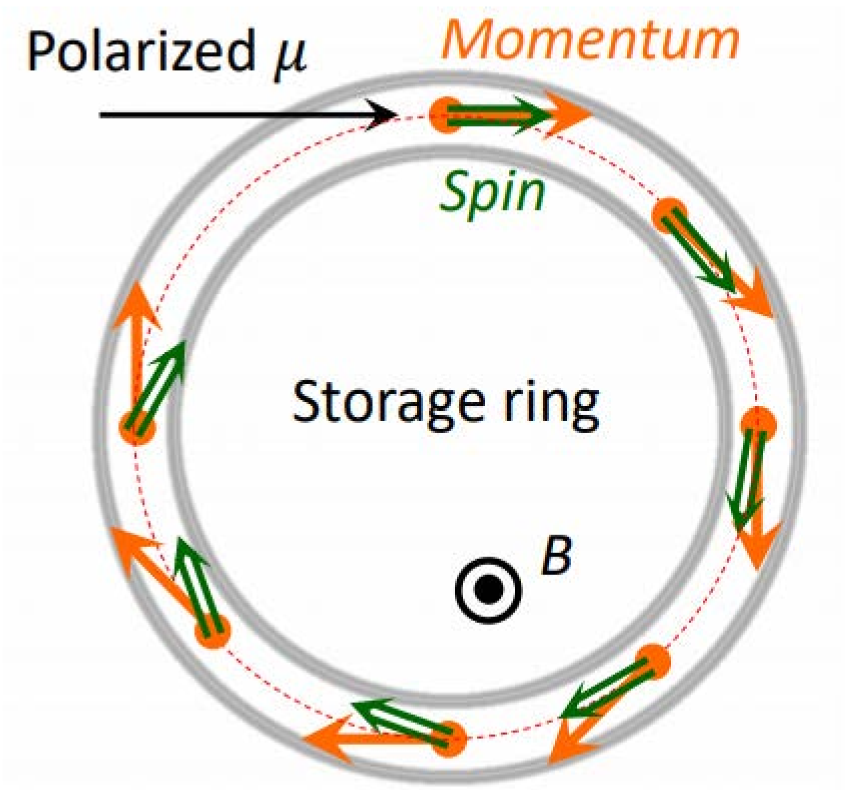
$$\simeq \frac{Q_\mu}{m_\mu} a_\mu \vec{B}$$

« magic » momentum
$p_\mu \approx 3.09\,\mathrm{GeV}$

Electron from P-violating muon decay is a spin-analyzer

$$\mu^- \to e^- + \nu_\mu + \bar{\nu}_e$$

boosted electron flies opposite
to the direction of muon spin



[Charpak+ 1962 → Bailey+ 1978] CERN
[Bennett+ 2006] BNL
[Abi+ 2021] FNAL

Measuring anomalous magnetic moments

B field measured from proton NMR in a water sample

$$\omega_p = 2\mu_p B$$

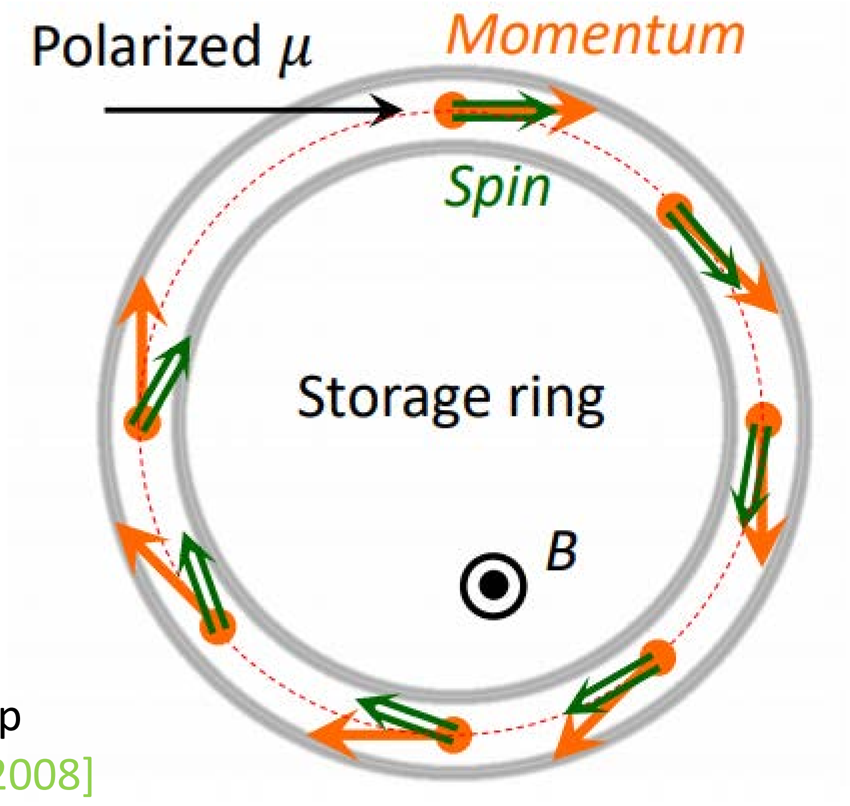
Muonium HFS
[Liu 1999]
+theory

$$a_\mu = \frac{\omega_a}{\omega_p}\frac{\mu_p}{\mu_e(H)}\frac{\mu_e(H)}{\mu_e}\frac{m_\mu}{m_e}\frac{g_e}{2}$$

CERN
BNL/FNAL

bound-state QED

Penning trap
[Gabrielse 2008]



Measurements history

	NEVIS	$a_\mu = 113(14) \times 10^{-5}$		[Garwin+ 1960]	consistent with Schwinger's $\alpha/2\pi$ muon is a heavy electron
	CERN-1	$a_\mu = 1162(5) \times 10^{-6}$		[Charpak+ 1962]	
	CERN-2	$a_\mu = 11661(3) \times 10^{-7}$		[Bailey+ 1968]	
	CERN-3	$a_\mu = 1165924(9) \times 10^{-9}$		[Bailey+ 1979]	
Muon g-2 collaboration	BNL	$a_\mu = 116592080(63) \times 10^{-11}$	$(0.54\,\mathrm{ppm})$	[Bennett+ 2006]	sensitive to weak interactions
	FNAL	$a_\mu = 116592040(54) \times 10^{-11}$	$(0.46\,\mathrm{ppm})$	[Abi+ 2021]	

BNL/FNAL combined $a_\mu = 116592061(41) \times 10^{-11}$ $(0.35\,\mathrm{ppm})$

Anomalous magnetic moments |Theory

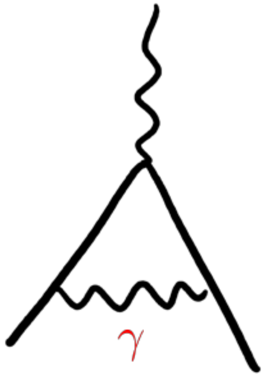
$$a_\ell = a_\ell^{\text{QED}} + a_\ell^{\text{weak}} + a_\ell^{\text{hadronic}} + a_\ell^{\text{BSM}}\,?$$

As a genuine quantum effect, it is sensitive to all fields, known *or not*
That's the reason why it is so interesting!

Not a new motivation
The first CERN measurement was looking for new physics beyond …QED!

Anomalous magnetic moments | Theory

$$a_\ell = a_\ell^{\mathrm{QED}} + a_\ell^{\mathrm{weak}} + a_\ell^{\mathrm{hadronic}}$$

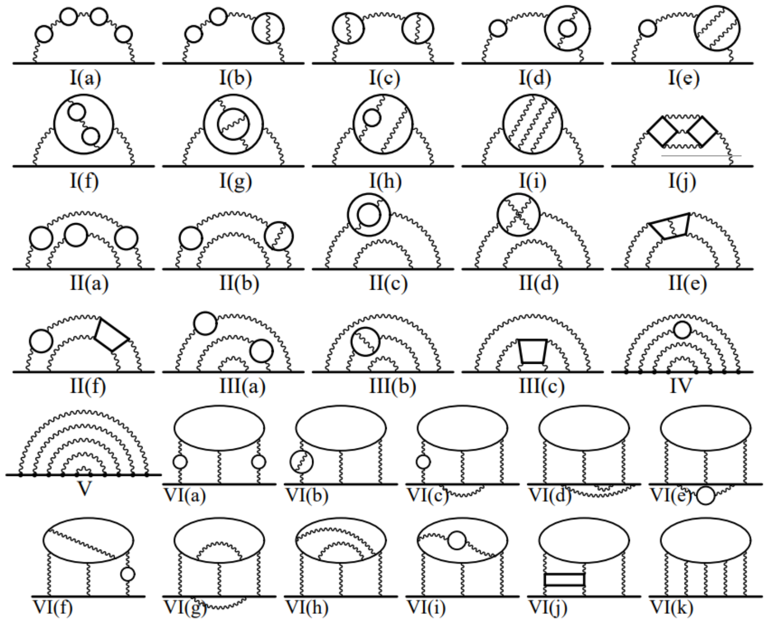


$$\frac{\alpha}{2\pi} \sim 10^{-3}$$

[Schwinger 1948]

$+ \cdots +$

2,3,4 loops known



12672 diagrams at $\mathcal{O}(\alpha^5)$

[Kinoshita+ 2006-12]

$+ \cdots$

uncalculated

$\mathcal{O}[(\alpha/\pi)^6] \sim 10^{-16}$

not needed (for now) given experimental precision

$$u(a_e) = 2.8 \times 10^{-13}$$

[Gabrielse+ 2008]

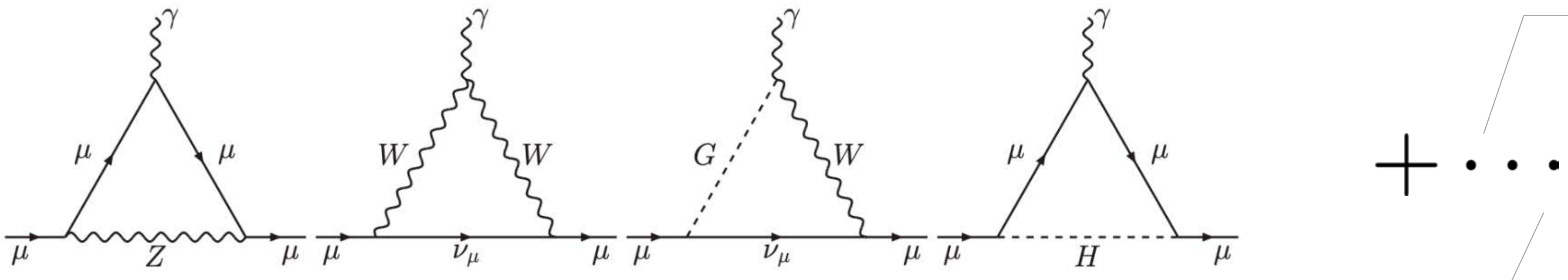
$$u(a_\mu) = 4.1 \times 10^{-10}$$

[BNL+FNAL 2021]

Anomalous magnetic moments |Theory

$$a_\ell = a_\ell^{\text{QED}} + a_\ell^{\text{weak}} + a_\ell^{\text{hadronic}}$$

very suppressed by the Fermi scale
but within experimental precision



two-loop
(hadronic included)
known
[Czarnecki 1996]

[Jackiw-Weinberg 1972]

$$\simeq \frac{5 G_F m_\ell^2}{24\sqrt{2}\pi^2}$$

three-loop
unknown beyond leading logs

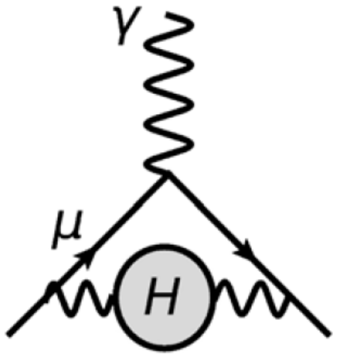
[Czarnecki 2006]

Anomalous magnetic moments |Theory

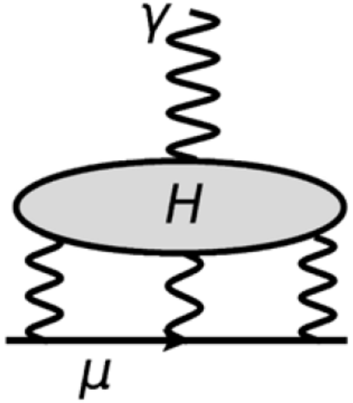
$$a_\ell = a_\ell^{\mathrm{QED}} + a_\ell^{\mathrm{weak}} + a_\ell^{\mathrm{hadronic}}$$

This is the *hardest* part,
because QCD is nonperturbative
at the lepton-mass scale

two classes
are relevant



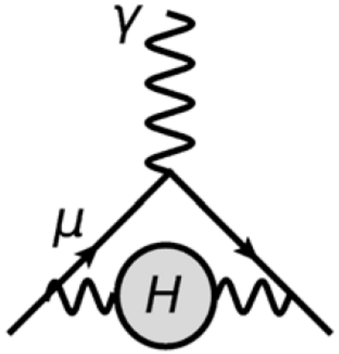
vacuum polarization
(N)LO-HVP $\mathcal{O}(\alpha^2)$



light-by-light scattering
LO-HLbL
$\mathcal{O}(\alpha^3)$

two ways to evaluate them:
R-ratio and lattice

Hadronic vacuum polarization



Data-driven approach [Bouchiat-Michel 1961] [Brodsky-de Raphael 1968]

similarly for HVP-NLO and HLbL-LO [Aoyama+ 2020]

$$a_\mu^{\mathrm{HVP-LO}} = \frac{\alpha^2}{3\pi^2}\int_{m_\pi^2}^{\infty} ds\, \frac{K(s)}{s} R(s)$$

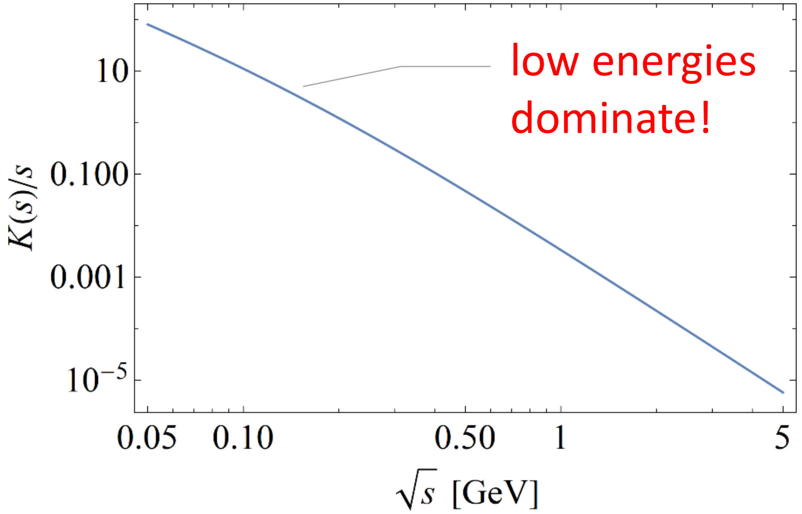
R-ratio from data

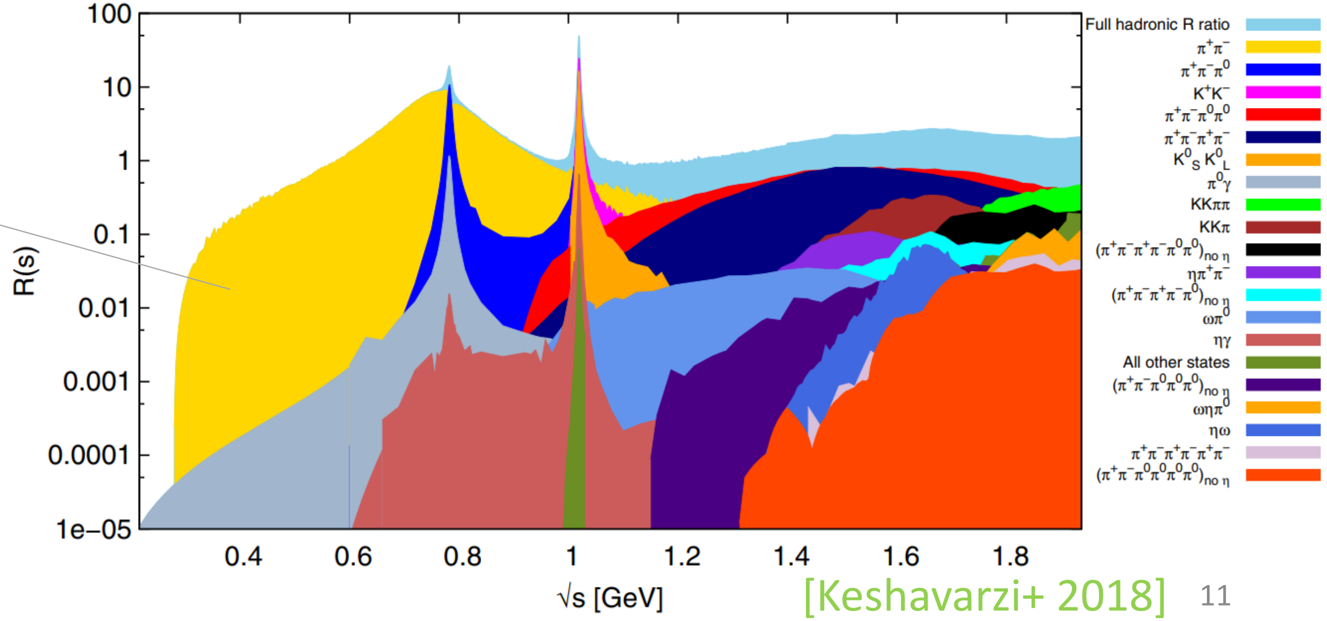
$$R(s) = \frac{3s}{4\pi\alpha^2}\sigma(e^+e^- \to \text{hadrons})$$

$$= 12\pi\, \mathrm{Im}[\Pi_\gamma(s)]$$

optical theorem

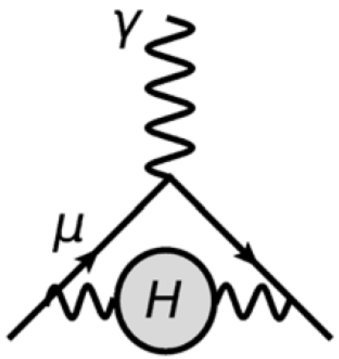
kernel function known from theory





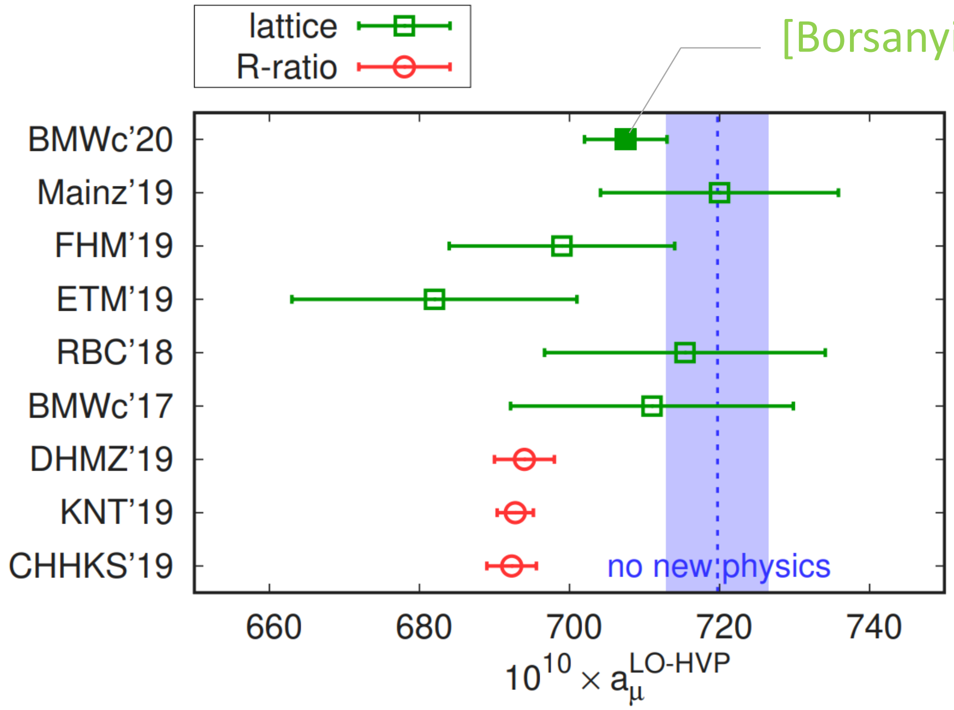
[Keshavarzi+ 2018]

Hadronic vacuum polarization



Lattice QCD ab-initio approach

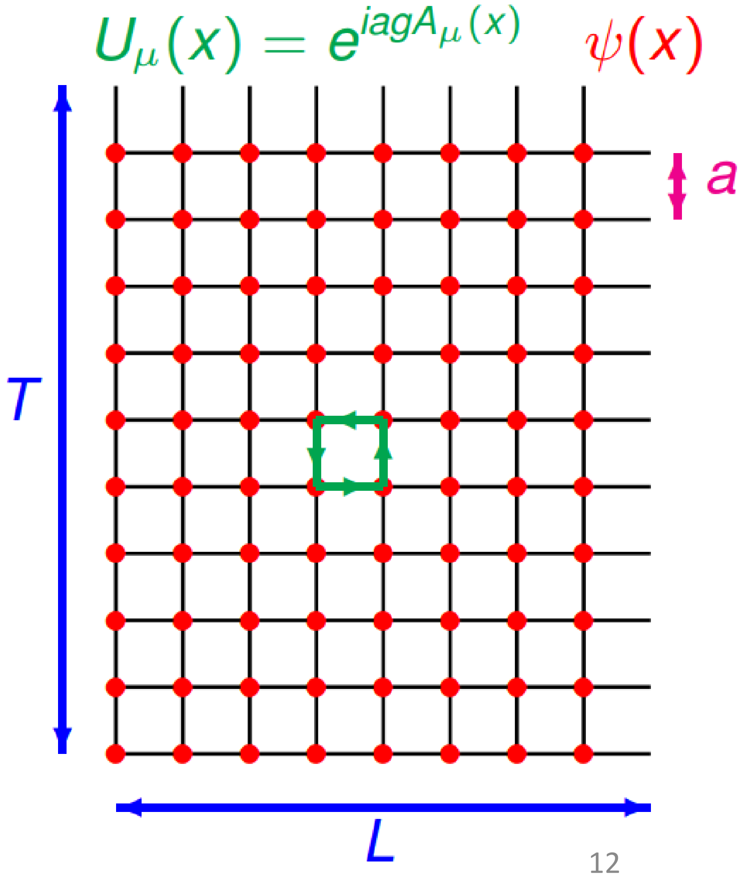
Sub-% precision requires small spacing, large (Euclidian) volume and physical pion masses → computationally challenging



[Borsanyi+ 2021]

consistent with previous results

3-fold reduced error due to
1) larger volume (L=11fm)
2) smaller numerical noise
3) many different a values



The muon g-2 puzzle(s)

[Lellouch's talk
GDR intensity frontier]

Why is the (R-ratio based) SM prediction 4.2σ smaller than experiment?

$$a_\mu^{\mathrm{BSM}} = 251(59) \times 10^{-11} \; ?$$

Why is the HVP-LO from the R-ratio ~2σ smaller than the lattice one?

99% of the literature addresses the first puzzle

The second one invites us to consider muon g-2 actually follows the SM