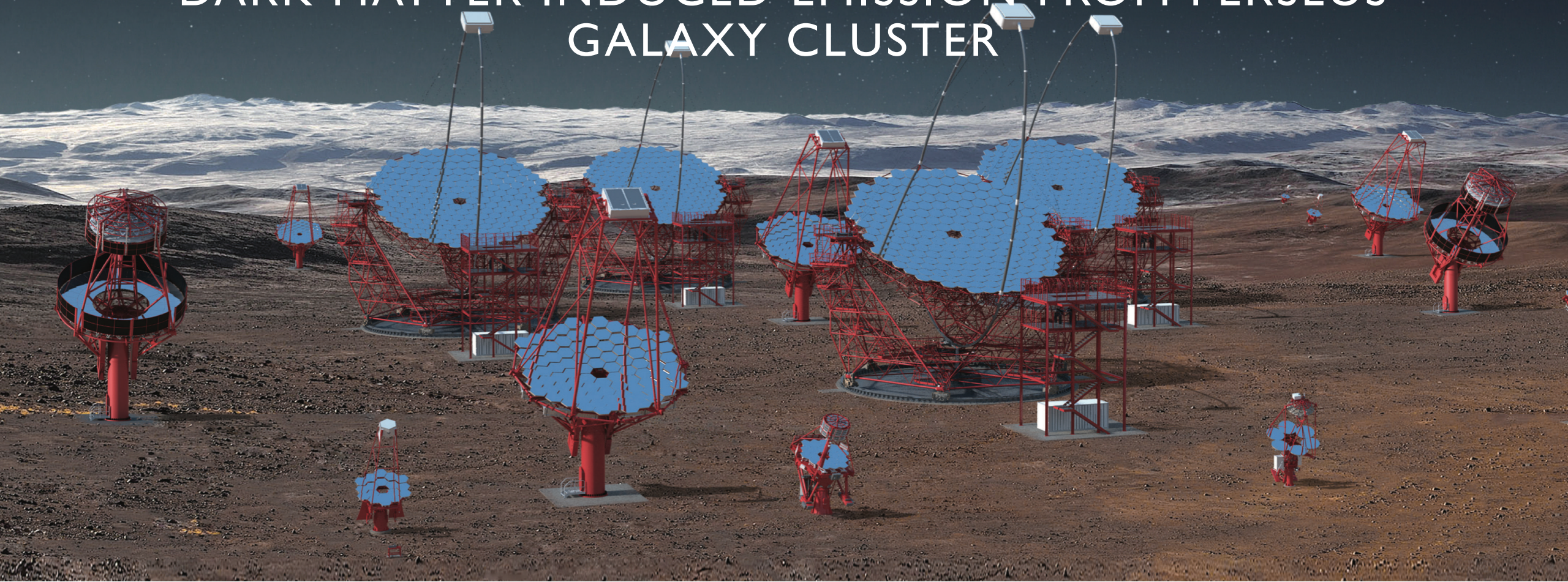


SENSITIVITY OF THE CHERENKOV TELESCOPE ARRAY TO DARK MATTER INDUCED EMISSION FROM PERSEUS GALAXY CLUSTER



cherekov
telescope
array



Séminaires LLR (CNRS & École Polytechnique)
29/11/2021

Judit Pérez-Romero

In collaboration with R.Adam, G. Brunetti, M. Hutten,
M.Á.Sánchez-Conde & "CTA Galaxy Clusters Task Force"

judit.perez@uam.es



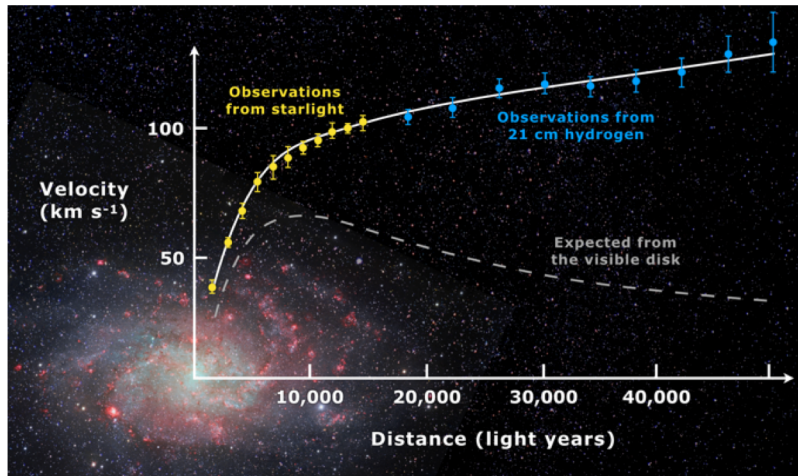
GOBIERNO
DE ESPAÑA

MINISTERIO
DE CIENCIA, INNOVACIÓN
Y UNIVERSIDADES

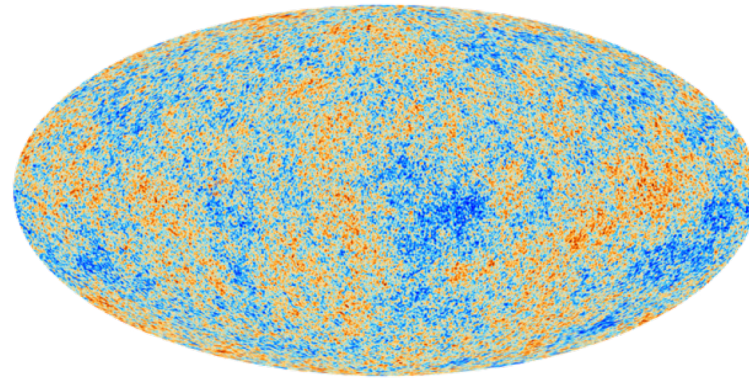


DARK MATTER EVIDENCE

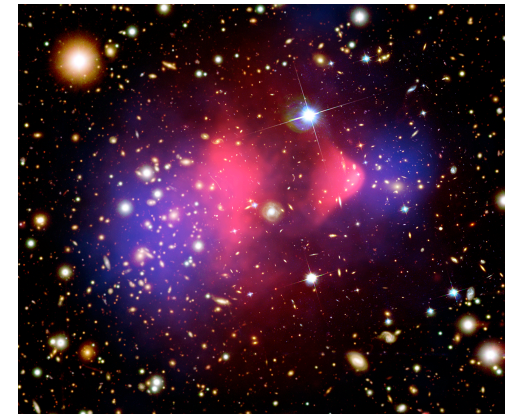
- Galactic rotational curves



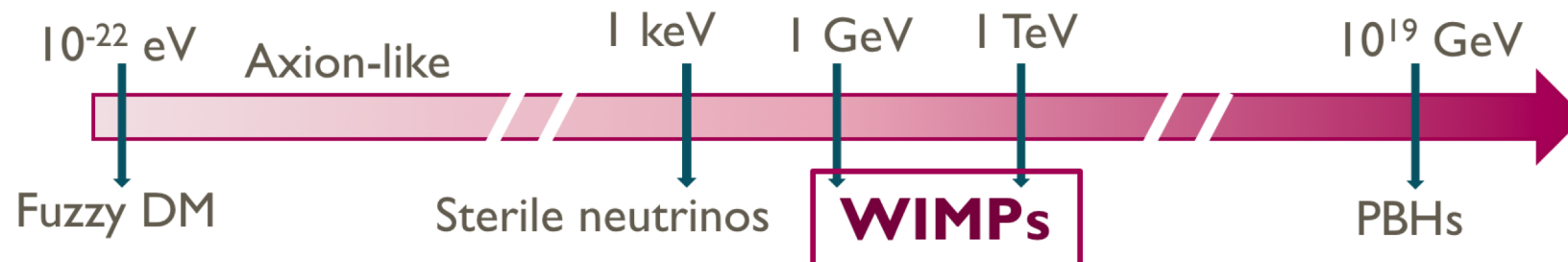
- CMB anisotropies



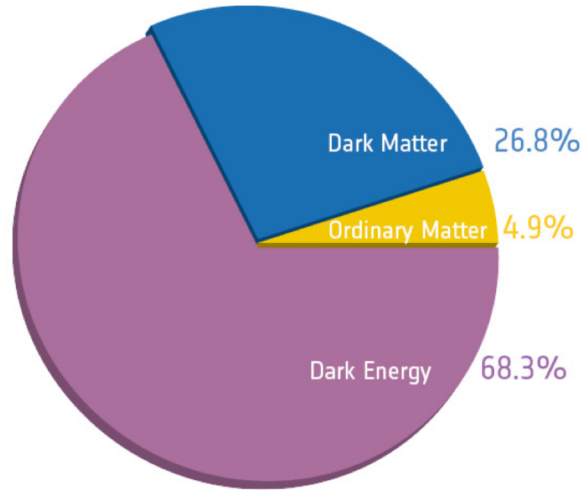
- Galaxy Clusters



- Different DM candidates, wide range of masses:



DARK MATTER IN Λ CDM COSMOLOGY



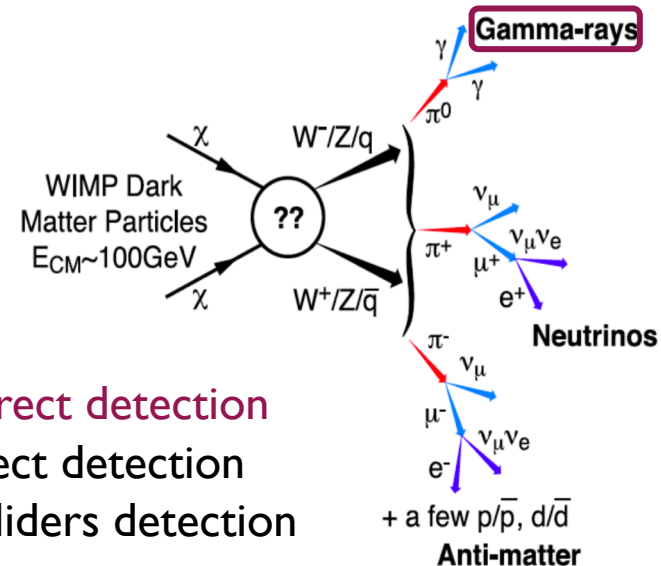
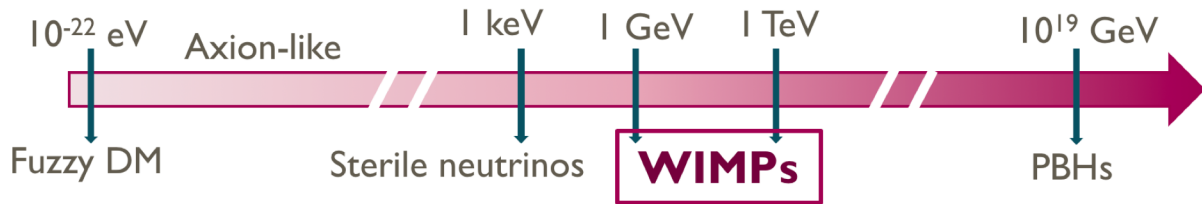
Observational Dark Matter evidences

Component of **Λ CDM Cosmology**

- Structure formation driven by DM
- Bottom-up scenario: smaller structures form first

DM distribution in Halos and Subhalos

• Different DM candidates:

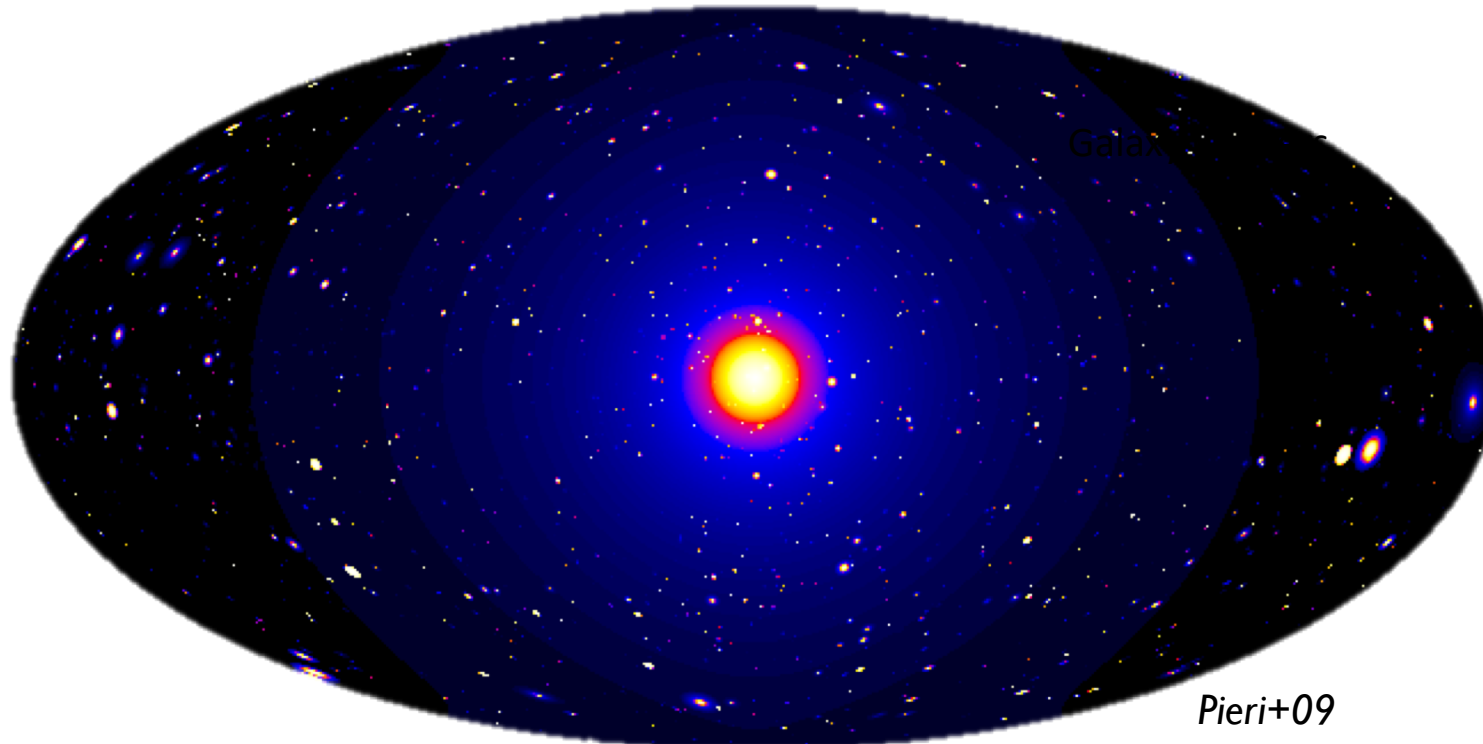


This γ -ray emission allows to perform Indirect DM Searches with current telescopes

- The search for the WIMP
 - Annihilation/Decay \rightarrow Indirect detection
 - Collision \rightarrow Direct detection
 - Production \rightarrow Colliders detection

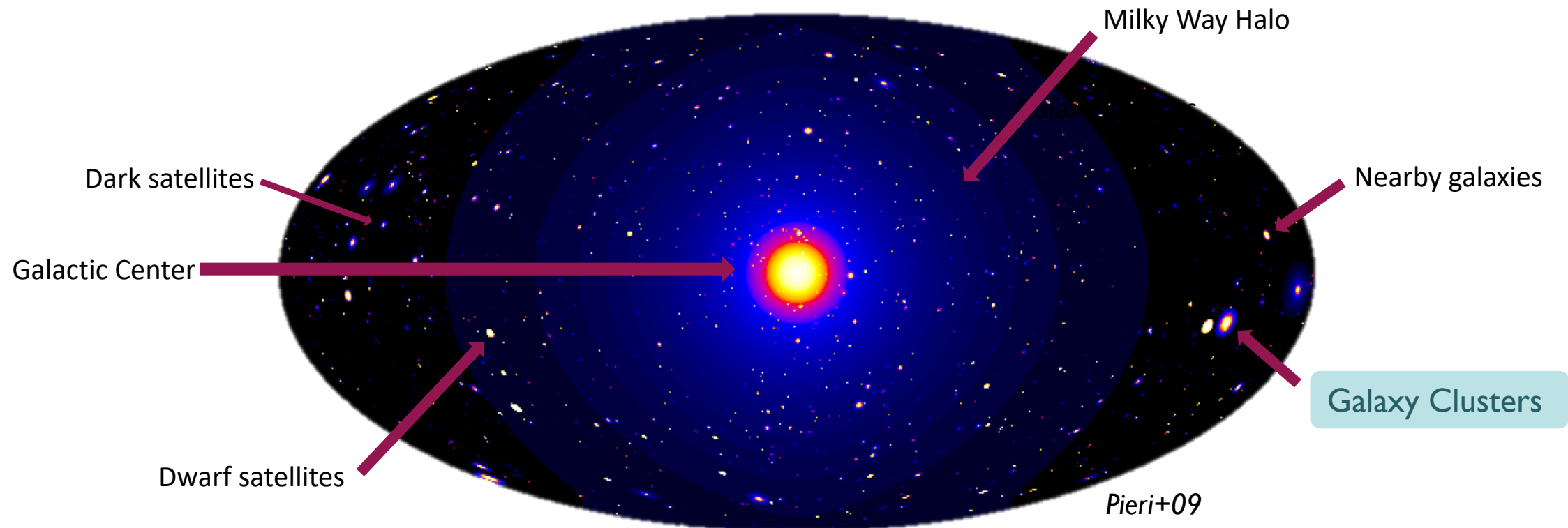
GAMMA-RAY DM SEARCHES

- Optimal conditions for indirect DM searches:
 - High DM density ($\phi_{\text{DM}} \propto \rho_{\text{DM}}^2$ for annihilation, $\phi_{\text{DM}} \propto \rho_{\text{DM}}$ for decay)
 - Massive nearby objects ($\phi_{\text{DM}} \propto M/d_{\text{Earth}}^2$)
 - Low astrophysical background



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 - Low astrophysical background



GAMMA-RAY EMISSION IN GALAXY CLUSTERS

- Largest **gravitationally bound** structures formed by gravitational collapse
- Masses of order $\sim 10^{14} - 10^{15} M_{\odot}$
- Components:
 - Baryonic Matter
 - Galaxies ($\sim 3\% - 5\%$)
 - ICM ($\sim 15\% - 17\%$)
 - **Dark Matter ($\sim 80\%$)**
- Even supposedly virialized objects, a lot of activity \longrightarrow Merger events
 - Feedback from galaxies and AGNs
 - Magnetic fields
 - Turbulence

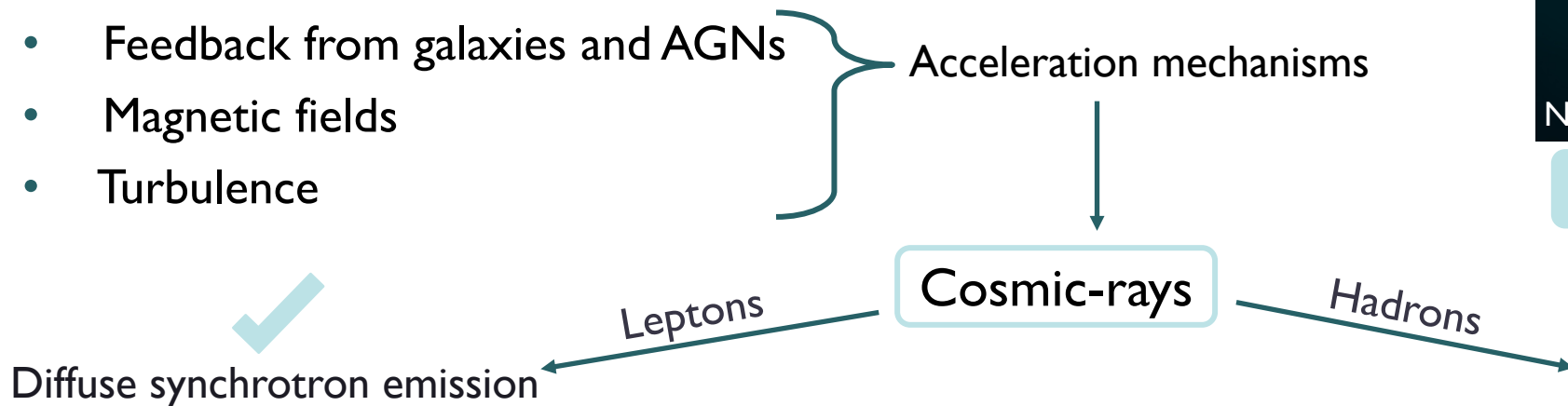


Chandra: NASA/CXC/SAO/Bulbul+14; XMM: ESA

NGC1275 in Perseus Galaxy Cluster

GAMMA-RAY EMISSION IN GALAXY CLUSTERS

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Chandra: NASA/CXC/SAO/Bulbul+14; XMM: ESA



Gamma-rays

No clear detection but
some hints claimed...

Ackermann+15 [Fermi-LAT Collab.], Xi+18,
Adam+21

GAMMA-RAY DM SEARCHES IN CLUSTERS?

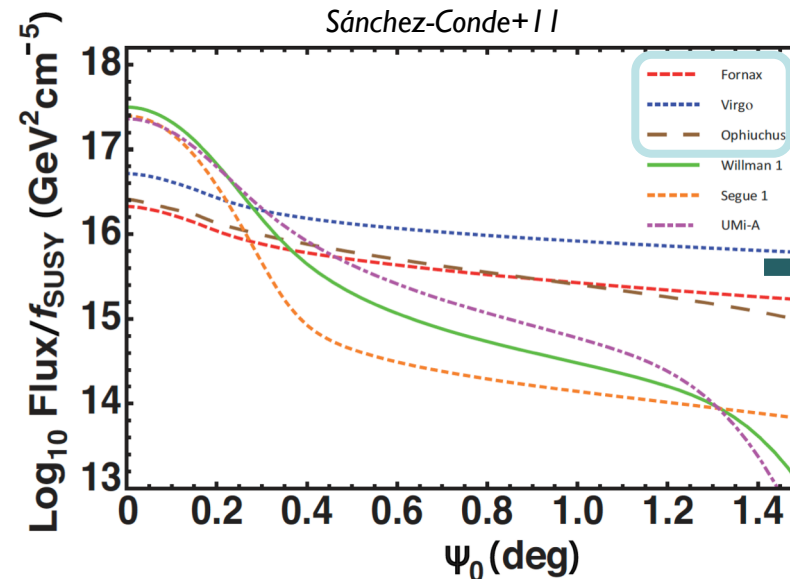
- Optimal conditions for indirect Dark Matter (DM) searches:
 - ✓ High DM density
 - ✓ Massive nearby objects
 - ? Low astrophysical background (Cosmic Rays - CR)

Decay

Annihilation

- Most massive known objects
- Located in the local Univers
- 80% of its mass is DM

Best possible targets to consider

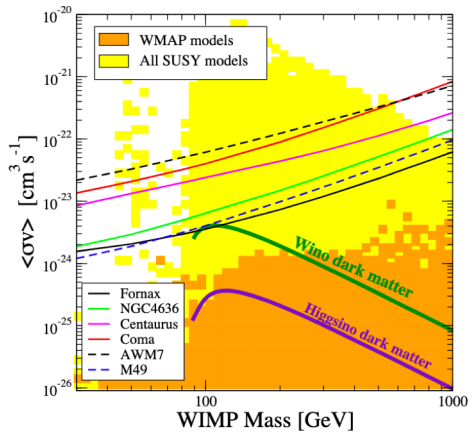


Competitive compared to other prime targets (e.g. dSphs) considering:

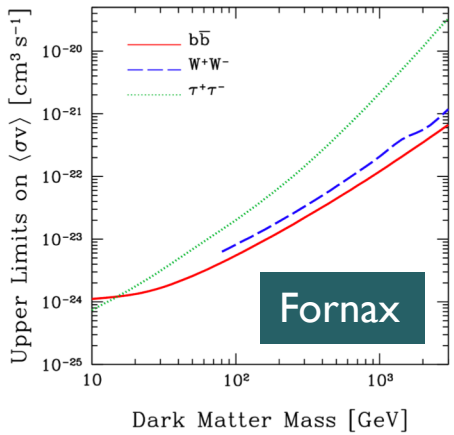
- Smooth DM halo component
- +
 - Halo substructure

PREVIOUS GAMMA-RAY DM SEARCHES IN GALAXY CLUSTERS

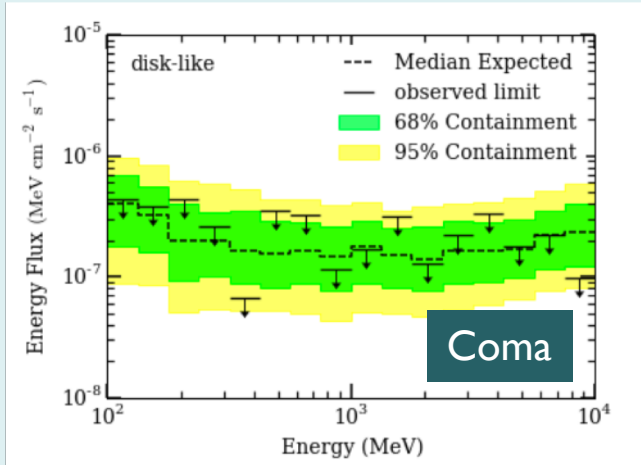
Fermi-LAT - Annihilation



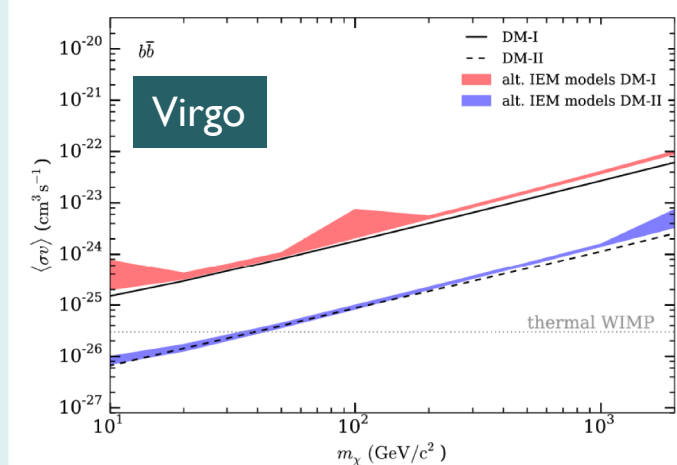
Ackermann+10 [Fermi-LAT Collab.]



Ando&Nagai 2



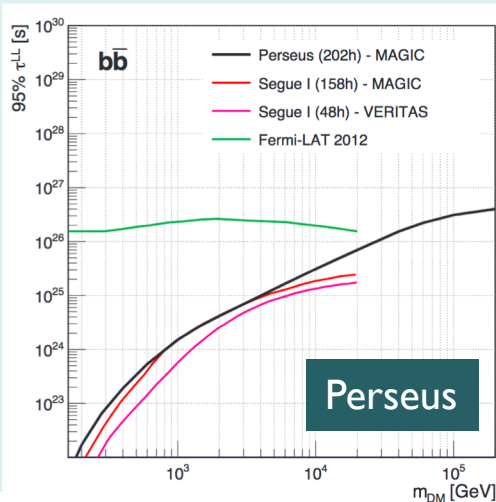
Ackermann+16 [Fermi-LAT Collab.]



Ackermann+15 [Fermi-LAT Collab.]

- Last word about gamma-ray searches in a big sample of galaxy clusters: CR focused (Ackermann+14 [Fermi-LAT Collab.])

MAGIC - Decay
Best constraints so far!



Acciarri+18 [MAGIC Collab.]

THE CHERENKOV TELESCOPE ARRAY (CTA)

- Future of ground-based Very High Energy (VHE) gamma-ray astronomy
- 2 arrays: Northern Array (La Palma, Spain) and Southern Array (Paranal, Chile)

The infographic features a central world map with red dotted lines connecting to various project components. On the left, two hexagonal images show the CTA North (Spain, La Palma) and CTA South (Chile, Paranal) sites. On the right, two hexagonal images show the SDMC DESY Campus in Zeuthen and the INAF Campus in Bologna. A central hexagon displays the statistics: 1,350 members, 210 institutes, and 32 countries. At the bottom right, three light blue boxes identify the telescope types and their energy ranges: SST (1-300 TeV), MST (100 GeV-10 TeV), and LST (20-200 GeV).

1,350 members

210 institutes

32 countries

CTA North
Spain, La Palma

CTA South
Chile, Paranal

SDMC
DESY Campus,
Zeuthen

Headquarters
INAF Campus,
Bologna

SST
1-300 TeV

MST
100 GeV-10 TeV

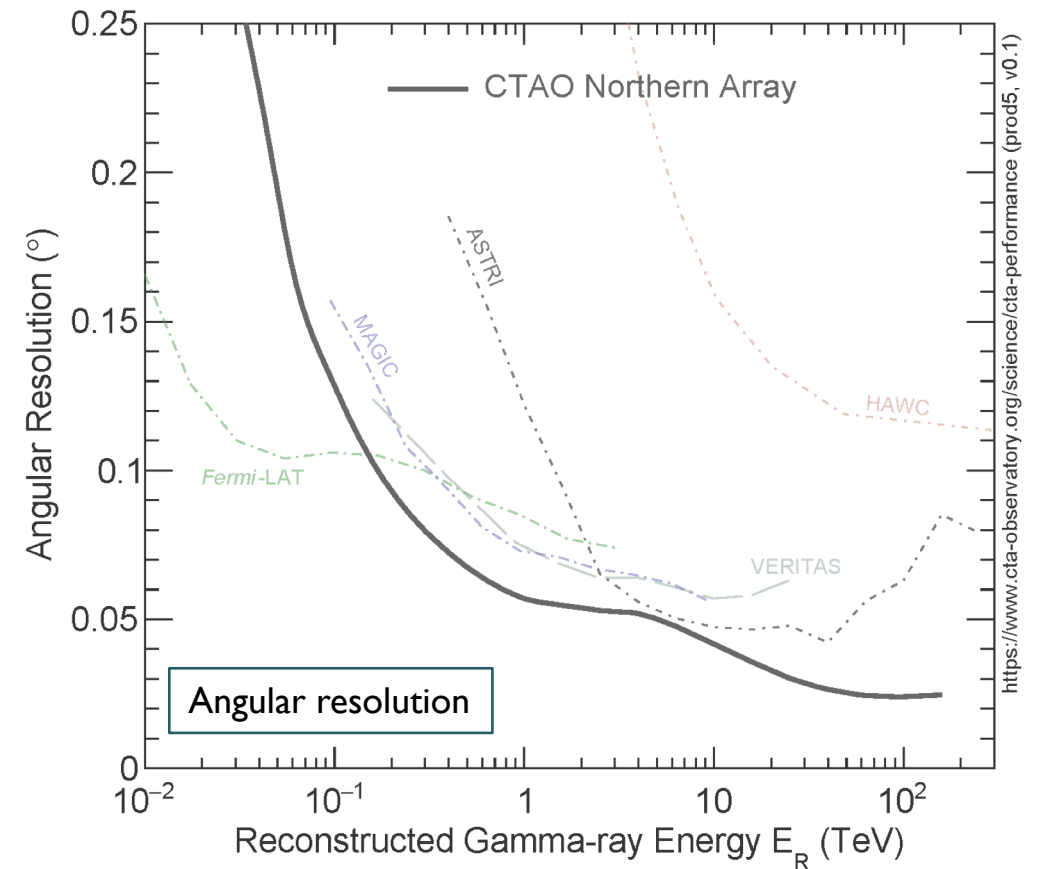
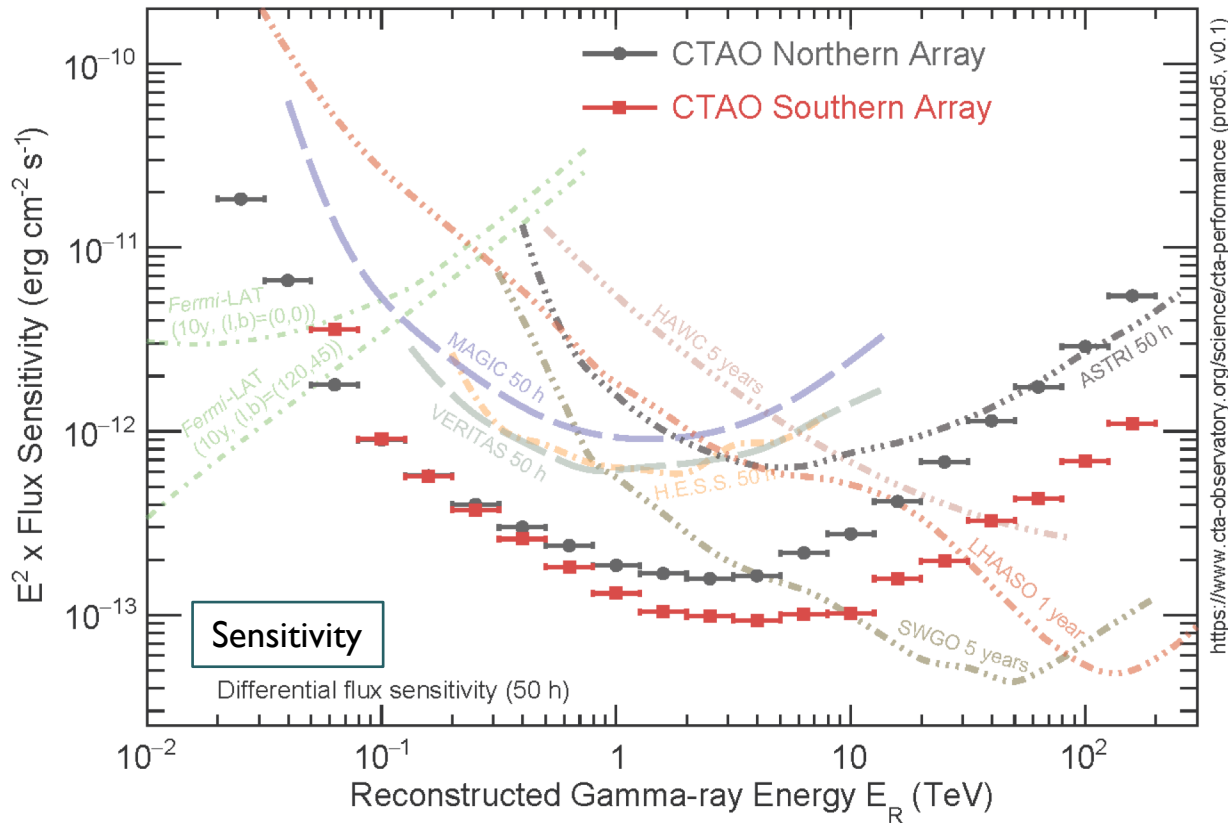
LST
20-200 GeV

<https://www.cta-observatory.org/>

CTA PERFORMANCE

Preliminary Performance Capabilities

<https://www.cta-observatory.org/>



CTA has superb capabilities for DM gamma-ray searches

KEY SCIENCE PROJECT: PERSEUS GALAXY CLUSTER WITH CTA

- Among local clusters, Perseus is the brightest in X-ray sky.

- Cool-cored, relaxed cluster

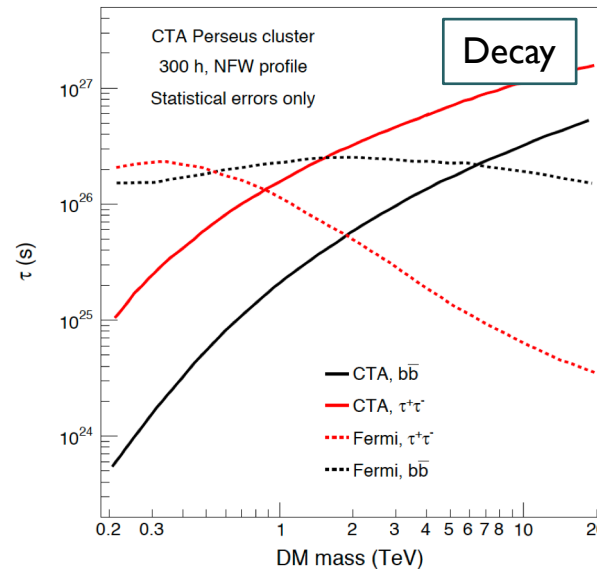
Object	l [deg]	b [deg]	d_L [Mpc]
Perseus	150.57	-13.26	75.01

- Host two AGNs, the BCG NGC1275 and IC310, both variable

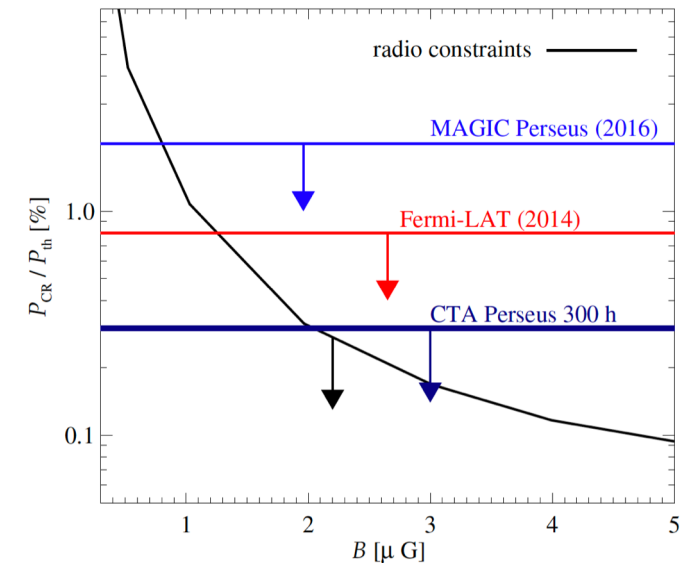
Object	l [deg]	b [deg]
NGC1275	150.58	-13.26
IC310	150.18	-13.74

BCG aligned with X-rays center

Optimal conditions for observation from the northern array



Acharya+17
[CTA Cons.]



Our goal: State-of-the-art study of the sensitivity of CTA to Dark Matter and Cosmic-Ray signals in Perseus cluster

We use the latest version of the CTA science tools with the latest Instrument Response Functions (IRFs) to perform the analysis

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Object
Perseus

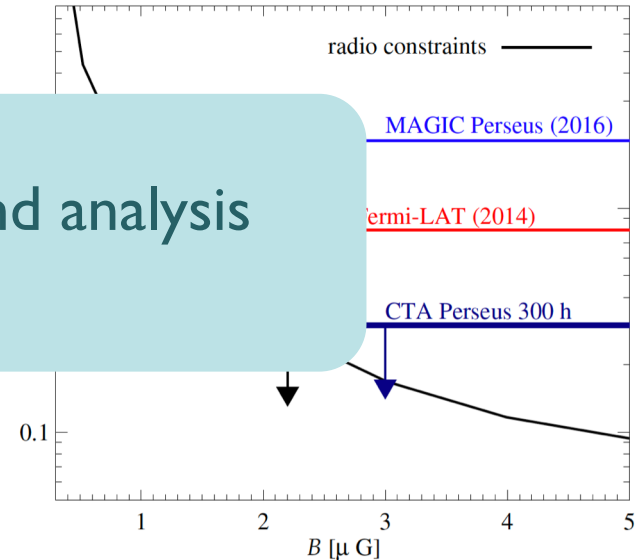
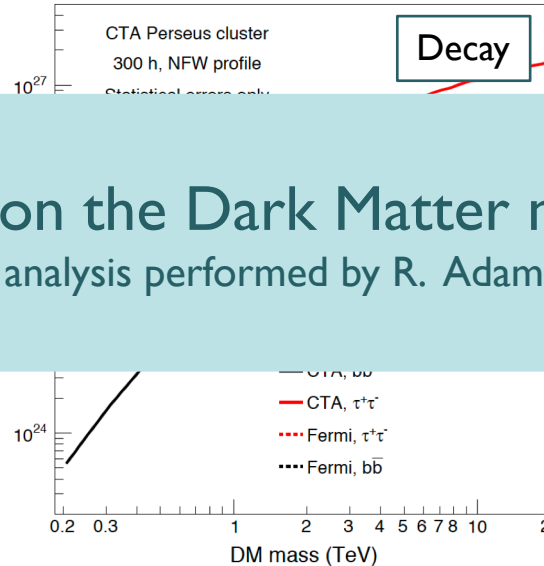
- Host two BCGs, NGC 1275 and IC310, both variable

Object	l [deg]	b [deg]
NGC1275	150.58	-13.26
IC310	150.18	-13.74

BCG aligned with X-rays center

Optimal conditions for observation from the northern array

Our goal today: Focus on the Dark Matter modelling and analysis
(CR modelling and analysis performed by R. Adam & collaborators)



Our goal: State-of-the-art study of the sensitivity of CTA to Dark Matter and Cosmic-Ray signals in Perseus cluster

Use latest version of CTA science tools with latest Instrument Response Functions (IRFs) to perform the analysis

DARK MATTER MODELLING

$$\frac{d\Phi_{DM}}{dE}(E, l.o.s, \Delta\Omega, z)$$

DM-induced γ -ray flux from an astrophysical object

$$\frac{d\phi}{dE}(E, z)$$

Particle Physics Model
Cirelli+12 (EW corrections)

Astrophysical factor



Charbonnier+12,
Bonnivard+15, Hütten+18

<https://clumpy.gitlab.io/CLUMPY/>

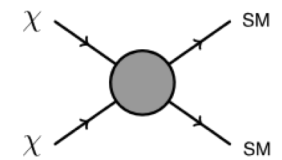


Annihilation

$$J(l.o.s, \Delta\Omega, z) = \int_{\Delta\Omega} \int_{l.o.s} \rho_{DM}^2(r) dr$$

DM density profile

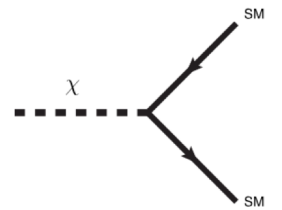
$$J \propto \frac{M_{200} c_{200}^3}{D_{Earth}^2}$$



Decay

$$D(l.o.s, \Delta\Omega, z) = \int_{\Delta\Omega} \int_{l.o.s} \rho_{DM}(r) dr$$

$$D \propto \frac{M_{200}}{D_{Earth}^2}$$



DARK MATTER MODELLING (I): MAIN HALO

Annihilation



$$J(l.o.s, \Delta\Omega, z) = \int_{\Delta\Omega} \int_{l.o.s} \rho_{DM}^2(r) dr$$

DM density profile

$$D(l.o.s, \Delta\Omega, z) = \int_{\Delta\Omega} \int_{l.o.s} \rho_{DM}(r) dr$$

Decay

MIN

Main halo without substructure

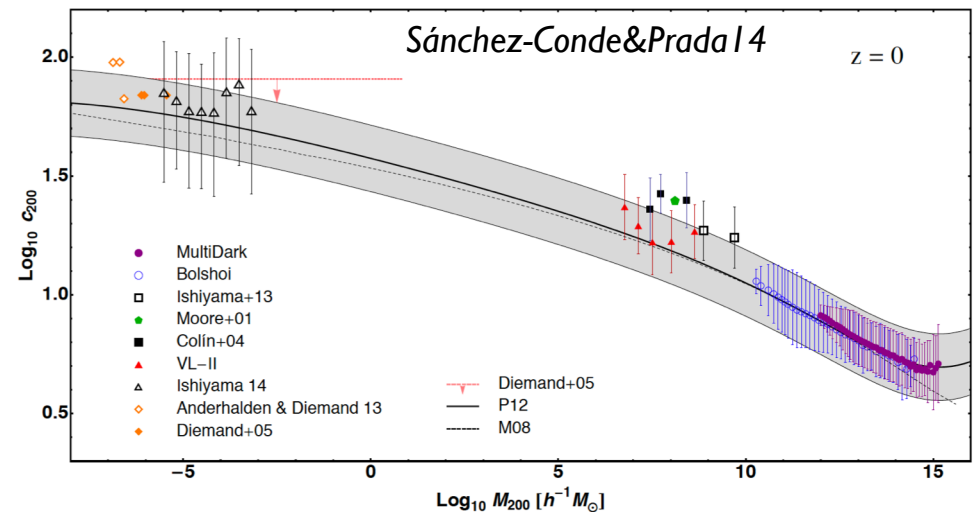
- State-of-the-art parametrization of the DM in galaxy clusters:

Assume density profile

$$\langle \rho_{\text{tot}} \rangle(r) = \rho_{\text{sm}}(r) + \langle \rho_{\text{subs}} \rangle(r) \longrightarrow \rho(r) = \frac{\rho_0}{\left(\frac{r}{r_s}\right) \left[1 + \frac{r}{r_s}\right]^2}$$

Navarro – Frenk – White (NFW)
Navarro+96, Navarro+97

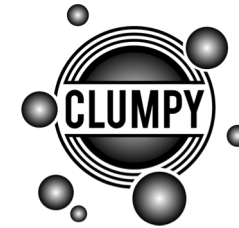
- To build the DM profile, we assume a concentration-mass relation ($c_{200} - M_{200}$):



DARK MATTER MODELLING (II): SUBSTRUCTURE

- Galaxy clusters are the most massive objects today, large amount of substructure expected
- Inclusion through ρ_{DM} using state-of-the-art subhalo models

$$\langle \rho_{\text{tot}} \rangle(r) = \rho_{\text{sm}}(r) + \langle \rho_{\text{subs}} \rangle(r) \longrightarrow \frac{d^3 N}{dV dM dc} = N_{\text{tot}} \frac{d\mathcal{P}_V}{dV}(r) \cdot \frac{d\mathcal{P}_M}{dM}(M) \cdot \frac{d\mathcal{P}_c}{dc}(M, c)$$



DM subhalo profile: NFW

$$\rho(r) = \frac{\rho_0}{\left(\frac{r}{r_s}\right) \left[1 + \frac{r}{r_s}\right]^2}$$

Subhalo Radial Distribution (SRD)

$$\rho_{\text{sub}}^{\text{VLII}}(R) = \frac{\rho_{\text{tot}}^{\text{VLII}}(R) (R/R_a)}{\left(1 + \frac{R}{R_a}\right)}$$

Via Lactea - II
Anti-biased relation
Diemand+08

Subhalo Mass Function (SHMF)

$$dN/dm = A/M(m/M)^{-\alpha}$$

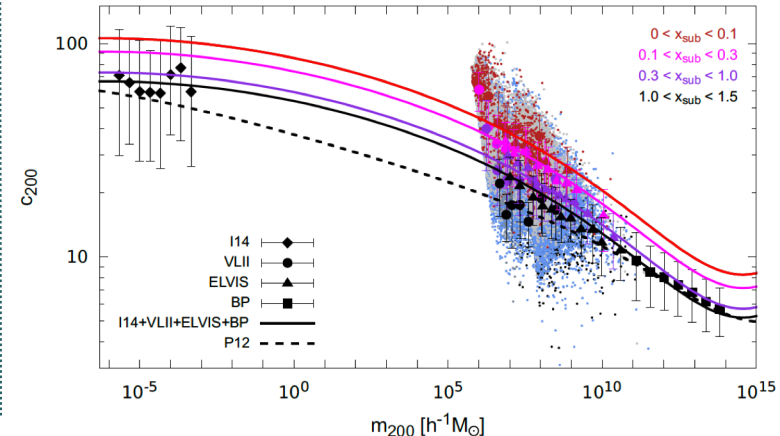
$\alpha = 1.9 \longrightarrow$ MED

Springel+08

$\alpha = 2.0 \longrightarrow$ MAX

Diemand+08

Subhalo Concentration-Mass relation ($c_{200}-M_{200}$)



Dependence on the subhalo position

$$c_{200}(m_{200}, x_{\text{sub}})$$

$$x_{\text{sub}} \equiv R_{\text{sub}}/R_{\Delta}$$

Moliné+17

EXPECTED DM SIGNAL

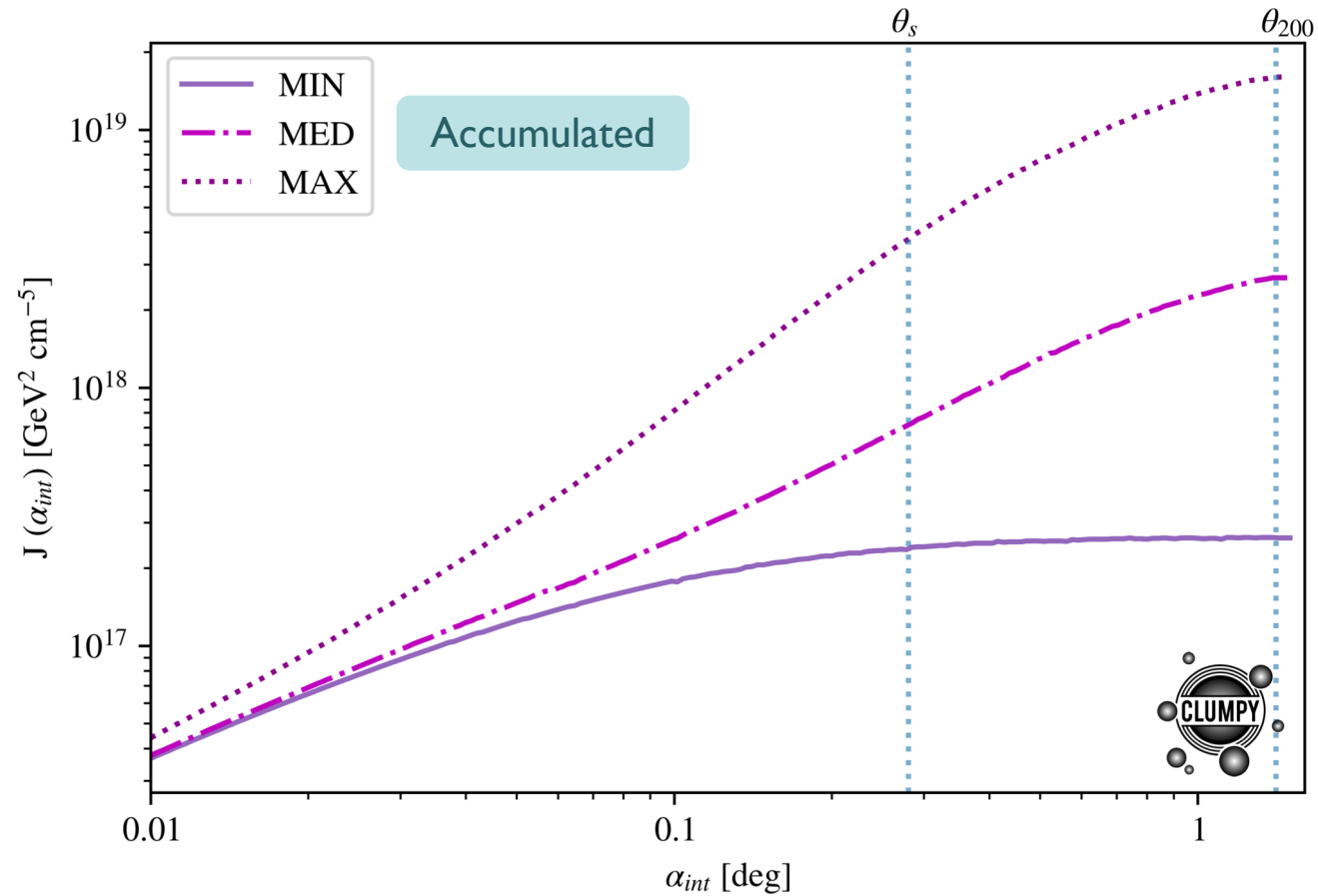
General parameters

Hitomi Coll. 18	z	0.017284	l, b	150.58 deg, -13.26 deg
Urban+14	M_{200}	$7.52 \times 10^{14} M_{\odot}$	R_{200}	1865.0 kpc
Sánchez-Conde & Prada 14	c_{200}	5.03	θ_{200}	1.42 deg
	r_s	370.82 kpc	θ_s	0.28 deg
Flat Λ CDM	d_L	75.01 Mpc	ρ_s	$299581 M_{\odot}/\text{kpc}^3$

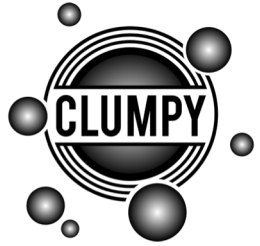
Apply modelling formalism

Annihilation	$\log_{10} J [\text{GeV}^2 \text{cm}^{-5}]$
MIN	17.42
MED	18.43
MAX	19.20
Decay	$\log_{10} D [\text{GeV cm}^{-2}]$
	19.20

Annihilation flux profile



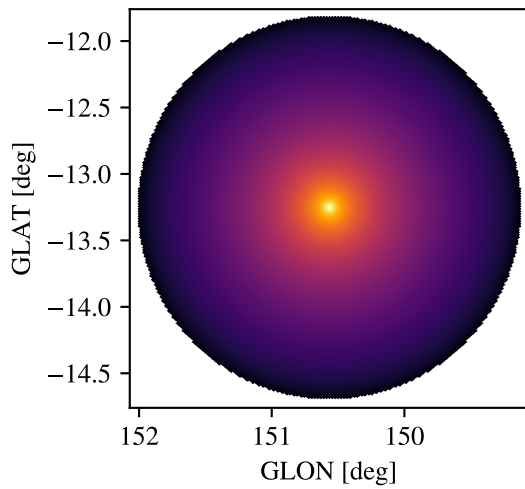
MORPHOLOGY OF DM SIGNAL



Skymaps of the differential J-factor

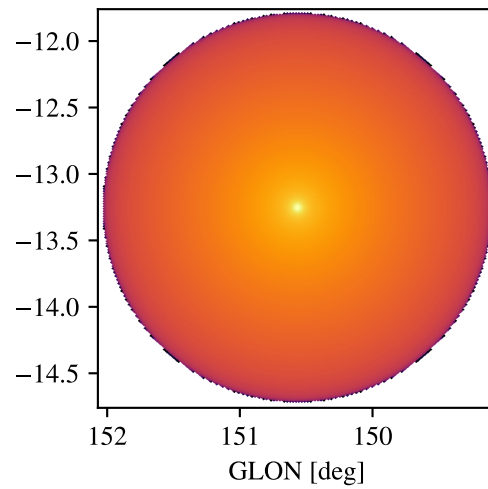
Annihilation

MIN



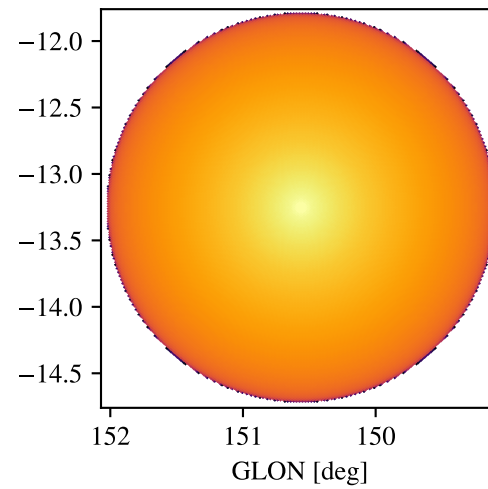
MIN

MED

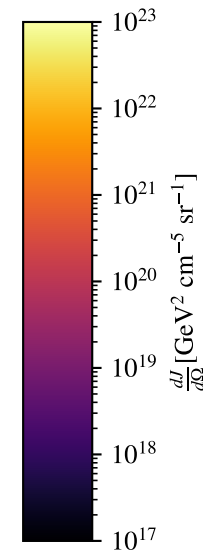


MED

MAX

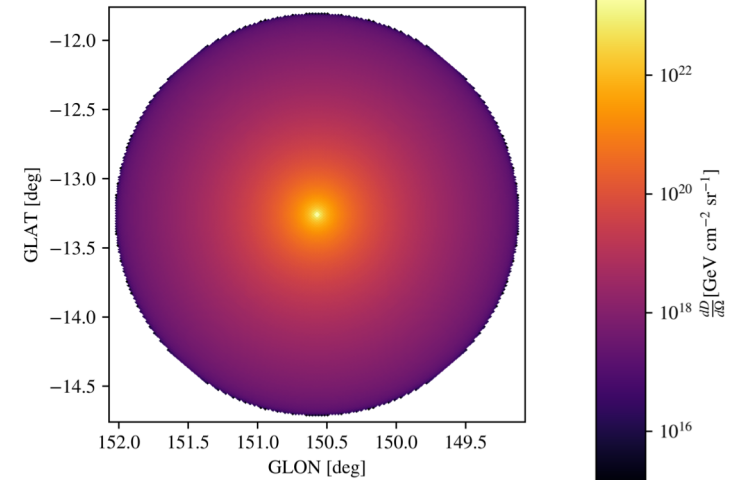


MAX



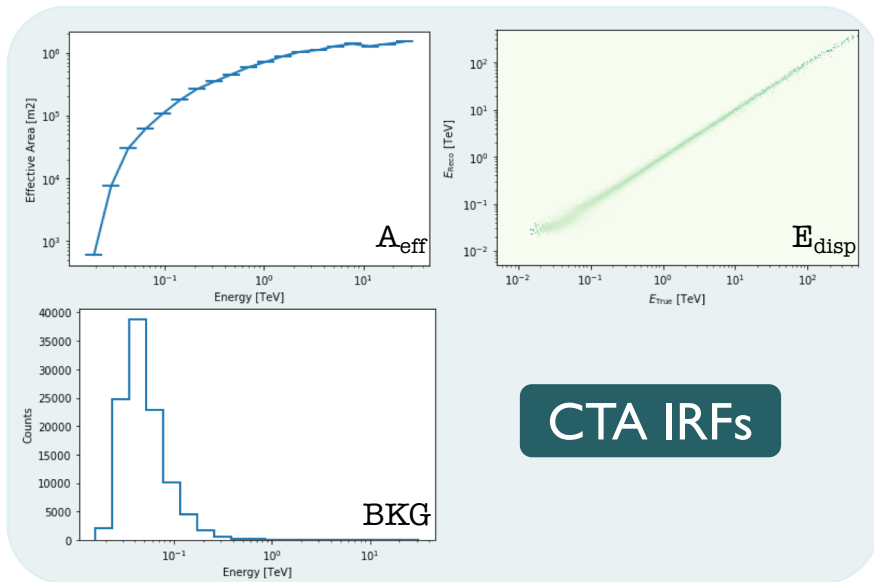
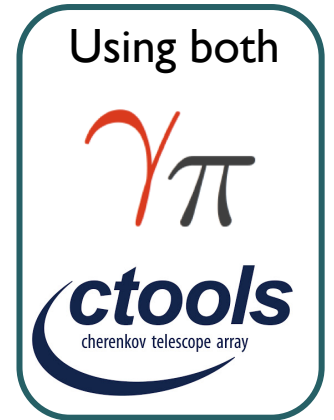
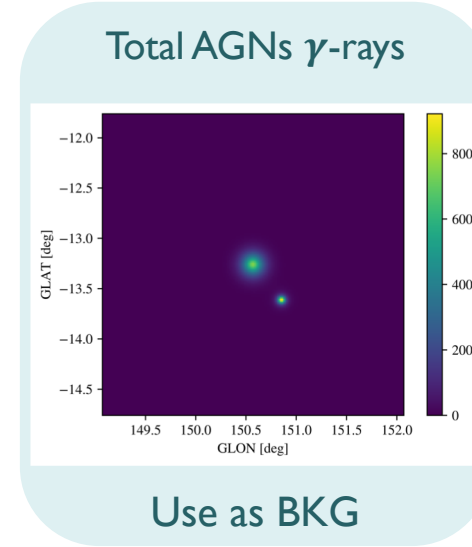
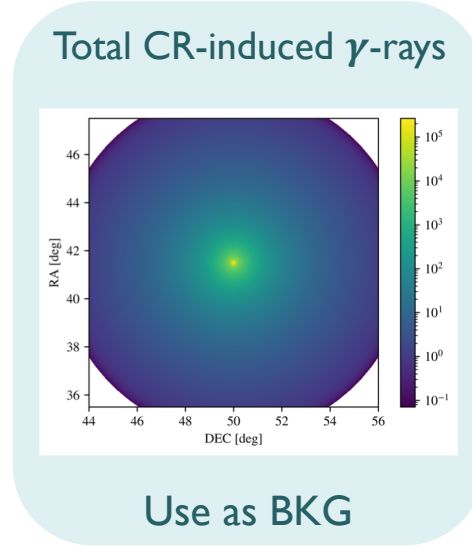
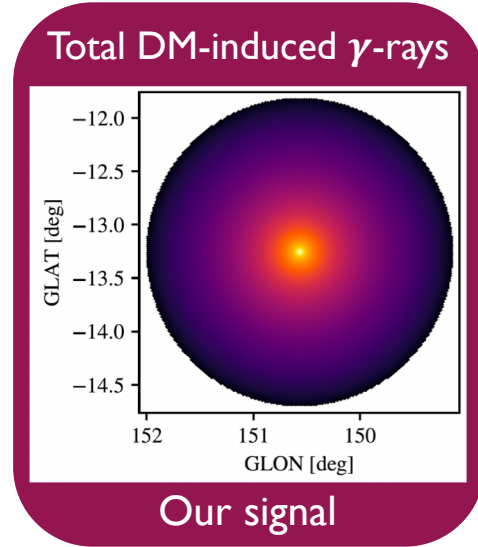
Skymap of the differential D-factor

Decay



CTA DM ANALYSIS ROADMAP

- Different gamma-ray sources in Perseus region:



Observation Simulation

If no signal found

Constraints on DM models

$$\frac{d\Phi_{DM}^{Annihil}}{dE} = \frac{\langle \sigma v \rangle dN}{8\pi m_{DM}^2 dE} \times J$$

$$\frac{d\Phi_{DM}^{Decay}}{dE} = \frac{1}{4\pi m_{DM} \tau_{DM}} \frac{dN}{dE} \times D$$

CTA ANALYSIS CONFIGURATION (I): ON/OFF ANALYSIS

- First analysis approach

- Only includes gamma-ray emission from DM and background from IRFs
- Assumes Perseus as a point-like source
- Historically used in Imaging Air Cherenkov Telescopes (IACTs) as MAGIC

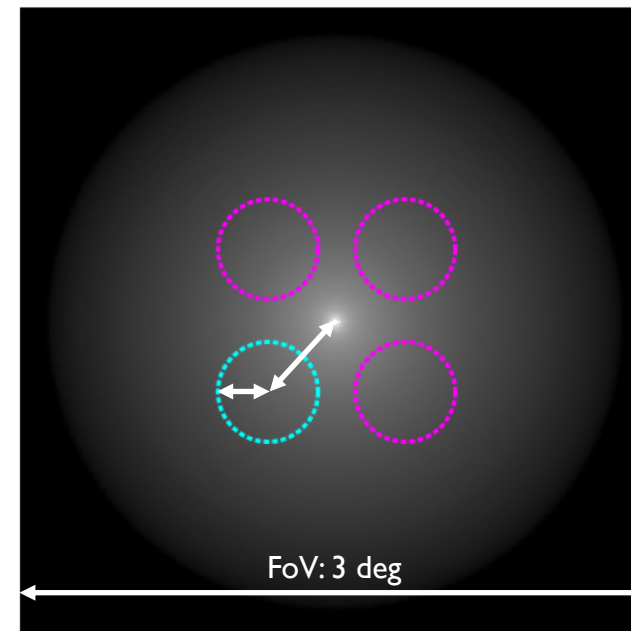
Lowest level of complexity,
more constraining results

Direct comparisons

- Different set-ups tested, best results for:

Regions	1 ON/ 3 OFF
Regions radius [deg]	0.5
Pointing (l , b) [deg]	(150.57, -13.26)
Offset [deg]	0.5

N_{obs}	50
T_{obs} [h]	300
IRFs	North_z20_50h, prod3b-v2
Energy range [TeV]	0.03 - 100

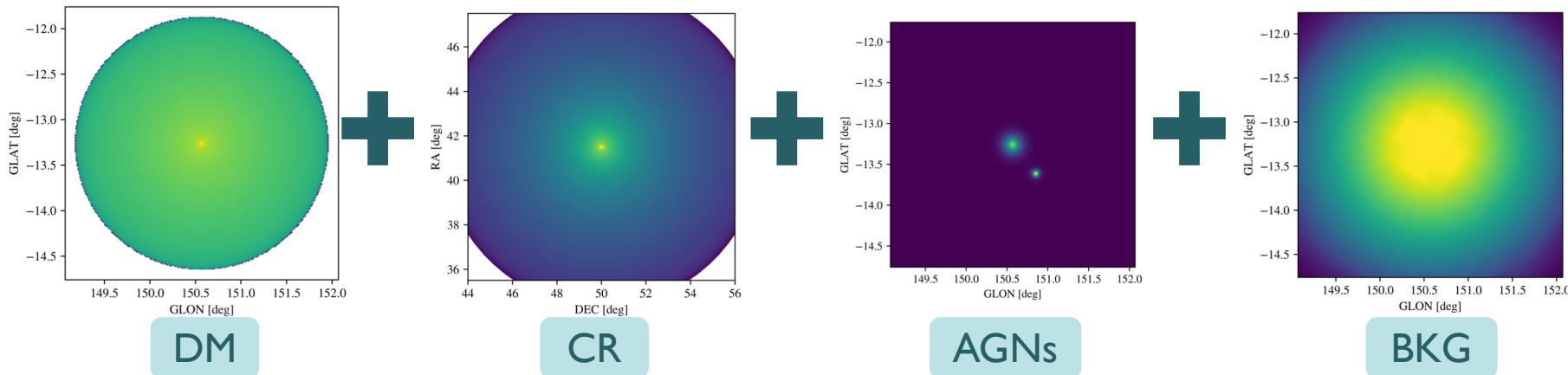


CTA ANALYSIS CONFIGURATION (II): TEMPLATE FITTING

On-going (in collab. with R.Adam)

- Final analysis goal:

- Includes all possible gamma-ray sources: DM + CRs + AGNs + BKG IFRs



More realistic physical scenario

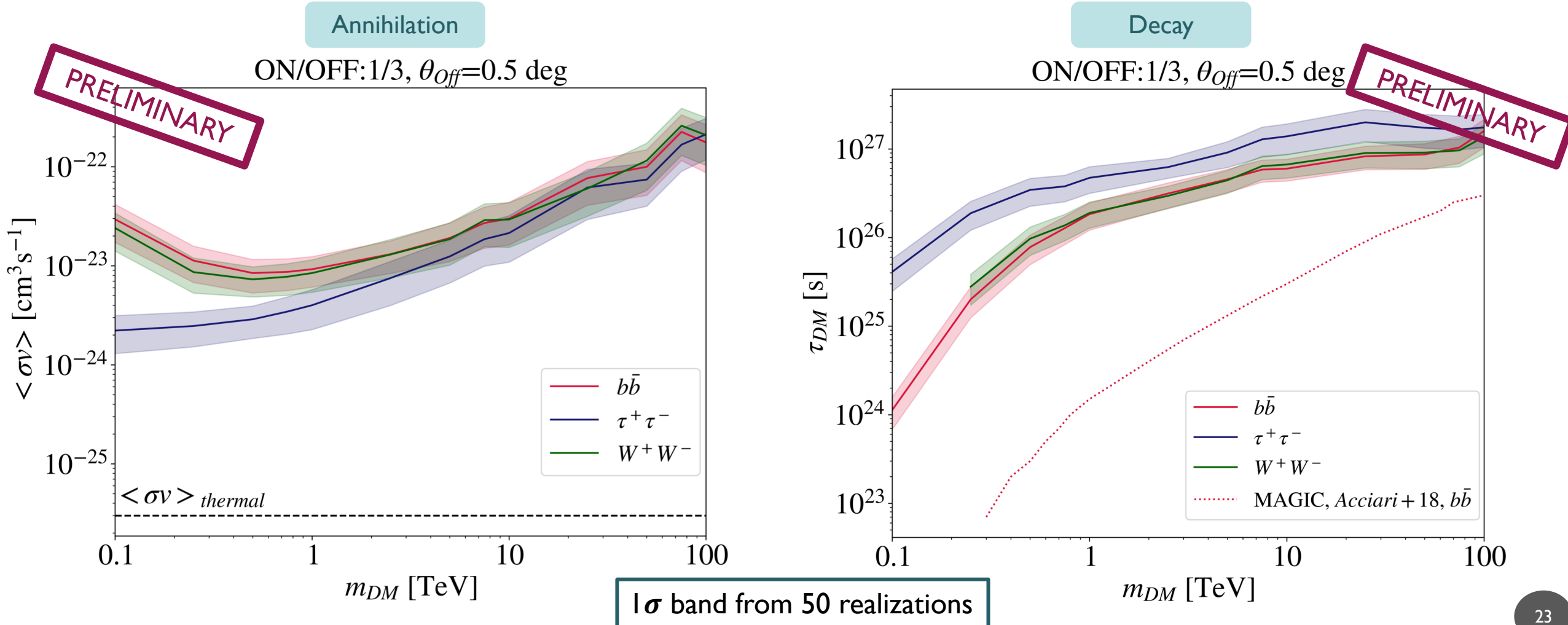
- Considers the different morphologies of each emission
- Allows to check correlations between components
- Historically used in Fermi-*LAT* analysis and in a recent CTA analysis (Acharyya+20 [CTA Cons.]

State-of-the-art analysis pipeline

ON/OFF RESULTS: DM CONSTRAINTS



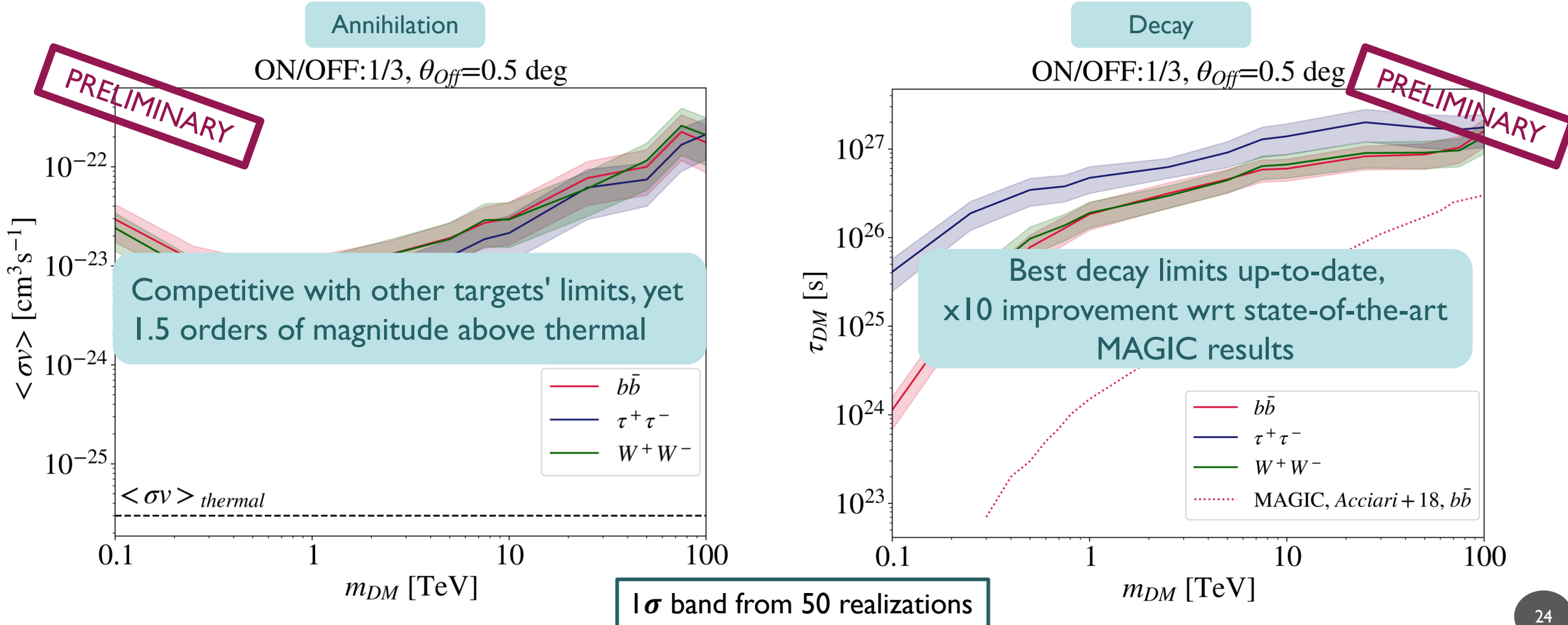
Limits for Perseus for MED annihilation model and decay
(point-like morphology & no J/D-factor uncertainties)



ON/OFF RESULTS: DM CONSTRAINTS



Limits for Perseus for MED annihilation model and decay
(point-like morphology & no J/D-factor uncertainties)



SUMMARY

- Galaxy clusters are excellent target for indirect DM searches (massive, closeby)
- Still no clear gamma-ray signal from clusters detected
- CTA is the future for VHE gamma-ray astronomy, with superb capabilities for WIMP searches
- Perseus Galaxy Cluster has optimal conditions for observation with CTA-North
- State-of-the-art DM modelling for Perseus including halo substructure
- Complete and comprehensive study of the different expected emissions: DM+CR+AGNs
- ON/OFF analysis for annihilation and decay main results:
 - Annihilation upper limits of $\sim O(10^{-23}) \text{ cm}^3 \text{ s}^{-1}$
 - Decay upper limits of $\sim O(10^{26}) \text{ s}$: will be the best limits
 - Most optimistic limits
- On-going template fitting analysis and inclusion of J/D-factor uncertainties for more realistic predictions

Thanks for your attention!

Back-up material

OBTENTION OF DM MODEL PARAMETERS

- State-of-the-art parametrization of the DM in galaxy clusters: $\langle \rho_{\text{tot}} \rangle(r) = \rho_{\text{sm}}(r) + \langle \rho_{\text{subs}} \rangle(r)$

1 Assume a DM profile $\rho(r) = \frac{\rho_0}{\left(\frac{r}{r_s}\right)\left[1 + \frac{r}{r_s}\right]^2}$ NFW

2 Assume a concentration-mass relation ($c_{200} - M_{200}$): *Sánchez-Conde&Prada 14* $c_{200}(M_{200}, z=0) = \sum_{i=0}^5 c_i \times \left[\ln \left(\frac{M_{200}}{h^{-1} M_{\odot}} \right) \right]^i$

3 Assume spherical collapse from an overdensity $\Delta = 200$ over the critical density $\Delta_{200} = \frac{3M_{200}}{4\pi R_{200}^3 \rho_{\text{crit}}}$

- 4 Compute remaining parameters

Scale density

$$\rho_0 = \frac{2\Delta_{200}\rho_{\text{crit}}c_{200}}{3F(c_{200})}$$

with

$$F(c_{200}) = \frac{2}{c_{200}} \left(\ln(1 + c_{200}) - \frac{c_{200}}{1 + c_{200}} \right)$$

Scale radius

$$c_{200} = \frac{R_{200}}{r_s}$$

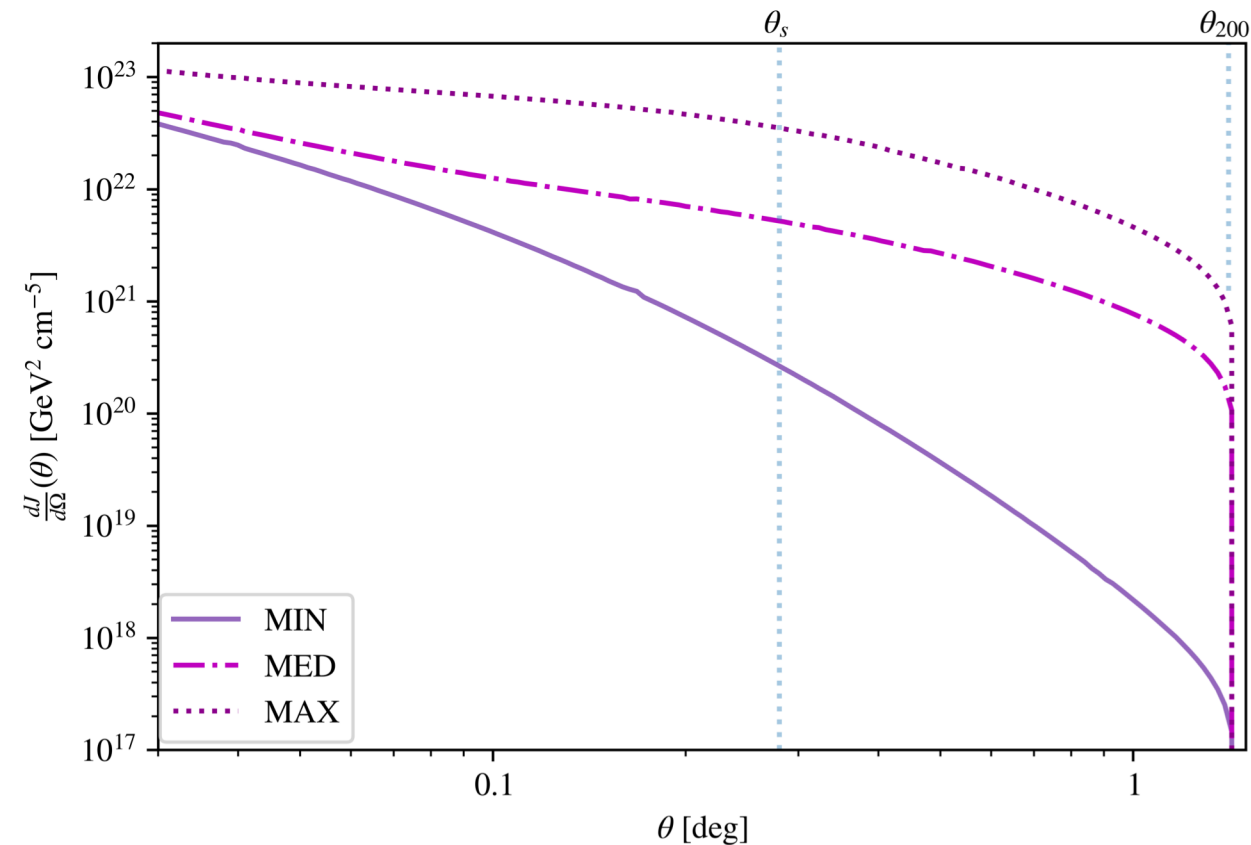
Angular extension

$$\theta_{200} = \tan \left(\frac{R_{200}}{d_L} \right)$$

DIFFERENTIAL ANNIHILATION FLUX PROFILE



Differential



General parameters

z	0.017284	l, b	150.58 deg, -13.26 deg
M_{200}	$7.52 \times 10^{14} M_{\odot}$	R_{200}	1865.0 kpc
c_{200}	5.03	θ_{200}	1.42 deg
r_s	370.82 kpc	θ_s	0.28 deg
d_L	75.01 Mpc	ρ_s	$299581 M_{\odot}/\text{kpc}^3$

CTA ANALYSIS ELEMENTS

- Likelihood ratio test:

$$TS = -2 \ln \frac{\mathcal{L}(\alpha; \hat{\nu} | \mathcal{D})}{\mathcal{L}(\hat{\alpha}; \hat{\nu} | \mathcal{D})}$$

ON/OFF analysis: Poisson likelihood for signal and background, *Wstat* statistics (*XSpec manual*)

$$W = 2(\mu_{sig} + (1+r)\mu_{bkg} - n_{ON} - n_{OFF} - n_{ON}(\ln(\mu_{sig} + r\mu_{bkg}) - \ln n_{ON}) - n_{OFF}(\ln \mu_{bkg} - \ln n_{OFF})) \quad r = \frac{N_{ON}}{N_{OFF}}$$

- $TS < 25 \rightarrow$ No signal

Template fitting: Poisson likelihood for each component, *Cash* statistics (*Cash 79*)

$$C = 2(\mu - n \ln \mu)$$

- Uncertainties in the J/D-factor enter through:

X-rays measurements



Urban+14
Mass modelling and extrapolations

- Masses from other methods
- Other X-rays measurements



$c(M) - M$ scatter

- $\sim O(0.3)$ dex for Sánchez-Conde & Prada 14



$$\mathcal{J}(J | J_{\text{obs}}, \sigma_J) = \frac{1}{\ln(10) J_{\text{obs}} \sqrt{2\pi} \sigma_J} \times e^{-\left(\log_{10}(J) - \log_{10}(J_{\text{obs}})\right)^2 / 2\sigma_J^2}$$

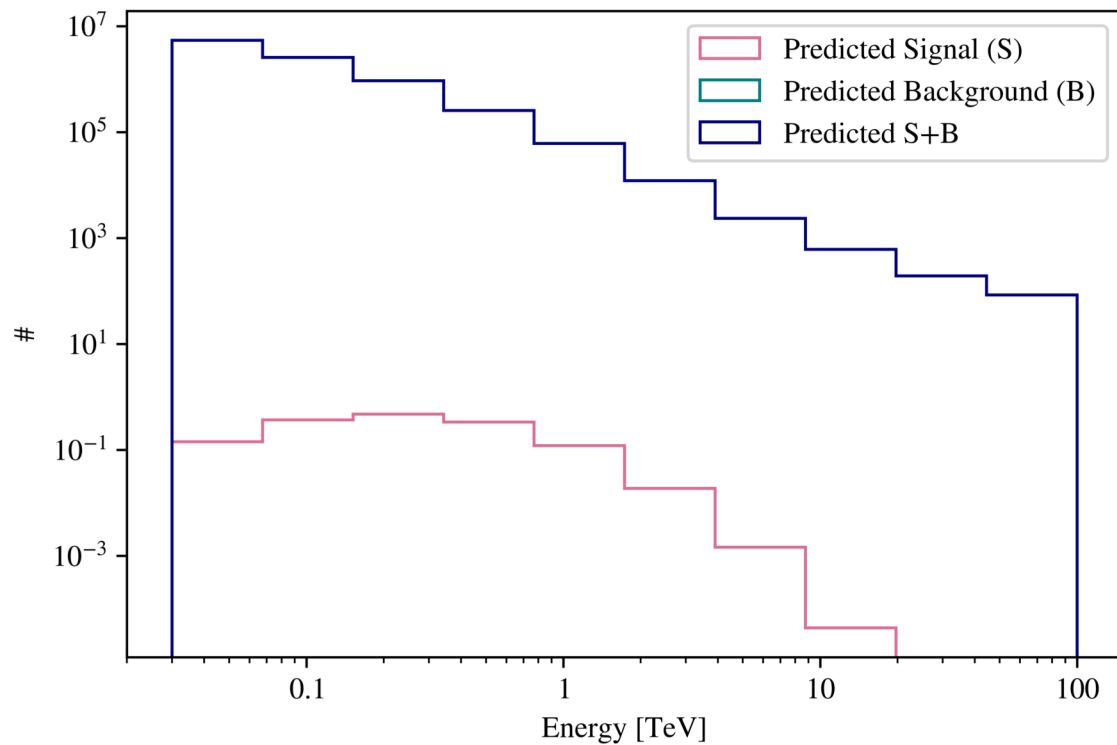
Gaussian prior

CHARACTERISTICS OF THE SIMULATIONS

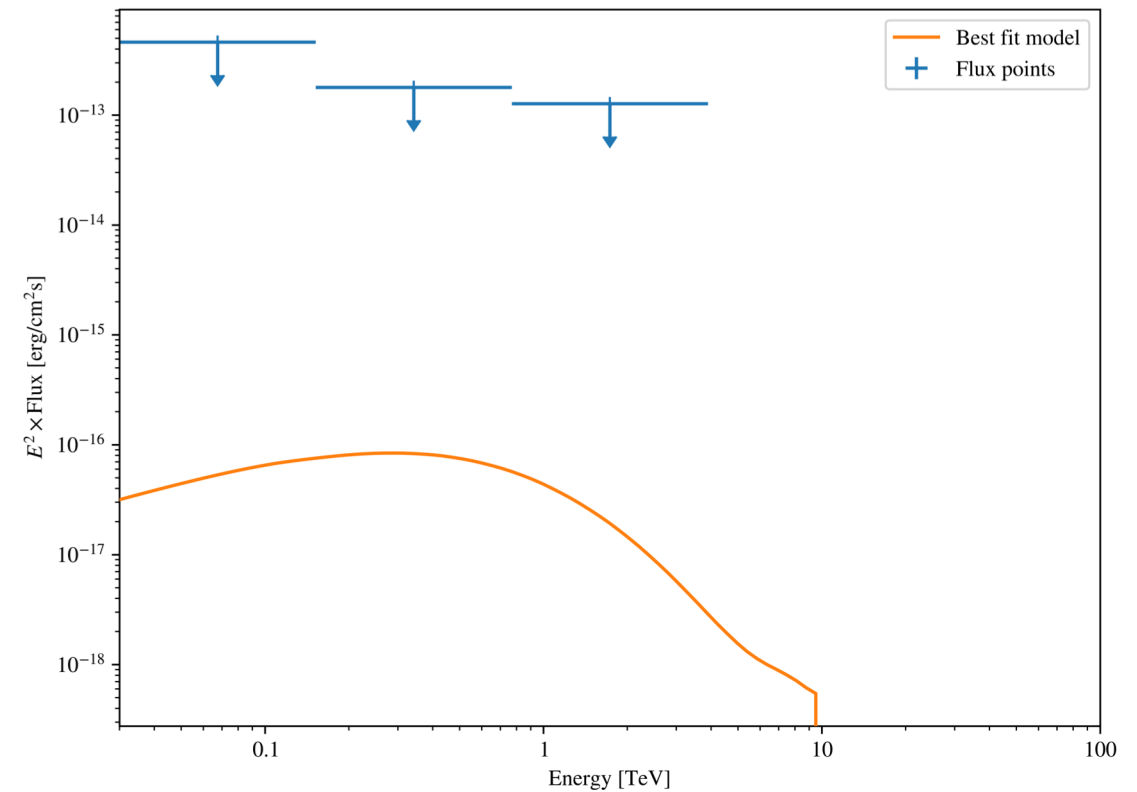


- One example simulation:
 - Annihilation
 - 10 TeV
 - b channel

Counts



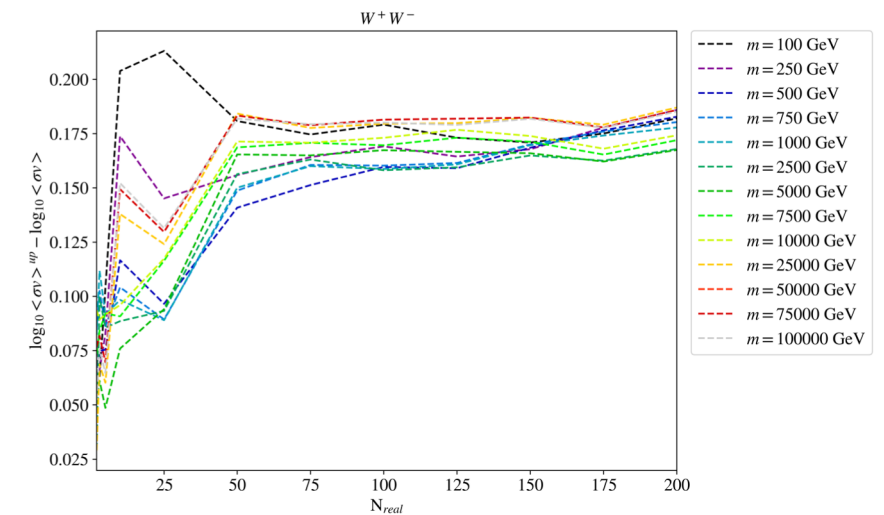
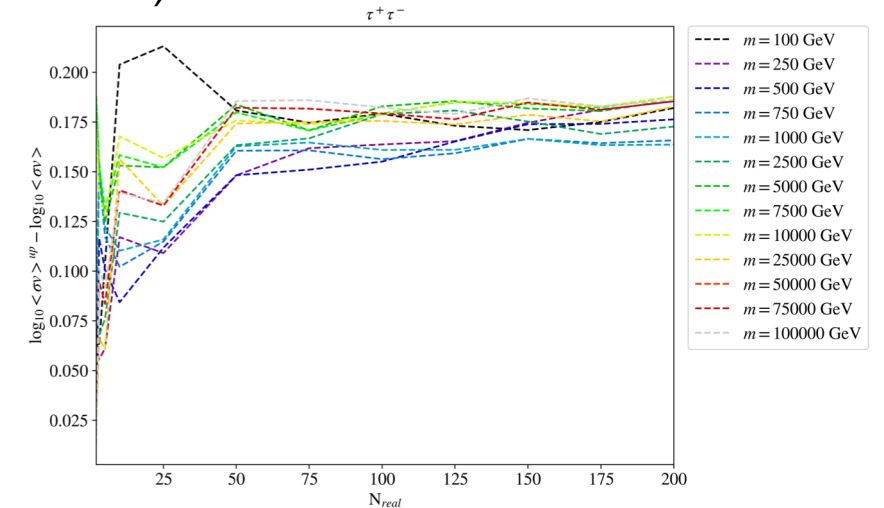
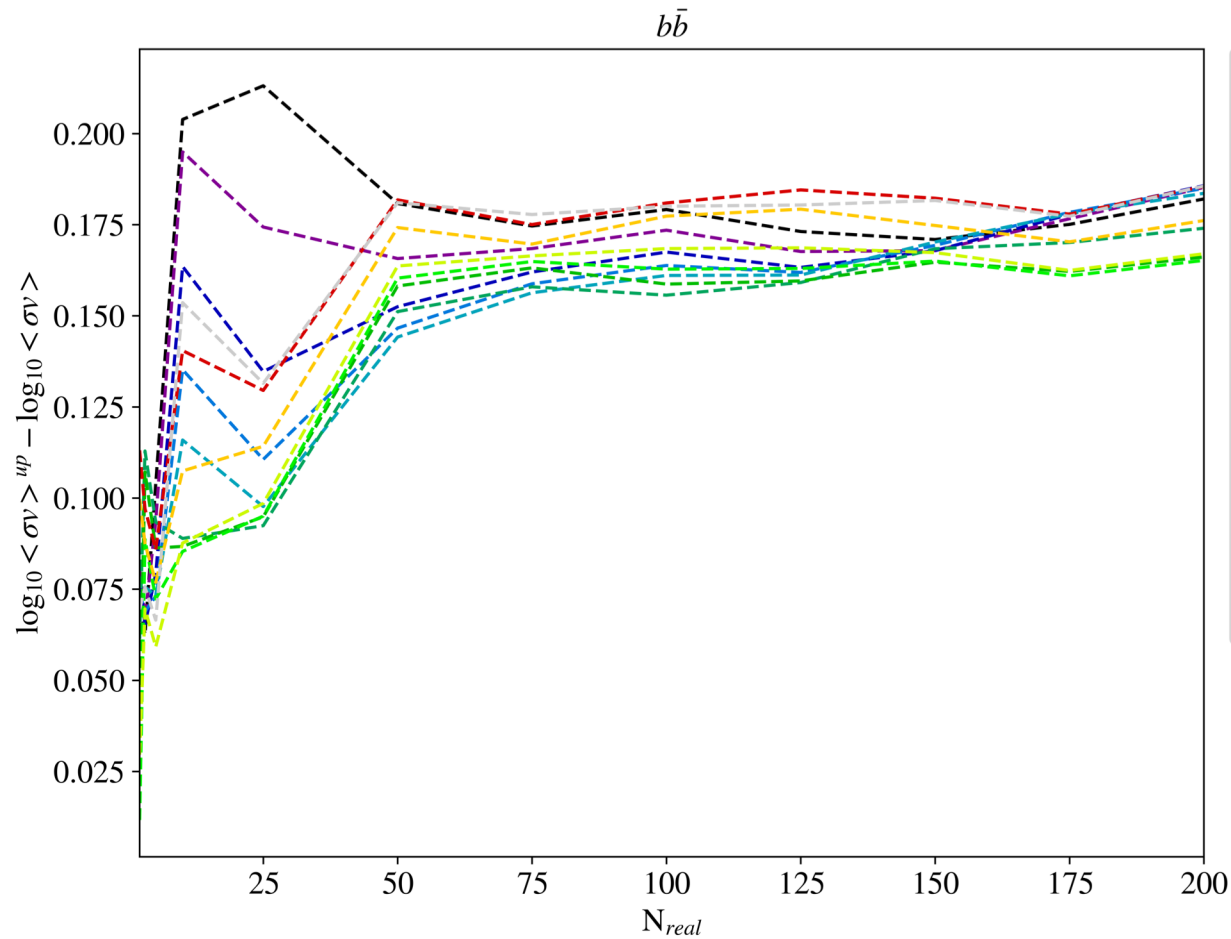
SED + ULs



DM CONSTRAINTS: 1σ BAND



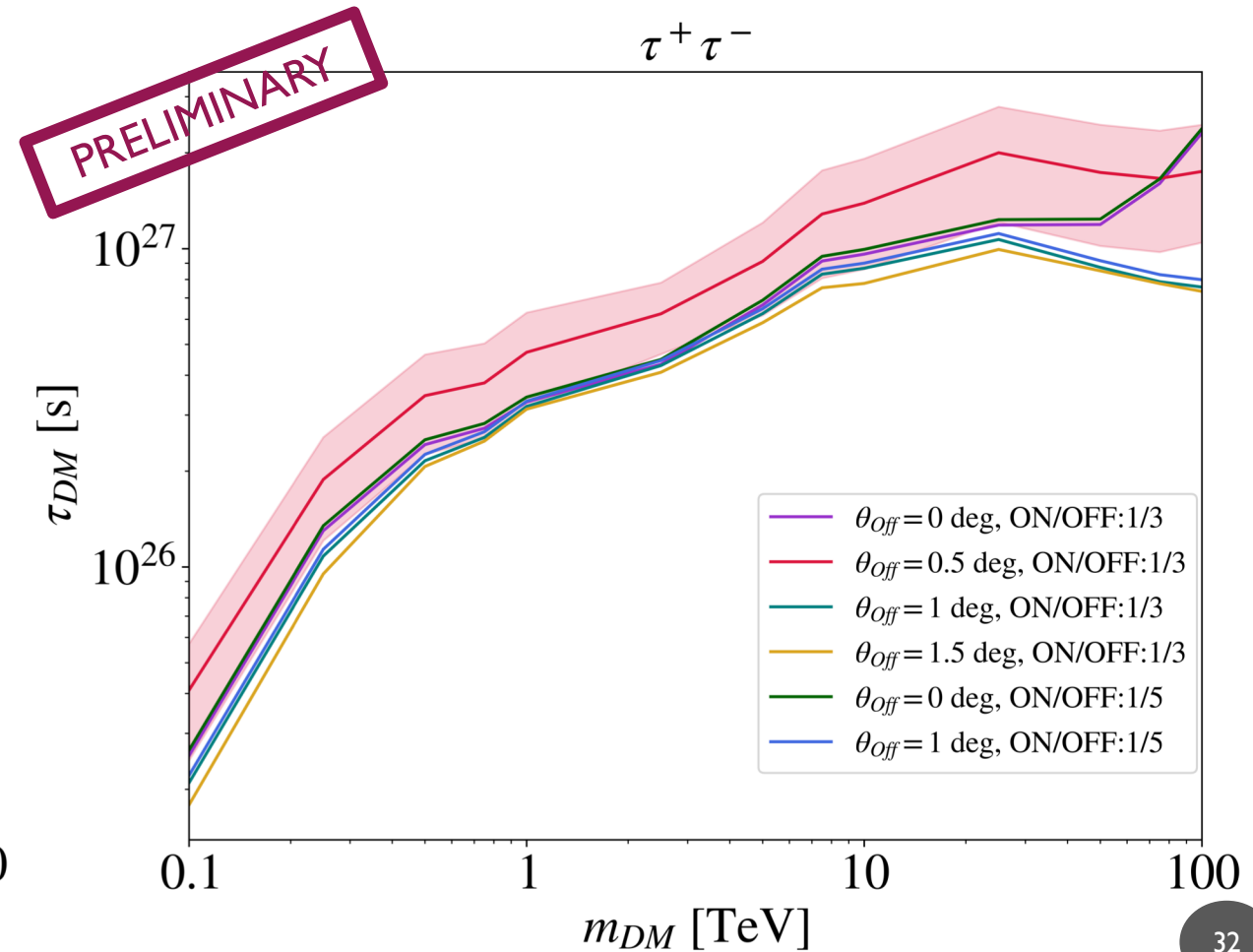
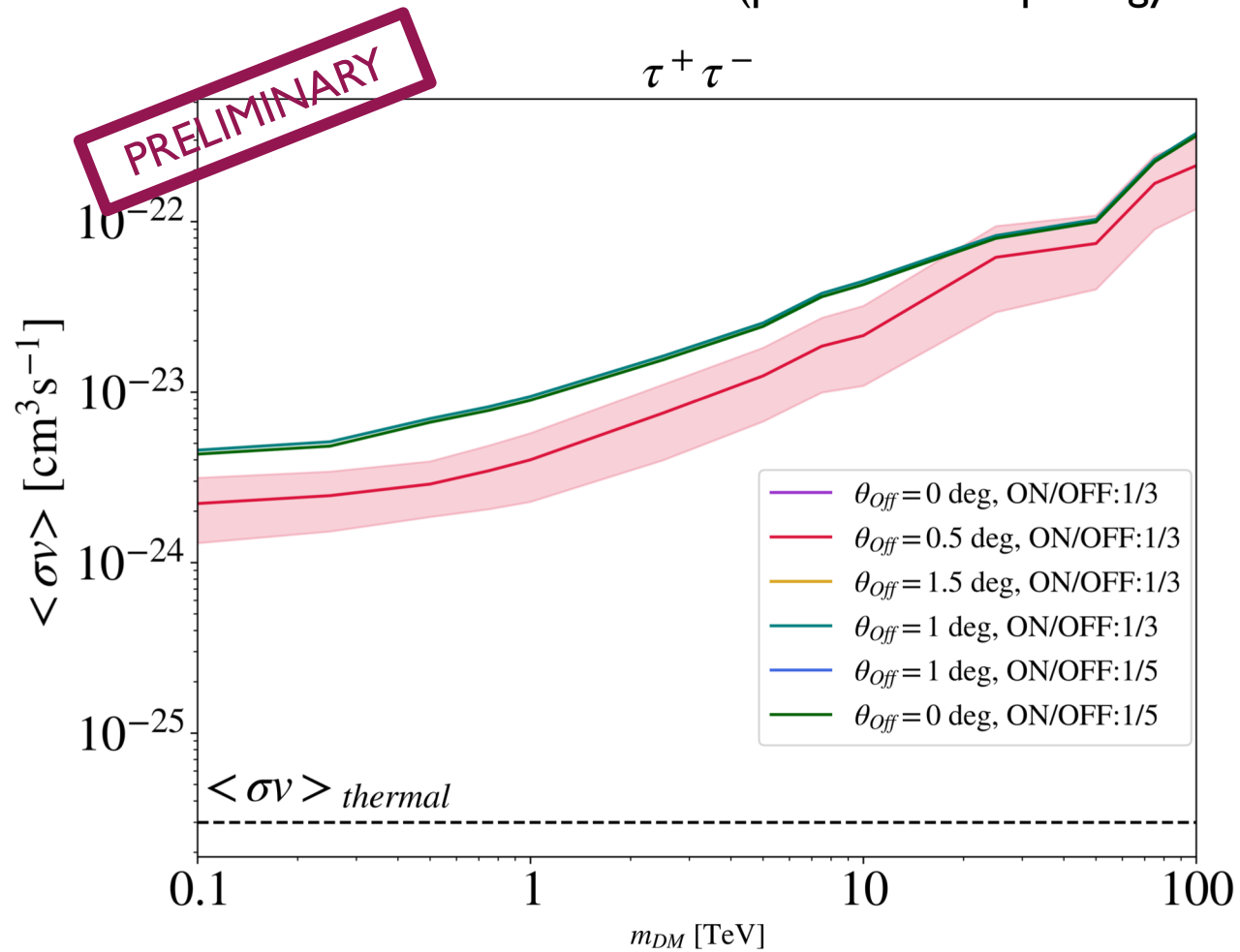
Limits for Perseus for MED annihilation model
(point-like morphology & no J-factor uncertainties)



DM CONSTRAINTS: ON/OFF SET-UPS

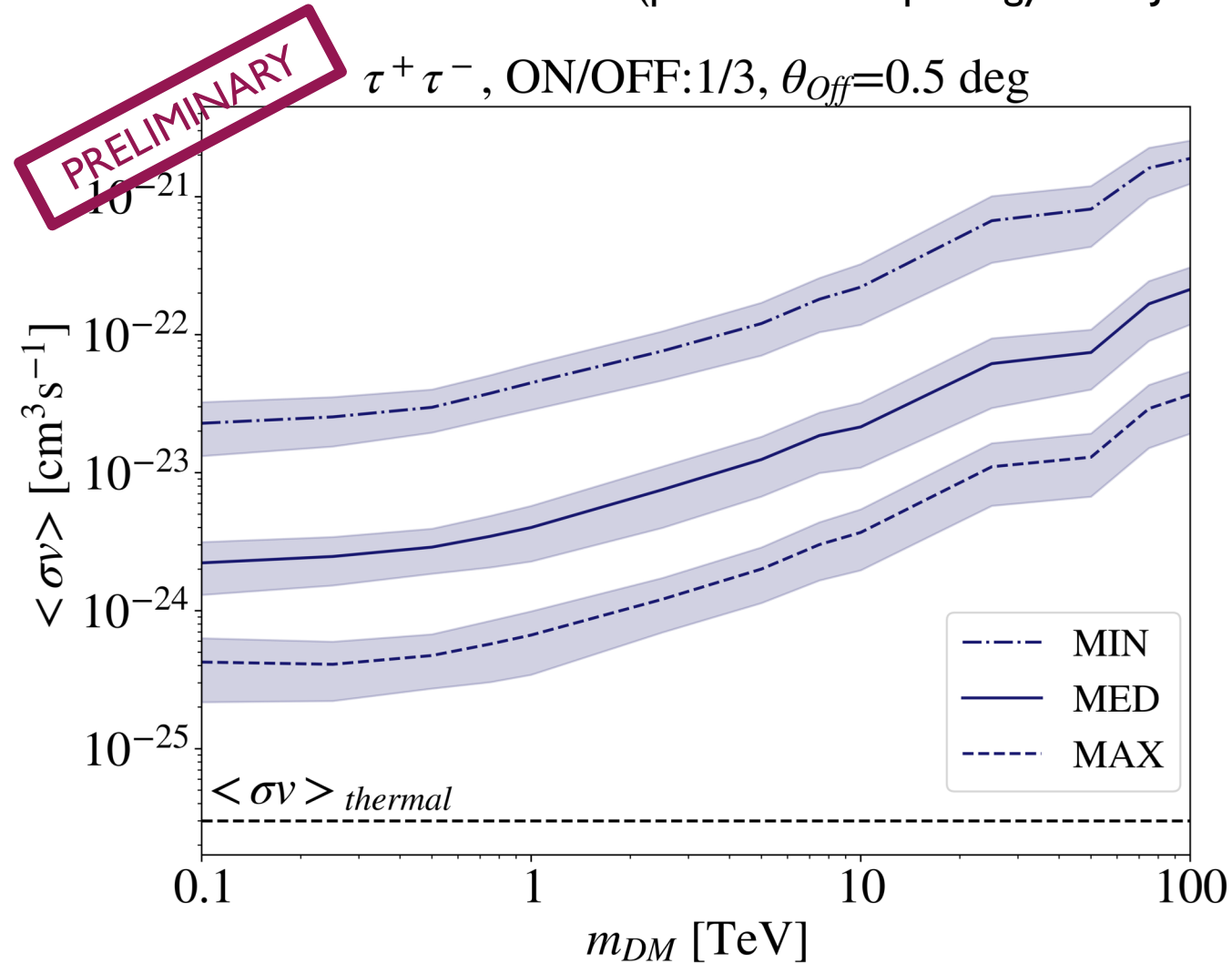


Limits for Perseus for $\tau^+\tau^-$ annihilation and decay models
(point-like morphology & no J/-D-factor uncertainties)



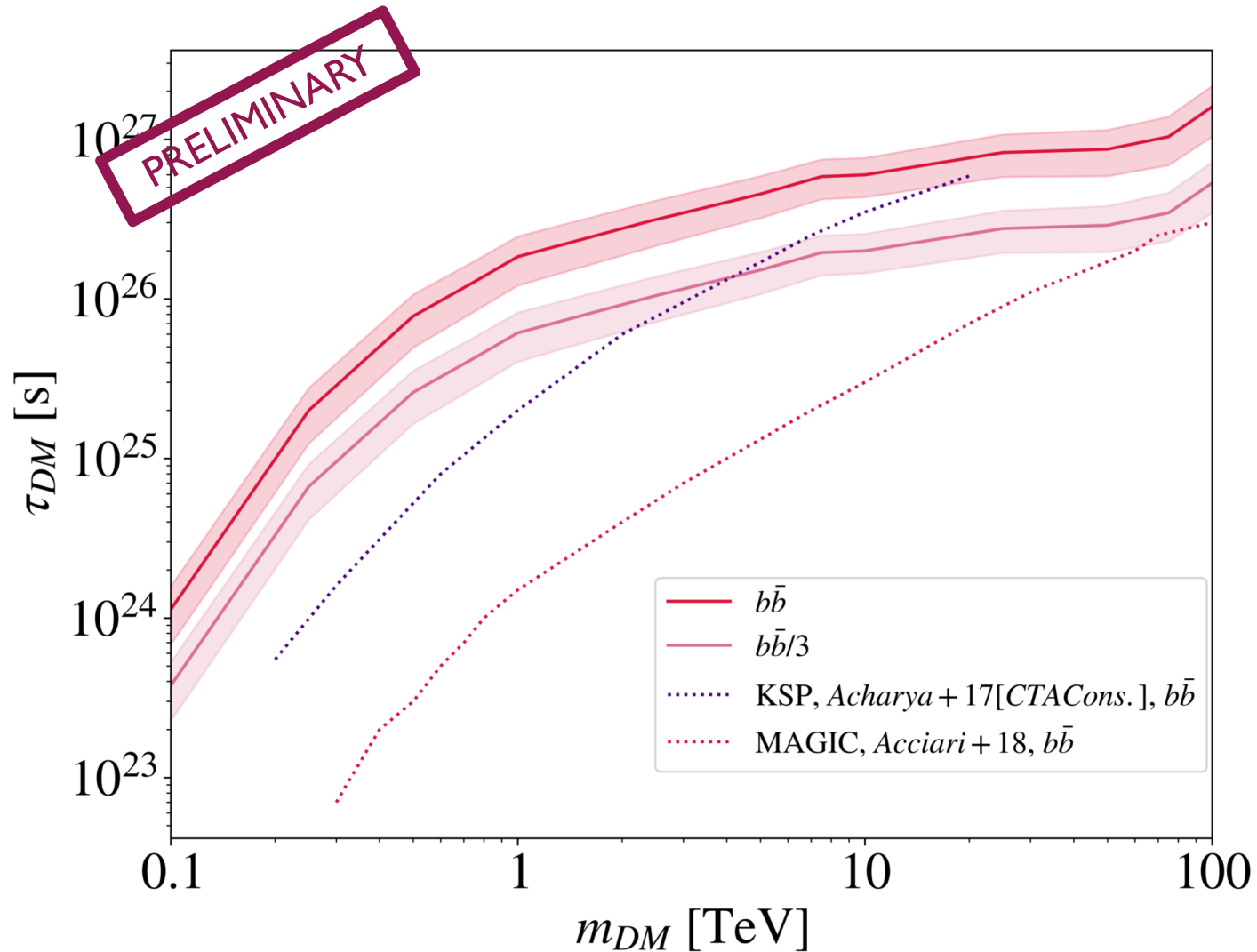
DM CONSTRAINTS: MIN-MED-MAX

Limits for Perseus for $\tau^+\tau^-$ annihilation model (point-like morphology & no J-factor uncertainties)



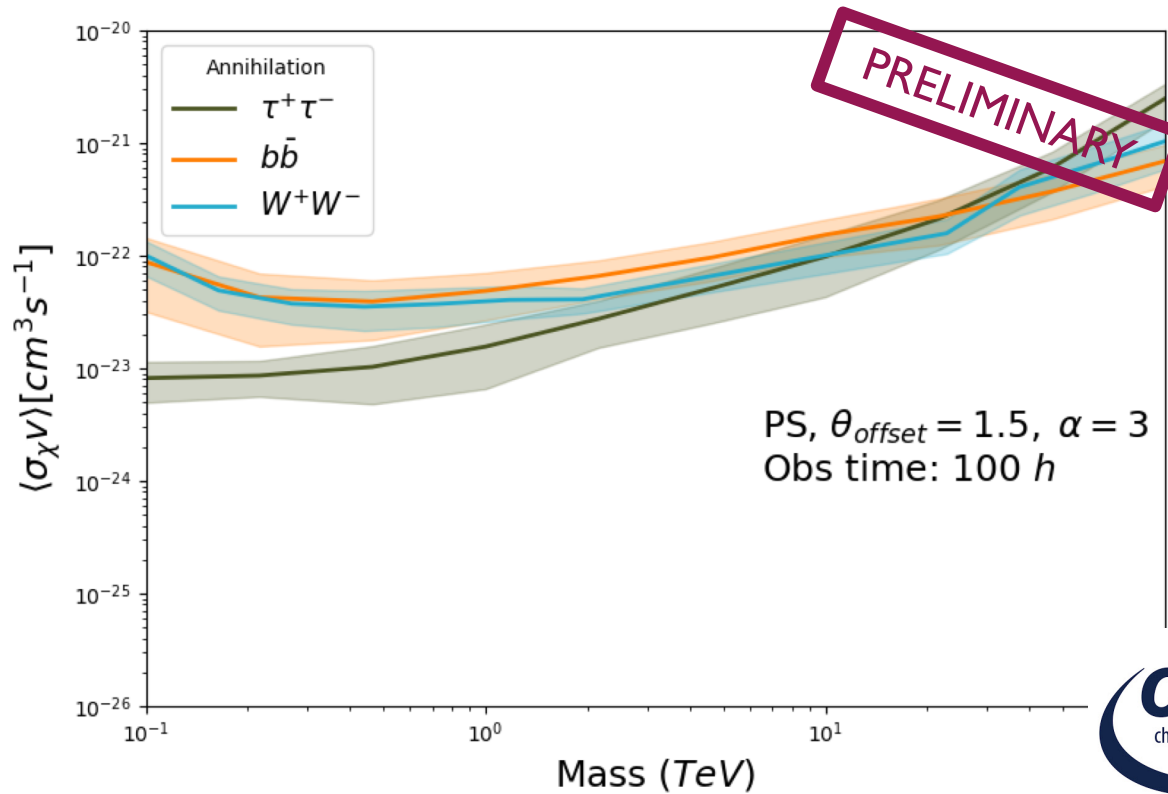
DM CONSTRAINTS: DECAY INSIGHT

Limits for Perseus for decay ON/OFF analysis (point-like morphology & no D-factor uncertainties)

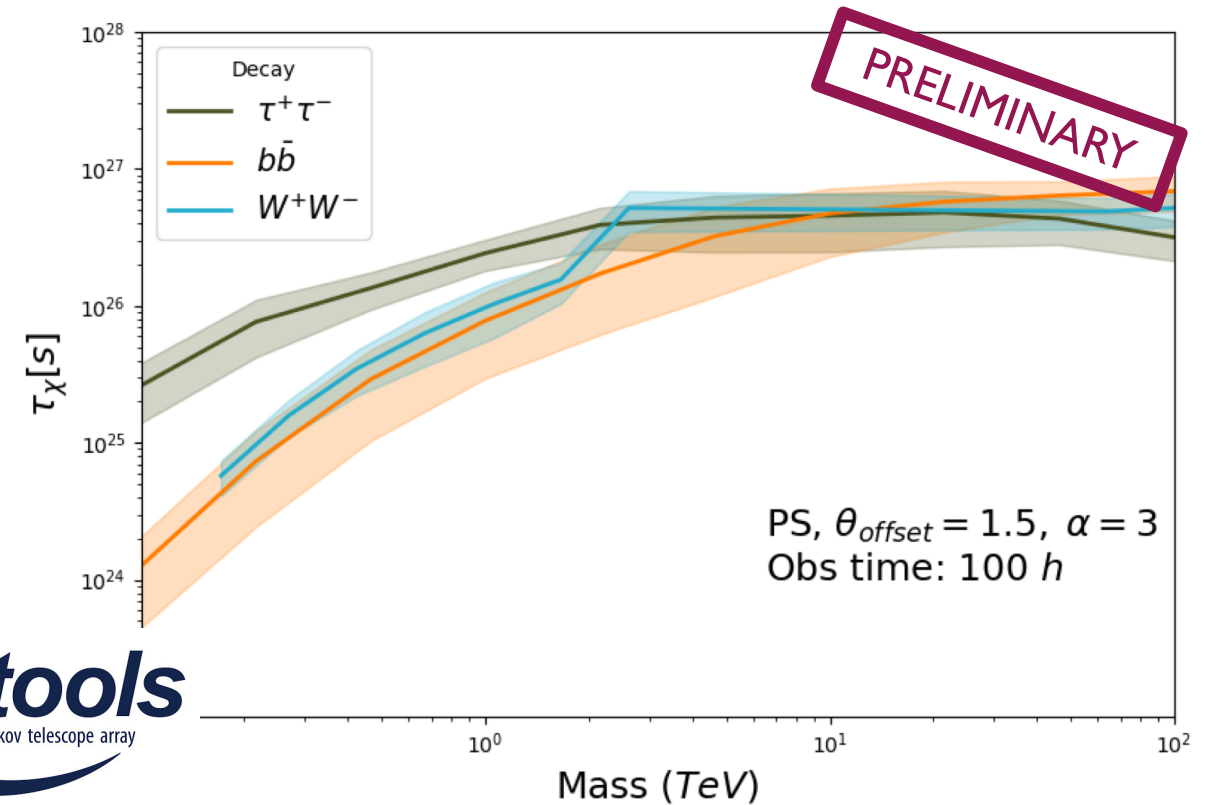


DM CONSTRAINTS: CTOOLS

Limits for Perseus for MED annihilation model and decay, assuming point-like morphology and no J/D-factor uncertainties



ctools
cherenkov telescope array



General agreement with gammapy results accounting for statistical errors and different configurations

BEYOND KSP: SAMPLE OF GALAXY CLUSTERS

- Search in catalogues for other interesting galaxy clusters to study in a DM context
- Natural extension of the KSP: why just focus on Perseus for DM searches?
- Built up of “gold” cluster sample for DM studies
- Will follow similar procedure than KSP, just applied to few other galaxy clusters and DM focused:

• Well-known M_{200} : from observations in X-rays using *Schellenberger&Reiprich 17*

• State-of-the-art parametrization of ρ_{DM}



• Local clusters: $z < 0.1$ (*Ando&Nagai 12*)

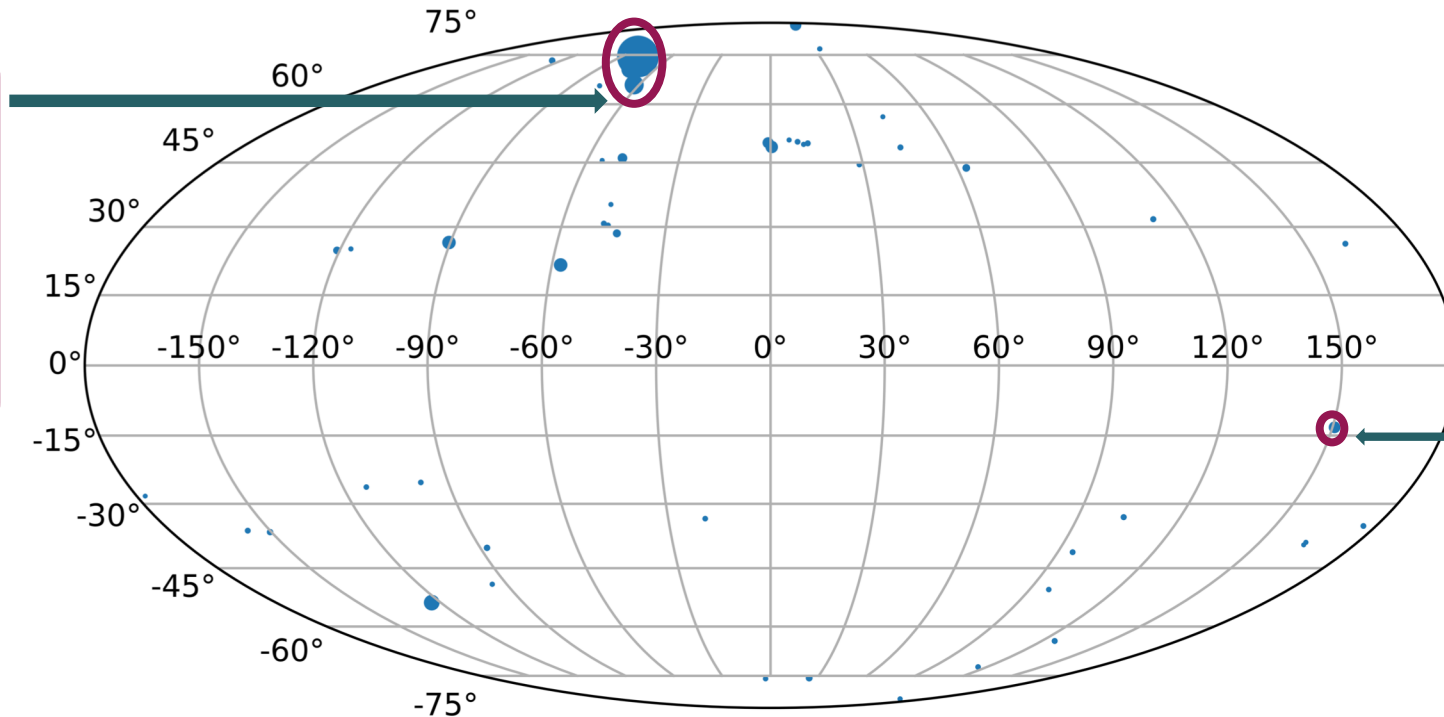


$$J \propto \frac{1}{d^2}$$

BEYOND KSP: TARGET SELECTION

Identification of best targets

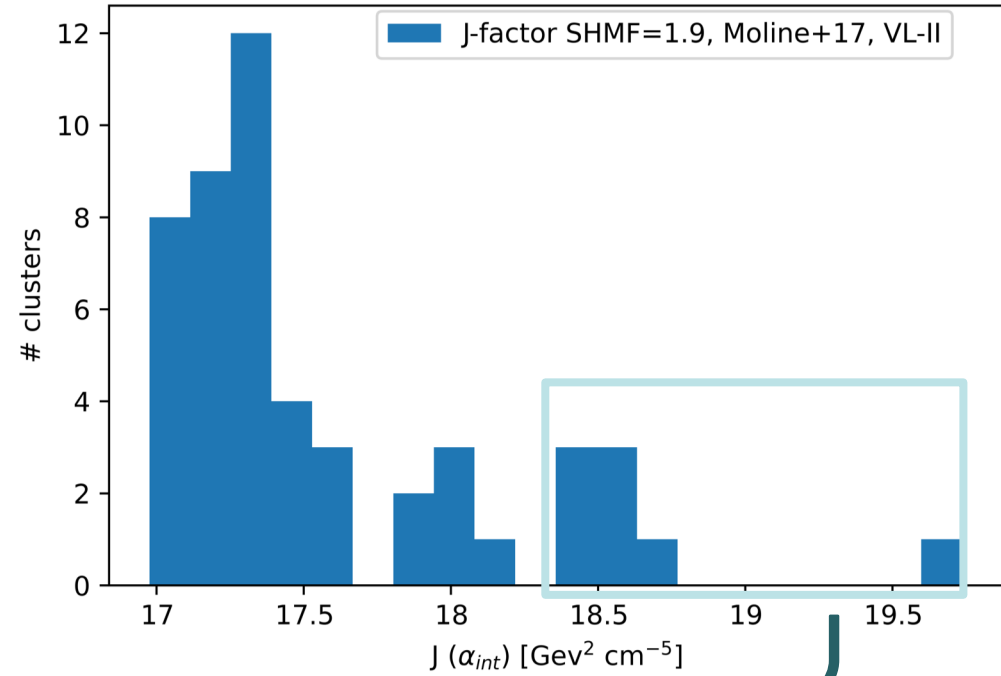
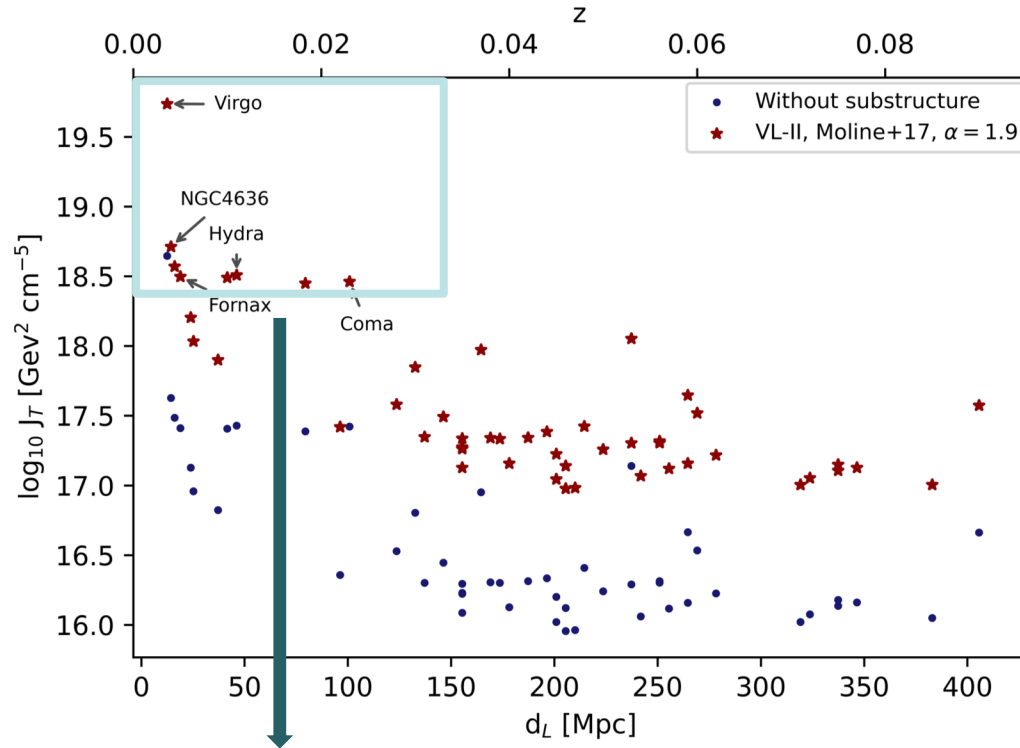
Separation of at least 2deg to account for cluster extension, except for M49 + Virgo



Mask $|b| < 20\text{deg}$, except Perseus, to avoid GDE

- Sample based on extended HIFLUGCS catalogue (*Reiprich&Borhinger02*), *Ackermann+10* [*Fermi-LAT Coll.*] and *Ackermann+14* [*Fermi-LAT Coll.*].
- 50 local clusters, $f_x \geq 1.7 \cdot 10^{-11} \text{ erg s}^{-1} \text{ cm}^{-2}$

BEYOND KSP: DM MODELLING



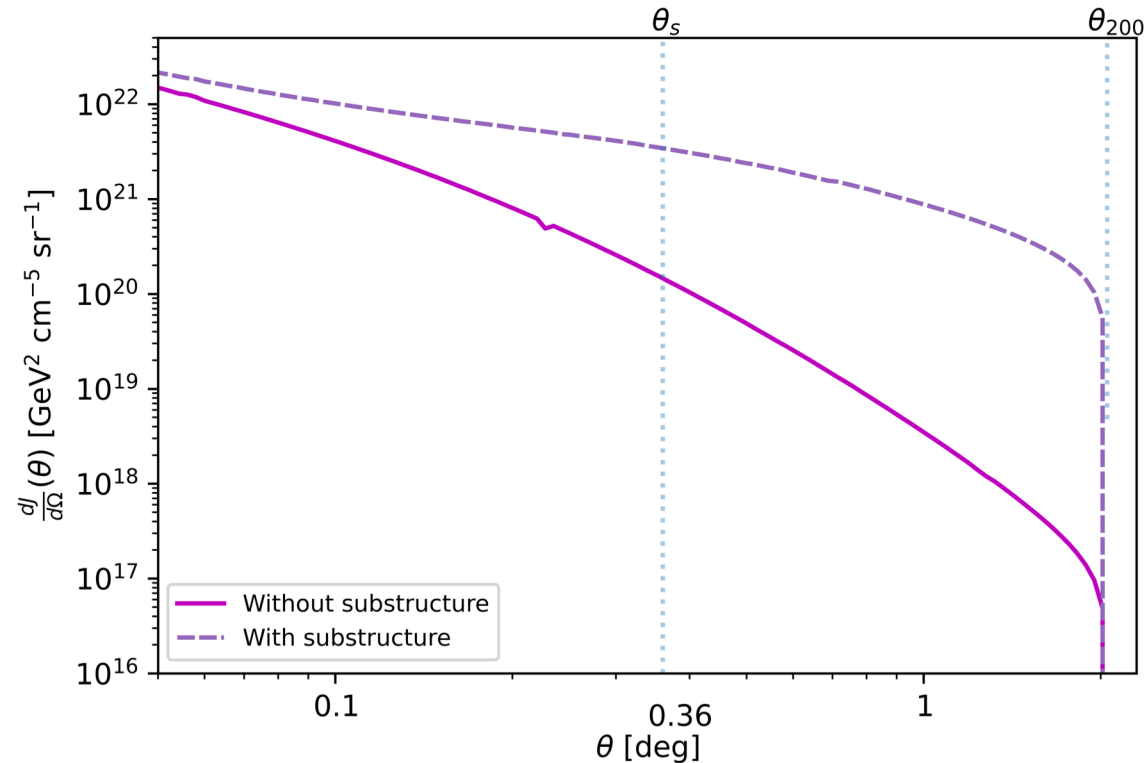
Object	z	M_{200} [$10^{14} M_{\odot}$]	R_{200} [kpc]	θ_{200} [deg]	J_{no-sub} [$10^{17} \text{GeV}^2 \text{cm}^{-5}$]	J_{subs} [$10^{18} \text{GeV}^2 \text{cm}^{-5}$]
Virgo	0.0036	5.600	1700	6.32	44.3	54.5
NGC3646	0.0040	0.534	777	2.60	4.24	5.15
M49	0.0044	0.464	741	2.26	3.06	3.72
A1060/Hydra	0.0110	2.966	1376	1.70	2.69	3.23
NGC1399/Fornax	0.0050	0.506	763	2.05	2.58	3.14
A3526/Centaurus	0.0100	2.266	1258	1.70	2.55	3.10
A1656/Coma	0.0230	13.158	2260	1.35	2.64	2.90
A0426/Perseus	0.0183	7.714	1892	1.41	2.44	2.81

- Two models:
 - Conservative: No substructure
 - Baseline: Conservative inclusion of substructure
- Substructure boosts $\mathcal{O}(10)$ for typical cluster masses (Sánchez-Conde+11, Sánchez-Conde+14, Moliné+17)

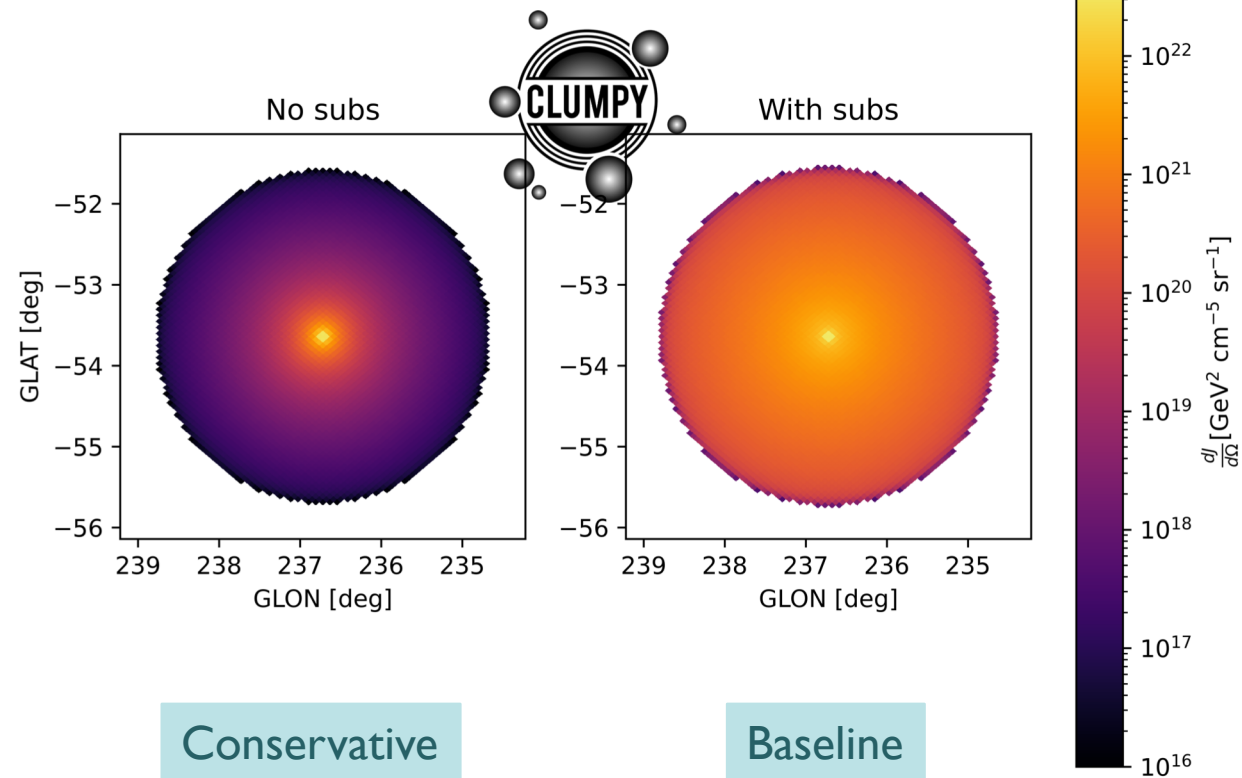
DARK MATTER MODELLING: FORNAX

Object	M_{200} [$10^{14} M_{\odot}$]	c_{200}	R_{200} [kpc]	θ_{200} [deg]	r_s [kpc]	θ_s [deg]
Fornax/NGC1399	0.506	5.79	763	2.05	132	0.36

Integration angle	J_{no-sub} [$10^{17} \text{GeV}^2 \text{cm}^{-5}$]	J_{subs} [$10^{17} \text{GeV}^2 \text{cm}^{-5}$]
θ_{200}	2.58	31.40



Skymaps of the differential J-factor for Fornax



- Effects of substructure:
 - Annihilation Boost = 11.2
 - Important in outskirts
- Adopt baseline DM model (substructure scenario) $\alpha=1.9$ for the slope of the sub-halo mass function