Non-perturbative effects on seven-brane Yukawa couplings

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In collaboration with Luca Martucci



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- ✤ F-theory: type IIB sugra + loc. sources → geometry (strong coupling)



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- One can understand their main features in terms of type IIB models
- The main new ingredients in F-theory w.r.t. type IIB models are
 - ◆ D7-branes \rightarrow (p,q) 7-branes
 - ← U(N) gauge group → U(N), SO(N), $E_{6,7,8}$ gauge group

F-theory can combine : • GUT's (heterotic) • Local models (D3-branes)

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Beasley, Heckman, Vafa'08 Donagi & Wijnholt'08



- In this framework, MSSM matter localizes on complex curves on S
- Yukawas couplings then arise from the triple intersection of these curves



 $\mathsf{SU(3)} \times \mathsf{SU(2)} \times \mathsf{U(1)} \times W_{MSSM} = \lambda_u^{ij} Q^i U^j H_u + \lambda_d^{ij} Q^i D^j H_d + \lambda_l^{ij} L^i E^j H_d$

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- The holomorphic piece of the Yukawas can be extracted from integrating the 8d Chern-Simons action
 Beasley, Heckman, Vafa'08

$$W = \int_{S} \operatorname{Tr}(F_{S}^{(0,2)} \wedge \Phi) = \int_{S} \operatorname{Tr}(\bar{\partial}A \wedge \Phi) + \int_{S} \operatorname{Tr}(A \wedge A \wedge \Phi)$$
$$\int_{A_{\bar{m}}}^{A_{\bar{m}}} = a_{\bar{m}} + \theta \psi_{\bar{m}}$$
$$\Phi_{12} = \varphi_{12} + \theta \chi_{12}$$
$$Y_{ij} = \int_{S} d\mu \, \psi_{\bar{1}}^{i} \psi_{\bar{2}}^{j} \varphi_{12}$$

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 curves → wavefunction overlap
 z_2

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$$\int_{V} \psi_{\bar{1}} \psi_{\bar{2}} \psi_{12} \psi_{\bar{1}} \psi_{\bar{2}} \psi_{\bar{1}} \psi_{\bar{1}} \psi_{\bar{2}} \psi_{\bar{1}} \psi_{\bar{1$$

U(1) symmetry: $z_i \rightarrow e^{i\alpha} z_i$

so only those integrands invariant under this symmetry will survive...



Heckman & Vafa'08 Font & Ibáñez'09

R²

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Heckman & Vafa'08 Font & Ibáñez'09

$$Y_{ij} = \int_S d\mu \,\psi_{\bar{1}}^i \psi_{\bar{2}}^j \varphi_{12}$$

Solving the eom for the matter fields we have

 $\psi_{\bar{1}}^{i}\psi_{\bar{2}}^{j}\varphi_{12} = f_{i}(z_{2})g_{j}(z_{1})G(z_{1},\bar{z}_{1},z_{2},\bar{z}_{2})$

 $f_i = z_2^{3-i}$ $g_j = z_1^{3-j}$

For constant fluxes G only depends on $|z_1|^2$ and $|z_1|^2$, so only $Y_{33} \neq 0$

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For general G one expects an expansion around the triple intersection, with terms of the form $(z_1\bar{z}_1)^a(z_2\bar{z}_2)^b(\bar{z}_1)^{3-j}(\bar{z}_1)^{3-i}$ that contribute to However all such latter contributions cancel out \Rightarrow Rank one Yukawa Cecotti, Cheng, Heckman Julaing problem

F-theory Recap

- The above setup provides an interesting framework to reproduce the flavor hierarchies present in the Standard Model
- One however needs a mechanism that modifies the above rank one Yukawa matrix by a small amount
- By the results of Cecotti et al.[0910.0477], such contribution does not arise from the seven-brane theory itself
 one must consider external effects that modify the superpotential
- It was proposed in Cecotti et al.[0910.0477] that
 - Such effects could be bulk fluxes $G_3 = F_3 \tau H_3$
 - Their contribution is encoded in a non-commutative deformation
- In the following, we will argue that such effects can only be of non-perturbative origin

Models with rank one Yukawas can also be obtained in the context of intersecting D6-branes on toroidal orbifolds *Cremades, Tháñez, 7.M. '03*



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$$Y^{U} \sim A \cdot \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} \cdot B, \qquad Y^{D} \sim A \cdot \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} \cdot \tilde{B}.$$



It was then proposed that this rank one problem could be solved by the one loop contribution of E2-instantons
Abel & Goodsell'06



Conditions: E2 rigid, O(1), and does not intersect the MSSM D6-branes

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- We can compute such superpotential
 - In the one-loop open string channel
 - ✤ In the tree-level closed string channel

Berg, Haack, Körs'04

Baumann et al. '06

In the latter case, one basically computes how, via its backreaction, the D3-brane position modifies the D7 gauge kinetic function

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- The result is

$$W^{\rm np} = M^3 e^{-f_{\rm D7}/n} = \mathcal{A} e^{-T_{\Sigma}/n} f(z_{\rm D3})^{1/n} \qquad n = \# {\rm D7's}$$

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$$W_{\rm D3}^{\rm np} = M^3 e^{-f_{\rm D7}/n} = \mathcal{A} e^{-[T_{\Sigma} - \text{Tr} \ln f(Z_{\rm D3})]/n} \quad n = \# \text{D7's}$$
$$= \mathcal{A} e^{-T_{\Sigma}/n} [\det f(Z_{\rm D3})]^{1/n}$$

- Let us now replace the D3-brane by a D7-brane wrapping a 4-cycle S4
- If this D7-brane is magnetized (F ≠ 0) it will carry an induced D3-brane charge and tension, given by N_{D3} = $\frac{1}{8\pi^2} \int_{S_4} \text{Tr}(F \land F)$
- ✤ Hence, it will also backreact on the warping and the RR field C₄.
 ⇒ The position moduli of S₄ should also couple to T_Σ
- In fact, if S₄ and Σ₄^{np} do not intersect, we can treat the D7 as N_{D3} smeared out D3-branes over S₄, with density $\rho = \text{Tr } F^2/8\pi^2$
- We then obtain

$$W_{\rm D7}^{\rm np} = \mathcal{A} e^{-T_{\Sigma}} \exp\left[\frac{1}{8\pi^2} \int_{\mathcal{S}_4} \operatorname{STr}(\ln f F \wedge F)\right]$$

• To extract the field dependence of this superpotential we must perform a non-Abelian Taylor expansion on the position field $\phi = 2\pi \alpha' w$

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$$\bigvee$$
$$W_{\text{D7}}^{\text{np}} = \mathcal{A} e^{-T_{\Sigma}} f|_{\mathcal{S}_4}^{N_{\text{D3}}} + \frac{1}{8\pi^2} \int_{\mathcal{S}_4} \theta \operatorname{Str}(\phi F \wedge F) + \dots$$
const.

$$\theta := 2\pi \alpha' \mathcal{A} e^{-T_{\Sigma}} (f^{N_{\mathrm{D}3}} \partial_w \log f)|_{\mathcal{S}_4}$$

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- ♦ We can perform the same expansion for the D7-brane tree-level superpotential $W_{\text{D7}}^{\text{tree}} = \int_{\Gamma_5} \text{Str} \, \Omega \wedge F$ Martucci^{'06}
- We obtain the full superpotential

$$W_{\text{D7}} = \int_{\mathcal{S}_4} \left[(\iota_w \Omega) \wedge \operatorname{tr}(\phi F) + \frac{1}{2} \,\theta \operatorname{Str}(\phi F^2) + \dots \right]$$
$$\int_{\mathcal{S}_4} \operatorname{Tr}(F_S^{(0,2)} \wedge \Phi) \qquad \text{dim. 5 correction}$$

Comments

- We have treated a magnetized D7-brane as a bunch of smeared out D3-branes but, actually, its backreaction is more involved
- Indeed, D7-branes also source the axio-dilaton τ. It is easy to see, however, that the action of a O(1) E3-instanton is independent of τ
 No further contributions to W^{np}
- We have found that the non-perturbative correction is given by a higher dim. operator. The contribution to the Yukawas can be extracted from terms involving only three fluctuations

$$\begin{split} \int_{\mathcal{S}_4} \theta \, \epsilon^{i\bar{j}k\bar{l}} \mathrm{Str} \left(\phi \, D_i A_{\bar{j}} D_k A_{\bar{l}} \right) \\ D_k &= \alpha'^{1/2} (\partial_k + i \langle A_k \rangle \wedge) \end{split} \tag{Froggat-Nielsen mech.}$$

... as well as from the modified eom

W^{np} Recap

- F-theory models can reproduce a flavor hierarchy, but without any external effect they suffer from a rank one Yukawa problem
- We have shown that such effect can be generated by non-perturbative dynamics, which correct the seven-brane tree-level superpotential
- Such correction exists if the seven-brane carries D3-brane charge.
 The corrections to W_{D3} and W_{D7} are closely related
- Here the corrected Yukawas are not perturbatively forbidden by anomalous global U(1) symmetries. The non-perturbative corrections are however relevant in the sense that they lift degeneracies.
- While we have worked in the type IIB limit, our results extend trivially to F-theory
- The non-perturbative correction to the Yukawas can be understood in terms of a dim. 5 operator multiplied by a function θ

This D7-brane superpotential

$$W_{\rm D7} = \int_{\mathcal{S}_4} \left[(\iota_w \Omega) \wedge \operatorname{tr}(\phi F) + \frac{1}{2} \,\theta \operatorname{Str}(\phi F^2) + \dots \right]$$

can also be generated in certain exotic type IIB vacua known as β-deformed backgrounds

These are naturally described in the language of generalized complex geometry, that encodes the Killing spinor of the background in terms of polyforms

$$\epsilon = \begin{pmatrix} \epsilon_1 \\ \epsilon_2 \end{pmatrix} \qquad \epsilon_i = \zeta_i \otimes \eta_i + \zeta_i^* \otimes \eta_i^*$$

$$\{\eta_1, \eta_2\} \quad \leftrightarrow \quad \{\Psi_1, \Psi_2\} \qquad \begin{cases} \Psi_1 = \psi_0 + \psi_2 + \psi_4 + \psi_6 \\ \Psi_2 = \psi_1 + \psi_3 + \psi_5 \end{cases}$$

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- These are naturally described in the language of generalized complex geometry, that encodes the Killing spinor of the background in terms of polyforms
 - + For a warped Kähler manifold $\Psi_2 = \Omega_3$
 - + For a β -deformed background $\Psi_2 = \psi_1 + \Omega_3$ $\psi_k = \beta^{ij} \Omega_{ijk}$

Ω

 ψ_1

It has been argued that the effect of a β-deformation on a D-brane worldvolume can be seen as a non-commutative deformation of the gauge theory

Kapustin'03 Pestun'06

This suggests the identification

np correction ⇔ nc deformation

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Indeed, in the β-deformed superpotential the F-flatness condition reads $(\iota_X \psi_1)|_{S_4} F^2 + 2(\iota_X \Omega)|_{S_4} \land F = 0 \qquad \forall X \in TM|_{S_4}$

and so $F^{(0,2)} \neq 0$. It can however be expressed as the condition

$$\hat{F}^{(0,2)} = 0$$

with \hat{F} constructed via the Seiberg-Witten map

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Destun'06

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In fact, one can map the nc superpotential

SW map

$$\hat{W}_{\mathrm{D7}} = \int_{\mathcal{S}_{4}} \mathrm{tr}(\hat{\varphi} \circledast \hat{F}) \qquad \hat{F}_{\alpha\beta} = F_{\alpha\beta} + \frac{1}{2} \Theta^{\gamma\delta} (F_{\alpha\gamma} F_{\delta\beta} + F_{\delta\beta} F_{\alpha\gamma}) \\ + \frac{1}{4} \Theta^{\gamma\delta} [A_{\gamma} (D_{\delta} F_{\alpha\beta} + \partial_{\delta} F_{\alpha\beta}) \\ + (D_{\delta} F_{\alpha\beta} + \partial_{\delta} F_{\alpha\beta}) A_{\gamma}] + \mathcal{O}(\theta^{2}) \\ \hat{\phi} = \phi + \frac{1}{4} \Theta^{\alpha\beta} [A_{\alpha} (\partial_{\beta} \phi + D_{\beta} \phi) \\ + (\partial_{\beta} \phi + D_{\beta} \phi) A_{\alpha}] + \mathcal{O}(\theta^{2}) \\ W_{\mathrm{D7}} = \int_{\mathcal{S}_{4}} \left[(\iota_{w} \Omega) \wedge \mathrm{tr}(\phi F) + \frac{1}{2} \theta \operatorname{Str}(\phi F^{2}) + \dots \right]$$

Conclusions

- Motivated by F-theory, we have shown that non-perturbative effects correct non-trivially the tree-level superpotential of seven-branes
- We have provided an explicit and simple expression for this correction, which allows to compute its effects even at a local level
- The correction can either be expressed in terms of a dim. 5 operator or as a non-commutative deformation, via the SW map

Conclusions

- Motivated by F-theory, we have shown that non-perturbative effects correct non-trivially the tree-level superpotential of seven-branes
- We have provided an explicit and simple expression for this correction, which allows to compute its effects even at a local level
- The correction can either be expressed in terms of a dim. 5 operator or as a non-commutative deformation, via the SW map
- It was argued by Cecotti et al.[0910.0477] that such nc deformed superpotential generically solve the rank one Yukawa problem. There, however, the origin of the nc deformation was advocated to bg fluxes
- Reversing our above discussion, one can see that the nc deformation for D7-branes has the same origin that a superpotential for D3-branes
- ✤ However, the no-scale condition for these wCY backgrounds implies that W_{D3} = 0 at tree-level ⇒ nc deformation must have a np origin!!