The early stages of high energy heavy ion collisions

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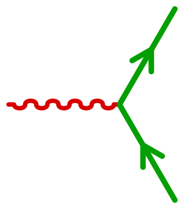
Outline

- Quark-gluon plasma and heavy ion collisions
- What have we learned from RHIC?
- AdS/CFT duality and the QGP
- Weak coupling approach

Quark-Gluon Plasma
Heavy Ion Collisions

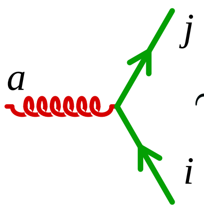
QCD : Quarks and gluons

- Electromagnetic interaction : Quantum electrodynamics
  - Matter : electron , interaction carrier : photon
  - Interaction :

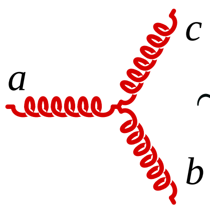


$\sim \; e$ (electric charge of the electron)

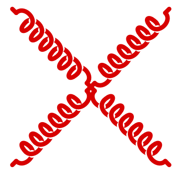
- Strong interaction : Quantum chromo-dynamics
  - Matter : quarks , interaction carriers : gluons
  - Interactions :



$\sim \; g\,(t^a)_{ij}$



$\sim \; g\,(T^a)_{bc}$

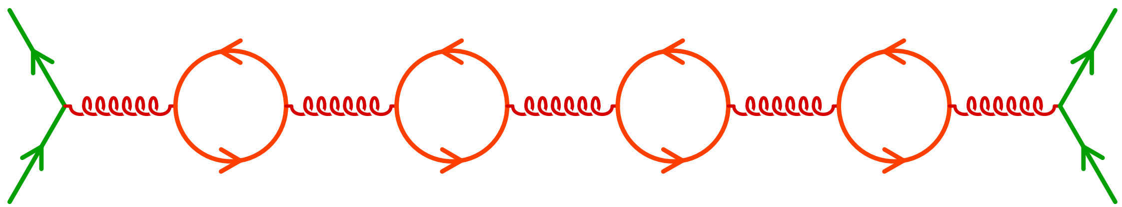


  - i, j : colors of the quarks (3 possible values)
  - a, b, c : colors of the gluons (8 possible values)
  - $(t^a)_{ij}$: 3×3 matrix , $(T^a)_{bc}$: 8×8 matrix

QCD : Asymptotic freedom

- Running coupling : $\alpha_s = g^2/4\pi$

$$\alpha_s(r) = \frac{2\pi N_c}{(11N_c - 2N_f)\log(1/r\Lambda_{QCD})}$$



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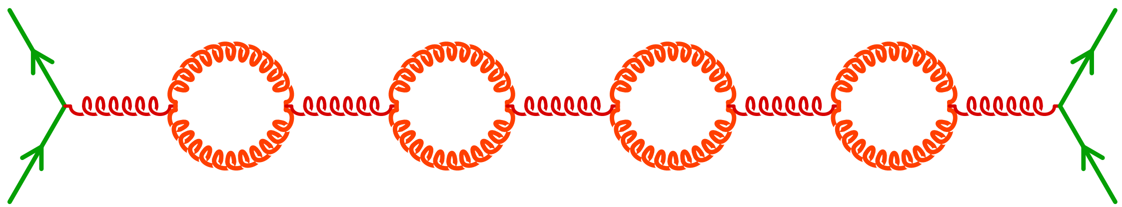
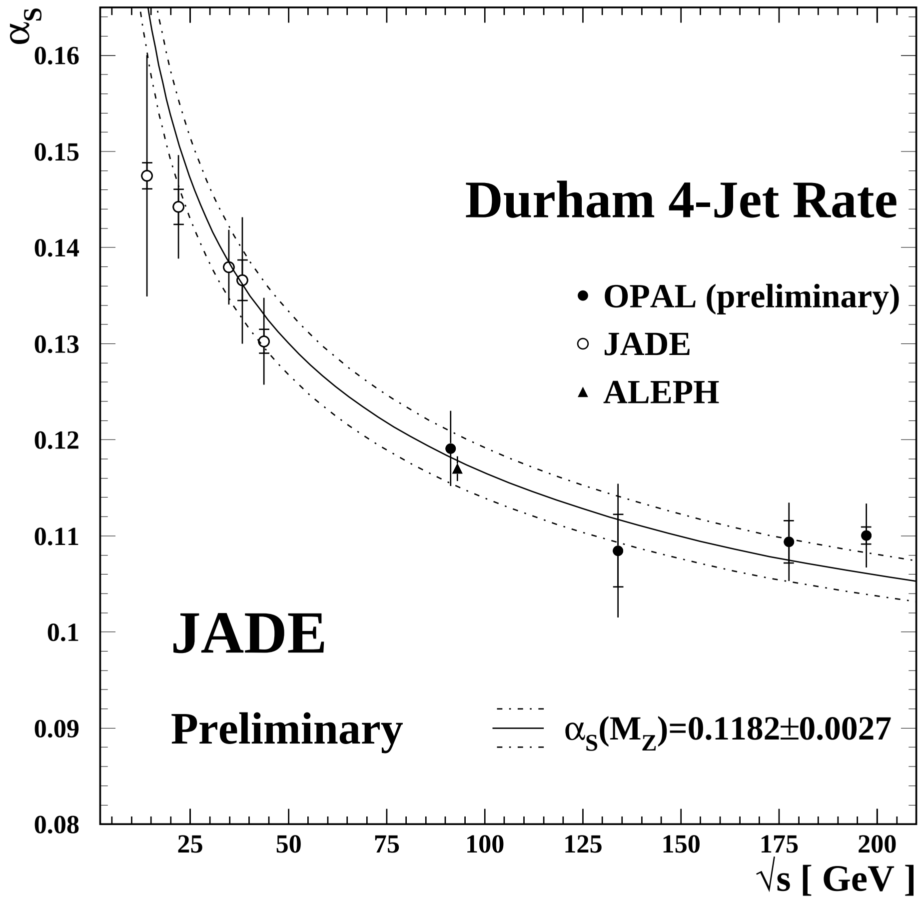


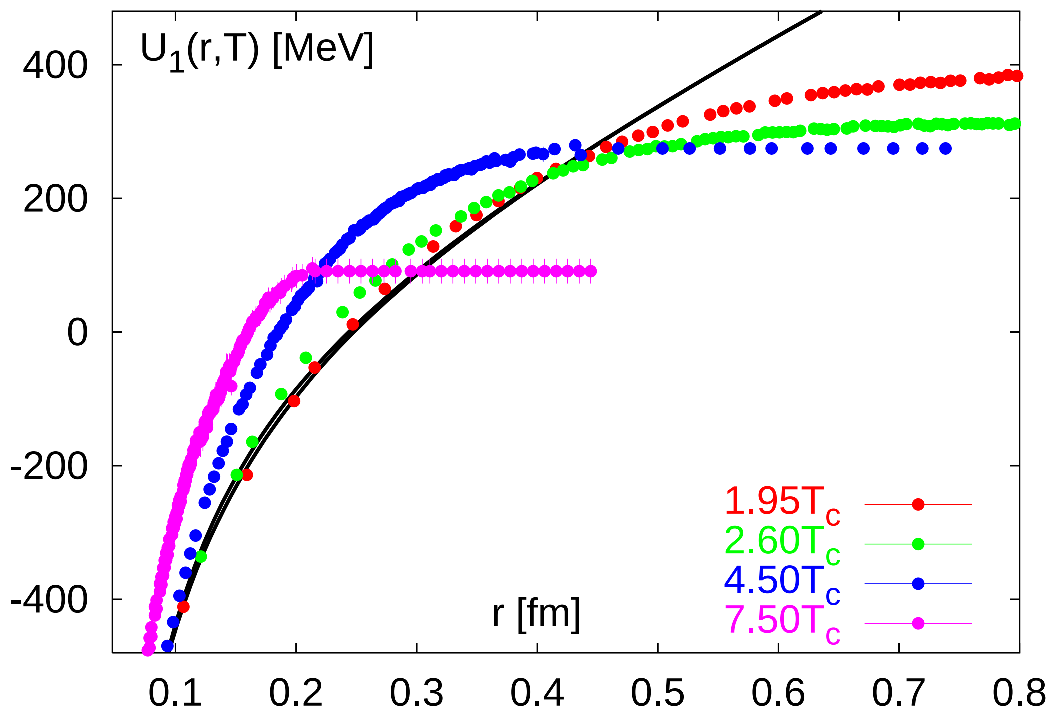
- The effective charge seen at large distance is screened by fermionic fluctuations (as in QED)
- But gluonic vacuum fluctuations produce an anti-screening (because of the non-abelian nature of their interactions)
- As long as $N_f < 11N_c/2 = 16.5$, the gluons win...

QCD : Asymptotic freedom



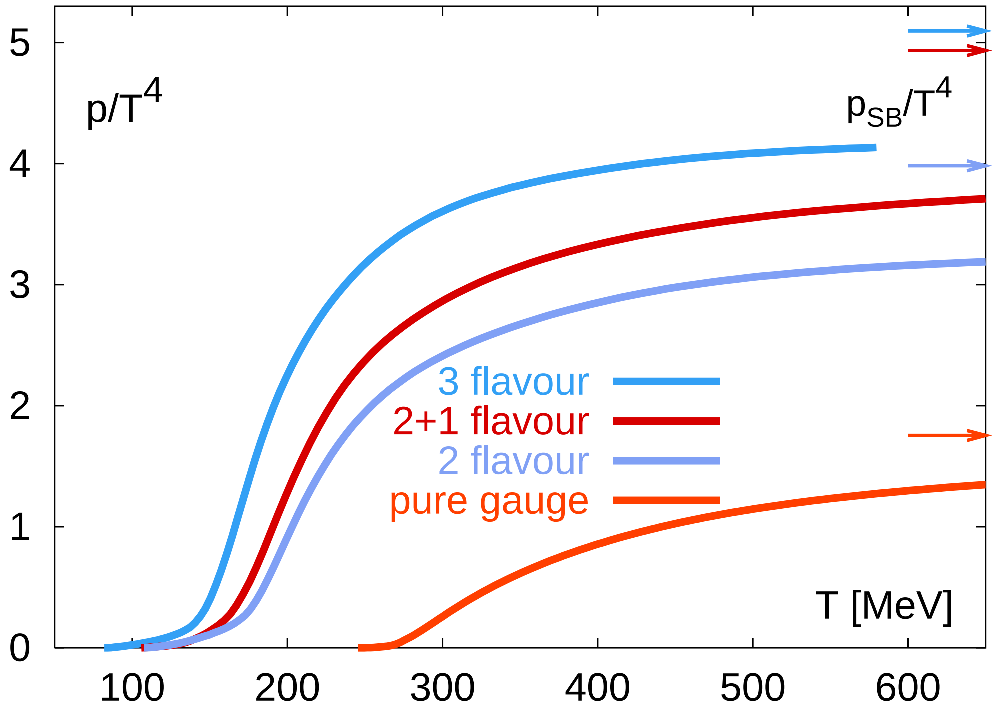
- The coupling constant is small at short distances
- At high density, a hadron gas may undergo deconfinement
  ▷ quark gluon plasma

Deconfinement at high T



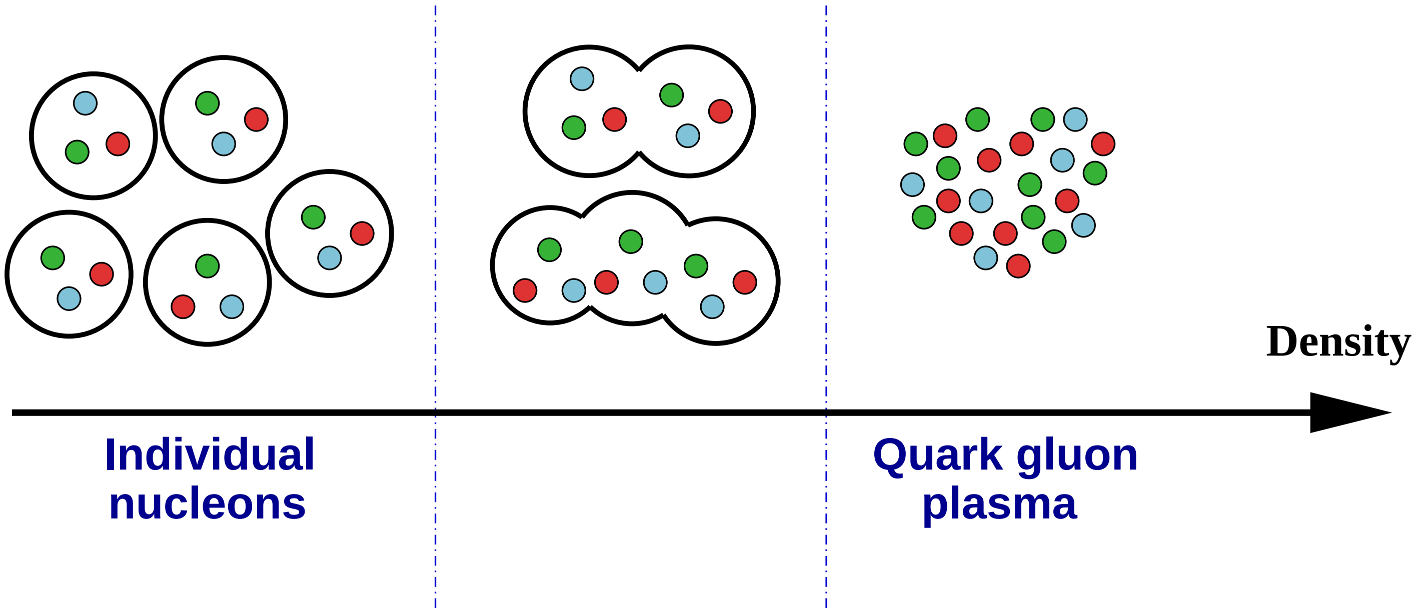
- Heavy quark potential :
  - confining at $T = 0$ (non-zero string tension)
  - at high T : becomes flat at large distance $\Rightarrow$ deconfinement

Deconfinement transition



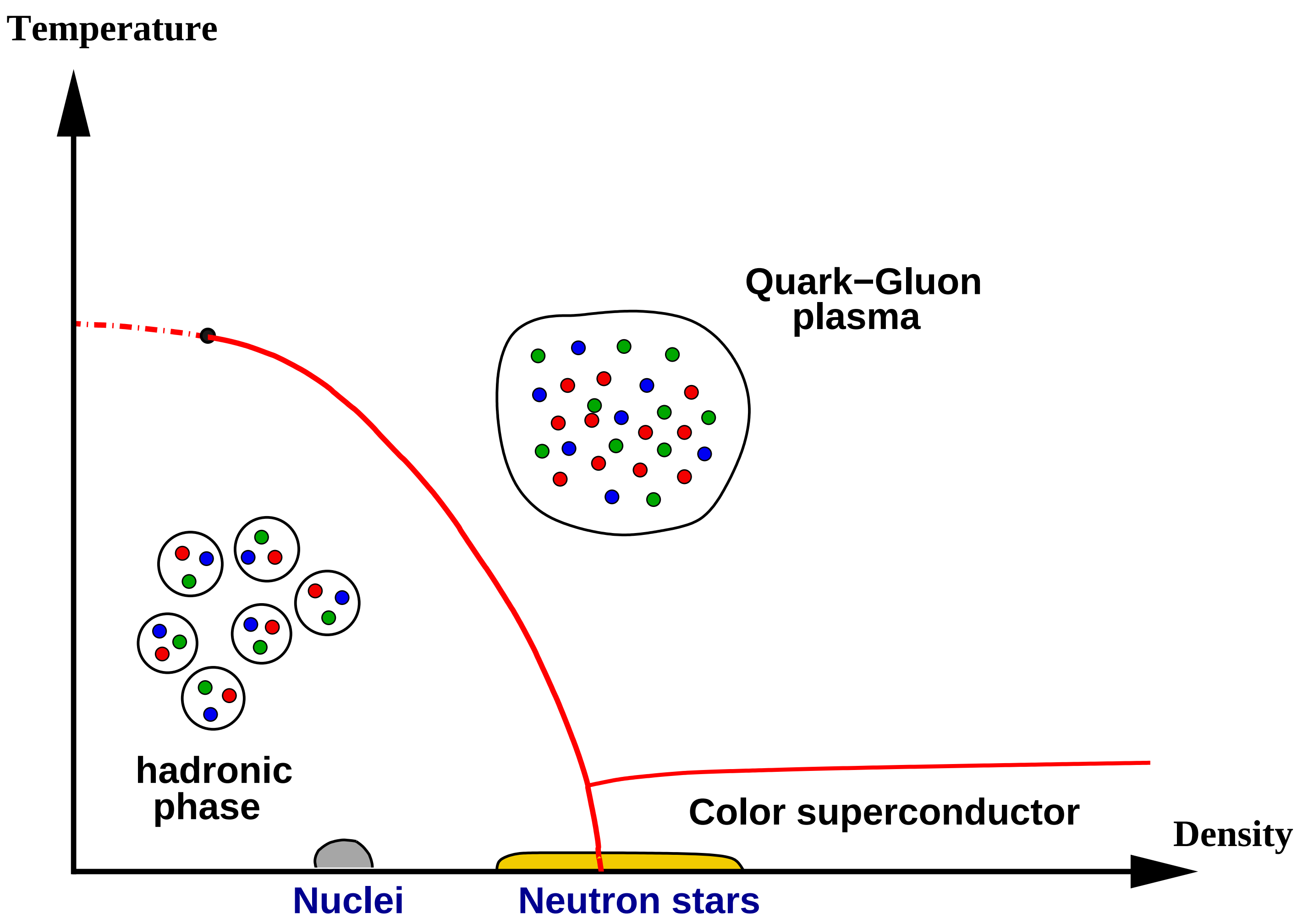
- Fast increase of the pressure :
  - at $T \sim 270$ MeV, if there are only gluons
  - at $T \sim 150$–170 MeV, depending on the number of light quarks

Deconfinement transition

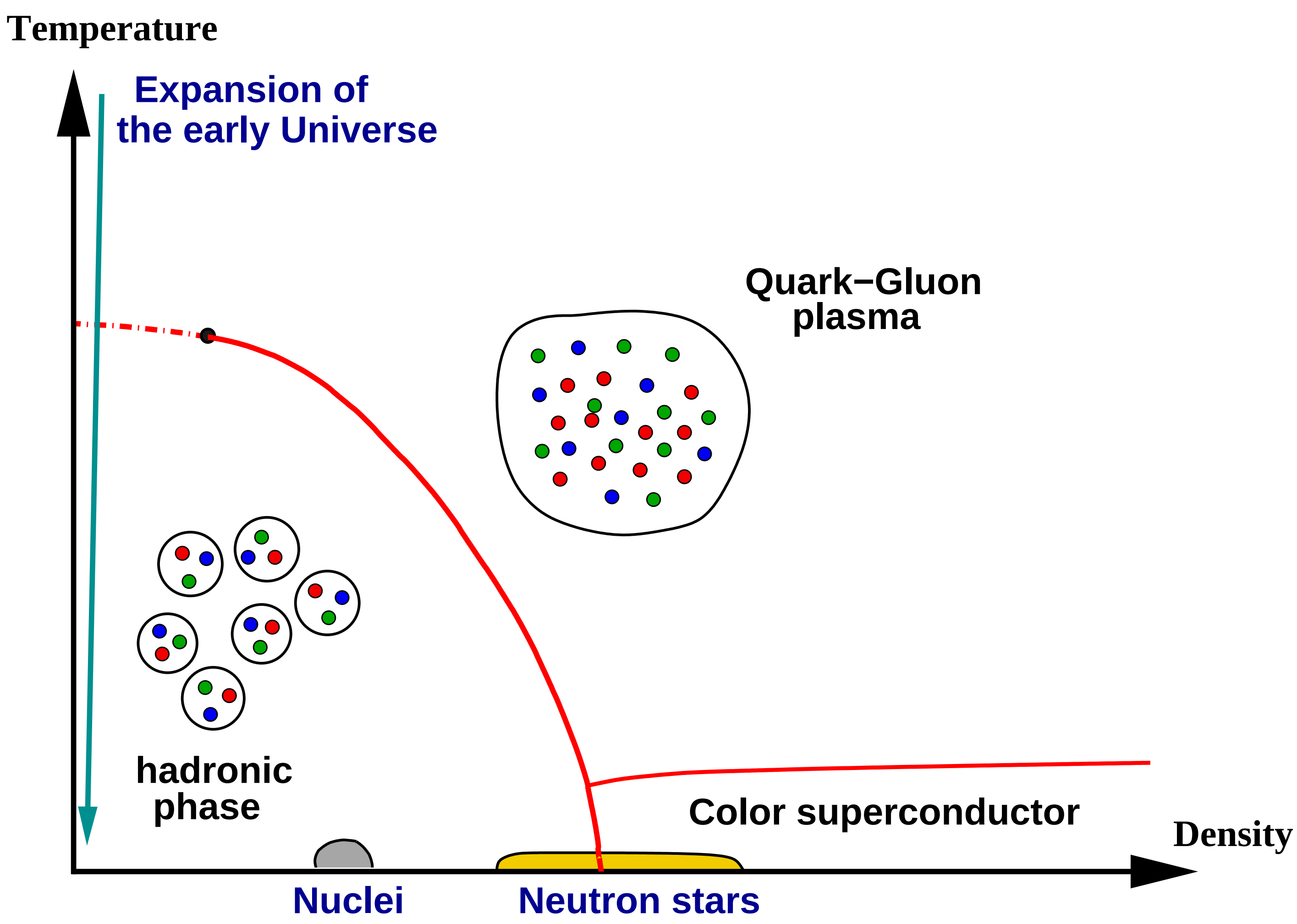


- When the nucleon density increases, they merge, enabling quarks and gluons to hop freely from a nucleon to its neighbors
- This phenomenon extends to the whole volume when the phase transition ends
- Note: if the transition is first order, it goes through a mixed phase containing a mixture of nucleons and plasma

QCD phase diagram



The QGP in the early universe



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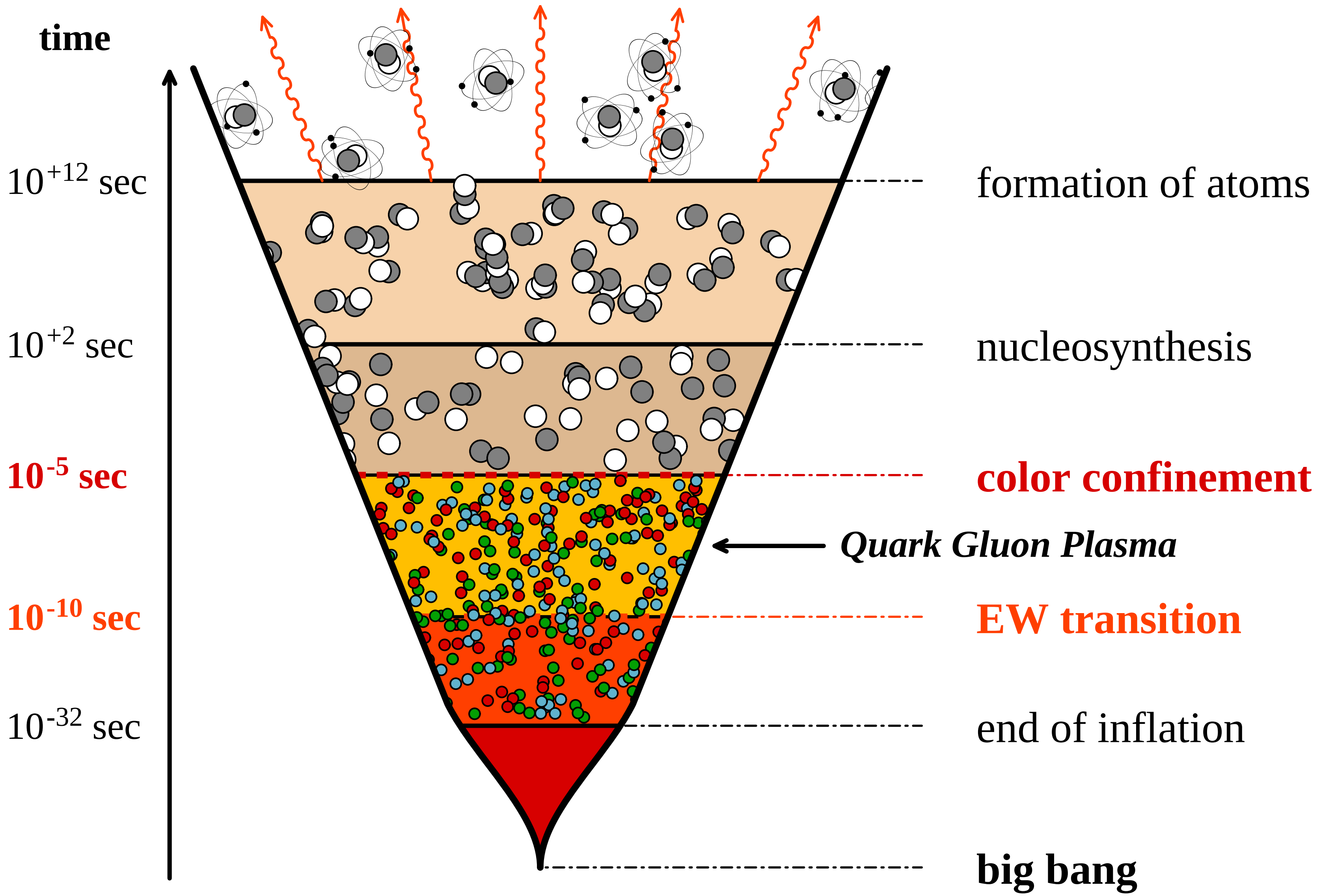
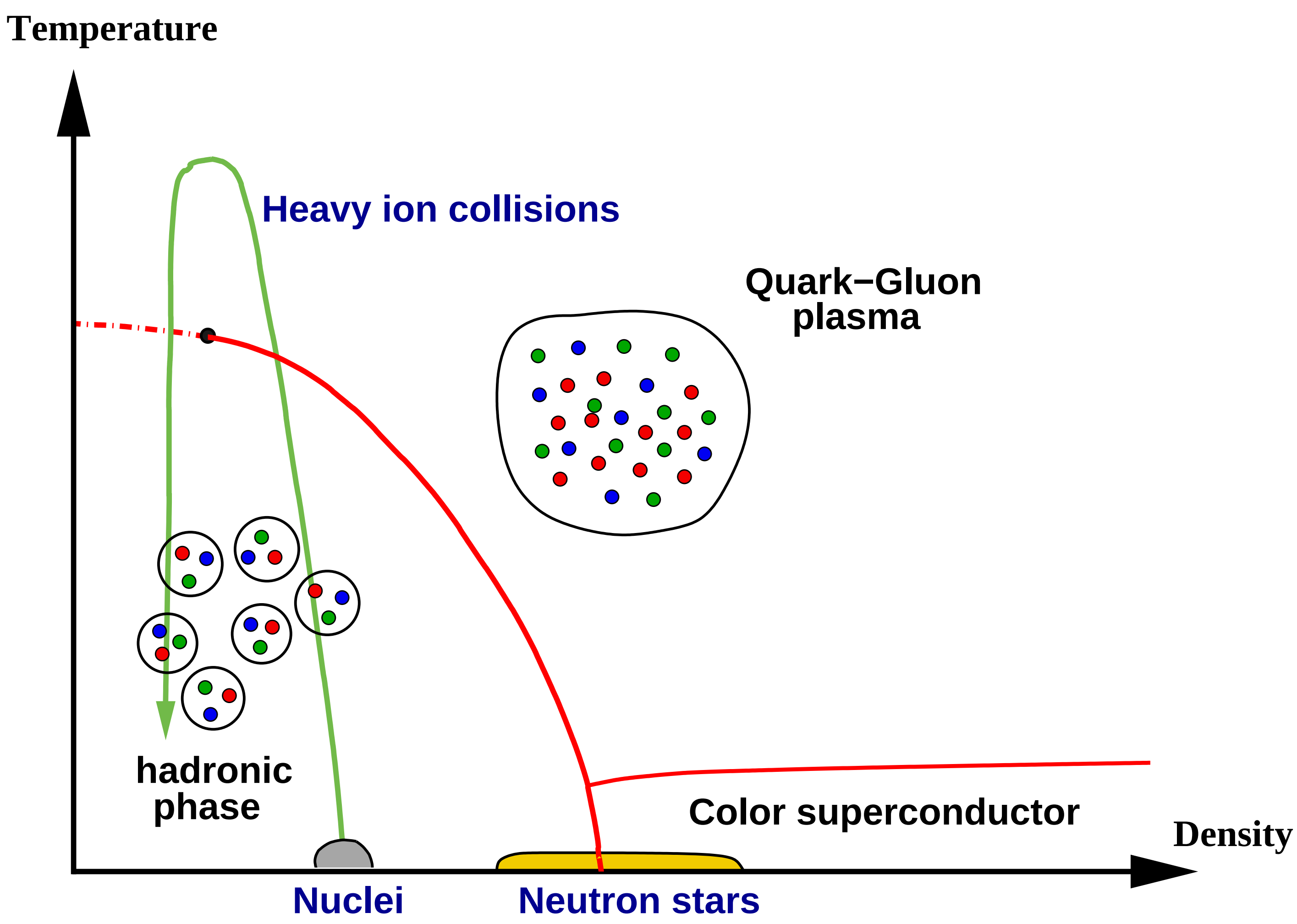
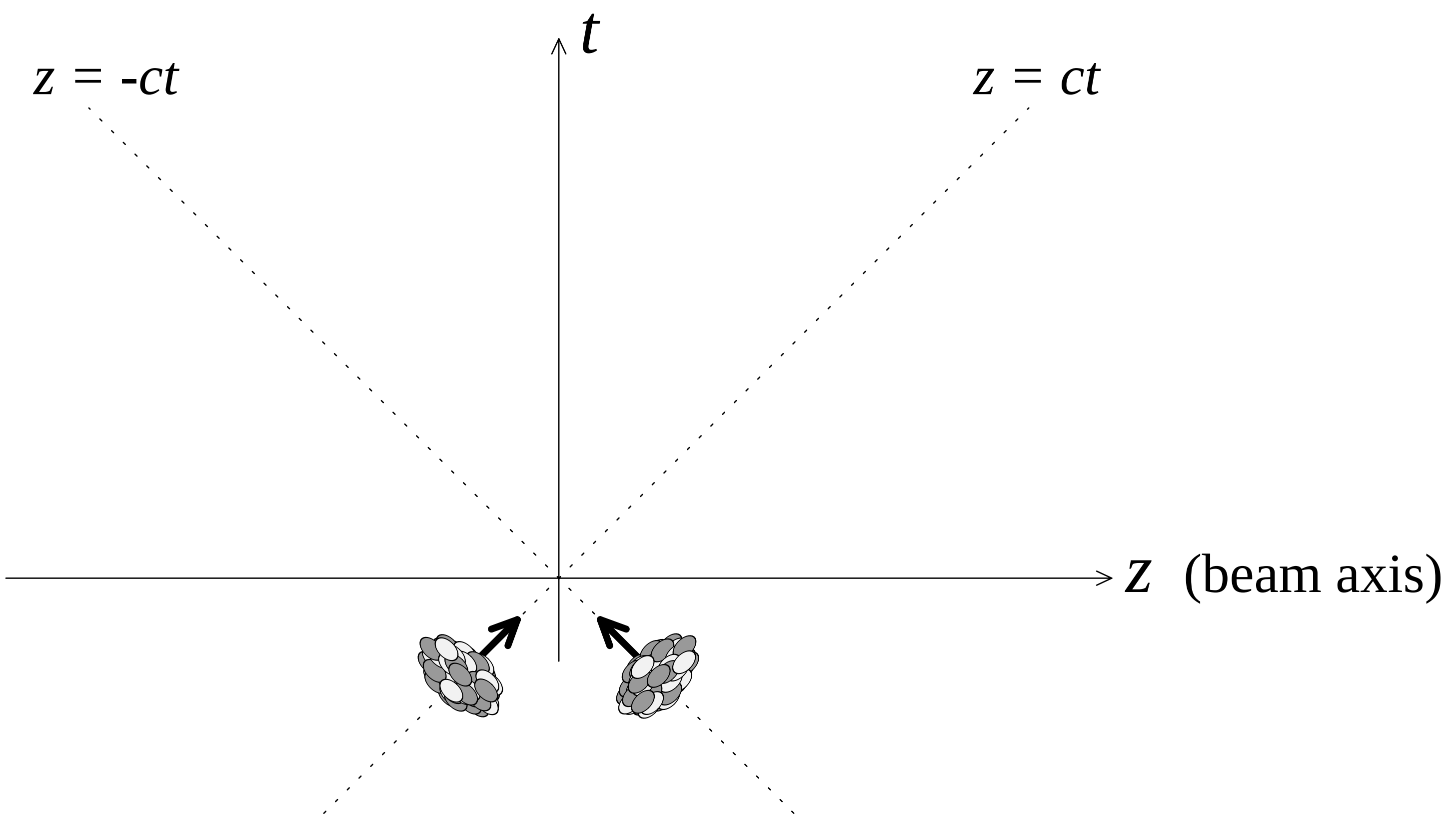


Heavy ion collisions



Stages of a nucleus-nucleus collision



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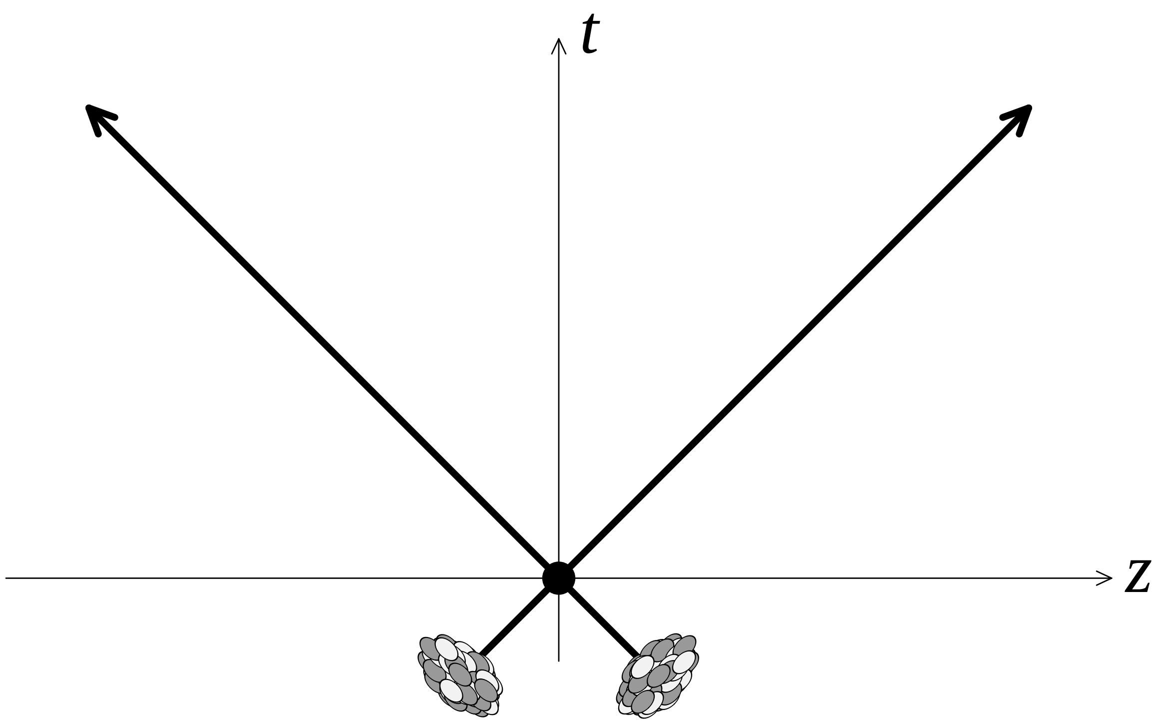
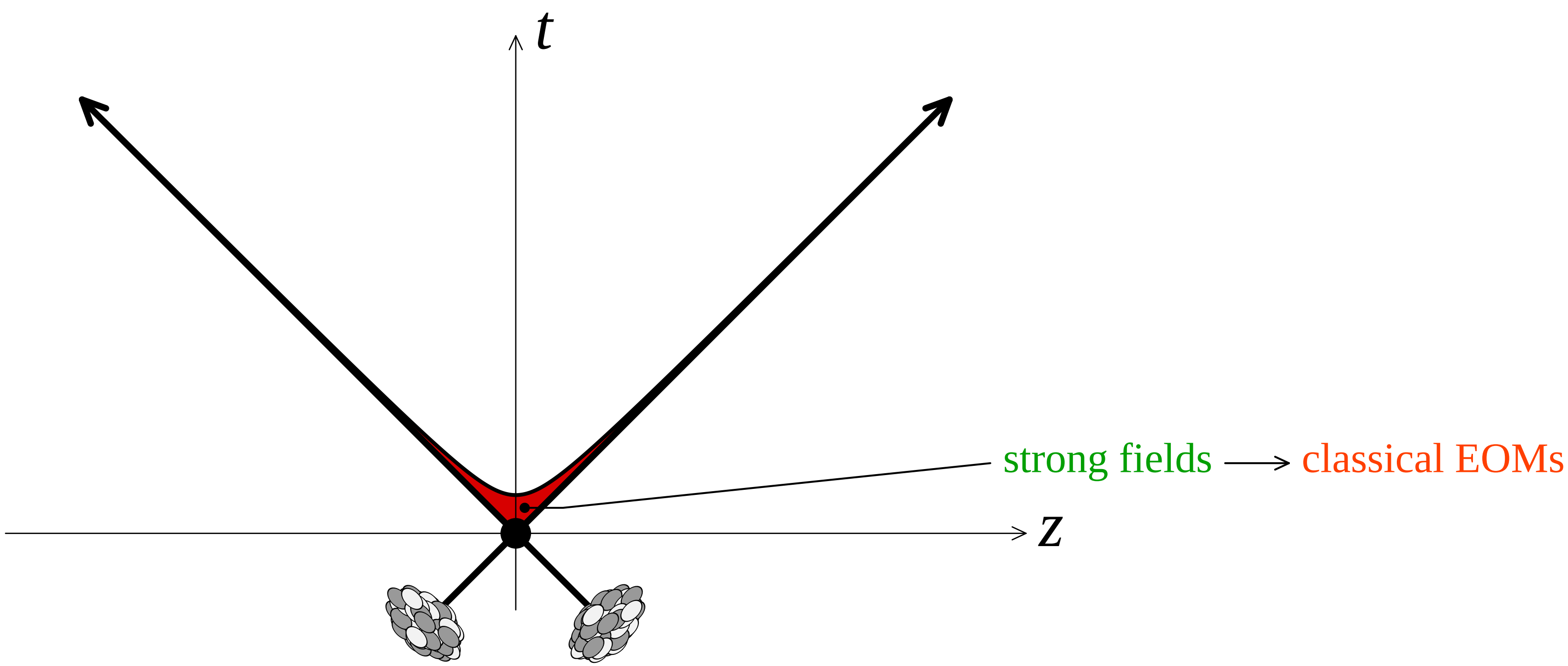


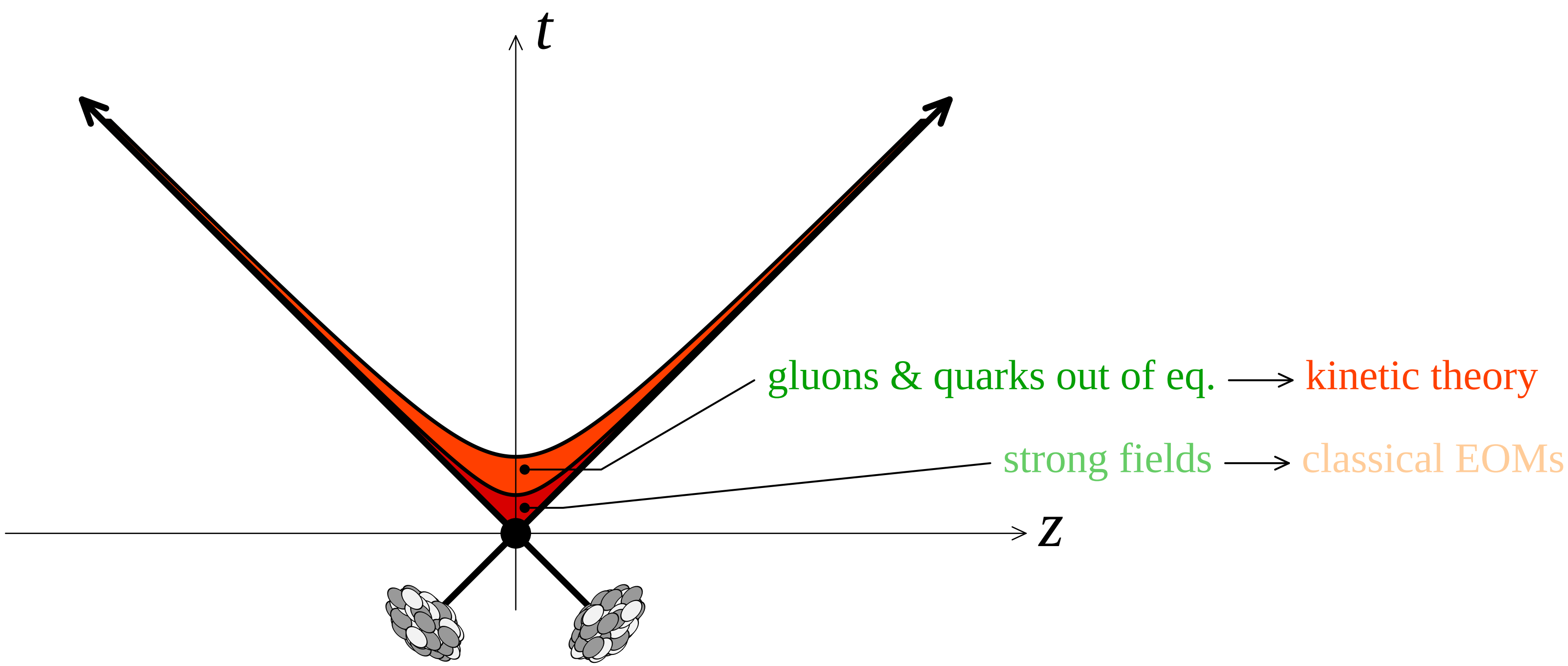
- $\tau \sim 0$ fm/c
- Production of hard particles :
  - jets, direct photons
  - heavy quarks
- calculable with perturbative QCD (leading twist)

Stages of a nucleus-nucleus collision



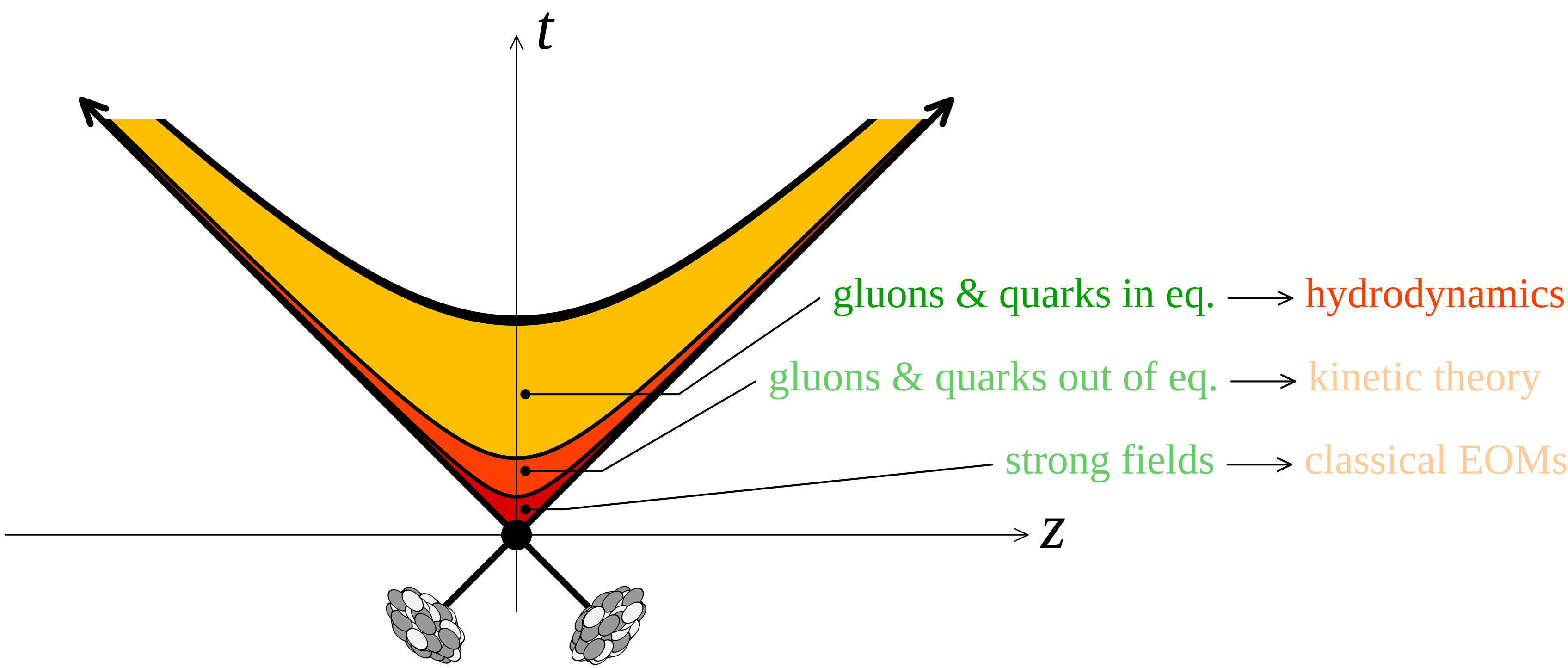
- $\tau \sim 0.2$ fm/c
- Production of semi-hard particles : gluons, light quarks
- relatively small momentum : $p_\perp \lesssim 2$–3 GeV
- make up for most of the multiplicity
- sensitive to the physics of saturation (higher twist)

Stages of a nucleus-nucleus collision



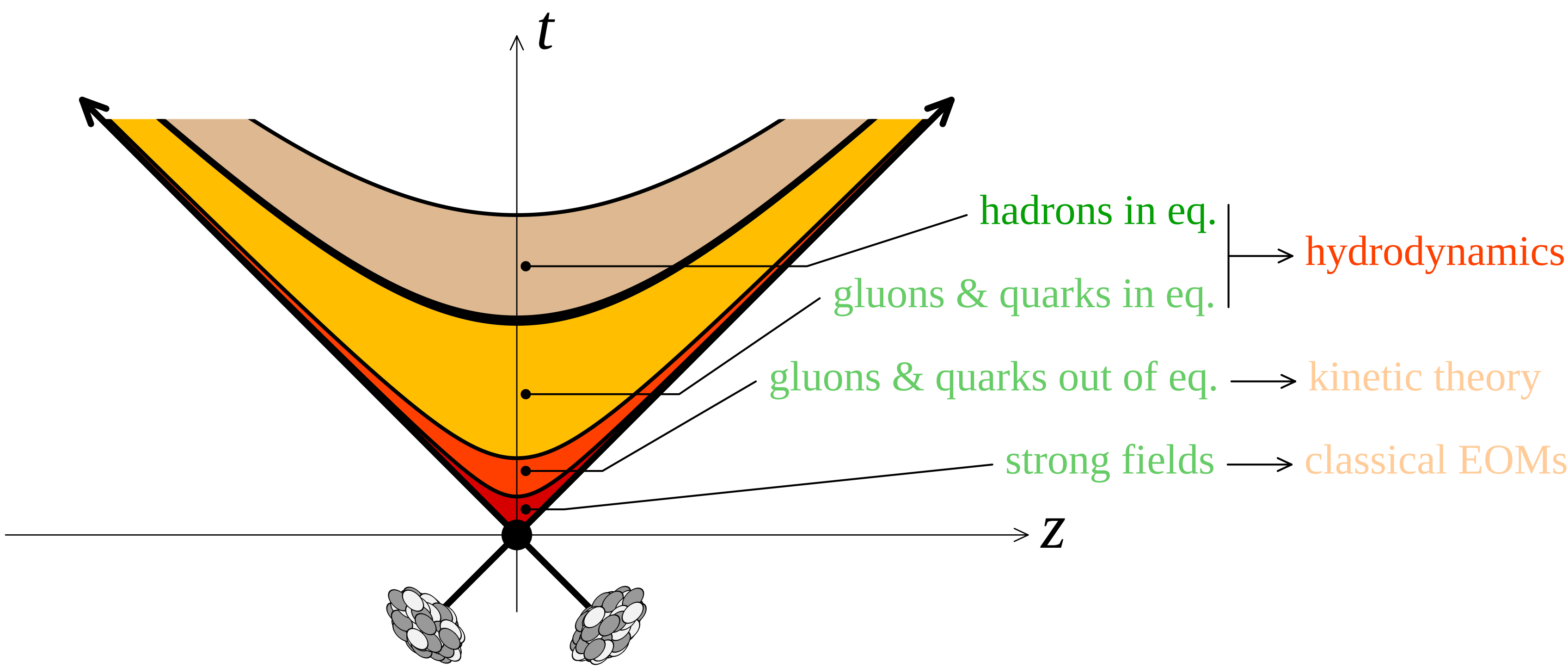
- $\tau \sim 1$–2 fm/c
- Thermalization
  - experiments suggest a fast thermalization
  - but this is still not understood from QCD

Stages of a nucleus-nucleus collision



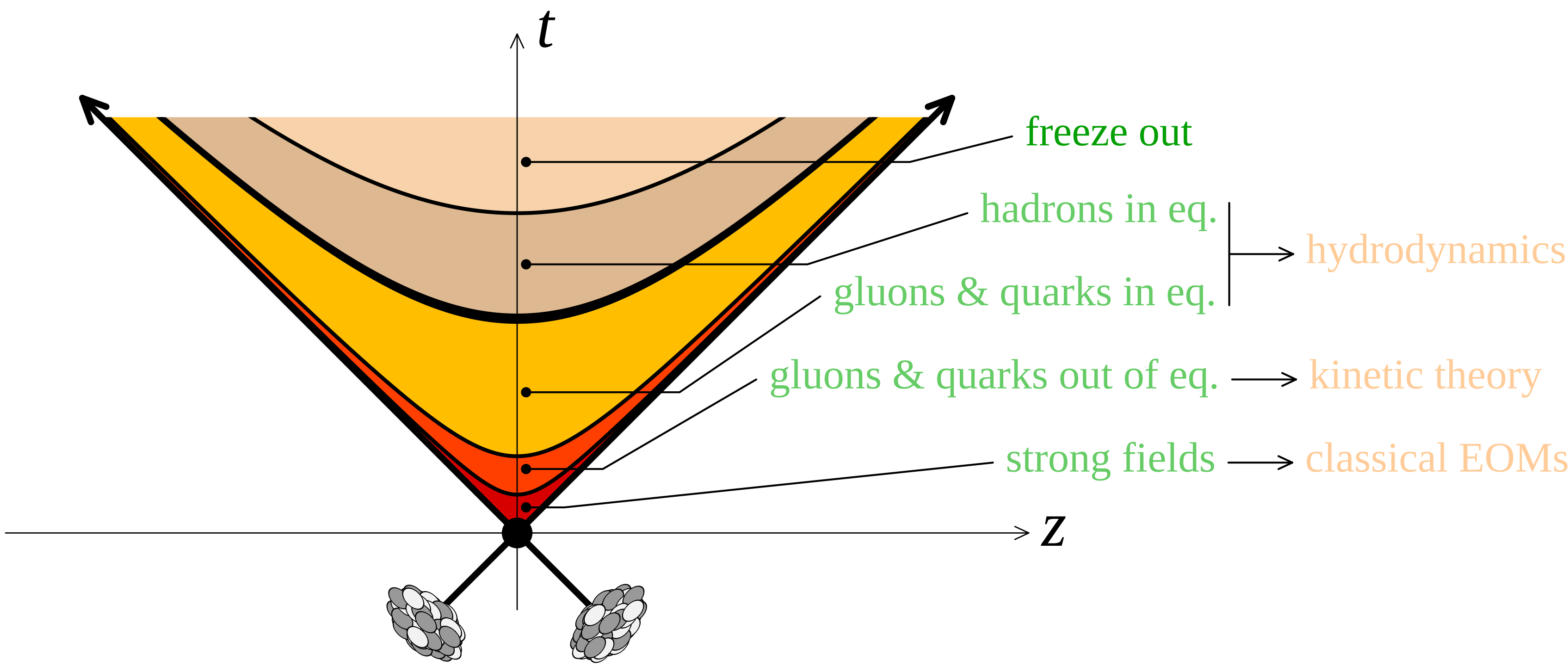
- $2 \le \tau \lesssim 5$ fm/c
- Quark gluon plasma

Stages of a nucleus-nucleus collision



- $5 \lesssim \tau \lesssim 10$ fm/c
- Hot hadron gas

Stages of a nucleus-nucleus collision

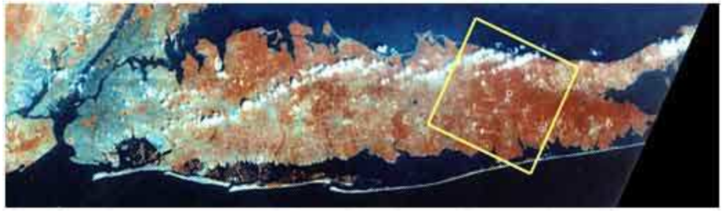


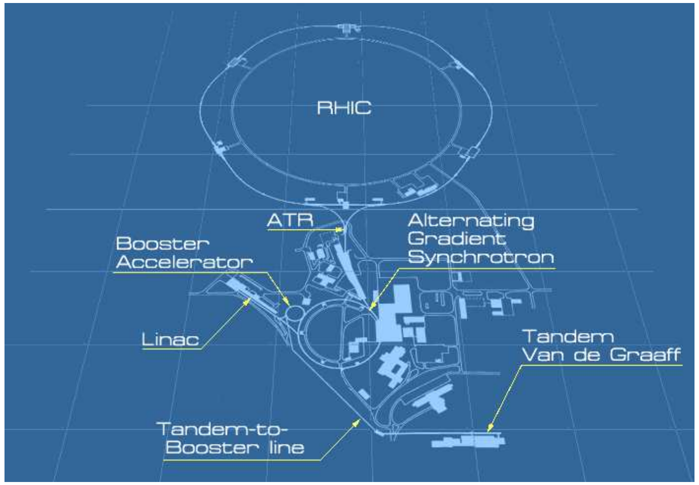
- $\tau \longrightarrow +\infty$
- Chemical freeze-out :
  density too small to have inelastic interactions
- Kinetic freeze-out :
  no more elastic interactions

What have we learned from RHIC?

(biased selection of results...)

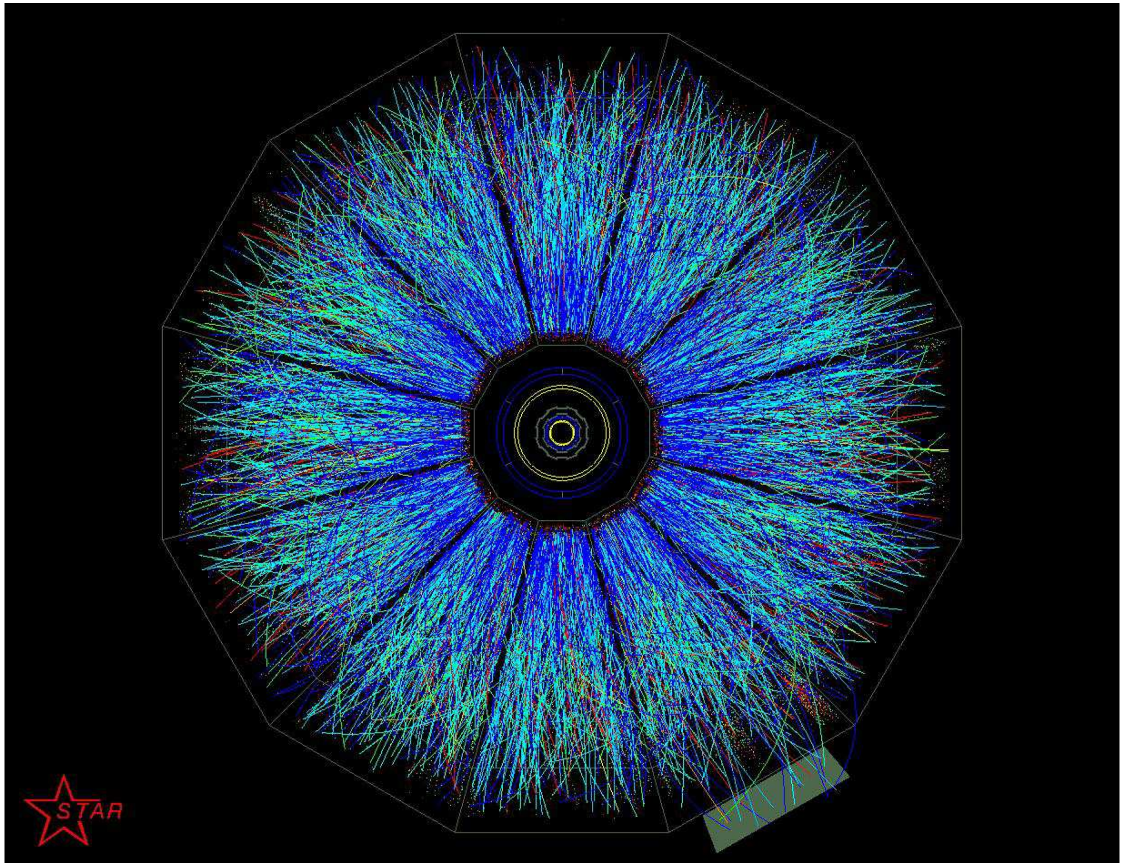
Since 2000 : RHIC





Since 2000 : RHIC

- Nucleus-nucleus collision seen in the STAR detector :



Initial energy density

- Bjorken estimate :

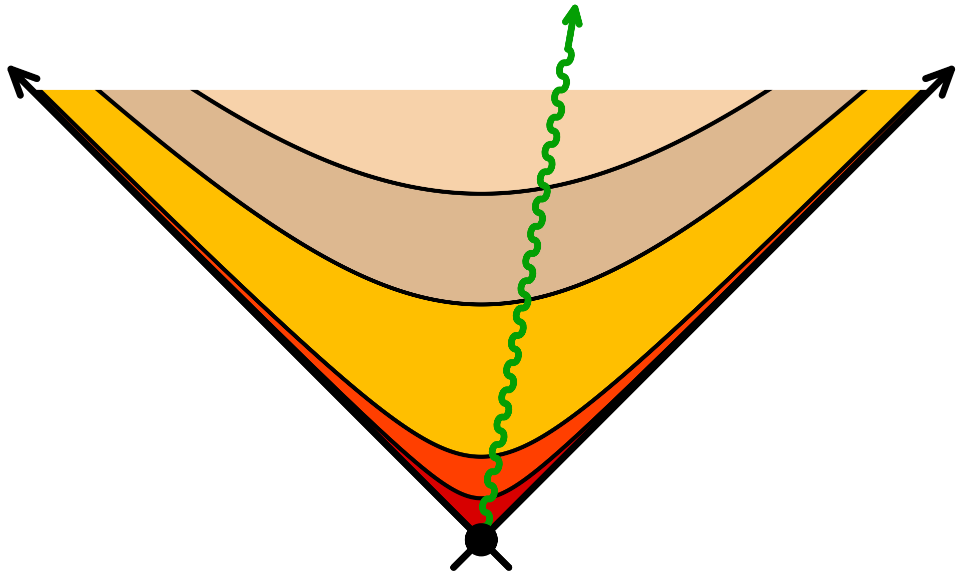
$$\epsilon_0 \approx \frac{1}{\tau_0 \boldsymbol{S}_\perp} \frac{dE_\perp}{dy}$$

- $dE_\perp/dy \approx 620$ GeV at RHIC ($\sqrt{s} = 200$ GeV, gold nuclei)
- $\boldsymbol{S}_\perp \approx 140$ fm^2 for central collisions
- $\tau_0 \approx 0.15$ fm

$$\triangleright \quad \epsilon_0 \approx 30 \text{ GeV/fm}^3$$

- Reminder : lattice QCD predicts deconfinement at $\epsilon_{\rm crit} \sim 1$ GeV/fm^3
- Note : things look less impressive in terms of the temperature since $\epsilon \sim T^4 \quad \Rightarrow \quad T/T_{\rm crit} \sim 30^{1/4} \sim 2.3$

QGP "opacity"



- High $p_\perp$ jets are produced at the initial impact
  - Not very interesting by themselves...

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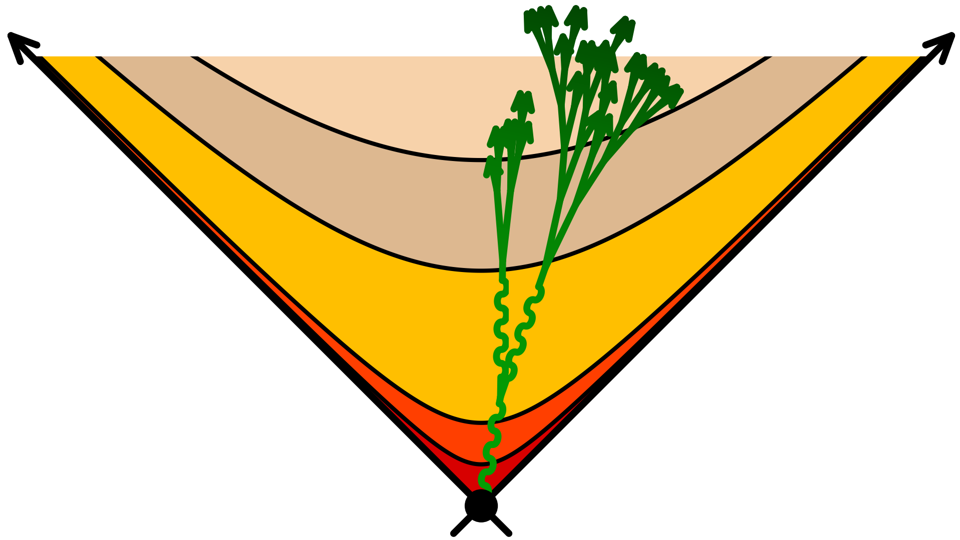
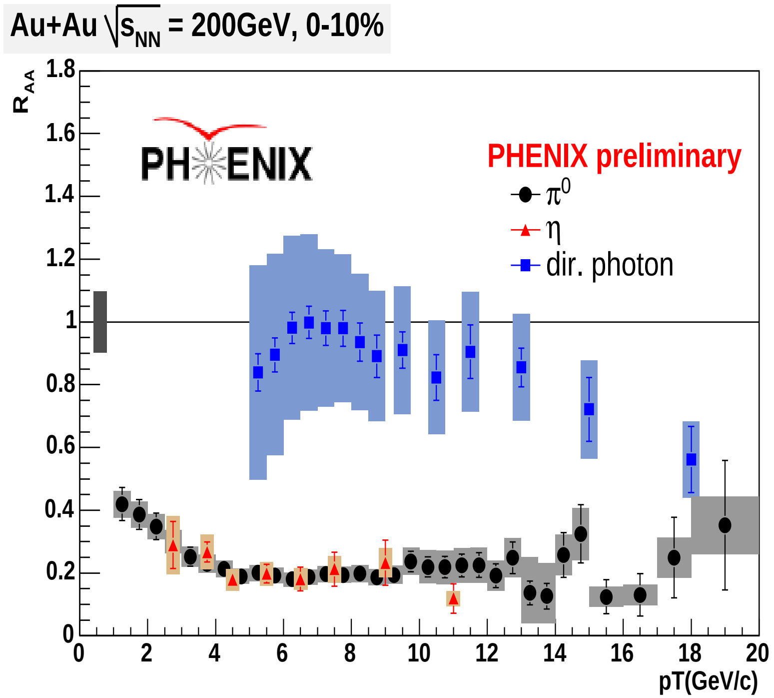
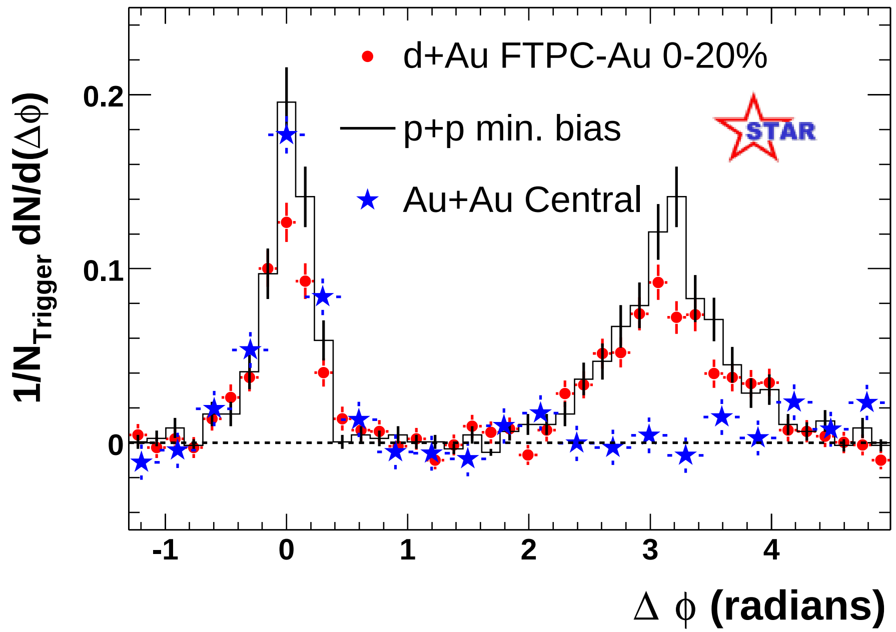


- High $p_\perp$ jets are produced at the initial impact
  - Not very interesting by themselves...
- Radiative energy loss when they travel through the QGP
  - Sensitive to the energy density of the medium
  - Depends on the path length as L^2
  - Important modification of the azimuthal correlations

QGP “opacity”

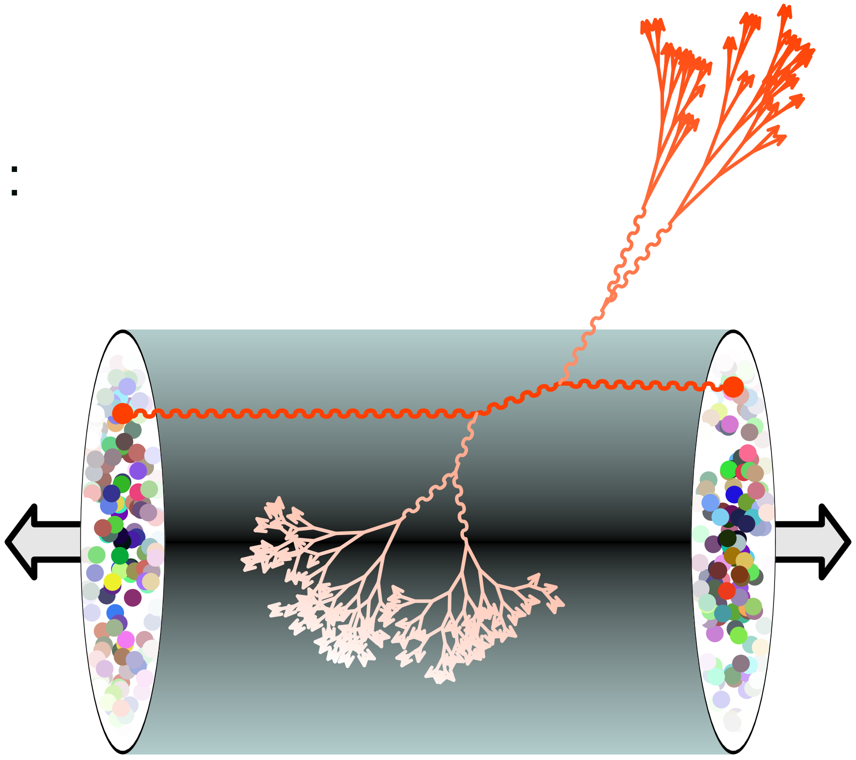




- Hadrons are strongly suppressed
  - Mesons involving heavy quarks (e.g. D) are also suppressed
  - Photons are not suppressed
- The correlation at 180^o disappears in AA collisions

QGP “opacity”

- Interpretation :

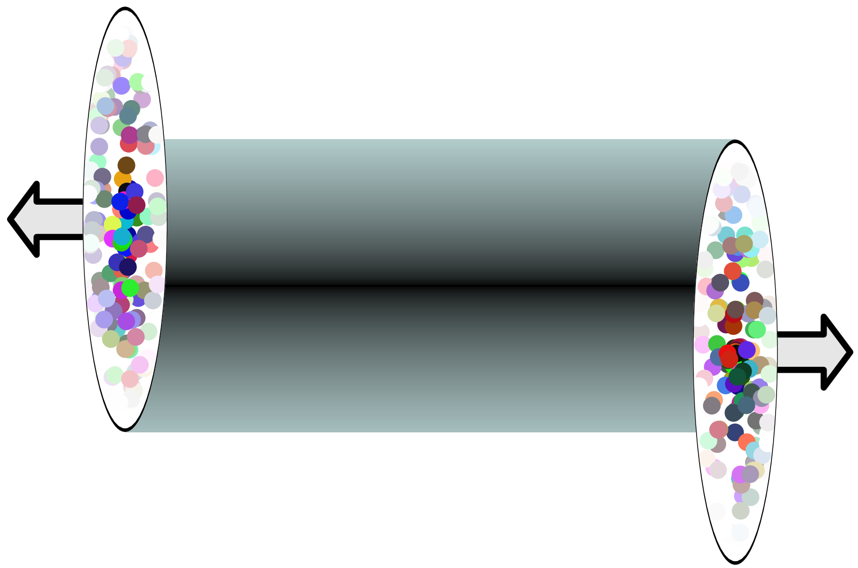


- Jets escape only if they are produced near the edge and are directed outwards
- The opposite jet is totally absorbed

▷ confirms the very large energy density

Collective flow

- Consider a non-central collision :



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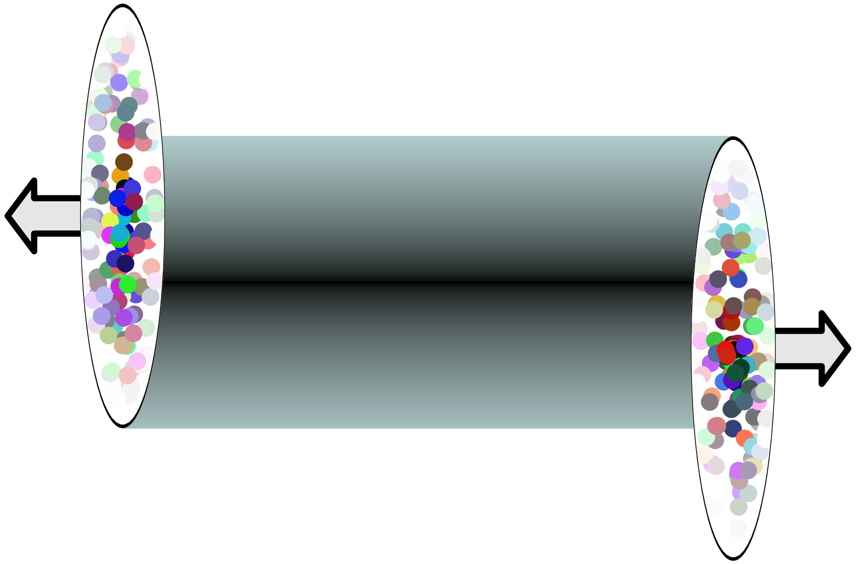

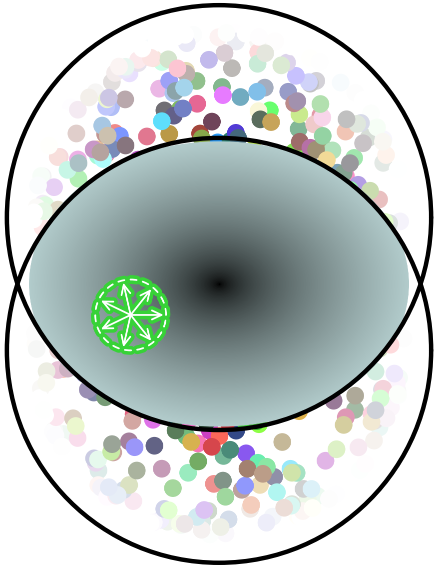

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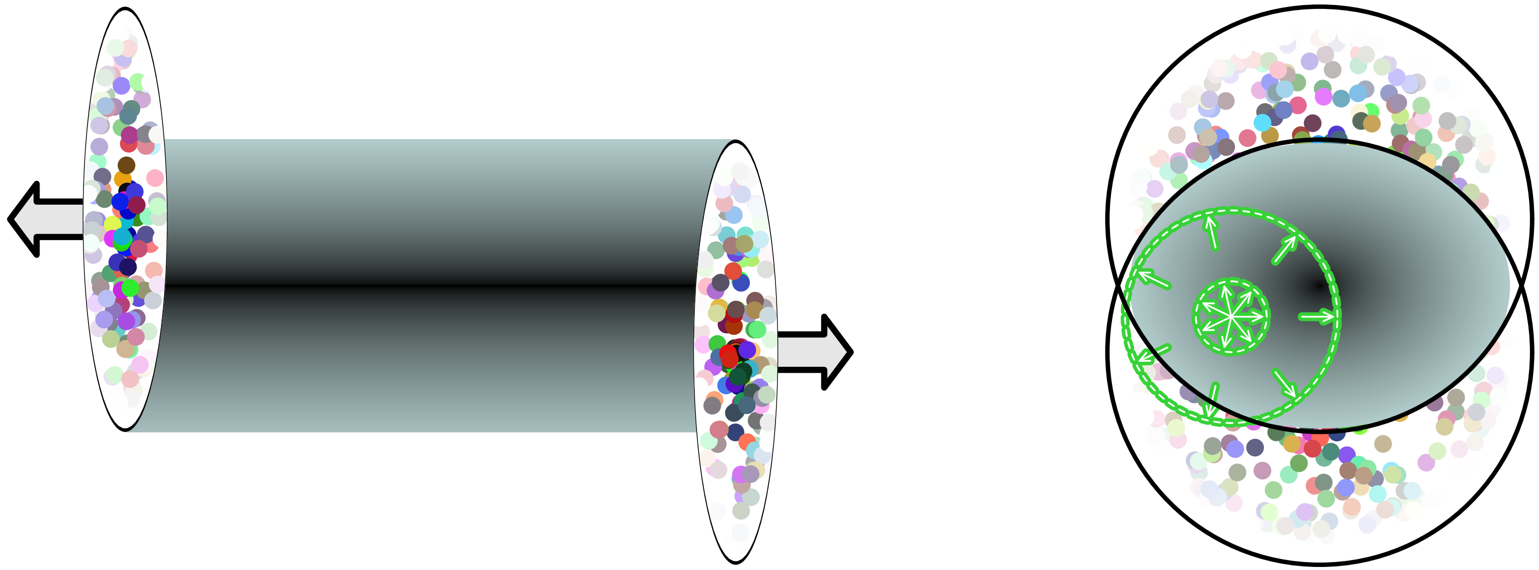

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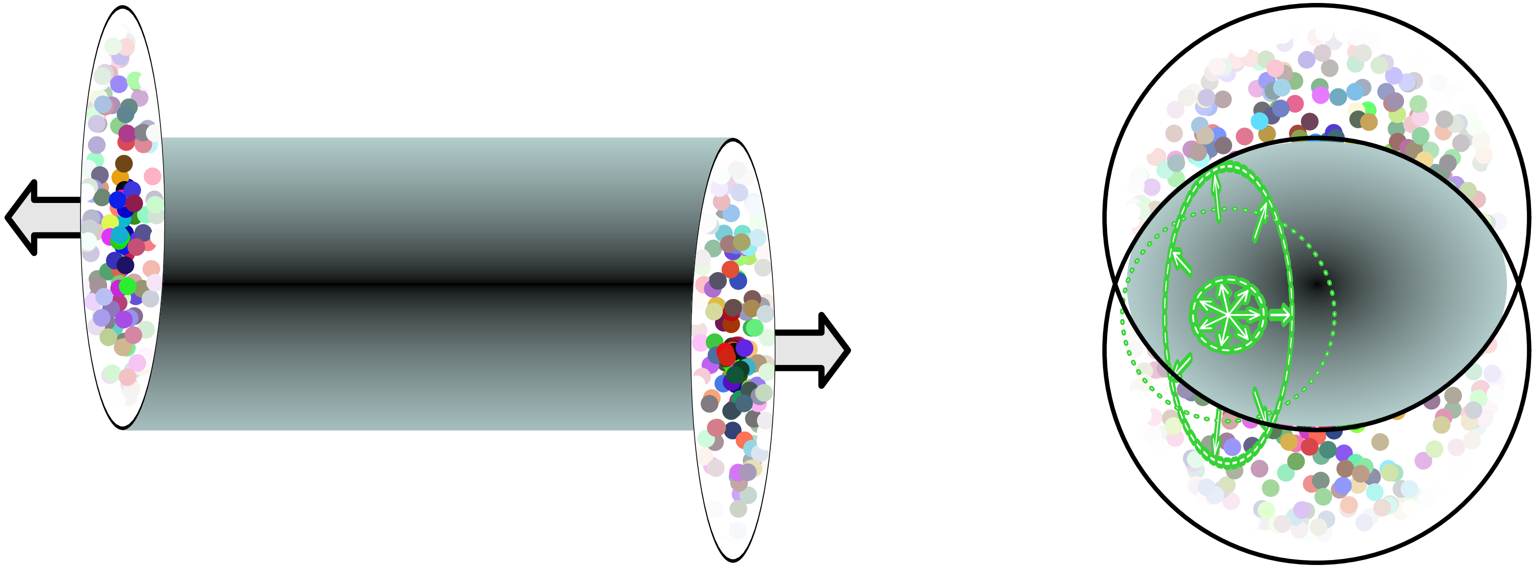


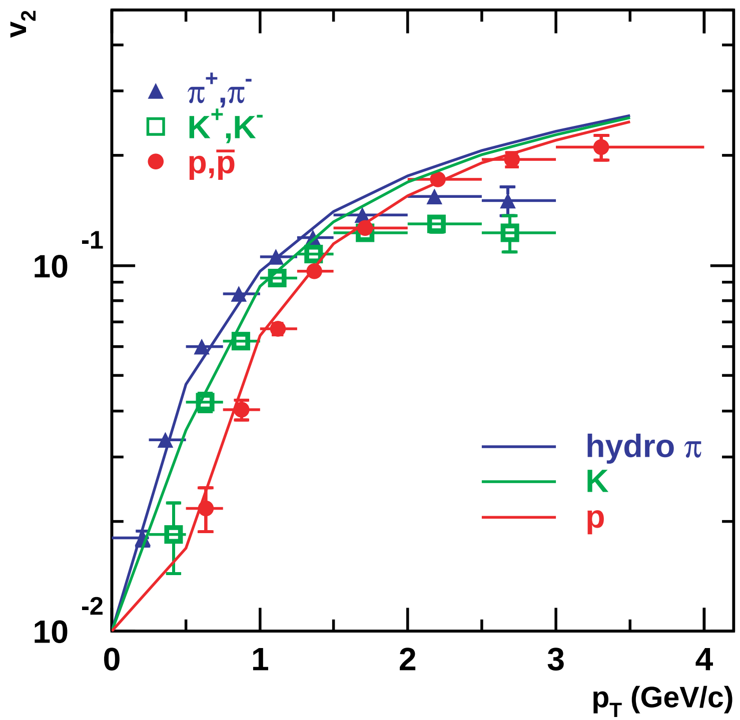
- Initially, the momentum distribution of particles is isotropic in the transverse plane, because their production comes from local partonic interactions
- If these particles were escaping freely, the distribution would remain isotropic at all times
- If the mean free path is small, pressure gradients are anisotropic and induce an anisotropy of the momentum distribution

Collective flow and ideal hydrodynamics

- Observable: 2^{nd} harmonic of the azimuthal distribution

$$dN/d\varphi \sim 1 + 2v_1\cos(\varphi) + 2{\color{red}v_2}\cos(2\varphi) + \cdots$$

▷ v_2 measures the ellipticity of the momentum distribution



Is the QGP a perfect fluid?

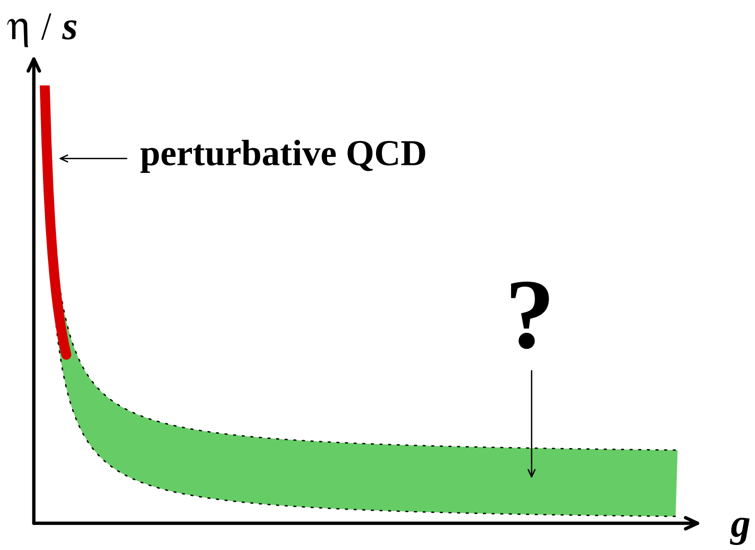
- The elliptic flow coefficient v_2 measured at RHIC well reproduced by ideal hydrodynamics (Euler equation), that has no viscosity
  - In Navier-Stokes equation of viscous hydrodynamics, the relevant parameter is the dimensionless ratio η/s of the shear viscosity to the entropy density
  - It has been concluded from there that the QGP must have a very small ratio η/s
- Problem: if the interactions are weak, η/s is large

AdS/CFT duality
Strongly coupled QGP

Weak coupling viscosity

- The shear viscosity/entropy ratio has been calculated in QCD at weak coupling, and it diverges when $g \to 0$:

$$\frac{\eta}{s} = \frac{5.12}{g^4 \ln\left(\frac{2.42}{g}\right)}$$



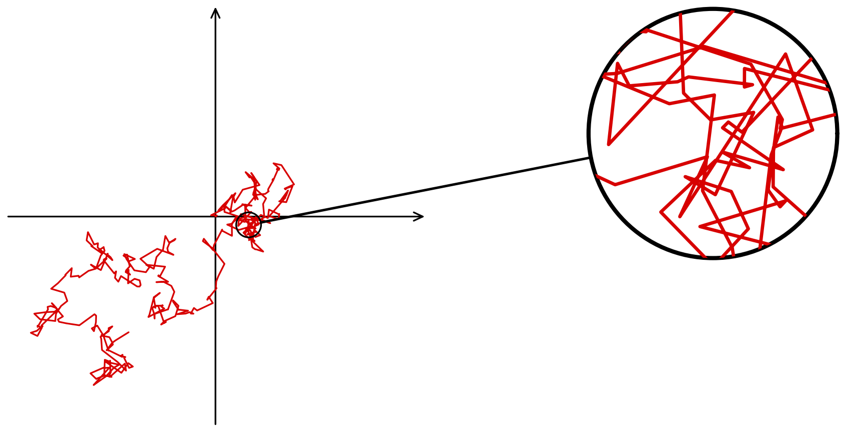
- However, η/s decreases quickly when the coupling increases $\triangleright$ one way to have a small viscosity is to have a large coupling. Problem : how to calculate it?

Uncertainty bound on η/s

- $\eta \sim \lambda\epsilon$ ($\lambda =$ mean free path, $\epsilon =$ energy density). Thus,

$$\frac{\eta}{s} \sim \lambda \underbrace{\frac{\epsilon}{s}}_{\text{energy per particle}}$$

- Heisenberg inequalities forbid the mean free path to be smaller than the De Broglie wavelength of the particles. Scatterings by an $\mathcal{O}(1)$ angle can occur only every $\lambda_{\rm Broglie}$ at most :



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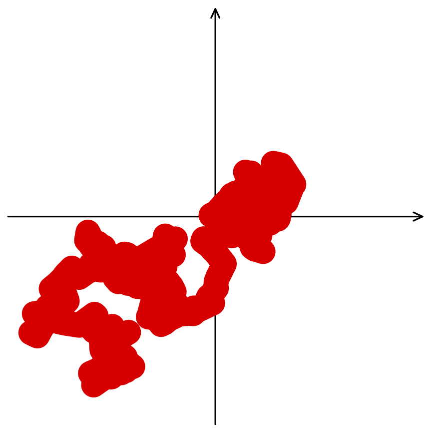


- Hence, $\frac{\eta}{s} \geq \mathcal{O}(1)$ [in a system of units were $\hbar = 1$]

AdS/CFT duality at T=0

- In QCD, we cannot compute the strong coupling limit
- Maximally super-symmetric $SU(N)$ Yang-Mills theories in the limit $g^2N \to +\infty$ are dual to classical super-gravity on an $\mathrm{AdS}_5 \times S_5$ manifold with metric

$$ds^2 = \frac{R^2}{z^2}(\underbrace{-dt^2 + d\vec{\boldsymbol{x}}^2}_{\text{we live here... (at z=0)}} + dz^2) + R^2 d\Omega_5^2$$

- If an operator $\mathcal{O}$ of our world is coupled on the boundary to a field ϕ that lives in the bulk, the duality states that :

$$e^{-S_{\rm cl}[\phi]} = \left\langle e^{\int_{\rm boundary} \mathcal{O}\ \phi(z=0)} \right\rangle$$

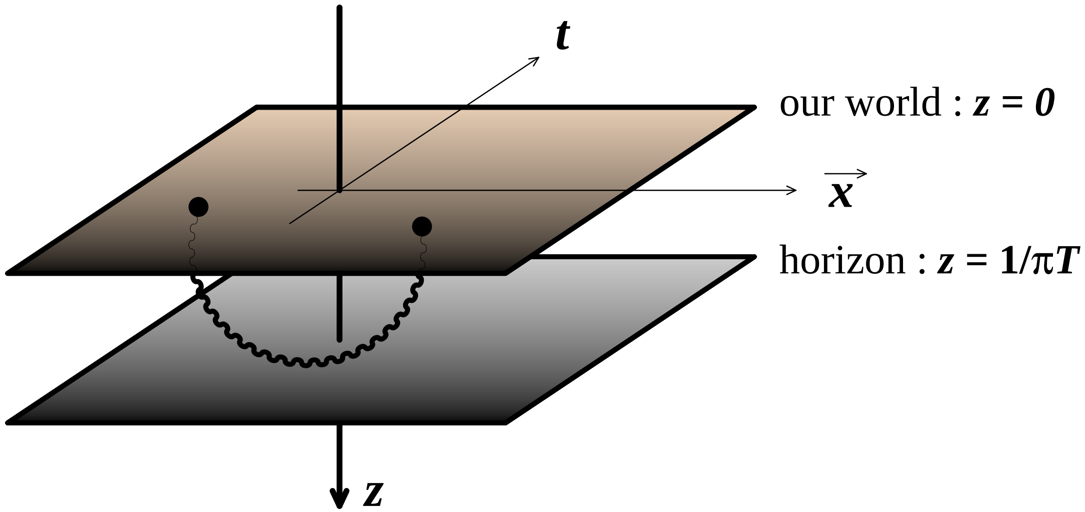
  - The right hand side is a generating functional for the correlators of operators $\mathcal{O}$ in the 4-dim super Yang-Mills theory
  - The left hand side is calculable in the gravity dual (solve the classical EOM for ϕ with the boundary condition $\phi(z=0)$)

AdS/CFT duality at high T

- At finite temperature T :

  $$-dt^2 + dz^2 \rightarrow -f(z)dt^2 + dz^2/f(z) \qquad \text{with} \qquad f(z) = 1 - (\pi z T)^4$$

- $f(z) = 0$ at $z = 1/\pi T \quad \Rightarrow$ black hole horizon



- Ordinary particles in 4-dimensions are the end points of open strings living in the bulk

Viscosity in SUSY Yang-Mills

Policastro, Son, Starinets (2001)

- The shear viscosity can be obtained from correlations of the energy-momentum tensor :

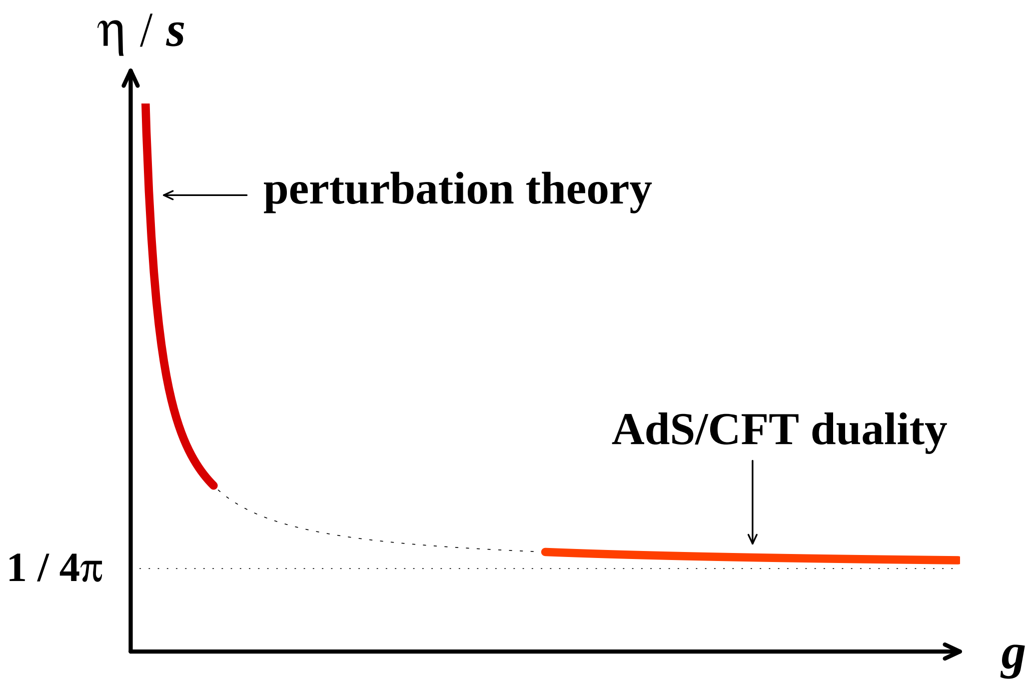
$$\eta \propto \int dt d^3\vec{\boldsymbol{x}} \, \left\langle T_{xy}(t,\vec{\boldsymbol{x}}) \, T_{xy}(0,\vec{\boldsymbol{0}}) \right\rangle$$

- In the dual theory, T_{xy} couples to metric perturbations, i.e. to the graviton. The above correlation function is also the absorption cross-section of a graviton (of zero frequency) by the black hole. Hence :

$$\eta \propto \sigma_{\rm abs}$$

- In the classical limit, $\sigma_{\rm abs}$ is the area of the horizon. Moreover, the area of a black-hole horizon is its entropy
- Combining everything, one obtains $\eta/s = 1/4\pi$

Viscosity in SUSY Yang-Mills



- Conjecture : $1/4\pi$ is the lowest possible value for η/s
- Note: all the known substances have a viscosity to entropy ratio (much) larger than the bound

Caveats of AdS/CFT: SUSY YM $\neq$ QCD

- AdS/CFT only applies to maximally super-symmetric Yang-Mills theories. Such theories are scale invariant, have no running coupling, no chiral symmetry breaking, and no confinement

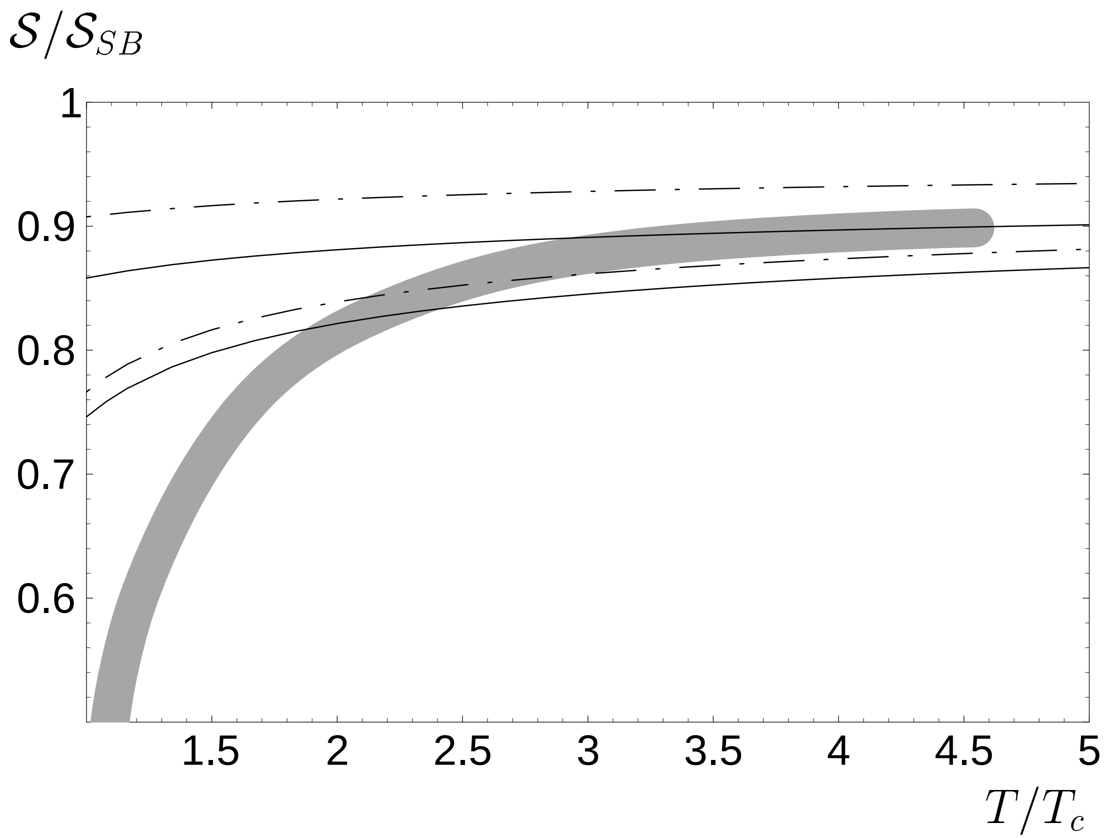
- Whether what we learn about these theories is accurate for QCD (that has broken scale invariance, running coupling, chiral symmetry breaking, confinement, and quite different matter fields...) is at best a wishful thinking

- Nevertheless an interesting playground in order to realize how wrong one's weak coupling prejudices may be...

- Note : in the strong coupling limit of any sensible field theory, η/s is probably close to the uncertainty principle limit

Caveats of AdS/CFT: is g really large?

- There are some dissenting views about whether the physics of the QGP at $T/T_{\rm crit} \sim 2-3$ is really strongly coupled. For quantities such as the entropy, perturbative techniques (+resummations) lead to accurate results in this region
Blaizot, Iancu, Rebhan (1999-2000)



Weak coupling approach

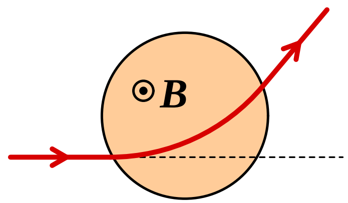
Small η/s in weak coupling ?

Asakawa, Bass, Muller (2006)

- Assume that $\alpha_s = \dfrac{g^2}{4\pi} \ll 1$
- Consider a domain of size Q_s^{-1}, in which the magnetic field is uniform and large, of order $B \sim Q_s^2/g$
- Let a particle of energy $E \sim Q_s$ go through this domain. The Lorenz force deflects its trajectory by an angle of order unity :

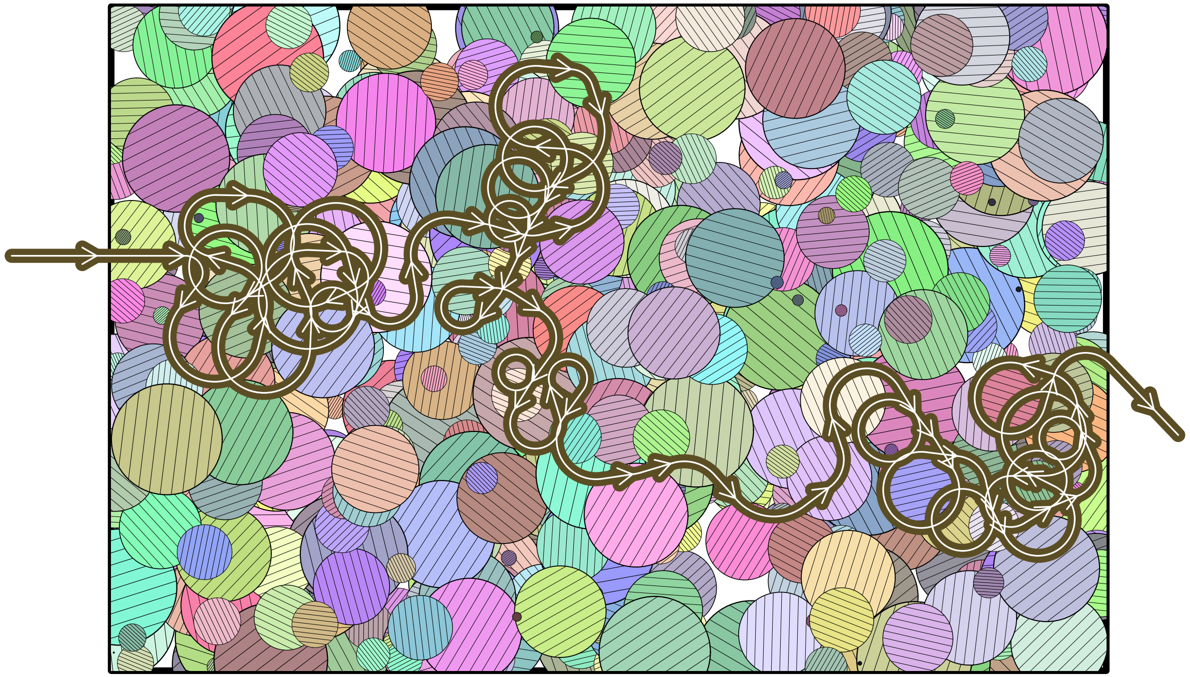
$$\frac{d\vec{\boldsymbol{p}}}{dt} = g\,\vec{\boldsymbol{v}} \times \vec{\boldsymbol{B}} \quad \Rightarrow \quad \dot{\theta} = \frac{gB}{E} \sim Q_s$$

time spent in the domain : $\delta\tau \sim Q_s^{-1}$



Small η/s in weak coupling ?

- Consider now a region filled with such domains, with random orientations for the magnetic field in each domain



▷ In such a medium, the mean free path of a particle of energy Q_s is of order Q_s^{-1}, i.e. as low as permitted by the uncertainty principle

▷ η/s must be close to the lower bound

Initial gluon production

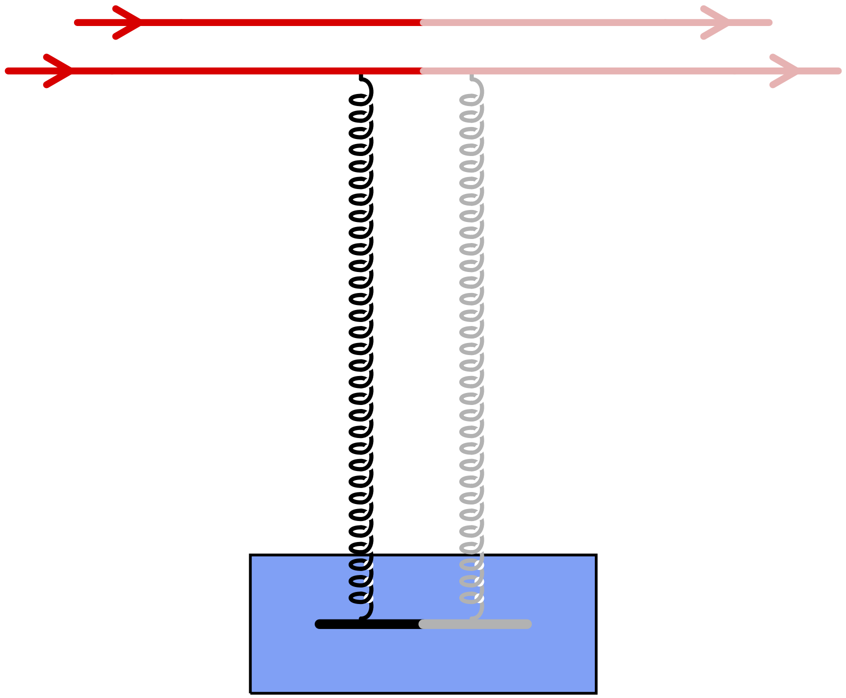
- Weak coupling QCD provides a mechanism for generating the turbulent magnetic fields that would lead to a small η/s

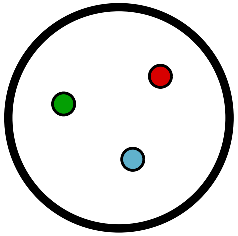
- Peculiarities of high energy heavy ion collisions :
  - 99% of the multiplicity below $p_\perp \sim 2$ GeV
  - The bulk of of particle production comes from gluons that carry a (very) small fraction x of the nucleus momentum

$$x \sim \frac{p_\perp}{\sqrt{s}}$$

  - Gluon distributions increase rapidly when $x \to 0$
    ▷ large gluon occupation number
    ▷ classical color fields

Gluon saturation



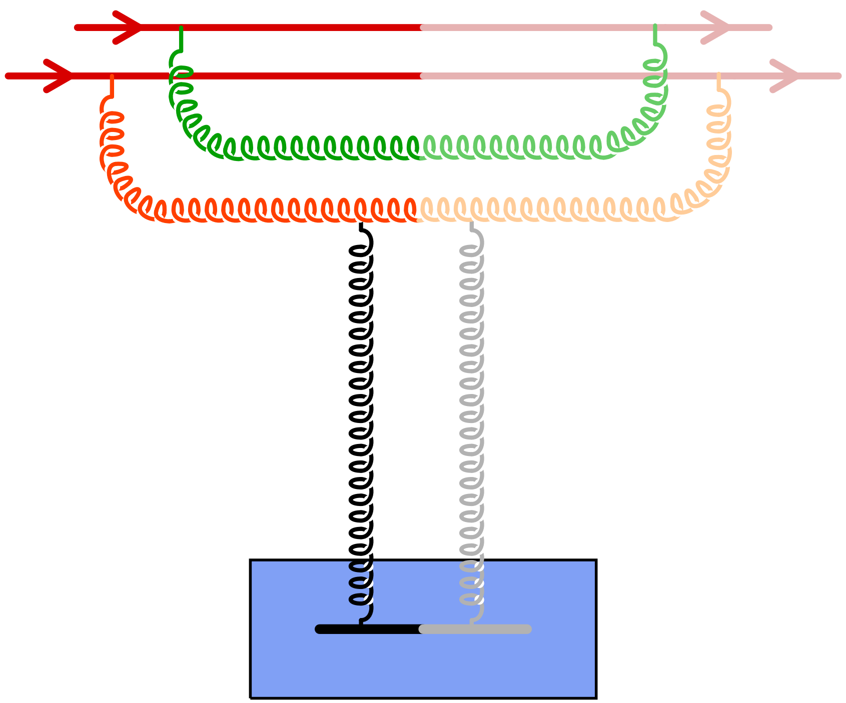


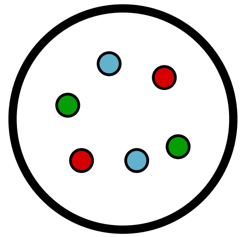
▷ assume that the projectile is big, e.g. a nucleus, and has many valence quarks (only two are represented)

▷ on the contrary, consider a small probe, with few partons

▷ at low energy, only valence quarks are present in the hadron wave function

Gluon saturation



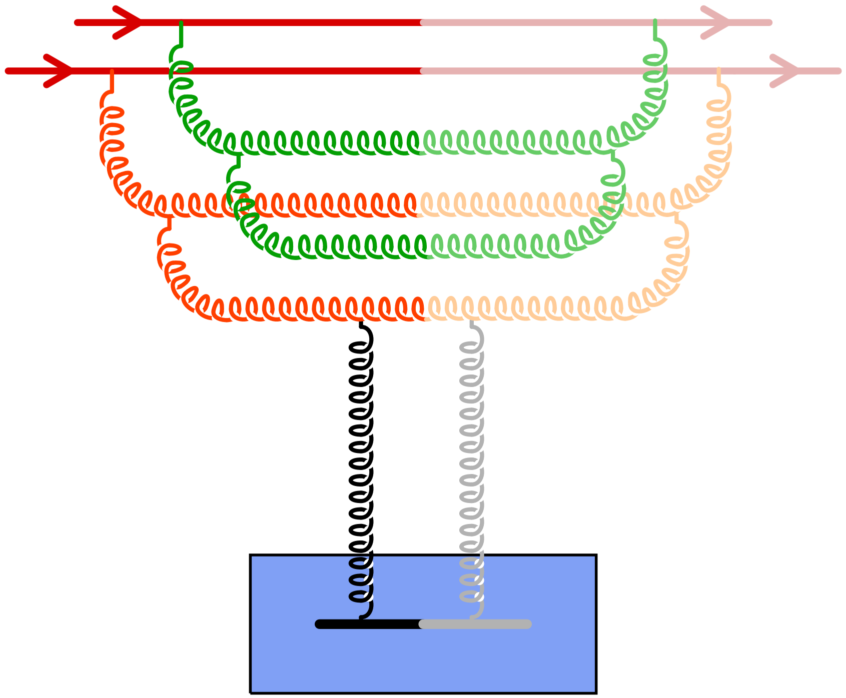


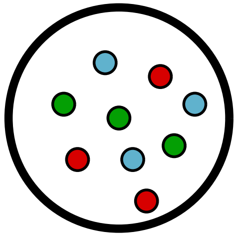
▷ when energy increases, new partons are emitted

▷ the emission probability is $\alpha_s \int \frac{dx}{x} \sim \alpha_s \ln(\frac{1}{x})$, with x the longitudinal momentum fraction of the gluon

▷ at small-x (i.e. high energy), these logs need to be resummed

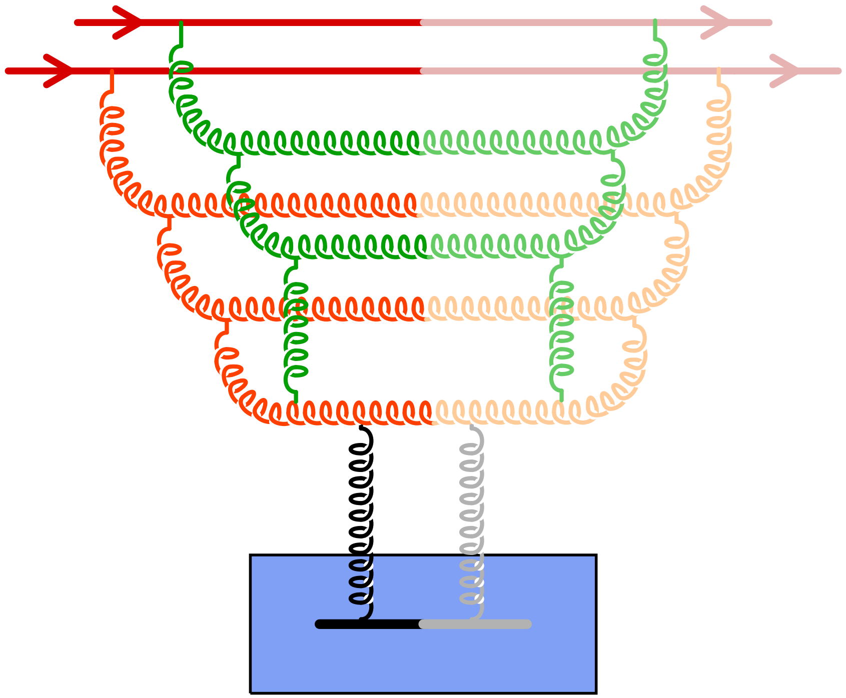
Gluon saturation

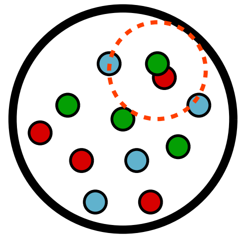




▷ as long as the density of constituents remains small, the evolution is linear: the number of partons produced at a given step is proportional to the number of partons at the previous step (BFKL)

Gluon saturation





- ▷ eventually, the partons start overlapping in phase-space
- ▷ parton recombination becomes favorable
- ▷ after this point, the evolution is non-linear:

  the number of partons created at a given step depends non-linearly on the number of partons present previously

Saturation criterion

Gribov, Levin, Ryskin (1983)

- Number of gluons per unit area:

$$\rho \sim \frac{xG_A(x,Q^2)}{\pi R_A^2}$$

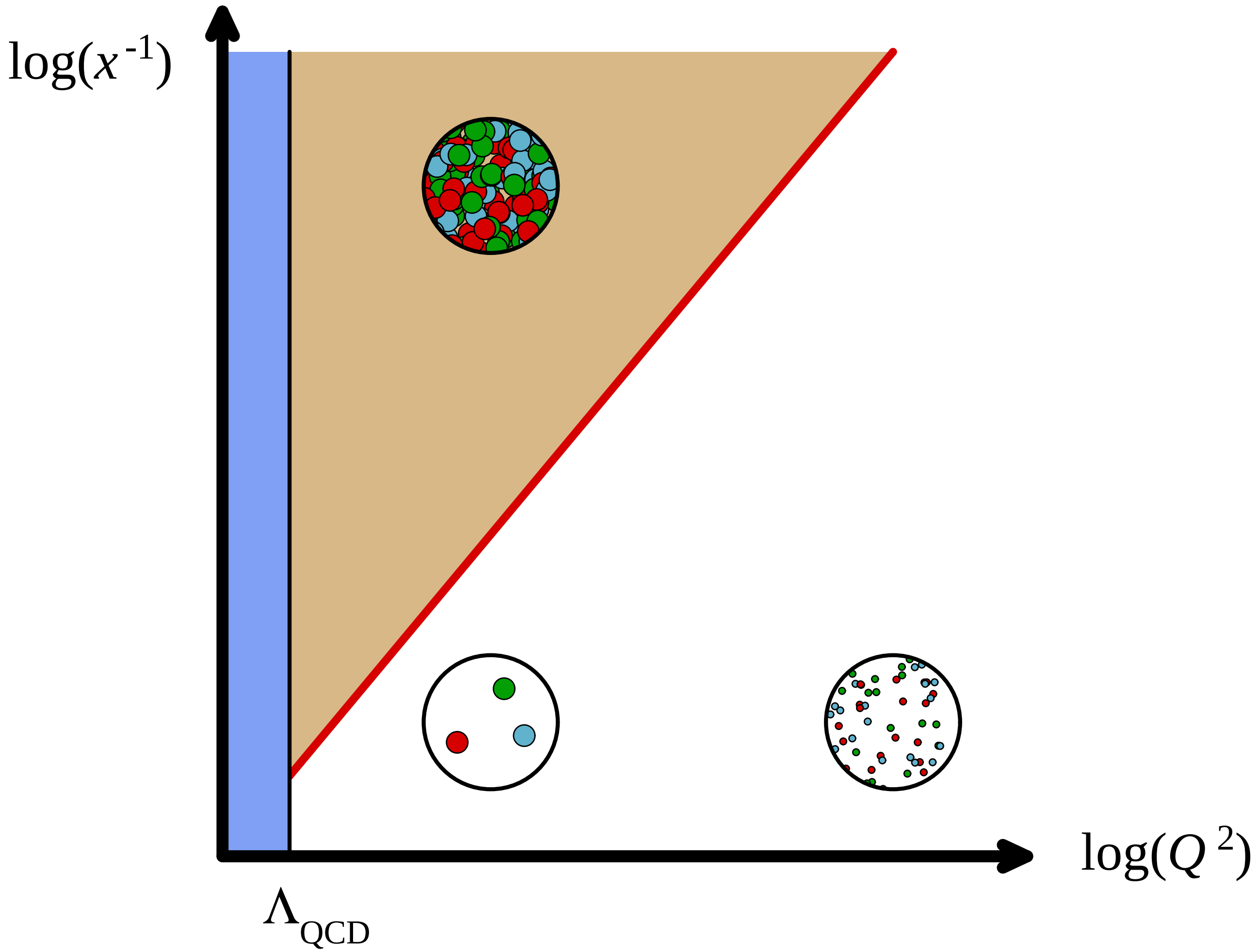
- Recombination cross-section:

$$\sigma_{gg\to g} \sim \frac{\alpha_s}{Q^2}$$

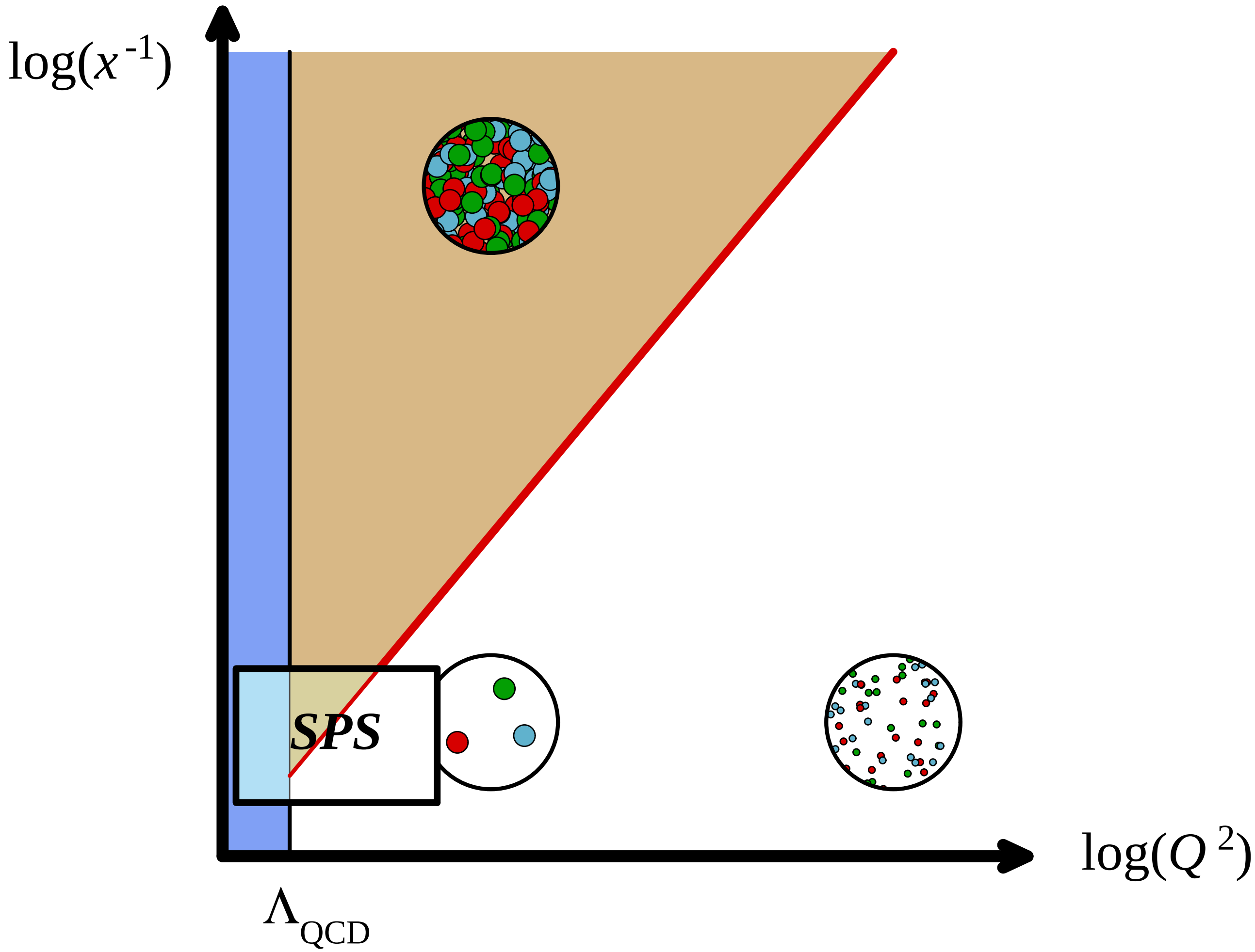
- Recombination happens if $\rho\sigma_{gg\to g} \gtrsim 1$, i.e. $Q^2 \lesssim Q_s^2$, with:

$$Q_s^2 \sim \frac{\alpha_s xG_A(x,Q_s^2)}{\pi R_A^2} \sim A^{1/3}\frac{1}{x^{0.3}}$$

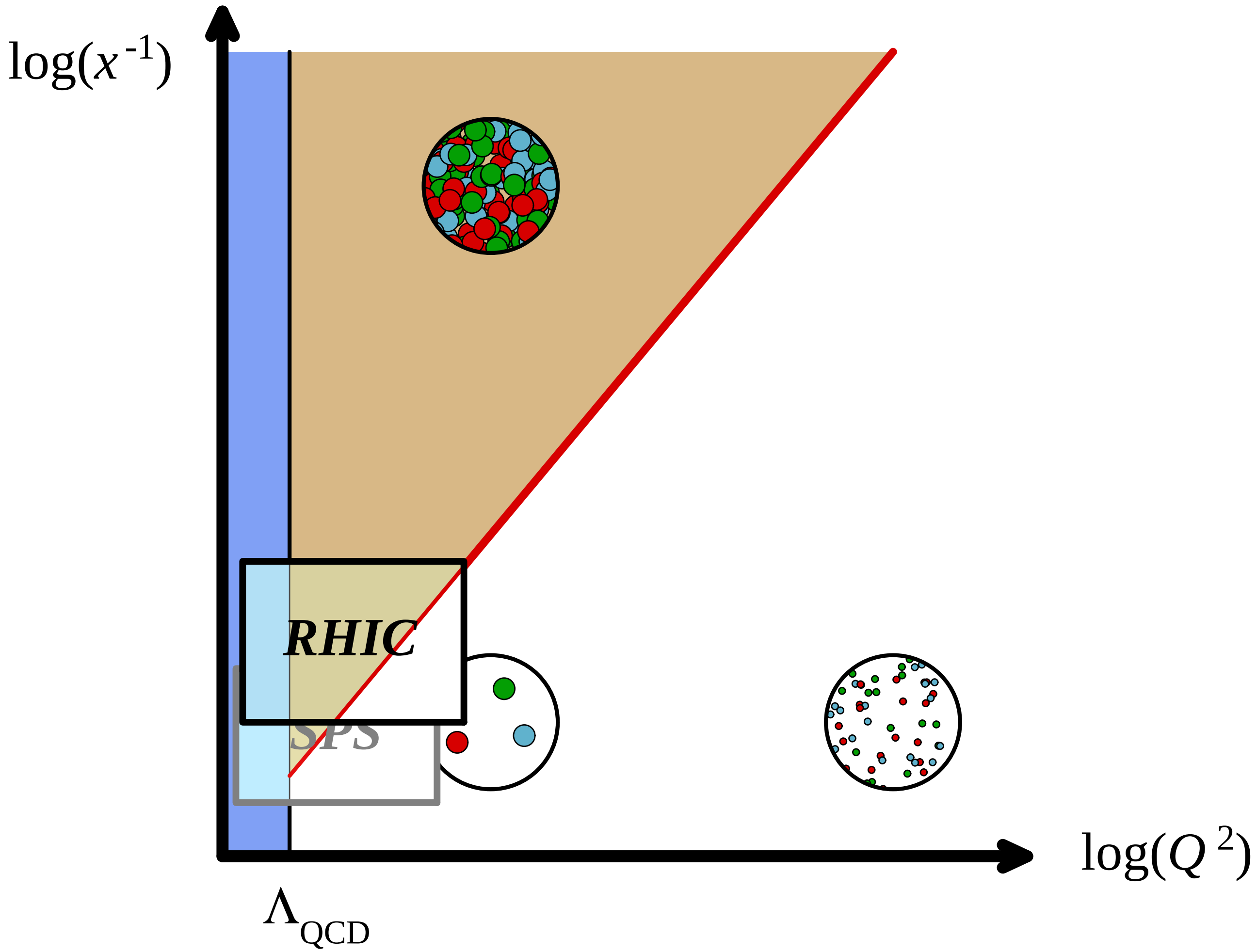
Saturation domain



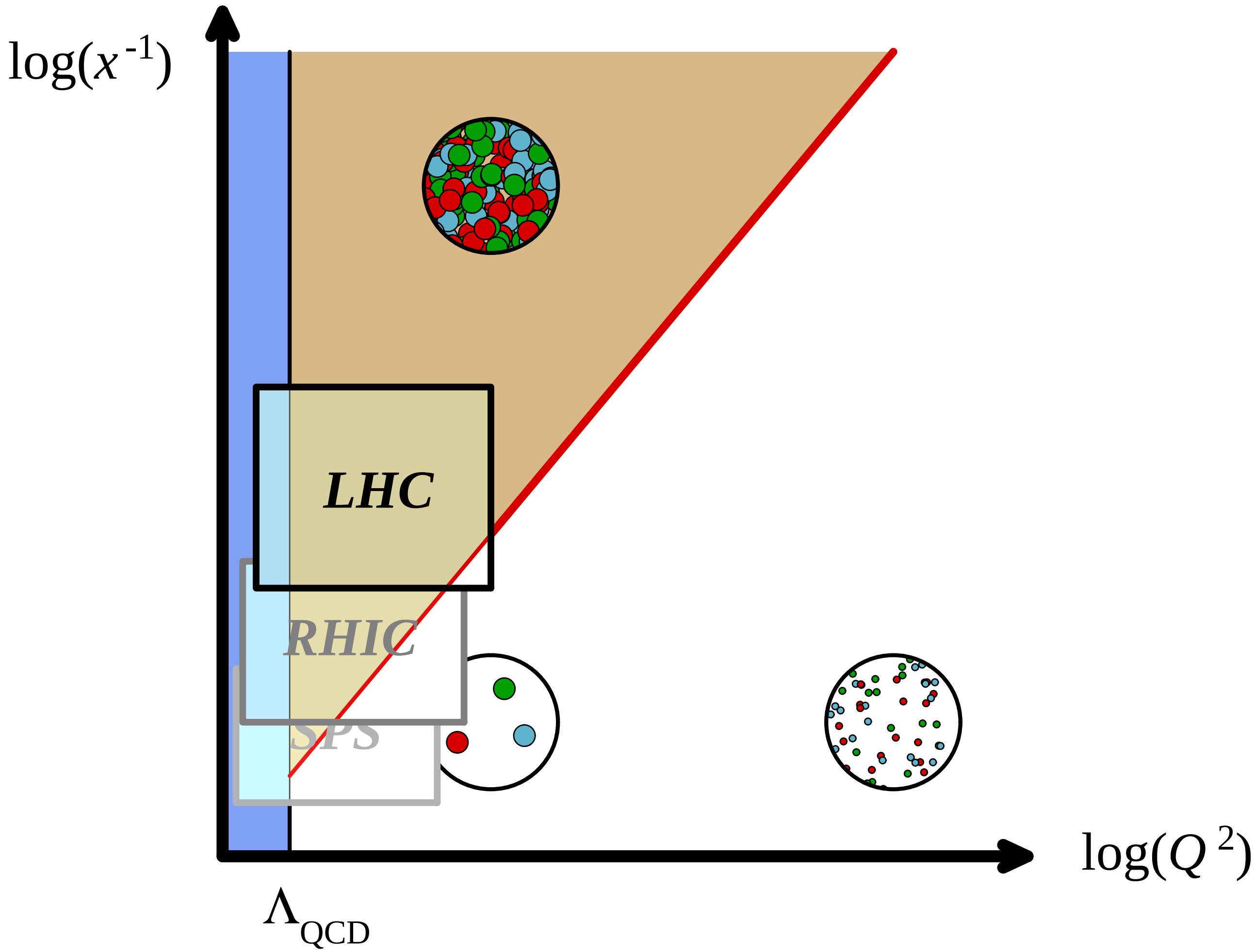
Saturation domain



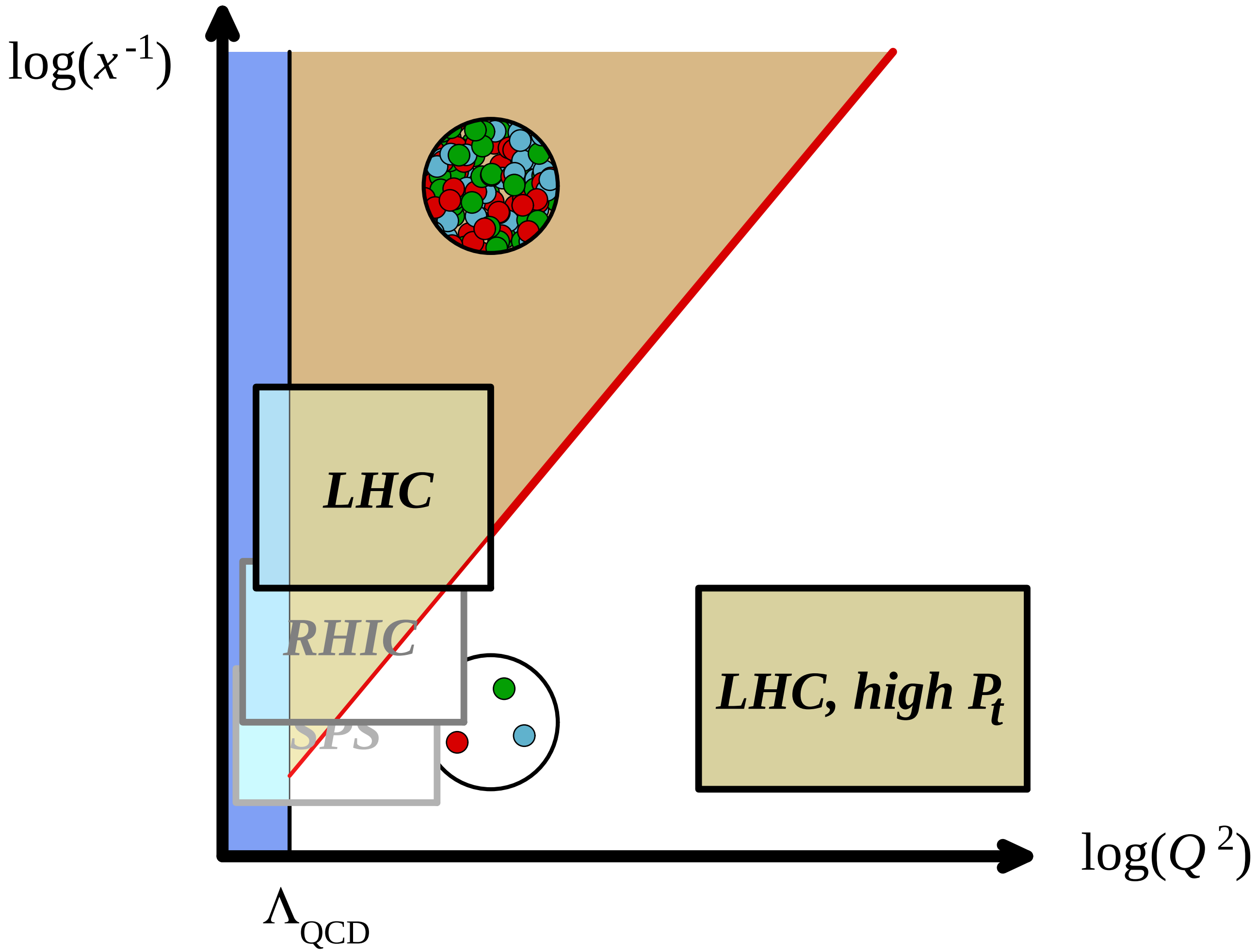
Saturation domain



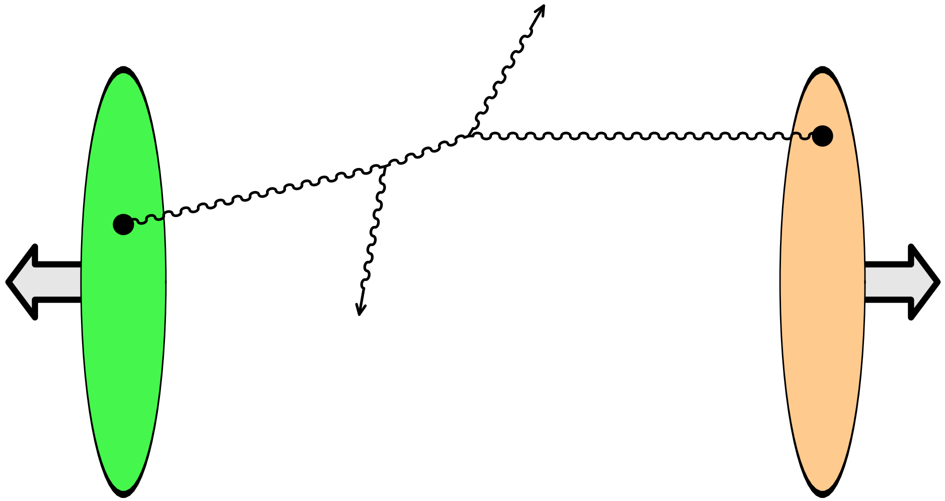
Saturation domain



Saturation domain

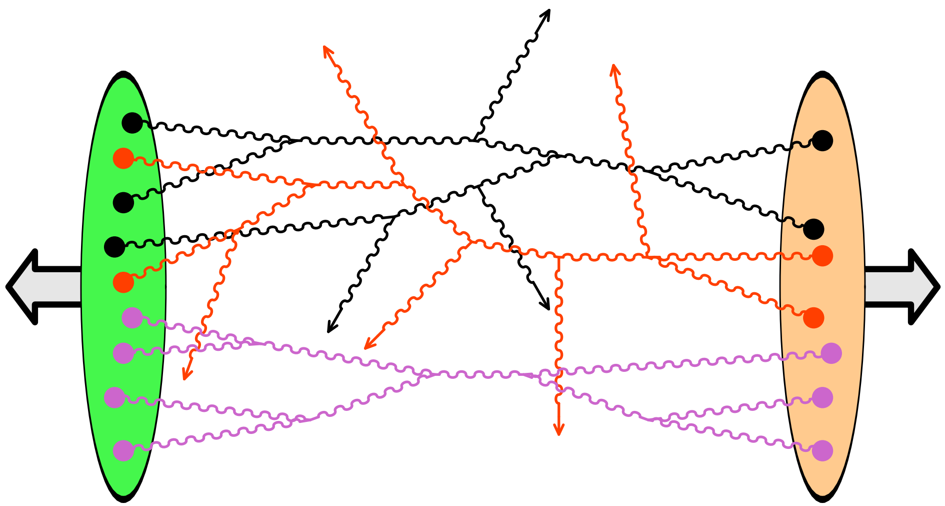


Initial gluon production



- Dilute regime : one parton in each projectile interact

Initial gluon production



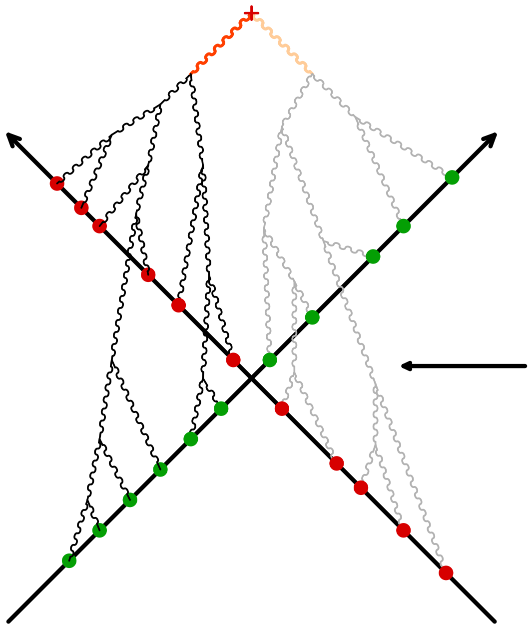
- Dilute regime : one parton in each projectile interact
- Dense regime : multiparton processes become crucial
  + pileup of many partonic scatterings in every AA collision

Initial gluon production at LO

Krasnitz, Nara, Venugopalan (1999 – 2001), Lappi (2003)

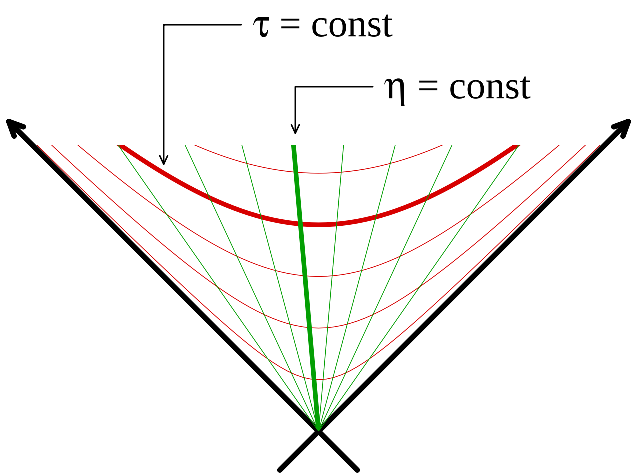
$$\frac{d\overline{N}_{LO}}{d^3\vec{p}} \propto \int_{x,y} e^{ip\cdot(x-y)} \quad \cdots \quad \mathcal{A}_\mu(x)\mathcal{A}_\nu(y)$$

- $\mathcal{A}^\mu(x) =$ classical solution of Yang-Mills equations with color sources ρ_1 and ρ_2 on the light-cone



only tree diagrams at LO

Initial gluon production at LO



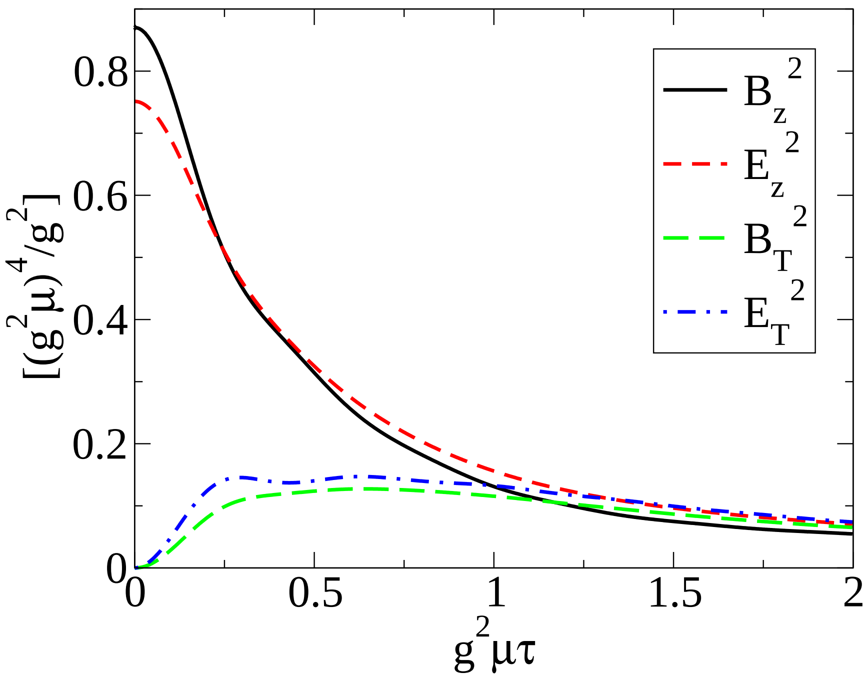
- The color field at $\tau = 0$ does not depend on the rapidity η

  ▷ it remains independent of η at all times
  (invariance under boosts in the z direction)

Initial color fields

Lappi, McLerran (2006)

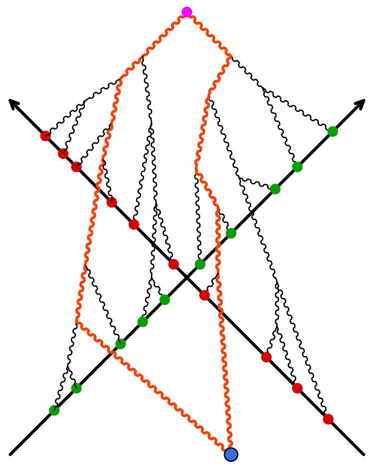
- Before the collision, the chromo-$\vec{E}$ and $\vec{B}$ fields are localized in two sheets transverse to the beam axis
- Immediately after the collision, the chromo-$\vec{E}$ and $\vec{B}$ fields have become longitudinal :



Quantum fluctuations

FG, Venugopalan (2006)

- Typical graph :



- Why is it important ?
  - Questions such as factorization can only be answered by looking at loop corrections
  - Instabilities in the classical solutions enhance the effect of loop corrections

Initial state factorization

FG, Lappi, Venugopalan (2008)

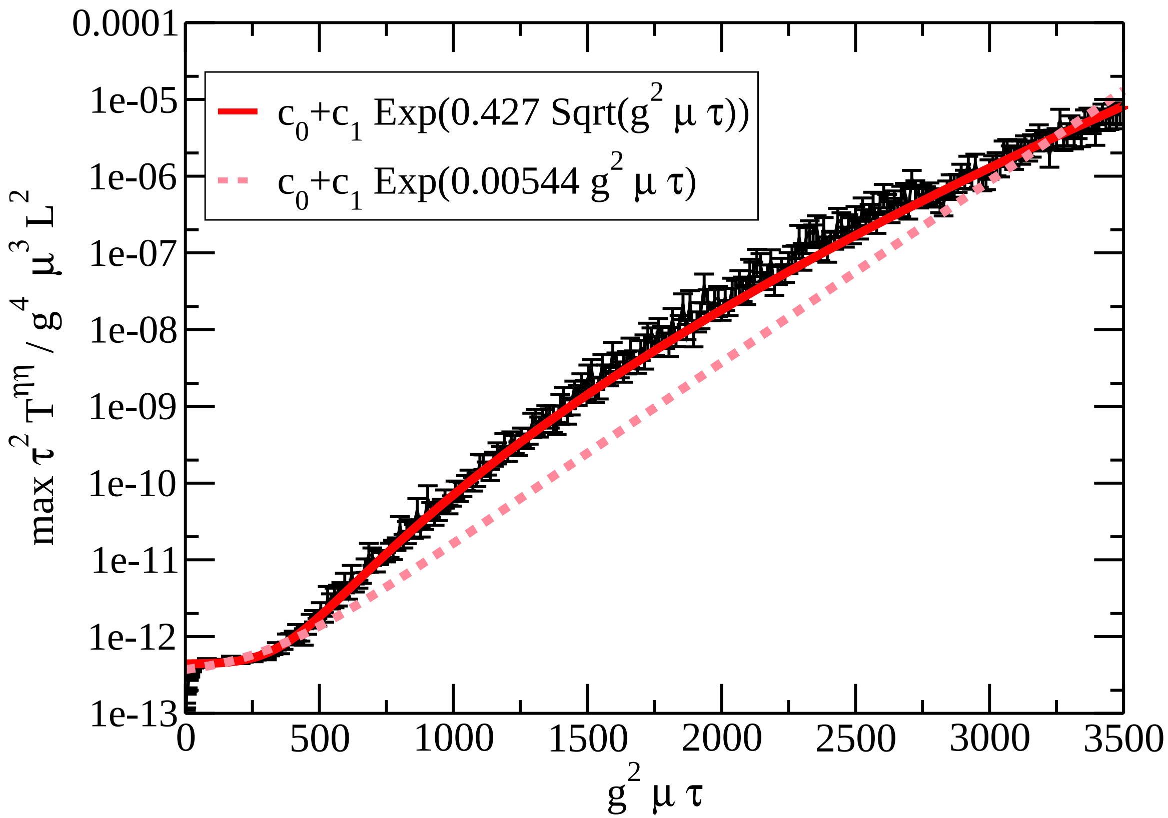
- Loop corrections produce large logarithms $\log(1/x_{1,2})$
- These large logs invalidate the naive perturbative expansion, because $\alpha_s \log(1/x_{1,2})$ may be large even if α_s is small
  - ▷ All the terms in $\left[\alpha_s \log(1/x_{1,2})\right]^n$ should be collected and resummed
- These large logs can be factorized into universal distributions of color sources $W[\rho_{1,2}]$:

$$\frac{dN}{dY\,d^2\vec{\boldsymbol{p}}_\perp} = \int \underbrace{[D\rho_1]\,[D\rho_2]\;W_1[\rho_1]\;W_2[\rho_2]}_{\text{projectiles source distributions}} \;\; \underbrace{\frac{dN[\rho_1,\rho_2]}{dY\,d^2\vec{\boldsymbol{p}}_\perp}}_{\text{gluon spectrum in fixed } \rho_{1,2}}$$

Classical field instability

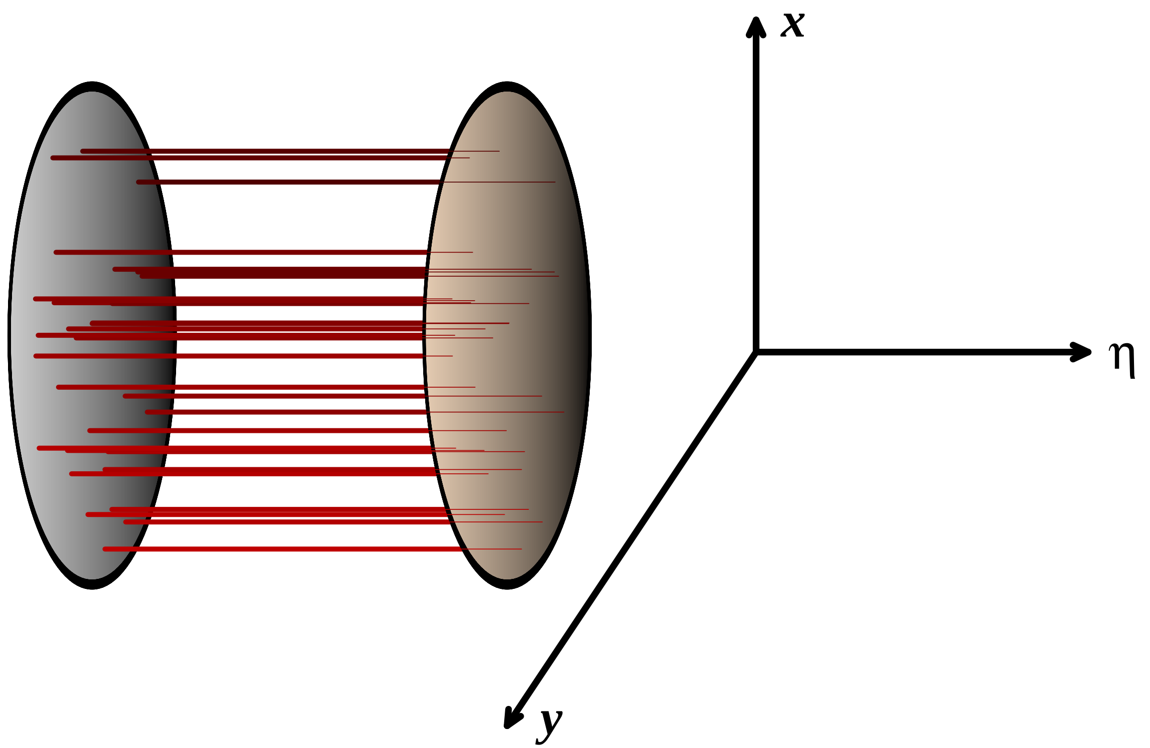
Romatschke, Venugopalan (2005)

- Rapidity dependent perturbations to the classical fields grow like $\exp(\sqrt{Q_s\tau})$ until the non-linearities become important :



Classical field instability

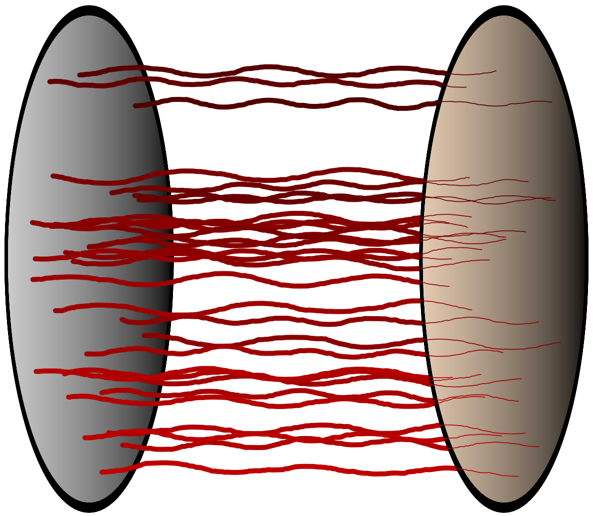
- Leading order magnetic fields at $\tau = 0^+$:



- At $\tau = 0^+$, the classical chromo-electric and chromo-magnetic fields are longitudinal
- They are also boost invariant (independent of η)

Classical field instability

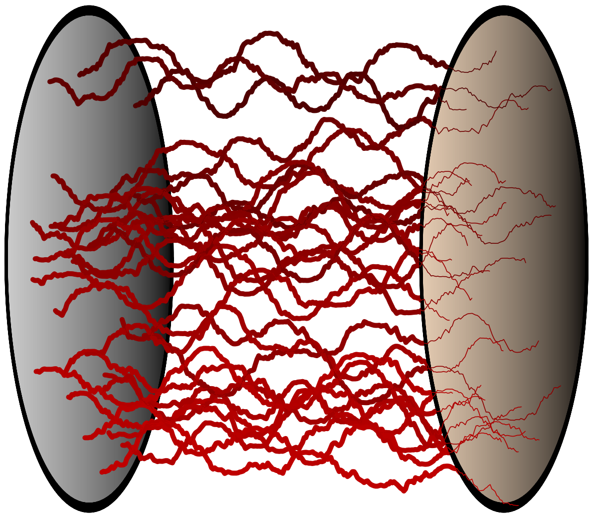
- Leading order + quantum fluctuations at $\tau = 0^+$:



  - Loop corrections bring quantum fluctuations in this picture
  - In the weak coupling regime, they are small corrections

Classical field instability

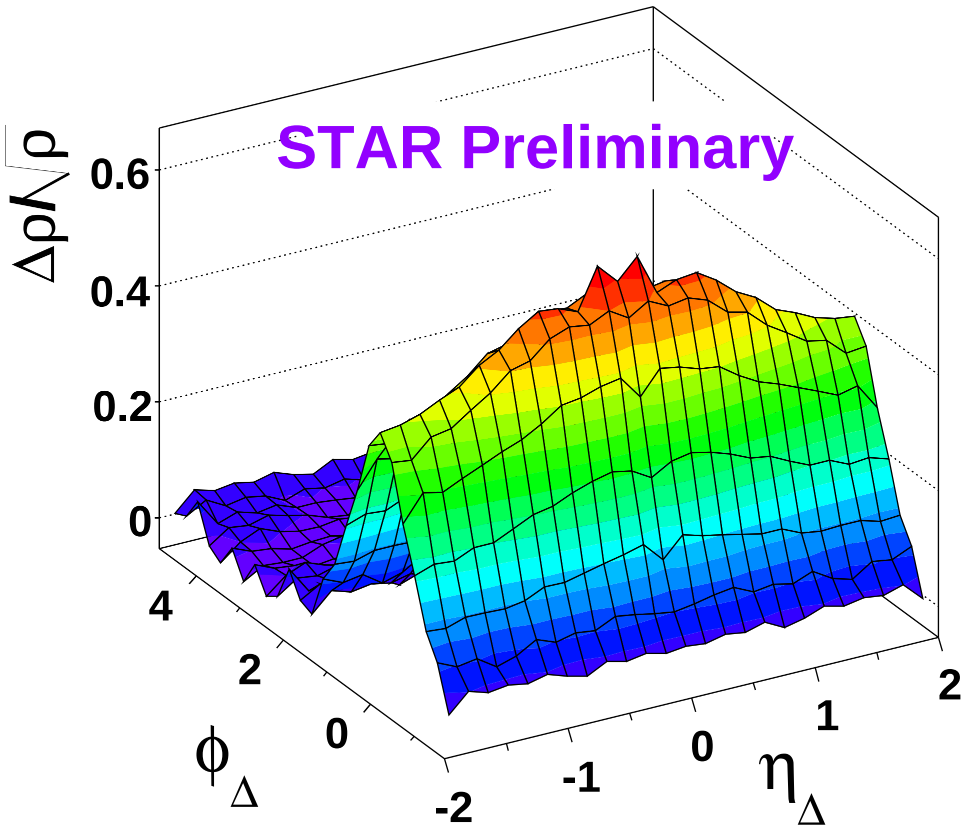
- Effect of the instability :



  - η-dependent perturbations grow exponentially in time
  - Outcome : disordered configurations of color fields

2-hadron correlations at RHIC

- 2-hadron correlation function in heavy ion collisions :

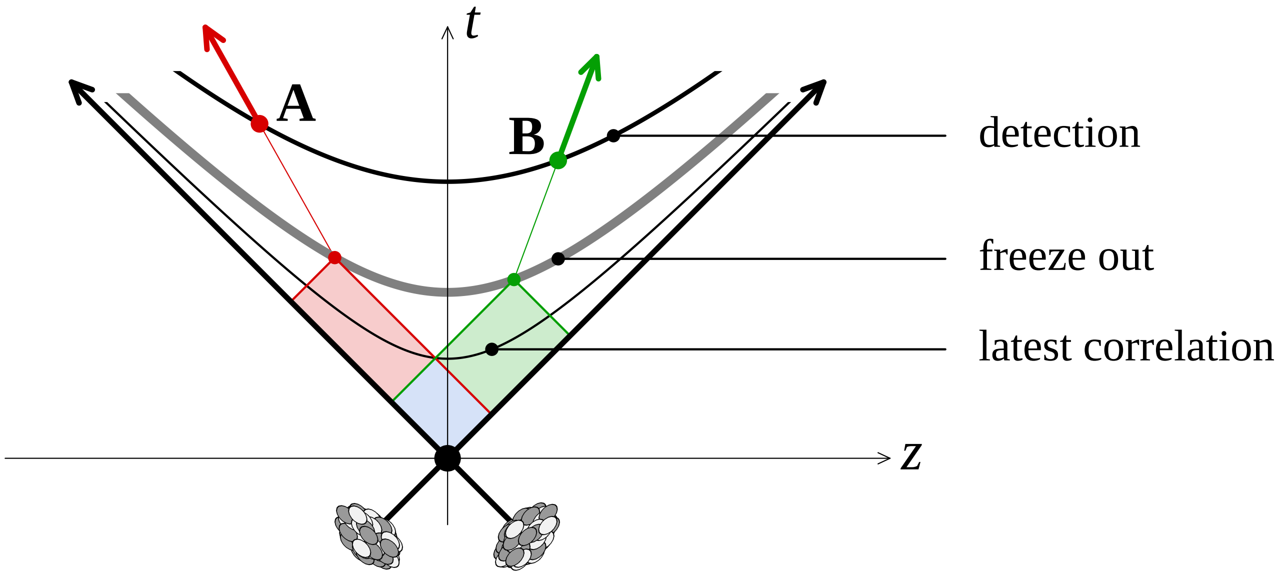


- Narrow correlation in $\Delta\varphi$
- Long range correlation in $\Delta\eta$

Super-horizon correlations

Dumitru, FG, McLerran, Venugopalan (2008)

- Long range correlations in rapidity probe early dynamics :

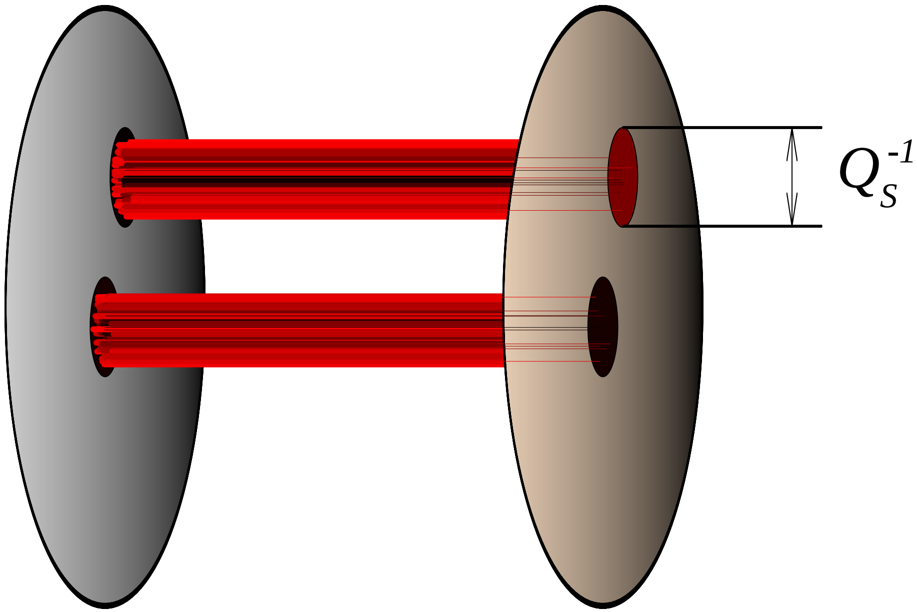


- If a correlation is measured between particles separated by ΔY, the process that caused the correlation must have occured before

$$\tau_{\rm max} = \tau_{\rm freeze\ out}\ \ e^{-\frac{1}{2}|\Delta Y|}$$

CGC interpretation

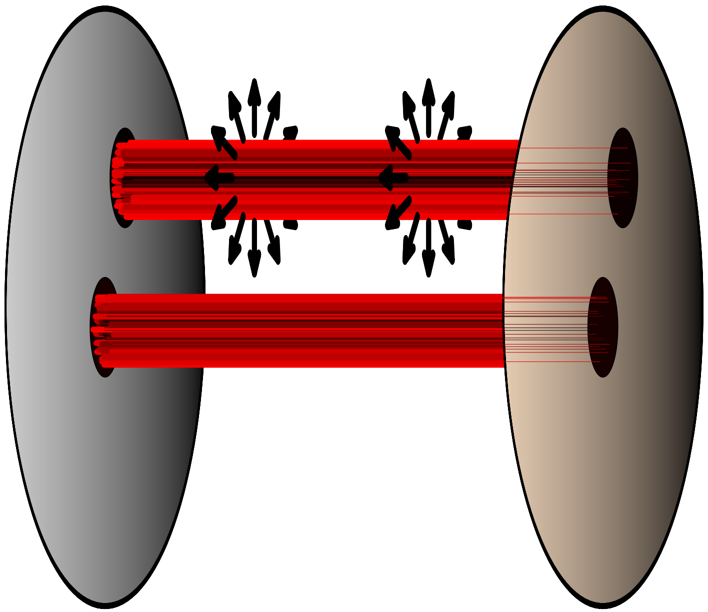
- Was there something independent of η at early times?
  - ▷ the chromo-$\vec{\boldsymbol{E}}$ and $\vec{\boldsymbol{B}}$ fields produced in the collision



- The color correlation length in the transverse plane is Q_s^{-1}
  - ▷ flux tubes of diameter Q_s^{-1}, filling up the transverse area

CGC interpretation

- η-independent fields lead to long range correlations in the 2-particle spectrum :



CGC interpretation

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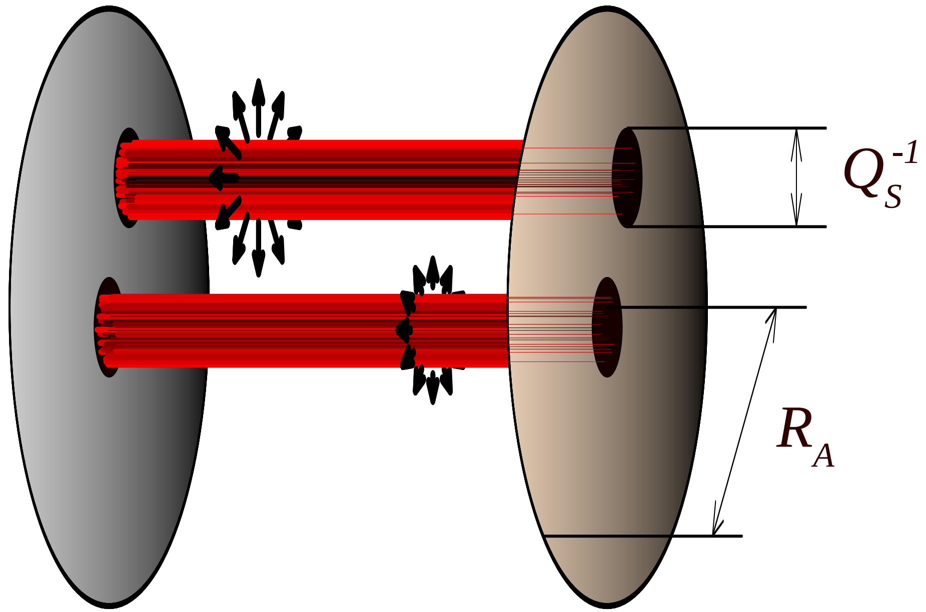

- Particles emitted by different flux tubes are not correlated. Therefore, $(R_A Q_s)^{-2}$ sets the strength of the correlation

CGC interpretation

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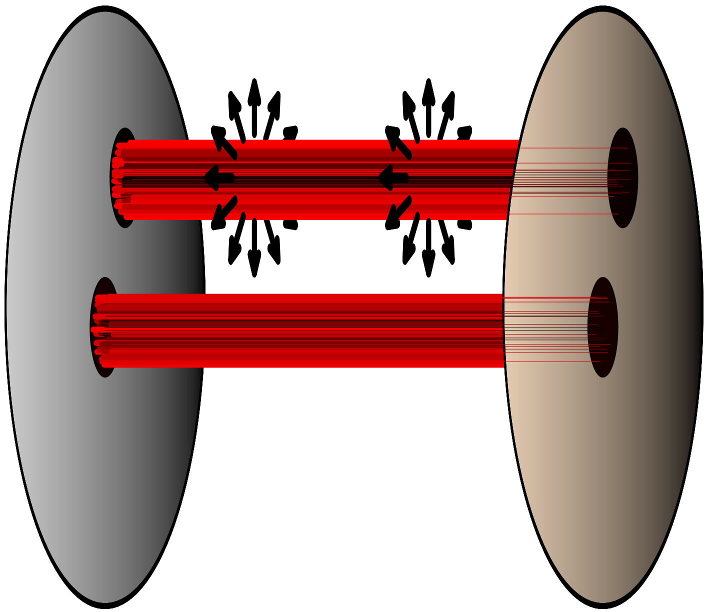

- Particles emitted by different flux tubes are not correlated. Therefore, $(R_A Q_s)^{-2}$ sets the strength of the correlation
- At early times, the correlation is flat in $\Delta\varphi$

CGC interpretation

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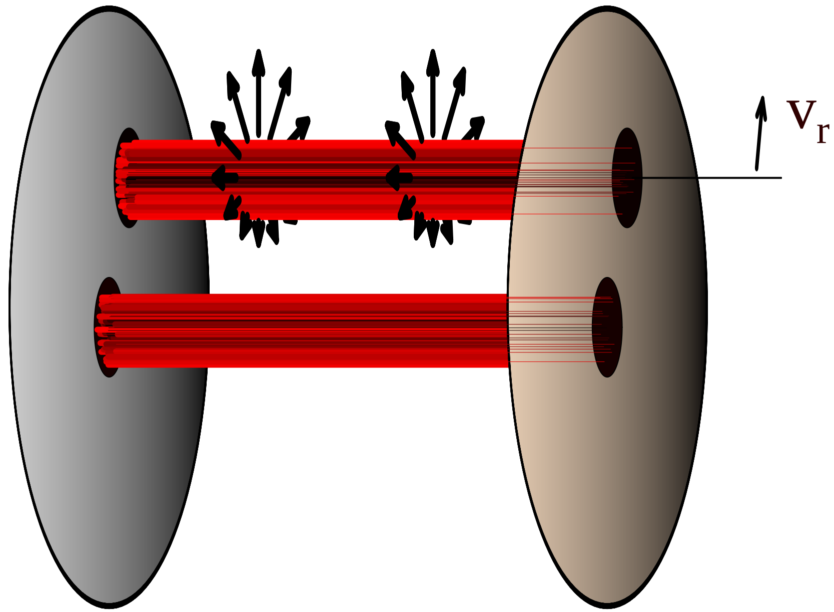
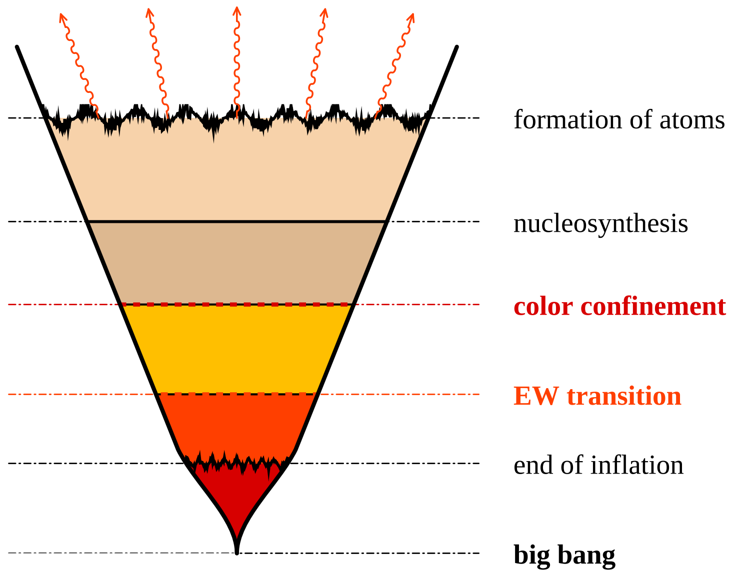


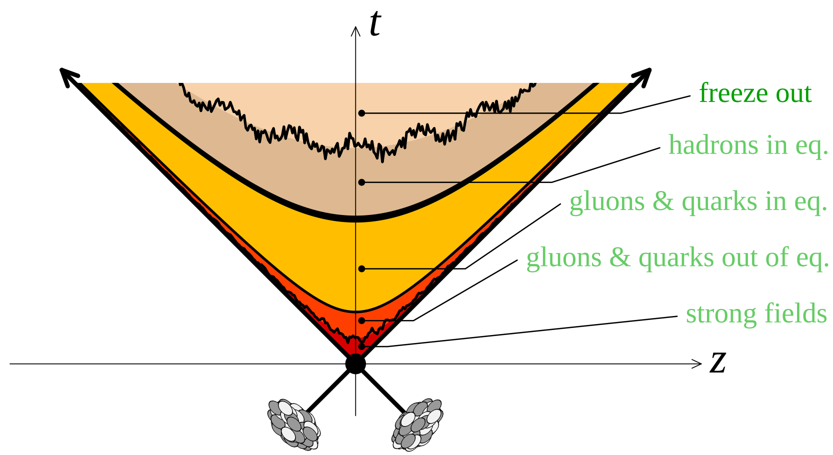
- Particles emitted by different flux tubes are not correlated. Therefore, $(R_A Q_s)^{-2}$ sets the strength of the correlation
- At early times, the correlation is flat in $\Delta\varphi$
  A collimation in $\Delta\varphi$ is produced later by radial flow

Analogies with CMB fluctuations



- Strong inflaton field
- Fluctuations of the inflaton field
- Instability, thermalization
- CMB temperature fluctuations

- Strong classical color field
- Fluctuations of the color field
- Instability, thermalization
- Fluctuations at freeze-out



Summary

Summary

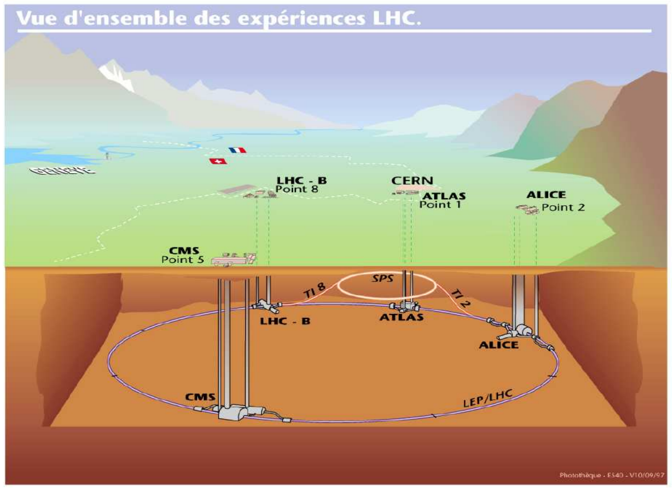
- Many experimental results from RHIC, that suggest :
  - extremely large energy density
  - low viscosity, almost perfect fluid (?)
  - early thermalization (?)
- The uncertainty principle gives a lower bound to η/s
- In the strong coupling limit of gauge theories, η/s is small

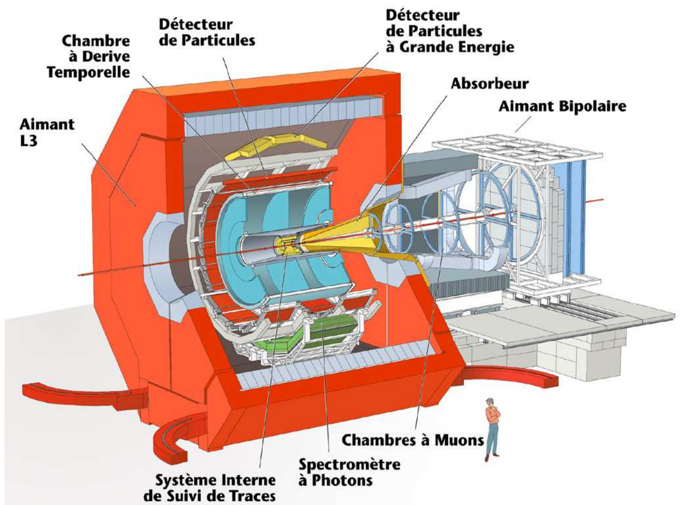
  For super-symmetric Yang-Mills theory, one can compute it explicitly in this limit by using the AdS/CFT correspondence
- One can also have a small η/s if the system has large disordered magnetic fields, even if the coupling is small

  In a weakly coupled QCD approach, field instabilities enhance quantum noise, which leads to such magnetic fields

  The observation of 2-particle correlations in rapidity may shed light on early dynamics

Near future : LHC / ALICE





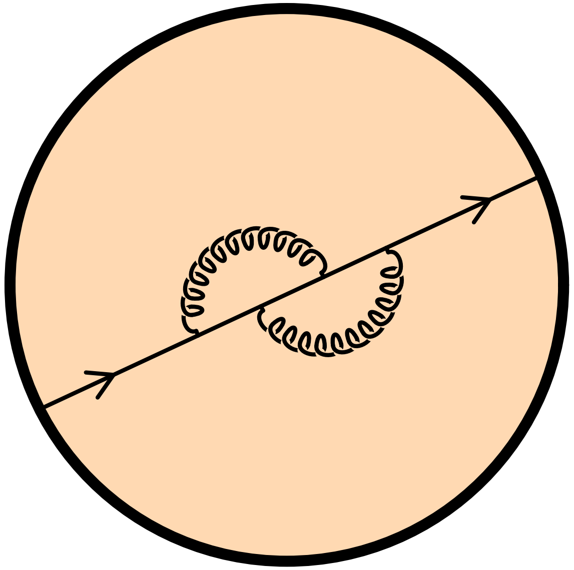
Extra bits

Length scales

- $1/T$: average distance between particles
- $1/gT$: typical distance for collective phenomena
  - Thermal masses of quasi-particles
  - Screening phenomena
  - Damping of plasma waves
- $1/g^2T$: distance between two small angle scatterings
  - Color transport
  - Photon emission
- $1/g^4T$: distance between two large angle scatterings
  - Momentum, electric charge transport
    - ▷ characteristic scale of hydrodynamic modes
- In the weak coupling limit ($g \ll 1$), there is a clear hierarchy between these scales

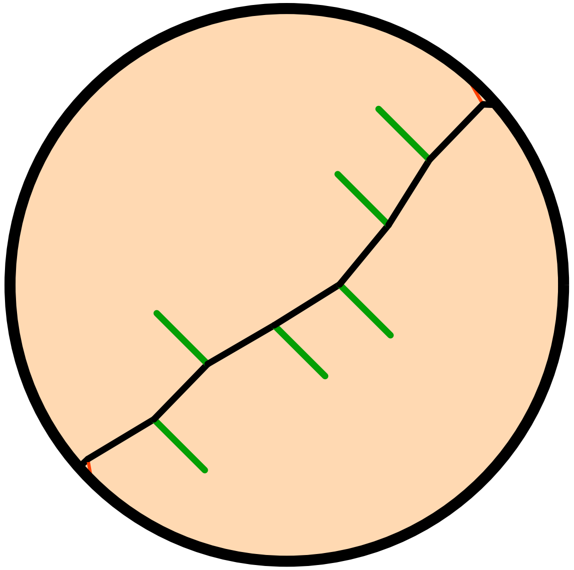
  ▷ Distinct effective descriptions according to the characteristic scale of the problem under study

Vacuum fluctuations



- At distances scales $\ell \lesssim 1/T$, medium effects are irrelevant
- At such scales the dynamics is simply described by the usual QCD in the vacuum

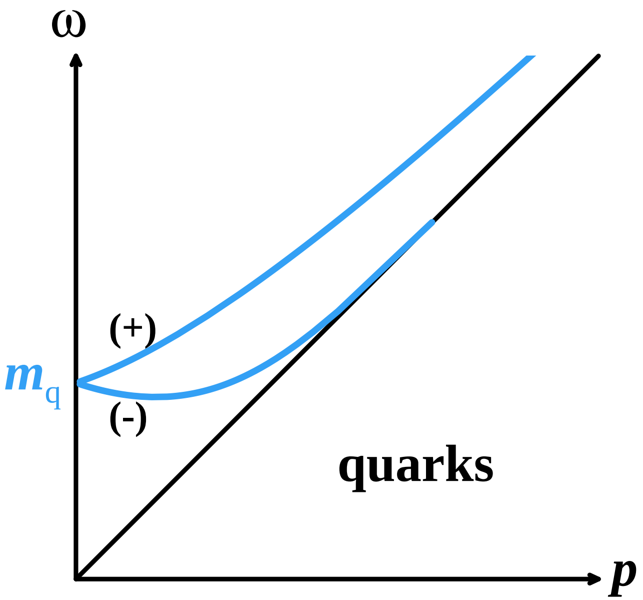
Thermal fluctuations

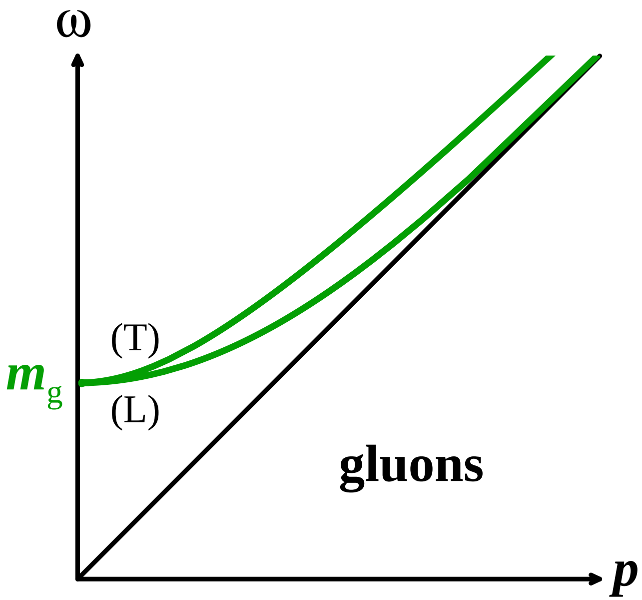


- Distance scales $1/T \lesssim \ell \lesssim 1/gT$ control the bulk thermodynamic properties. The system can be studied by QCD at finite temperature
- The leading thermal effects can be treated by an effective theory that encompasses the main collective effects, and that has the form of a collision-less Vlasov equation

Quasi-particles

- Dispersion curves of particles in the plasma :



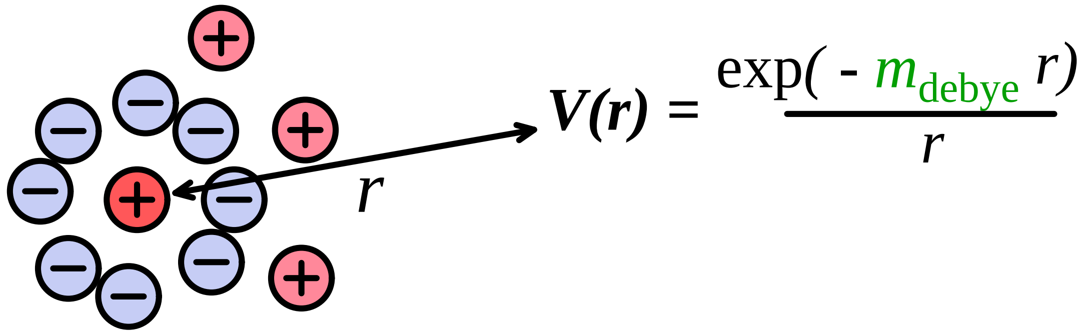


- Thermal masses due to interactions with the other particles in the plasma :

$$m_q \sim m_g \sim gT$$

Debye screening

- A test charge polarizes the particles of the plasma in its vicinity, in order to screen its charge :

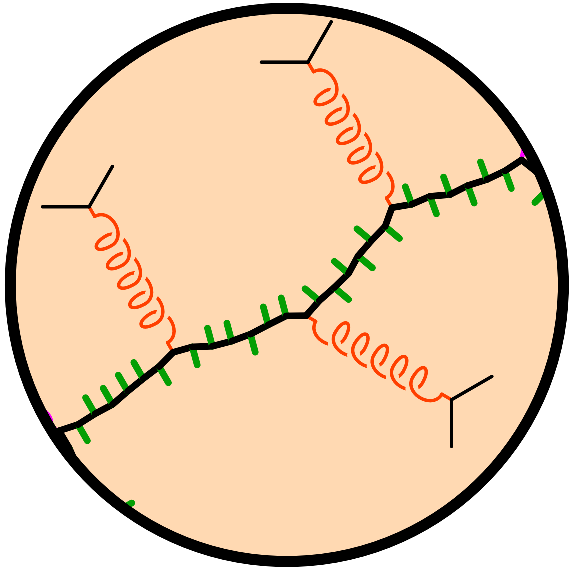


- The Coulomb potential of the test charge decreases exponentially at large distance. The effective interaction range is :

$$\ell \sim 1/m_{\rm debye} \sim 1/gT$$

- Note : leads to a suppression of heavy quark bound states

Small angle scatterings

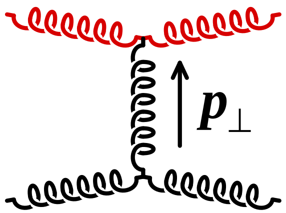


- When we follow a plasma particle over distances $1/g^2T \lesssim \ell$, it is necessary to account for soft (small angle) collisions with other particles of the plasma
- This can be done simply by adding a collision term to the previous Vlasov equation

Collision rate

- Collision rate :

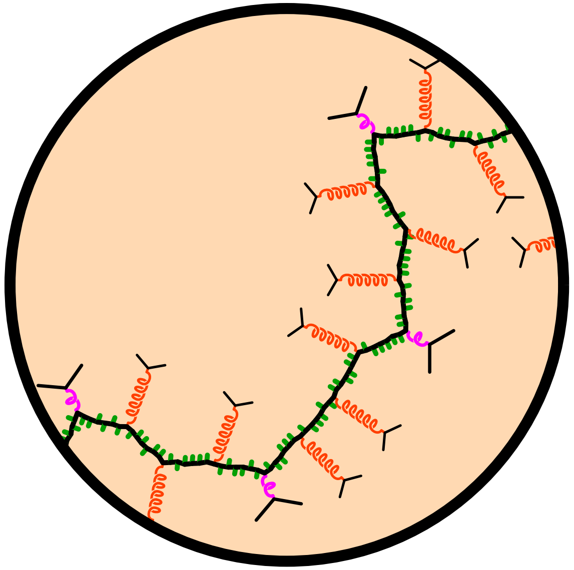
$$\Gamma_{\rm coll} = \left| \quad \right|^2 \sim g^4\, T^3 \int_{m_{\rm debye}} \frac{d^2\vec{\boldsymbol{p}}_\perp}{p_\perp^4} \sim g^2 T$$



- $\lambda \equiv 1/\Gamma_{\rm coll}$ is the mean free path between two small angle scatterings ($\theta \sim g$)

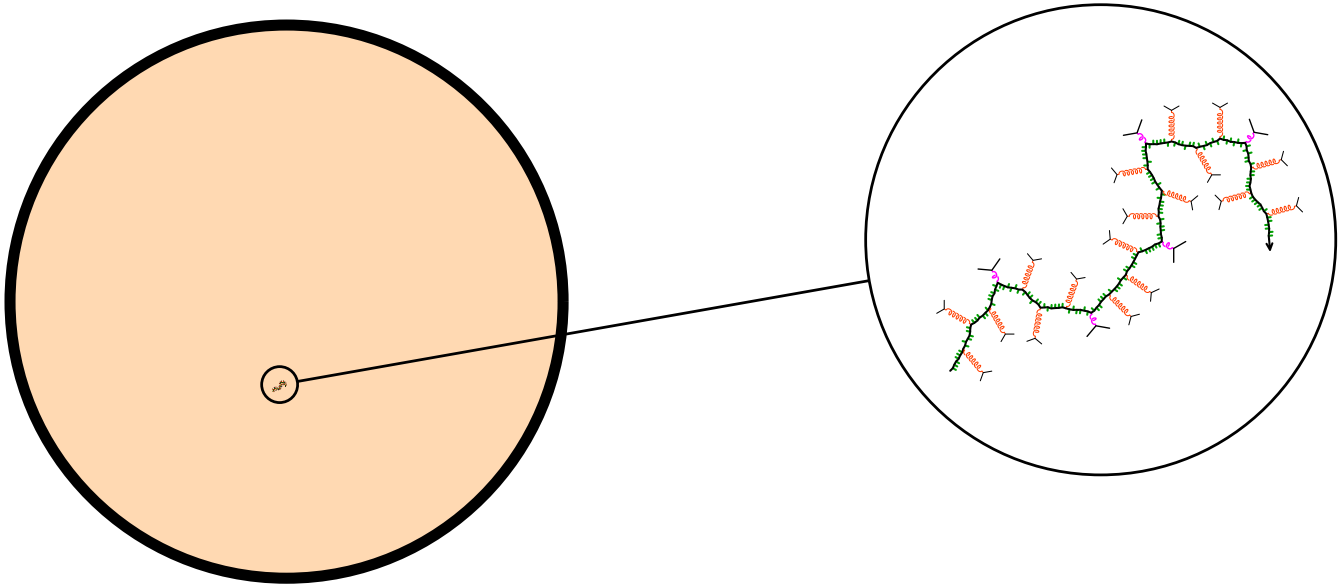
- Note : the mean free path between two large angle scatterings ($\theta \sim 1$) is $\sim 1/g^4 T$

Large angle scatterings



- Over distance scales $\ell \sim 1/g^4 T$, one must take into account the large angle collisions, that change significantly the direction of motion of the particle (this is necessary e.g. for calculating transport coefficients)
- The most efficient way to describe the system over these scales is via a Boltzmann equation for color/spin averaged particle distributions

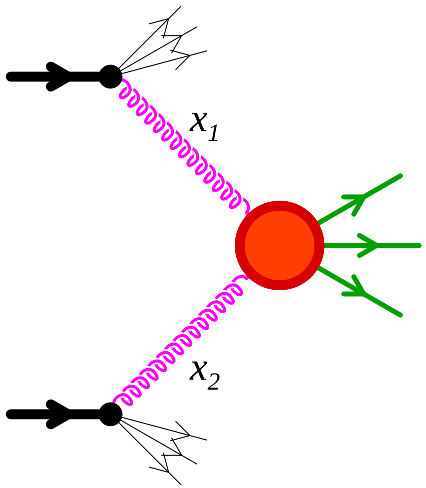
Hydrodynamical regime



- The hydrodynamical regime is reached for length scales that are much larger than the mean free path : $1/g^4T \ll \ell$
- In order to describe the system at such scales, one needs :
  - Hydrodynamical equations (Euler, Navier-Stokes)
  - Conservation equations for the various currents
  - Equation of state, viscosity

Logs of 1/x

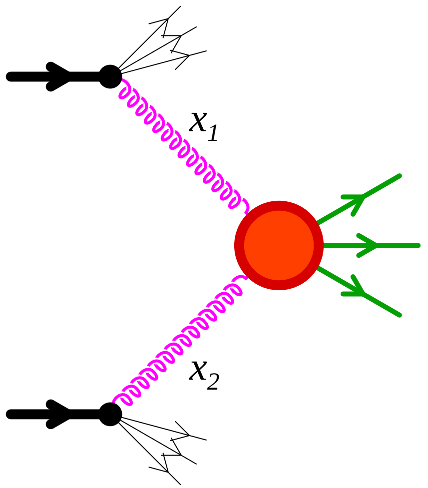
- Calculation of some process at LO :



$$\left.\right\} (M_\perp\,, Y) \qquad \left\{ \begin{aligned} x_1 &= M_\perp\, e^{+Y}/\sqrt{s} \\ x_2 &= M_\perp\, e^{-Y}/\sqrt{s} \end{aligned} \right.$$

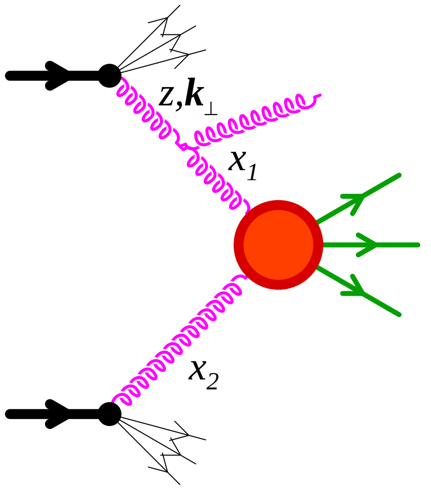
Logs of 1/x

- Calculation of some process at LO :



$$\Big\} \; (M_\perp\,, Y) \qquad \begin{cases} x_1 = M_\perp\, e^{+Y}/\sqrt{s} \\ x_2 = M_\perp\, e^{-Y}/\sqrt{s} \end{cases}$$

- Radiation of an extra gluon in the initial state :



$$\Big\} \; (M_\perp\,, Y) \qquad \Longrightarrow \qquad \alpha_s \int_{x_1} \frac{dz}{z} \int^{M_\perp} \frac{d^2\vec{\boldsymbol{k}}_\perp}{\boldsymbol{k}_\perp^2}$$

Logs of 1/x

- Large $\log(1/x_1)$ when $x_1 \ll 1$

  ▷ this log can compensate the additional α_s, and void the naive application of perturbation theory

  ▷ a resummation is necessary

- When only single scatterings are important, the logs of x_1 are resummed by the BFKL equation :
  - promote the gluon distribution $g(x_1)$ to a non integrated distribution $\varphi(x_1, \vec{\boldsymbol{k}}_\perp)$
  - let $\varphi(x_1, \vec{\boldsymbol{k}}_\perp)$ evolve with x_1 according to the BFKL equation

$$\frac{\partial \varphi(x, k_\perp)}{\partial \ln(1/x)} = \alpha_s \int \frac{d^2\vec{\boldsymbol{p}}_\perp}{(2\pi)^2} K(\vec{\boldsymbol{k}}_\perp, \vec{\boldsymbol{p}}_\perp) \otimes \varphi(x, \vec{\boldsymbol{p}}_\perp)$$

($k_\perp$-factorization)

Multiple scatterings

- Power counting : rescattering corrections are suppressed by inverse powers of the typical mass scale in the process :

$$\left[\frac{\mu^2}{M_\perp^2}\right]^n$$

- The parameter μ^2 has a factor of α_s, and a factor proportional to the gluon density
  ▷ rescatterings are important at high parton density
- Relative order of magnitude :

$$\frac{\text{2 scatterings}}{\text{1 scattering}} \sim \frac{Q_s^2}{M_\perp^2} \quad \text{with} \quad Q_s^2 \sim \alpha_s \frac{xG(x,Q_s^2)}{\pi R^2} \sim A^{1/3}\frac{1}{x^{0.3}}$$

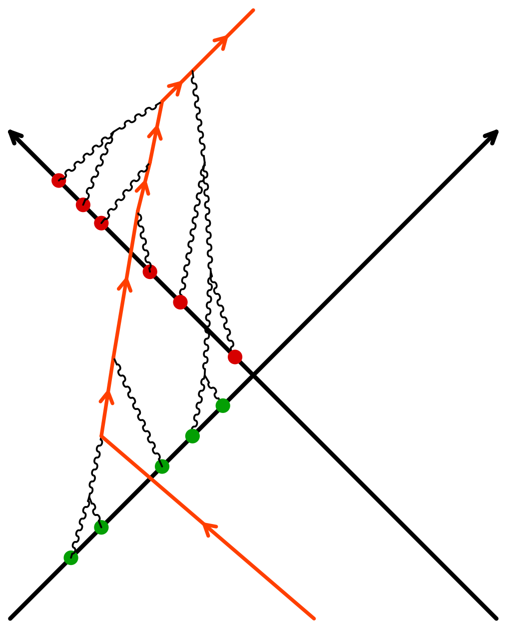
- When this ratio becomes ~ 1, all the rescattering corrections become important ▷ parton saturation
- These effects are not accounted for in BFKL

Quark production

FG, Kajantie, Lappi (2004, 2005)

$$E_{\boldsymbol{p}} \frac{d\langle n_{\text{quarks}} \rangle}{d^3\vec{\boldsymbol{p}}} = \frac{1}{16\pi^3} \int_{x,y} e^{ip\cdot(x-y)}\, \partial\!\!\!/_x \partial\!\!\!/_y \, \langle \overline{\psi}(x)\psi(y) \rangle$$

- Dirac equation in the classical color field :



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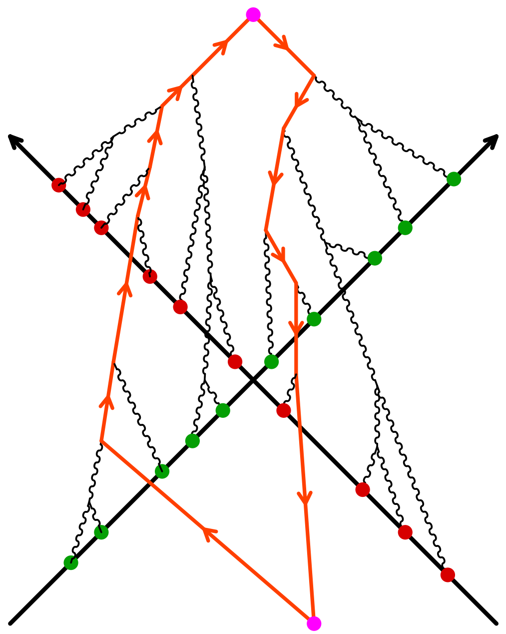
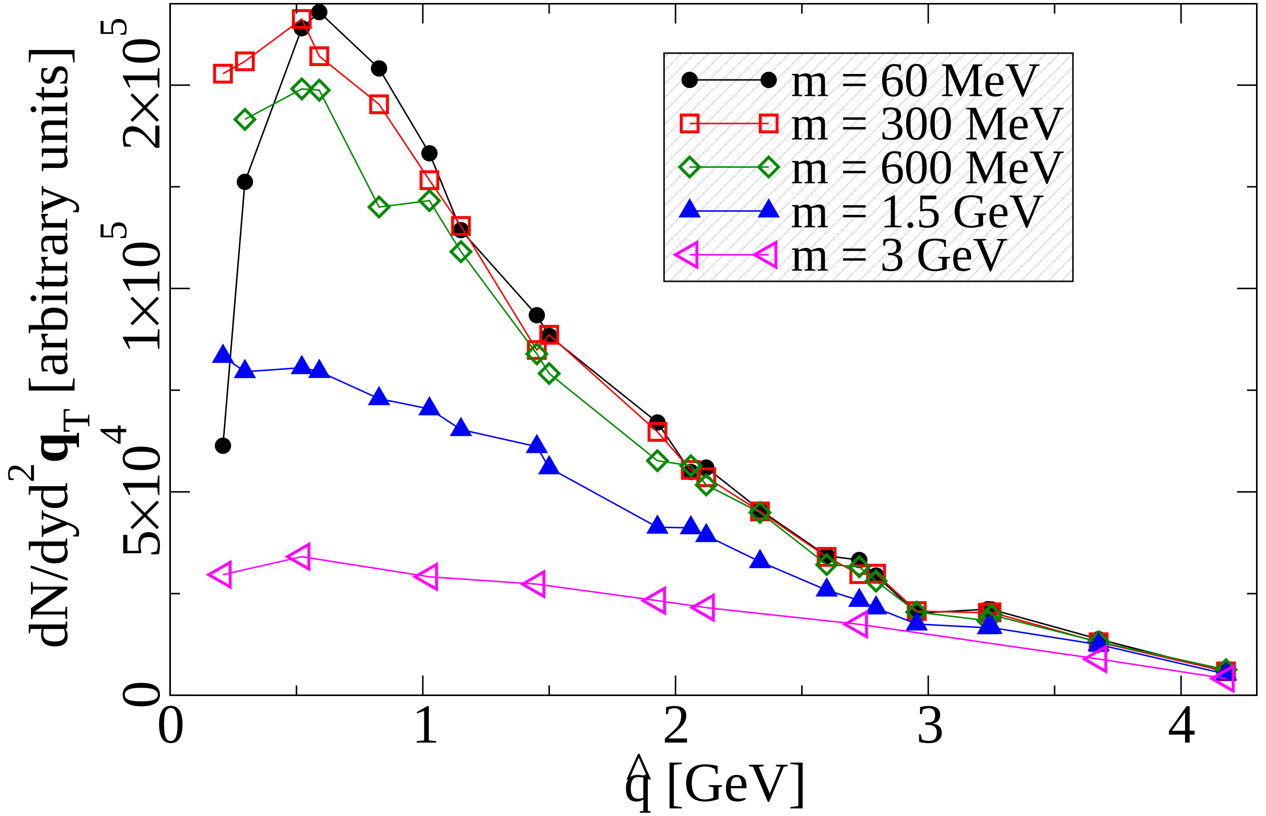
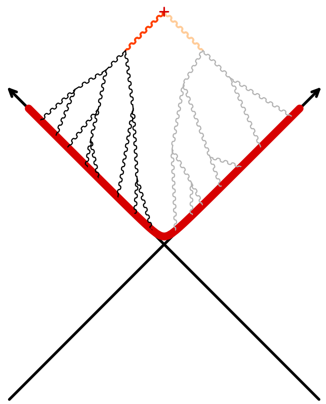


Spectra for various quark masses



1-loop corrections to N

- The 1-loop correction to $\overline{N}$ can be written as a perturbation of the initial value problem encountered at LO :



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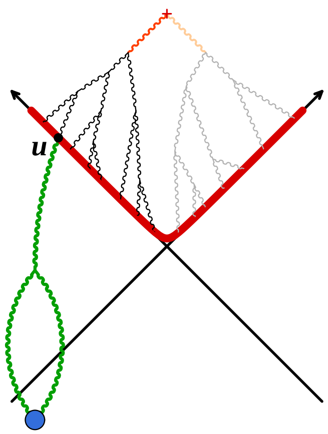

$$\delta\overline{N} = \left[\int_{\vec{u} \in \text{light cone}} \delta\mathcal{A}_{\text{in}}(\vec{u}) \; \boldsymbol{T}_{\vec{u}} \right] \overline{N}_{LO}$$

- $\overline{N}_{LO}$ is a functional of the initial fields $\mathcal{A}_{\text{in}}(\vec{u})$ on the light-cone
- $\boldsymbol{T}_{\vec{u}}$ is the generator of shifts of the initial condition at the point $\vec{u}$ on the light-cone, i.e. : $\boldsymbol{T}_{\vec{u}} \sim \delta/\delta\mathcal{A}_{\text{in}}(\vec{u})$

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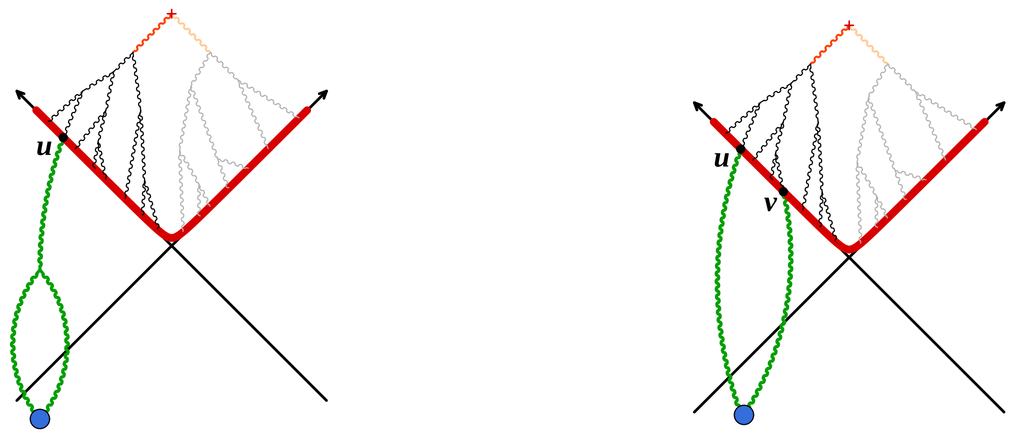

$$\delta\overline{N} = \left[\int_{\vec{u}\in \text{light cone}} \delta\mathcal{A}_{\text{in}}(\vec{u})\ \boldsymbol{T}_{\vec{u}} + \int_{\vec{u},\vec{v}\in \text{light cone}} \frac{1}{2}\ \boldsymbol{\Sigma}(\vec{u},\vec{v})\ \boldsymbol{T}_{\vec{u}}\,\boldsymbol{T}_{\vec{v}} \right] \overline{N}_{LO}$$

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- $\delta\mathcal{A}_{\text{in}}(\vec{u})$ and $\boldsymbol{\Sigma}(\vec{u},\vec{v})$ are in principle calculable analytically

Divergences

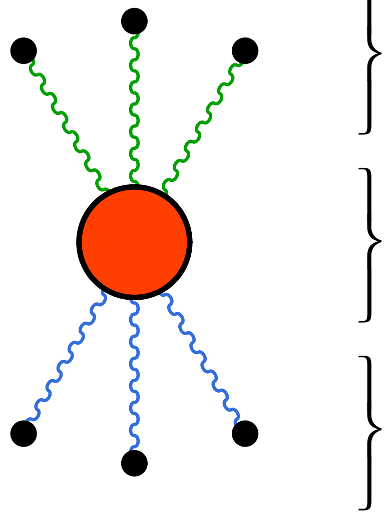
- If taken at face value, this 1-loop correction is plagued by several divergences :
  - The two coefficients $\delta\mathcal{A}_{\rm in}(\vec{\boldsymbol{x}})$ and $\boldsymbol{\Sigma}(\vec{\boldsymbol{x}},\vec{\boldsymbol{y}})$ are infinite, because of an unbounded integration over a rapidity variable
  - At late times, $\boldsymbol{T}_{\vec{\boldsymbol{x}}}\mathcal{A}(\tau,\vec{\boldsymbol{y}})$ diverges exponentially,

$$\boldsymbol{T}_{\vec{\boldsymbol{x}}}\mathcal{A}(\tau,\vec{\boldsymbol{y}}) \underset{\tau\to+\infty}{\sim} e^{\sqrt{Q_s\tau}}$$

because of an instability of the classical solution of Yang-Mills equations under rapidity dependent perturbations (Romatschke, Venugopalan (2005))

Initial state factorization

- Anatomy of the full calculation :



$$\mathbf{W}_{\mathbf{Y}_{\mathrm{beam}}\mathbf{-Y}}[\rho_1]$$

$$\mathbf{N}[\ \mathbf{A}_{\mathrm{in}}(\rho_1\ ,\ \rho_2)\ ]$$

$$\mathbf{W}_{\mathbf{Y}_{\mathrm{beam}}\mathbf{+Y}}[\rho_2]$$

Initial state factorization

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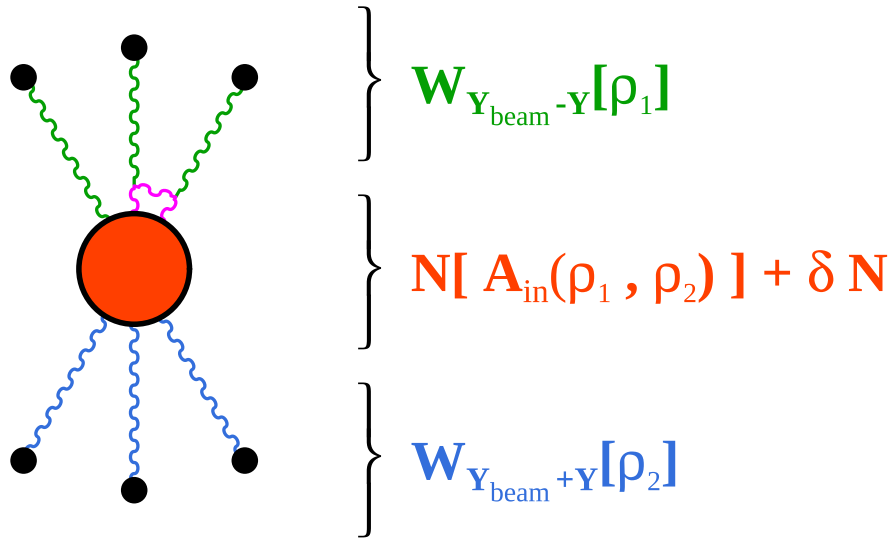

- When the observable $\overline{N}[\mathcal{A}_{\rm in}(\rho_1, \rho_2)]$ is corrected by an extra gluon, one gets divergences of the form $\alpha_s \int dY$ in $\delta\overline{N}$
  ▷ one would like to be able to absorb these divergences into the Y dependence of the source densities $W_Y[\rho_{1,2}]$

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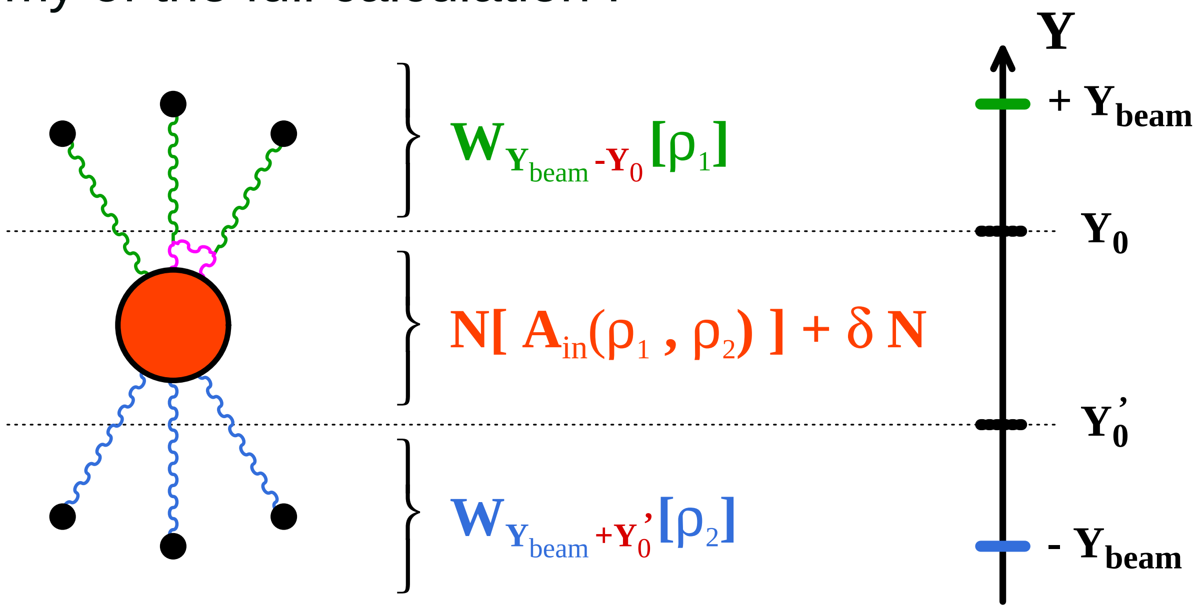

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  ▷ one would like to be able to absorb these divergences into the Y dependence of the source densities $W_Y[\rho_{1,2}]$
- Equivalently, if one puts some arbitrary frontier Y_0 between the “observable” and the “source distributions”, the dependence on Y_0 should cancel between the various factors

Initial state factorization

- The two kind of divergences don't mix, because the divergent part of the coefficients is boost invariant.

  Given their structure, the divergent coefficients seem related to the evolution of the sources in the initial state

- In order to prove the factorization of these divergences in the initial state distributions of sources, one needs to establish :

$$\left[\delta\overline{N}\right]_{\substack{\text{divergent}\\\text{coefficients}}} = \left[(Y_0 - Y)\,\mathcal{H}^\dagger[\rho_1] + (Y - Y_0')\,\mathcal{H}^\dagger[\rho_2]\right]\ \overline{N}_{LO}$$

where $\mathcal{H}[\rho]$ is the Hamiltonian that governs the rapidity dependence of the source distribution $W_Y[\rho]$:

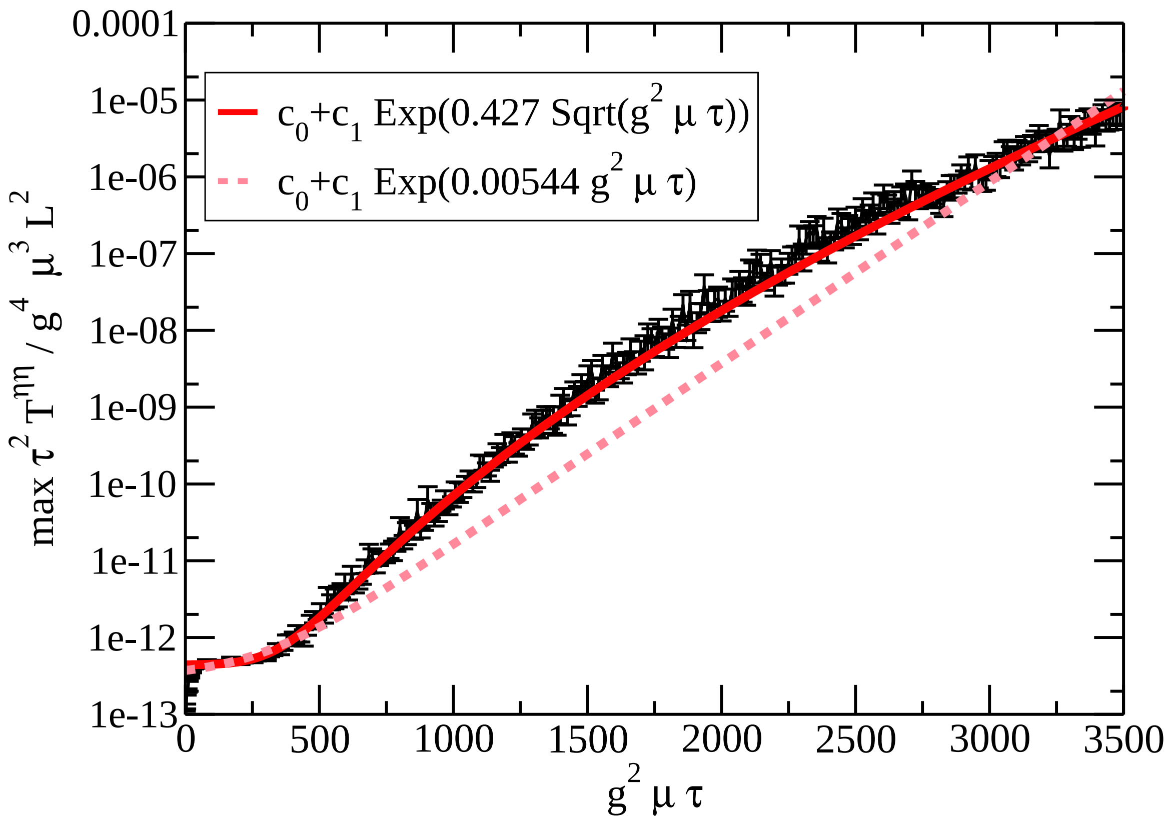
$$\frac{\partial W_Y[\rho]}{\partial Y} = \mathcal{H}[\rho]\ W_Y[\rho]$$

FG, Lappi, Venugopalan (work in progress)

Unstable modes

Romatschke, Venugopalan (2005)

- Rapidity dependent perturbations to the classical fields grow like $\exp(\#\sqrt{\tau})$ until the non-linearities become important :



Unstable modes

- The coefficient $\delta\mathcal{A}_{\rm in}(\vec{\boldsymbol{x}})$ is boost invariant.
  Hence, the divergences due to the unstable modes all come from the quadratic term in $\delta\overline{N}$:

$$\Big[\delta\overline{N}\Big]_{\substack{\text{unstable}\\ \text{modes}}} = \left\{\frac{1}{2}\int_{\vec{\boldsymbol{x}},\vec{\boldsymbol{y}}} \boldsymbol{\Sigma}(\vec{\boldsymbol{x}},\vec{\boldsymbol{y}})\,\boldsymbol{T}_{\vec{\boldsymbol{x}}}\boldsymbol{T}_{\vec{\boldsymbol{y}}}\right\}\overline{N}_{LO}[\mathcal{A}_{\rm in}(\rho_1,\rho_2)]$$

- When summed to all orders, this becomes a certain functional $Z[\boldsymbol{T}_{\vec{\boldsymbol{x}}}]$:

$$\Big[\delta\overline{N}\Big]_{\substack{\text{unstable}\\ \text{modes}}} = Z[\boldsymbol{T}_{\vec{\boldsymbol{x}}}]\;\overline{N}_{LO}[\mathcal{A}_{\rm in}(\rho_1,\rho_2)]$$

Unstable modes

- This can be arranged in a more intuitive way :

$$\Big[\delta\overline{N}\Big]_{\substack{\text{unstable}\\ \text{modes}}} = \int [Da]\ \widetilde{Z}[a(\vec{x})]\ e^{i\int_{\vec{x}} a(\vec{x})\,T_{\vec{x}}}\ \overline{N}_{LO}[\mathcal{A}_{\rm in}(\rho_1,\rho_2)]$$

$$= \int [Da]\ \widetilde{Z}[a(\vec{x})]\ \overline{N}_{LO}[\mathcal{A}_{\rm in}(\rho_1,\rho_2)+a]$$

▷ summing these divergences simply requires to add fluctuations to the initial condition for the classical problem
▷ the fact that $\delta\mathcal{A}_{\rm in}(\vec{x})$ does not contribute implies that the distribution of fluctuations is real

- Interpretation :

Despite the fact that the fields are coupled to strong sources, the classical approximation alone is not good enough, because the classical solution has unstable modes that can be triggered by the quantum fluctuations

Unstable modes

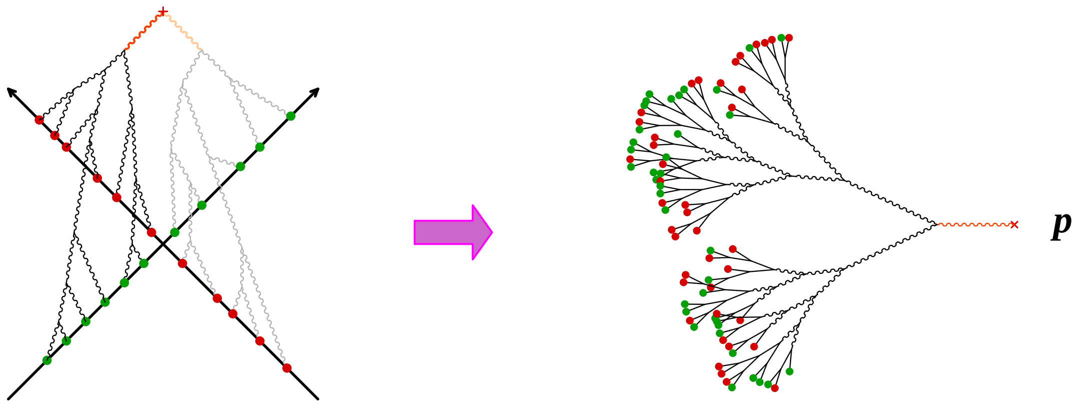
- Combining everything, one should write

$$\frac{d\overline{N}}{dY\,d^2\vec{\boldsymbol{p}}_\perp} = \int [D\rho_1]\,[D\rho_2]\; W_{Y_{\rm beam}-Y}[\rho_1]\, W_{Y_{\rm beam}+Y}[\rho_2] \times \int [Da]\;\; \widetilde{Z}[a]\; \frac{d\overline{N}[\mathcal{A}_{\rm in}(\rho_1,\rho_2)+a]}{dY\,d^2\vec{\boldsymbol{p}}_\perp}$$

▷ This formula resums (all?) the divergences that occur at one loop, both in the initial and final state

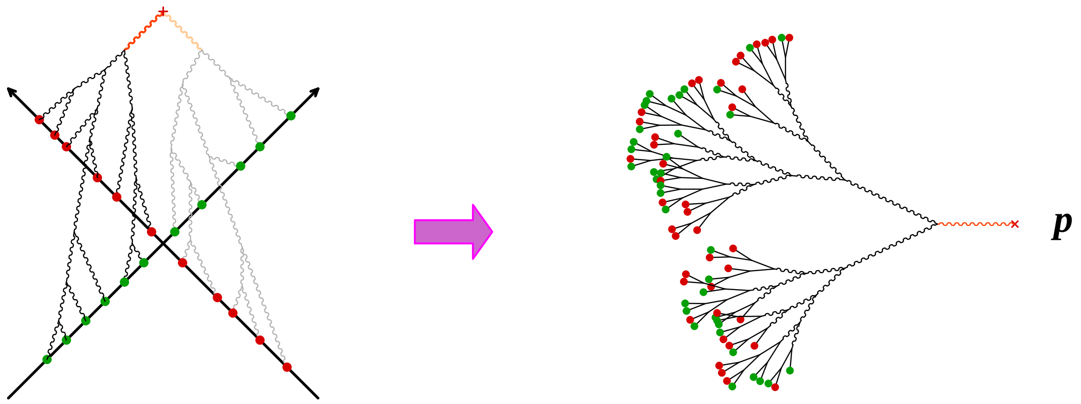
Unstable modes

- Tree level :

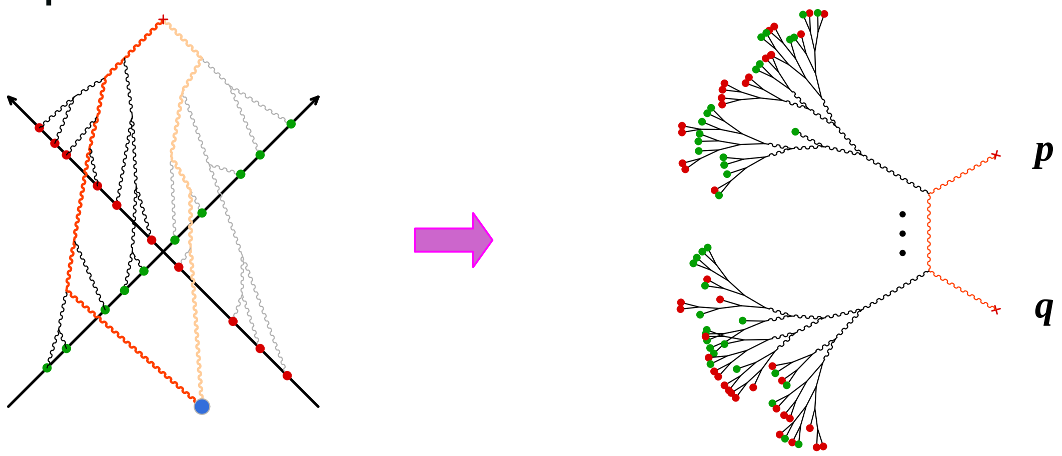


Unstable modes

- Tree level :



- One loop :



▷ The momentum $\vec{q}$ is integrated out

▷ If $\alpha_s^{-1} \lesssim \left|y_{\boldsymbol{p}} - y_{\boldsymbol{q}}\right|$, the correction is absorbed in $W[\rho_{1,2}]$

▷ If $\left|y_{\boldsymbol{p}} - y_{\boldsymbol{q}}\right| \lesssim \alpha_s^{-1}$: gluon splitting in the final state

Digressions on v4

- It can be shown that if ideal hydrodynamics applies, then the fourth harmonic coefficient should be $v_4 = v_2^2/2$ at large $p_\perp$ (Borghini, Ollitrault (2005))
- However, at RHIC, the measured v_4 is about twice larger than this prediction, which seems to suggest that there are less collisions per particle than expected. This would imply :
  - larger mean free path
  - smaller cross-sections
  - higher viscosity and less perfect thermalization
- How to accommodate the fact that ideal hydrodynamics reproduces well the measured v_2 ?
  - Viscous hydrodynamics could lead to the correct v_2 provided one starts from a slightly higher initial eccentricity
  - Relativistic viscous hydro codes are being developed, and this possibility will soon be assessed quantitatively...