Local maxima of white noise spectrograms and interlacing of zeros in higher Landau Levels

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Der Wissenschaftsfonds.

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3-Landau Levels: $\sigma(L_z) = \{n : n \in \mathbb{N}_0\}$

$$L_z := -\partial_z \partial_{\overline{z}} + \overline{z} \partial_{\overline{z}} = -\nabla_z^+ \nabla_z^-$$

- Eigenspaces with reproducing kernel:

$$K_n(z, w) = L_n(|z - w|^2) e^{z\overline{w}}$$

Critical points of white noise spectrograms

- Gaussian Entire Function:

$$f_0(z) = \sum_{k=0}^{\infty} \zeta_k \frac{z^k}{\sqrt{k!}}$$

- Spectrogram of white noise (Bardenet, Flamant, Chainais)

$$Spec_{h_0}\mathcal{N}(z) = \left| V_{h_0}\mathcal{N}(\frac{\bar{z}}{\pi}) \right|^2 = \left| e^{-|z|^2/2} f_0(z) \right|^2$$

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$$0 = \nabla \log \left| f_0(z) e^{-\frac{|z|^2}{2}} \right| = \frac{\partial_z f_0(z)}{f_0(z)} + \frac{\partial_{\overline{z}} f_0(z)}{f_0(z)} - \overline{z} = \frac{\partial_z f_0(z)}{f_0(z)} - \overline{z}$$

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- Critical points equation (LL raising operator)

$$\partial_z f_0(z) - \overline{z} f_0(z) = 0 \Longleftrightarrow \nabla_z^+ f_0(z) = 0$$

Notation for 1-point intensities

- 1-point intensities: expected number of points in $\Omega \subset \mathbb{C}$:

$$\mathbb{E}\left(\mathcal{X}\left(\Omega\right)\right) = \int_{\Omega} \rho^{(1)}(z) \frac{dz}{\pi}.$$

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- Example: zeros of Gaussian Entire Function

$$\rho^{(1)}_{zeros,0}(z) = 1$$

Local maxima density

Theorem (A., 2020)

$\rho^{(1)}_{critical,0}(z) = 5/3$, $\rho^{(1)}_{loc\max,0}(z) = 1/3$ *and* $\rho^{(1)}_{saddle,0}(z) = 4/3$.

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Proof idea At a critical point z_c,

$$|f_0(z_c)|^2 \det D^{Hess}(\log \left| f_0(z)e^{-\frac{|z|^2}{2}} \right|^2)(z_c) = Jac(\nabla_z^+ f_0)(z_c)$$

- $Jac(\nabla_z^+ f_0)(z_c) > 0 \Rightarrow z_c$ maximum (minima are zeros)
- $Jac(\nabla_z^+ f_0)(z_c) < 0 \Rightarrow z_c$ saddle point
- Kac-Rice formulas:

$$\rho^{(1)}_{loc\max,0}(z) = \mathbb{E}(\left| Jac(\nabla_z^+ f_0) : Jac(\nabla_z^+ f_0) > 0 \right| \delta_0(\nabla_z^+ f_0)) = \frac{1}{3}$$

$$\rho^{(1)}_{saddle,0}(z) = \mathbb{E}(\left| Jac(\nabla_z^+ f_0) : Jac(\nabla_z^+ f_0) < 0 \right| \delta_0(\nabla_z^+ f_0)) = \frac{4}{3}$$

Flandrin's honeycomb

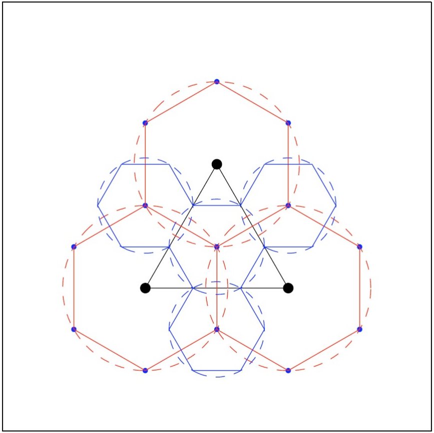

Figure: Flandrin's honeycomb: the area of the hexagons centered at local maxima (big dots) is three times the area of the hexagons centered at zeros (small dots)

Gaussian function in first Landau Level

- Gaussian Entire Function:

$$f_0(z) = \sum_{k=0}^{\infty} \frac{\zeta_k}{\sqrt{k!}} z^k$$

- Gaussian function in first Landau Level:

$$f_1(z, \overline{z}) = \nabla_z^+ f_0(z) = \sum_{k=0}^{\infty} \zeta_k \frac{k z^{k-1} - z^k \overline{z}}{\sqrt{k!}}$$

- Zeros:

$$\rho^{(1)}_{zeros,1}(z) = \rho^{(1)}_{critical,0}(z) = \frac{5}{3}$$

- Relation with spectrogram of white noise:

$$Spec_{h_1}\mathcal{N}(z) = \left| V_{h_1}\mathcal{N}(\frac{\bar{z}}{\pi}) \right|^2 = \left| e^{-|z|^2/2} f_1(z, \overline{z}) \right|^2$$

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- Interlacing property of local maxima

Maxima of $Spec_{h_0}\mathcal{N}(z)$ *separate maxima of* $Spec_{h_1}\mathcal{N}(z)$

Gaussian functions in higher Landau Levels

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$$f_0(z) = \sum_{k=0}^{\infty} \zeta_k \frac{z^k}{\sqrt{k!}}$$

- Gaussian (Polyanalytic) Function in higher Landau levels:

$$f_n(z, \overline{z}) := \frac{1}{\sqrt{n!}} (\nabla_z^+)^n f_0(z) = \sum_{k=0}^{\infty} \zeta_k \frac{(\nabla_z^+)^n z^k}{\sqrt{n!k!}}$$

- Orthogonal Gaussian functions:

$$\int_{\mathbb{C}} f_n(z, \overline{z}) \overline{f_{n'}(z, \overline{z})} e^{-|z|^2} dz = \delta_{nn'}.$$

- Correlation kernels

$$K_n(z, w) = L_n(|z - w|^2) e^{z\overline{w}}$$

Complex Hermite Polynomials

- Gaussian (Polyanalytic) Function in higher Landau levels:

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- $H_{k,n}(z,\overline{z})$ are the complex Hermite polynomials:

$$H_{k,n}(z,\overline{z}) = \begin{cases} \sqrt{\frac{n!}{k!}} z^{k-n} L_n^{k-n}\left(|z|^2\right), & k > n \geq 0, \\ (-1)^{n-k}\sqrt{\frac{k!}{n!}} \overline{z}^{n-k} L_k^{n-k}\left(|z|^2\right), & 0 \leq k \leq n, \end{cases}$$

- Doubly-indexed orthogonality relation

$$\int_{\mathbb{C}} H_{k,n}(z,\overline{z})\overline{H_{k',n'}(z,\overline{z})}e^{-|z|^2}dz = \delta_{kk'}\delta_{nn'}$$

- Orthogonality in concentric discs

$$\int_{D(0,R)} H_{k,n}(z,\overline{z})\overline{H_{k',n}(z,\overline{z})}e^{-|z|^2}dz = C_{n,k,k'}(R)\delta_{kk'}$$

Zeros in higher Landau Levels

- Recall:

$$\nabla_z^+ f_0(z) = f_1(z, \overline{z}) \implies \rho^{(1)}_{zeros,1}(z) = \rho^{(1)}_{critical,0}(z) = \frac{5}{3}$$

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- Gaussian Weyl-Heisenberg function

Theorem (Haimi, Koliander, Romero)

$\rho^{(1)}_{zeros,n}(z) = n + 1/2 + \frac{1}{4n+2}$

Critical points in higher Landau Levels

Theorem (A., Shirai, 2022)

$$\rho^{(1)}_{critical,n}(z) \quad = \quad \frac{1}{2}(2n^2 + 4n + 3 + \frac{1}{2n^2 + 4n + 3})$$

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\rho^{(1)}_{critical,n}(z) &= \frac{1}{2}(2n^2+4n+3+\frac{1}{2n^2+4n+3}) \\
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Proof Sketch Critical points equation

$$0=\nabla\log\left|f_n(z,\overline{z})e^{-\frac{|z|^2}{2}}\right|=\frac{\partial_z f_n(z,\overline{z})}{f_n(z,\overline{z})}+\frac{\partial_{\overline{z}} f_n(z,\overline{z})}{f_n(z,\overline{z})}-\overline{z}$$

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- STFT of white noise (Gaussian Weyl-Heisenberg function)

$$\nabla_z^+ f_n(z,\overline{z}) + \nabla_z^- f_n(z,\overline{z}) = e^{ix\xi}e^{-|z|^2/2}V_{\sqrt{n}h_{n-1}-\sqrt{n+1}h_{n+1}}\mathcal{N}\left(\frac{\bar{z}}{\sqrt{\pi}}\right)$$

Bayran-Baraniuk (1996): Multitaper estimator

- Average of Hermite spectrograms of single realization of $\mathcal{S}$

$$\frac{1}{N}\sum_{n=0}^{N-1} Spec_{h_n}\mathcal{S}(z) = \frac{1}{N}\sum_{n=0}^{N-1}\left|V_{h_n}\mathcal{S}(\frac{\bar{z}}{\pi})\right|^2$$

- White noise case (conjecture):

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- Reminiscent of Pythagoras theorem

$$\sin^2 x + \cos^2 x = 1$$

Pythagoras theorem

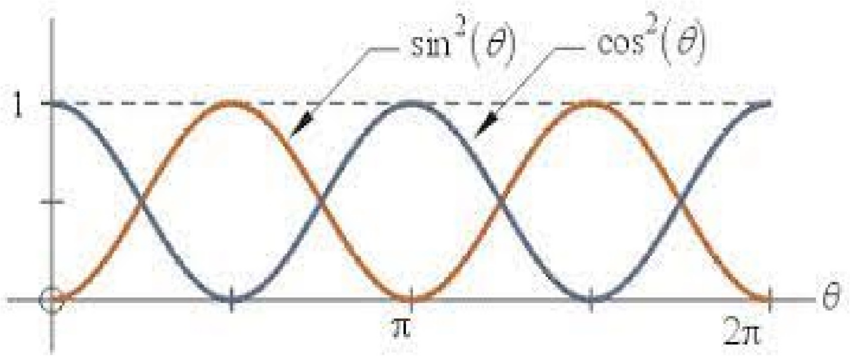

Figure: Energy allocation of sin^2x in the line complements energy allocation of cos^2x; Maxima and zeros of $\sin x$ and $\cos x$ are interlaced.

Pythagoras theorem

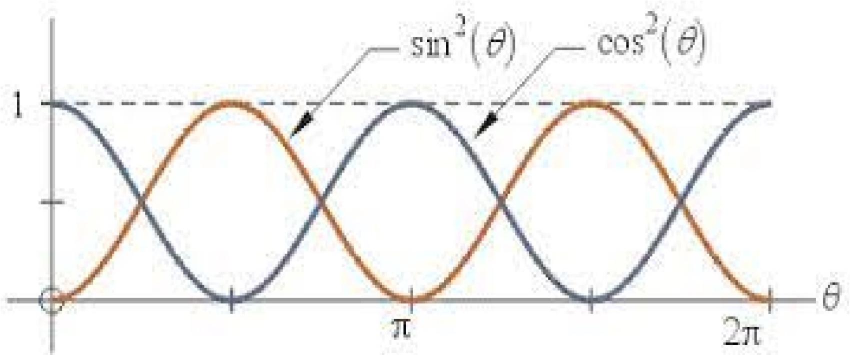

Figure: Energy allocation of $sin^2 x$ in the line complements energy allocation of $cos^2 x$; Maxima and zeros of $\sin x$ and $\cos x$ are interlaced.

- Do maxima and zeros of $Spec_{h_n}\mathcal{N}(z)$ and $Spec_{h_{n+k}}\mathcal{N}(z)$ tend to interlace?

Interlacing of Hermite spectrogram energy

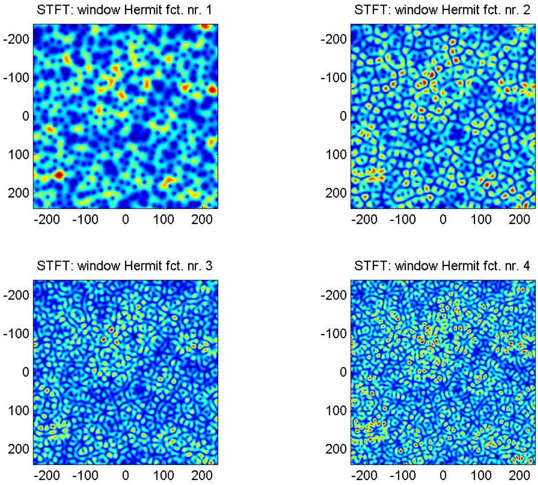

Figure: Orange: high energy (loud); Dark blue: low energy (silent)

Hermite spectrogram maxima (Flandrin)

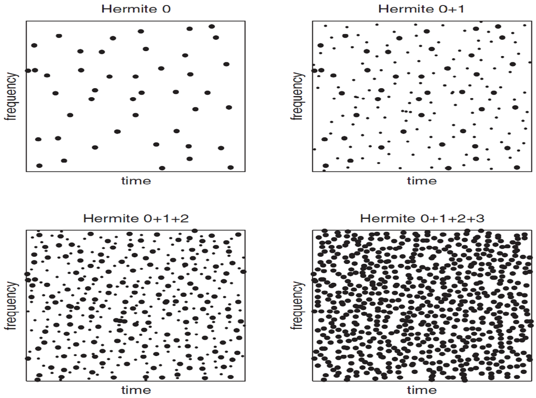

Figure: Maxima become more numerous and occur in different regions

- Maxima seem to interlace (seems hopeless to prove)

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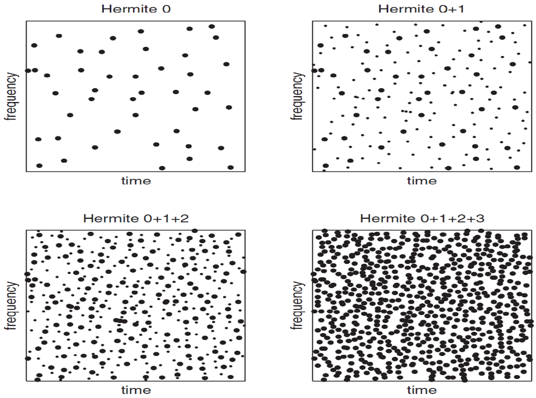

Figure: Maxima become more numerous and occur in different regions

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- What about zeros? Do zeros interlace? Repulsion?

Correlations between zeros of different Landau levels

- How strong is the orthogonality?

$$\int_{\mathbb{C}} V_{h_n}\mathcal{W}(z)\overline{V_{h_{n+k}}\mathcal{W}(z)}dz = \delta_{n,n+k}$$

- Energy of $Spec_{h_n}\mathcal{N}(z)$ and $Spec_{h_{n+k}}\mathcal{N}(z)$ placed in complementary regions? (repulsion?)

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- $\rho^{(2)}_{n,n+k}(z,w) =$ correlation zeros $f_n(z,\overline{z})$, zeros of $f_{n+k}(z,\overline{z})$.
- *Pair correlation function* $g_{n,n+1}(z,w)$:

$$g_{n,n+1}(z,w) = \frac{\rho^{(2)}_{n,n+k}(z,w)}{\rho^{(1)}_{zeros,n}(z)\rho^{(1)}_{zeros,n+k}(z)} = \begin{cases} < 1 \rightarrow & \text{Repulsion} \\ = 1 \rightarrow & \text{Neutrality} \\ > 1 \rightarrow & \text{Attraction} \end{cases}$$

Repulsion of zeros in consecutive levels

Theorem (Short range correlations A., Shirai 2022)

As $w \to z$,

$$g_{n,n+1}(z,w) \sim \frac{n(n+2)}{(n+1/2+\frac{1}{4n+2})(n+3/2+\frac{1}{4n+4})} < 1$$

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- Repulsion between zeros of $V_{h_n}\mathcal{N}$ and $V_{h_{n+1}}\mathcal{N}$.
- Zeros of $V_{h_n}\mathcal{N}$ and $V_{h_{n+1}}\mathcal{N}$ tend to interlace.
- For $k > 1$, zeros of $V_{h_n}\mathcal{N}$ ‘don’t feel’ zeros of $V_{h_{n+k}}\mathcal{N}$.

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- Landau Levels too far apart?
- Inherance of interlacing properties of h_n and h_{n+1}?

Ideas in the proof

- For the proof, we use the formulas

$$\nabla^+ f_n = \sqrt{n+1} f_{n+1}, \quad \nabla^- f_n = -\sqrt{n} f_{n-1}.$$

- And rewrite the 6-dimensional Gaussian vector

$$(f_n(z), (\nabla^+)^k f_n(w), \nabla^+ f_n(z), \nabla^- f_n(z), (\nabla^+)^{k+1} f_n(w), \nabla^- (\nabla^+)^k f_n(w)$$

$$\begin{aligned} = \quad & (f_n(z), \sqrt{(n+1)_k} f_{n+k}(w), \sqrt{n+1} f_{n+1}(z), -\sqrt{n} f_{n-1}(z), \\ & \sqrt{(n+1)_{k+1}} f_{n+k+1}(w), -\sqrt{n+k}\sqrt{(n+1)_k} f_{n+k-1}(w)) \end{aligned}$$

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$$(f_n(z), (\nabla^+)^k f_n(w), \nabla^+ f_n(z), \nabla^- f_n(z), (\nabla^+)^{k+1} f_n(w), \nabla^-(\nabla^+)^k f_n(w)$$

$$\begin{aligned} = \; & (f_n(z), \sqrt{(n+1)_k} f_{n+k}(w), \sqrt{n+1} f_{n+1}(z), -\sqrt{n} f_{n-1}(z), \\ & \sqrt{(n+1)_{k+1}} f_{n+k+1}(w), -\sqrt{n+k}\sqrt{(n+1)_k} f_{n+k-1}(w)) \end{aligned}$$

- The resulting Kac-Rice formulas are intractable for general (z, w)
- As $w \to z$ several entries of the correlation matrix become 0

Long Range correlations

- No surprise in the long range correlations:

Theorem (Long range correlations A., Shirai 2022)

As $r \to \infty$,

$$g_{n,n+1}(r,0) \to 1.$$

- zeros of $V_{h_n}\mathcal{N}$ 'don't feel' zeros of $V_{h_{n+1}}\mathcal{N}$ when they are asymptotically far from each other.

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Thanks!

Thanks!