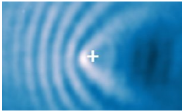
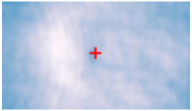
Quantum fluids

From Ohm's law to the Tonks-Girardeau gas

Mathias ALBERT[1]

[1]Institut de Physique de Nice - Nice





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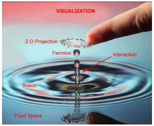
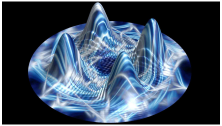
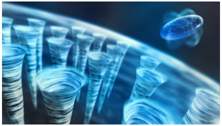
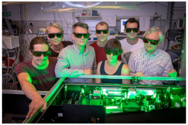
PhoQuS
Photons for Quantum Simulation

One can find serious things on the first page...









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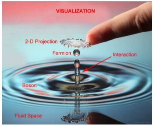
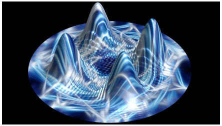
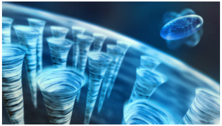
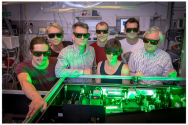


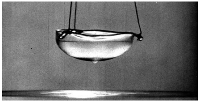




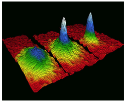


and more exotic ones if one scrolls a little bit

- Quantum ATF premium transmission fluid: "Quality automatic transmission fluid that the pro use"
- TheraZinc Spray Quantum Liquid: "is the tastiest way to deliver ionizable Zinc to your throat"
- Quantum blood and fluid warming system
- Quantum Christ, a secret fluid that contains your soul



Liquid Helium

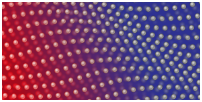


BEC

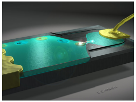
Superconductors

Conductors

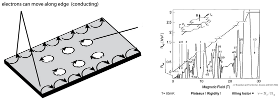
Lasers



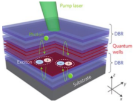
Phonons



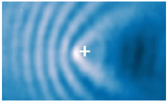
Mesoscopic conductors



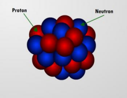
Quantum Hall liquids



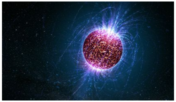
Polaritons



Light in nonlinear materials



Atomic nucleus



Neutron star

- Basic notions on quantum fluids
- Example 1: the waiting time distribution in coherent conductors #Determinant
- Example 2: the Tonks-Girardeau gas #Permanant
- Conclusion

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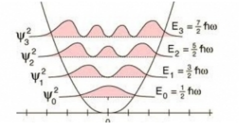
Wave-particle duality

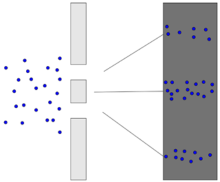
$\lambda = h/p$
$\Delta x \Delta p \geq \hbar$
$[\hat{x}, \hat{p}] = i\hbar$

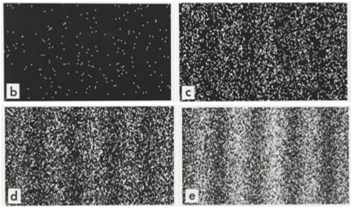


Schrödinger equation

$$i\hbar \frac{\partial}{\partial t}\psi(\vec{r},t) = -\frac{\hbar^2}{2m}\Delta\psi(\vec{r},t) + V(\vec{r})\psi(\vec{r},t)$$

$|\psi(\vec{r},t)|^2$ is the probability density to find the particle

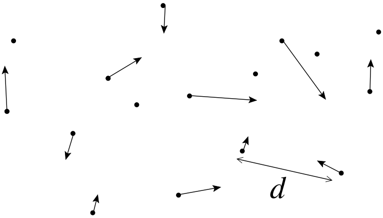




Classical limit: $\lambda \to 0$, similar to geometrical optics

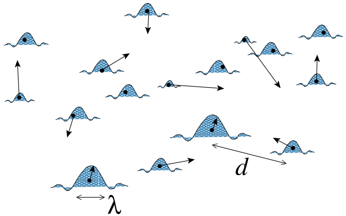
Even without interaction, many particle quantum physics is not trivial

Classical gas of identical particles



$\langle E \rangle = \frac{3}{2} kT, \quad \langle p \rangle \sim \sqrt{kT/m}$

Quantum gas of identical particles



$\lambda = h/p \sim 1/\sqrt{T}$

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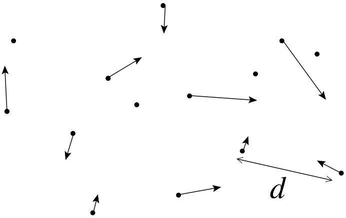

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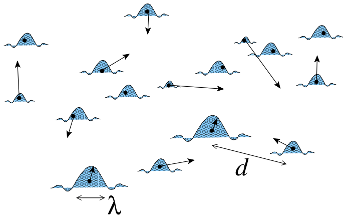
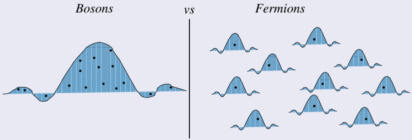
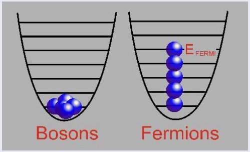


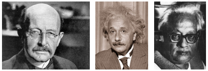
$\lambda = h/p \sim 1/\sqrt{T}$

When $\lambda \sim d \rightarrow$ "quantum identity crisis" Jean Dalibard

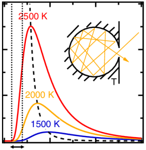




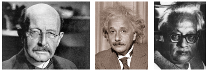
Macroscopic coherent state vs Incompressible state



- Energy is quantised: photon
- Photoelectric effect
- Light is a quantum gas of photons



Light is a quantum field $\hat{\mathcal{E}}$: integer number of photons occupy ≠ modes of the EM field



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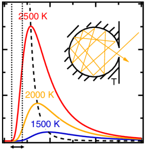

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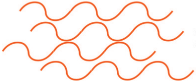
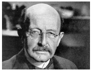
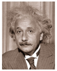
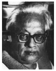
Perfect Laser
macroscopic occupation of a single mode

Phase coherence

Thermal light
statistical mixture of modes
$n(\omega) = \frac{1}{\exp[\hbar\omega/kT]-1}$



no phase coherence

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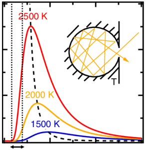

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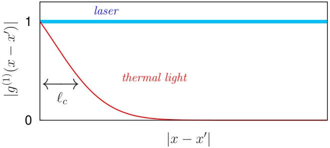

- Photon statistics
- Field coherence
$$g^{(1)}(x,x') = \frac{\langle \mathcal{E}^{\dagger}(x,t)\mathcal{E}(x',t)\rangle}{\langle |\mathcal{E}(x,t)|^2\rangle}$$

3 aspects of quantum fluids: granularity, wave coherence, quantum fluctuations

Field coherence is measured by interferometry

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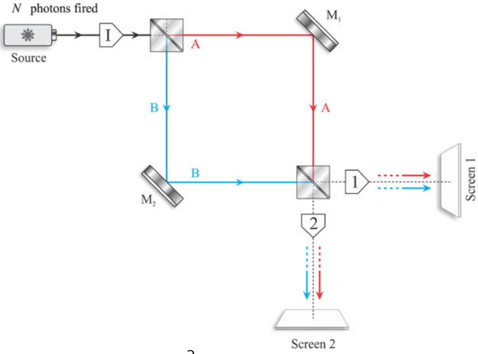
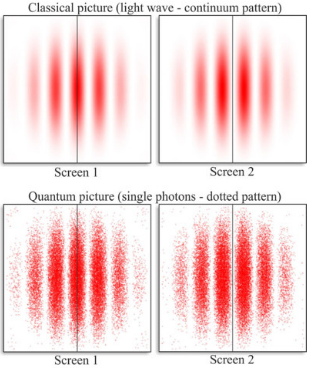
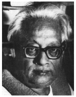
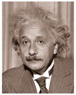


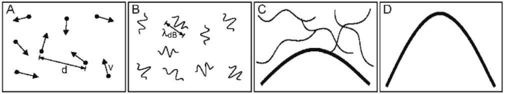


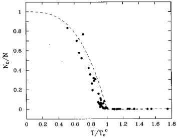
Light Intensity $I = \langle |\mathcal{E}(x,t)|^2\rangle$

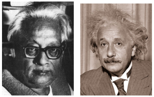
$$I_2 = \langle |\mathcal{E}_A + \mathcal{E}_B|^2\rangle = I_A + I_B + 2\,\mathrm{Re}[\langle \mathcal{E}_A^\dagger \mathcal{E}_B\rangle]$$

1925, below T_c: phase transition in a non interacting gas of bosons, macroscopic occupation of the ground state







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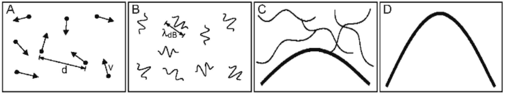
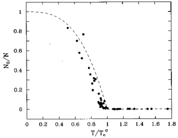
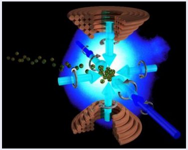




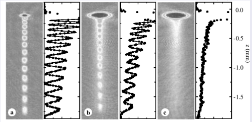
How do we make it?

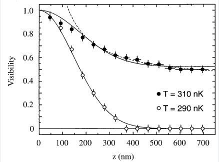


- laser cooling
- magnetic trap
- First observation 1995

Cornell PRL 1995
Bloch J. Mod. Optics 2000

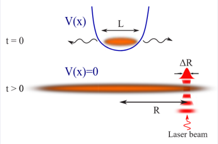
Signature in x space

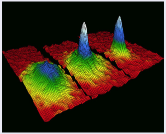




$$\rho_1(\vec{r},\vec{r}') = \langle \hat{\Psi}^\dagger(\vec{r})\hat{\Psi}(\vec{r}')\rangle \to n_0$$
when $|\vec{r}-\vec{r}'| \to +\infty$

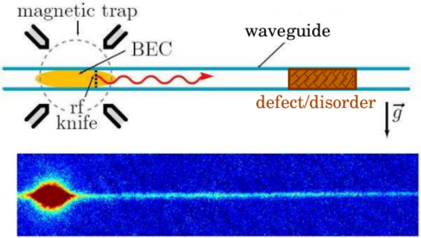
Signature in k space





$$n(\vec{k}) = \mathcal{F}[\rho_1(\vec{r},\vec{r}')] = n_0\delta(\vec{k}) + n_T(\vec{k})$$

Coherence properties of a BEC suggests to use it as an atomic laser



W. Guerin *et al* PRL 2006

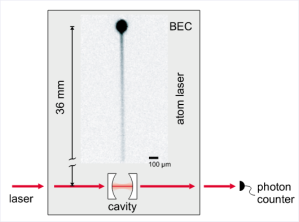
Matter field $\hat{\Psi}(\vec{r},t)$

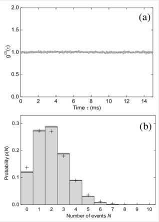
atomic density $\langle\hat{\Psi}^\dagger(\vec{r},t)\hat{\Psi}(\vec{r},t)\rangle$

$\leftrightarrow$

EM field $\hat{\mathcal{E}}(\vec{r},t)$

light intensity $\langle\hat{\mathcal{E}}^\dagger(\vec{r},t)\hat{\mathcal{E}}(\vec{r},t)\rangle$





- Same coherence properties
- Same counting statistics

Öttl *et al* PRL 2005

What is the fate of BEC in the presence of interactions at $T = 0$?

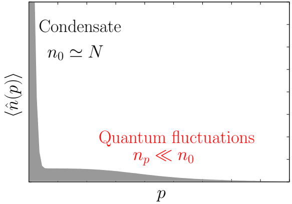
$$\hat{H} = \int d\vec{r}\, \hat{\Psi}^\dagger(\vec{r}) [\underbrace{-\frac{\hbar^2}{2m}\Delta}_{\text{Kinetic}} + \underbrace{V(\vec{r})}_{\text{Trap}} + \underbrace{\frac{g}{2}\hat{\Psi}^\dagger(\vec{r})\hat{\Psi}(\vec{r})}_{\text{Interaction}}] \hat{\Psi}(\vec{r})$$

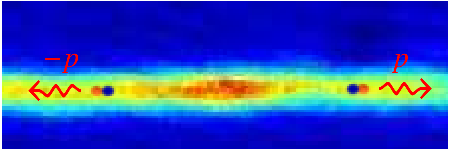
Penrose-Onsager $\rho_1(\vec{r},\vec{r}\,') = \langle \hat{\Psi}^\dagger(\vec{r})\hat{\Psi}(\vec{r}\,')\rangle \to n_0 \quad \leftrightarrow \quad \langle \hat{\Psi}(\vec{r})\rangle \neq 0$

when $|\vec{r} - \vec{r}\,'| \to +\infty$ Ginzburg-Landau 1950, Penrose 1951, Penrose-Onsager 1956

Bose-Einstein condensation in **3D** is **stable** against **weak** interaction Bogoliubov 1947

$$\hat{\Psi}(\vec{r}) = \underbrace{\langle \hat{\Psi}(\vec{r})\rangle}_{\text{Condensate}} + \underbrace{\delta\hat{\Psi}(\vec{r})}_{\text{Gas of quasi-particles}}$$





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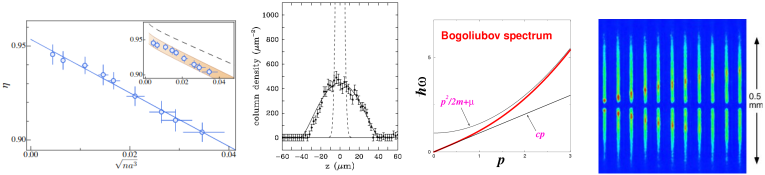


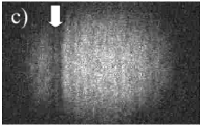
Hadzibabic PRL 2017, Hau et al PRA 1998, Ketterle PRL 1997

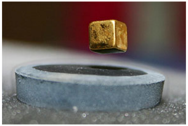
Gross-Pitaeskii equation for the condensate $\psi = \langle\hat{\Psi}\rangle$

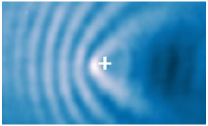
$$i\hbar\frac{\partial}{\partial t}\psi(\vec{r},t) = \left[-\frac{\hbar^2}{2m}\Delta + V(\vec{r})\right]\psi(\vec{r},t) + gN|\psi(\vec{r},t)|^2\,\psi(\vec{r},t)$$

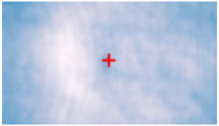
Macroscopic wave function $\langle\hat{\Psi}\rangle = \sqrt{n(\vec{r})}\exp[i\theta(\vec{r})] \rightarrow$ **lockstep motion** $\vec{V} = \frac{\hbar}{m}\vec{\nabla}\theta(\vec{r})$

Superfluid flow and superconductivity (critical velocity $V_c = f(g)$)









Engels PRL 2007

Michel Nature Com 2018

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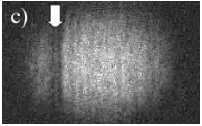

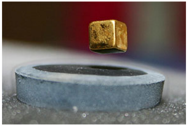

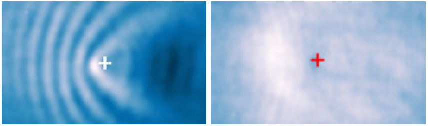

Engels PRL 2007

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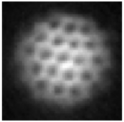
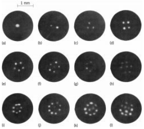
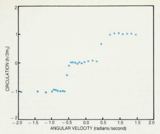
Quantised Vortices



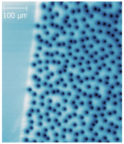
BEC (Dalibard 2000)



Liquid Helium 4



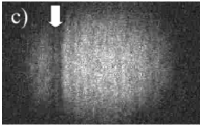
^3He (Zieve 1993)

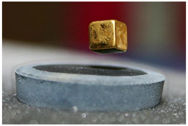


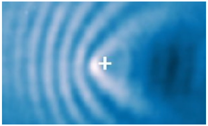
Superconductor

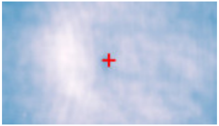
Macroscopic wave function $\langle\hat{\Psi}\rangle = \sqrt{n(\vec{r})}\exp[i\theta(\vec{r})] \rightarrow$ **lockstep motion** $\vec{V} = \frac{\hbar}{m}\vec{\nabla}\theta(\vec{r})$

Superfluid flow and superconductivity (critical velocity $V_c = f(g)$)









Engels PRL 2007

Michel Nature Com 2018

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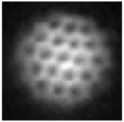

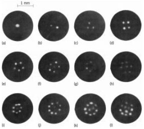

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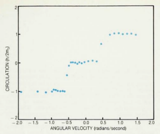
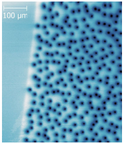
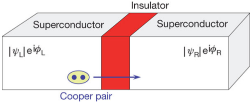
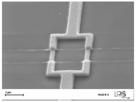


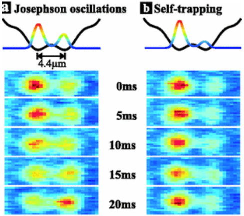


^3He (Zieve 1993) Superconductor

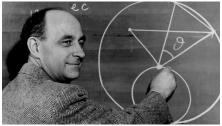
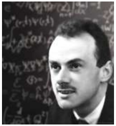
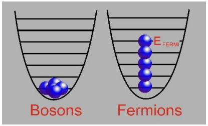
Josephson effect

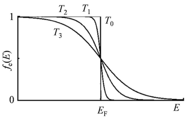




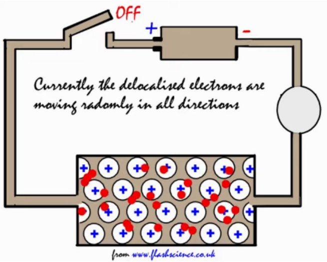


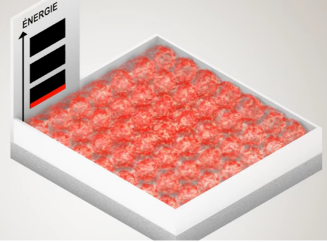
BEC (Oberthaler PRL 2005)



- **Electrons** from the conduction band are delocalised → transport.
- Degenerate gas of electrons even at room temperature $T_F \sim 10\,000\,K$
- Macroscopic conductor = diffusive motion of "electrons" due to impurities and phonons (charge carriers are actually quasi particles and not electrons)

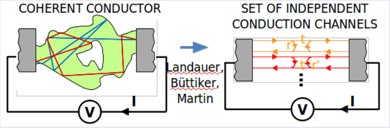




Fermi statistics is essential to understand conduction properties of macroscopic systems. Wave coherence does not play any role.

Wave behavior of electrons in conductors? Yes but at very low T (mK) and for **small conductors** ($\sim 10\mu m$)

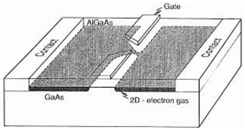
When the coherence length of electrons becomes larger than the system size

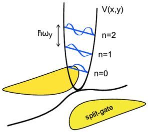


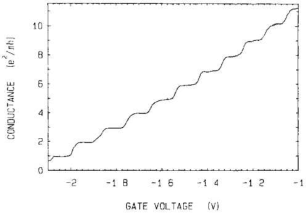
Conductance depends on

- Scattering amplitudes t_n
- Number of channels

$G = \frac{e^2}{h} \sum_n |t_n|^2$, Landauer 1957, Büttiker 80's







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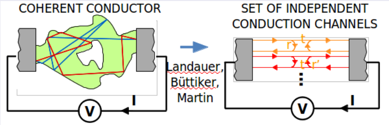

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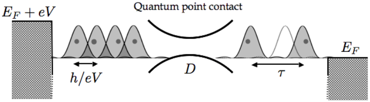
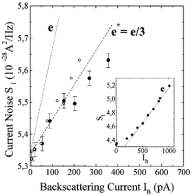
Single quantum channel

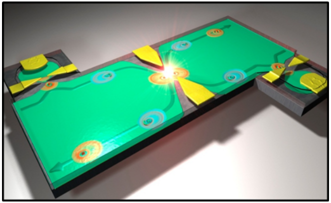




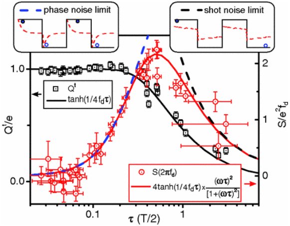
Average current $I = eD/(h/eV) = D\frac{e^2}{h}V$ **Shot noise** $S = \int \langle I(t)I(0)\rangle dt = 2q\,I$, for $D \ll 1$.
Granularity of I is not always the elementary charge! Glattli 1997, $\nu = 1/3$, $q = e/3$

The **noise** is sensitive to : wave coherence, granularity but also quantum fluctuations!
"The noise is the signal" Rolf Landauer

Single electron source of electrons and holes

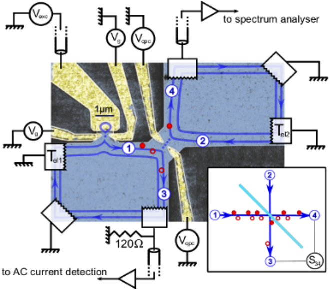


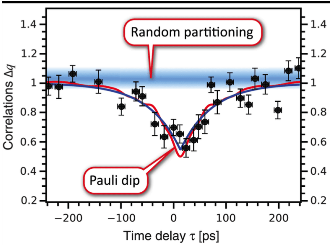
G. Fève LPA ENS Paris



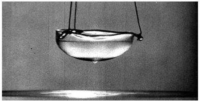
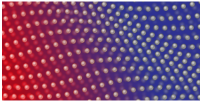
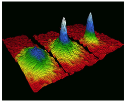
Albert *et al* PRB 2010

Hong-Ou-Mandel experiment



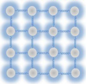
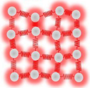


Bocquillon *et al* Science Express 2014

- Quantum fluids can be made of: matter, energy, both of them (quasi-particles)
- They usually exist at very low temperature (counter examples: electrons in metals, neutron stars...)
- They may present macroscopic coherence (BEC, superconductors, lasers...) and superfluidity
- They are rarely described in terms of their constituants: non trivial ground state + quantised eigenmodes (gas of quasi-particles)
- Bosons vs Fermions

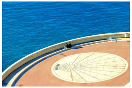
Poor man quantum field theory

o ground state
o small perturbations: eigenmodes
o quantisation: quasi-particles

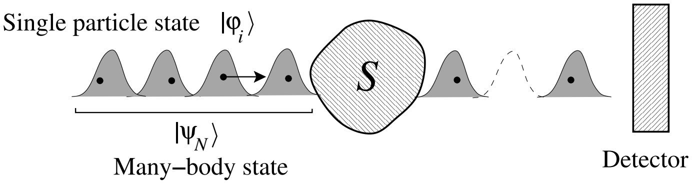
$$\hat{\Psi}(x) = \sum_q \hat{b}_q \phi_q(x), \quad [\hat{\Psi}(x), \hat{\Psi}^\dagger(x)] = \delta(x - x')$$

Classical limits

o continuous field $\hat{\Psi} \rightarrow \psi(x)$
o decoherence $\langle \hat{\Psi}(x)^\dagger \hat{\Psi}(x') \rangle \rightarrow 0$
destructive infterference, interaction, temperature, disorder

- Basic notions on quantum fluids
- Example 1: the waiting time distribution in coherent conductors #Determinant
- Example 2: the Tonks-Girardeau gas #Permanant
- Conclusion



- Quantum particles are described by wave packets.
  → Quantum jitter!
- Classical measurement → detect a spike.
- The WTD probes both the structure of the many-body state and the fluctuations generated by the scatterer.

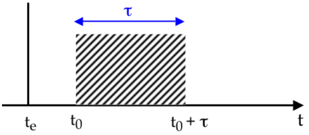
Problems!

- Energy-time Heisenberg inequality.
- The vaccum is not "empty": Fermi sea.
- We have to include the detection process in the theory.

First attempt to solve the problem

Ideal situation: $T = 0$, free fermions, no Fermi-Sea, time-independent scatterer, stationary process and ideal detector.

- Idle time probability $\Pi(\tau)$: prob to detect nothing in a time slot τ.



$$\Pi(\tau) = \frac{1}{\langle\tau\rangle}\int_{t_e}^{\infty} dt_0 \left(1 - \int_{t_e}^{t_0+\tau} w(t)dt\right)$$

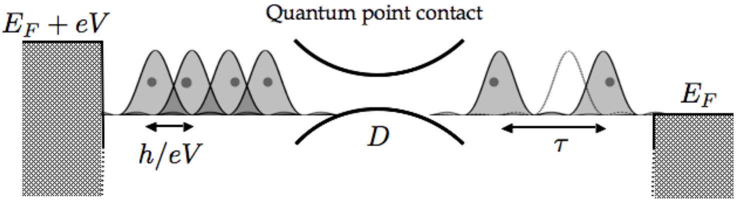
$$\boxed{w(\tau) = \langle\tau\rangle \frac{d^2\Pi(\tau)}{d\tau^2}}$$

- Transmission operator over a finite time window $\mathbf{Q}_\tau = \int_0^{v_F\tau} |x\rangle\langle x|\, dx$
  - For 1 electron: $\Pi(\tau) = \langle\varphi|\mathbb{1} - \mathbf{Q}_\tau|\varphi\rangle$
  - For N electrons: $\Pi(\tau) = \langle\Psi_N|\prod_1^N[\mathbb{1} - \mathbf{Q}_\tau]|\Psi_N\rangle$

If $|\Psi_N\rangle$ is a Slater determinant we get the determinant formula:

$$\boxed{\Pi(\tau) = \det\langle\varphi_n|\mathbb{1} - \mathbf{Q}_\tau|\varphi_m\rangle} = \langle: e^{-\mathbf{Q}_\tau} :\rangle$$

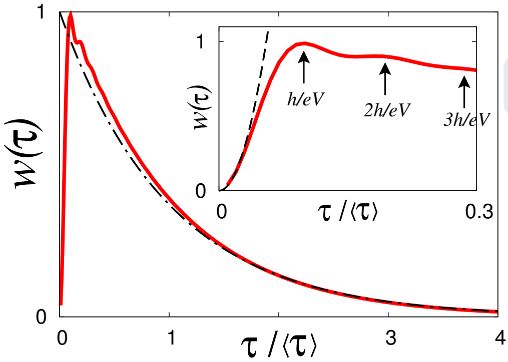
We take the $\mathbf{N} \to \infty$ limit to mimic a stationary process. Hassler *et al* PRB 2008.



- Quantum mechanical time scale h/eV.
- Intuitive picture: The Pauli principle leads to the formation of a train of wave packets. T. Martin and R. Landauer PRB 1992.

- FCS in the long time limit: Binomial process with time step h/eV and probability D. Levitov and Lesovik JETP 1993.
  For $D \ll 1$: poissonian statistics (uncorrelated transport).
- The noise $S(\omega) = \int e^{i\omega t}\langle \delta I(t)\delta I(0)\rangle dt = 0$ at $D = 1$!!!
  → No access to the structure of the wave function!

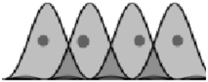
The WTD should exhibit the quantum jitter!

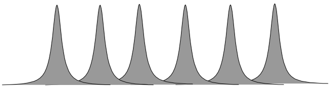


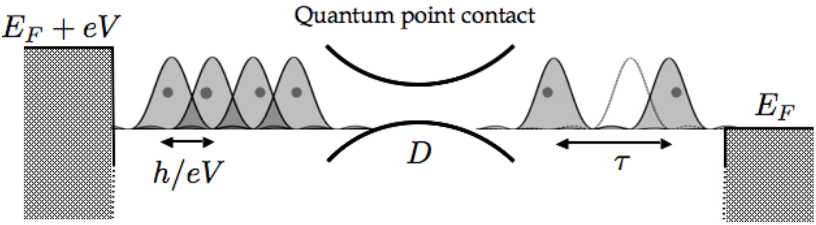
$$D = 0.1\,, \quad \langle\tau\rangle = \frac{h}{eVD}$$

- Almost uncorrelated $\rightarrow$ exponential WTD.
- Pauli exclusion principle $\rightarrow$ hole at $\tau = 0$.
- Quantum oscillations with period h/eV!!

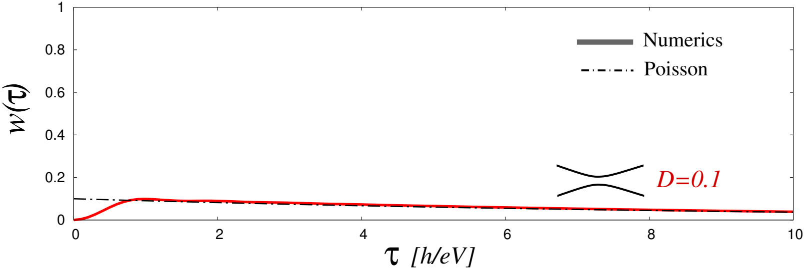
- Liquid like correlations due to the strong overlap of the wave packets. Here the particles have to fill the quantum channel.
- Solid like correlations would be observable with a triggered source. J. Keeling, I. Klich and L. Levitov PRL 2006.



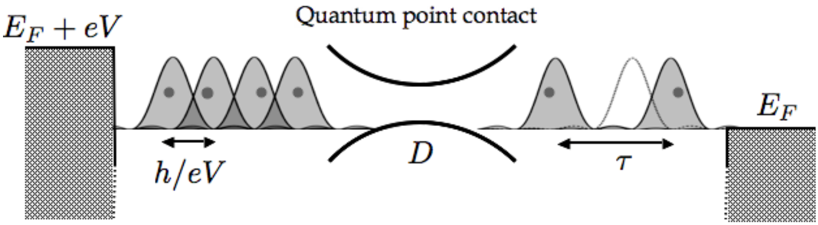


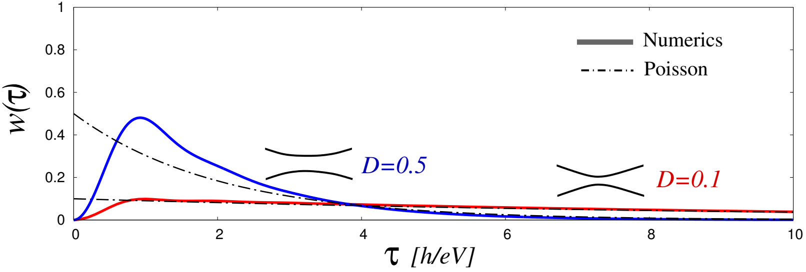


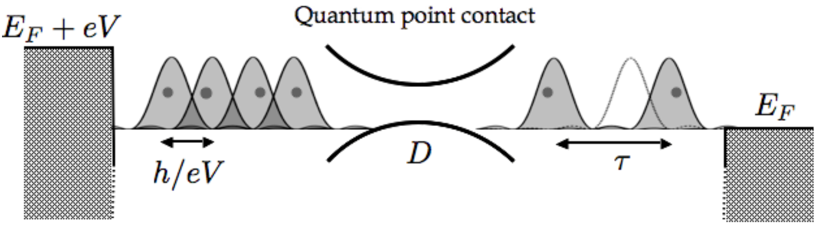
$E_F + eV$
Quantum point contact
E_F
h/eV
D
τ

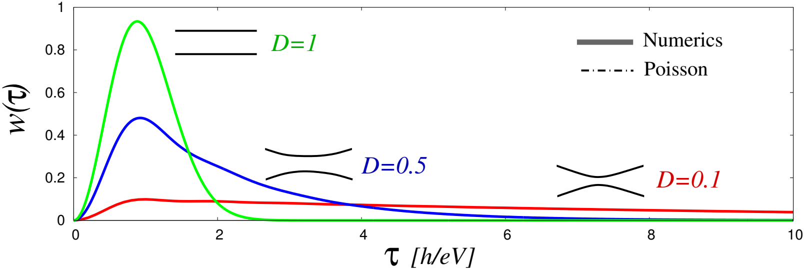


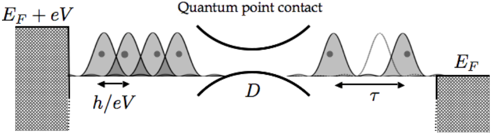
$w(\tau)$
τ [h/eV]
Numerics
Poisson
D=0.1
1
0.8
0.6
0.4
0.2
0
0
2
4
6
8
10

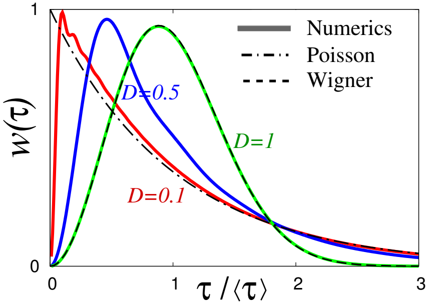

$E_F + eV$
Quantum point contact
E_F
h/eV
D
τ


1
0.8
0.6
0.4
0.2
0
$w(\tau)$
0
2
4
6
8
10
τ [h/eV]
Numerics
Poisson
D=0.5
D=0.1

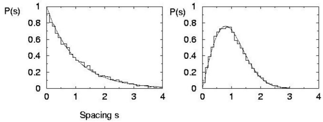

$E_F + eV$
Quantum point contact
E_F
h/eV
D
τ


1
0.8
0.6
0.4
0.2
0
$w(\tau)$
D=1
D=0.5
D=0.1
Numerics
Poisson
0
2
4
6
8
10
τ [h/eV]





- Average waiting time $\langle\tau\rangle = \frac{h}{eVD}$.
- Crossover from Poisson to Wigner-Dyson (GUE).

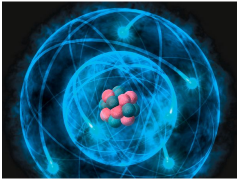


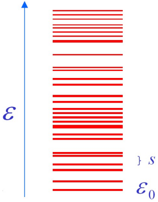
$p(s) = e^{-s}$ $\quad$ $p(s) = \frac{32}{\pi^2} s^2 e^{-\frac{4}{\pi} s^2}$

- Large fluctuations even at $D = 1$.
  $\rightarrow$ Quantum jitter!

Albert *et al*, Phys. Rev. Lett. **108**, 186806 (2012)

Connection with Random Matrix Theory (RMT) S. Majumdar's lecture





$$\mathbf{H} = \begin{pmatrix} H_{11} & \cdots & H_{1N} \\ \vdots & \ddots & \vdots \\ H_{N1} & \cdots & H_{NN} \end{pmatrix}$$

$$P(\mathbf{H}) \sim \exp\left[-\mathrm{Tr}\, V(\mathbf{H})\right]$$

$$P(E_1, \ldots, E_N) \sim \prod_{n>m} |E_n - E_m|^{\beta} \exp\left[-\sum_n V(E_n)\right] = |\Psi(E_1, \ldots, E_N)|^2$$

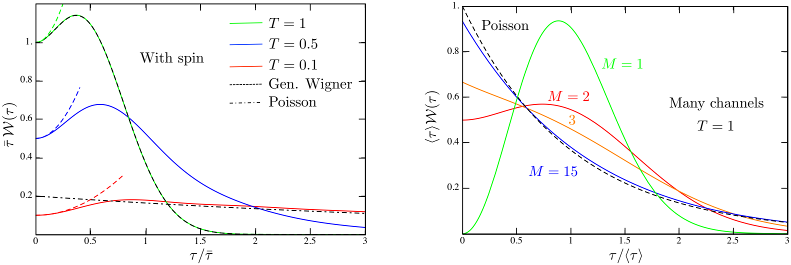
The level repulsion $(E_n - E_m)^{\beta}$ depends on symmetries ($\beta = 1, 2, 4$ for orthogonal, unitary and symplectic ensembles).

$\Psi(E_1, \ldots, E_N)$ is the ground state of the Calogero-Sutherland Model:

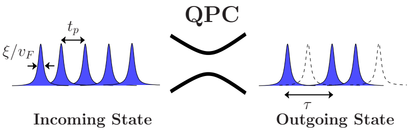
$$\hat{H} = -\frac{1}{2}\sum_n \frac{\partial^2}{\partial E_n^2} + \frac{\beta}{2}\left(\frac{\beta}{2} - 1\right)\sum_{n>m} \frac{1}{(E_n - E_m)^2}$$

$\beta = 2$: free fermions $\Rightarrow$ mapping between RMT and free fermions in 1D. All the correlation functions are identical. $E_n \leftrightarrow x_n$

- Single quantum channel with spin 1/2 $\rightarrow$ $M = 2$
- Quasi-one dimensional quantum wire $M \sim S/\lambda_F^2$

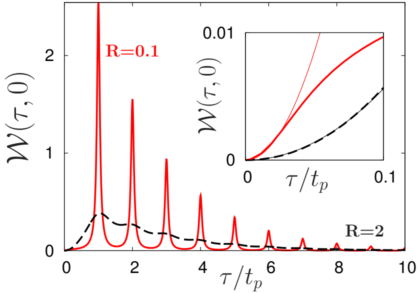


- non zero probability to have **two** electrons at the **same time**
- Crossover from Wigner-Dyson to Poisson when $M \rightarrow \infty$



Lorentzian pulses with $n = 1$

J. Keeling, I. Klich and L. Levitov PRL 2006.
J. Dubois *et al* Nature 2013.



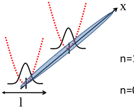
$R = \frac{\xi}{v_F t_p}$ $\quad$ $D = 0.4$

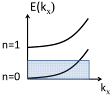
Albert & Devillard PRB 2014, Dasenbrook *et al* PRL 2014.

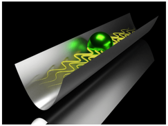
- Tunable aspect ratio: liquid to solid crossover.
- $\xi/v_F \ll t_p$: thin peaks reflecting the shape of the wave packet.
- $\xi/v_F \gg t_p$: constant bias limit $eV = hv_F/\xi$.
- $D = 1$ is no longer given by RMT.

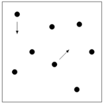
- Basic notions on quantum fluids
- Example 1: the waiting time distribution in coherent conductors #Determinant
- Example 2: the Tonks-Girardeau gas #Permanant
- Conclusion

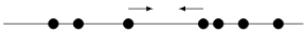
There is no Bose-Einstein condensation transition in **1D**.
Even at $T = 0$, the condensate is destroyed by weak interactions. Why?





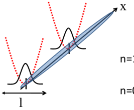


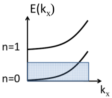


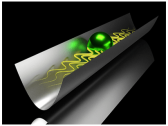
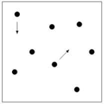
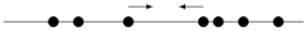


In one dimension, interactions are **impossible to avoid !**

There is no Bose-Einstein condensation transition in **1D**.
Even at $T = 0$, the condensate is destroyed by weak interactions. Why?



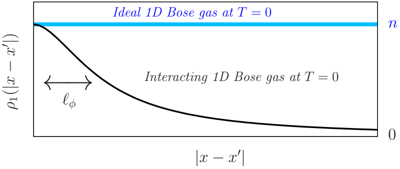


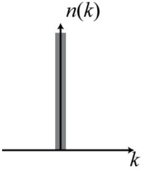




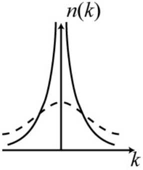
In one dimension, interactions are **impossible to avoid !**

Long range order is destroyed

$$\rho_1(x,x') = \langle \hat{\Psi}^\dagger(x)\hat{\Psi}(x')\rangle \sim C|x-x'|^{-\sqrt{\gamma}/2\pi}, \quad \ell_\phi = \xi\, e^{2\pi/\sqrt{\gamma}}$$



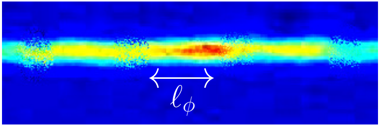


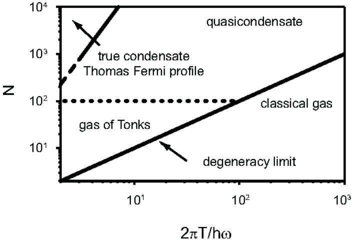


γ is the interaction parameter. When $\gamma \to 0$, $\ell_\phi \to +\infty$.

The **phase diagram** is established by comparing ℓ_ϕ with the system size

Quasi condensate



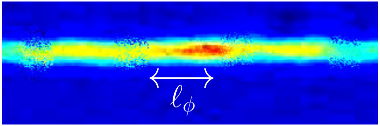


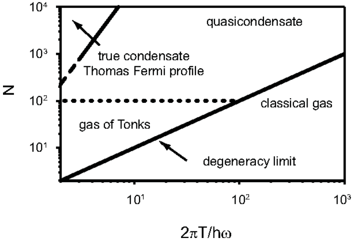
In a trap: $L \sim \sqrt{N}$, $n \sim \sqrt{N}$, $\gamma \sim 1/\sqrt{N}$
Petrov PRL 2000

ℓ_ϕ also decreases with T

The **phase diagram** is established by comparing ℓ_ϕ with the system size

Quasi condensate





In a trap: $L \sim \sqrt{N}$, $n \sim \sqrt{N}$, $\gamma \sim 1/\sqrt{N}$
Petrov PRL 2000

ℓ_ϕ also decreases with T

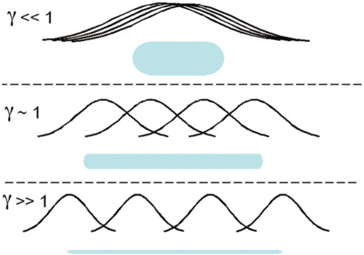
What is the **Tonks gas**?

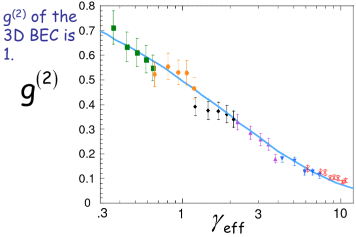
$$\hat{H} = \int dx \hat{\Psi}^\dagger(x)\left[-\frac{\hbar^2}{2m}\frac{d^2}{dx^2} + V(x) + \frac{g}{2}\hat{\Psi}^\dagger(x)\hat{\Psi}(x)\right]\hat{\Psi}(x)$$

Effect of interaction: $d \sim 1/n$, $E_{\mathrm{kin}} \sim \frac{\hbar^2}{2md^2} = \frac{\hbar^2}{2m}n^2$, $E_{\mathrm{int}} \sim gn$

$$\Rightarrow \boxed{\gamma = \frac{E_{\mathrm{int}}}{E_{\mathrm{kin}}} = \frac{mg}{\hbar^2 n}}$$

Interactions are stronger at small density!!

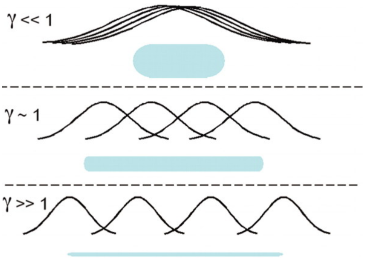


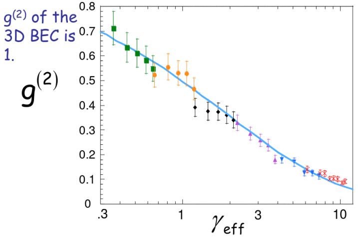


Weiss Science 2004, PRL 2005

1d bosons at strong repulsion ($\gamma \to \infty$) **exclude** each other → FERMIONISATION

$$\psi_B(x_1,...,x_N) = |\psi_F(x_1,...,x_N)|$$ Girardeau 1960





Weiss Science 2004, PRL 2005

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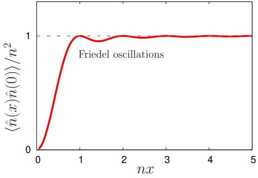
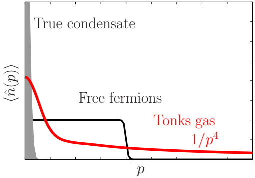
- Same density correlations
- Different phase correlations

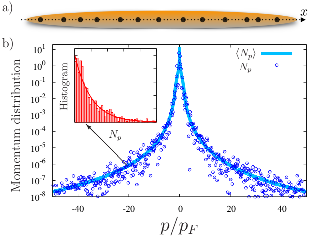


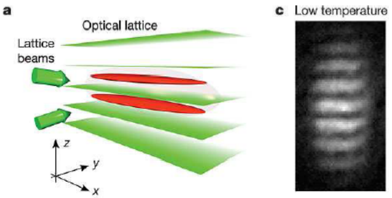
density-density



momentum distribution

Fluctuations of the number of atoms with momentum p, $\hat{n}(p) = \hat{a}_p^\dagger \hat{a}_p$





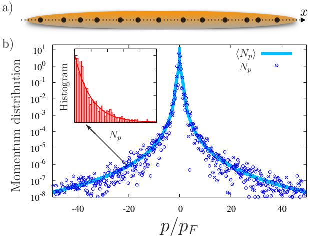
- Shot to shot fluctuations in time of flight images
- Shot to shot fluctuations in interference contrast
- Quantum fluctuations of the quasi-condensate $\hat{n}_0$

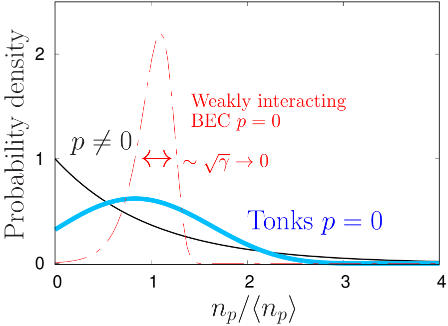
Probe high order coherence function $\langle \hat{\Psi}^\dagger(x_1)\cdots\hat{\Psi}^\dagger(x_n)\cdots\hat{\Psi}(y_1)\cdots\hat{\Psi}(y_n)\rangle$

because $\hat{a}_p = \int dx \hat{\Psi}(x) e^{ipx/\hbar}$

Fluctuations of the number of atoms with momentum p $\Rightarrow$ $P(n_p)$

Similar to photon statistics





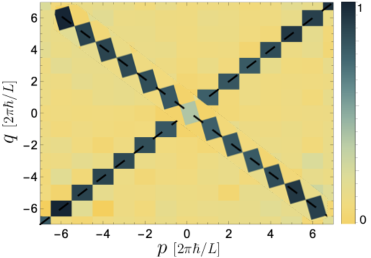
Crossover from exponential distribution to a single mode distribution for $p = 0$

- **Weak interaction:** Gumbel distribution with $\langle n_0 \rangle \sim N$ and $\Delta n_0 \to 0$
  Lovas *et al* PRA 95, 053621 (2017)
- **Tonks regime:** sub poissonian "half Gaussian" with $\langle n_0 \rangle \sim \sqrt{N}$ and $\Delta n_0 = 0.58 \langle n_0 \rangle$
  Devillard, Chevallier, Vignolo and Albert, PRA 101, 063604 (2020)

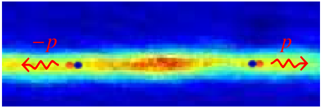
What about **correlations** between excitations with different momentum?

$$\frac{\langle \hat{n}_p \hat{n}_q \rangle - \langle \hat{n}_p \rangle \langle \hat{n}_q \rangle}{\langle \hat{n}_p \rangle \langle \hat{n}_q \rangle}$$

Weak interactions

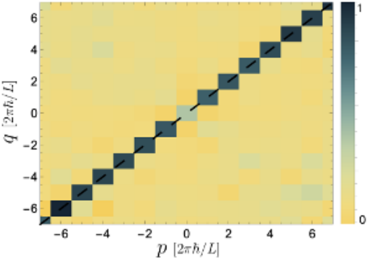


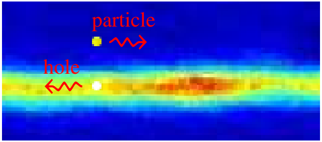
Pairs of Bogoliubov quasi-particles



Bouchoule, PRA 86, 043626 (2012)

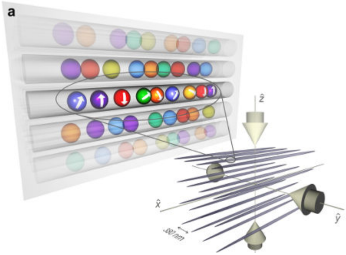
Tonks regime



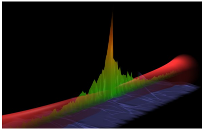


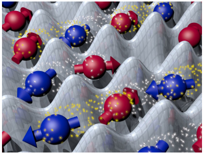
Devillard, PRA 101, 063604 (2020)

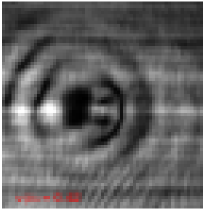
- Quantum fluids are various in nature
- One dimensional systems exists and are very pecular



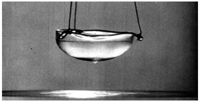
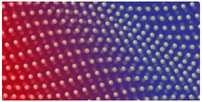
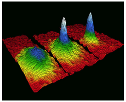
Pagano, Nature Physics 2014



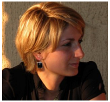




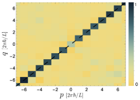
- disorder : weak localisation, Anderson localisation, Bose glass
- mixtures with $SU(N)$ symmetry
- thermalisation, quenches, quantum turbulence
- Casimir effect, Hawking radiation
- quantum simulators

Thank you for your attention and thanks to all my collaborators!

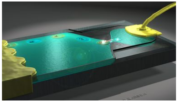
Michel *et al*, Nature Comm. **9**, 2108 (2018)



Devillard *et al*, Phys. Rev. A **101**, 063604 (2020)

Devillard *et al*, Phys. Rev. A **104**, 053306 (2021)

Parmentier *et al*, Phys. Rev. B **85**, 165438 (2012)

Albert *et al*, Phys. Rev. Lett. **108**, 186806 (2012)

Properties of the fermionized regime

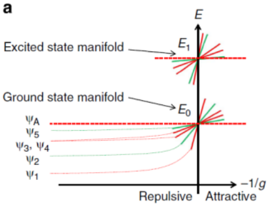
- For a r-component Fermi gas, large degeneracy of the ground state:
$$\frac{N!}{N_1!N_2!\dots N_r!}$$
- Mapping onto an ideal Fermi gas with $N = N_1 + N_2 + \dots N_r$ fermions
- the ideal-Fermi gas wavefunction has the right nodes
- when exchanging two fermions belonging to the same component, the wavefunction takes a minus sign
- one needs to fix the phase of the wavefunction when exchanging two fermions belonging to different components: origin of the degeneracy

Exact wavefunction in the fermionized regime

- Generalization of Girardeau's wavefunction for impenetrable bosons [Volosniev et al., Nat. Phys. 2015]

$$\Psi(x_1,\dots,x_N) = \sum_{P\in S_N} a_P \xi(x_{P(1)} < \cdots < x_{P(N)})\Psi_A(x_1,\dots,x_N)$$

- the coefficients a_P are determined minimizing the energy: g^{-1} expansion $\Rightarrow$ $\min(\partial E/\partial g^{-1})$



$$E = E_\infty - \frac{K}{g_{1D}}$$

K are the eigenvalues of the effective Hamiltonian in the degenerate many-fold (Sutherland Hamiltonian)