Quantum computational advantages with light

Mattia Walschaers

DPP Fermions 2022

Overview

- Modes and states in optics
- Boson sampling
- Continuous-variable approach
- Requirements for quantum computational advantage

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Lecture 1

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Lecture 2

Resources

Resources

REVIEWS OF MODERN PHYSICS, VOLUME 92, JULY–SEPTEMBER 2020

Modes and states in quantum optics

C. Fabre* and N. Treps†

Laboratoire Kastler Brossel, Sorbonne Université, ENS, CNRS, Collège de France, Campus Pierre et Marie Curie, 75005 Paris, France

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PRX QUANTUM **2**, 030204 (2021)

Tutorial

Non-Gaussian Quantum States and Where to Find Them

Mattia Walschaers*

Laboratoire Kastler Brossel, Sorbonne Université, CNRS, ENS-Université PSL, Collège de France, 4 place Jussieu, Paris F-75252, France

(Received 2 May 2021; published 28 September 2021)

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IOP Publishing

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Journal of Physics B: Atomic, Molecular and Optical Physics

https://doi.org/10.1088/1361-6455/ab5c30

PhD Tutorial

Signatures of many-particle interference

Mattia Walschaers

Laboratoire Kastler Brossel, Sorbonne Université, CNRS, ENS-PSL Research University, Collège de France, France

Overview

- Modes and states in optics
- Boson sampling
- Continuous-variable approach
- Requirements for quantum computational advantage

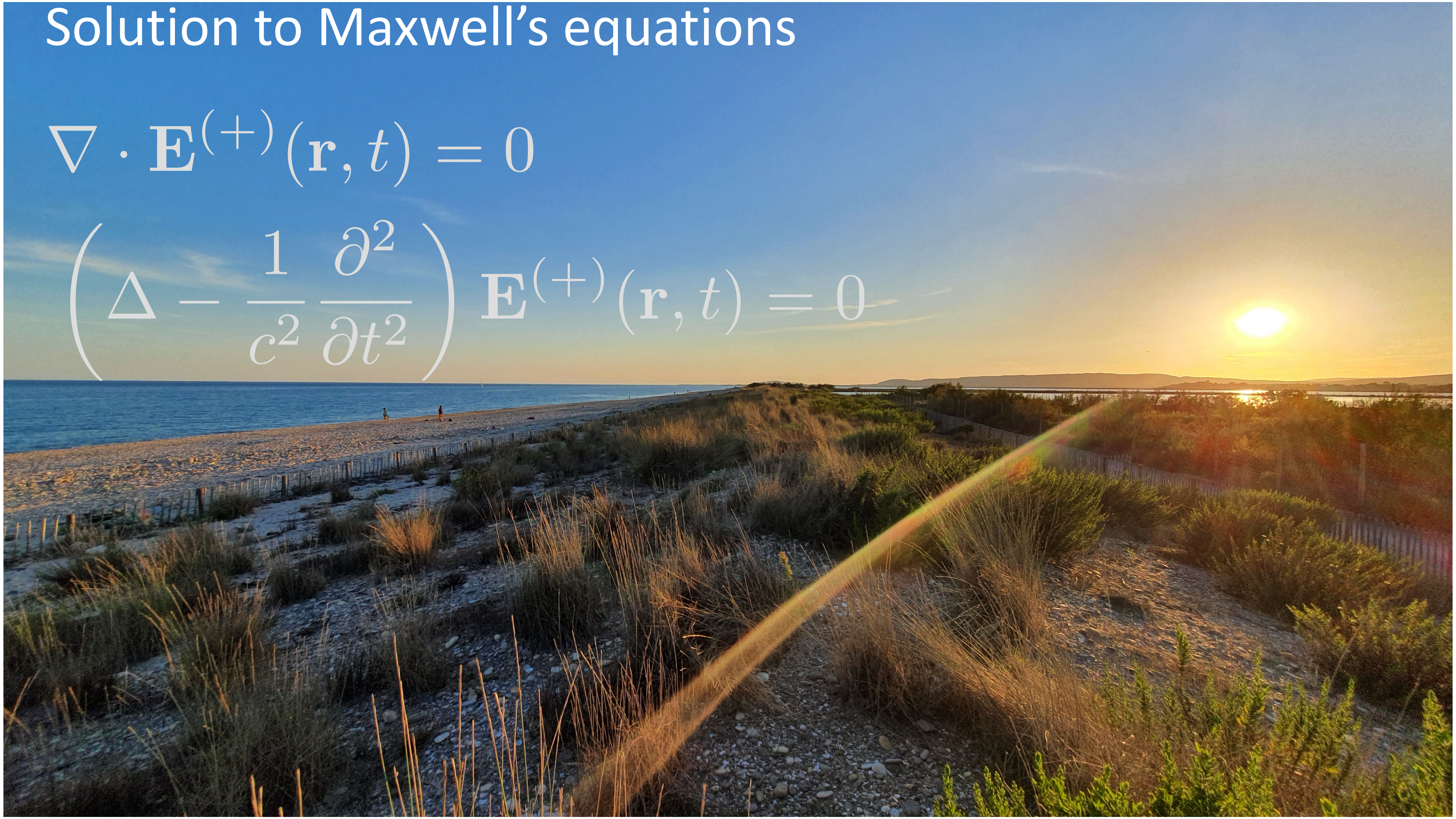
Light

Light

Solution to Maxwell's equations

$$\nabla \cdot \mathbf{E}^{(+)}(\mathbf{r}, t) = 0$$

$$\left(\Delta - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right)\mathbf{E}^{(+)}(\mathbf{r}, t) = 0$$



Modes in free space

Jeff Lundeen @LundeenOttawa · Nov 3

What is an “optical mode”?

Please reply if you have any insight or opinion.

I keep confusing myself about this. There seem to be many different but subtly related definitions.

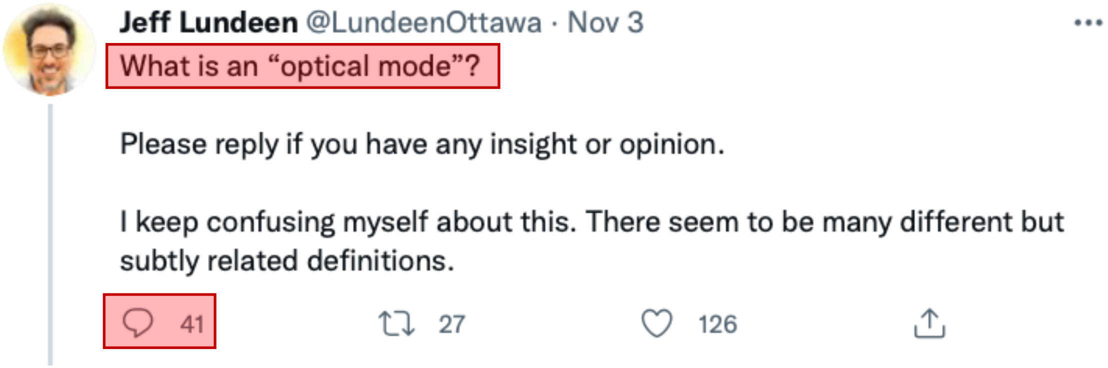
41 27 126

Modes in free space

Optical **modes** form an orthonormal basis of solutions to Maxwell's equations

$$\nabla \cdot \mathbf{u}_i(\mathbf{r}, t) = 0$$

$$\left(\Delta - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right)\mathbf{u}_i(\mathbf{r}, t) = 0$$



Modes in free space

$$\frac{1}{V}\int_V u_j^*(\mathbf{r},t)u_k(\mathbf{r},t)d^3\mathbf{r} = \delta_{j,k}$$

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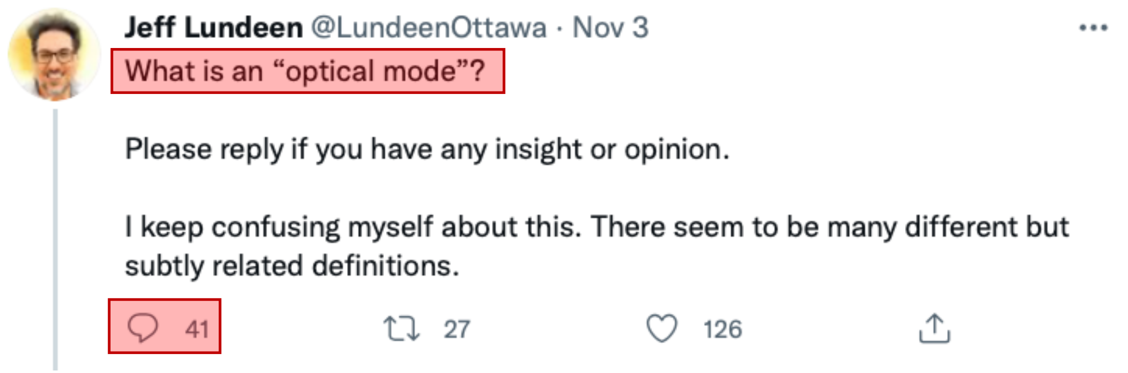

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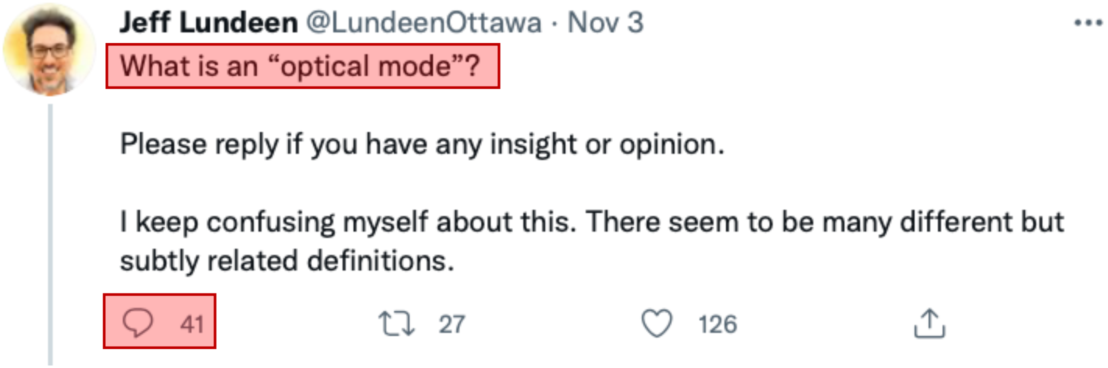

Light is then described as a superposition of optical modes

$$\mathbf{E}^{(+)}(\mathbf{r},t) = \sum_{j=1}^{\infty} \mathcal{E}_j \alpha_j \mathbf{u}_j(\mathbf{r},t)$$

Modes in free space

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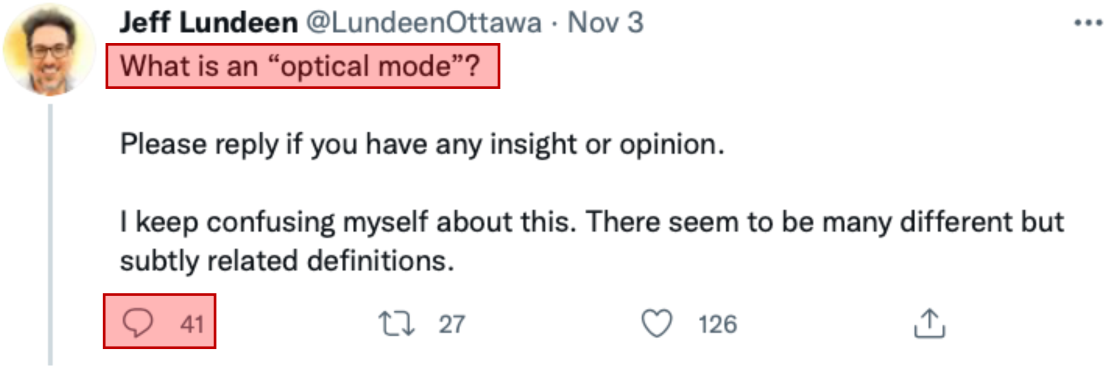

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Complex amplitude

Modes in free space

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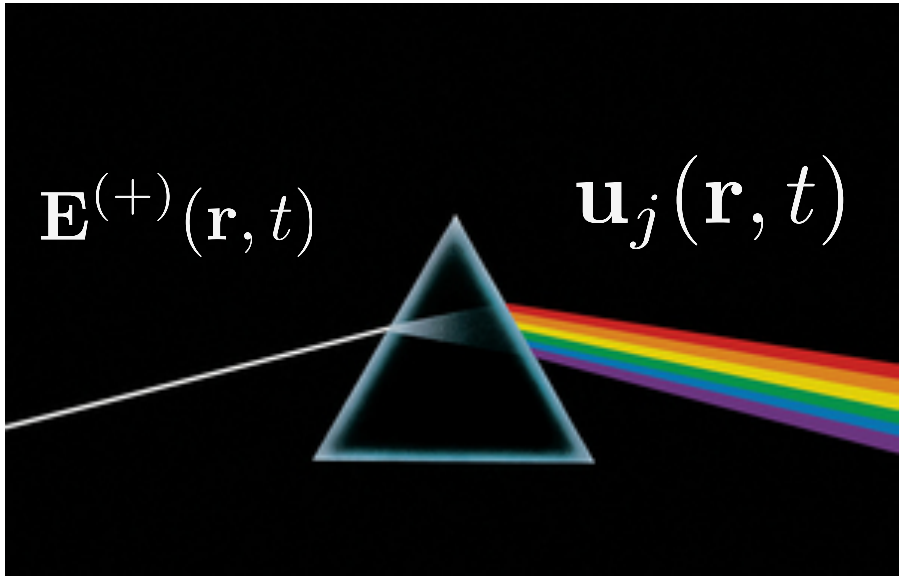

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Complex amplitude

Kinds of modes: Spectral

$$\mathbf{E}^{(+)}(\mathbf{r},t)=\sum_{j=1}^{\infty}\mathcal{E}_j\alpha_j\mathbf{u}_j(z/c-t)\mathbf{v}(\mathbf{r})$$

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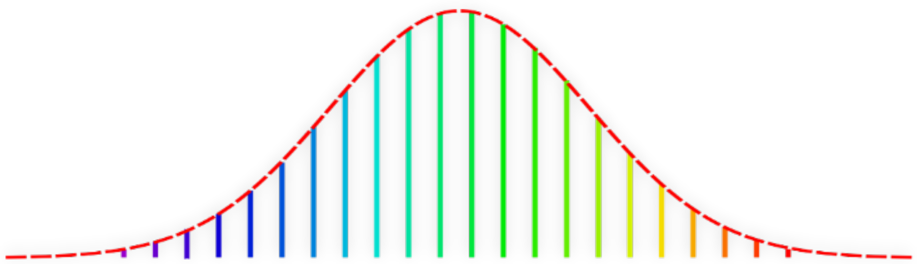
Frequency domain



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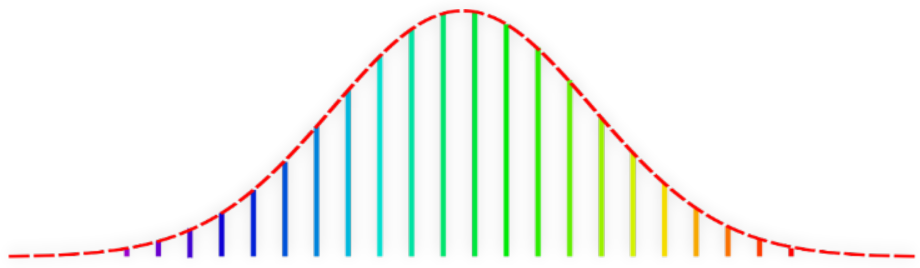

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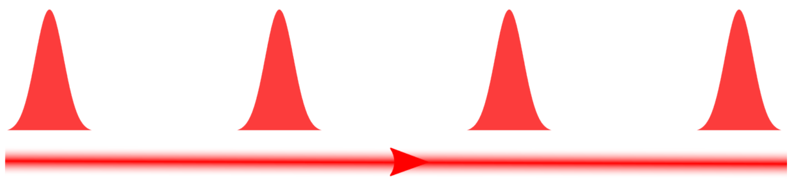
Time domain



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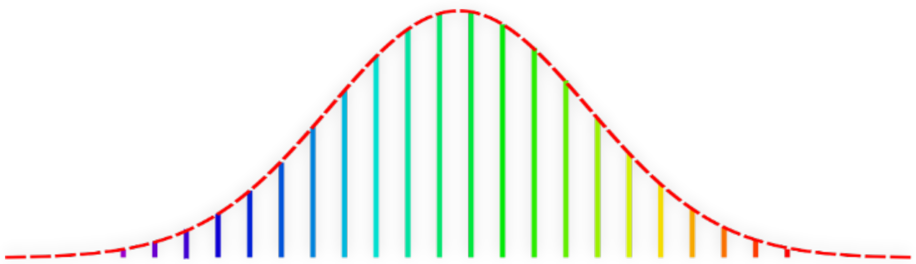

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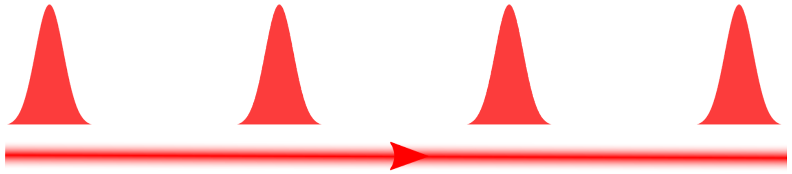
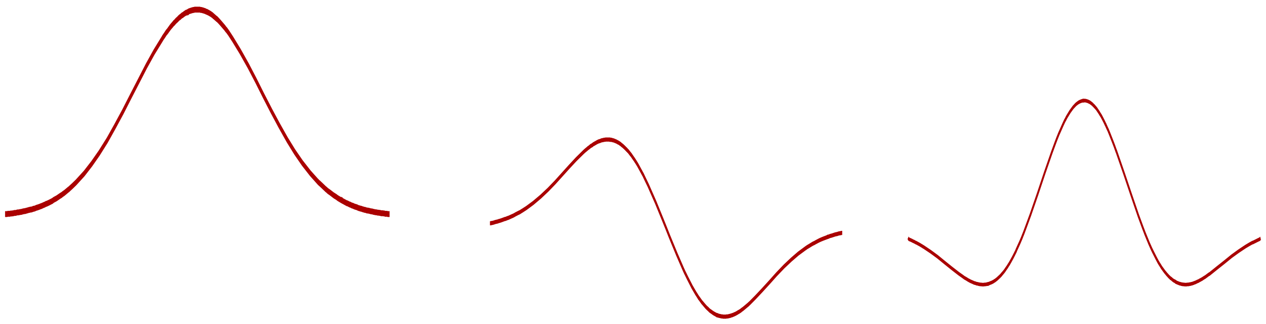


Hermite-Gauss



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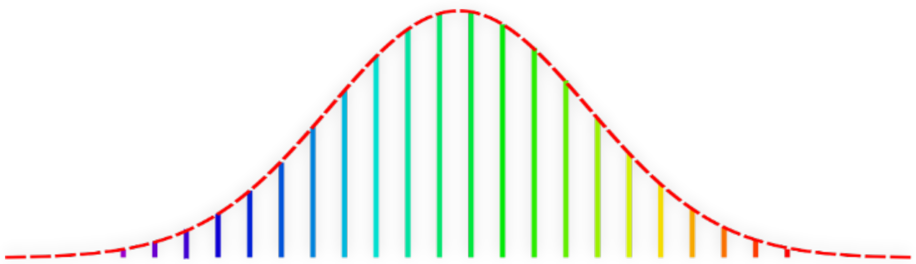

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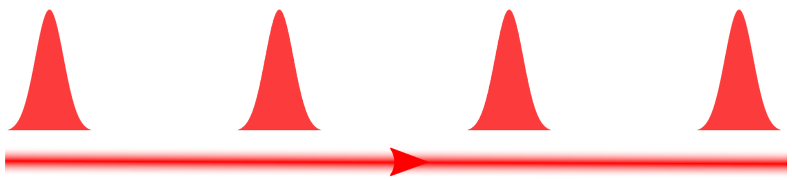

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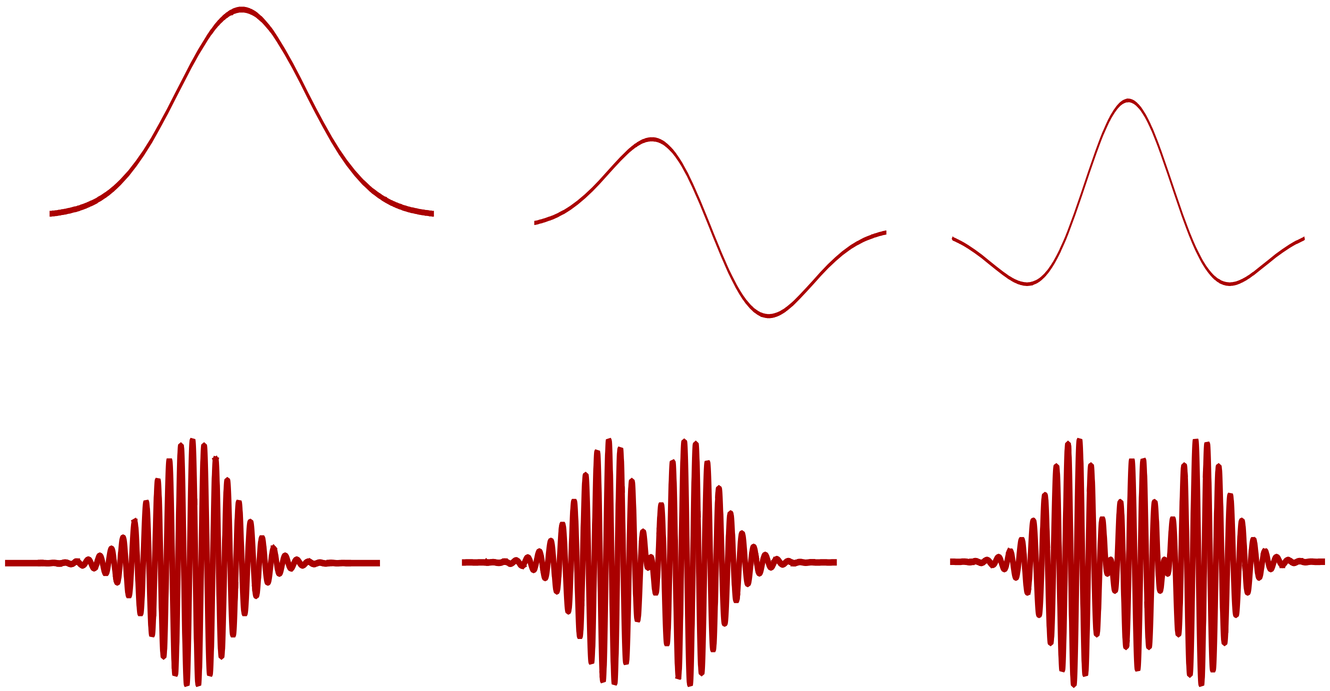

Kinds of modes: Spatial

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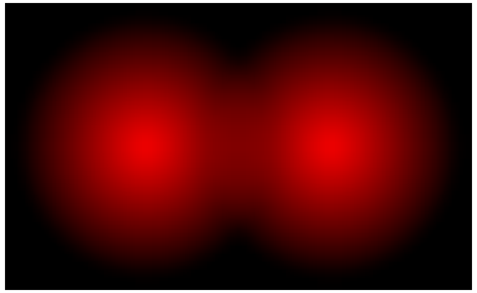

Two light beams in image plane

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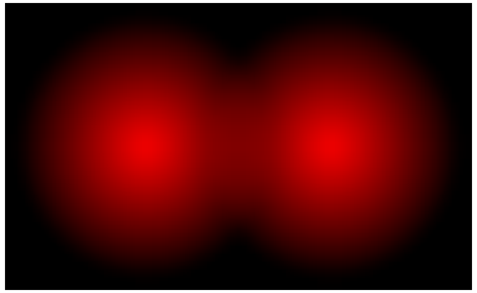

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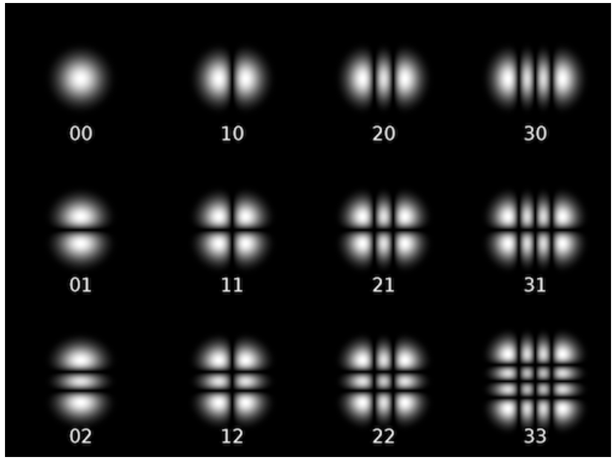

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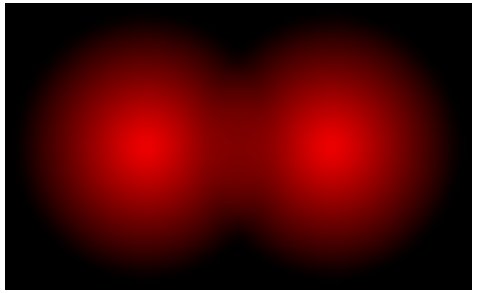

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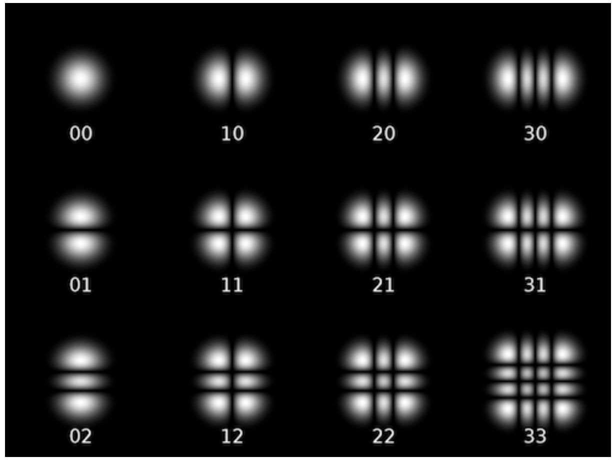
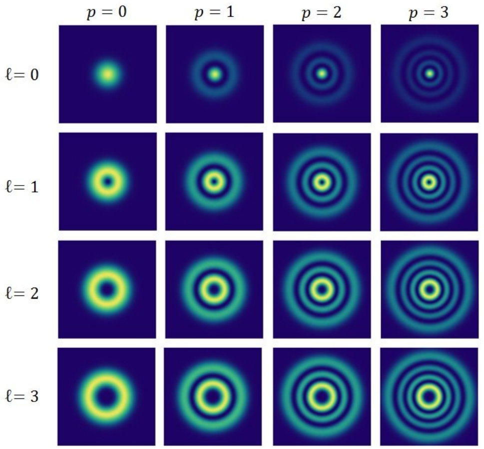


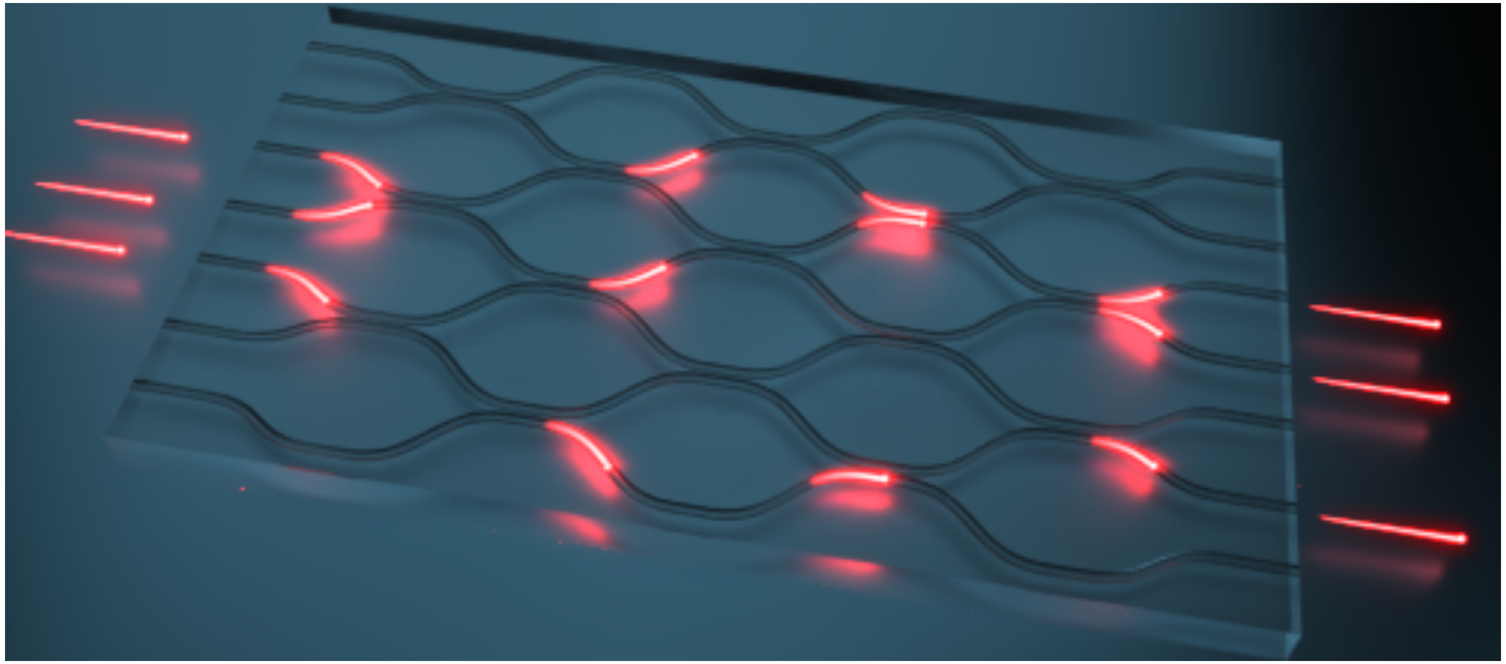
Laguerre-Gauss



Kinds of modes: Spatial

$$\mathbf{E}^{(+)}(\mathbf{r},t)=\sum_{j=1}^{\infty}\mathcal{E}_j\alpha_j\mathbf{u}_j(\mathbf{r})\mathbf{v}(z,t)$$

Guided modes (will play a crucial role in Boson Sampling)



Modes in free space

Light is then described as a superposition of optical modes

$$\mathbf{E}^{(+)}(\mathbf{r},t) = \sum_{j=1}^{\infty} \mathcal{E}_j \alpha_j \mathbf{u}_j(\mathbf{r},t)$$

Complex amplitude

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Complex amplitude

The analytical signal is related to the real electric field is given by

$$\mathbf{E}(\mathbf{r},t) = \mathbf{E}^{(+)}(\mathbf{r},t) + \left(\mathbf{E}^{(+)}\right)^{*}(\mathbf{r},t)$$

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The energy in the light is given by

$$H = \frac{\epsilon_0}{2}\int_V d^3\mathbf{r}\left[|\mathbf{E}(\mathbf{r},t)|^2 + c^2|\mathbf{B}(\mathbf{r},t)|^2\right]$$

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By using Maxwell's equations and some Fourier-style analysis, we find

$$H = 2\epsilon_0 V \sum_j |\mathcal{E}_j|^2 |\alpha_j|^2$$

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Energy per mode

Into the quantum realm

Light is then described as a superposition of optical modes

$$\mathbf{E}^{(+)}(\mathbf{r},t)=\sum_{j=1}^{\infty}\mathcal{E}_j\alpha_j\mathbf{u}_j(\mathbf{r},t)$$

Hamiltonian of the systems

$$H=2\epsilon_0 V\sum_j|\mathcal{E}_j|^2|\alpha_j|^2$$

Into the quantum realm

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$$\hat{\mathbf{E}}^{(+)}(\mathbf{r},t) = \sum_{j=1}^{\infty} \mathcal{E}_j \hat{a}_j \mathbf{u}_j(\mathbf{r},t)$$

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Satisfy canonical commutation relation

$$[\hat{a}_j, \hat{a}_k^\dagger] = \delta_{j,k}$$

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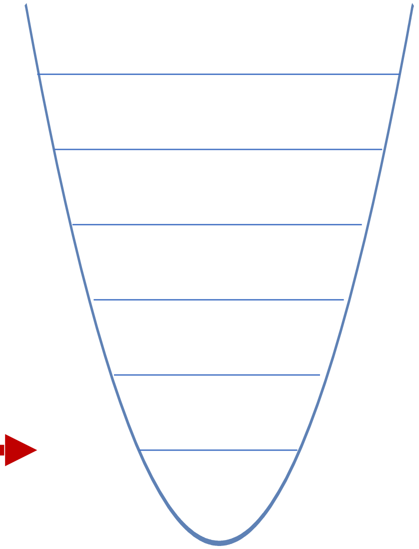

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Modes and photons

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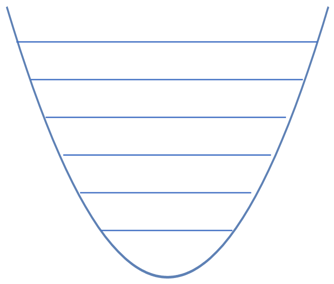

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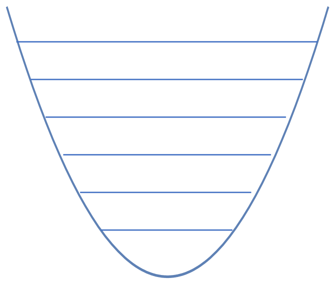

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$\hat{a}_j$ is the annihilation operator of a **photon** in mode $\mathbf{u}_j(\mathbf{r},t)$

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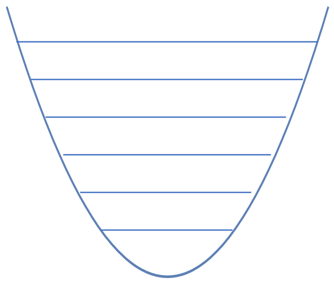

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Bases are not unique, we could describe the same light with a different mode basis

$$\hat{\mathbf{E}}^{(+)}(\mathbf{r},t)=\sum_{j}\mathcal{E}_j\hat{b}_j\mathbf{v}_j(\mathbf{r},t)$$

Modes and photons

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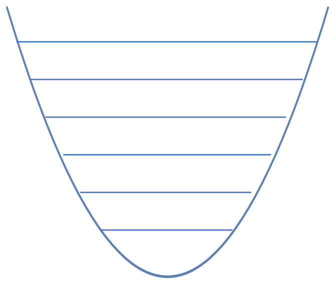
$$\hat{\mathbf{E}}^{(+)}(\mathbf{r},t) = \sum_{j=1}^{\infty} \mathcal{E}_j \hat{a}_j \mathbf{u}_j(\mathbf{r},t) \Longleftrightarrow \hat{\mathbf{E}}^{(+)}(\mathbf{r},t) = \sum_{j} \mathcal{E}_j \hat{b}_j \mathbf{v}_j(\mathbf{r},t)$$



How do we change mode basis?

Modes and photons

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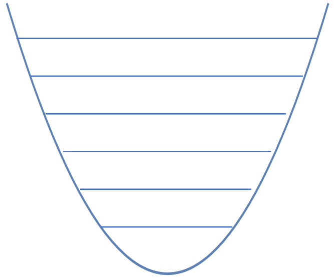

How do we change mode basis?

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Because *u*-modes form a basis

Modes and photons

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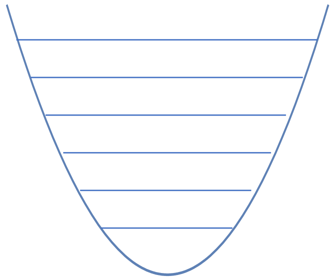

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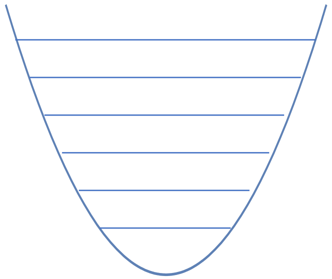
Because *u*-modes form a basis

$$\Rightarrow \quad \hat{a}_k=\sum_j U_{jk}\hat{b}_j$$

Modes and photons

Light is then described as a superposition of optical modes

$$\hat{\mathbf{E}}^{(+)}(\mathbf{r},t) = \sum_{j=1}^{\infty} \mathcal{E}_j \hat{a}_j \mathbf{u}_j(\mathbf{r},t) \Longleftrightarrow \hat{\mathbf{E}}^{(+)}(\mathbf{r},t) = \sum_{j} \mathcal{E}_j \hat{b}_j \mathbf{v}_j(\mathbf{r},t)$$



How do we change mode basis?

$$\mathbf{v}_j(\mathbf{r},t) = \sum_k U_{jk} \mathbf{u}_k(\mathbf{r},t) \quad \text{with} \quad U_{jk} = \frac{1}{V}\int_V \mathbf{u}_j^*(\mathbf{r},t)\mathbf{v}_k(\mathbf{r},t) d^3\mathbf{r}$$

Because *u*-modes form a basis

$$\Rightarrow \quad \hat{a}_k = \sum_j U_{jk} \hat{b}_j \quad \Rightarrow \quad \hat{b}_j^\dagger = \sum_k U_{jk} \hat{a}_k^\dagger$$

The beamsplitter

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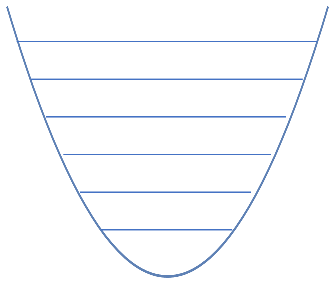

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Example

The beamsplitter

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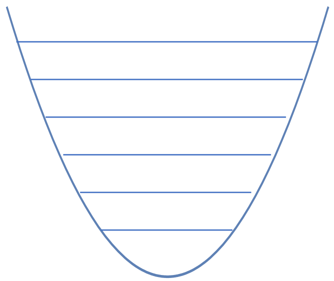

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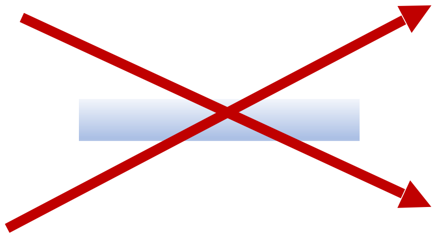

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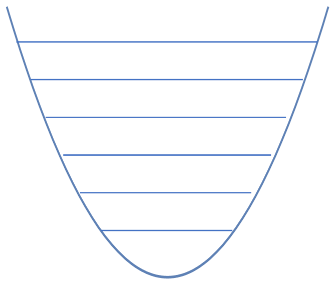

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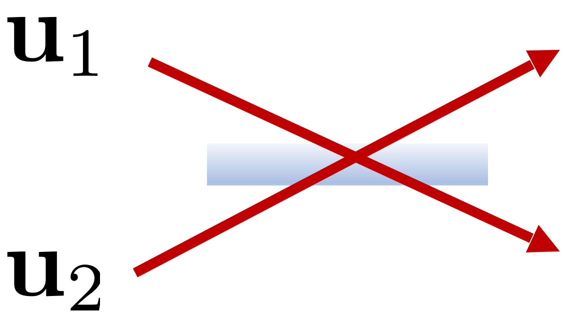

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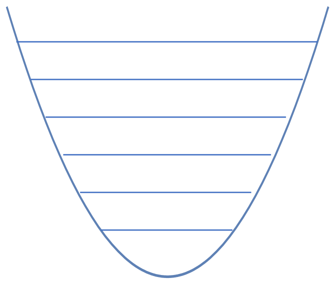

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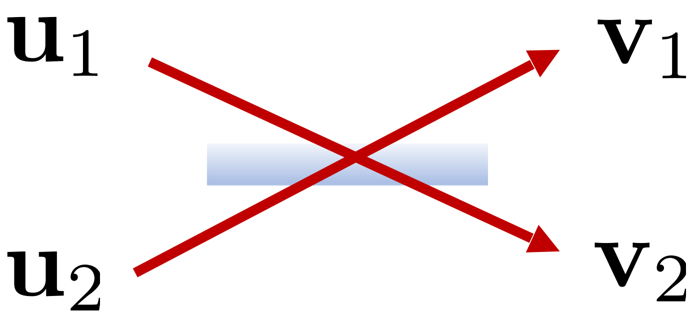

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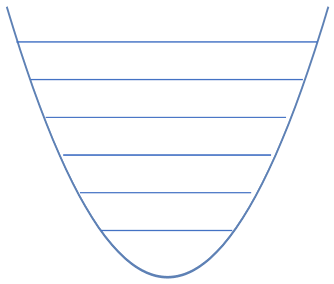

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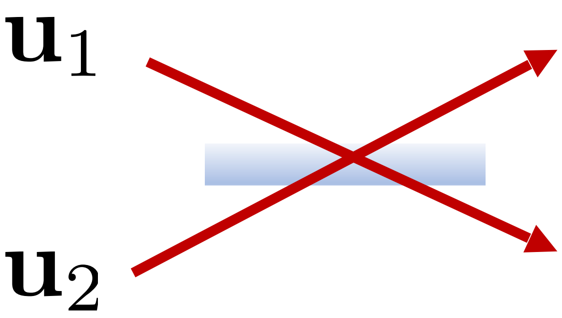

$$\mathbf{v}_1 = \frac{1}{\sqrt{2}}(\mathbf{u_1} + \mathbf{u_2})$$

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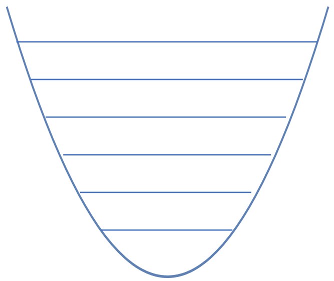

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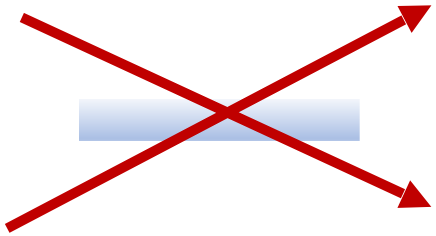
Example

$\mathbf{u}_1$



$\mathbf{u}_2$

$$\mathbf{v}_1 = \frac{1}{\sqrt{2}}(\mathbf{u_1}+\mathbf{u_2})$$

$$\mathbf{v}_2 = \frac{1}{\sqrt{2}}(\mathbf{u_1}-\mathbf{u_2})$$

$$U = \frac{1}{\sqrt{2}}\begin{pmatrix}1 & 1\\ 1 & -1\end{pmatrix}$$

Modes and photons

Superpositions of modes form new modes

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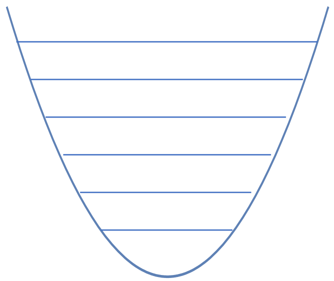

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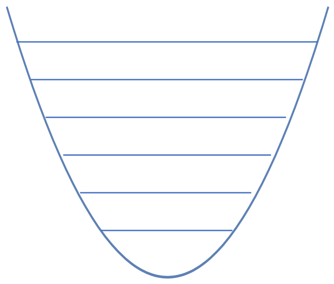

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$$\hat{a}^\dagger(f) = \sum_k f_k \hat{a}_k^\dagger$$

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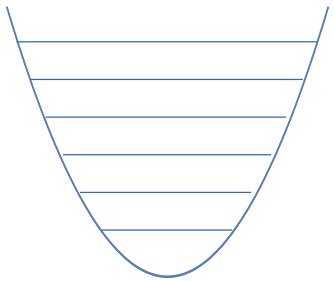

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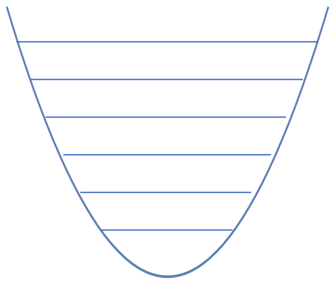

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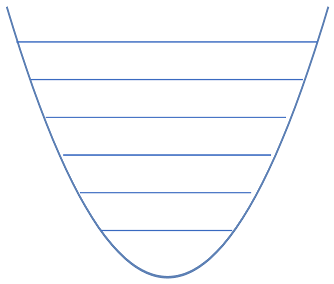

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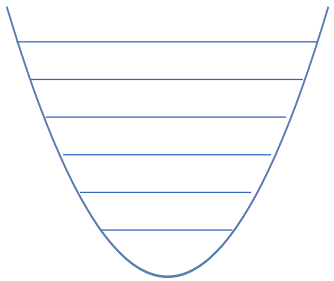

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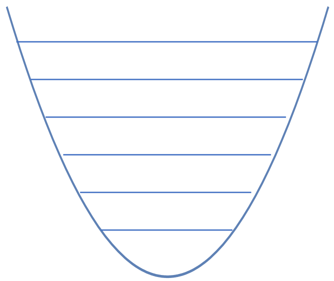

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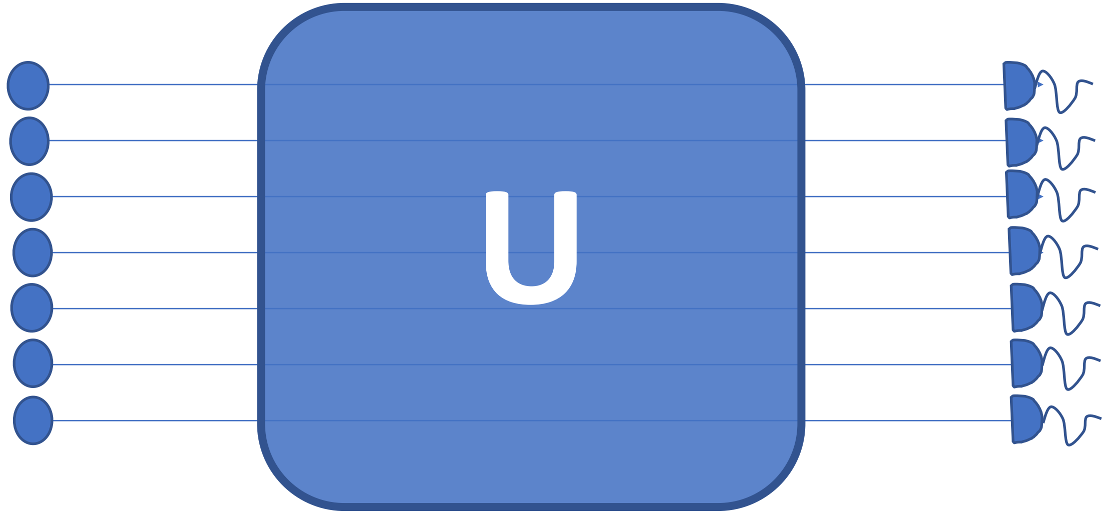
Just a short-hand notation

Modes and sampling problems

Changes of mode basis are a brick in a typical sampling problem



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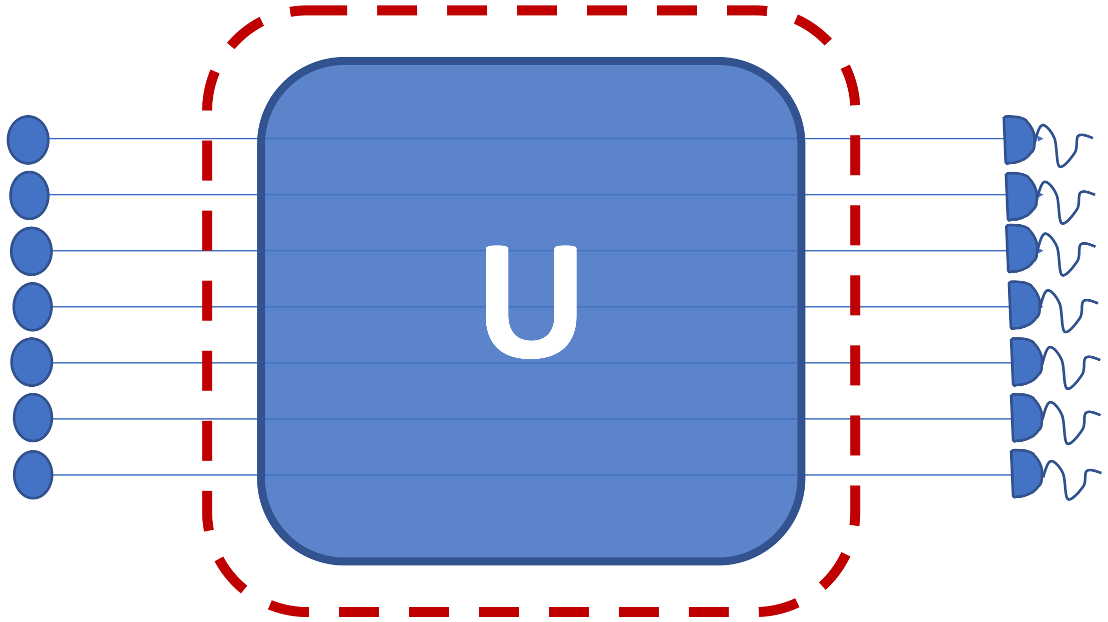

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What remains to be understood is the states

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... and the measurement

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$$\forall m, n, f_1, \dots, f_n$$

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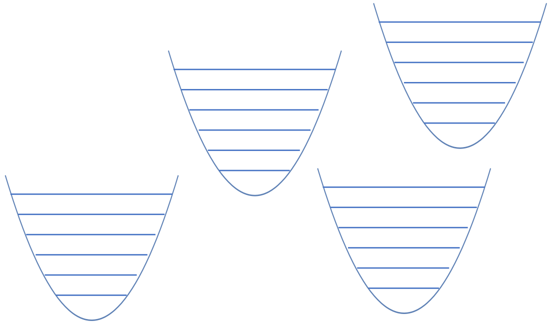

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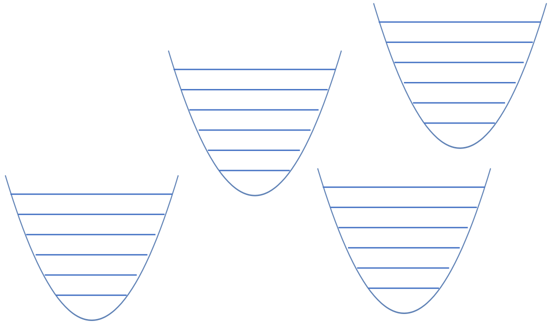

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Fock states are created by adding a finite number of photons to the system in arbitrary modes **f**$_1$, …, **f**$_{n.}$

$$|\Phi\rangle = \frac{1}{\mathcal{N}}\hat{a}^\dagger(f_1)\ldots\hat{a}^\dagger(f_n)|0\rangle$$

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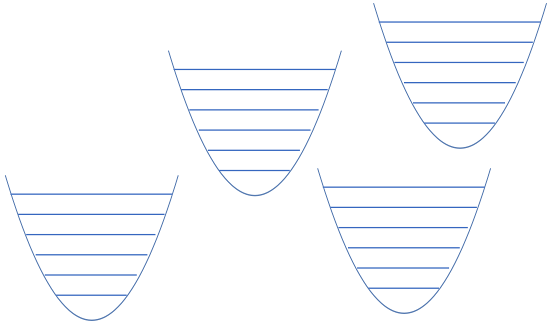

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The full Hilbert space of the bosonic system[1] is generated by the closure of the span of these states.

[1] to be exact, they generate the Fock representation of the algebra of observables

Modes and photons

Key idea: we prepare a state in one mode basis and we measure it in a different mode basis

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The measurement?

Number of photons

A key observable is the **number operator** in mode **f**

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Use $[\hat{a}(f), \hat{a}^\dagger(f)] = (f, f) = 1$

to show

$$\hat{n}(f)\frac{1}{\sqrt{n!}}\hat{a}^\dagger(f)^n |0\rangle = n\frac{1}{\sqrt{n!}}\hat{a}^\dagger(f)^n |0\rangle$$

Furthermore, when $(f, g) = 0$

$$\hat{n}(f)\frac{1}{\sqrt{n'!}}\hat{a}^\dagger(g)^{n'} |0\rangle = 0$$

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The number operator for the full system is

$$\hat{N} = \sum_k \hat{n}(u_k)$$

Some mode basis

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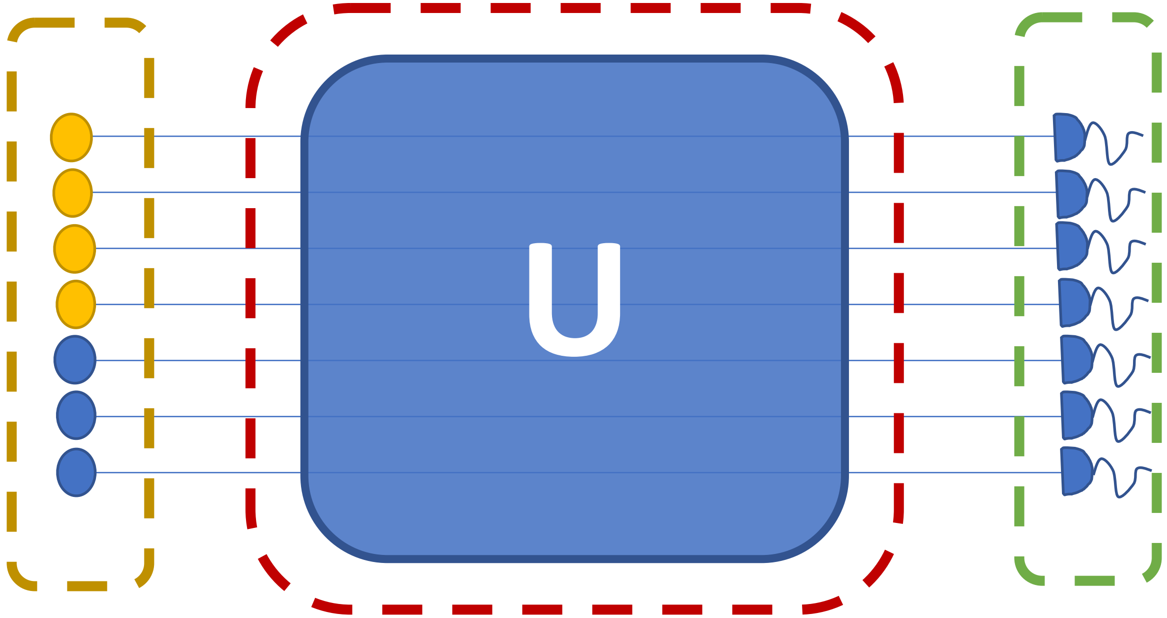
Fock states are energy eigenstates and measuring the energy in a specific mode projects on Fock states

Modes and photons

Prepare a state in one mode basis and we measure it in a different mode basis

Our states are of the form $\hat{a}^\dagger(e_1)\ldots\hat{a}^\dagger(e_n)\,|0\rangle$

Detection events with at most one photon per detector $\langle 0|\,\hat{a}(e'_{j_1})\ldots\hat{a}(e'_{j_n})$



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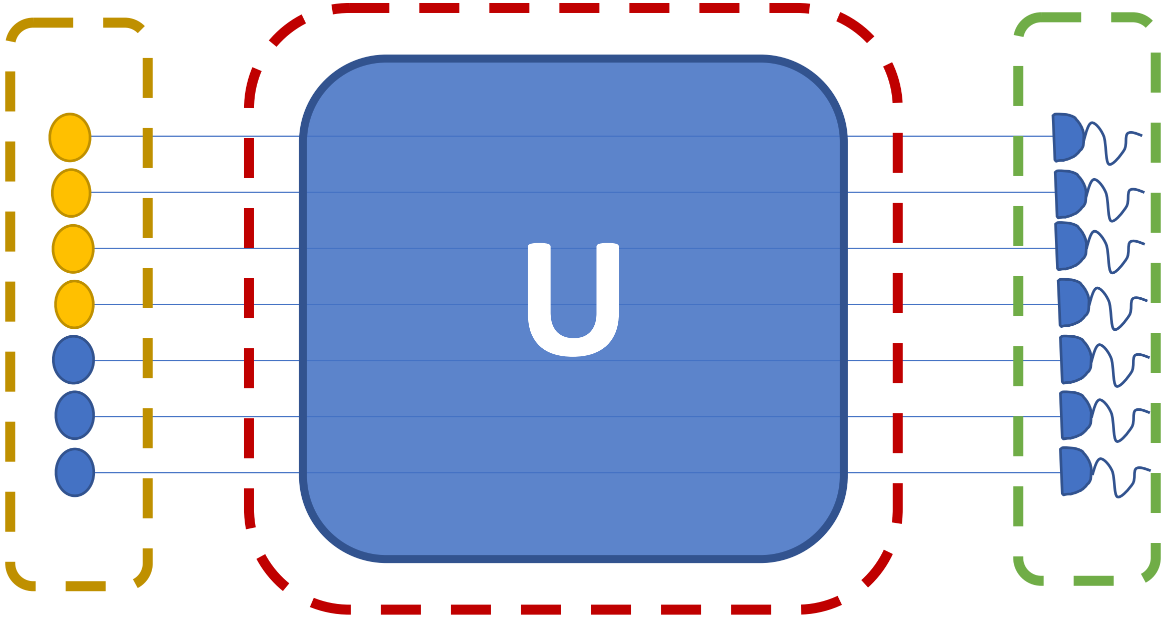
Recap

Input mode basis
$\{\mathbf{e}_k(\mathbf{r},t)|k=1,\ldots,m\}$

Output mode basis
$\{\mathbf{e}'_k(\mathbf{r},t)|k=1,\ldots,m\}$

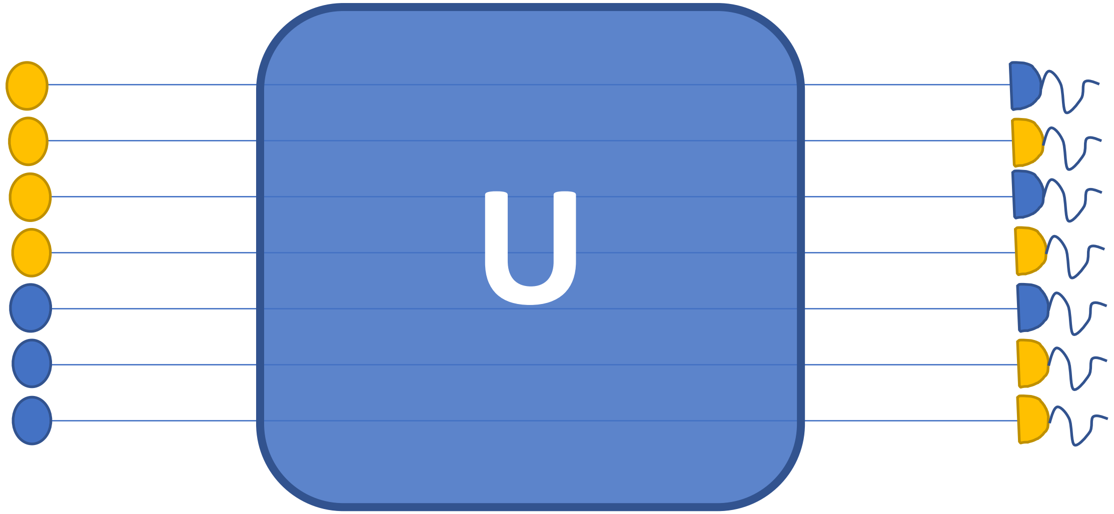
$$U_{k,l}=\frac{1}{V}\int_V \mathbf{e}^*_k(\mathbf{r},t)\mathbf{e}'_l(\mathbf{r},t)d^3\mathbf{r}$$



Overview

- Modes and states in optics
- Boson sampling
- Continuous-variable approach
- Requirements for quantum computational advantage

Transfer probability



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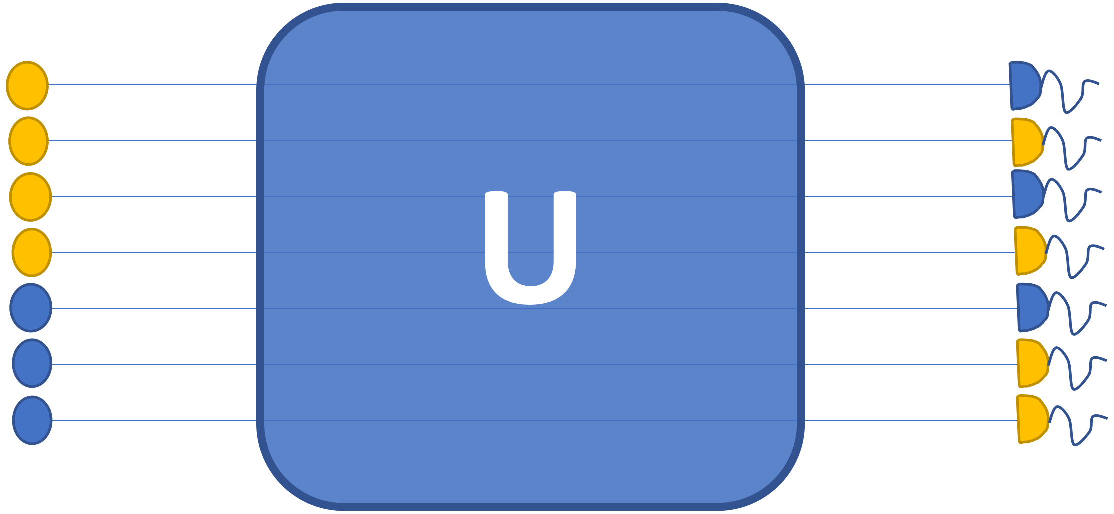

What is the probability to find photons in *output* modes j_1, …, j_n, given that we started with photon in *input* modes 1, …, n?

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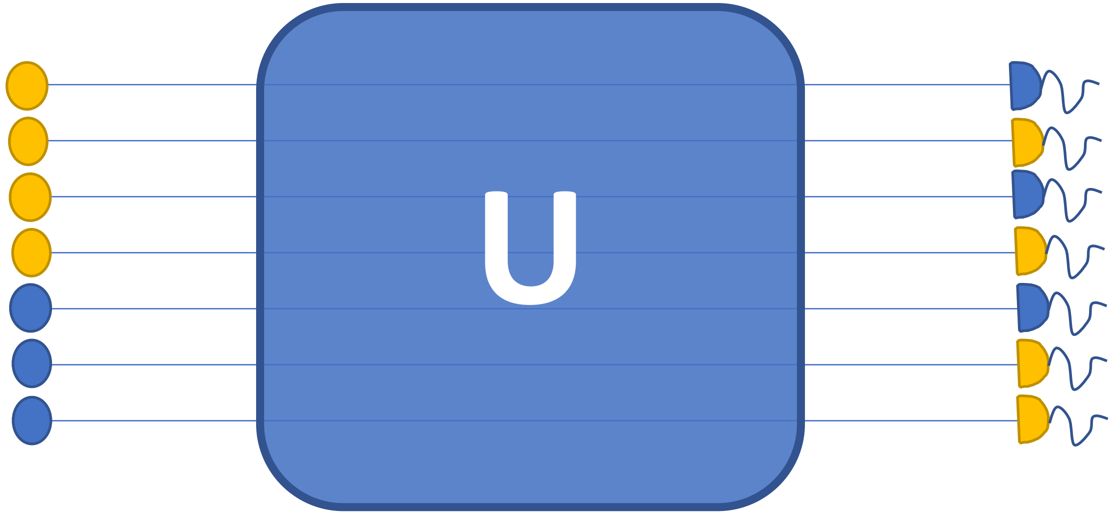

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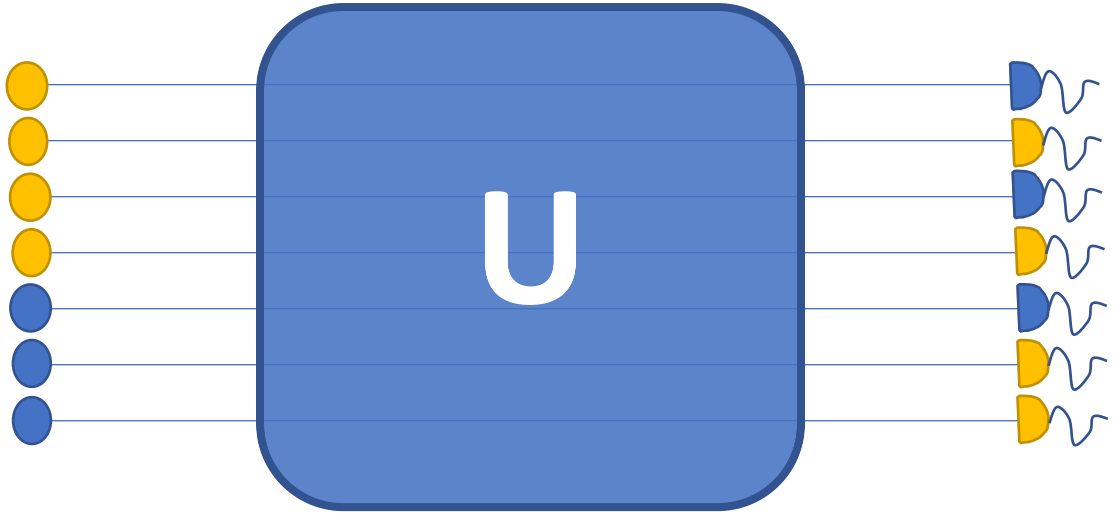
Calculate the overlap with the measurement state to find

$$P(e'_{j_1}, \dots e'_{j_n} | e_1, \dots e_n) = \left| \langle 0 | \hat{a}(e'_{j_1}) \dots \hat{a}(e'_{j_n}) \hat{a}^\dagger(e_1) \dots \hat{a}^\dagger(e_n) | 0 \rangle \right|^2$$

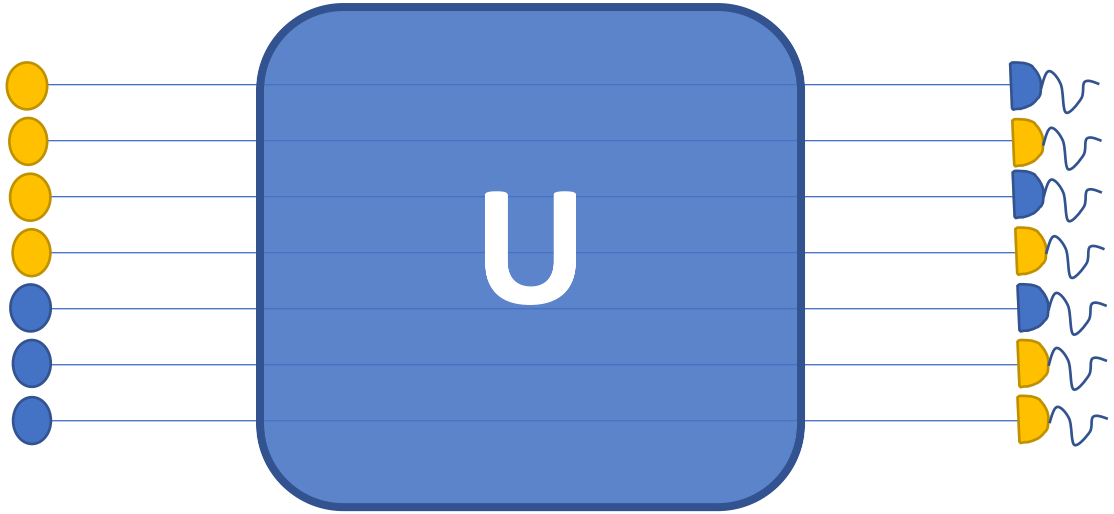
Transfer probability



What is the probability to find photons in *output* modes j_1, …, j_n, given that we started with photon in *input* modes 1, …, n?

$$P(e'_{j_1},\dots e'_{j_n}|e_1,\dots e_n) = \left|\langle 0|\,\hat{a}(e'_{j_1})\dots\hat{a}(e'_{j_n})\hat{a}^\dagger(e_1)\dots\hat{a}^\dagger(e_n)\,|0\rangle\right|^2$$

Transfer probability



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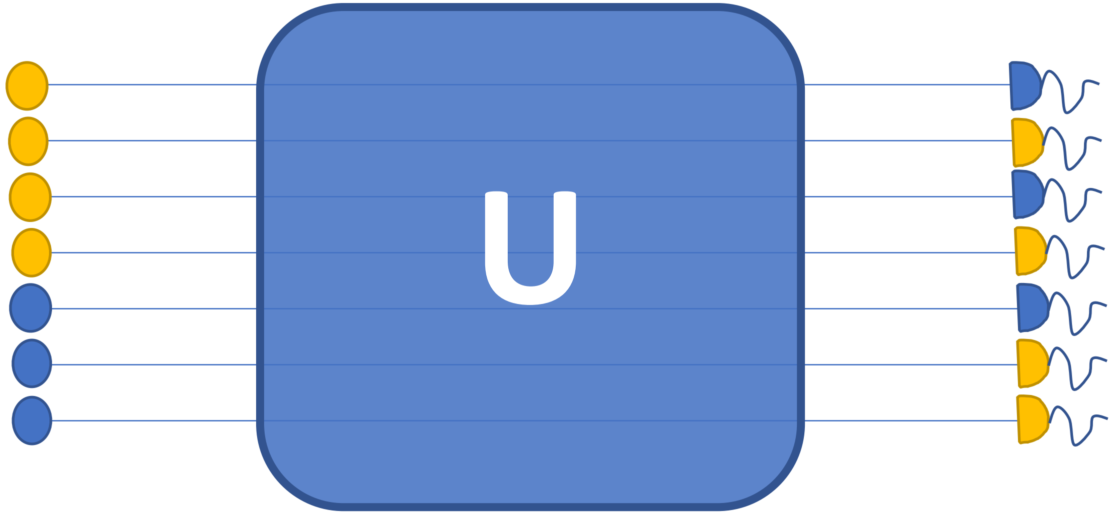
Use canonical commutation relation $[\hat{a}(e'_k),\hat{a}^\dagger(e_l)] = \frac{1}{V}\int_V \mathbf{e}'^*_k(\mathbf{r},t)\mathbf{e}_l(\mathbf{r},t)d^3\mathbf{r}$

Transfer probability



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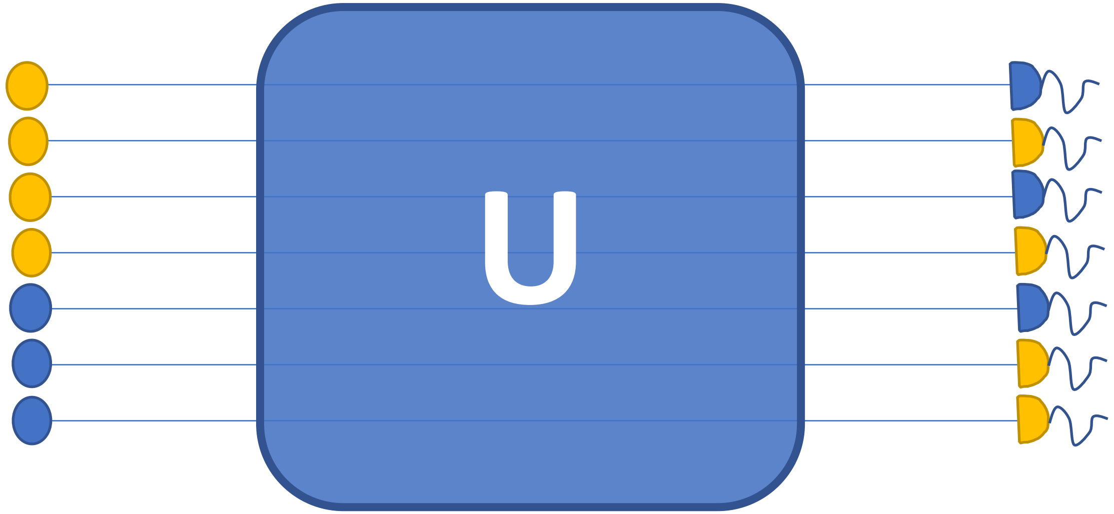
Transfer probability



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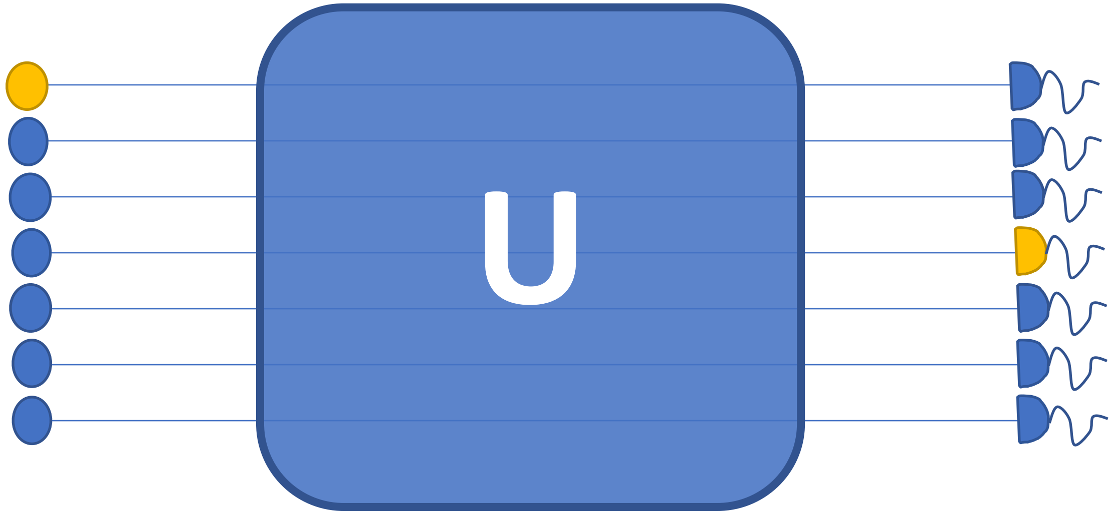
+ Wick contractions

Example: n = 1



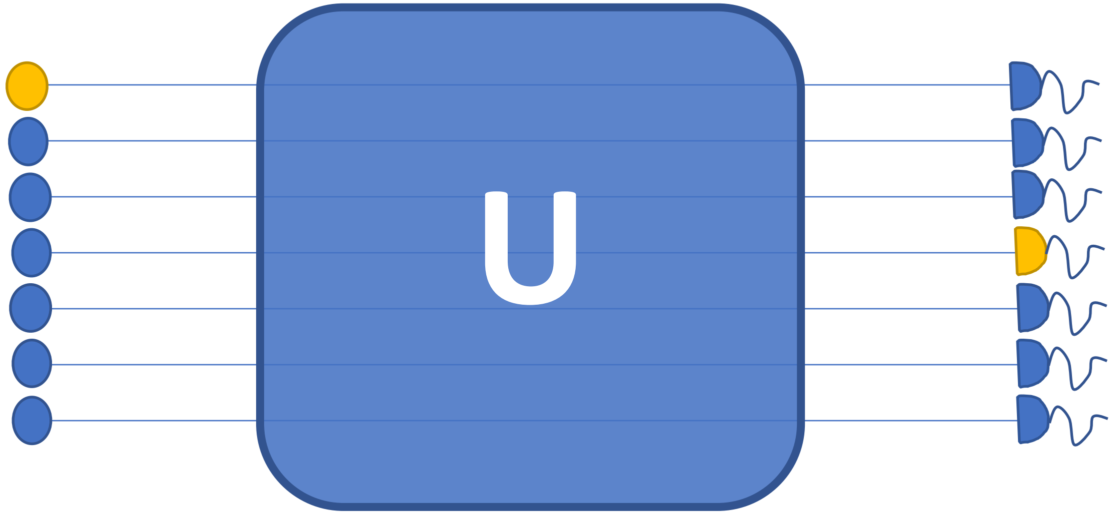
Use canonical commutation relation

$$[\hat{a}(e'_k), \hat{a}^\dagger(e_l)] = U_{kl}$$

What is the probability to find photons in *output* modes j, given that we started with photon in *input* modes 1?

$$P(e'_j|e_1) = \left|\langle 0|\, \hat{a}(e'_j)\hat{a}^\dagger(e_1)\,|0\rangle\right|^2$$

Example: n = 1



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What is the probability to find photons in *output* modes j, given that we started with photon in *input* modes 1?

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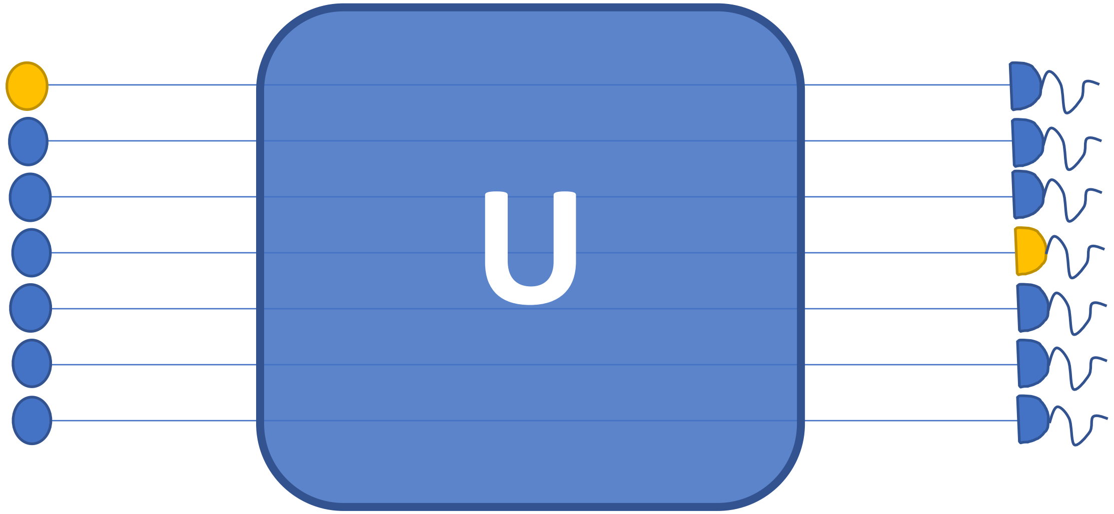
$$= \left| \langle 0| \left[\hat{a}^\dagger(e_1)\hat{a}(e'_j) + U_{j1}\right] |0\rangle \right|^2$$

Example: n = 1



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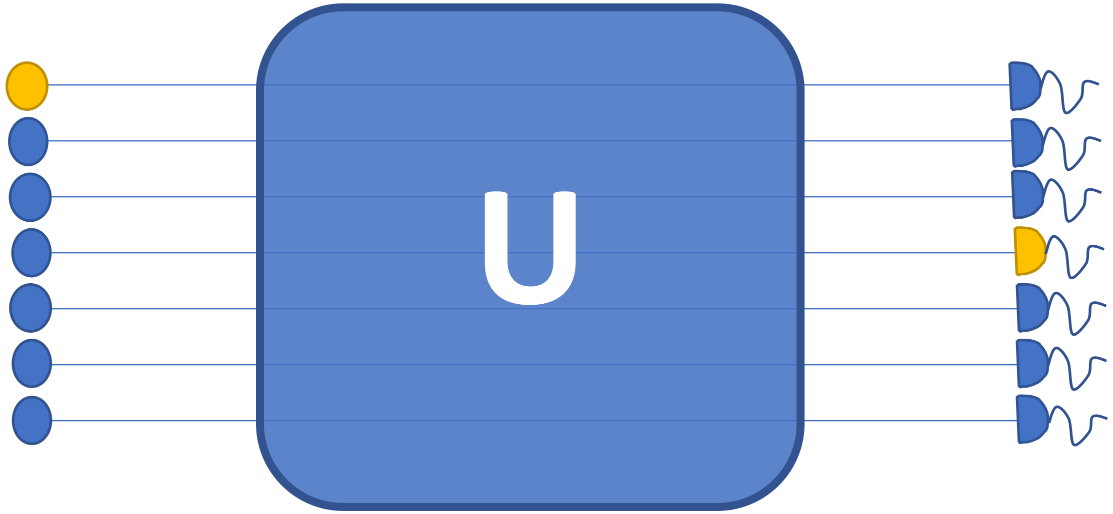

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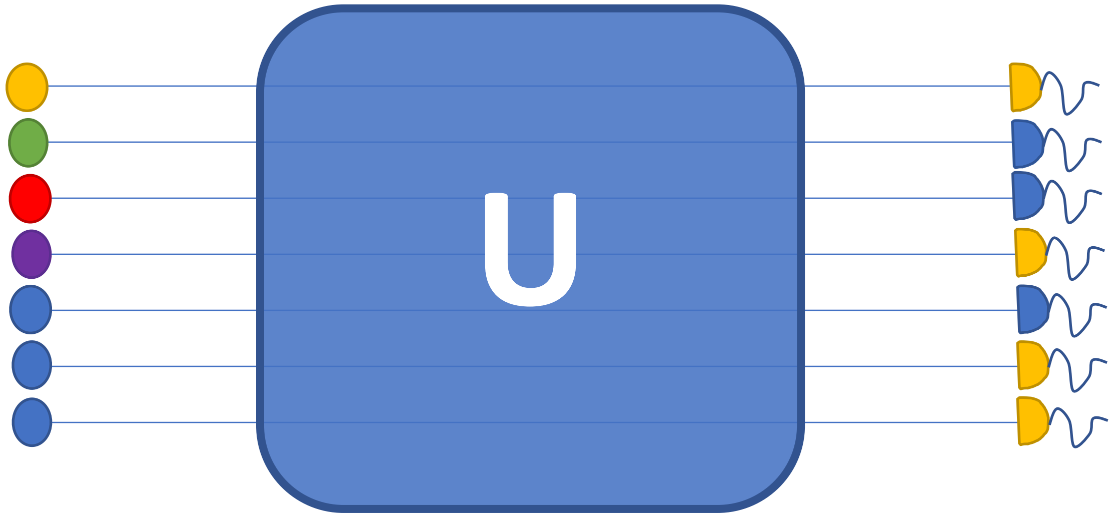
$$= \left| \langle 0| \left[\cancel{\hat{a}^\dagger(e_1)\hat{a}(e'_j)} + U_{j1} \right] |0\rangle \right|^2$$

$$= \left| U_{j1} \right|^2$$

Classical particles

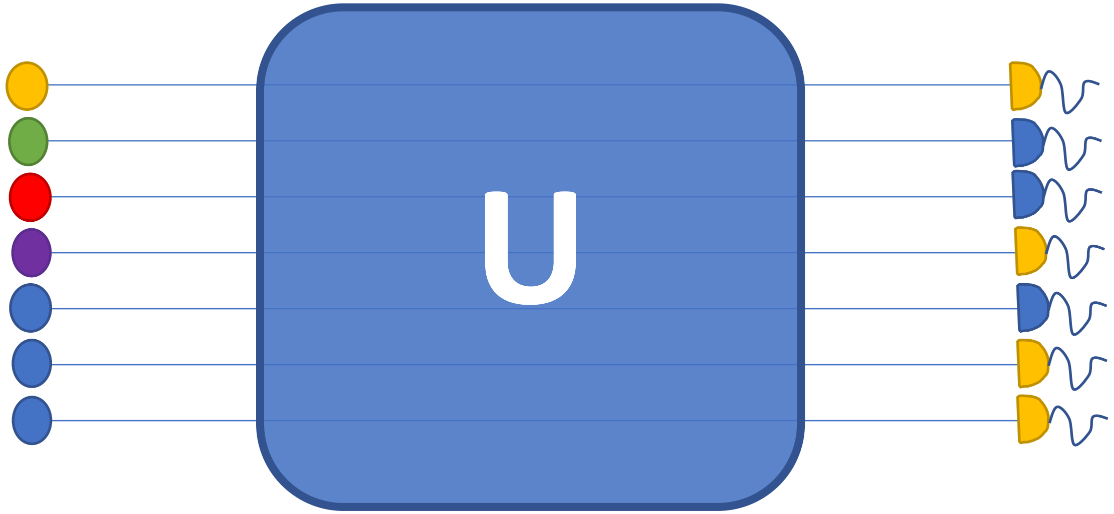


Use canonical commutation relation

$$[\hat{a}(e'_k), \hat{a}^\dagger(e_l)] = U_{kl}$$

What is the probability to find classical particles in *output* modes j_1, …, j_n, given that we started with classical particles in *input* modes 1, …, n?

Classical particles



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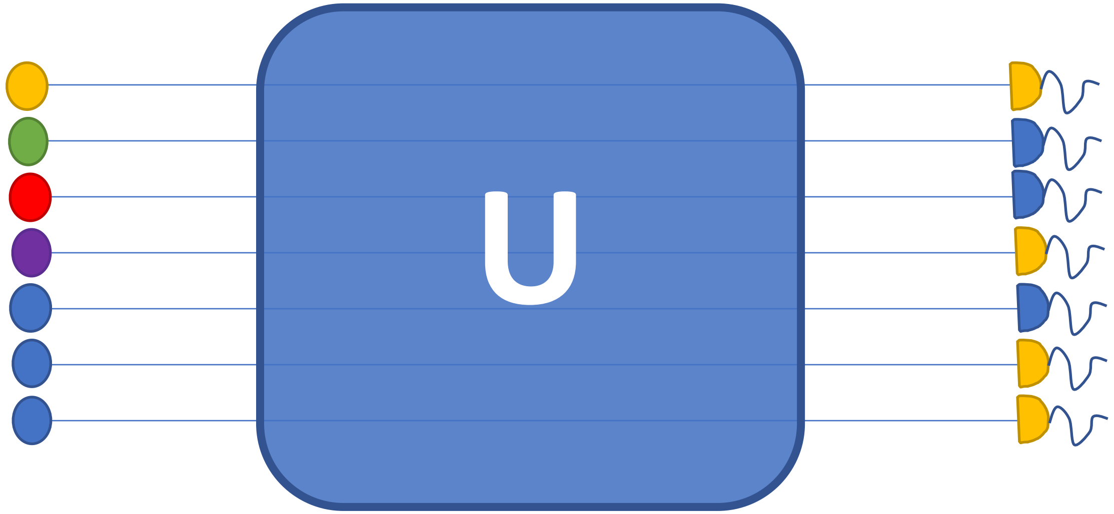
$$[\hat{a}(e'_k), \hat{a}^\dagger(e_l)] = U_{kl}$$

What is the probability to find classical particles in *output* modes j_1, …, j_n, given that we started with classical particles in *input* modes 1, …, n?

We assume that the detectors do not see the difference between these particles

$$P(e'_{j_1}, \ldots e'_{j_n} | e_1, \ldots e_n) = \sum_{\sigma \in S_n} P(e'_{j_1} | e_{\sigma(1)}) \ldots P(e'_{j_n} | e_{\sigma(n)})$$

Classical particles



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What is the probability to find classical particles in *output* modes $j_1, \ldots, j_n$, given that we started with classical particles in *input* modes $1, \ldots, n$?

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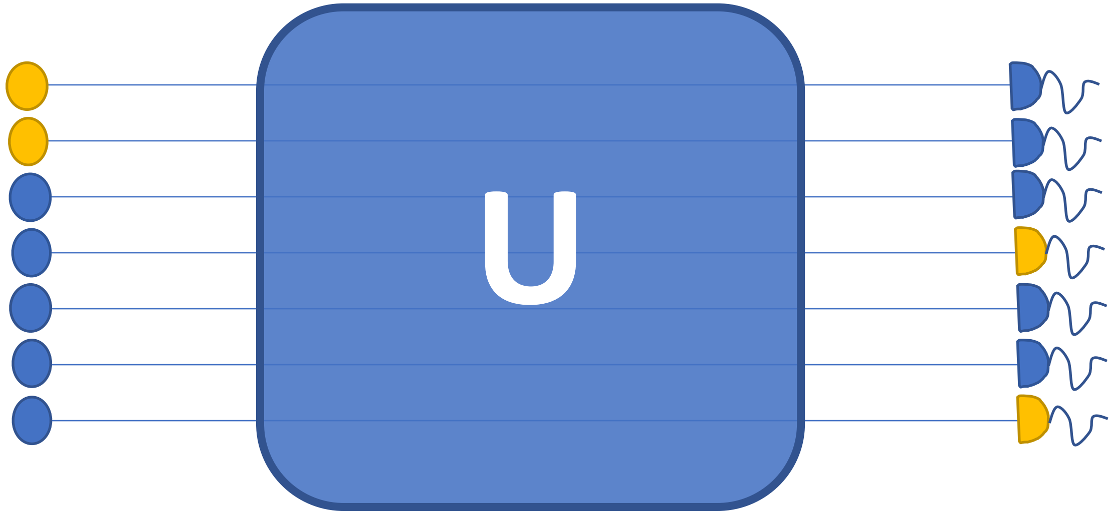
$$P(e'_{j_1}, \ldots e'_{j_n} | e_1, \ldots e_n) = \sum_{\sigma \in S_n} P(e'_{j_1} | e_{\sigma(1)}) \ldots P(e'_{j_n} | e_{\sigma(n)})$$

$$= \sum_{\sigma \in S_n} \left| U_{j_1 \sigma(1)} \right|^2 \ldots \left| U_{j_n \sigma(n)} \right|^2$$

Example: n = 2



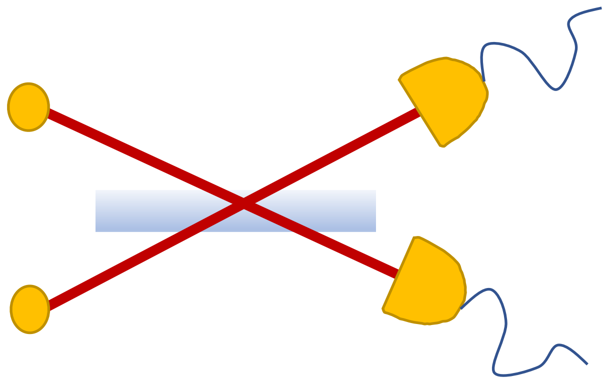
Use canonical commutation relation

$$[\hat{a}(e'_k), \hat{a}^\dagger(e_l)] = U_{kl}$$

What is the probability to find photons in *output* modes j_1 and j_2, given that we started with photon in *input* modes 1 and 2?

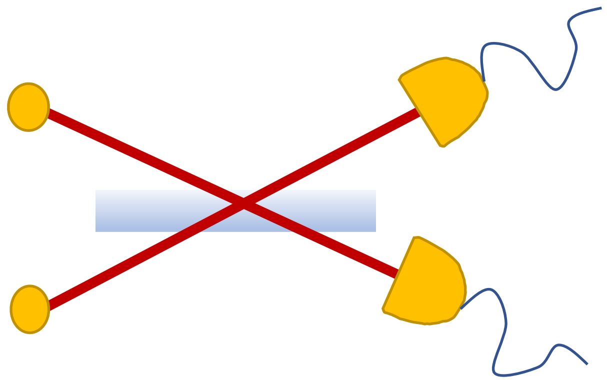
$$\begin{aligned}
P(e'_{j_1}, e'_{j_2}|e_1, e_2) &= \left|\langle 0|\,\hat{a}(e'_{j_1})\hat{a}(e'_{j_2})\hat{a}^\dagger(e_1)\hat{a}^\dagger(e_2)\,|0\rangle\right|^2 \\
&= \left|\langle 0|\,\hat{a}(e'_{j_1})[\hat{a}^\dagger(e_1)\hat{a}(e'_{j_2}) + U_{j_2 1}]\hat{a}^\dagger(e_2)\,|0\rangle\right|^2 \\
&= \left|\langle 0|\,\hat{a}(e'_{j_1})\hat{a}^\dagger(e_1)\hat{a}(e'_{j_2})\hat{a}^\dagger(e_2)\,|0\rangle + U_{j_2 1}\,\langle 0|\,\hat{a}(e'_{j_1})\hat{a}^\dagger(e_2)\,|0\rangle\right|^2 \\
&= \left|\langle 0|\,\hat{a}(e'_{j_1})\hat{a}^\dagger(e_1)\hat{a}(e'_{j_2})\hat{a}^\dagger(e_2)\,|0\rangle + U_{j_2 1}U_{j_1 2}\right|^2 \\
&= \left|U_{j_1 1}\,\langle 0|\,\hat{a}(e'_{j_2})\hat{a}^\dagger(e_2)\,|0\rangle + U_{j_2 1}U_{j_1 2}\right|^2 \\
&= \left|U_{j_1 1}U_{j_2 2} + U_{j_2 1}U_{j_1 2}\right|^2
\end{aligned}$$

Example: simple Hong-Ou-Mandel



What is the probability to find photons in *output* modes 1 and 2, given that we started with photon in *input* modes 1 and 2?

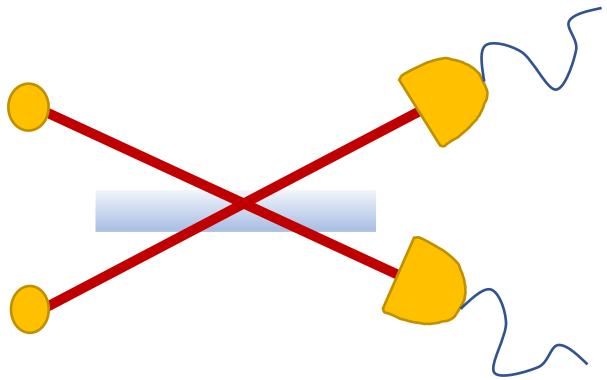
Example: simple Hong-Ou-Mandel



What is the probability to find photons in *output* modes 1 and 2, given that we started with photon in *input* modes 1 and 2?

$$P(e_1', e_2' | e_1, e_2) = |U_{11}U_{22} + U_{21}U_{12}|^2$$

Example: simple Hong-Ou-Mandel



$$U = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$$

What is the probability to find photons in *output* modes 1 and 2, given that we started with photon in *input* modes 1 and 2?

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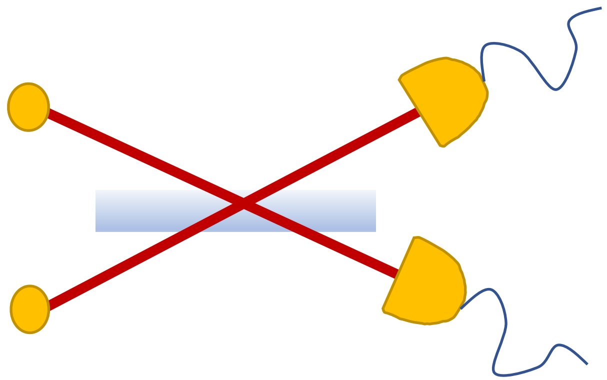

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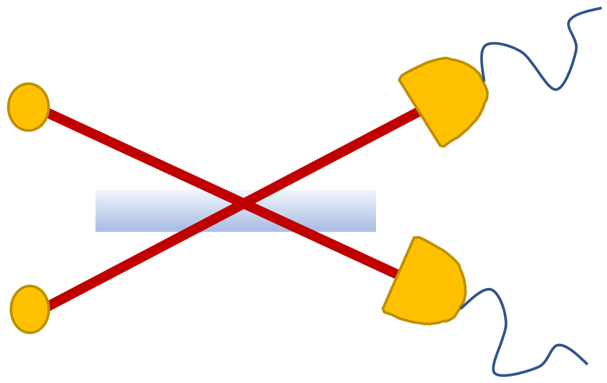

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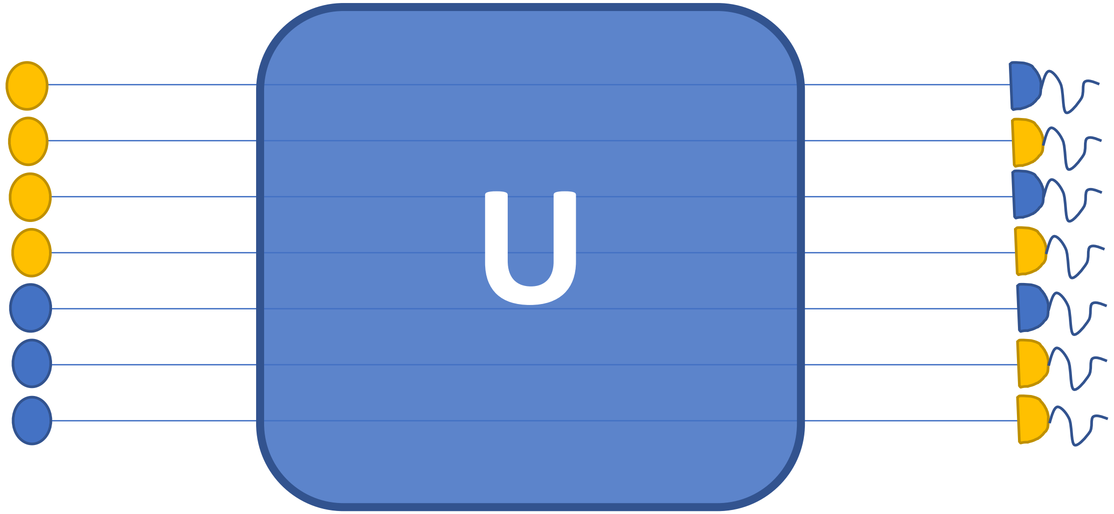
$$= 0$$

We never detect one photon in each output mode, they **always bunch together**

Boson sampling



$$[\hat{a}(e'_k), \hat{a}^\dagger(e_l)] = U_{kl}$$

What is the probability to find photons in *output* modes j_1, …, j_n, given that we started with photon in *input* modes 1, …, n?

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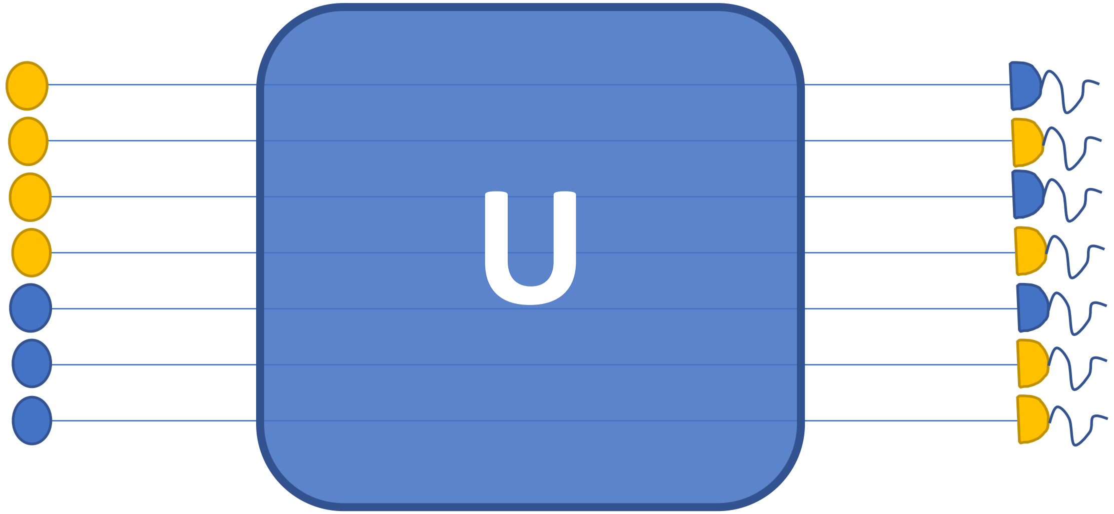
Boson sampling



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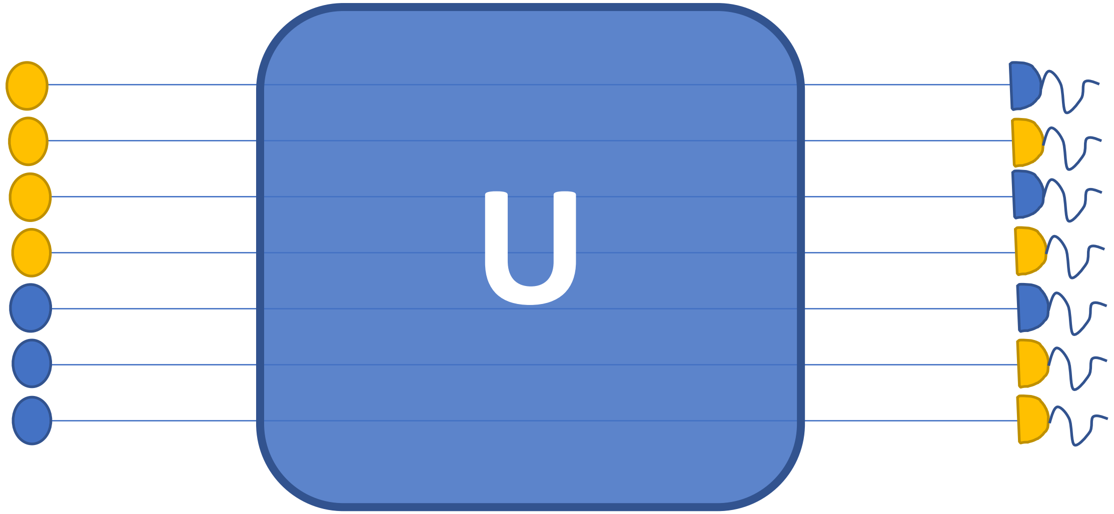
$$\Rightarrow \quad P(e'_{j_1}, \ldots, e'_{j_n} | e_1, \ldots, e_n) = \left| \sum_{\sigma \in S_n} U_{\sigma(1) j_1} \ldots U_{\sigma(n) j_n} \right|^2$$

Boson sampling



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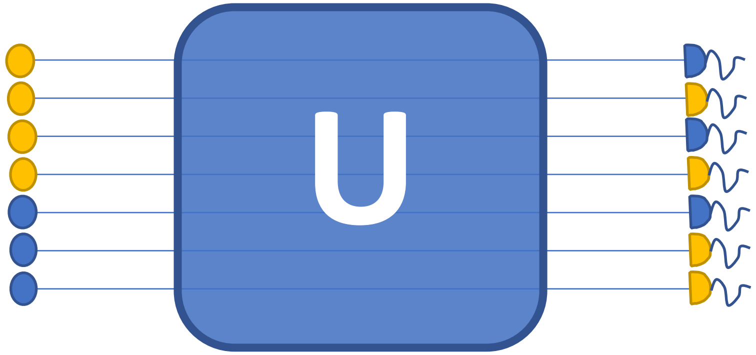
$$\Rightarrow \quad P(e'_{j_1}, \dots, e'_{j_n} | e_1, \dots, e_n) = \left| \mathrm{prem}\, U_{\mathrm{sub}} \right|^2$$

Hardness of boson sampling

[Aaronson & Arkhipov arXiv:1011.3245]



Why are people so excited about boson sampling?

$$P(e'_{j_1}, \ldots, e'_{j_n} | e_1, \ldots, e_n) = \left|\mathrm{prem}\, U_{\mathrm{sub}}\right|^2$$

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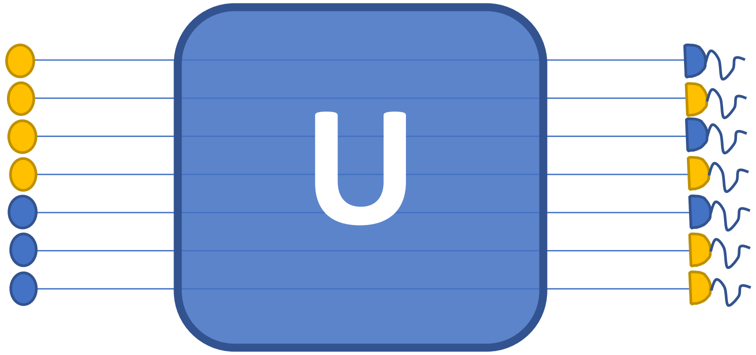

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Permanents are ***typically*** hard (#P) to calculate

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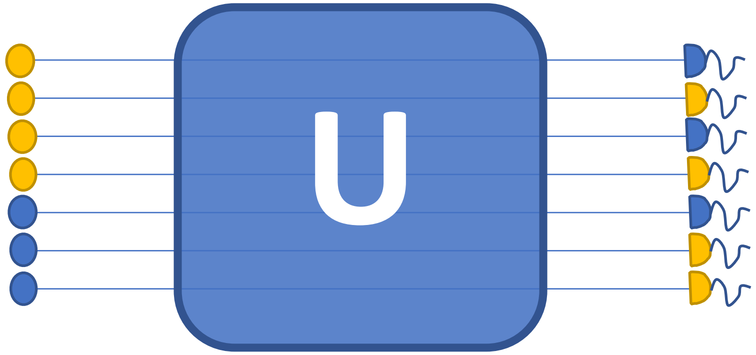

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Idea of Aaronson and Arkhipov: take U to be random (Haar measure) with number of photons *n* >> number of modes *m*

$$U_{\text{sub}} \sim \text{Ginibre ensemble}$$

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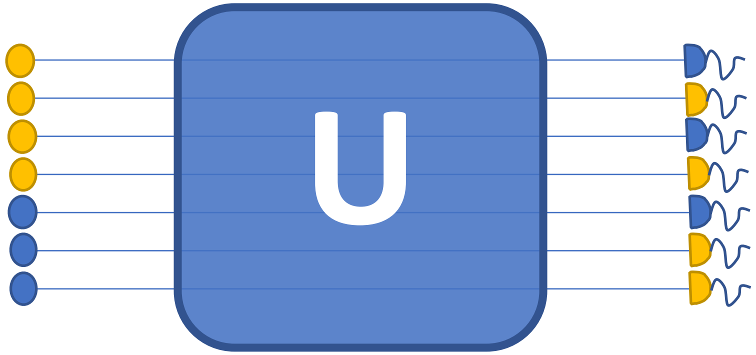

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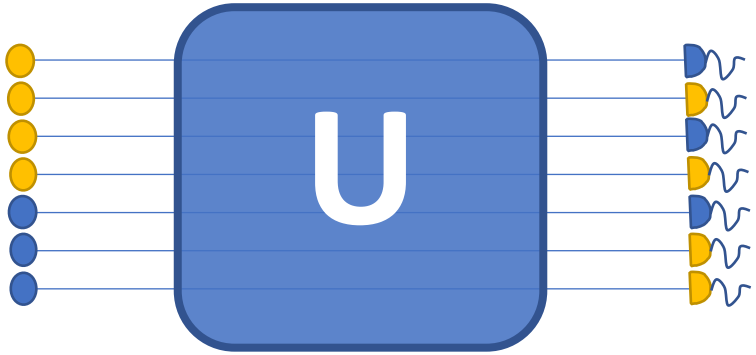
Conjecture: It is highly probable that permanents of these matrices are hard to calculate

Hardness of boson sampling

[Aaronson & Arkhipov arXiv:1011.3245]



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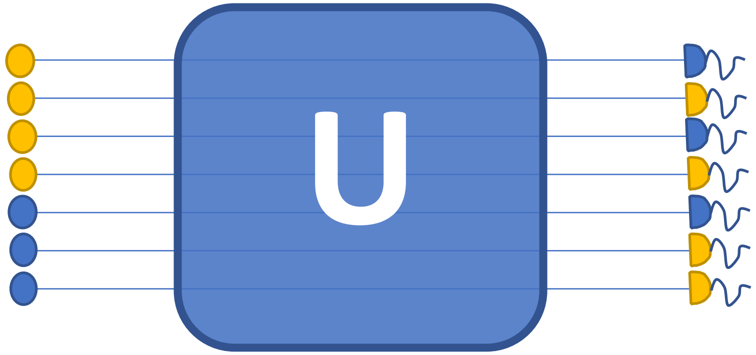

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Permanents are ***typically*** hard (#P) to calculate

Second idea from Aaronson and Arkhipov: sampling from any distribution that is sufficiently close to $P(e'_{j_1},\ldots e'_{j_n}|e_1,\ldots e_n)$ is computationally hard.

Conjecture 6 (Permanent Anti-Concentration Conjecture) *There exists a polynomial p such that for all n and $\delta > 0$,*

$$\Pr_{X\sim\mathcal{N}(0,1)^{n\times n}_{\mathbb{C}}}\left[|\mathrm{Per}(X)| < \frac{\sqrt{n!}}{p(n,1/\delta)}\right] < \delta.$$

The physics of boson sampling

The physics of boson sampling

People in quantum optics: *“Well, here’s the best I can do for now”*

The physics of boson sampling

People in quantum optics: *“Well, here’s the best I can do for now”*

REPORT

Boson Sampling on a Photonic Chip

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+ See all authors and affiliations

Science 15 Feb 2013:
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REPORT

Photonic Boson Sampling in a Tunable Circuit

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nature photonics LETTERS
PUBLISHED ONLINE: 26 MAY 2013 | DOI: 10.1038/NPHOTON.2013.112

Integrated multimode interferometers with arbitrary designs for photonic boson sampling

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LETTERS nature photonics
PUBLISHED ONLINE: 12 MAY 2013 | DOI: 10.1038/NPHOTON.2013.102

Experimental boson sampling

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The physics of boson sampling

People in quantum optics: *“Well, here’s the best I can do for now”*

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Between 2 and 4 photons

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LETTERS
PUBLISHED ONLINE: 12 MAY 2013 | DOI: 10.1038/NPHOTON.2013.102
nature photonics

Experimental boson sampling

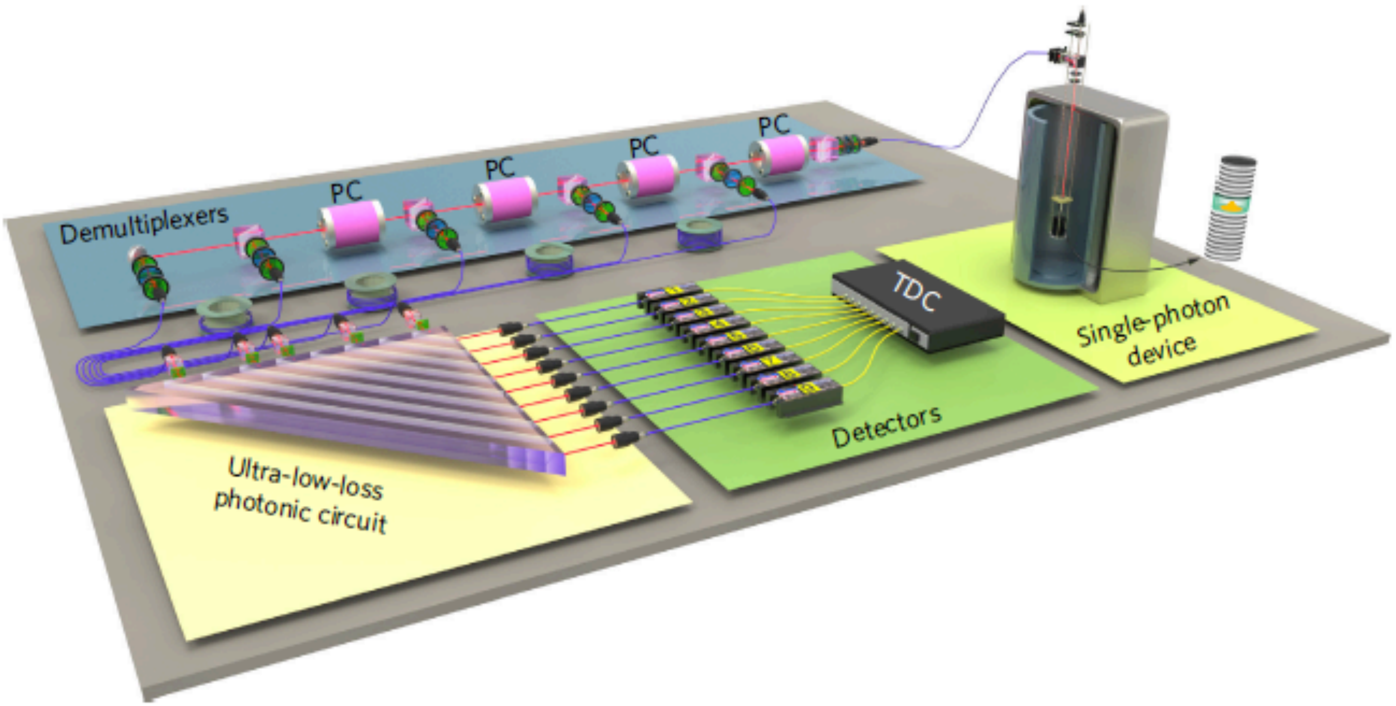
Max Tillmann[1,2*], Borivoje Dakić[1], René Heilmann[3], Stefan Nolte[3], Alexander Szameit[3] and Philip Walther[1,2*]

The physics of boson sampling

nature photonics
ARTICLES
PUBLISHED ONLINE: 1 MAY 2017 | DOI: 10.1038/NPHOTON.2017.63

High-efficiency multiphoton boson sampling

Hui Wang[1,2†], Yu He[1,2†], Yu-Huai Li[1,2†], Zu-En Su[1,2], Bo Li[1,2], He-Liang Huang[1,2], Xing Ding[1,2], Ming-Cheng Chen[1,2], Chang Liu[1,2], Jian Qin[1,2], Jin-Peng Li[1,2], Yu-Ming He[1,2,3], Christian Schneider[3], Martin Kamp[3], Cheng-Zhi Peng[1,2], Sven Höfling[1,3,4], Chao-Yang Lu[1,2*] and Jian-Wei Pan[1,2*]



Idea:

On demand photons from a quantum dot source

Up to five photons, but should be scalable

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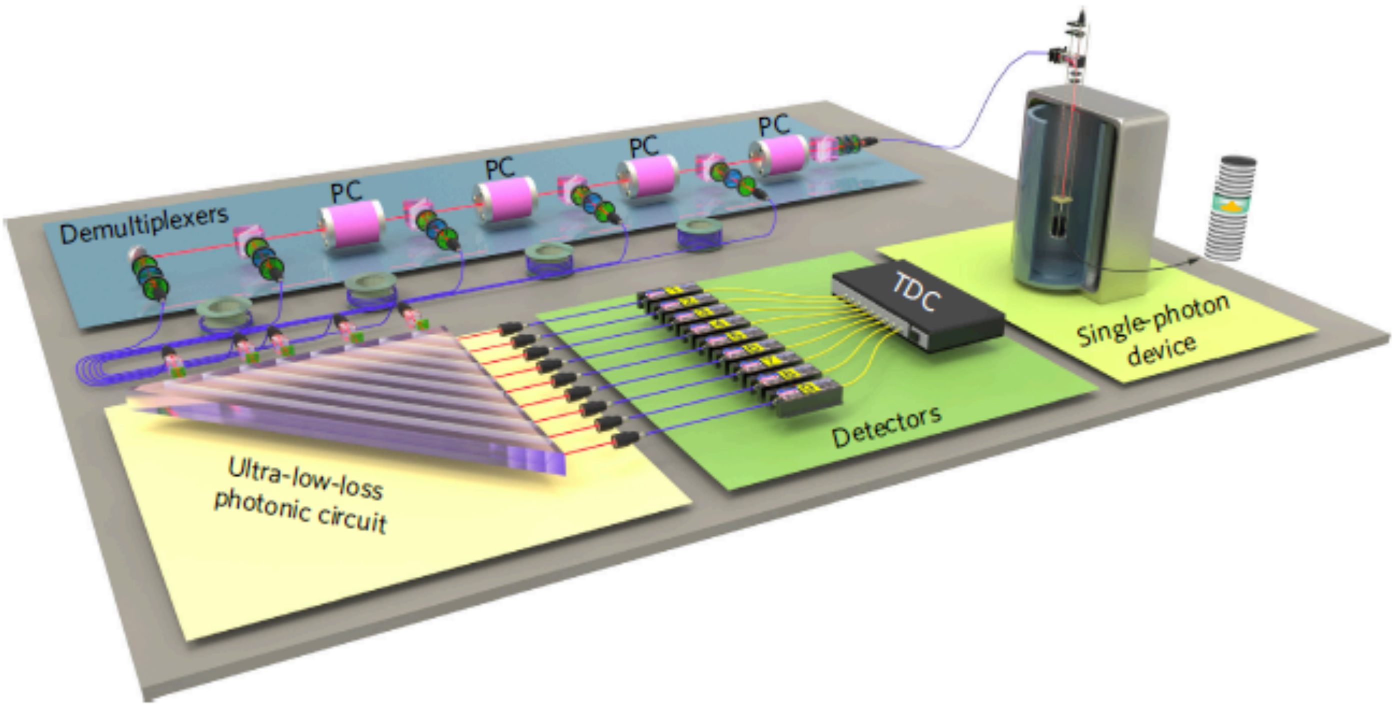

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High-efficiency multiphoton boson sampling

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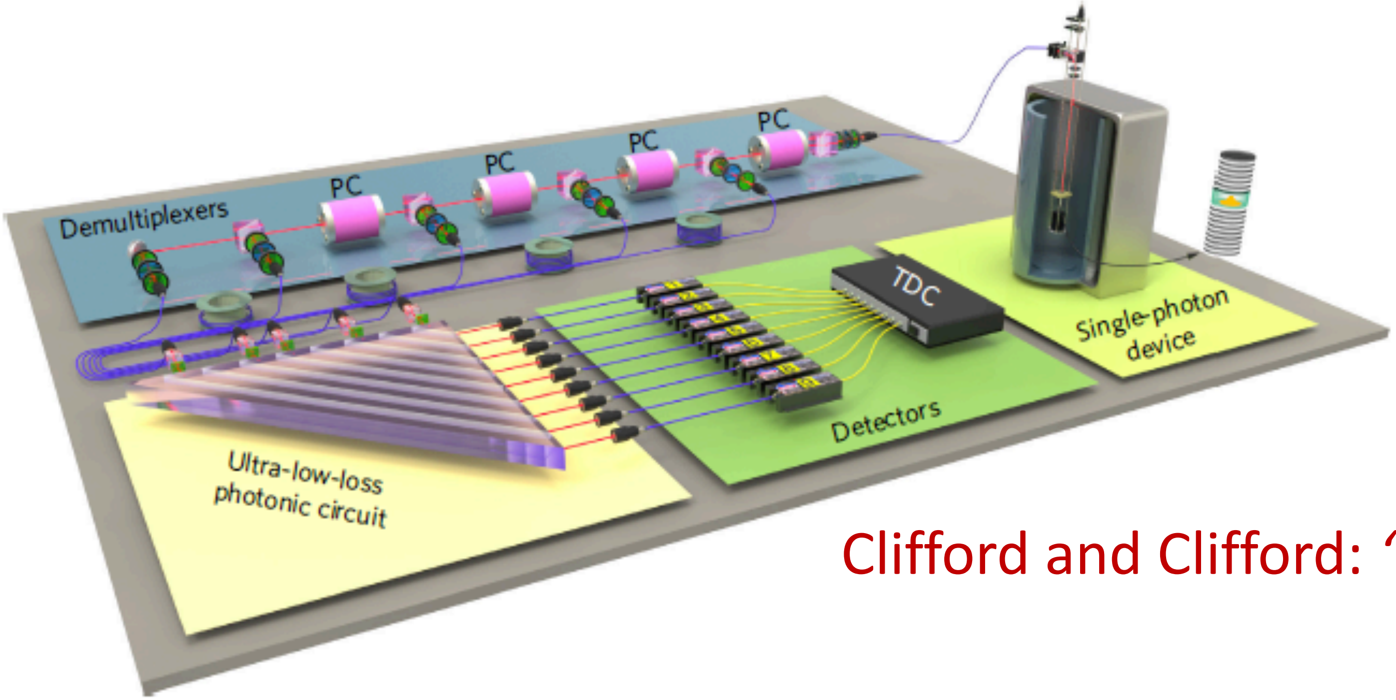

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On demand photons from a quantum dot source

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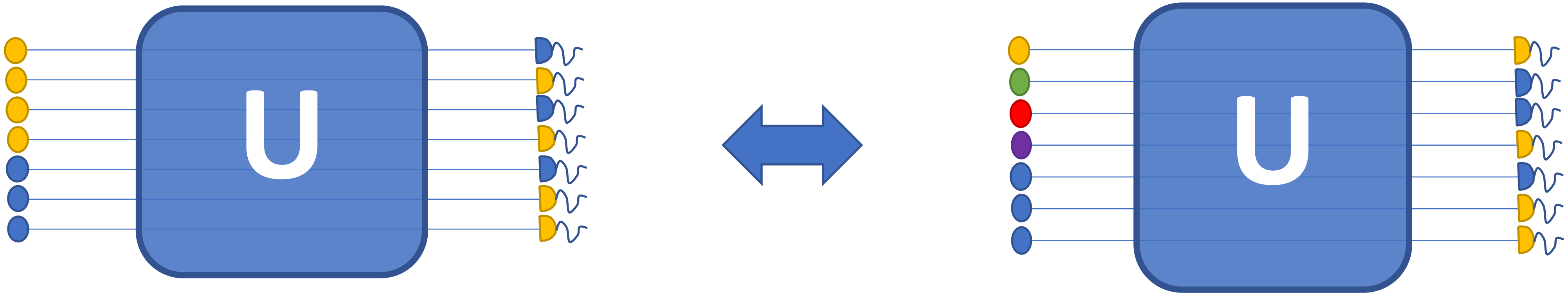
How many photons do we need for this to be hard?

Clifford and Clifford: *"let's say around 50" [1706.01260]*

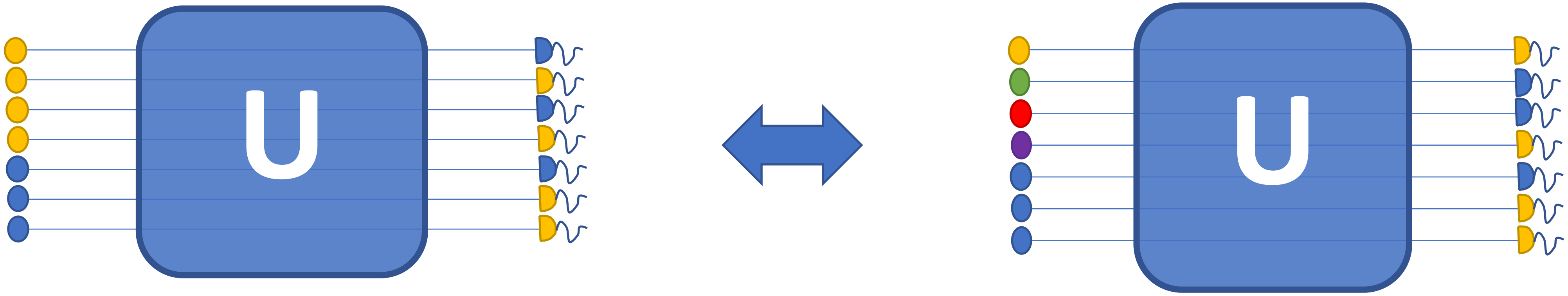
Distinguishability



When and how do quantum particles become classical?

$$P(e'_{j_1}, \ldots, e'_{j_n} | e_1, \ldots, e_n) = \left| \sum_{\sigma \in S_n} U_{\sigma(1)j_1} \ldots U_{\sigma(n)j_n} \right|^2$$

Distinguishability



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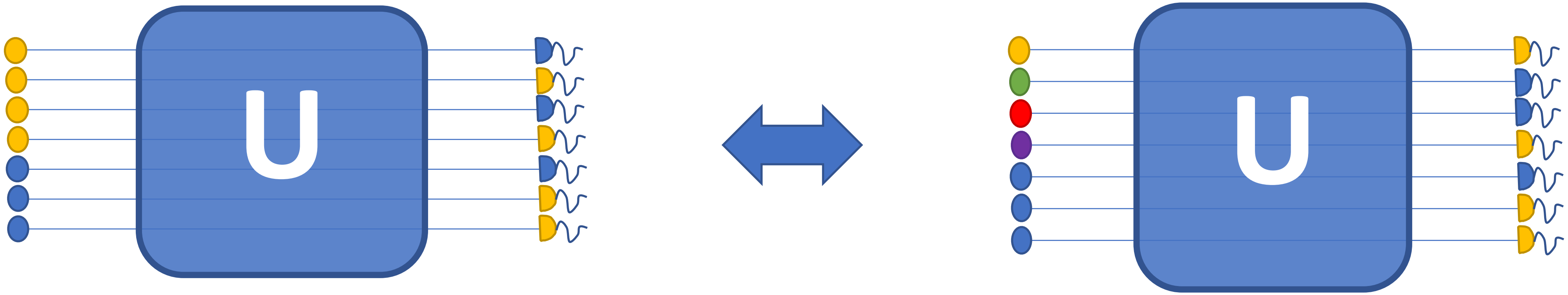
The devil is in the details…

$$U_{k,l} = \frac{1}{V}\int_V \mathbf{e}_k^*(\mathbf{r},t)\mathbf{e}'_l(\mathbf{r},t)d^3\mathbf{r}$$

Distinguishability



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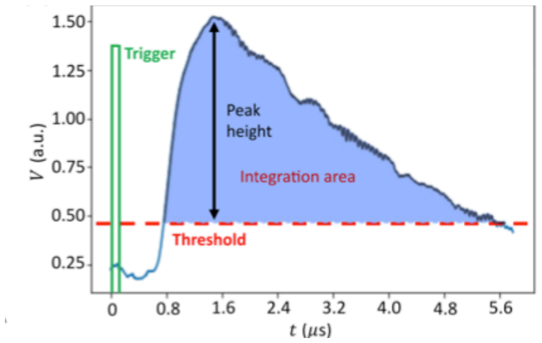
We assume implicitly that our detector is perfectly resolving the mode of the particle. Including in the time frequency domain…



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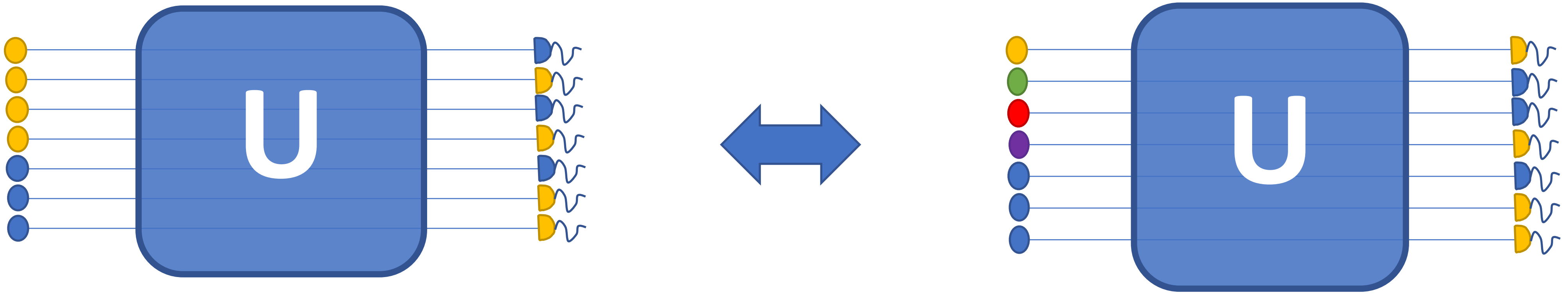

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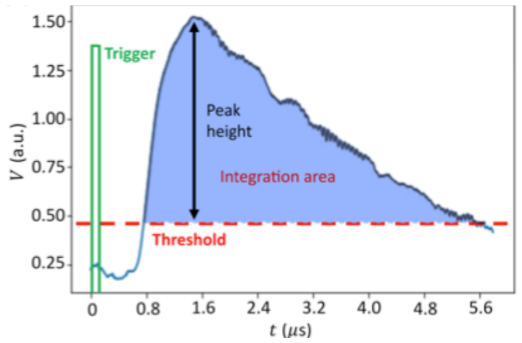
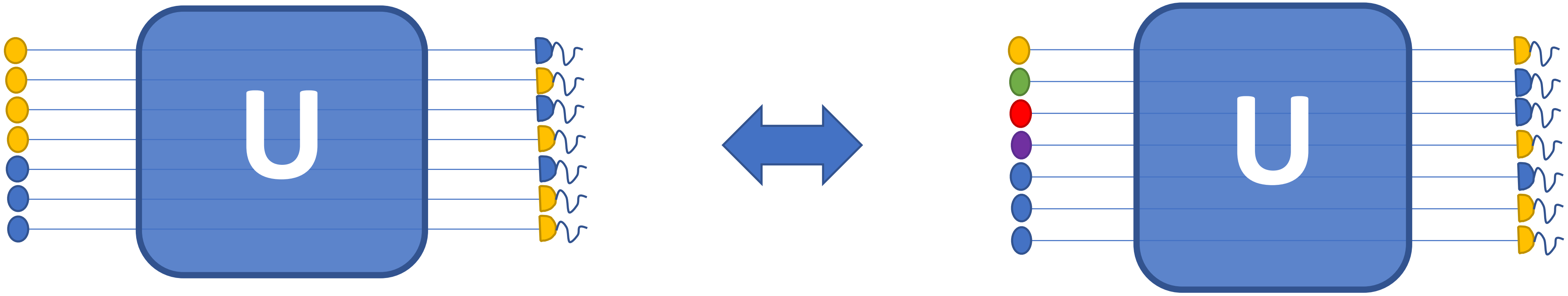


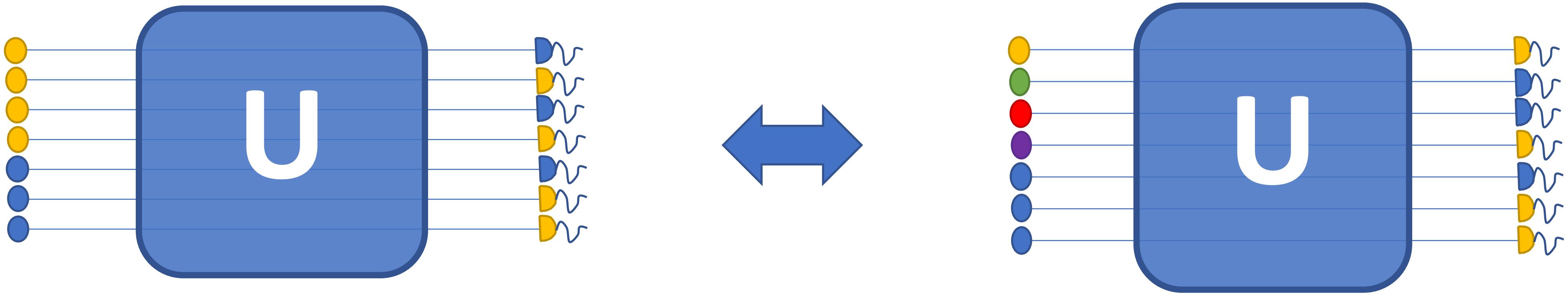
We need a better model for our detector…

Distinguishability



Let us assume some structure on the modes of the input photons

Distinguishability



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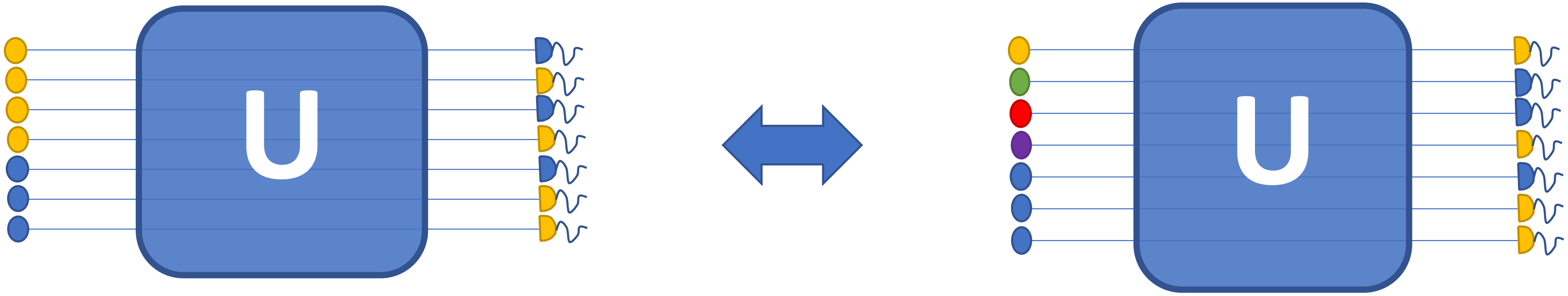
$$\mathbf{e}_k(\mathbf{r}, t) = \mathbf{e}_k(x, y)\psi_k(z/c - t)$$

Distinguishability



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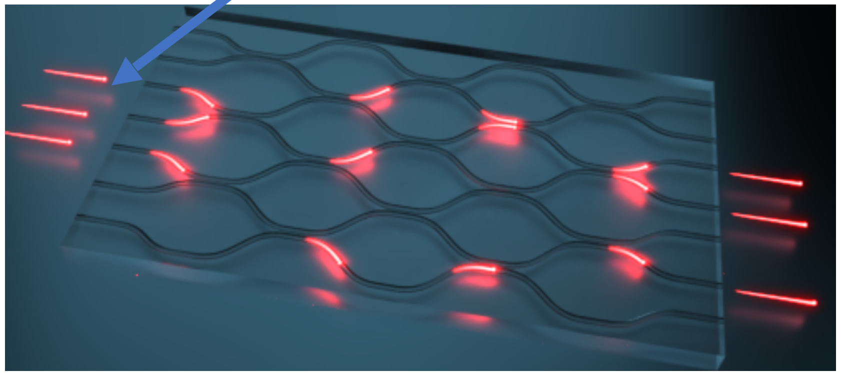

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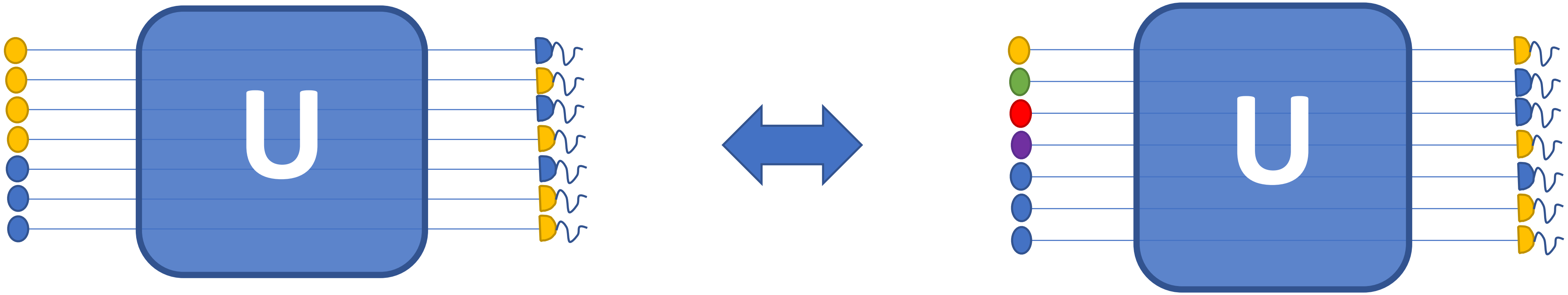

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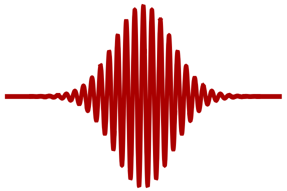

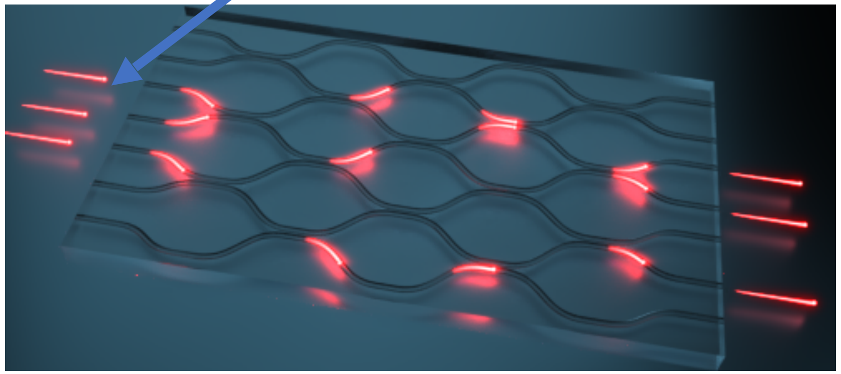

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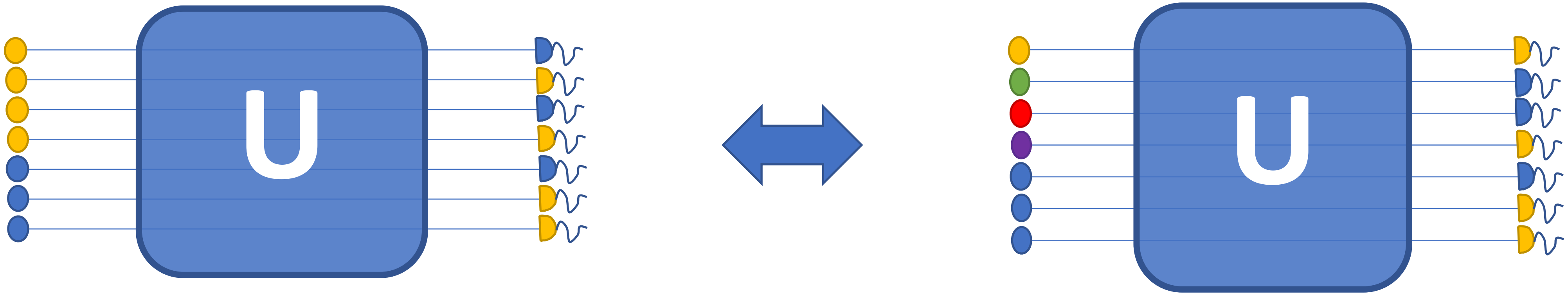

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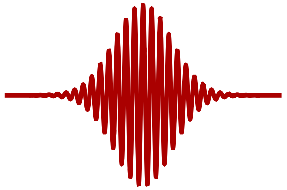
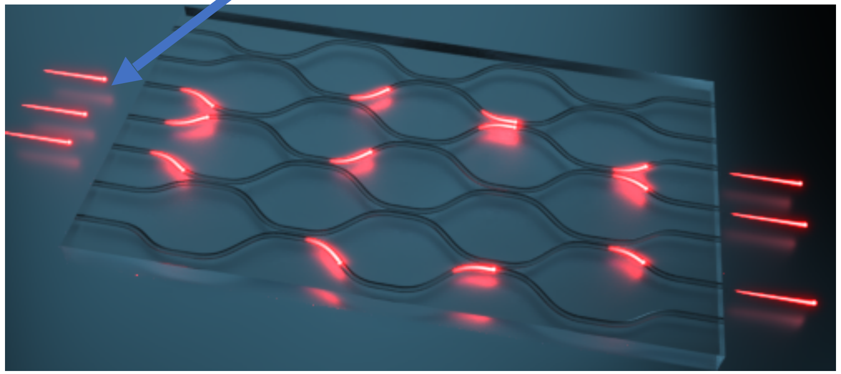
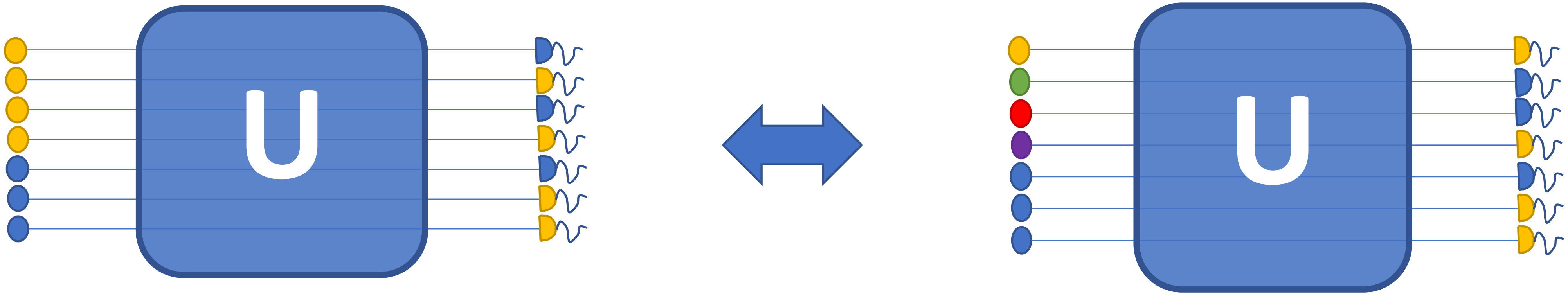




… and for the detectors

$$\mathbf{e}'_k(\mathbf{r},t) = \mathbf{e}'_k(x,y)\eta_k^{(l_k)}(z/c-t)$$

Distinguishability



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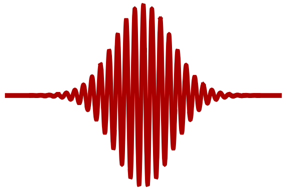

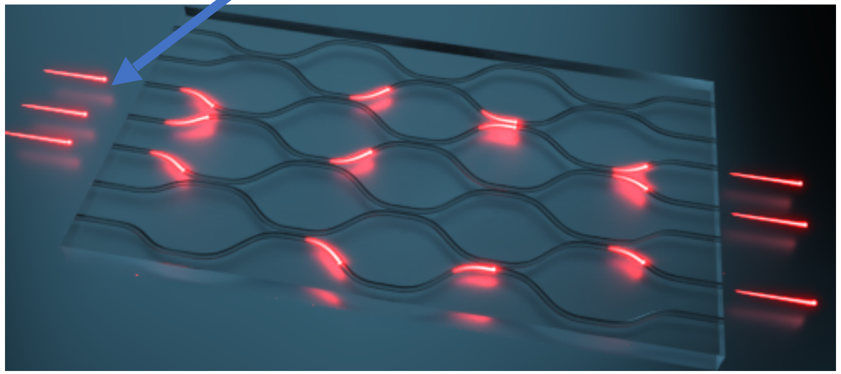

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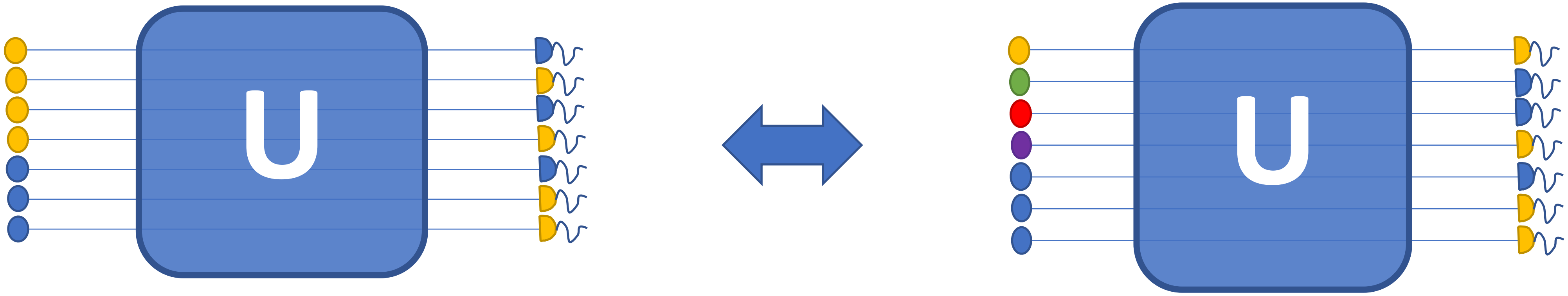
$$\mathbf{e}'_k(\mathbf{r},t) = \mathbf{e}'_k(x,y)\eta_k^{(l_k)}(\overbrace{z/c-t}^{=\tau})$$

Distinguishability



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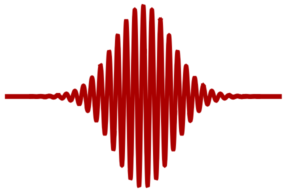
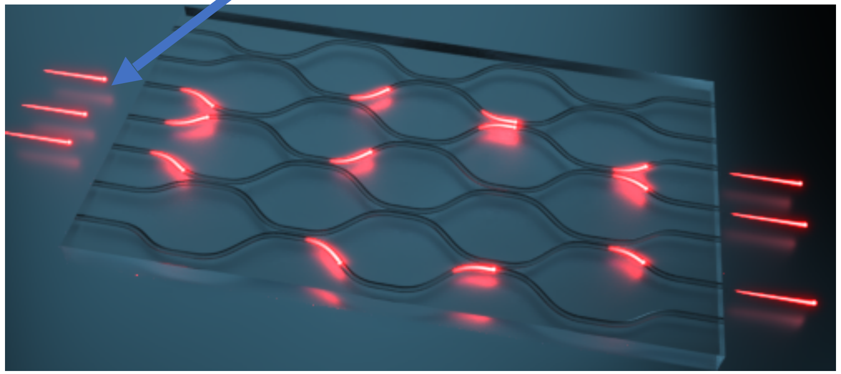


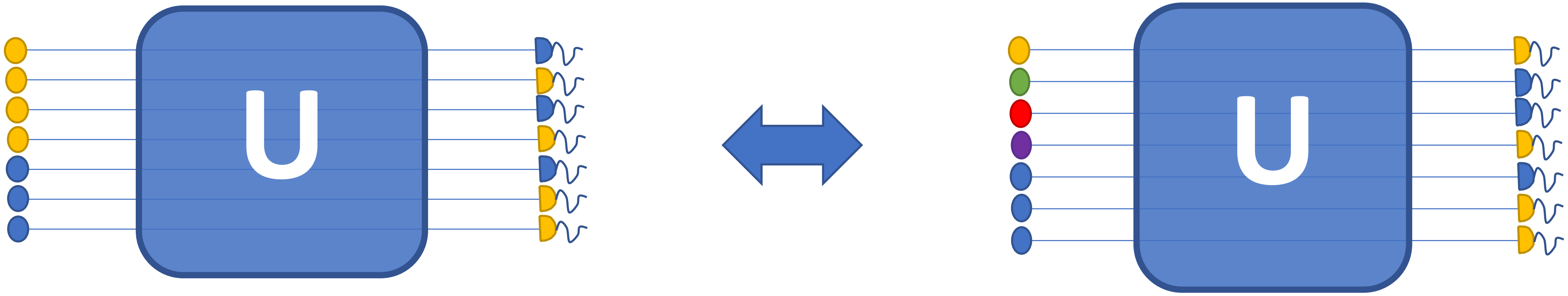


… and for the detectors

$$\mathbf{e}'_k(\mathbf{r},t) = \mathbf{e}'_k(x,y)\boxed{\eta_k^{(l_k)}\overbrace{(z/c-t)}^{=\,\tau}}$$

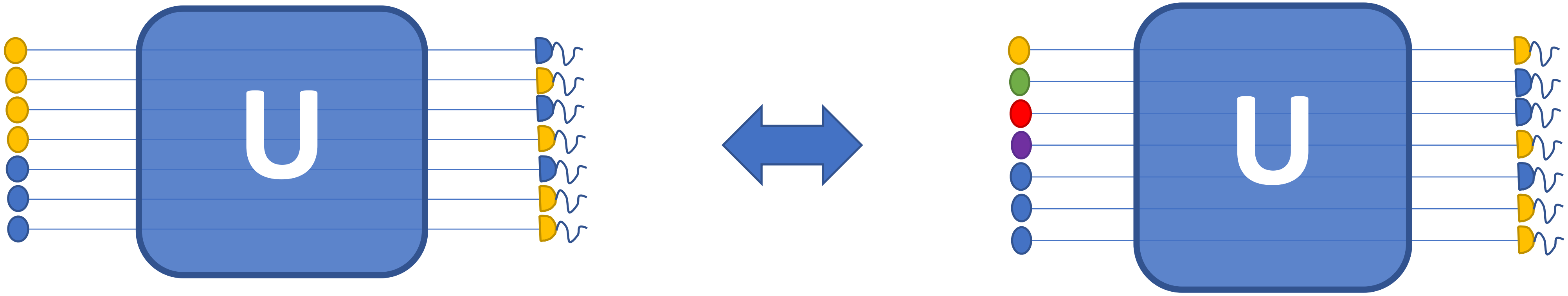
$$\sum_{l_k}[\eta_k^{(l_k)}]^*(\tau)\eta_k^{(l_k)}(\tau') = \delta(\tau-\tau')$$

Distinguishability



Detectors that do not resolve temporal structure

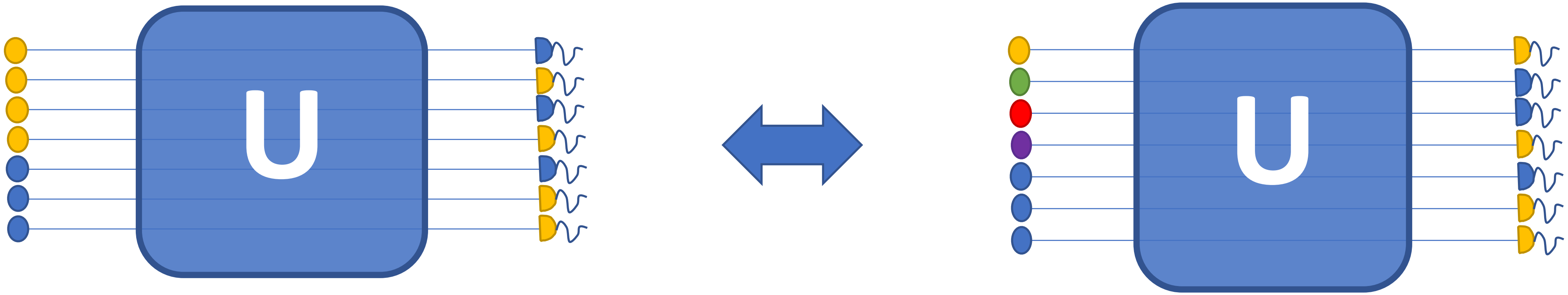
Distinguishability



Detectors that do not resolve temporal structure

$$P(e'_{j_1},\dots,e'_{j_n}|e_1,\dots,e_n) = \sum_{l_1,\dots,l_n} \left|\langle 0|\,\hat{a}(e'_{j_1})\dots\hat{a}(e'_{j_n})\hat{a}^\dagger(e_1)\dots\hat{a}^\dagger(e_n)\,|0\rangle\right|^2$$

Distinguishability



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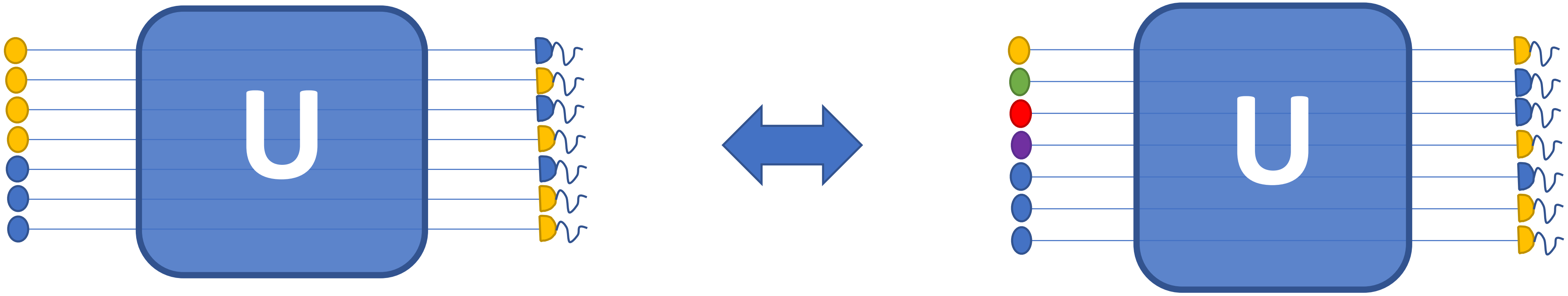

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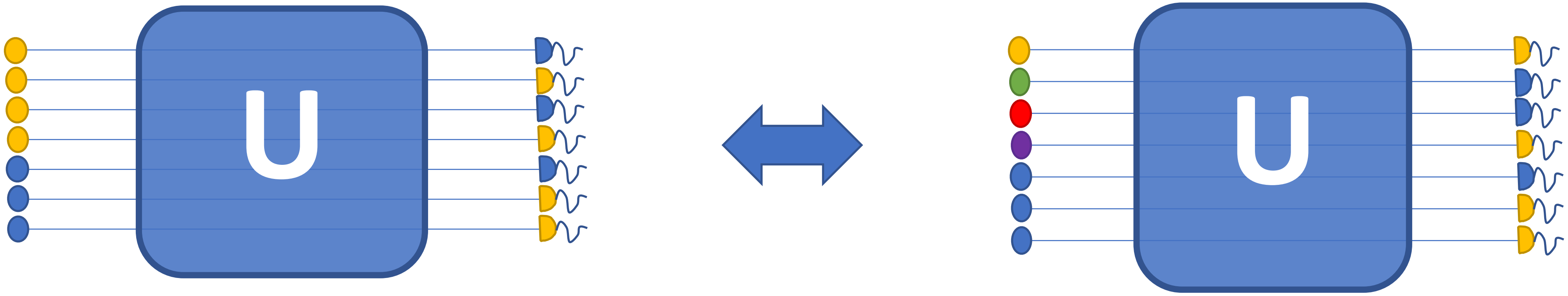
$$= \sum_{\sigma,\sigma'\in S_n} U_{j_1\sigma(1)}\dots U_{j_n\sigma(n)}U^*_{j_1\sigma'(1)}\dots U^*_{j_n\sigma'(n)}$$

Distinguishability



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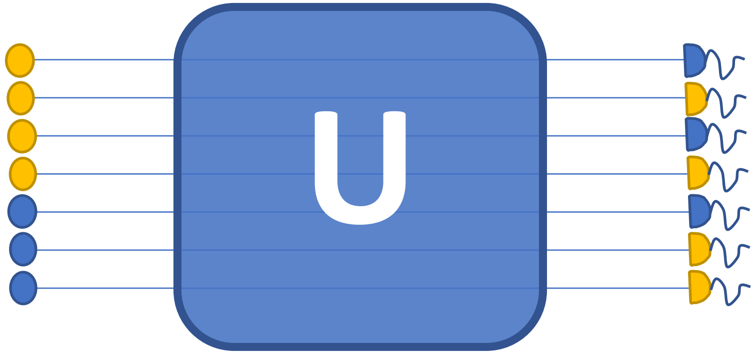
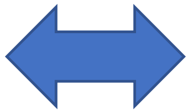
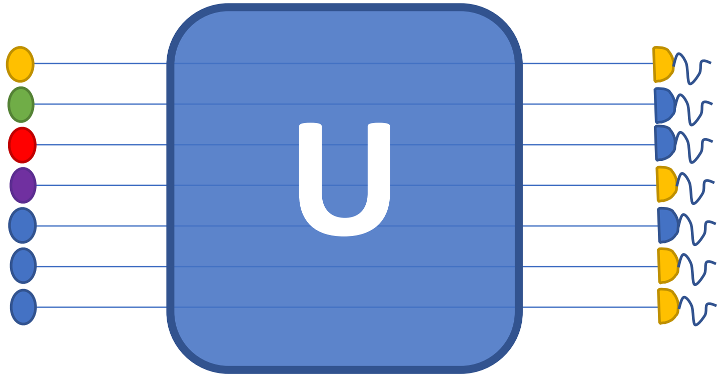
Note the appearance of terms $U_{j_k\sigma(k)}U^*_{j_k\sigma'(k)}$

Distinguishability



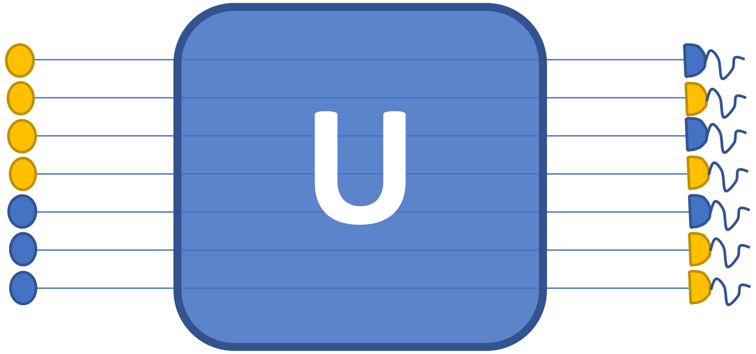


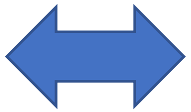


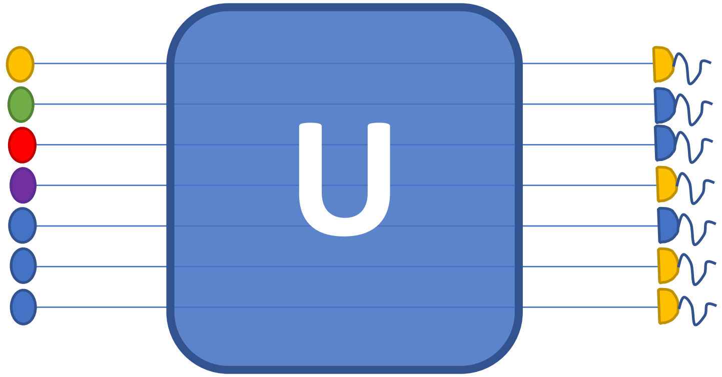
Now let's impose our temporal structure

$$\sum_{l_k} U_{j_k\sigma(k)} U^*_{j_k\sigma'(k)}$$

Distinguishability







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$$\sum_{l_k} U_{j_k\sigma(k)}U^*_{j_k\sigma'(k)}$$

Recall $U_{k,l} = \frac{1}{V}\int_V \mathbf{e}^*_k(\mathbf{r},t)\mathbf{e}'_l(\mathbf{r},t)d^3\mathbf{r}$

$$\mathbf{e}_k(\mathbf{r},t) = \mathbf{e}_k(x,y)\psi_k(\tau)$$

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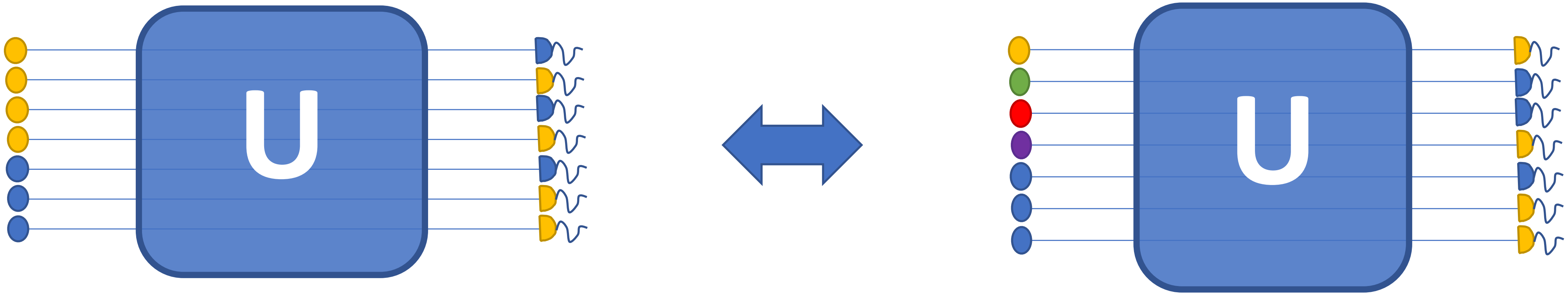
Distinguishability



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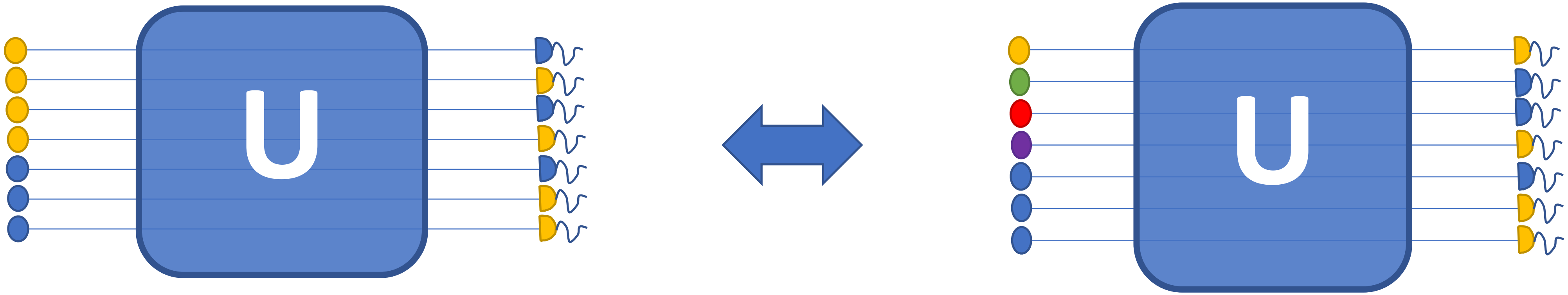
$$U_{kl} = \frac{1}{L^2}\int_{L^2} \mathbf{e}_k^*(x,y)\mathbf{e}_k'(x,y)dxdy\frac{c}{L}\int_T \psi_k^*(\tau)\eta_k^{(l_k)}(\tau)d\tau$$

Distinguishability



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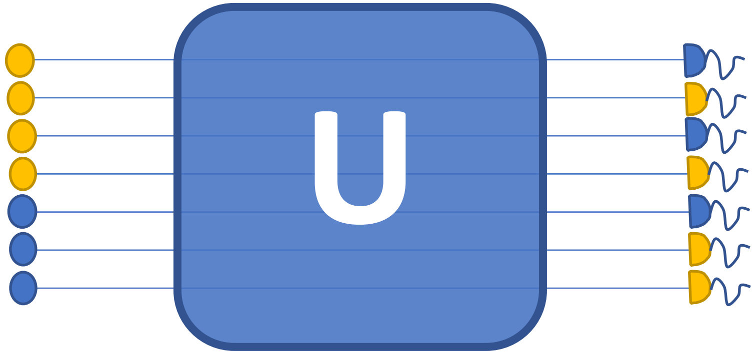

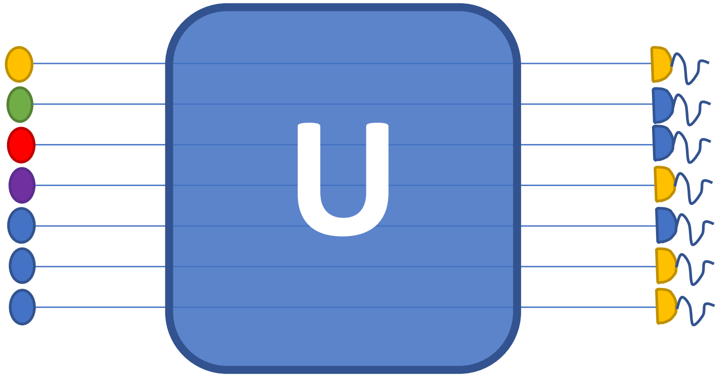

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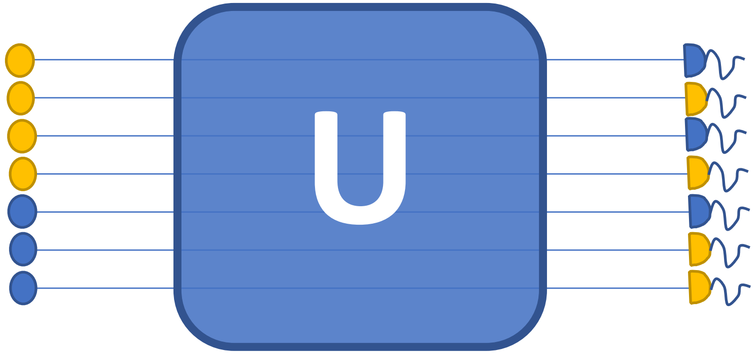
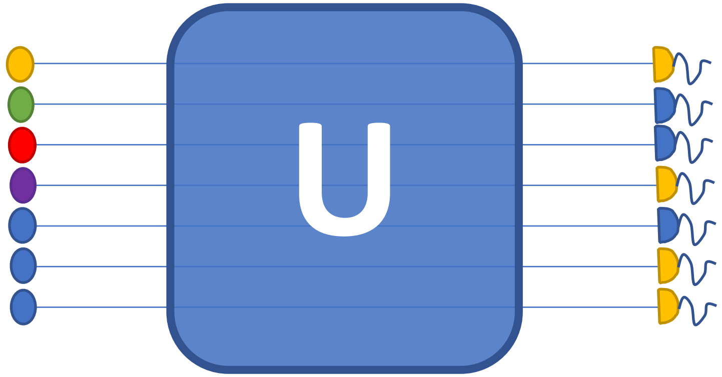
Distinguishability





Now let's impose our temporal structure

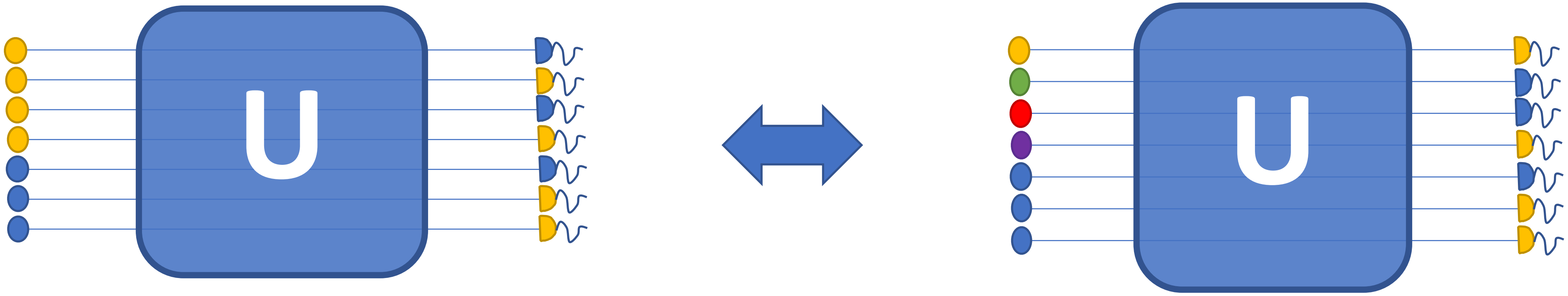
$$\sum_{l_k} U_{j_k\sigma(k)}U^*_{j_k\sigma'(k)}$$
$$= \sum_{l_k} \mathcal{U}_{j_k\sigma(k)}\mathcal{U}^*_{j_k\sigma(k)}(\psi_{\sigma(k)},\eta^{(l_k)}_{j_k})(\eta^{(l_k)}_{j_k},\psi_{\sigma'(k)})$$

Recall $U_{k,l} = \frac{1}{V}\int_V \mathbf{e}^*_k(\mathbf{r},t)\mathbf{e}'_l(\mathbf{r},t)d^3\mathbf{r}$

$$\mathbf{e}_k(\mathbf{r},t) = \mathbf{e}_k(x,y)\psi_k(\tau)$$
$$\mathbf{e}'_k(\mathbf{r},t) = \mathbf{e}'_k(x,y)\eta^{(l_k)}_k(\tau)$$

$$U_{kl} = \underbrace{\frac{1}{L^2}\int_{L^2}\mathbf{e}^*_k(x,y)\mathbf{e}'_k(x,y)dxdy}_{=\mathcal{U}_{kl}}\underbrace{\frac{c}{L}\int_T \psi^*_k(\tau)\eta^{(l_k)}_k(\tau)d\tau}_{=(\psi_k,\eta^{(l_k)}_k)}$$

Distinguishability



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$$\sum_{l_k} U_{j_k\sigma(k)} U^*_{j_k\sigma'(k)} = \mathcal{U}_{j_k\sigma(k)} \mathcal{U}^*_{j_k\sigma(k)} \sum_{l_k} (\psi_{\sigma(k)}, \eta^{(l_k)}_{j_k})(\eta^{(l_k)}_{j_k}, \psi_{\sigma'(k)})$$

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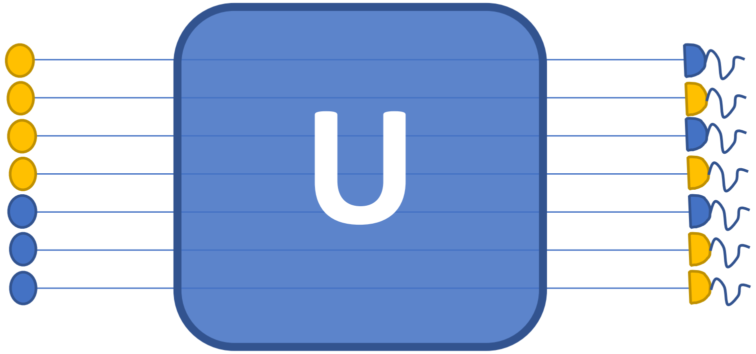

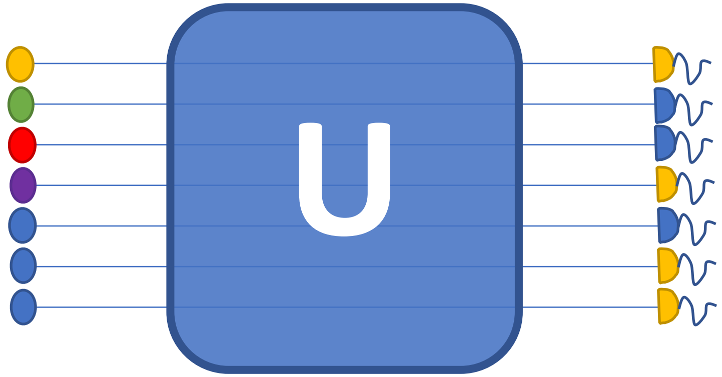

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Recall $\sum_{l_k} [\eta_k^{(l_k)}]^*(\tau)\eta_k^{(l_k)}(\tau') = \delta(\tau - \tau')$

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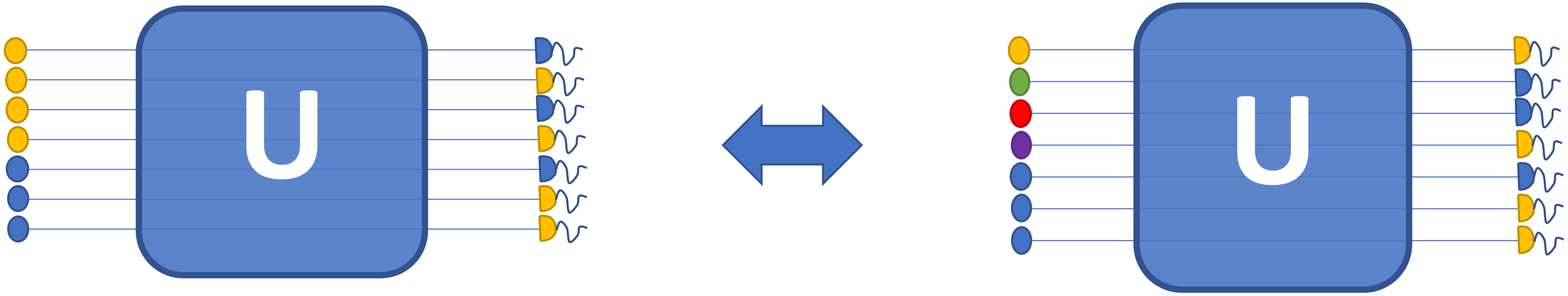

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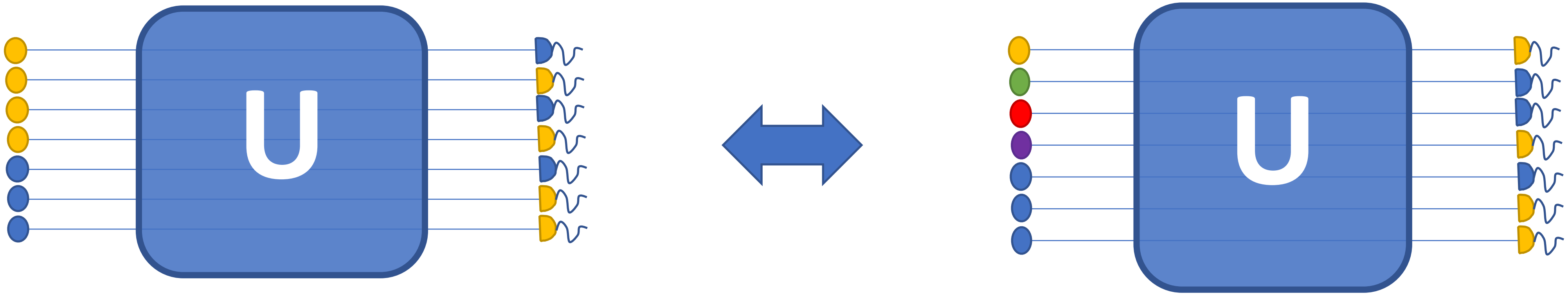

When and how do quantum particles become classical?

$$P(e'_{j_1},\ldots,e'_{j_n}|e_1,\ldots,e_n) = \sum_{\sigma,\sigma'\in S_n} U_{j_1\sigma(1)}\ldots U_{j_n\sigma(n)}U^*_{j_1\sigma'(1)}\ldots U^*_{j_n\sigma'(n)}$$

Some degree of freedom not resolved by detectors

$$= \sum_{\sigma,\sigma'\in S_n} (\psi_{\sigma(1)},\psi_{\sigma'(1)})\ldots(\psi_{\sigma(n)},\psi_{\sigma'(n)})\mathcal{U}_{j_1\sigma(1)}\ldots\mathcal{U}_{j_n\sigma(n)}\mathcal{U}^*_{j_1\sigma'(1)}\ldots\mathcal{U}^*_{j_n\sigma'(n)}$$

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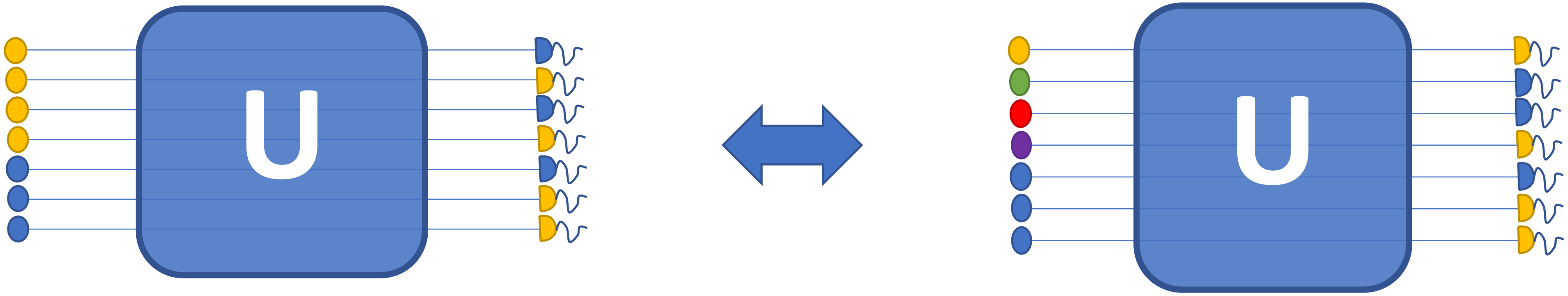

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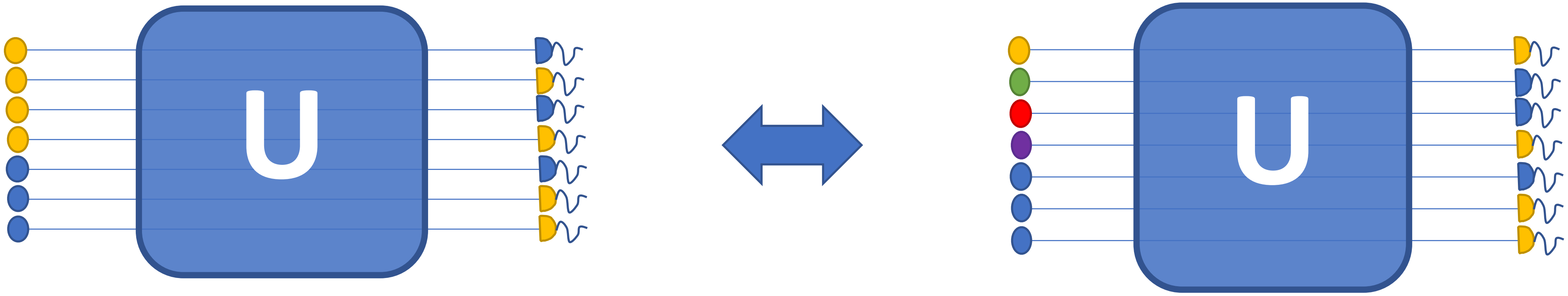
Distinguishability



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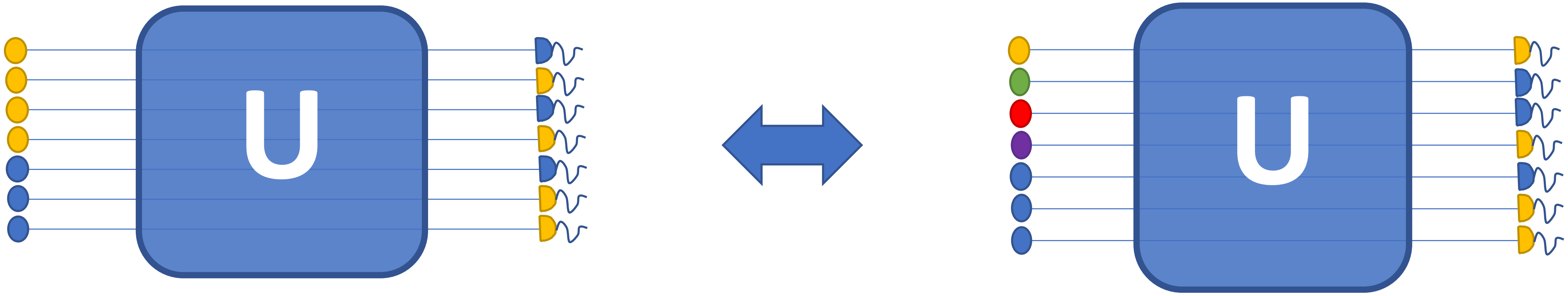

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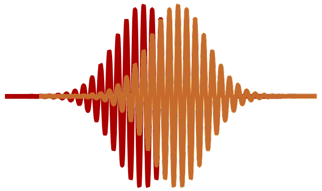
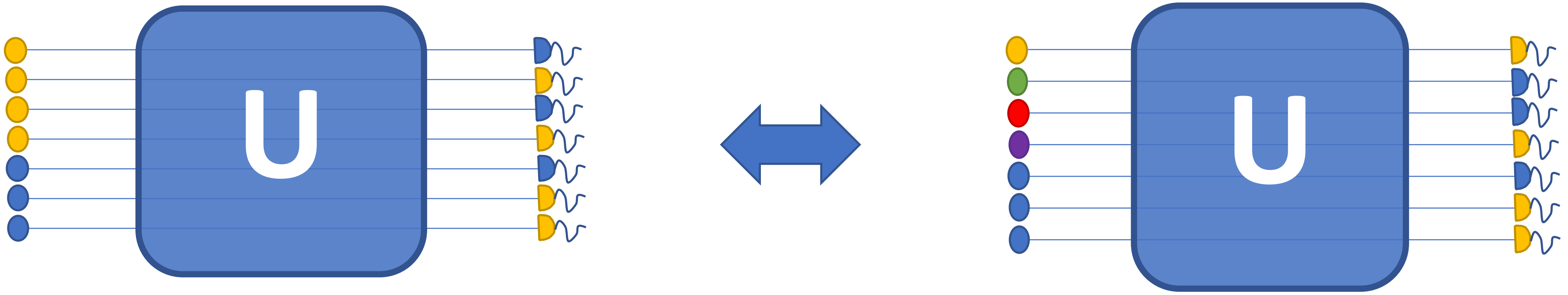


Ability to tell particles apart using other degrees of freedom

Interfering degrees of freedom

Distinguishability



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$$P(e'_{j_1},\dots,e'_{j_n}|e_1,\dots,e_n) = \sum_{\sigma,\sigma'\in S_n} (\psi_{\sigma(1)},\psi_{\sigma'(1)})\dots(\psi_{\sigma(n)},\psi_{\sigma'(n)})\mathcal{U}_{j_1\sigma(1)}\dots\mathcal{U}_{j_n\sigma(n)}\mathcal{U}^*_{j_1\sigma(1)}\dots\mathcal{U}^*_{j_n\sigma(n)}$$

$$= \sum_{\sigma\in S_n} \left|\mathcal{U}_{j_1\sigma(1)}\right|^2\dots\left|\mathcal{U}_{j_n\sigma(n)}\right|^2$$

$$+ \sum_{\sigma\neq\sigma'\in S_n} (\psi_{\sigma(1)},\psi_{\sigma'(1)})\dots(\psi_{\sigma(n)},\psi_{\sigma'(n)})\mathcal{U}_{j_1\sigma(1)}\dots\mathcal{U}_{j_n\sigma(n)}\mathcal{U}^*_{j_1\sigma'(1)}\dots\mathcal{U}^*_{j_n\sigma'(n)}$$

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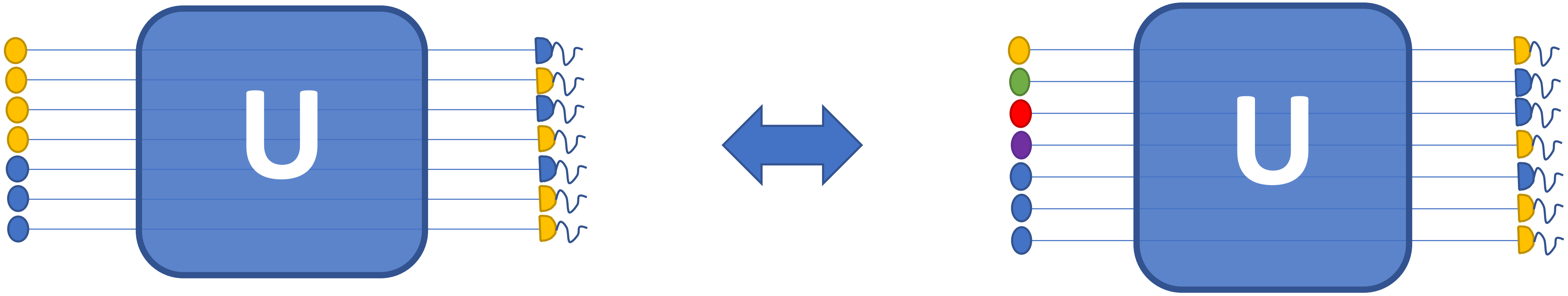

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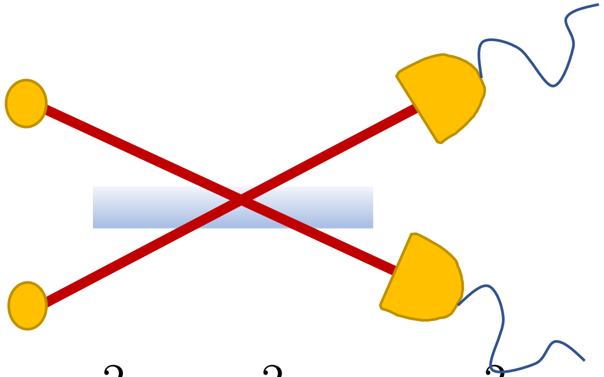
Distinguishability leads to some form of decoherence

Example: Actual Hong-Ou-Mandel



$$\mathcal{U} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$$

$$P(e'_1, e'_2 | e_1, e_2) = |\mathcal{U}_{11}|^2 \, |\mathcal{U}_{22}|^2 + |\mathcal{U}_{12}|^2 \, |\mathcal{U}_{21}|^2 + |(\psi_1, \psi_2)|^2 \left[\mathcal{U}_{11}\mathcal{U}_{22}\mathcal{U}^*_{12}\mathcal{U}^*_{21} + \mathcal{U}_{12}\mathcal{U}_{21}\mathcal{U}^*_{11}\mathcal{U}^*_{22}\right]$$

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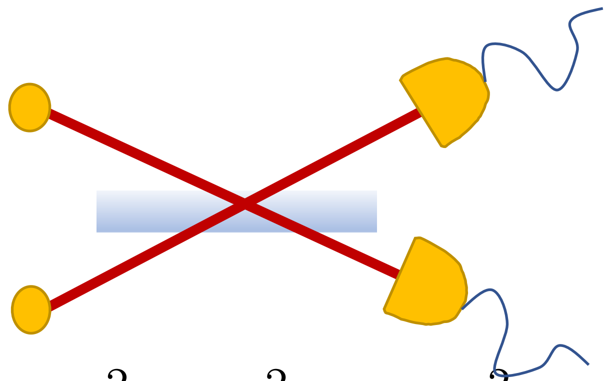

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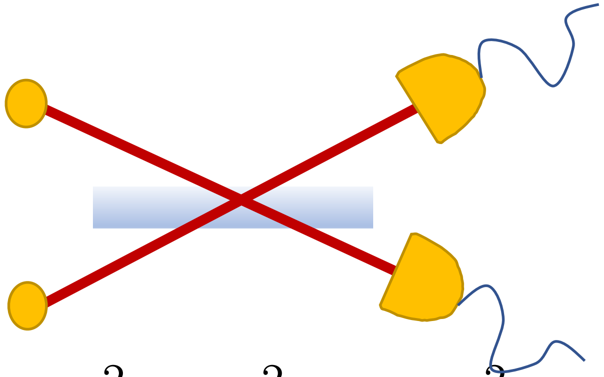

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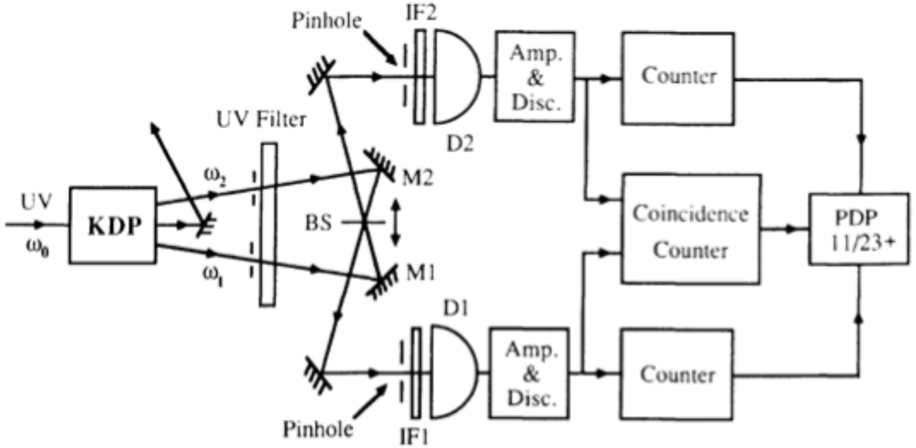

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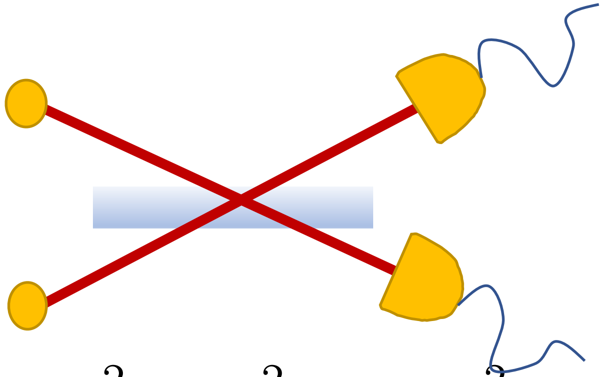

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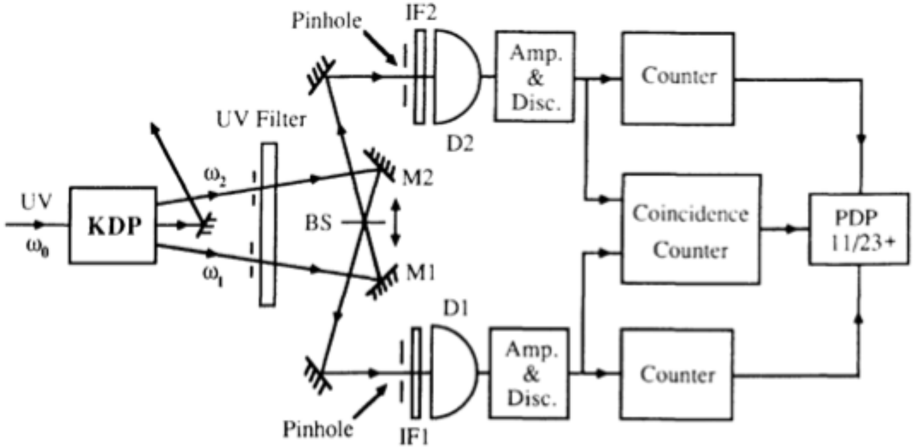

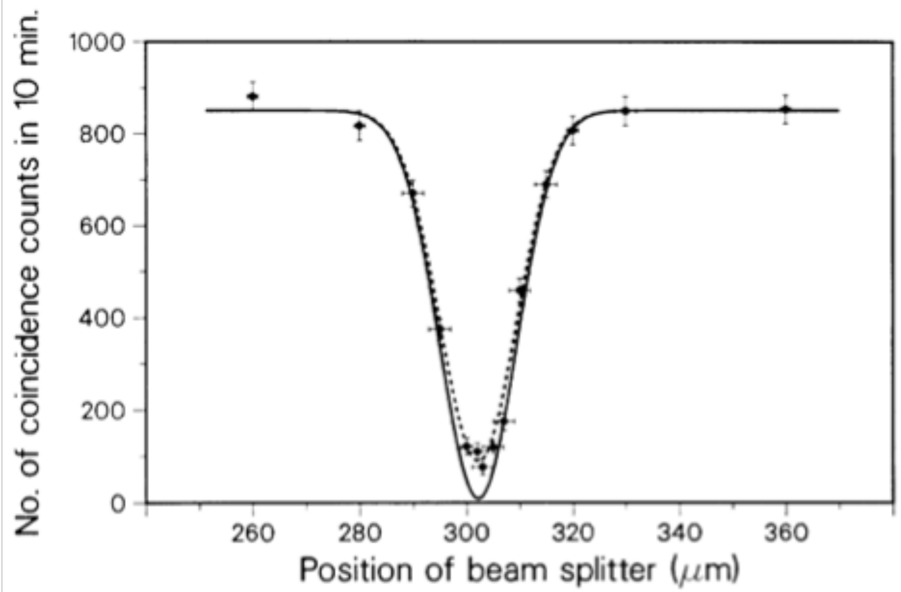

The end of boson sampling?

One does not just need many photons, one also needs good quality photons and circuits!

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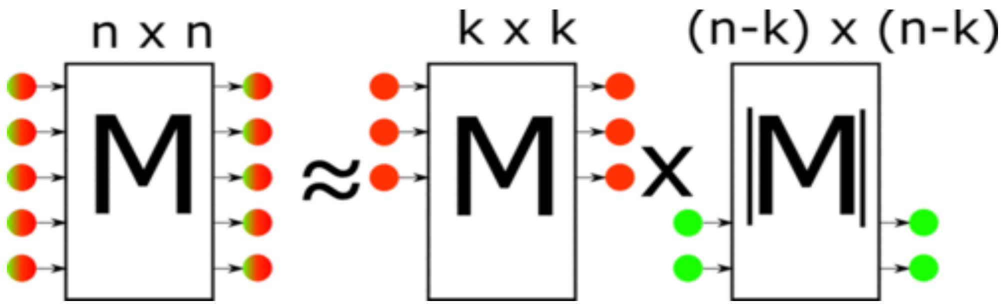
Efficient Classical Algorithm for Boson Sampling with Partially Distinguishable Photons

J. J. Renema, A. Menssen, W. R. Clements, G. Triginer, W. S. Kolthammer, and I. A. Walmsley
Phys. Rev. Lett. **120**, 220502 – Published 30 May 2018



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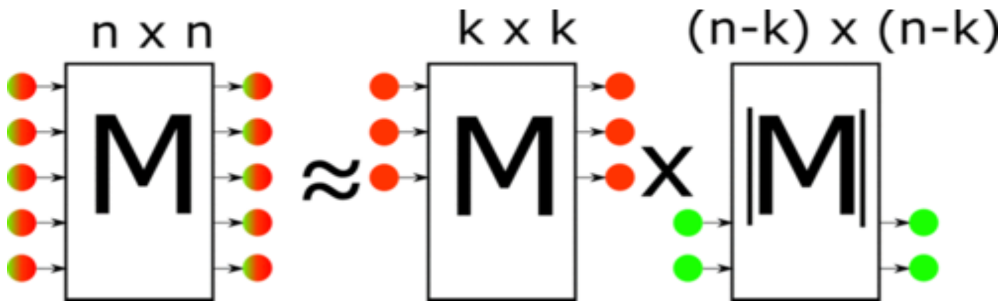

PAPER

Classically simulating near-term partially-distinguishable and lossy boson sampling

Alexandra E Moylett[6,1,2,3], Raúl García-Patrón[4], Jelmer J Renema[5] and Peter S Turner[1]

Published 26 November 2019 •

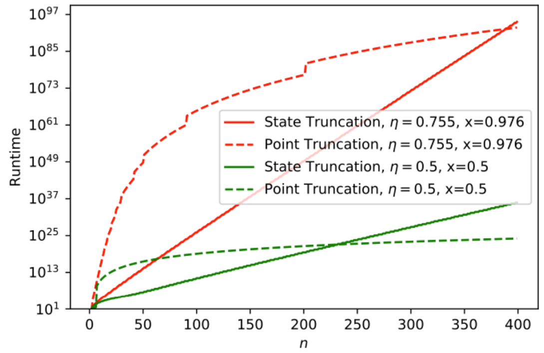

FIG. 2. Approximated runtime in terms of number of operations to simulate n-photon Boson Sampling with chosen values of η and x up to 10% error ($\epsilon = 0.1$) via state (solid) or point (dashed) truncation.

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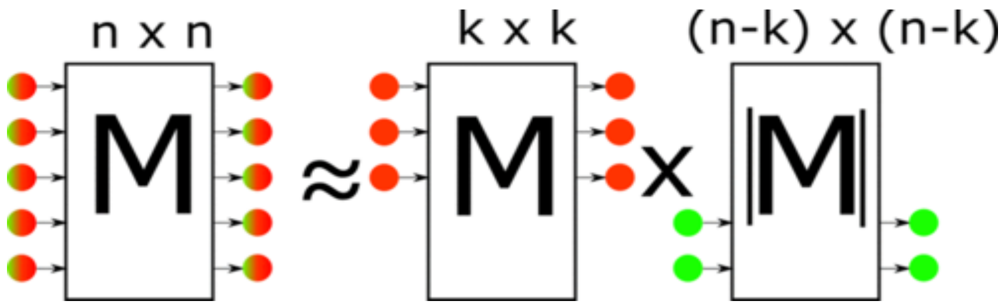
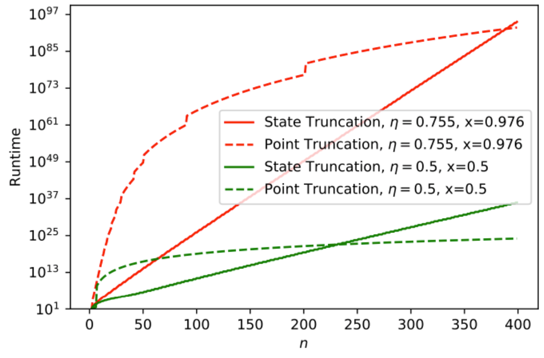


People in quantum optics: 😭 😭 😭 😭



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Summary

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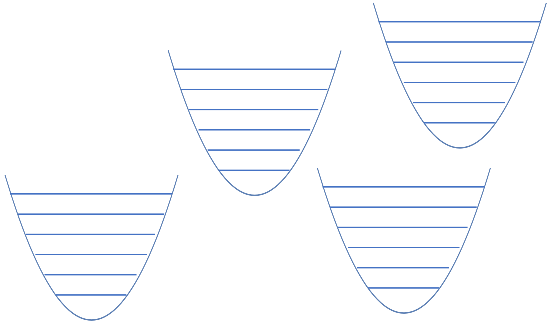
Multimode light is in essence a set of quantum harmonic oscillators

$$\hat{\mathbf{E}}^{(+)}(\mathbf{r},t) = \sum_{j=1}^{\infty} \mathcal{E}_j \hat{a}_j \mathbf{u}_j(\mathbf{r},t)$$



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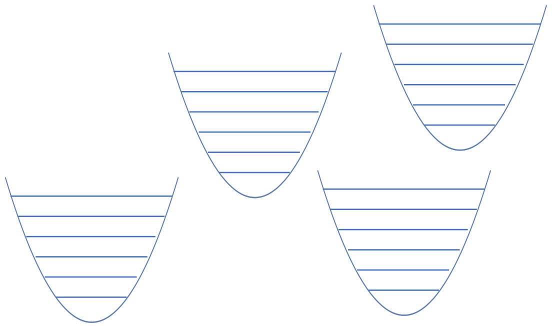

Boson sampling boils down to preparing a Fock states in one mode basis and measuring it in another

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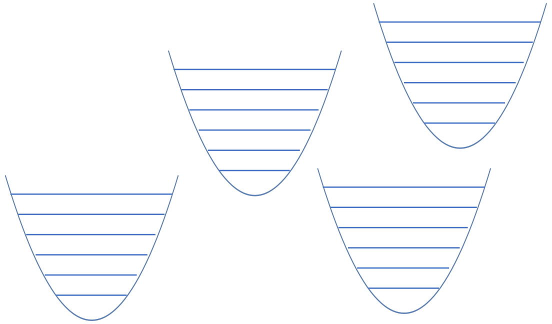

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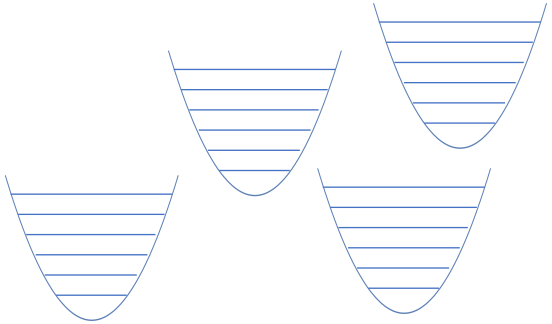

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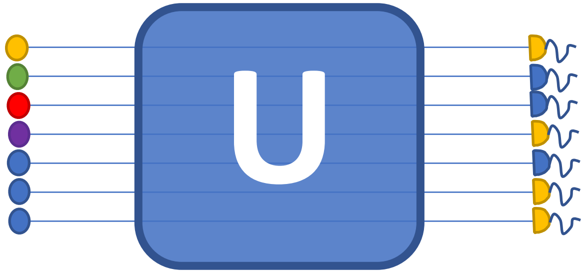
Temporal structure of the photons destroys this hardness



Preview of the next lecture…

Is there a future for bosonic sampling problems in quantum computation?

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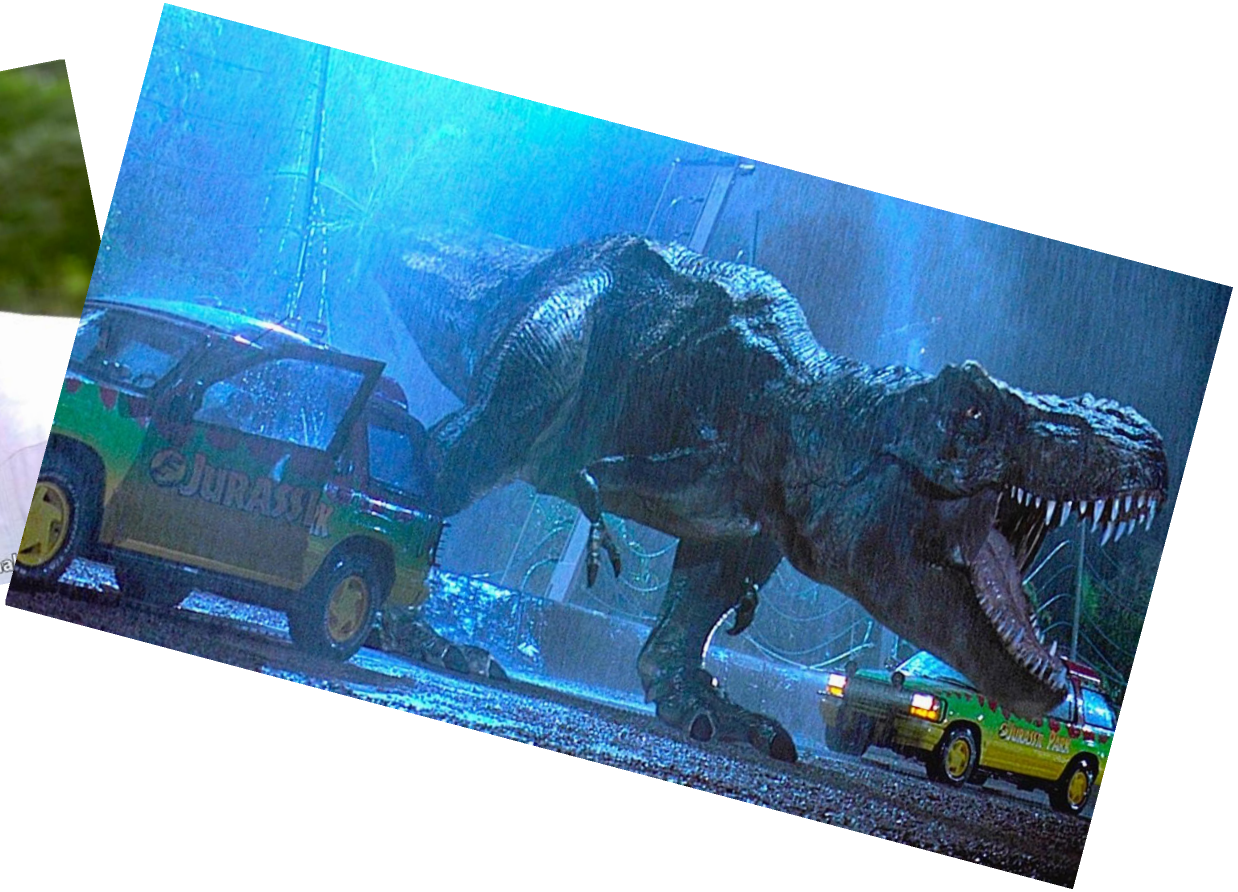

Extra slide: Photon detection

A key observable is the **number operator** in mode $\mathbf{e_j}$

$$H = 2\epsilon_0 V \sum_{j=0}^{m} |\mathcal{E}_j|^2 \left(\boxed{\hat{a}^\dagger(e_j)\hat{a}(e_j)} + \frac{1}{2} \right)$$

Extra slide: Photon detection

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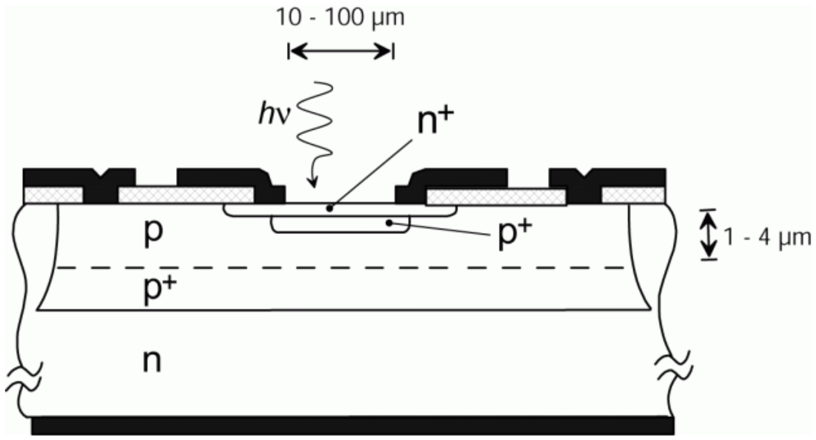
Detect the presence of at least one photon



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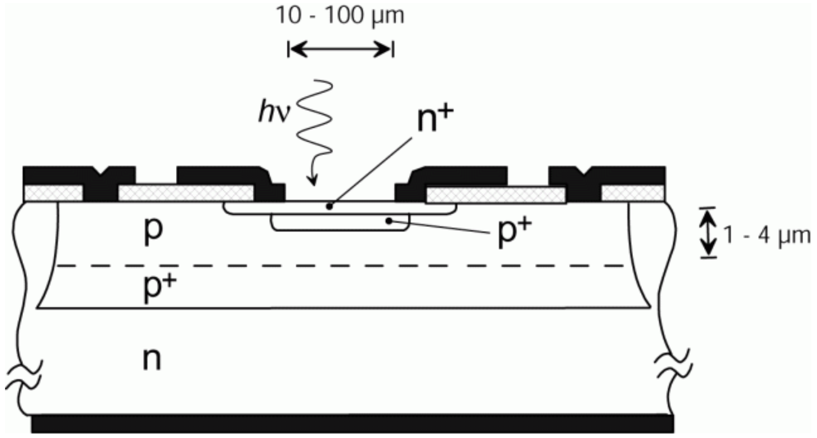
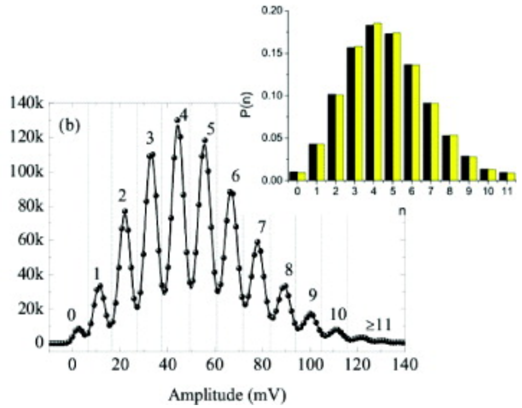
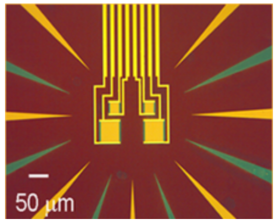
Count number of photons

Detect the presence of at least one photon





Brida et al
NJP **14** 085001 (2012)



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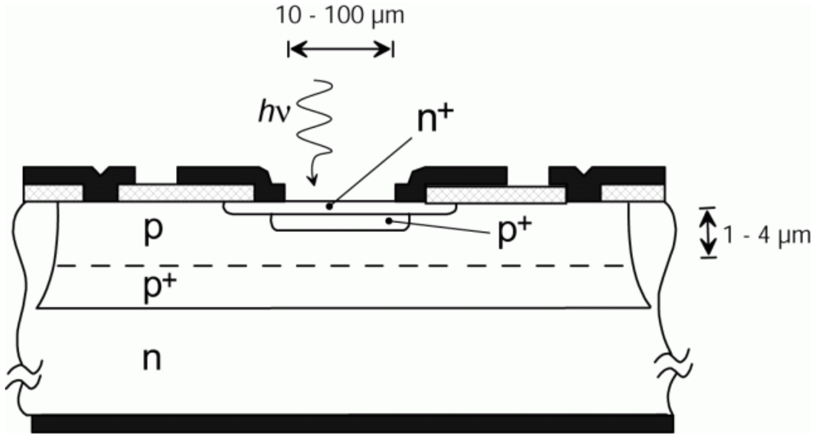
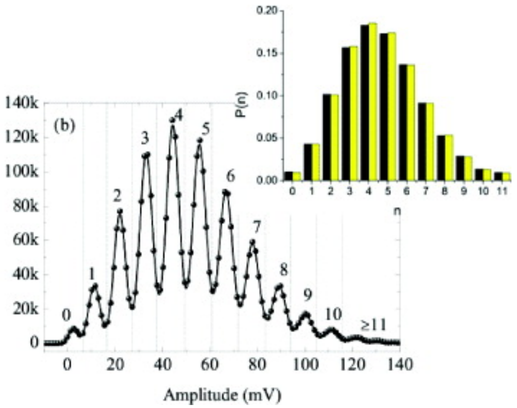
A key observable is the **number operator** in mode $\mathbf{e_j}$

$$H = 2\epsilon_0 V \sum_{j=0}^{m} |\mathcal{E}_j|^2 \left(\boxed{\hat{a}^\dagger(e_j)\hat{a}(e_j)} + \frac{1}{2} \right)$$

Detect the presence of at least one photon



Count number of photons



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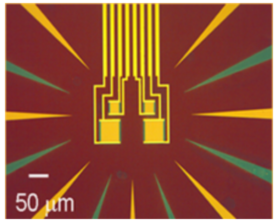
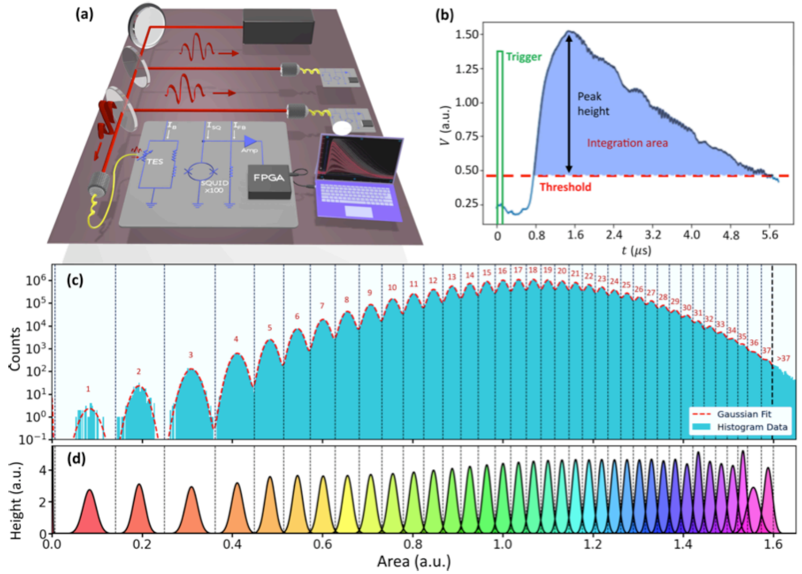
NJP **14** 085001 (2012)





Eaton et al
arXiv:2205.01221