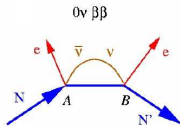


Towards reliable nuclear matrix elements for neutrinoless $\beta\beta$ decay

Frédéric Nowacki



GDR Deep Underground Physics plenary meeting
LPNHE Paris, November 30th 2021

Nuclear physics and neutrinoless $\beta\beta$ decay

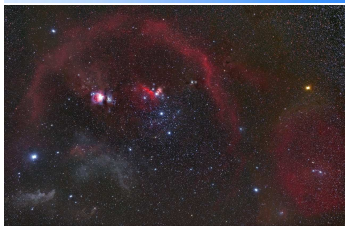
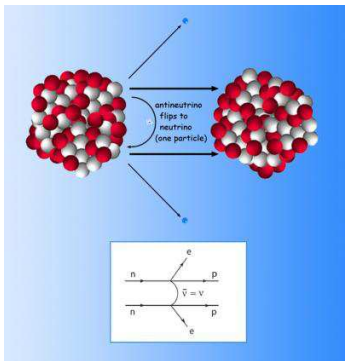
Neutrinos, dark matter studied in experiments using nuclei

Nuclear matrix elements depend on nuclear structure crucial to anticipate reach and fully exploit experiments

$$0\nu\beta\beta \text{ decay: } [T_{1/2}^{0\nu}]^{-1} \propto |M^{0\nu}|^2 \langle m_\nu \rangle^2$$

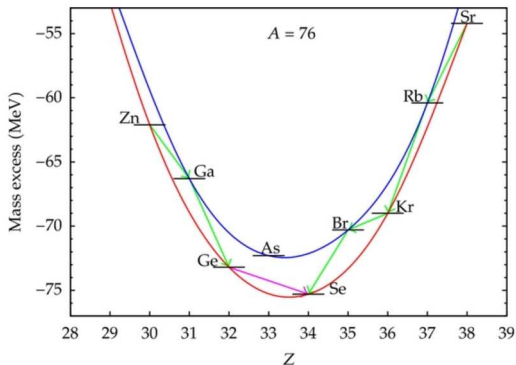
$$\text{Dark matter: } \frac{d\sigma_{\chi\mathcal{N}}}{dq^2} \propto |\sum_i c_i \zeta_i \mathcal{F}_i|^2$$

$M^{0\nu}$: Nuclear matrix element
 \mathcal{F}_i : Nuclear structure factor



Neutrinoless $\beta\beta$ decay

Lepton-number violation, Majorana nature of neutrinos
Second order process only observable in rare cases with
 β -decay energetically forbidden or hindered by ΔJ

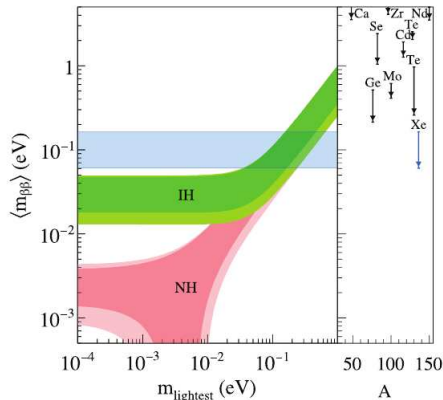
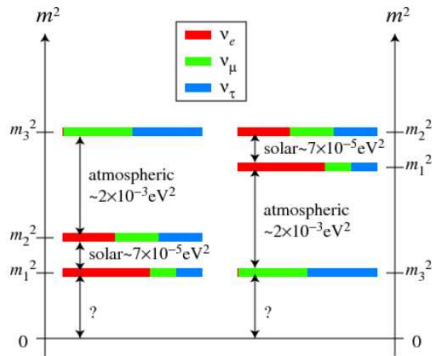


Present best limits $T_{1/2}^{0\nu} \gtrsim 10^{25}$ y.:

^{76}Ge (GERDA, Majorana), ^{130}Te (CUORE), ^{136}Xe (EXO, KamLAND-Zen)

Next generation experiments: inverted hierarchy

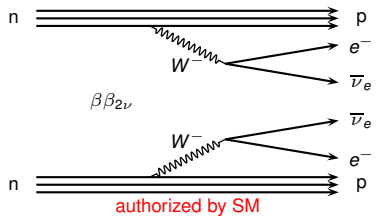
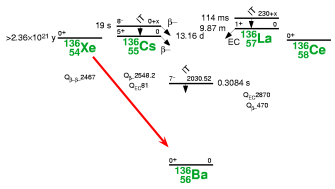
The decay lifetime is $[T_{1/2}^{0\nu}(0^+ \rightarrow 0^+)]^{-1} = G_{0\nu} |M^{0\nu}|^2 \langle m_{\nu}^{\beta\beta} \rangle^2$
 sensitive to absolute neutrino masses, $\langle m_{\nu}^{\beta\beta} \rangle = \sum_i U_{ei}^2 m_i$



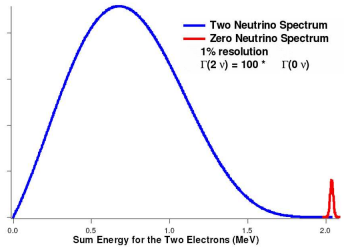
KamLAND-Zen, PRL117 082503 (2016)

Matrix elements needed to make sure next generation ton-scale experiments fully explore “inverted hierarchy”

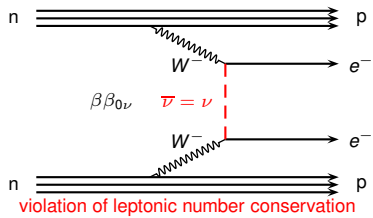
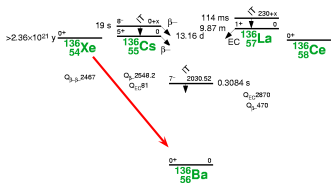
$\beta\beta$ decay



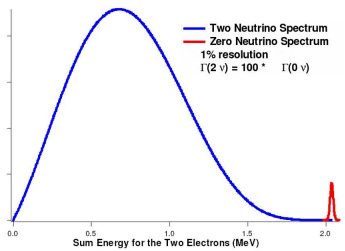
Transition	$Q_{\beta\beta}$ (keV)	Abundance ($^{232}\text{Th} = 100$)
$^{110}\text{Pd} \rightarrow ^{110}\text{Cd}$	2013	12
$^{76}\text{Ge} \rightarrow ^{76}\text{Se}$	2040	8
$^{124}\text{Sn} \rightarrow ^{124}\text{Te}$	2288	6
$^{136}\text{Xe} \rightarrow ^{136}\text{Ba}$	2479	9
$^{130}\text{Te} \rightarrow ^{130}\text{Xe}$	2533	34
$^{116}\text{Cd} \rightarrow ^{116}\text{Sn}$	2802	7
$^{82}\text{Se} \rightarrow ^{82}\text{Kr}$	2995	9
$^{100}\text{Mo} \rightarrow ^{100}\text{Ru}$	3034	10
$^{96}\text{Zr} \rightarrow ^{96}\text{Mo}$	3350	3
$^{150}\text{Nd} \rightarrow ^{150}\text{Sm}$	3667	6
$^{48}\text{Ca} \rightarrow ^{48}\text{Ti}$	4271	0.2



$\beta\beta$ decay



Transition	$Q_{\beta\beta}$ (keV)	Abundance ($^{232}\text{Th} = 100$)
$^{110}\text{Pd} \rightarrow ^{110}\text{Cd}$	2013	12
$^{76}\text{Ge} \rightarrow ^{76}\text{Se}$	2040	8
$^{124}\text{Sn} \rightarrow ^{124}\text{Te}$	2288	6
$^{136}\text{Xe} \rightarrow ^{136}\text{Ba}$	2479	9
$^{130}\text{Te} \rightarrow ^{130}\text{Xe}$	2533	34
$^{116}\text{Cd} \rightarrow ^{116}\text{Sn}$	2802	7
$^{82}\text{Se} \rightarrow ^{82}\text{Kr}$	2995	9
$^{100}\text{Mo} \rightarrow ^{100}\text{Ru}$	3034	10
$^{96}\text{Zr} \rightarrow ^{96}\text{Mo}$	3350	3
$^{150}\text{Nd} \rightarrow ^{150}\text{Sm}$	3667	6
$^{48}\text{Ca} \rightarrow ^{48}\text{Ti}$	4271	0.2



Specificity of $(\beta\beta)_{0\nu}$:

NO EXPERIMENTAL DATA !!!

prediction for m_ν very **difficult**
easier for $m_\nu(A)/m_\nu(A')$

Specificity of $(\beta\beta)_{0\nu}$:

NO EXPERIMENTAL DATA !!!

prediction for m_ν very **difficult**
easier for $m_\nu(A)/m_\nu(A')$

What is the best isotope to observe $(\beta\beta)_{0\nu}$ decay ?

Specificity of $(\beta\beta)_{0\nu}$:

NO EXPERIMENTAL DATA !!!

prediction for m_ν very **difficult**
easier for $m_\nu(A)/m_\nu(A')$

What is the best isotope to observe $(\beta\beta)_{0\nu}$ decay ?

What is the influence of the structure of the nucleus on $(\beta\beta)_{0\nu}$ matrix elements ?

Calculating nuclear matrix elements

Nuclear matrix elements needed to study fundamental symmetries

$$\langle \text{Final} | \mathcal{L}_{\text{leptons-nucleus}} | \text{Initial} \rangle = \langle \text{Final} | dx j^\mu(x) J_\mu(x) | \text{Initial} \rangle$$

- Nuclear structure calculation of the initial and final states:

Shell model Retamosa, Caurier, FN...

Energy-density functional Rodriguez, Yao...

QRPA Vogel, Faessler, Simkovic, Suhonen...

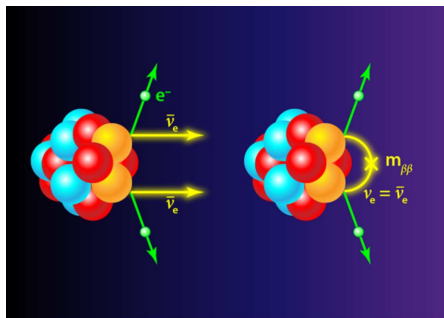
Interacting boson model Iachello, Barea...

Ab Initio many-body methods

Green's Function MC, Coupled-Cluster, IM-SRG

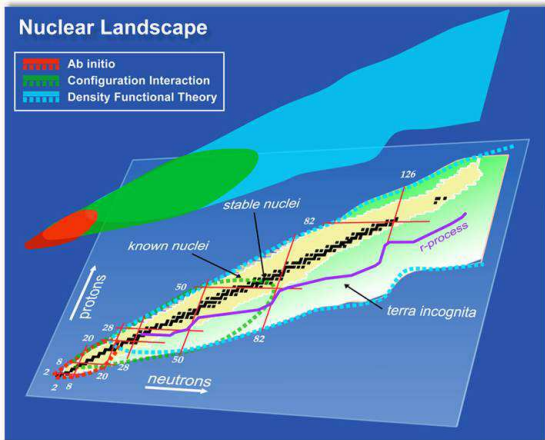
- Lepton-nucleus interaction:

Study hadronic current in nucleus:
phenomenological approaches, effective theory of QCD



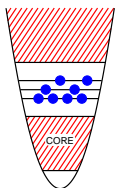
Nuclear many-body problem

The number of nucleons in nuclei is too large for an exact solution of A -body Schrödinger equation. Still, it is much too small for statistical methods



- Ab initio Methods
- Nuclear Shell Model (SM) / Configuration Interaction (CI)
- Density Functional Theory (DFT)

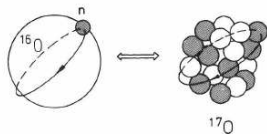
Shell Model Problem



- Define a valence space
- Derive an effective interaction

$$\mathcal{H}\Psi = E\Psi \rightarrow \mathcal{H}_{\text{eff}}\Psi_{\text{eff}} = E\Psi_{\text{eff}}$$

- Build and diagonalize the Hamiltonian matrix.



In general, effective operators also have to be introduced to account for the restrictions of the Hilbert space

$$\langle \Psi | \mathcal{O} | \Psi \rangle = \langle \Psi_{\text{eff}} | \mathcal{O}_{\text{eff}} | \Psi_{\text{eff}} \rangle$$

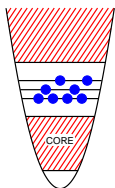
In principle, all the spectroscopic properties are described simultaneously (Rotational band **AND** β decay half-life).

- A valence space can be adequate to describe some properties and completely wrong for others

${}^{48}_{24}\text{Cr}_{24}$	$(f_7)_{\frac{7}{2}}^8$	$(f_7 p_3)_{\frac{7}{2} \frac{3}{2}}^8$	$(f_7 f_5)_{\frac{7}{2} \frac{5}{2}}^8$	$(fp)^8$
$\langle n_{f_7/2} \rangle$	8	7.21	7.60	6.55
$E(2^+)$	0.55	0.42	1.17	0.74
$Q(2^+)$	0.0	-26	-0.03	-29.5
$BE_2(2^+ \rightarrow 0^+)$	77	150	82	215
$B(GT)$	0.80	0.96	4.54	4.25

- For the quadrupole properties $f_7 p_3$ is a good space whereas for magnetic and Gamow-Teller processes the presence of the spin orbit partners is compulsory.

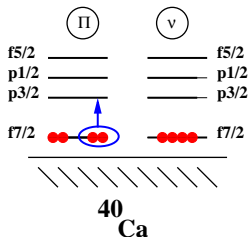
Shell Model Problem



- Define a valence space
- Derive an effective interaction

$$\mathcal{H}\Psi = E\Psi \rightarrow \mathcal{H}_{\text{eff}}\Psi_{\text{eff}} = E\Psi_{\text{eff}}$$

- Build and diagonalize the Hamiltonian matrix.



In general, effective operators also have to be introduced to account for the restrictions of the Hilbert space

$$\langle \Psi | \mathcal{O} | \Psi \rangle = \langle \Psi_{\text{eff}} | \mathcal{O}_{\text{eff}} | \Psi_{\text{eff}} \rangle$$

In principle, all the spectroscopic properties are described simultaneously (Rotational band **AND** β decay half-life).

- A valence space can be adequate to describe some properties and completely wrong for others

$^{48}\text{Cr}_{24}$	$(f_{7/2})^8$	$(f_{7/2} p_{3/2})^8$	$(f_{7/2} f_{5/2})^8$	$(fp)^8$
$\langle n_{f_{7/2}} \rangle$	8	7.21	7.60	6.55
$E(2^+)$	0.55	0.42	1.17	0.74
$Q(2^+)$	0.0	-26	-0.03	-29.5
$\text{BE}_2(2^+ \rightarrow 0^+)$	77	150	82	215
$\text{B}(\text{GT})$	0.80	0.96	4.54	4.25

- For the quadrupole properties $f_{7/2} p_{3/2}$ is a good space whereas for magnetic and Gamow-Teller processes the presence of the spin orbit partners is compulsory.

Two neutrinos mode

The theoretical expression of the half-life of the 2ν mode can be written as:

$$[T_{1/2}^{2\nu}]^{-1} = G_{2\nu} |M_{GT}^{2\nu}|^2,$$

with

$$M_{GT}^{2\nu} = \sum_m \frac{\langle 0_f^+ || \vec{\sigma} t_- || 1_m^+ \rangle \langle 1_m^+ || \vec{\sigma} t_- || 0_i^+ \rangle}{E_m + E_0}$$

- $G_{2\nu}$ contains the phase space factors and the axial coupling constant g_A

Two neutrinos mode

The theoretical expression of the half-life of the 2ν mode can be written as:

$$[T_{1/2}^{2\nu}]^{-1} = G_{2\nu} |M_{GT}^{2\nu}|^2,$$

with

$$M_{GT}^{2\nu} = \sum_m \frac{\langle 0_f^+ || \vec{\sigma} t_- || 1_m^+ \rangle \langle 1_m^+ || \vec{\sigma} t_- || 0_i^+ \rangle}{E_m + E_0}$$

- $G_{2\nu}$ contains the phase space factors and the axial coupling constant g_A
- summation over intermediate states

Two neutrinos mode

The theoretical expression of the half-life of the 2ν mode can be written as:

$$[T_{1/2}^{2\nu}]^{-1} = G_{2\nu} |M_{GT}^{2\nu}|^2,$$

with

$$M_{GT}^{2\nu} = \sum_m \frac{\langle 0_f^+ || \vec{\sigma} t_- || 1_m^+ \rangle \langle 1_m^+ || \vec{\sigma} t_- || 0_i^+ \rangle}{E_m + E_0}$$

- $G_{2\nu}$ contains the phase space factors and the axial coupling constant g_A
- summation over intermediate states
- to quench or not to quench ? ($\sigma\tau_{eff}$.)

Two neutrinos mode

The theoretical expression of the half-life of the 2ν mode can be written as:

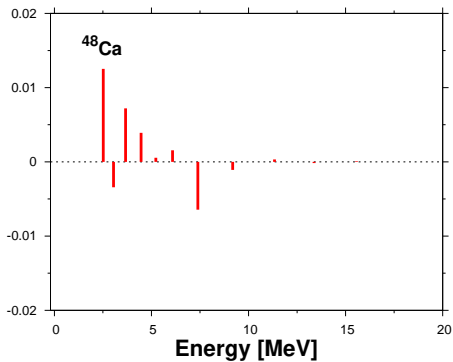
$$[T_{1/2}^{2\nu}]^{-1} = G_{2\nu} |M_{GT}^{2\nu}|^2,$$

with

$$M_{GT}^{2\nu} = \sum_m \frac{\langle 0_f^+ || \vec{\sigma} t_- || 1_m^+ \rangle \langle 1_m^+ || \vec{\sigma} t_- || 0_i^+ \rangle}{E_m + E_0}$$

- $G_{2\nu}$ contains the phase space factors and the axial coupling constant g_A
- summation over intermediate states
- to quench or not to quench ? ($\sigma\tau_{eff}$.)
- does a good 2ν ME guarantee a good 0ν ME ?

2ν half-lives



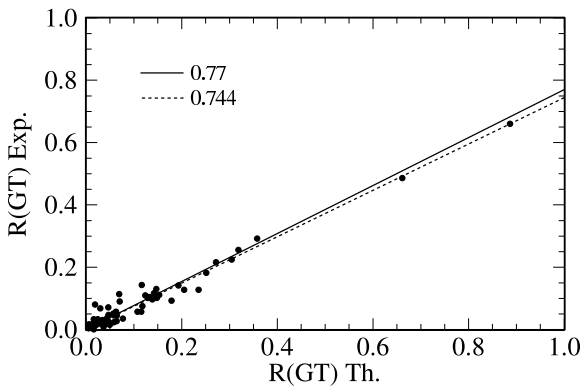
2ν strength function in ^{48}Ca , ^{130}Te and ^{136}Xe

Parent nuclei	^{48}Ca	^{76}Ge	^{82}Se	^{130}Te	^{136}Xe
$T_{1/2}^{2\nu}(g.s.)$ th.	$3.7E19$	$1.15E21$	$3.4E19$	$4E20$	$6E20$
$T_{1/2}^{2\nu}(g.s.)$ exp	$4.2E19$	$1.4E21$	$8.3E19$	$2.7E21$	$2.38E21$

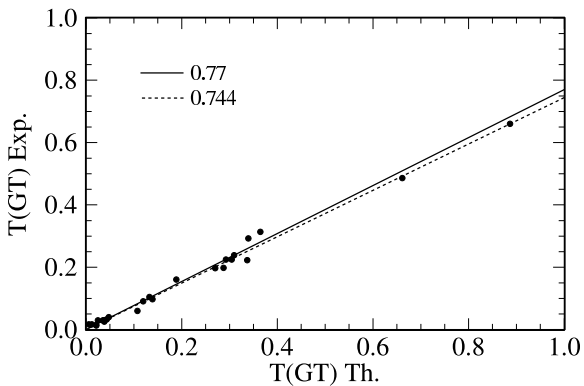
Quenching of GT operator in the *pf*-shell

Nucleus	Uncorrelated	Correlated		Expt.
		Unquenched	$Q = 0.74$	
^{51}V	5.15	2.42	1.33	1.2 ± 0.1
^{54}Fe	10.19	5.98	3.27	3.3 ± 0.5
^{55}Mn	7.96	3.64	1.99	1.7 ± 0.2
^{56}Fe	9.44	4.38	2.40	2.8 ± 0.3
^{58}Ni	11.9	7.24	3.97	3.8 ± 0.4
^{59}Co	8.52	3.98	2.18	1.9 ± 0.1
^{62}Ni	7.83	3.65	2.00	2.5 ± 0.1

Quenching of GT strength in the pf -shell

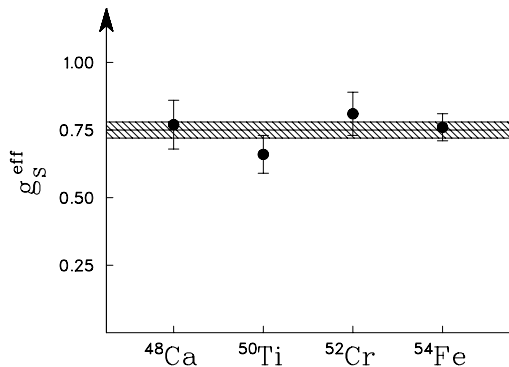


Quenching of GT strength in the pf -shell



Quenching of M1 operator in the pf -shell

KB3 interaction

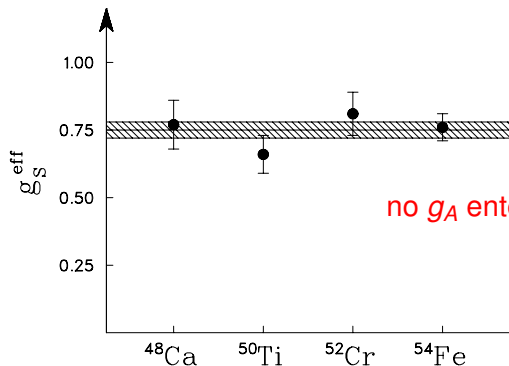


Neumann-Cosel et al.

Phys. Lett. **B433** 1 (1998)

Quenching of M1 operator in the pf -shell

KB3 interaction



no g_A entering in the operator !

Neumann-Cosel et al.

Phys. Lett. **B433** 1 (1998)

Quenching of M1 operator in the pf -shell

KR3 interaction

g_S^{eff}

Definitions

See also: "Value of the axial-vector coupling strength in β and $\beta\beta$ decays: A review" published in *Frontiers in Physics* 5 (2017) 55.

Nucleon weak current in a nucleus:

$$j_N^\mu = g_V \gamma^\mu - g_A \gamma^\mu \gamma^5$$

Quenching:

$$q = g_A / g_A^{\text{free}}$$

Free value of g_A (Particle Data Group 2016) from the decay of free neutron:

$$g_A^{\text{free}} = 1.2723(23)$$

Effective value of g_A :

$$g_A^{\text{eff}} = q g_A^{\text{free}}$$

Jouni Suhonen (JYFL, Finland)

MEDEX'19 7 / 31

From J. Suhonen, MEDEX 2019, Prague

Ne
Phys.

Quenching of GT operator in the pf -shell

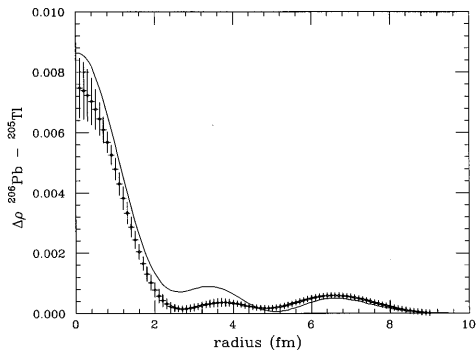
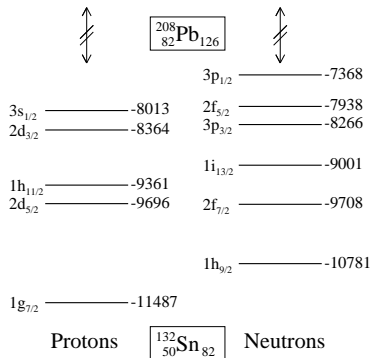


FIG. 3. Density difference between ^{206}Pb and ^{205}Tl . The experimental result of Cavendon *et al.* (1982) is given by the error bars; the prediction obtained using Hartree-Fock orbitals with adjusted occupation numbers is given by the curve. The systematic shift of 0.0008 fm^{-3} at $r \leq 4 \text{ fm}$ is due to deficiencies of the calculation in predicting the core polarization effect.

V. R. Pandharipande, I. Sick and P. K. A. deWitt
Huberts, *Rev. mod. Phys.* **69** (1997) 981



Quenching of GT operator in the pf -shell

If we write


$$|\hat{i}\rangle = \alpha|0\hbar\omega\rangle + \sum_{n \neq 0} \beta_n |n\hbar\omega\rangle,$$

$$|\hat{f}\rangle = \alpha'|0\hbar\omega\rangle + \sum_{n \neq 0} \beta'_n |n\hbar\omega\rangle$$

then

$$\langle \hat{f} | \mathcal{T} | \hat{i} \rangle^2 = \left(\alpha\alpha' T_0 + \sum_{n \neq 0} \beta_n \beta'_n T_n \right)^2,$$

- $n \neq 0$ contributions negligible
- $\alpha \approx \alpha'$

 projection of the physical wavefunction in the $0\hbar\omega$ space is $Q \approx \alpha^2$

 transition quenched by Q^2

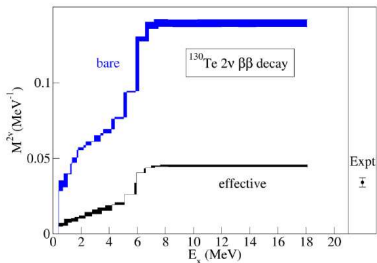
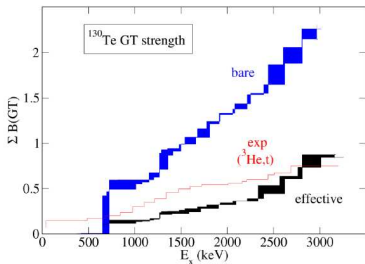
Renormalisation of the GT operator by MBPT

PHYSICAL REVIEW C **95**, 064324 (2017)



Calculation of Gamow-Teller and two-neutrino double- β decay properties for ^{130}Te and ^{136}Xe with a realistic nucleon-nucleon potential

L. Coraggio,^{1,*} L. De Angelis,¹ T. Fukui,¹ A. Gargano,¹ and N. Itaco^{1,2}



Renormalisation of the GT by Many-Body Perturbation Theory

$$\langle \Psi | O | \Psi \rangle = \langle \Psi_{\text{eff}} | O_{\text{eff}}^{(1)} + O_{\text{eff}}^{(1,2)} | \Psi_{\text{eff}} \rangle$$

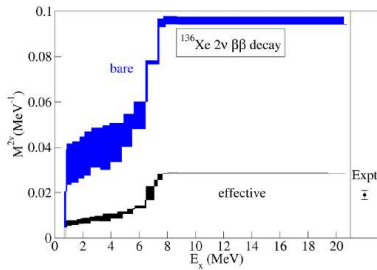
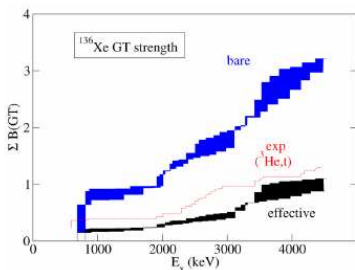
Renormalisation of the GT operator by MBPT

PHYSICAL REVIEW C **95**, 064324 (2017)



Calculation of Gamow-Teller and two-neutrino double- β decay properties for ^{130}Te and ^{136}Xe with a realistic nucleon-nucleon potential

L. Coraggio,^{1,*} L. De Angelis,¹ T. Fukui,¹ A. Gargano,¹ and N. Itaco^{1,2}



Renormalisation of the GT by Many-Body Perturbation Theory

$$\langle \Psi | O | \Psi \rangle = \langle \Psi_{eff} | O_{eff}^{(1)} + O_{eff}^{(1,2)} | \Psi_{eff} \rangle$$

Towards reliable predictions of NMEs for $(\beta\beta)_{0\nu}$ within SSQRPA

- A Gamow-Teller (GT)-type term is the dominant contribution in NMEs
- Even if the two processes have of course a different nature,) NMEs predictions would be more trustworthy if the employed many-body model is able to provide GT spectra in agreement with experiment (well-known problem of the missing strength: the operators are quenched by hand to reproduce data)
- Incoherence and open problem: available many-body models often use by-hand-quenched operators in GT, quenched axial-vector coupling constant g_A value in single β decay, and the bare value of g_A in $(\beta\beta)_{0\nu}$!!
- **Promising direction:** with the Subtracted Second Random-Phase Approximation (SSRPA) an important amount of GT strength is naturally pushed at higher energies in agreement with the data. The experimental spectra are reproduced without altering excitation operator. This model can be safely used for computing NMEs with bare g_A

$$[T_{1/2}^{0\nu}(0^+ \rightarrow 0^+)]^{-1} = G_{0\nu} |M^{0\nu}|^2 \langle m_\nu^{\beta\beta} \rangle^2$$

PHYSICAL REVIEW LETTERS 125, 212501 (2020)

Gamow-Teller Strength in ^{48}Ca and ^{78}Ni with the Charge-Exchange Subtracted Second Random-Phase Approximation

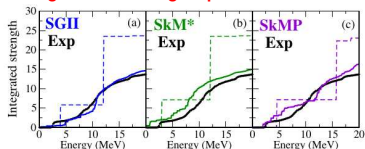
D. Gambacorta¹, M. Grasso², and J. Engel³

¹INFN-LNS, Laboratori Nazionali del Sud, 95123 Catania, Italy

²Université Paris-Saclay, CNRS/SN2P3, IJCLab, 91405 Orsay, France

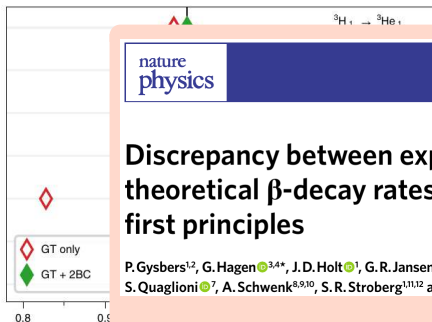
³Department of Physics and Astronomy, CB 3255, University of North Carolina, Chapel Hill, North Carolina 27599-3255, USA

Integrated GT Strength up to 20 MeV in ^{48}Ca



For the first time, the GT spectrum of ^{48}Ca is reproduced without resorting to quenching
Collaboration with LNS Catania, Italy and North Carolina University, US

- Incorporation of 2-body



nature
physics

LETTERS

<https://doi.org/10.1038/s41567-019-0450-7>

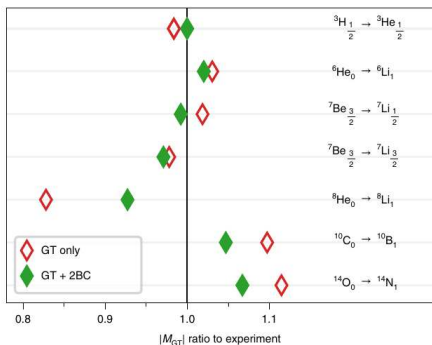
Discrepancy between experimental and theoretical β -decay rates resolved from first principles

P. Gysbers^{1,2}, G. Hagen^{3,4*}, J. D. Holt⁵, G. R. Jansen^{3,5}, T. D. Morris^{3,4,6}, P. Navrátil⁶, T. Papenbrock^{3,4}, S. Quaglioni⁷, A. Schwenk^{8,9,10}, S. R. Stroberg^{11,12} and K. A. Wendt⁷

P. Gysbers et al., Nature Physics (2020)

$$\langle \Psi | O | \Psi \rangle = \langle \Psi_{eff} | O_{eff}^{(1)} + O_{eff}^{(1,2)} | \Psi_{eff} \rangle$$

Recent Ab-initio EFT approaches



- Incorporation of 2-body currents (2BC) operators in Effective Field Theory context

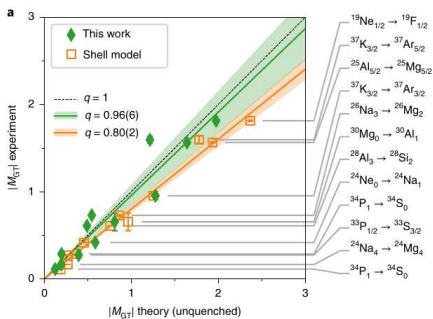
- Ab-Initio NCSM and VS-IMSRG many-body methods with 2N and 3N interactions

- **No need for an extra quenching factor**

P. Gysbers et al., Nature Physics (2020)

$$\langle \Psi | \mathcal{O} | \Psi \rangle = \langle \Psi_{\text{eff}} | \mathcal{O}_{\text{eff}}^{(1)} + \mathcal{O}_{\text{eff}}^{(1,2)} | \Psi_{\text{eff}} \rangle$$

Recent Ab-initio EFT approaches



- Incorporation of 2-body currents (2BC) operators in Effective Field Theory context

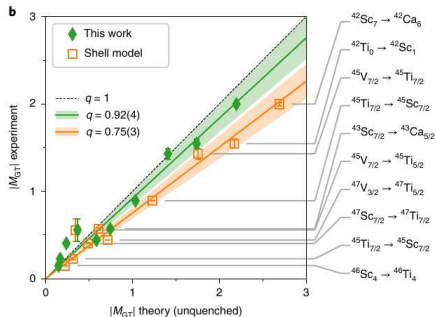
- Ab-Initio NCSM and VS-IMSRG many-body methods with 2N and 3N interactions

- **No need for an extra quenching factor**

P. Gysbers et al., Nature Physics (2020)

$$\langle \Psi | O | \Psi \rangle = \langle \Psi_{eff} | O_{eff}^{(1)} + O_{eff}^{(1,2)} | \Psi_{eff} \rangle$$

Recent Ab-initio EFT approaches



- Incorporation of 2-body currents (2BC) operators in Effective Field Theory context

- Ab-Initio NCSM and VS-IMSRG many-body methods with 2N and 3N interactions

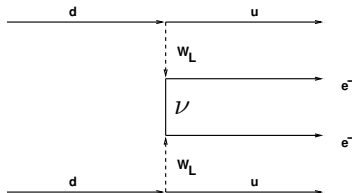
- **No need for an extra quenching factor**

P. Gysbers et al., Nature Physics (2020)

$$\langle \Psi | O | \Psi \rangle = \langle \Psi_{eff} | O_{eff}^{(1)} + O_{eff}^{(1,2)} | \Psi_{eff} \rangle$$

Neutrinoless mode:

Exchange of a light neutrino, only left-handed currents



The theoretical expression of the half-life of the 0ν mode can be written as:

$$[T_{1/2}^{0\nu}(0^+ \rightarrow 0^+)]^{-1} = G_{0\nu} |M^{0\nu}|^2 \langle m_\nu \rangle^2$$


CLOSURE APPROXIMATION then

$$\langle \Psi_f | \mathcal{O}^{(K)} | \Psi_i \rangle \quad \text{with} \quad \mathcal{O}^{(K)} = \sum_{ijkl} W_{ijkl}^{\lambda, K} \left[(a_i^\dagger a_j^\dagger)^\lambda (\tilde{a}_k \tilde{a}_l)^\lambda \right]^K$$

CLOSURE APPROXIMATION then

$$\langle \Psi_f || \mathcal{O}^{(K)} || \Psi_i \rangle \quad \text{with} \quad \mathcal{O}^{(K)} = \sum_{ijkl} W_{ijkl}^{\lambda, K} \left[(a_i^\dagger a_j^\dagger)^\lambda (\tilde{a}_k \tilde{a}_l)^\lambda \right]^K$$

two-body operator



CLOSURE APPROXIMATION then

$$\langle \Psi_f || \mathcal{O}^{(K)} || \Psi_i \rangle \quad \text{with} \quad \mathcal{O}^{(K)} = \sum_{ijkl} W_{ijkl}^{\lambda, K} \left[(a_i^\dagger a_j^\dagger)^\lambda (\tilde{a}_k \tilde{a}_l)^\lambda \right]^K$$

two-body operator

We are left with a “standard” nuclear structure problem

$$M^{(0\nu)} = M_{GT}^{(0\nu)} - \left(\frac{g_V}{g_A} \right)^2 M_F^{(0\nu)} - M_T^{(0\nu)}$$

SM results for $(\beta\beta)_{0\nu}$ with the bare operator

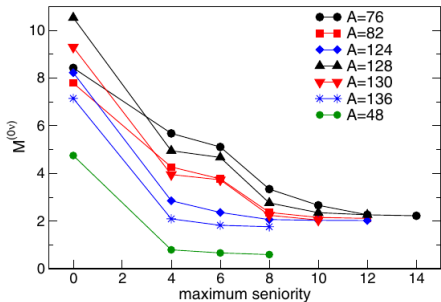
emitter	$\langle m_\nu \rangle$ ($T_{\frac{1}{2}} = 10^{25}$ y.)	$M_{0\nu}^{tot}$ (UCOM)
^{48}Ca	0.63	0.85
^{76}Ge	0.72	2.81
^{82}Se	0.37	2.64
^{96}Zr		
^{100}Mo		
^{110}Pd		
^{116}Cd	0.46	1.60
^{124}Sn	0.37	2.62
^{128}Te	1.32	2.88
^{130}Te	0.28	2.65
^{136}Xe	0.38	2.19
^{150}Nd	heavy and deformed !	

Pairing correlations and $0\nu\beta\beta$ decay

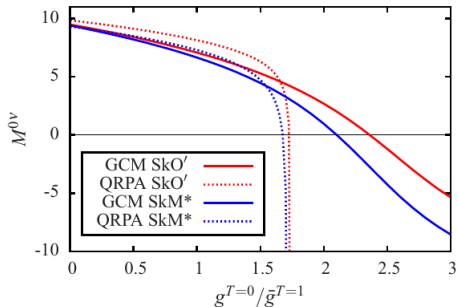
$0\nu\beta\beta$ decay favoured by proton-proton, neutron-neutron pairing, but it is disfavored by proton-neutron pairing

Ideal case: superfluid nuclei reduced with high-seniorities

Addition of isoscalar pairing reduces matrix element value



E. Caurier et al., PRL100 052503 (2008)



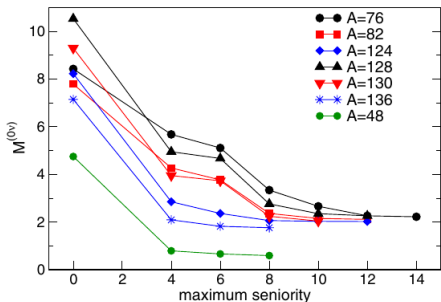
Hinohara, Engel, PRC 90 031301 (2014)

Related to approximate SU(4) symmetry of the $\sum H(r)\sigma_i\sigma_j\tau_i\tau_j$ operator

Pairing correlations and $0\nu\beta\beta$ decay

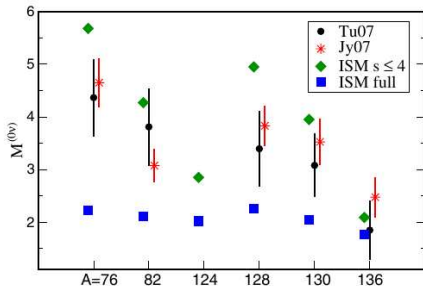
$0\nu\beta\beta$ decay favoured by proton-proton, neutron-neutron pairing,
but it is disfavored by proton-neutron pairing

Ideal case: superfluid nuclei
reduced with high-seniorities



E. Caurier et al., PRL100 052503 (2008)

Addition of isoscalar pairing
reduces matrix element value

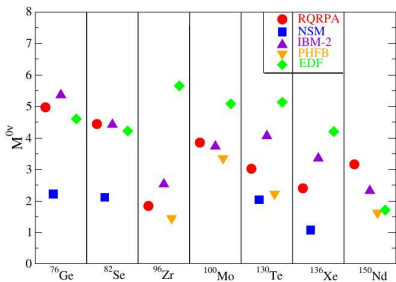


E. Caurier et al., PRL100 052503 (2008)

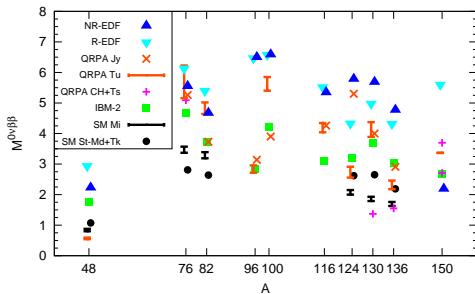
Related to approximate SU(4) symmetry of the $\sum H(r)\sigma_i\sigma_j\tau_i\tau_j$ operator

$0\nu\beta\beta$ matrix elements: last 10 years

Comparison of nuclear matrix elements calculations: 2012 vs 2017



P. Vogel, J. Phys. G39 124002 (2012)



J. Engel, Rep. Prog. Phys.80 046301 (2017)

What have we learned in the last 10 years ?

Shell model configuration space: two shells

For ^{48}Ca enlarge configuration space

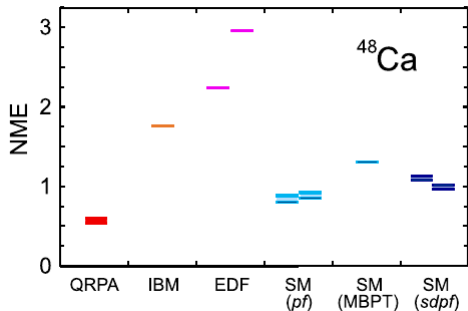
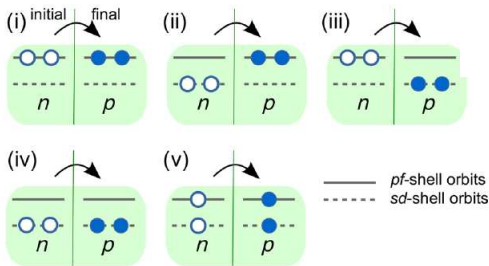
from pf to $sdpf$

4 to 7 orbitals, dimension 10^5 to 10^9

increases matrix elements

but only moderately 30%

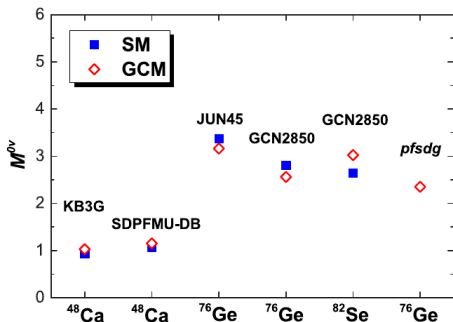
Iwata et al. PRL116 112502 (2016)



Contributions dominated by pairing
2 particle - 2 hole excitations
enhance the $\beta\beta$ matrix element,
Contributions dominated by
1 particle - 1 hole excitations
suppress the $\beta\beta$ matrix element

^{76}Ge matrix element in two shells: approximate

Large configuration space calculations in 2 major oscillator shells include all relevant correlations: isovector/isoscalar pairing, deformation
Many-body approach: Generating Coordinate Method (GCM)



- GCM approximates shell model calculation

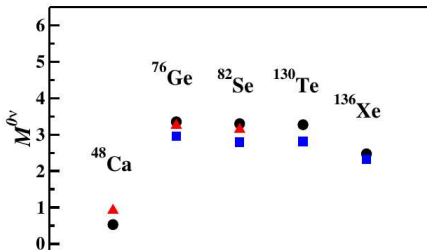
- Degrees of freedom, or generating coordinates validated against exact shell model in restricted configuration space

Jiao et al., PRC96 054310 (2017)

^{76}Ge nuclear matrix elements in 2 major shells
very similar to shell model nuclear matrix element in 1 major shell

Calculation of the neutrinoless double- β decay matrix element within the realistic shell model

L. Coraggio,¹ A. Gargano,¹ N. Itaco^{2,1}, R. Mancino^{2,1} and F. Nowacki^{3,2}

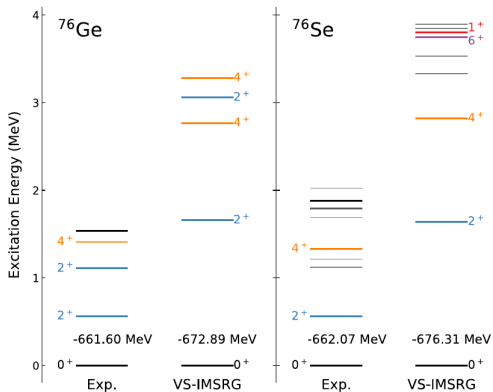


- MBPT applied to the neutrinoless operator
- calculations for emitters ranging from $A=48$ to $A=136$
- Collaboration IPHC - INFN/Naples University

$$\langle \Psi | \mathcal{O} | \Psi \rangle = \langle \Psi_{\text{eff}} | \mathcal{O}_{\text{eff}}^{(1)} + \mathcal{O}_{\text{eff}}^{(1,2)} | \Psi_{\text{eff}} \rangle$$

The reduction of nuclear matrix elements for the neutrinoless mode
much smaller than for the 2-neutrinos mode

Valence Ab-Initio calculations



• A. belley, et al.
 Phys. Rev. Lett. 126, 042502 (2021)

• Ab-initio calculations for
 ^{48}Ca , ^{76}Ge and ^{82}Se

• bad spectroscopy reproduction

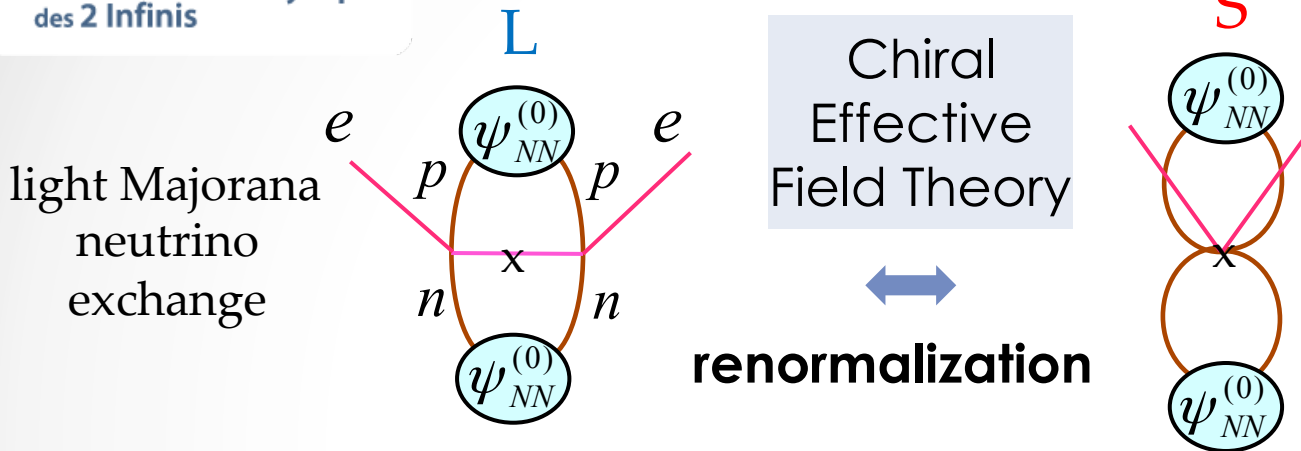
• strong sensitivity to 3N incorporation

	^{48}Ca			^{76}Ge			^{82}Se		
	HO	HF	IMSRG	HO	HF	IMSRG	HO	HF	IMSRG
<i>GT</i>	0.51(1)	0.46(1)	0.54(1)	4.2(2)	3.5(2)	2.04(10)	3.39(1)	2.76(1)	1.19(5)
<i>F</i>	0.13(1)	0.13(1)	0.16(1)	0.47(1)	0.42(1)	0.46(2)	0.39(1)	0.35(1)	0.39(1)
<i>T</i>	-0.07(1)	-0.08(1)	-0.12(1)	-0.04(1)	-0.02(1)	-0.37(2)	-0.04(1)	-0.02(1)	-0.33(1)
Total	0.57(1)	0.51(1)	0.58(1)	4.6(2)	3.9(2)	2.14(9)	3.77(1)	3.09(1)	1.24(5)

A NEW LEADING MECHANISM FOR NEUTRINOLESS DOUBLE-BETA DECAY

F. Cirigliano, ..., U. van Kolck, *Phys. Rev. Lett.* **122** (2019) 143001; *Phys. Rev. C* **100** (2019) 055504

short-range exchange



- eventually calculable in lattice QCD
- *estimated* from charge-independence breaking

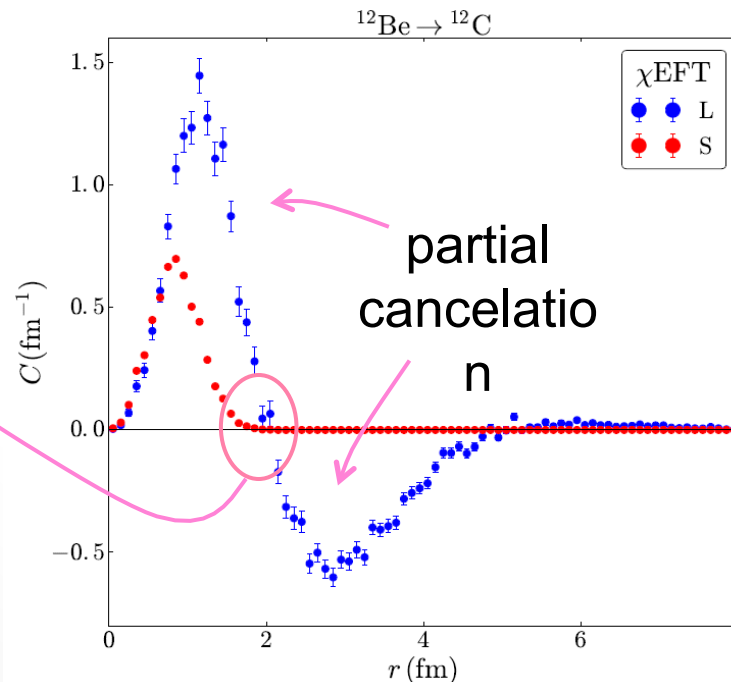
... but neglected in all existing calculations

nuclear matrix element:

$$A_{0\nu 2\beta} = \int dr C(r)$$

$$\frac{A_{0\nu 2\beta}^{(S)}}{A_{0\nu 2\beta}^{(L)}} \approx 0.8$$

SIGNIFICANT!



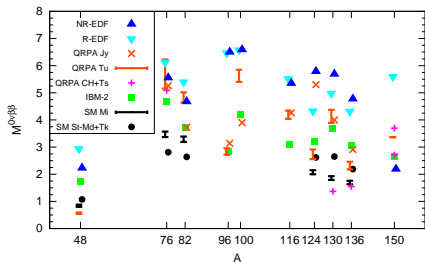
Variational Monte Carlo

orthogonality initial/final states



feature of realistic transitions

$0\nu\beta\beta$ matrix elements: critical assessment



List of criteria for a critical assessment

- reproduce low-lying states spectroscopy in parent and daughter nuclei
- reproduce ElectroMagnetic properties
- reproduce single Gamow-Teller properties
- reproduce $(\beta\beta)_{2\nu}$ properties

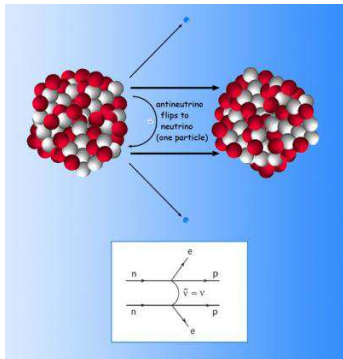
J. Engel, Rep. Prog. Phys.80

046301 (2017)

Not much calculations left ...

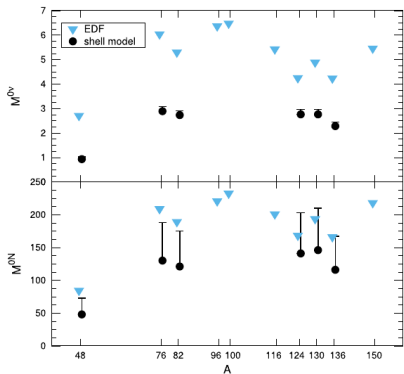
Reliable nuclear matrix elements needed to plan and fully exploit impressive experiments looking for neutrinoless double-beta decay

- Matrix elements differences between present calculations, factor 2-3 besides additional “quenching” ?
- ^{48}Ca and ^{76}Ge matrix elements in larger configuration space increase $\approx 30\%$, missing correlations introduced in IBM, EDF
- Ab-initio calculations of β decays do not need additional “quenching”, Ab-initio matrix elements for ^{48}Ca (several approaches), ^{76}Ge and ^{82}Ge
- $2\nu\beta\beta$ decay, μ -capture/ ν -nucleus scattering and double Gamow-Teller transitions can give insight on $0\nu\beta\beta$ matrix elements



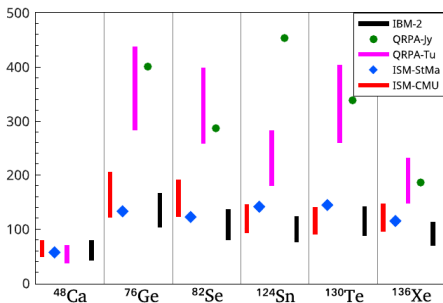
Heavy-neutrino exchange nuclear matrix elements

Contrary to light-neutrino exchange, for heavy-neutrino exchange decay shell model, IBM and EDF matrix elements agree reasonably!



J. Menendez, JPG 45 014003

(2018)



A. Neacsu et al., PRC 93 024308 (2016)

Suggests differences in treating
longer-range nuclear correlations
dominant in light-neutrino exchange

Heavy-neutrino matrix element

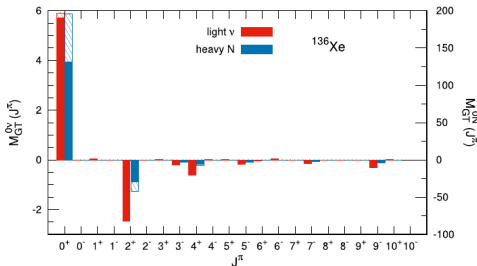
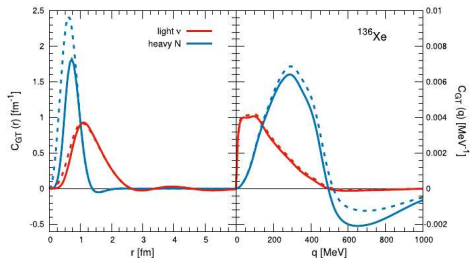
Compared to
light-neutrino exchange

heavy neutrino exchange
dominated by shorter inter-
nucleon range,
larger momentum transfers

heavy neutrino exchange
contribution
from $J > 0$ pairs smaller:
pairing most relevant

Long-range correlations
(except pairing)
not under control

J. Menendez, JPG 45 014003 (2018)



$(\beta\beta)_{0\nu}$ matrix elements

$$\begin{aligned}M_{GT}^{(0\nu)} &= \langle 0_f^+ \| \sum_{n,m} h(\sigma_n \cdot \sigma_m) t_{n-} t_{m-} \| 0_i^+ \rangle, & \chi_F &= \langle 0_f^+ \| \sum_{n,m} h t_{n-} t_{m-} \| 0_i^+ \rangle \left(\frac{g_V}{g_A} \right)^2 / M_{GT}^{(0\nu)}, \\ \chi'_{GT} &= \langle 0_f^+ \| \sum_{n,m} h'(\sigma_n \cdot \sigma_m) t_{n-} t_{m-} \| 0_i^+ \rangle / M_{GT}^{(0\nu)}, & \chi'_F &= \langle 0_f^+ \| \sum_{n,m} h' t_{n-} t_{m-} \| 0_i^+ \rangle \left(\frac{g_V}{g_A} \right)^2 / M_{GT}^{(0\nu)}, \\ \chi_{GT}^\omega &= \langle 0_f^+ \| \sum_{n,m} h_\omega(\sigma_n \cdot \sigma_m) t_{n-} t_{m-} \| 0_i^+ \rangle / M_{GT}^{(0\nu)}, & \chi_F^\omega &= \langle 0_f^+ \| \sum_{n,m} h_\omega t_{n-} t_{m-} \| 0_i^+ \rangle \left(\frac{g_V}{g_A} \right)^2 / M_{GT}^{(0\nu)}, \\ \chi_T &= \langle 0_f^+ \| \sum_{n,m} h' [(\sigma_n \cdot \hat{r}_{n,m})(\sigma_m \cdot \hat{r}_{n,m}) - \frac{1}{3} \sigma_n \cdot \sigma_m] t_{n-} t_{m-} \| 0_i^+ \rangle / M_{GT}^{(0\nu)}, \\ \chi_P &= \langle 0_f^+ \| i \sum_{n,m} h' \left(\frac{r_{+n,m}}{2r_{n,m}} \right) [(\sigma_n - \sigma_m) \cdot (\hat{r}_{n,m} \times \hat{r}_{+n,m})] t_{n-} t_{m-} \| 0_i^+ \rangle \frac{g_V}{g_A} / M_{GT}^{(0\nu)}, \\ \chi_R &= \frac{1}{6} (g_{-\frac{1}{2}}^s - g_{\frac{1}{2}}^s) \langle 0_f^+ \| \sum_{n,m} h_R(\sigma_n \cdot \sigma_m) t_{n-} t_{m-} \| 0_i^+ \rangle \frac{g_V}{g_A} / M_{GT}^{(0\nu)}.\end{aligned}$$

back

$(\beta\beta)_{0\nu}$ matrix elements

$$h(r, \langle\mu\rangle) = \frac{R_0}{r} \phi(\langle\mu\rangle m_e r),$$

$$h'(r, \langle\mu\rangle) = h + \langle\mu\rangle m_e R_0 h_0(\langle\mu\rangle r),$$

$$h_\omega(r, \langle\mu\rangle) = h - \langle\mu\rangle m_e R_0 h_0(\langle\mu\rangle r),$$

$$h_R(r, \langle\mu\rangle) = -\frac{\langle\mu\rangle m_e}{M_j} \left(\frac{2}{\pi} \left(\frac{R_0}{r} \right)^2 - \langle\mu\rangle m_e R_0 h \right) + \frac{4\pi R_0^2}{M_p} \delta(r),$$

$$h_0(x) = -\frac{d\phi}{dx}(x),$$

$$\phi(x) = \frac{2}{\pi} [\sin(x) C_{int}(x) - \cos(x) S_{int}(x)],$$

$$\frac{d\phi}{dx} = \frac{2}{\pi} [\sin(x) C_{int}(x) + \cos(x) S_{int}(x)].$$

$S_{int}(x)$ and $C_{int}(x)$ being the integral sinus and cosinus functions,

$$S_{int}(x) = -\int_x^\infty \frac{\sin(\zeta)}{\zeta} d\zeta, \quad C_{int}(x) = -\int_x^\infty \frac{\cos(\zeta)}{\zeta} d\zeta$$

back