Modifications of gravity and restoring general relativity

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+work in progress

with D. Langlois

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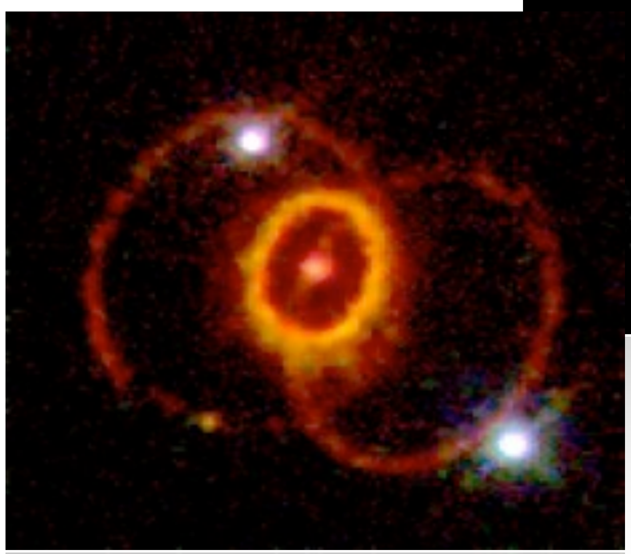
OUTLINE

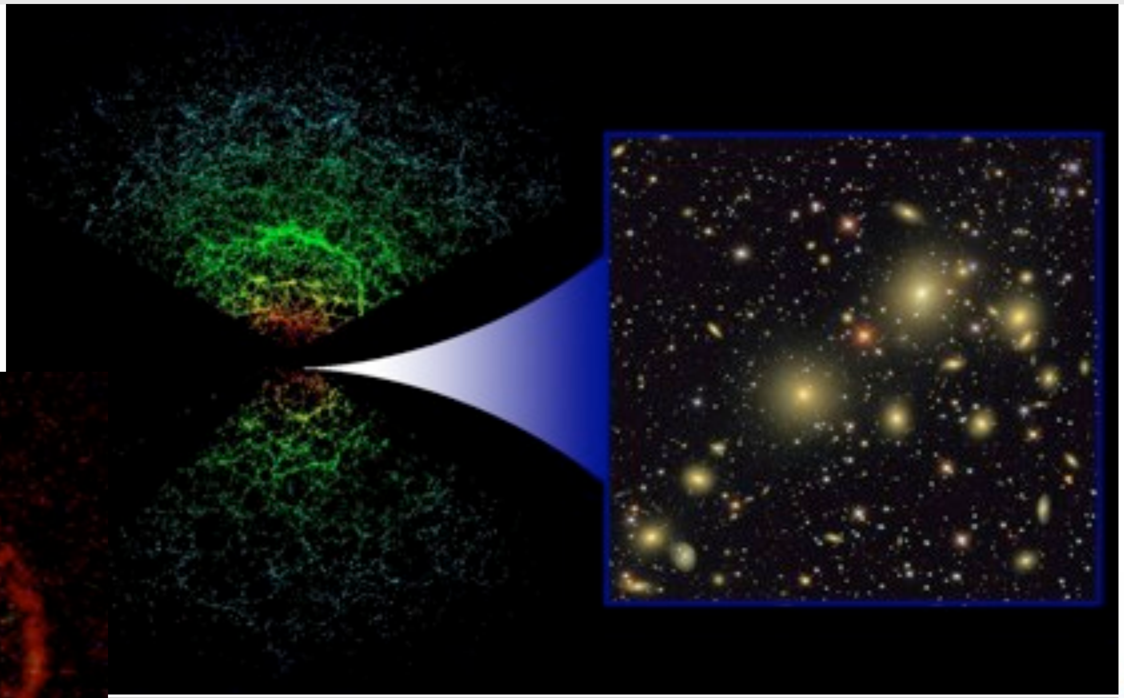
- INTRODUCTION and MOTIVATIONS
- f(R) GRAVITY
  - history of f(R)
  - chameleon mechanism
  - curvature singularity and (non)-existence of simple solutions
  - solution of the problem
- MASSIVE GRAVITY and VAINSHTEIN MECHANISM
  - massive gravity and the idea of Vainshtein
  - decoupling limit
  - solutions of the full system
- K-MOUFLAGE GRAVITY
- CONCLUSION

INTRODUCTION and MOTIVATIONS

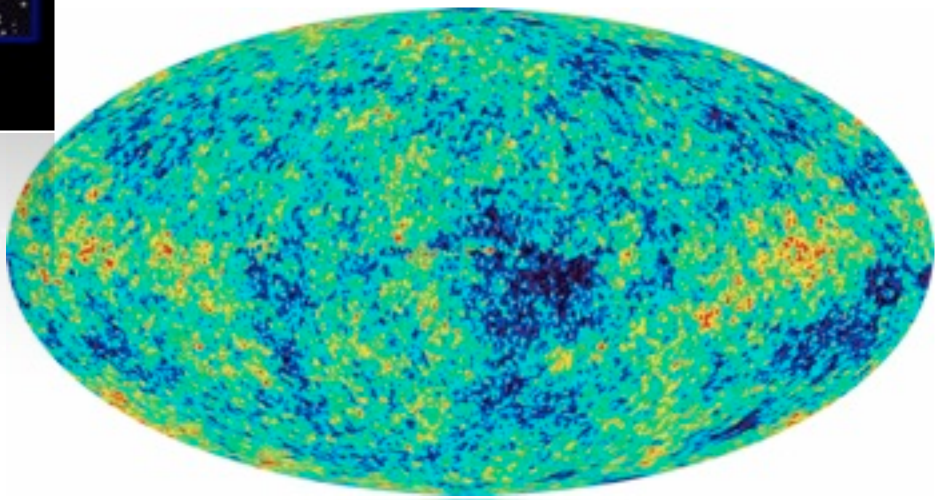
acceleration of the universe



SN Ia



BAO



CMB

- Very different probes: SN Ia, BAO, CMB

They indicate that the Universe is accelerating!

source of acceleration?

- ✦ Einstein equations

$$G_{\mu\nu} = 8\pi G\, T_{\mu\nu}^{(m)} - \Lambda g_{\mu\nu}$$

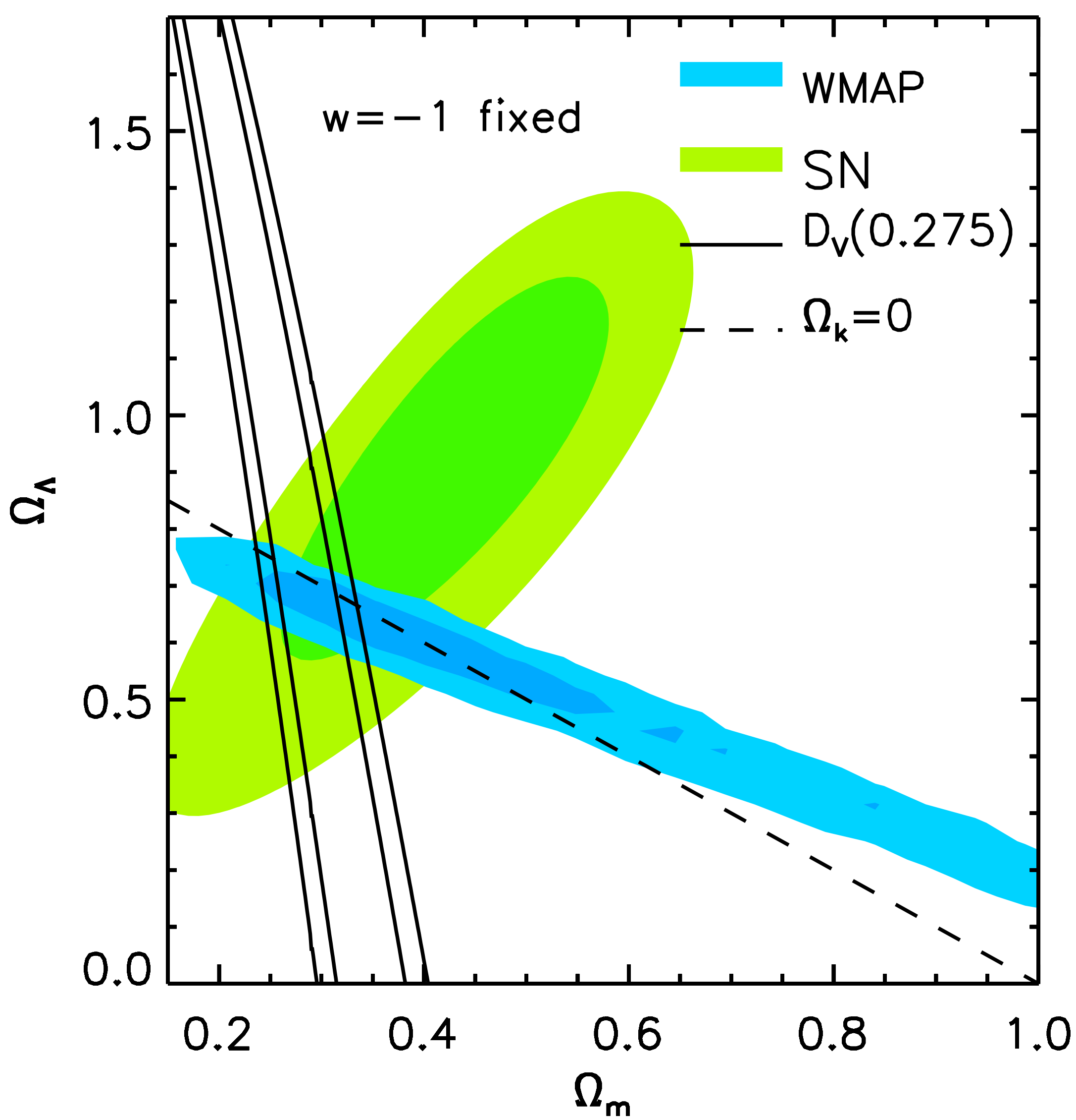
Geometry

Matter

Dark Energy

Modify gravity?

Add exotic matter?



Percival et al. '09

modifying gravity (I)

Aim: to generalize Einstein equations to explain the acceleration of the Universe without explicit introduction of extra fields (Dark Energy)

Problem: we want to recover GR at short distances (inside the solar system) while modifying gravity at large distances

Mechanisms for recovering GR:

➡Chameleon mechanism

➡Vainshtein mechanism

modifying gravity (II)

➡Chameleon mechanism — Khoury & Weltman '03

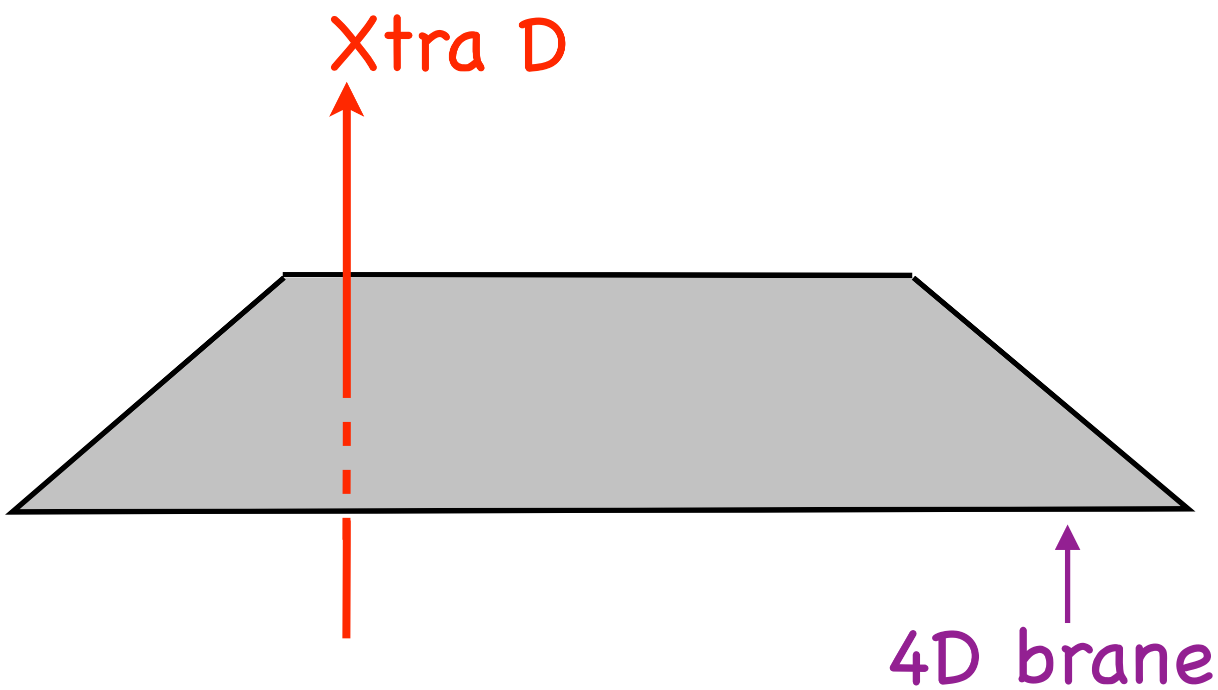
✦ scalar-tensor theories

✦ f(R) gravity

➡Vainshtein's mechanism — Vainshtein '72

✦ massive gravity

✦ models with extra dimensions



Dvali,Gabadadze,Porrati'00

✓ matter stays on the brane

✓ 5D + 4D gravity

✓ can provide an explanation for the cosmic acceleration

✓ 5D massless graviton = infinite tower of 4D gravitons

f(R) and scalar-tensor theories

- A way to get acceleration of the Universe (Dark Energy):

$$S = \frac{M_P^2}{2} \int d^4x \sqrt{-g}\, R + S_m[\Phi_m; g_{\mu\nu}],$$

$$S = \frac{M_P^2}{2} \int d^4x \sqrt{-g}\, f(R) + S_m[\Phi_m; g_{\mu\nu}],$$

so the dynamics is different compared to GR

$$S = \int d^4x \sqrt{-g} \left[\frac{M_P^2}{2} R - \frac{1}{2} (\nabla\phi)^2 - V(\phi) \right] + S_m[\Psi_m; e^{-\phi/(\sqrt{6}M_P)} g_{\mu\nu}]$$

f(R) and scalar-tensor: problems

- ✦ An extra scalar degree of freedom -- how to avoid local gravity constrains?

  Chameleon mechanism - the mass of the extra s.d.f depends on background density of matter

  [Khoury & Weltman '03]

- ✦ Only singular solutions for relativistic objects...

  Analytic understanding + try harder to find solutions

  [EB & Langlois '09]

massive gravity: basic idea... and ghosts

Naive idea: give a mass to the graviton with $m \sim H_0$

Pathologies:

- ✦ Hamiltonian unbounded from below (ghosts) [Boulware & Deser'72; Creminelli et al.'05; Deffayet & Rombouts'05]

However:

- ✦ Other models with massive gravitons assume the Vainshtein mechanism to work (E.g. Nair, Randjbar-Daemi, V. Rubakov'08).
- ✦ Massive gravity -- basic ingredient for models with extra-dimensions (DGP)

massive gravity: problems

- Extra degrees of freedom -- how to avoid local gravity constrains?

  In non-perturbative regime GR is restored (conjecture)

  [Vainshtein' 72]

  [EB,C.Deffayet,R.Ziour'09]

- Singular solutions [Damour et al.'03]

  Analytic understanding + try harder to find solutions

  [EB,C.Deffayet,R.Ziour'09]

f(R) GRAVITY AND CHAMELEON MODEL

f(R) models

$$R \to R + \frac{R^2}{M^2}$$ inflation [Starobinsky' 80]

$$R \to R - \frac{M^{2(n+1)}}{R^n}$$ Dark Energy [Carroll et al' 03]

- Fast tachyonic instability, [Dolgov & Kawasaki' 03]
- $\gamma = 1/2$ (need $\gamma = 1$) [Chiba' 03]

Simple models of f(R) do not work !

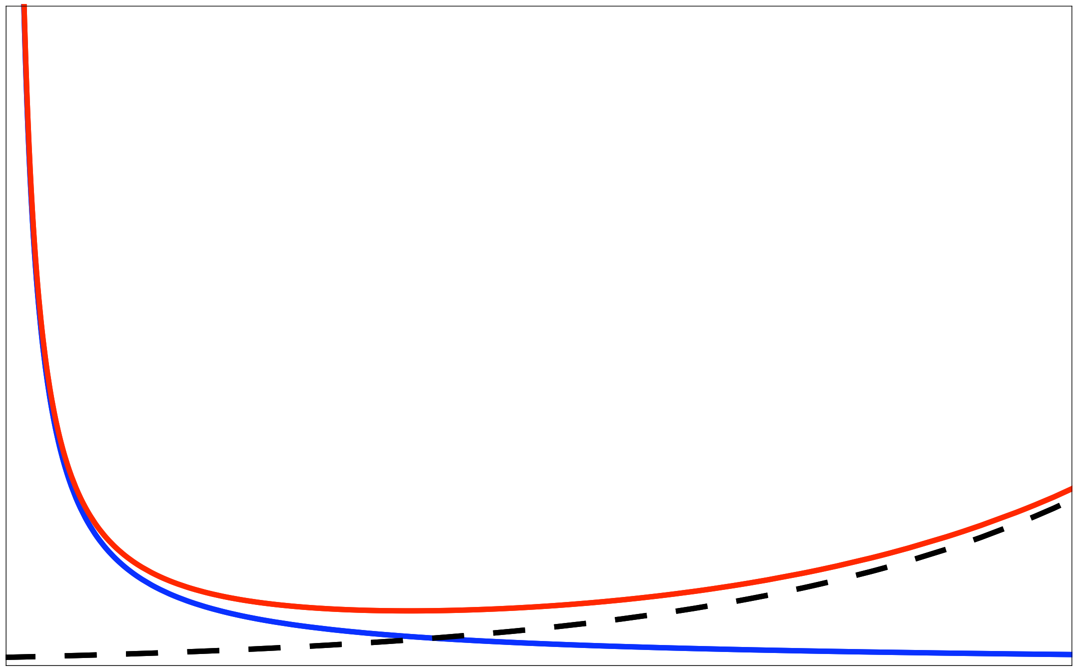
chameleon mechanism

- Chameleon effect in scalar-tensor theory, [Khoury & Weltman' 03]

$$S = \int d^4x \sqrt{-g}\left[\frac{M_P^2}{2}R - \frac{1}{2}(\nabla\phi)^2 - V(\phi)\right] + S_m[\Psi_m; e^{Q\phi/M_P} g_{\mu\nu}]$$

scalar field moves
in the effective potential

$$V_{\rm eff} = V + \frac{1}{4}e^{4Q\phi/M_P}(\tilde{\rho} - 3\tilde{P})$$



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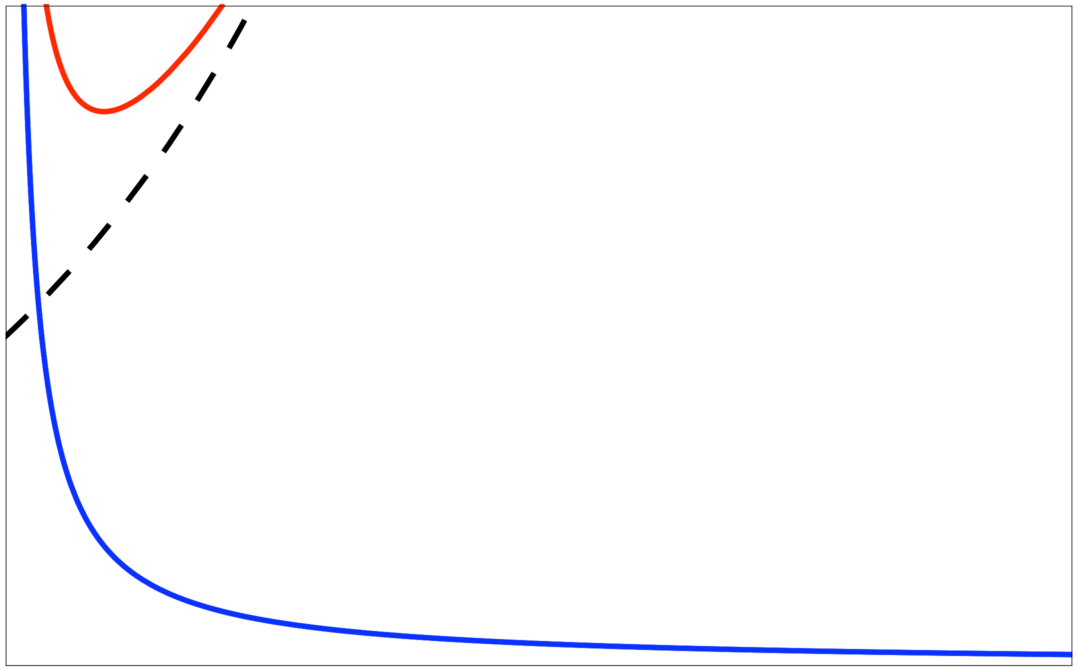
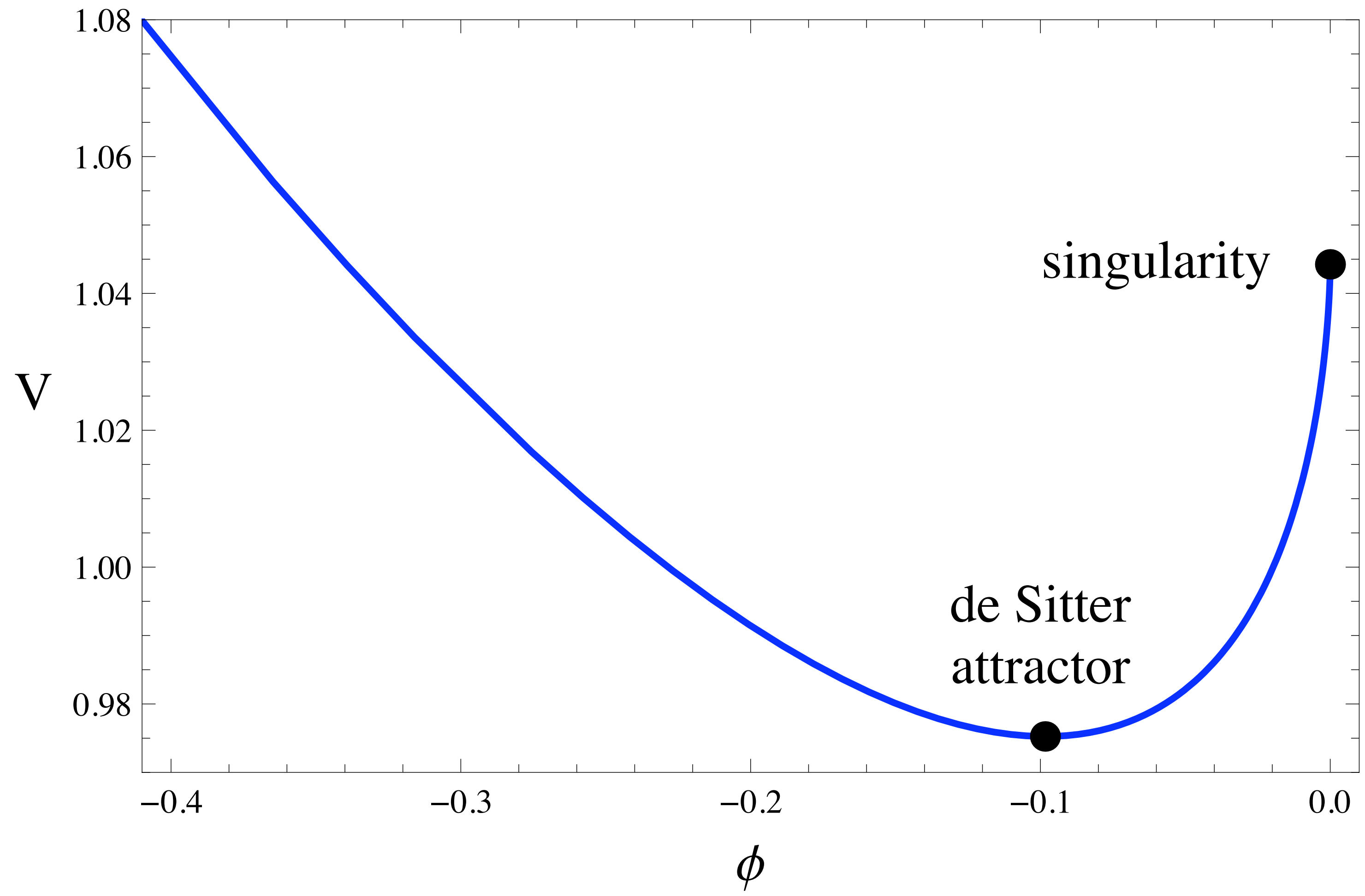


because of f(R) <=> scalar-tensor models,
can do the same for f(R)

curvature singularity in f(R)

- Curvature singularity problem - curvature singularity can be easily accessed for generic infrared modified f(R) theories (Frolov'08)



non-existence of simple solutions?

- No neutron stars for generic f(R) models [Starobinsky and Hu-Sawicky models] (Kobayashi&Maeda'08)
- No neutron stars for higher curvature modified f(R) models (Kobayashi&Maeda'08)
- No solutions for highly relativistic objects in the case of Chameleon field (Tsujikawa et al.'09)
- Instability associated with huge effective of s.d.f. and "fine-tuning problem" (Thongkool et al.'09)

?

action and eoms

- action

$$S = \int d^4x \sqrt{-g} \left[\frac{M_P^2}{2} R - \frac{1}{2} (\nabla\phi)^2 - V(\phi) \right] + S_m \left[\Psi_m ; \Omega^2(\phi) g_{\mu\nu} \right]$$

matter coupled to $\tilde{g}_{\mu\nu} = \Omega^2(\phi) g_{\mu\nu}$

- equations of motion,

$$R_{\mu\nu} - \frac{1}{2} R = M_P^{-2} \left[T^{(m)}_{\mu\nu} + \partial_\mu\phi\partial_\nu\phi - \frac{1}{2} g_{\mu\nu} \partial^\sigma\phi\partial_\sigma\phi - V g_{\mu\nu} \right]$$

$$\nabla_\sigma \nabla^\sigma \phi = -\frac{dV}{d\phi} - \frac{\Omega'}{\Omega} T^{(m)}$$

$$T^{(m)} \equiv g^{\mu\nu} T^{(m)}_{\mu\nu} = -\rho + 3P$$

equations of motion

- Static spherically symmetric ansatz,

$$ds^2 = -e^{\nu} dt^2 + e^{\lambda} dr^2 + r^2 \left(d\theta^2 + \sin^2\theta \, d\phi^2\right)$$
$$e^{-\lambda} = 1 - 2m(r)/r$$

tt component	$m' = \frac{r^2}{2M_P^2}\left[\Omega^4 \tilde{\rho} + \frac{1}{2} e^{-\lambda} \phi'^2 + V(\phi)\right],$
rr component	$\nu' = e^{\lambda}\left[\frac{2m}{r^2} + \frac{r}{M_P^2}\left(\frac{1}{2} e^{-\lambda}\phi'^2 - V(\phi)\right) + \frac{r\Omega^4 \tilde{P}}{M_P^2}\right],$
conservation	$\tilde{P}' = -\frac{1}{2}\left(\tilde{\rho} + \tilde{P}\right)\left(\nu' + 2\frac{\Omega'}{\Omega}\phi'\right),$
eq of state	$\tilde{P} = \tilde{P}(\tilde{\rho}),$
Klein-Gordon eq	$\phi'' + \left(\frac{2}{r} + \frac{1}{2}(\nu' - \lambda')\right)\phi' = e^{\lambda}\left[\frac{dV}{d\phi} + \Omega^3 \Omega' (\tilde{\rho} - 3\tilde{P})\right].$

instability for high-density star

- constant density star in GR, the higher density the higher pressure in the center,

$$\rho - 3P < 0 \quad \text{for} \quad \Phi_* \equiv \frac{\mathrm{GM}}{\mathrm{r}_*} > \frac{5}{18}$$

$$\phi'' + \left(\frac{2}{r} + \frac{1}{2}(\nu' - \lambda')\right)\phi' = e^{\lambda}\left[\frac{dV}{d\phi} + \Omega^3\Omega_\phi(\tilde{\rho} - 3\tilde{P})\right].$$

Tachyon instability

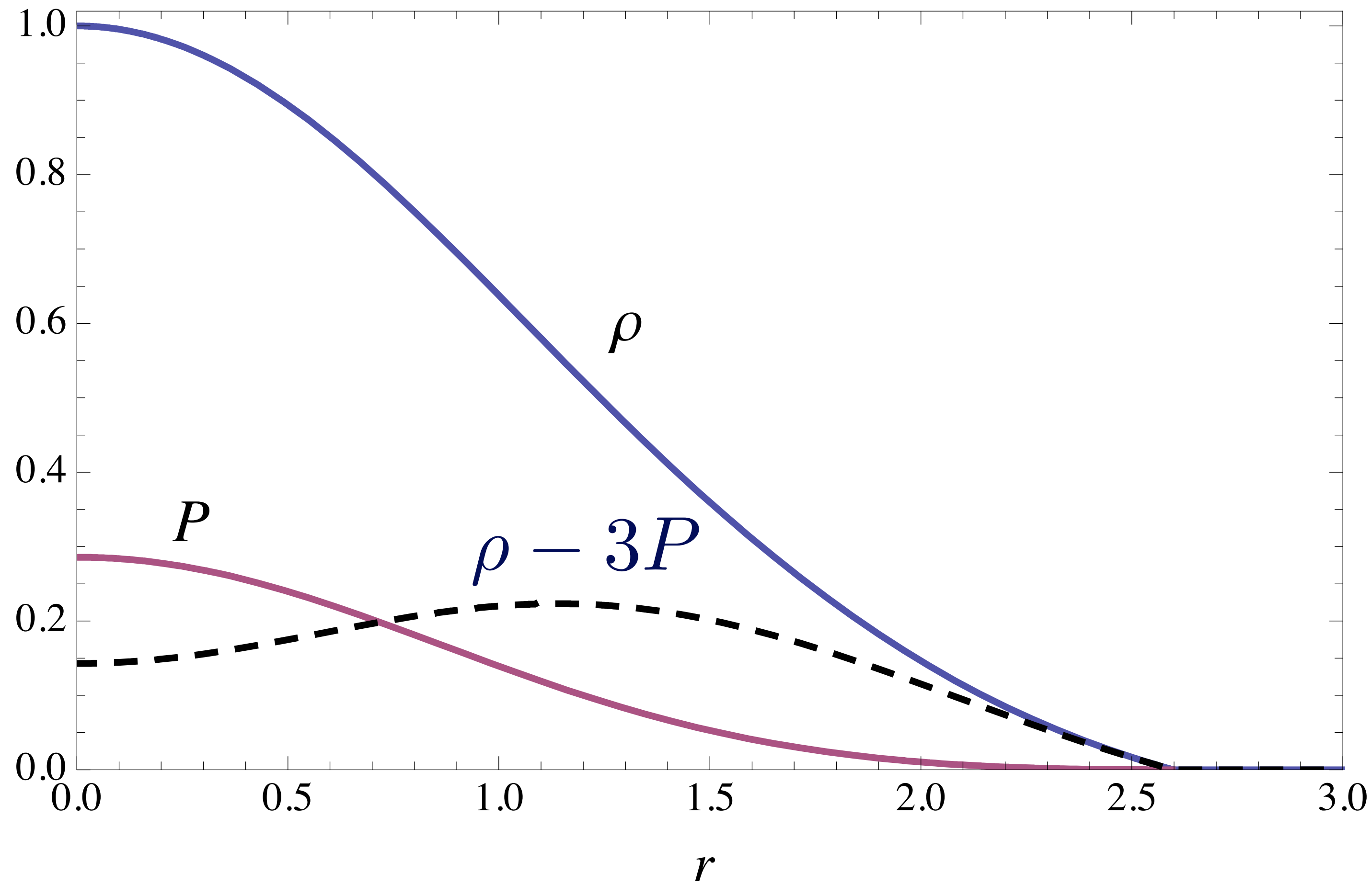
- explains the results of Tsujikawa et.al'09, no solutions for the Chameleon for $\Phi > 0.3$

realistic neutron star

- For a realistic EoS of NS $\rho - 3P > 0$

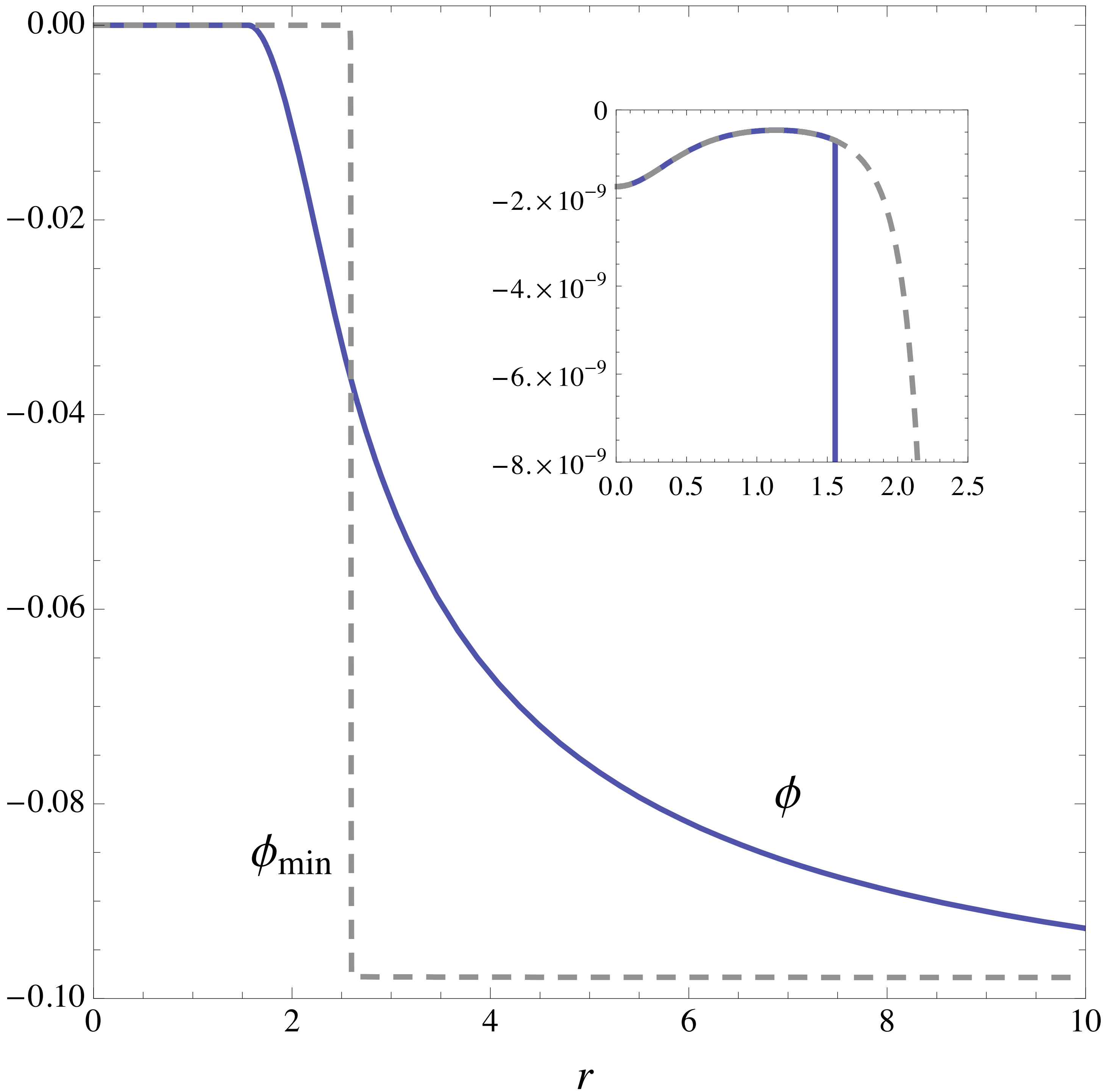
$$\tilde{\rho}(\tilde{n}) = m_B \left(\tilde{n} + K \frac{\tilde{n}^2}{n_0} \right), \quad \tilde{P}(\tilde{n}) = K m_B \frac{\tilde{n}^2}{n_0},$$

$$m_B = 1.66 \times 10^{-27}\mathrm{kg},\; n_0 = 0.1\,\mathrm{fm}^{-1} \text{ and } K = 0.1.$$



$$\begin{aligned} \tilde{n}_c &= 0.4\ \mathrm{fm}^{-3}, \\ |\Phi_*| &\simeq 0.25 \end{aligned}$$

numerical solution



$$v_0 = \frac{M_P^2 R_0}{\tilde{\rho}_c}$$

Massive gravity
and
the Vainshtein mechanism

what is massive gravity?

- The quadratic action for the massive graviton:

f : background metric (often flat) $H_{\mu\nu}$: spin 2 excitation over f

$$S = \frac{M_P^2}{2}\int d^4x \left(\text{“}H\partial^2 H + \ldots\text{''} - \frac{m^2}{4}\left[H_{\mu\nu}H^{\mu\nu} - (H^\mu_\mu)^2\right]\right) + \int d^4x \, \frac{1}{2} T_{\mu\nu}H^{\mu\nu}$$

Kinetic term   Pauli-Fierz Mass term   Matter coupling

- Non-linear completion: dynamical metric $g = f + H$

$$S = \frac{M_P^2}{2}\int d^4x \left(\sqrt{-g}R[g] - \frac{m^2}{4}\mathcal{V}^{(a)}[\mathbf{g}^{-1}\mathbf{f}]\right) + S_m[g]$$

Scalar density

- Examples:

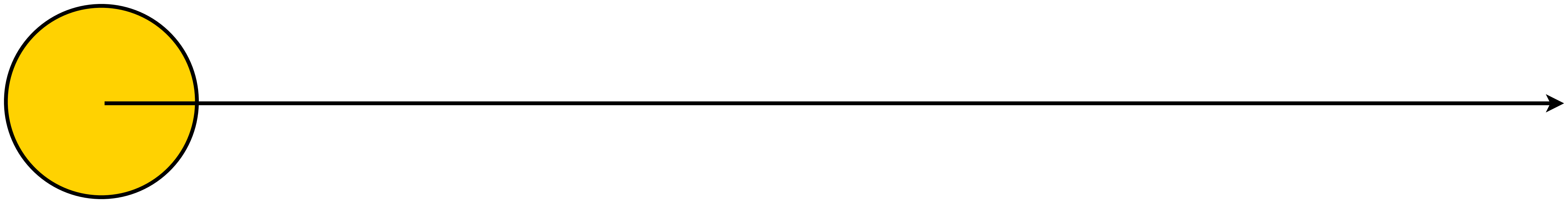
$$\mathcal{V}^{(BD)}[\mathbf{g}^{-1}\mathbf{f}] = \sqrt{-f}\, H_{\mu\nu}H_{\sigma\tau}\left(f^{\mu\sigma}f^{\nu\tau} - f^{\mu\nu}f^{\sigma\tau}\right)$$

Boulware & Deser'72

$$\mathcal{V}^{(AGS)}[\mathbf{g}^{-1}\mathbf{f}] = \sqrt{-g}\, H_{\mu\nu}H_{\sigma\tau}\left(g^{\mu\sigma}g^{\nu\tau} - g^{\mu\nu}g^{\sigma\tau}\right)$$

Arkani-Hamed, Georgi, Schwartz'03

static spherically symmetric solutions



- Schwarzschild Gauge (bi-diagonal, asymptotically flat)

$$g_{\mu\nu}dx^\mu dx^\nu = -e^{\nu(R)}dt^2 + e^{\lambda(R)}dR^2 + R^2 d\Omega^2 \quad \text{: “Schwarzschild” like}$$

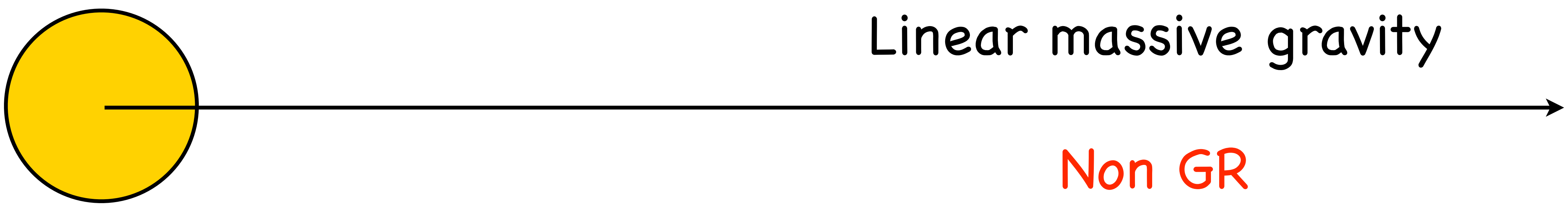
$$f_{\mu\nu}dx^\mu dx^\nu = -dt^2 + \left(1 - \frac{R\mu'(R)}{2}\right)^2 e^{-\mu(R)}dR^2 + e^{-\mu(R)}R^2 d\Omega^2 \quad \text{: flat}$$

- Equations of motion:

$$\begin{aligned} e^{\nu-\lambda}\left(\frac{\lambda'}{R} + \frac{1}{R^2}(e^\lambda - 1)\right) &= 8\pi G_N \left(T^g_{tt} + \rho e^\nu\right), \\ \frac{\nu'}{R} + \frac{1}{R^2}\left(1 - e^\lambda\right) &= 8\pi G_N \left(T^g_{RR} + P e^\lambda\right), \\ \nabla^\mu T^g_{\mu R} &= 0. \end{aligned}$$

Is it possible to find a solution regular everywhere?

Solutions far from source (I)



- Expansion in the Newton's constant

$$\lambda \quad = \quad \lambda_0 + \lambda_1 + \ldots \quad \text{etc.}\,, \quad \text{with} \quad \lambda_i, \nu_i, \mu_i \propto G_N^{i+1}$$

- Linearized solutions

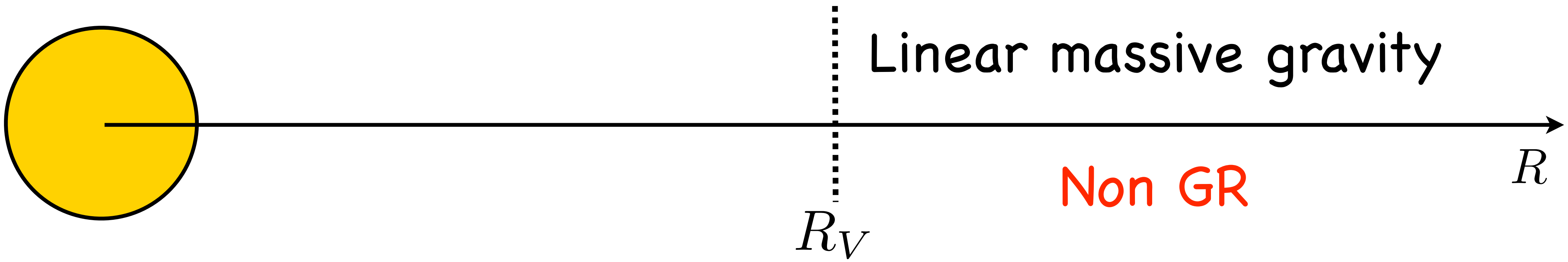
$$\begin{aligned}
\nu_0 &= -\mathcal{C} \times \frac{R_S}{R}\, e^{-mR} \\
\lambda_0 &= \mathcal{C} \times \frac{R_S}{2R}(1+mR)e^{-mR} \\
\mu_0 &= \mathcal{C} \times \frac{R_S}{2(mR)^2 R}\left(1 + mR + (mR)^2\right) e^{-mR}
\end{aligned}$$

- For $R \ll m^{-1}\,:\;\; \lambda_0 = \frac{C_1}{2R},\;\; \nu_0 = -\frac{C_1}{R},\;\; \mu_0 = \frac{1}{(mR)^2}\frac{C_1}{2R}\,.$

vDVZ discontinuity

Iwasaki'70, van Dam, Veltman'70, Zakharov'70

Solutions far from source (II)



✦ Non-linear corrections

Vainshtein '72

$$\nu = -\frac{2}{3}\frac{R_S}{R} + \frac{R_S^2}{R^2}\frac{n_1}{(mR)^4} + \mathcal{O}(R_S^3)$$

$$\lambda = \frac{1}{3}\frac{R_S}{R} + \frac{R_S^2}{R^2}\frac{l_1}{(mR)^4} + \mathcal{O}(R_S^3)$$

$$\mu = \frac{1}{3(mR)^2}\frac{R_S}{R} + \frac{R_S^2}{R^2}\frac{m_1}{(mR)^6} + \mathcal{O}(R_S^3)$$

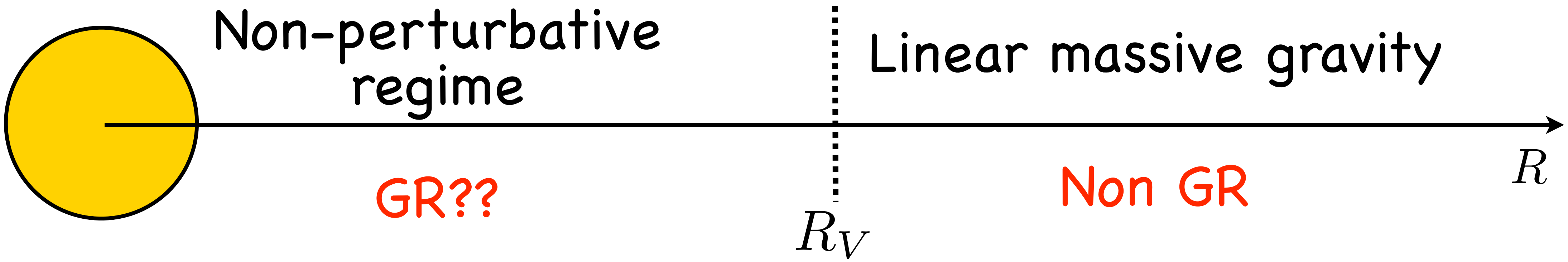
Relevant at

$$R_V = \left(\frac{R_S}{m^4}\right)^{1/5}$$

New scale: Vainshtein's radius

What happens inside the Vainshtein radius?

Solution close to the source



- Expand λ, ν, μ in m: $f(R) = \sum_{n=0}^{\infty} m^{2n} f_n(R)$
- Order 0 = GR
- Expansion

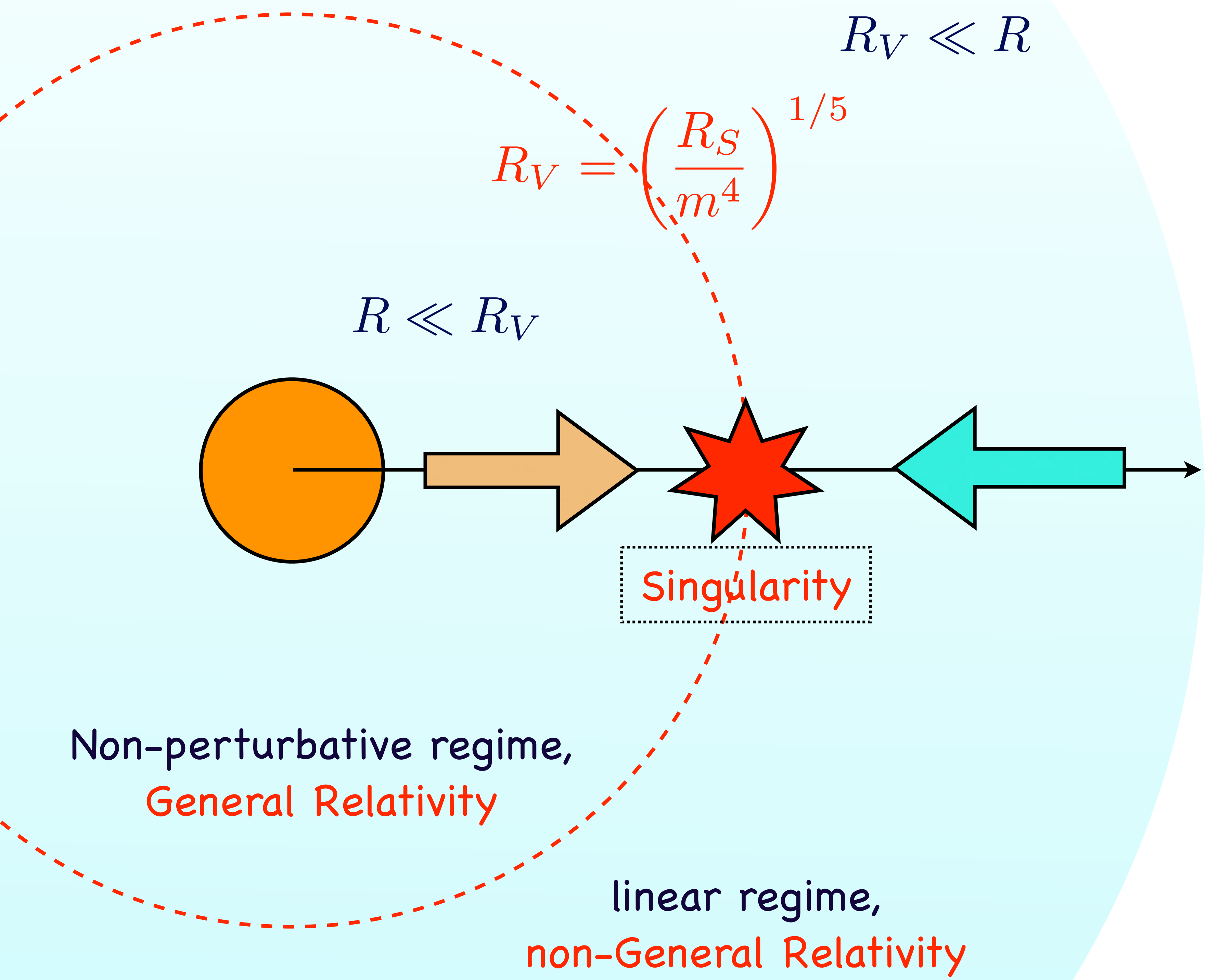
Vainshtein '72

$$
\begin{aligned}
\nu &= -\frac{R_S}{R} + n_1 \, (mR)^2 \sqrt{\frac{R_S}{R}} + \mathcal{O}\left(m^4\right) \\
\lambda &= \frac{R_S}{R} + l_1 \, (mR)^2 \sqrt{\frac{R_S}{R}} + \mathcal{O}\left(m^4\right) \\
\mu &= m_0 \sqrt{\frac{R_S}{R}} + m_1 \, (mR)^2 + \mathcal{O}\left(m^4\right)
\end{aligned}
$$

Relevant at

$$R_V = \left(\frac{R_S}{m^4}\right)^{1/5}$$

Existence of static spherically symmetric solution?



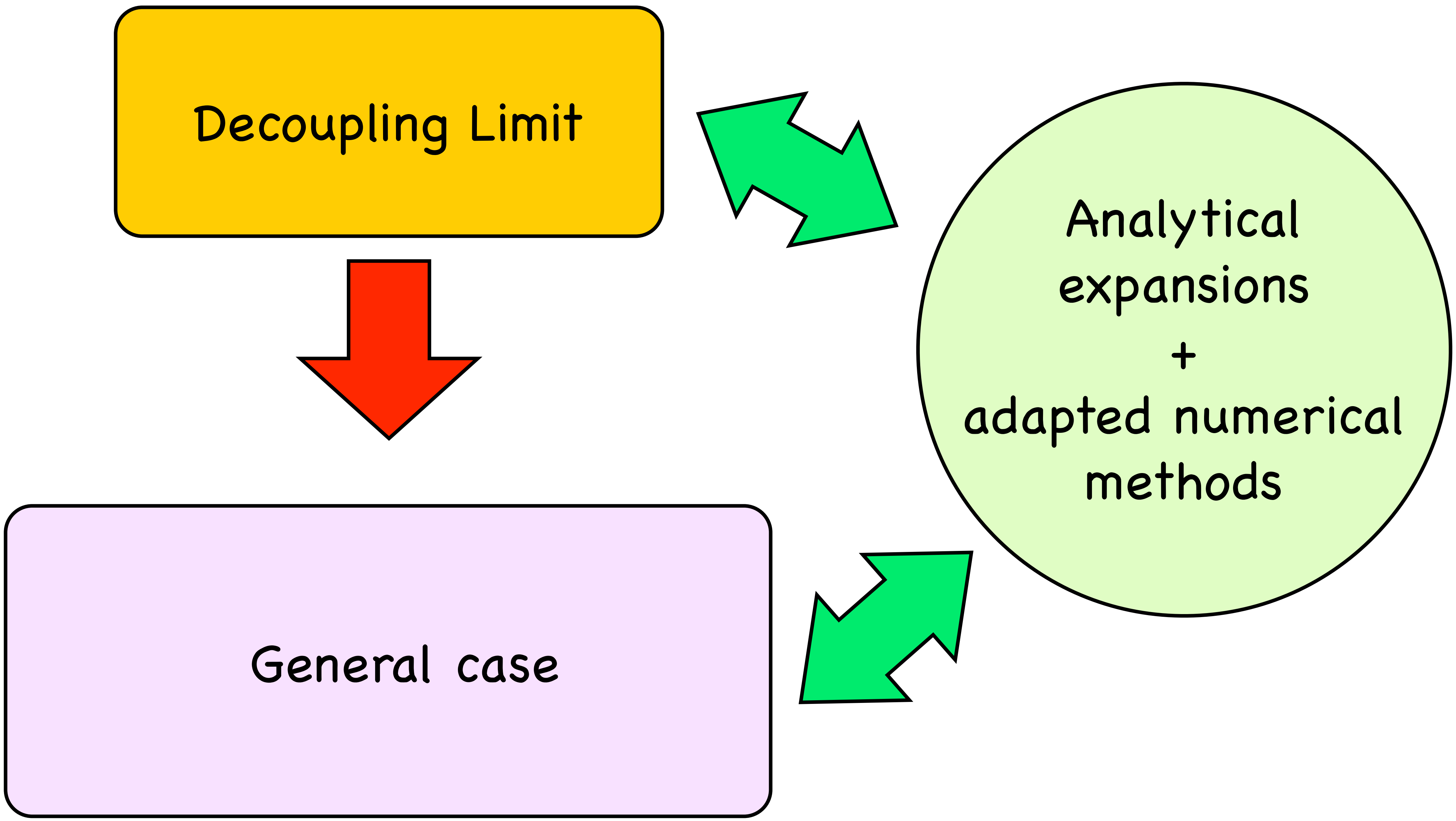
Is the Vainshtein mechanism valid?

Is it possible to find a solution regular everywhere? (and asymptotically flat)

Vainsthein '72,
Boulware & Deser '72,
Jun & Kang '86

Damour, Kogan, Papazoglou '03

our approach



two way to get decoupling limit

$$S \sim \int R + \mathcal{V}$$

Stuckelberg field

↓

demixing of kinetic terms

$$S \sim \int h\Box h + A\Box A + \phi\Box\phi + \ldots$$

dominant higher-order term

$$S \sim \int \phi\Box\phi + \frac{1}{m^4 M_P}(\Box\phi)^3$$

focus on physics around R_V

- ★ Remove the GR nonlinearities, $M_P \to \infty, \; M/M_P \sim const$
- ★ Remove Yukawa decay, $m \to 0$
- ★ Keep R_V constant,

$$R_V = (R_S m^{-4})^{1/5} \sim \left(\frac{M}{M_P}\frac{1}{M_P m^4}\right)^{1/5} \sim \text{const}$$

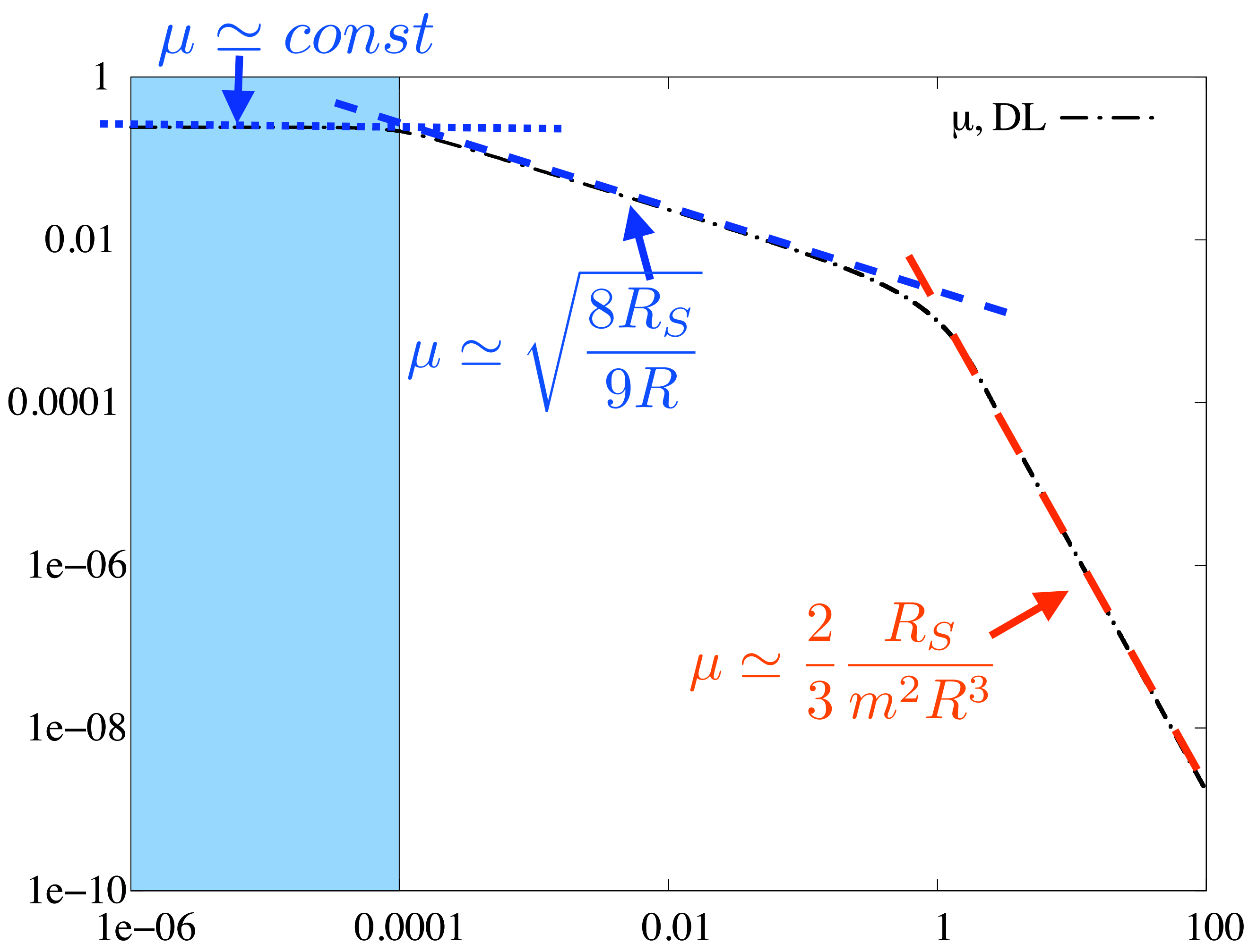
Decoupling limit

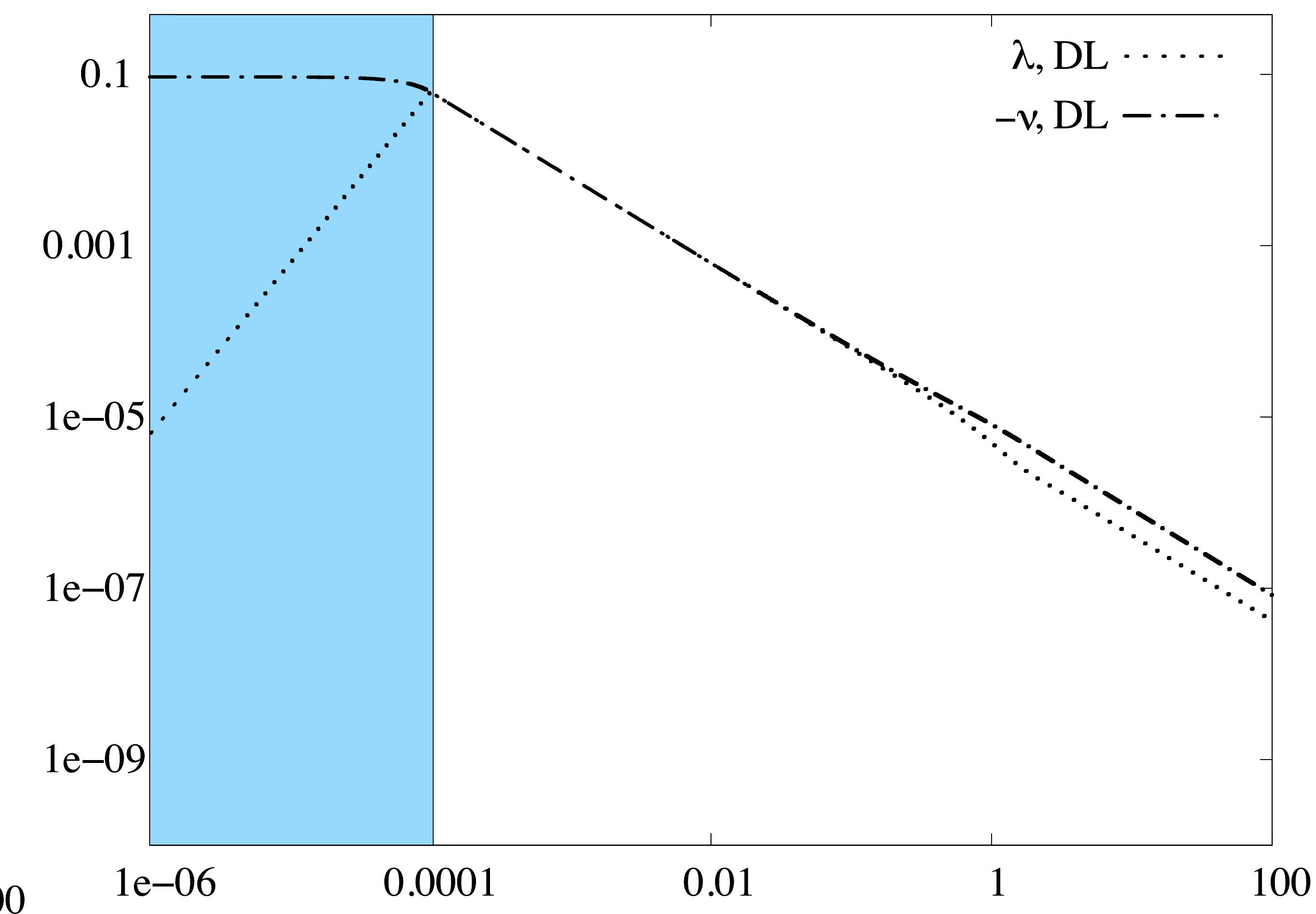
$$\begin{aligned} M_P &\to \infty \\ m &\to 0 \\ \Lambda \equiv (M_P m^4)^{1/5} &\sim const \\ T_{\mu\nu}/M_P &\sim const \end{aligned}$$

$$2Q(\mu, \mu'\mu'') + \frac{3}{2}\, m^2 \mu = \frac{R_S}{R^3}$$

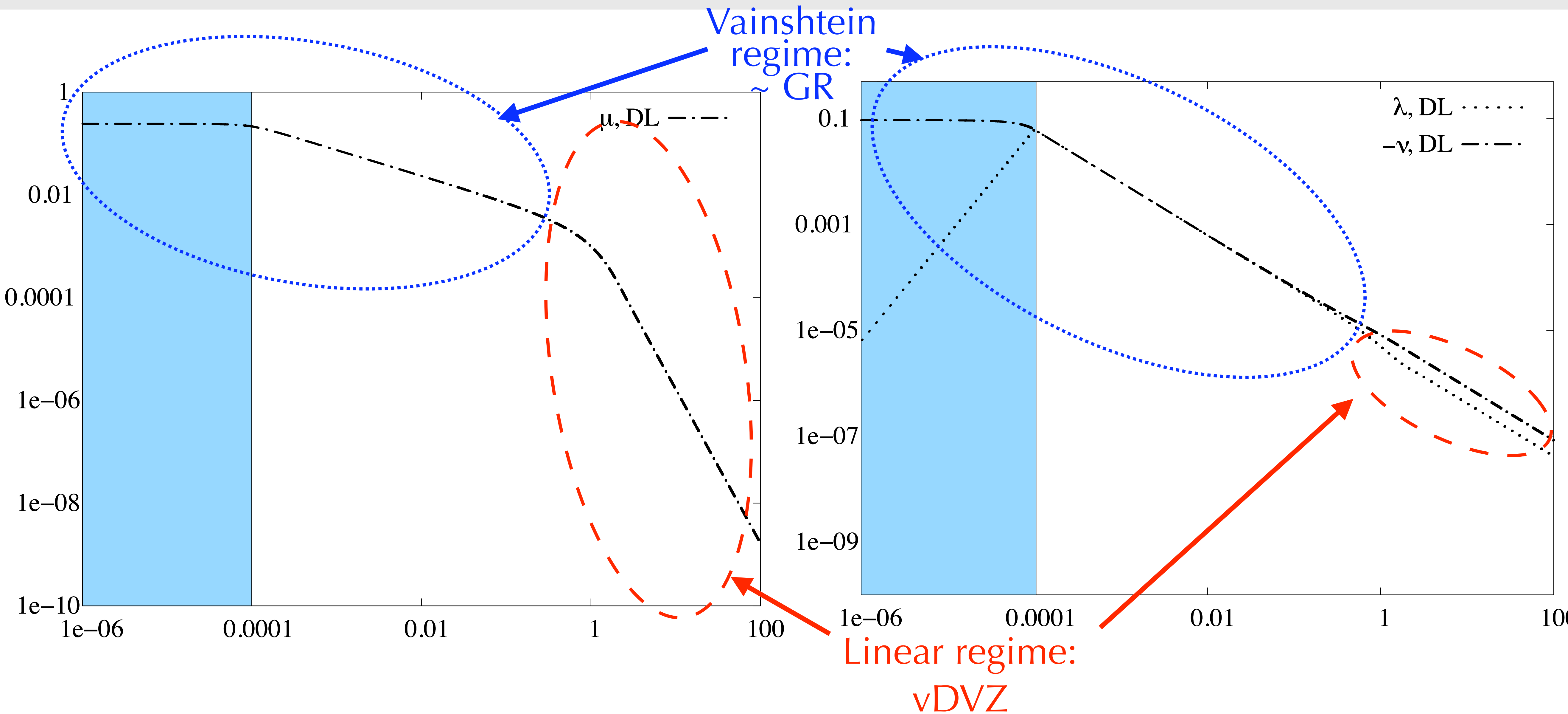
EB,Deffayet,Ziour'09

decoupling limit: solutions

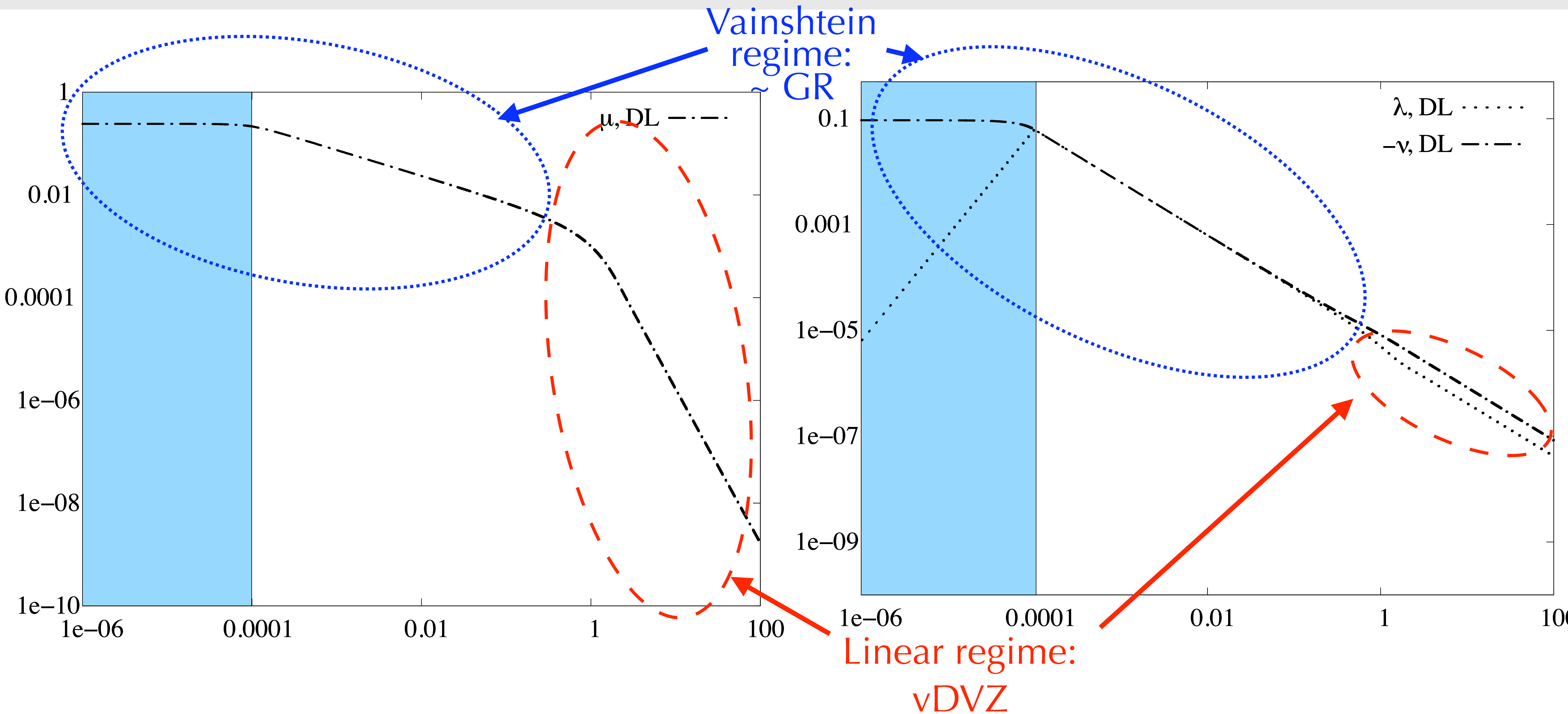




decoupling limit: solutions



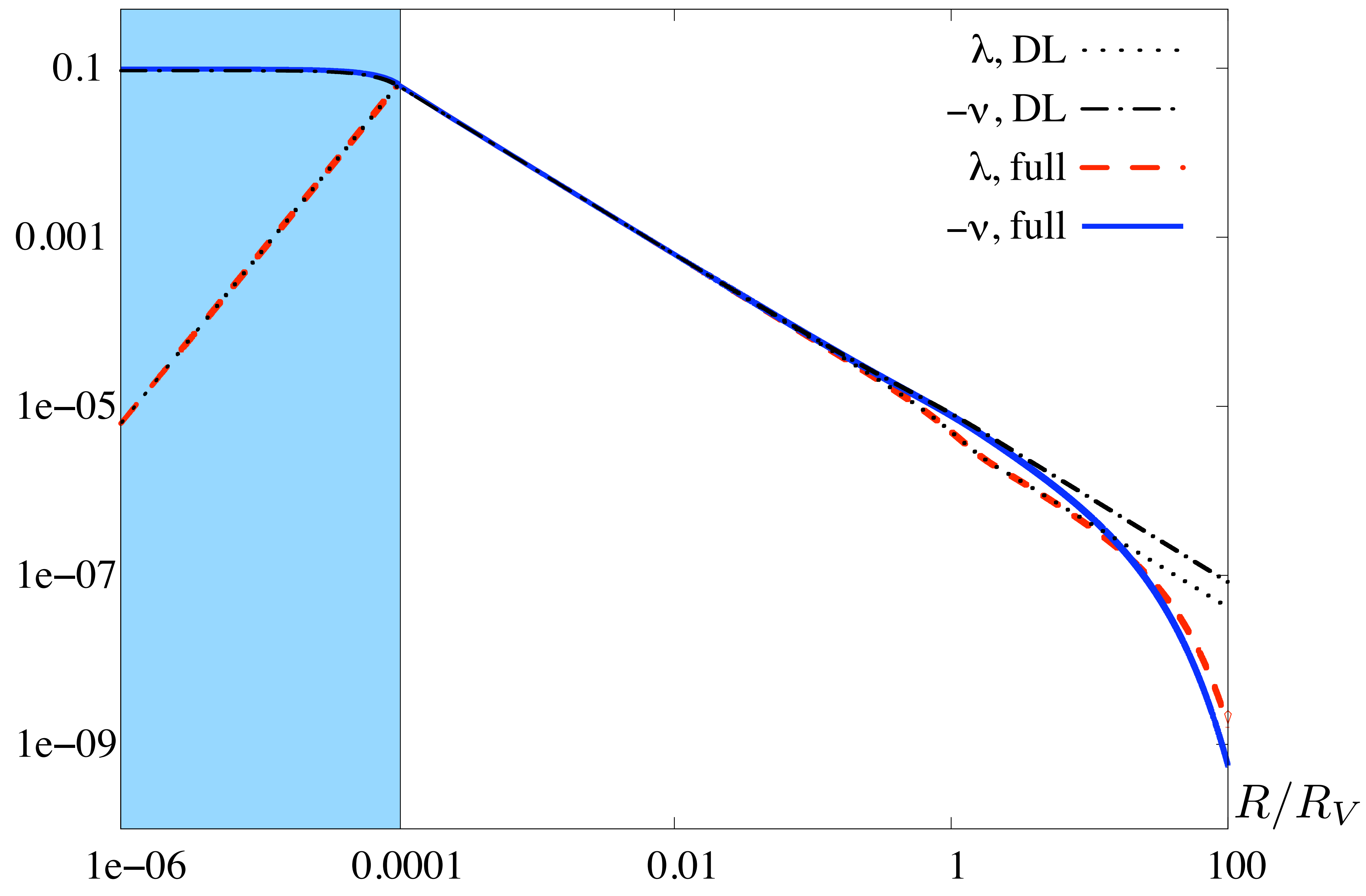
decoupling limit: solutions



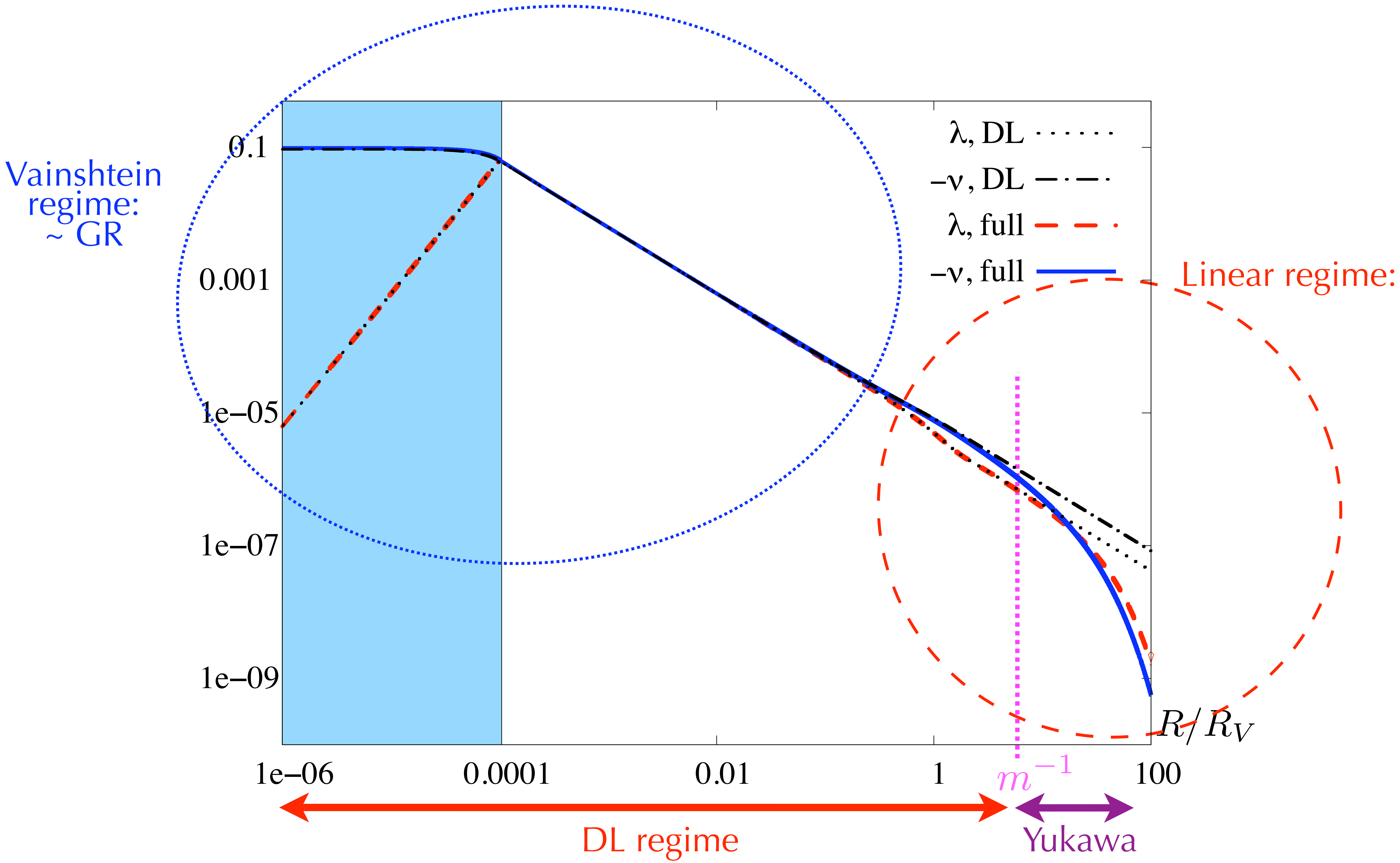
✦ Role of the nonlinearites not taken into account by the DL?

what about the full system?

solutions of the full system (I)



solutions of the full system (I)



analytic and numerical tools

✦ Series expansions: far and inside the source

✦ Cauchy problem at infinity: evidence for the existence of infinite number of solutions with the same asymptotic behavior (collaboration with J. Ecalle)

✦ Simplified model (weak field)

✦ Numerics: relaxation method
- ✓ Decoupling Limit
- ✓ Full system (Decoupling Limit as a starting guess)

✦ Numerics: shooting method

All the methods agree: robustness of the results

K-mouflage gravity

from massive gravity to k-mouflage (I)

✦ k-Mouflage models

EB,Deffayet,Ziour'09

$$S = M_P^2 \int d^4x \sqrt{-g} \left(\frac{R}{2} + \frac{\gamma}{2} m^2 \phi R + m^2 H(\phi) \right) + S_m$$

✦ schematically

$$S = M_P^2 \int d^4x \left(\text{"}h\Box h\text{"} + m^2 \phi \text{"}\Box h\text{"} + m^2 F(\phi) \right) + S_m$$

✦ equations of motion, schematically

$$\text{"}\Box h\text{"} + m^2 \text{"}\Box\phi\text{"} = 0$$

$$\text{"}\Box h\text{"} + \frac{\delta F(\phi)}{\delta \phi} = 0$$

=> $$m^2 \text{"}\Box\phi\text{"} + \frac{\delta F(\phi)}{\delta \phi} = 0$$

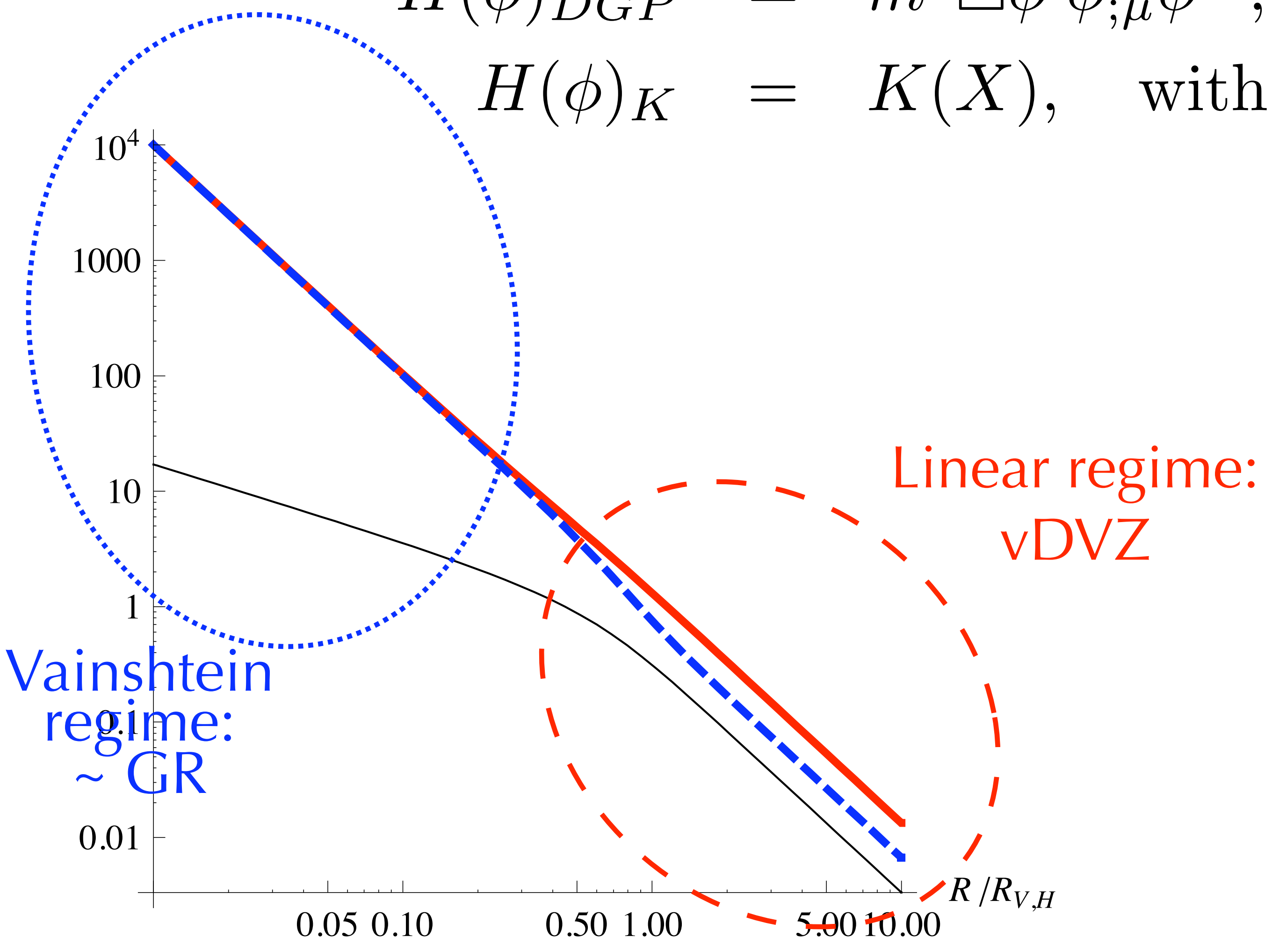
from massive gravity to k-mouflage (II)

✦ k-Mouflage models

$$S = M_P^2 \int d^4x\sqrt{-g}\left(\frac{R}{2} + \frac{\gamma}{2}m^2\phi R + m^2 H(\phi)\right) + S_m$$

with

$$\begin{aligned} H(\phi)_{MG} &= \frac{\alpha}{2}(\Box\phi)^3 + \frac{\beta}{2}(\Box\phi\, \phi_{;\mu\nu}\phi^{;\mu\nu}), \\ H(\phi)_{DGP} &= m^2\Box\phi\, \phi_{;\mu}\phi^{;\mu}, \\ H(\phi)_K &= K(X), \quad \text{with} \quad X = m^2\phi_{;\mu}\phi^{;\mu} \end{aligned}$$



The Vainshtein mechanism is valid for a broad class of models

conclusions

✦ Modified gravity models - a way to get accelerated Universe

✦ General relativity must be restored locally,

- chameleon mechanism for scalar-tensor and f(R) models
- Vainshtein mechanism for massive gravities and models with extra dimensions

✦ f(R) and scalar-tensor theories:

- static spherically symmetric configurations exist in the relativistic regime, in particular neutron stars exists,
- tachyon instability is generic if EoS for matter $\rho - 3P < 0$

✦ Massive gravity:

- Existence of non-singular solutions
- validity of the Vainshtein mechanism and recovery of general relativity from massive gravity

✦ New model: K-mouflage gravity

✦ Many interesting things to do: compact sources in massive gravity, stability of found solutions, cosmology and phenomenology of K-mouflage model