Theory of spectral distortions

Cyril Pitrou (IAP)

What I remember about CMB

~~Theory of~~ spectral distortions

Cyril Pitrou (IAP)

What is a non-distorted spectrum?

* Thermal equilibrium is enforced by Compton collisions

$$\gamma + e^- \leftrightarrow \gamma + e^-$$

$$C[f(p)] \propto \int \ldots |M|^2 \, [f'g'(1+f)(1-g) - fg(1+f')(1-g')]$$

* Equilibrium is reached when

$$\frac{f}{1+f}\frac{g}{1-g} = \frac{f'}{1+f'}\frac{g'}{1-g'}$$

* Condition : Fermi Dirac or Bose Einstein

photons

$$f(E) = \frac{1}{\mathrm{e}^{(E+\mu)/T} - 1}$$

$$\frac{f}{1+f} = \frac{g}{1-g} = e^{-(E+\mu)/T}$$

If only thermal equilibrium, chemical potential is allowed $\Rightarrow \mu_\gamma + \mu_{e-} = \mu_\gamma + \mu_{e-}$

Chemical equilibrium

*Double Compton (DC) $\gamma + e^- \leftrightarrow \gamma + \gamma + e^-$

$$\mu_\gamma + \mu_{e^-} = 2\mu_\gamma + \mu_{e^-} \rightarrow \mu_\gamma = 0$$

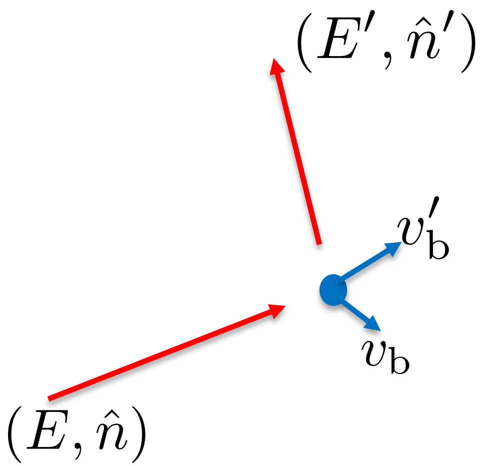
*Rayonnement continu de freinage or Bremsstrahlung (BS)

$$p^+ + e^- \rightarrow p^+ + e^- + \gamma$$

For photons, we consider that a non distorted spectrum is at both thermal and chemical equilibrium. Planck spectrum

$$f(E) = \frac{1}{\mathrm{e}^{E/T} - 1}$$

Compton Scattering



In electron rest frame

$$E' = E(1 + \mathcal{O}(E/m_e))$$

Thomson approximation

Compton correction

Maxwellian distribution of electron velocities

Kompaneets equation

$$\dot{f} \propto \frac{n_e \sigma_T}{E^3} D[E^3(T_e D f + E f(1+f))]$$

$$D \equiv \frac{\partial}{\partial \ln E}$$

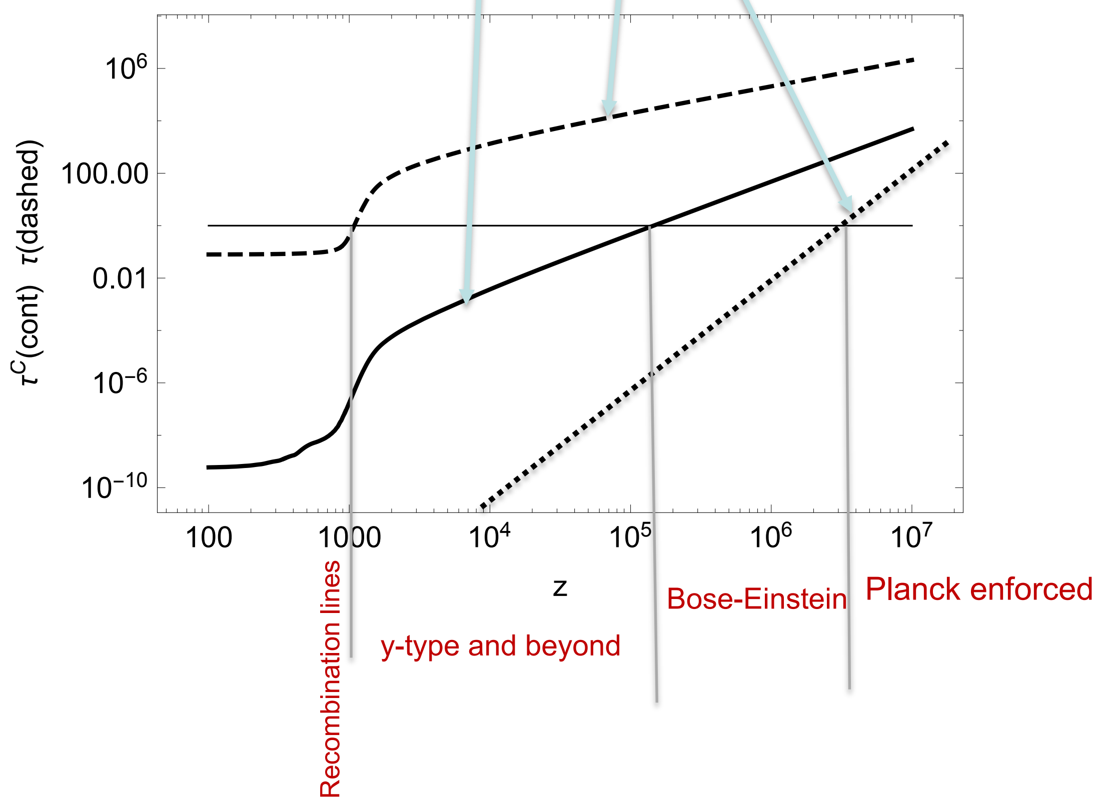
Conservation of $n \propto \int f(E) E^3 \mathrm{d} \ln E$

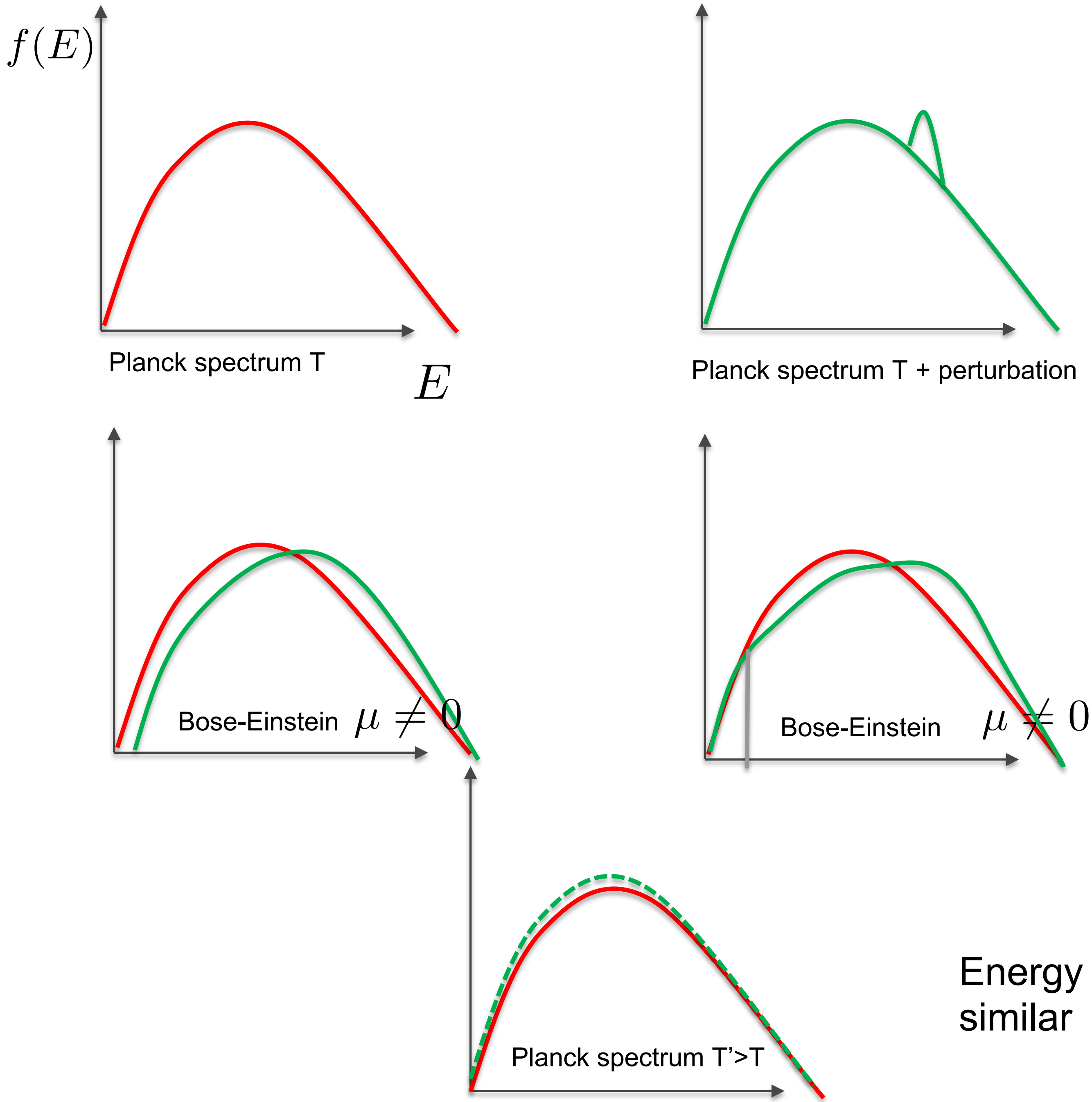
Collisional time scales

- Optical depth (anisotropies) τ
- Compton depth τ_C
- Photon creating time scale. τ_μ

$$\tau' = n_e \sigma_T$$

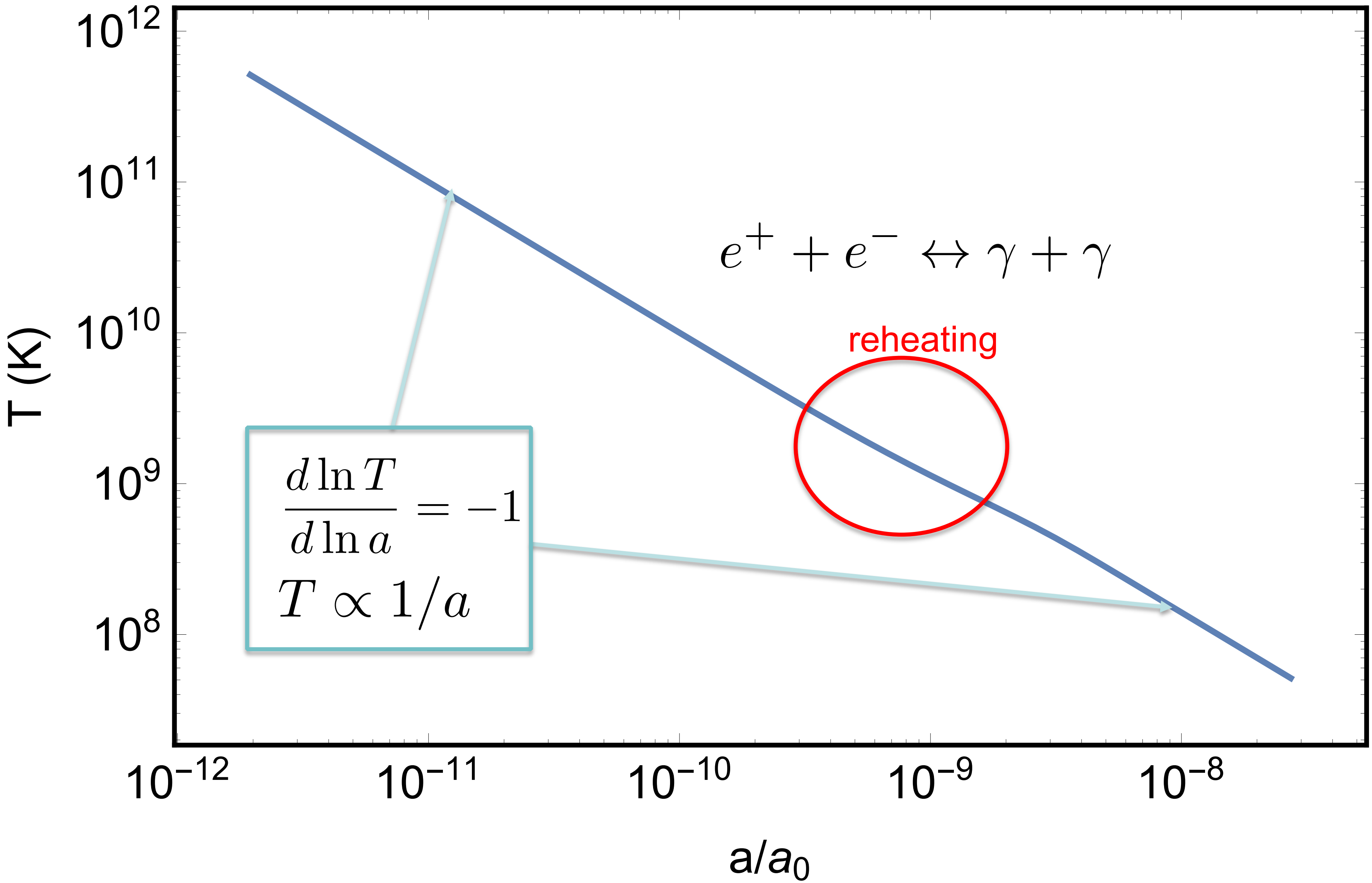
$$\tau'_C = n_e \sigma_T \frac{T}{m_e}$$

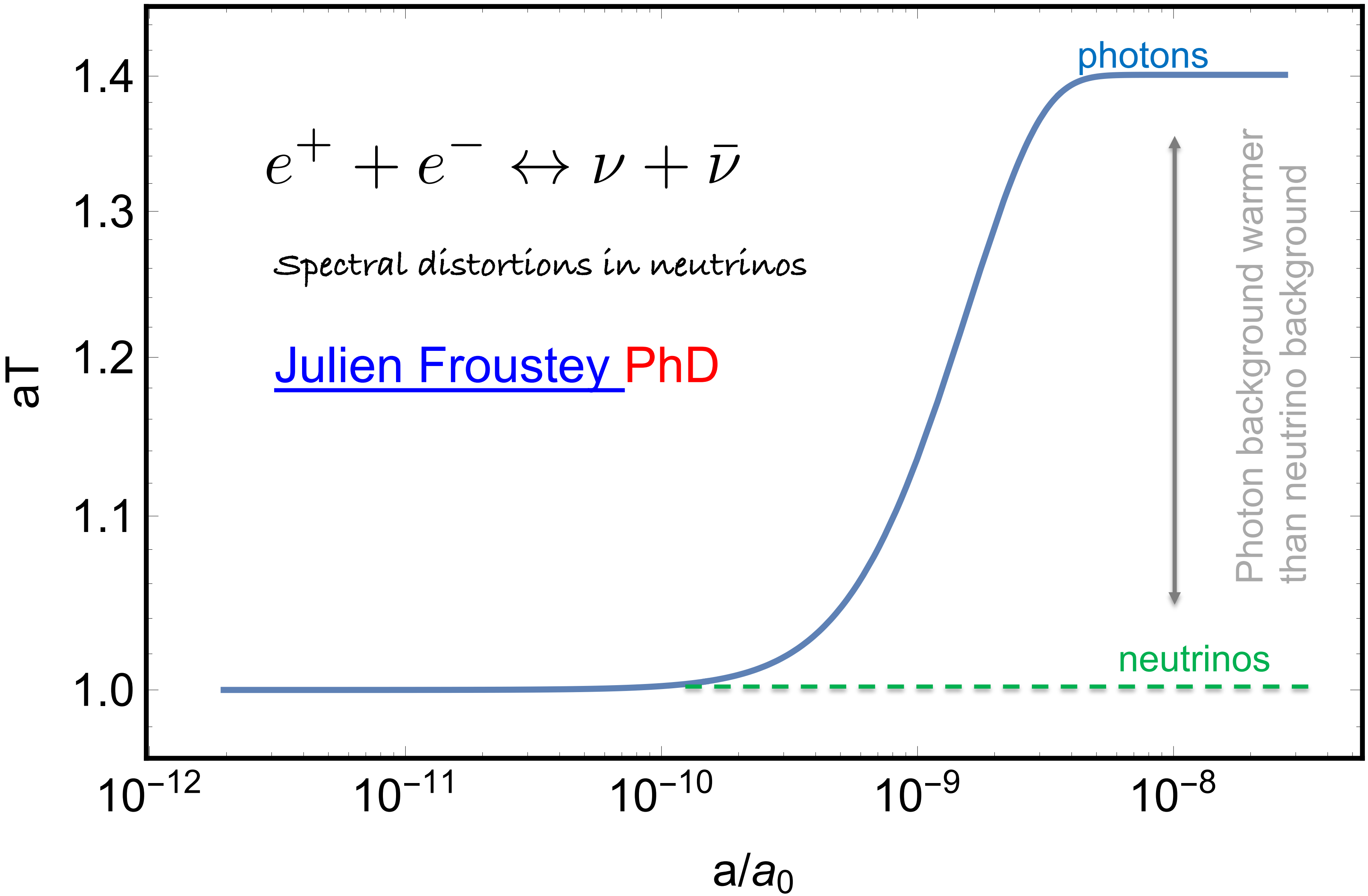




Energy injection in baryons is similar

When was the CMB distorted ?





$e^+ + e^- \leftrightarrow \nu + \bar{\nu}$
Spectral distortions in neutrinos
Julien Froustey PhD
photons
neutrinos
Photon background warmer than neutrino background
aT
1.4
1.3
1.2
1.1
1.0
10^{-12}
10^{-11}
10^{-10}
10^{-9}
10^{-8}
a/a_0

Distortions from cold baryons

Cosmological expansion $p \propto 1/a$

$$\frac{1}{2} m v^2 \propto k_B T \quad \Longrightarrow \quad T_b \propto 1/a^2$$

Baryons steal energy from photons

$$\frac{\delta(a^4 \rho_\gamma)}{a^4 \rho_\gamma} = -\frac{1}{2} \left. \frac{n_B T_\gamma}{\rho_\gamma} \right|_{\rm today} \int {\rm d} \ln V$$

Mu-era (negative) injection

10^6

$$\frac{\delta \rho_\gamma}{\rho_\gamma} \simeq -6.1 \times 10^{-10} \ln \left[\frac{1 + z_{\rm max}}{1 + z_{\rm min}} \right] \simeq -5.6 \times 10^{-9} \quad \Longrightarrow \quad \frac{\mu}{T_\gamma} \simeq -2.5 \times 10^{-9}$$

10^5

Heat capacities interpretation

Baryons $$C_b = \frac{3}{2} n k_B \ \propto n_b$$

Photons $$C_r = 16 \frac{\sigma_S}{c} T^3 \ \propto n_\gamma$$

$$\frac{C_b}{C_r} = \mathrm{Cte} \propto \eta_0 \simeq 6 \times 10^{-10}$$

Distortions from boost ?

Take a massive particle (e.g. neutrinos, electrons). $E^2 = p^2 + m^2$

Fermi Dirac distribution $n(E) = \frac{1}{e^{E/T} + 1}$

In a different frame (or for a different observer)

$$n'(E') = n(E)$$

$$E' = \gamma(E - \mathbf{p} \cdot \mathbf{v}) \neq \alpha E$$

Not Fermi-Dirac anymore !

Now if the particles are massless $p = E$

$$E' = \gamma E(1 - \mathbf{n} \cdot \mathbf{v}) = \alpha E$$

$$n'(E') = \frac{1}{e^{E'/(\alpha T)} + 1}$$

$$\Rightarrow T' = \alpha T$$

No spectral distortions for massless particles

Distortions from gravitational effects ?

Geodesic equation for massive particles

$$\dot{E} = \frac{p^2}{E}(\mathrm{metric}) + p(\mathrm{metric})$$

For massive particles we have distortions

For massless particles

$$\dot{E} \propto E(\mathrm{metric}) \qquad \frac{\mathrm{d}\ln E}{\mathrm{d}t} \propto \mathrm{metric}$$

Again variations of energies can be reabsorbed in variation of temperature

Describing distortions: the temperature transform

Stebbins 2007

If not a simple blackbody, then at least it is approximately a sum of blackbodies

$$n(E,\ldots) = \int_0^\infty \mathrm{d}T\, p(T,\ldots) \mathcal{N}\left(\frac{E}{T}\right)$$

$$\mathcal{N}(x) = 1/(e^x - 1)$$

This allows to define averages over this distribution p(T)

$$\bar{T}^n \equiv \int T^n p(T) dT$$

n=1 Rayleigh Jeans temperature

n=3 Number density temperature

n=4 Energy density temperature (bolometric temperature)

But the good temperature definition is

$$\log \bar{T} \equiv \int \log T p(T) \mathrm{d}T = \int \log T q(\log T) \mathrm{d}\log T$$

With logs, the blackbody distribution is $\mathcal{N}(\log E - \log T)$

Boost, propagation are of the form $\log E \to \log E + \mathrm{Cte}$

So propagation and observer dependence amount to a shift in the distribution of blackbodies

$$\boxed{q(\log T) \to q(\log T + \mathrm{Cte})}$$

Its shape is conserved, and its shape is precisely the distortion !

Moments of distribution

Two types of moments

Moments around a fiducial temperature (a la *Sunyaev*)

$$d_n \equiv < (\log T - \log T_0)^n >$$

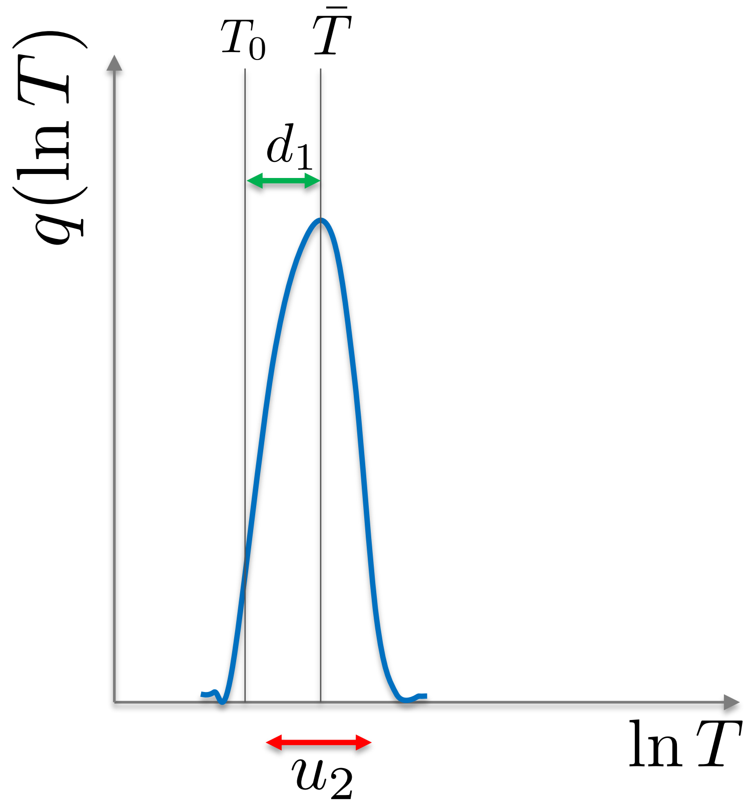
$$\log \bar{T} = d_1 + \log T_0$$

Moments centered around the average value (a la *Stebbins*)

$$u_n \equiv < (\log T - \log \bar{T})^n >$$

$$u_1 = 0$$

By construction these moments are observer independent !



Boosts and gravity only induce a shape preserving horizontal shift.

Interpretation of the moments

$$n(E) = \sum_{m=0}^{\infty} \frac{d_m}{m!} D^m \mathcal{N}\left(\frac{E}{T_0}\right) = \sum_{m=0}^{\infty} \frac{u_m}{m!} D^m \mathcal{N}\left(\frac{E}{\bar{T}}\right)$$

$$D^m \mathcal{N}(x) \equiv (-1)^m \mathrm{d}^m \mathcal{N}(x)/\mathrm{d}\ln(x)^m$$

Observer independent method

$$Y = (D^2 + 3D)\mathcal{N}$$

$$u_2 = 2y$$

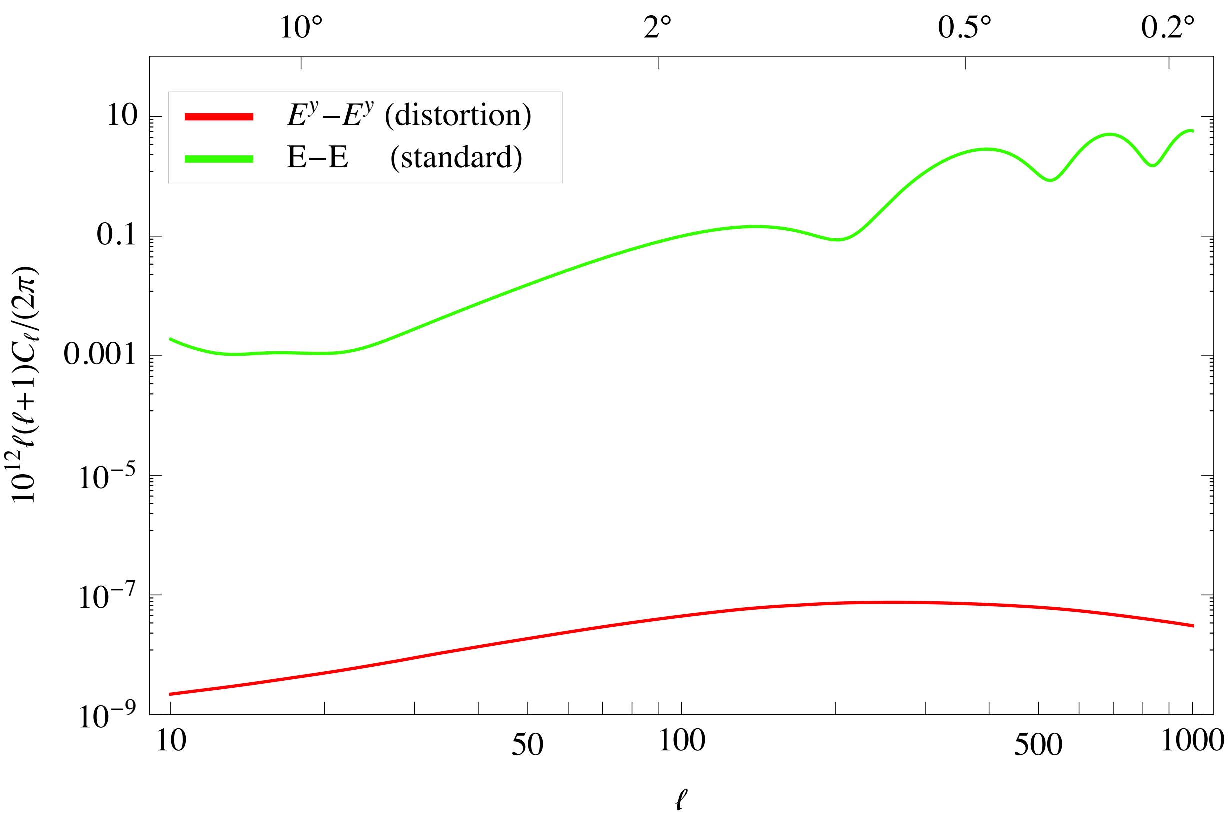
$$u_2 = d_2 - \boxed{d_1^2}$$

We should not omit that term

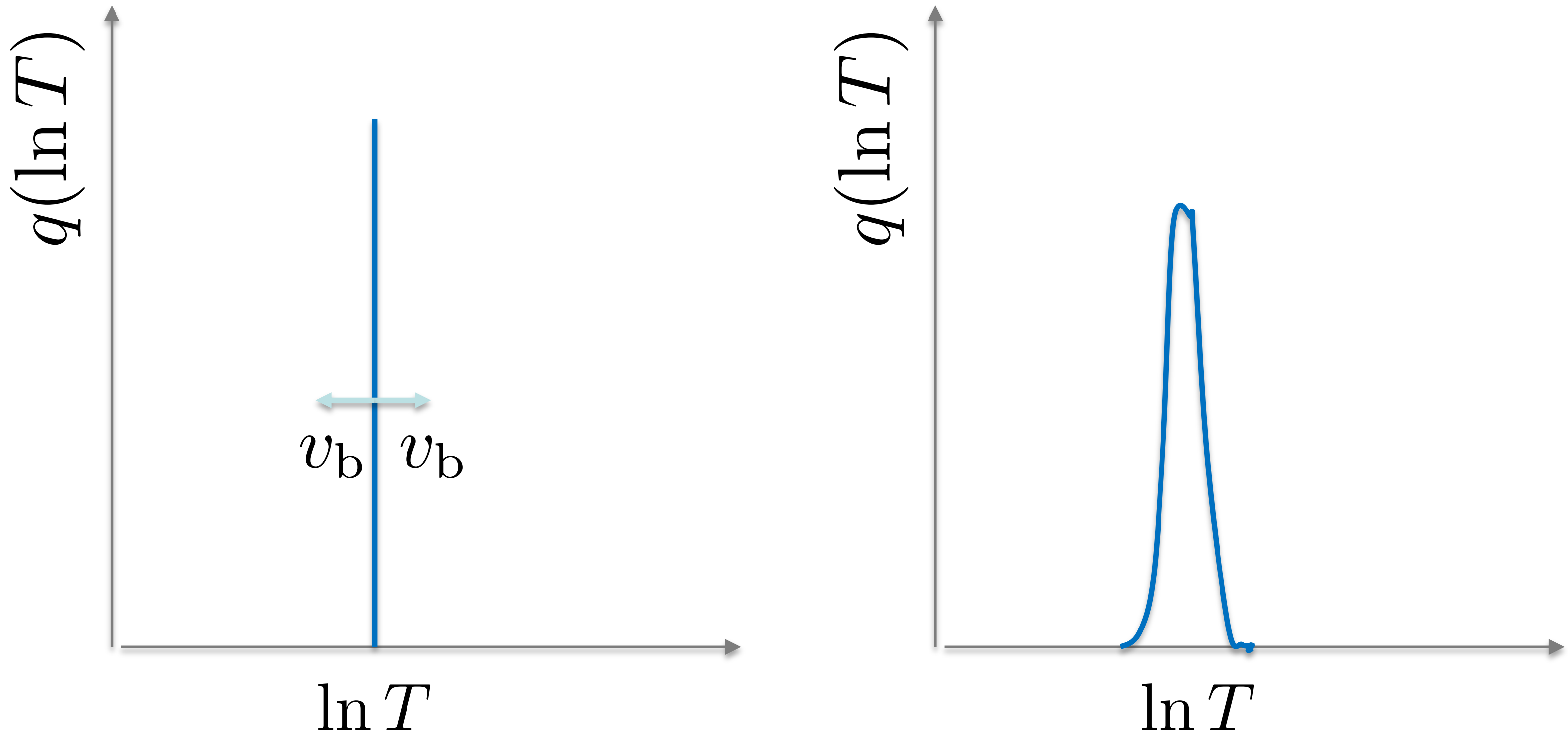
Distortions in polarization

$$f^{ab} = \frac{1}{2}\begin{pmatrix} I+Q & U-\mathrm{i}V \\ U+\mathrm{i}V & I-Q \end{pmatrix}$$

$$f^{ab}(E) = \int_{-\infty}^{\infty} \mathrm{d}\mathcal{T} q^{ab}(\mathcal{T})\ \mathcal{B}\left(E\, e^{-\mathcal{T}}\right)$$



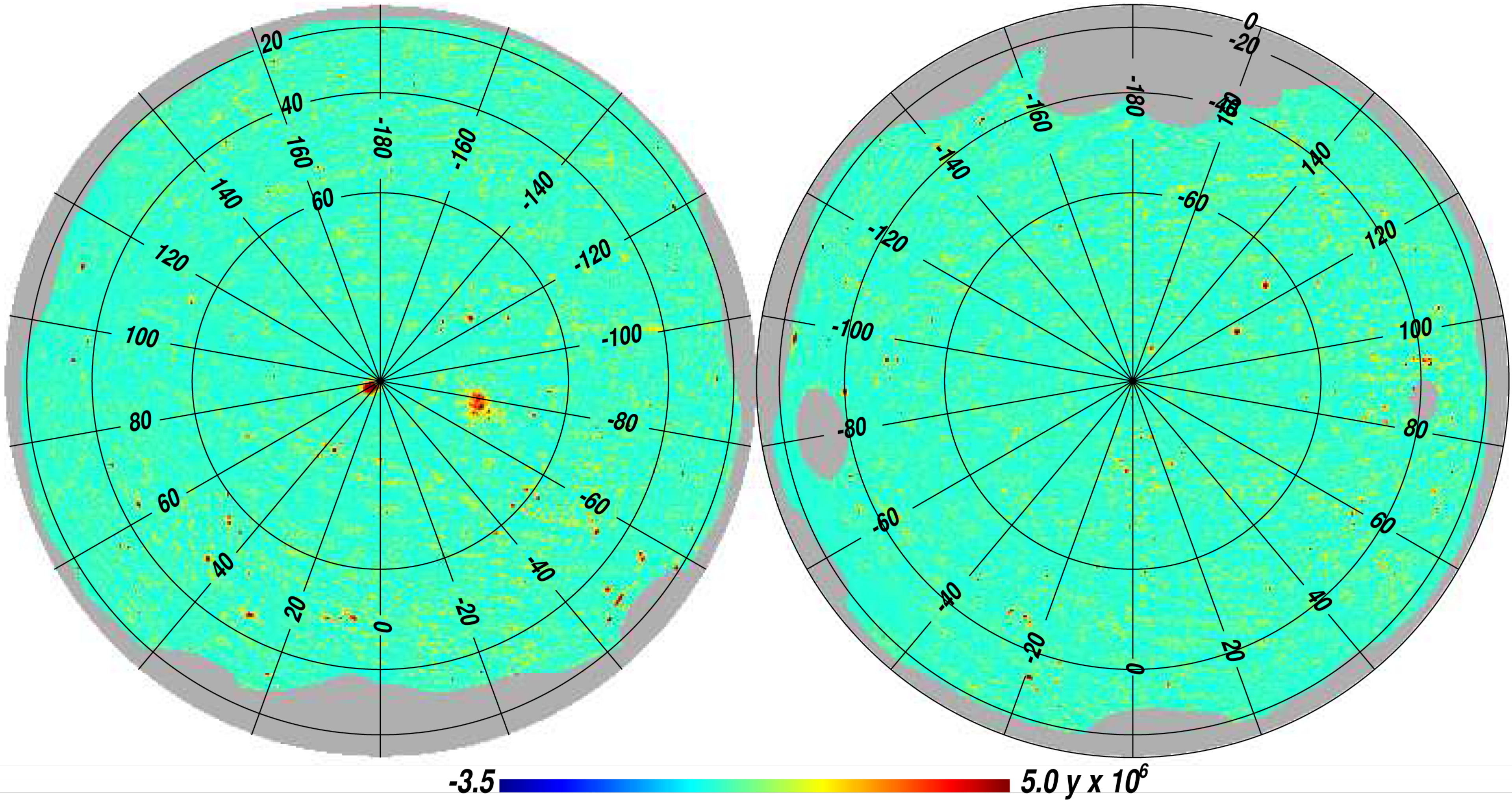
Baryon velocity induced distortions



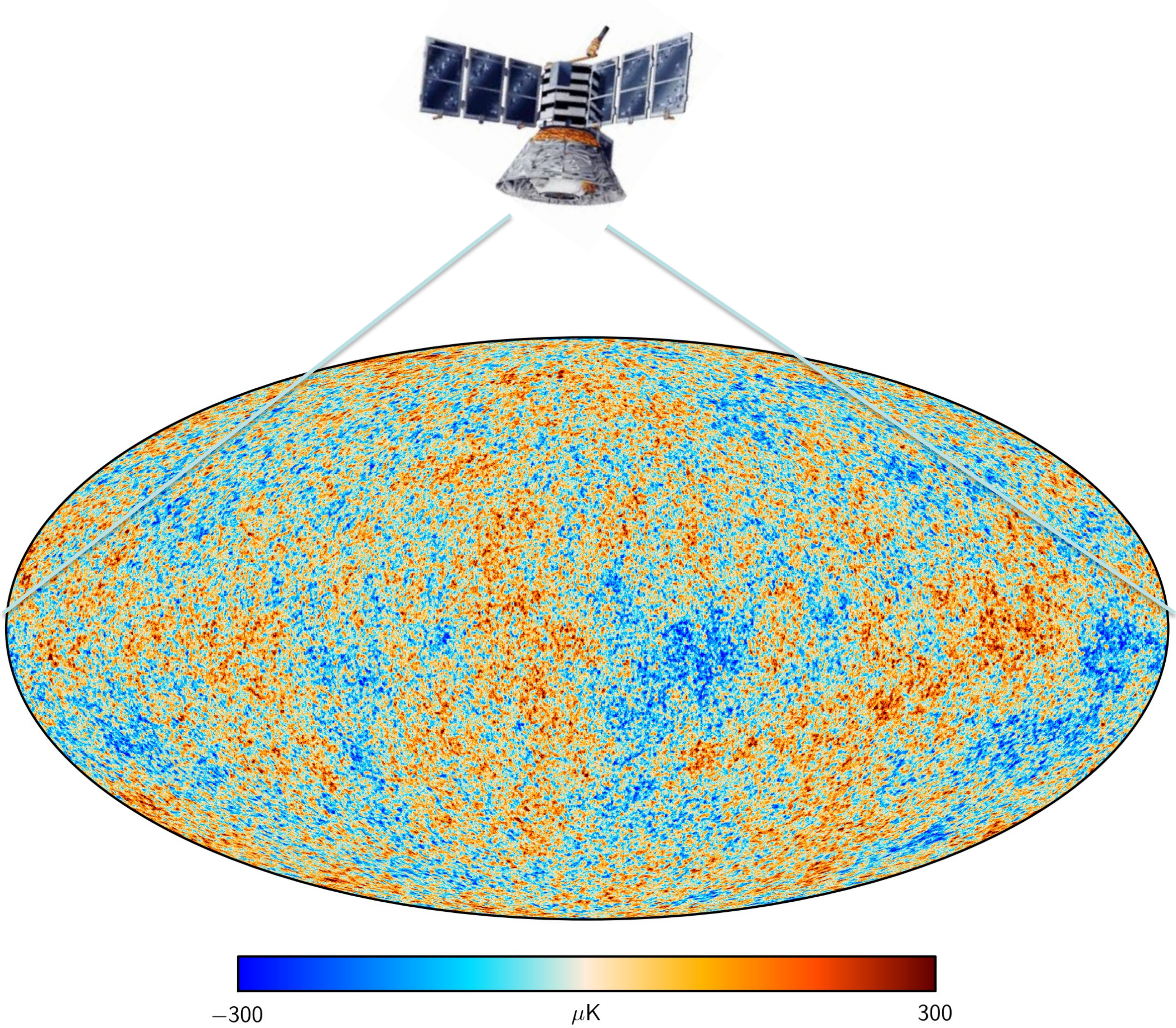
$$\langle v_i v_j \rangle = V_i V_j + \delta_{ij} \frac{k_B T}{m}$$

Bulk
Non-linear kSZ

Thermal (e.g. SZ effect)



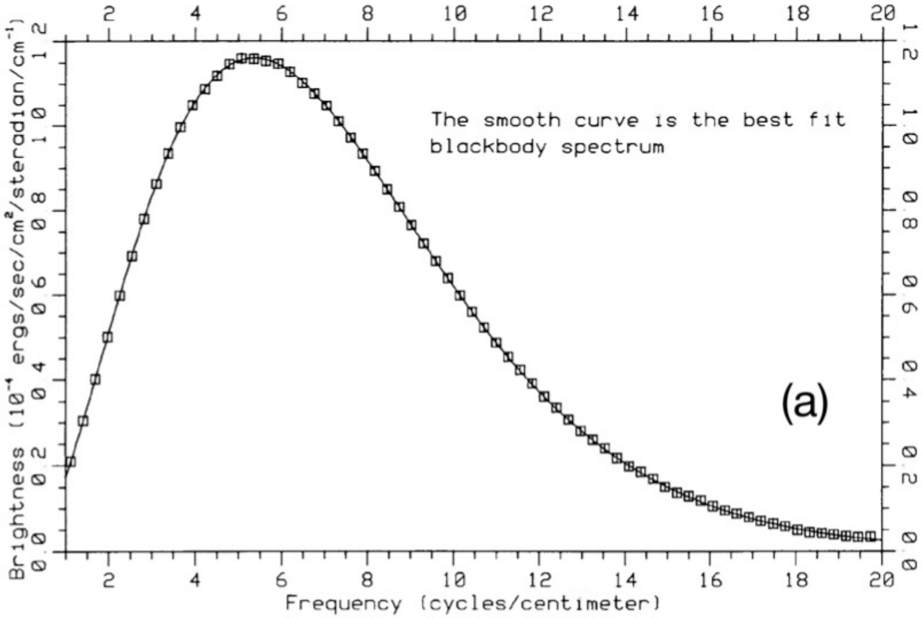
Diffuse thermal component dominates.
Non-linear kSZ from reionization subdominant.



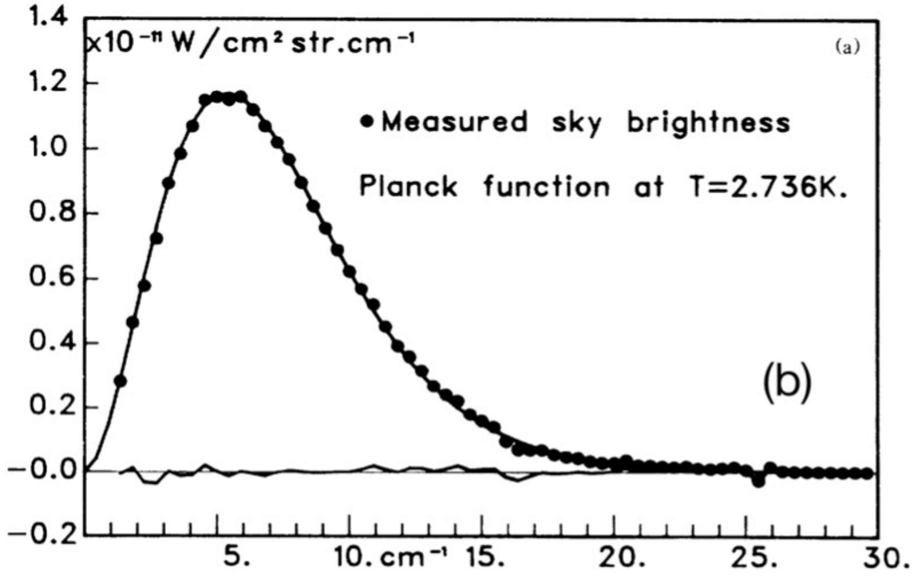
−300
μK
300

COBE

COBRA



The smooth curve is the best fit blackbody spectrum
(a)
Brightness (10⁻⁴ ergs/sec/cm²/steradian/cm⁻¹)
Frequency (cycles/centimeter)

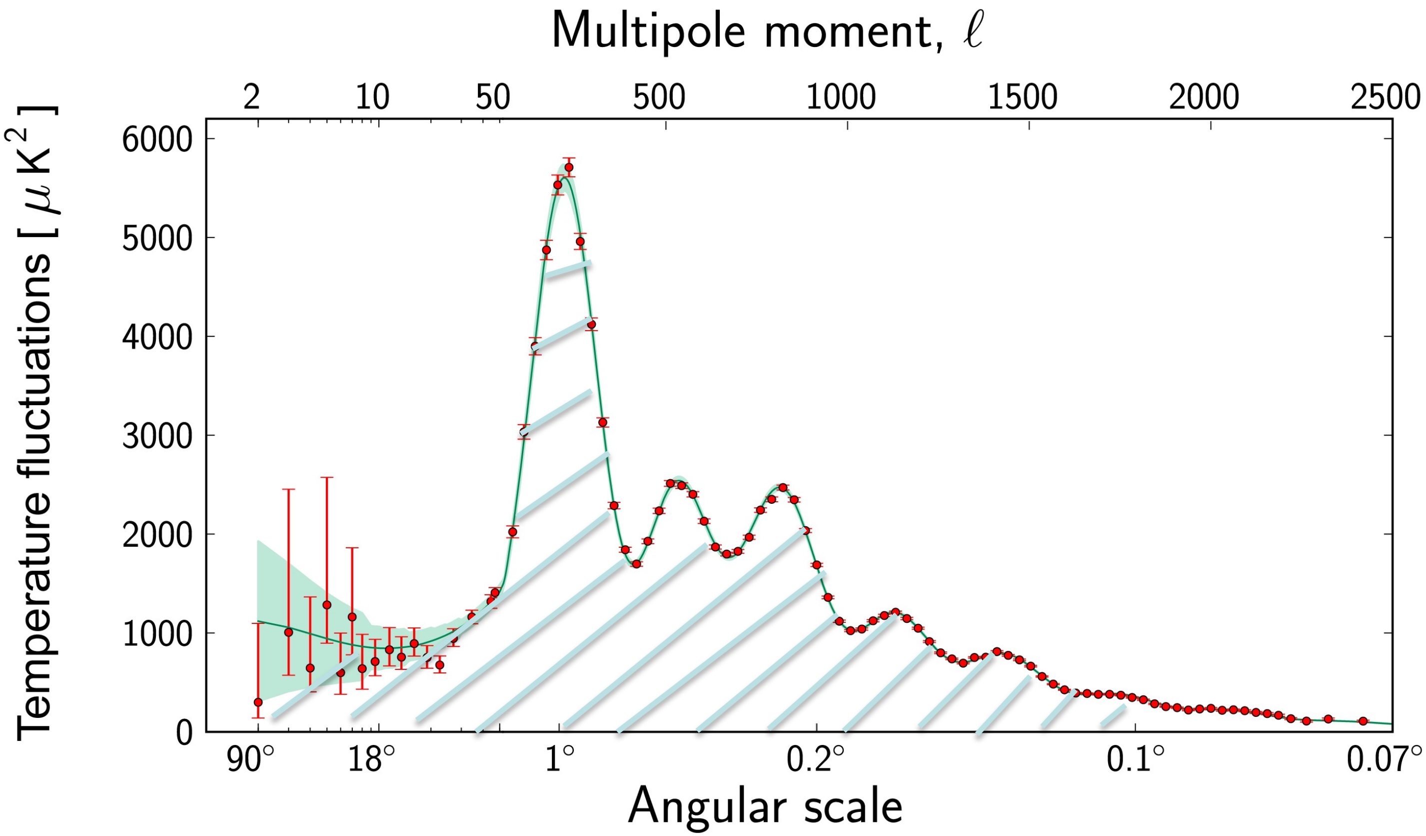


x10⁻¹¹ W/cm² str.cm⁻¹
(a)
● Measured sky brightness
Planck function at T=2.736K.
(b)
10. cm⁻¹

How much time did it take for COBRA to take data ?

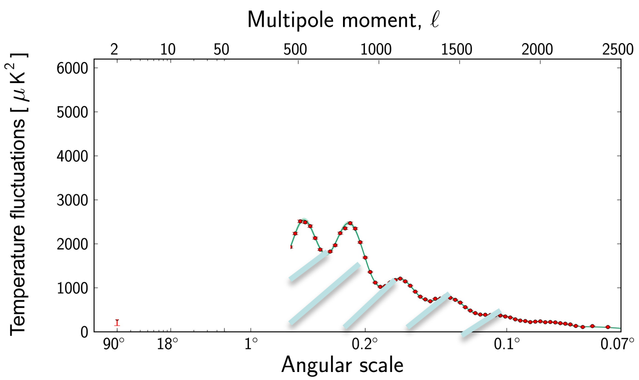
- 5 years
- 5 months
- 5 days
- 5 minutes

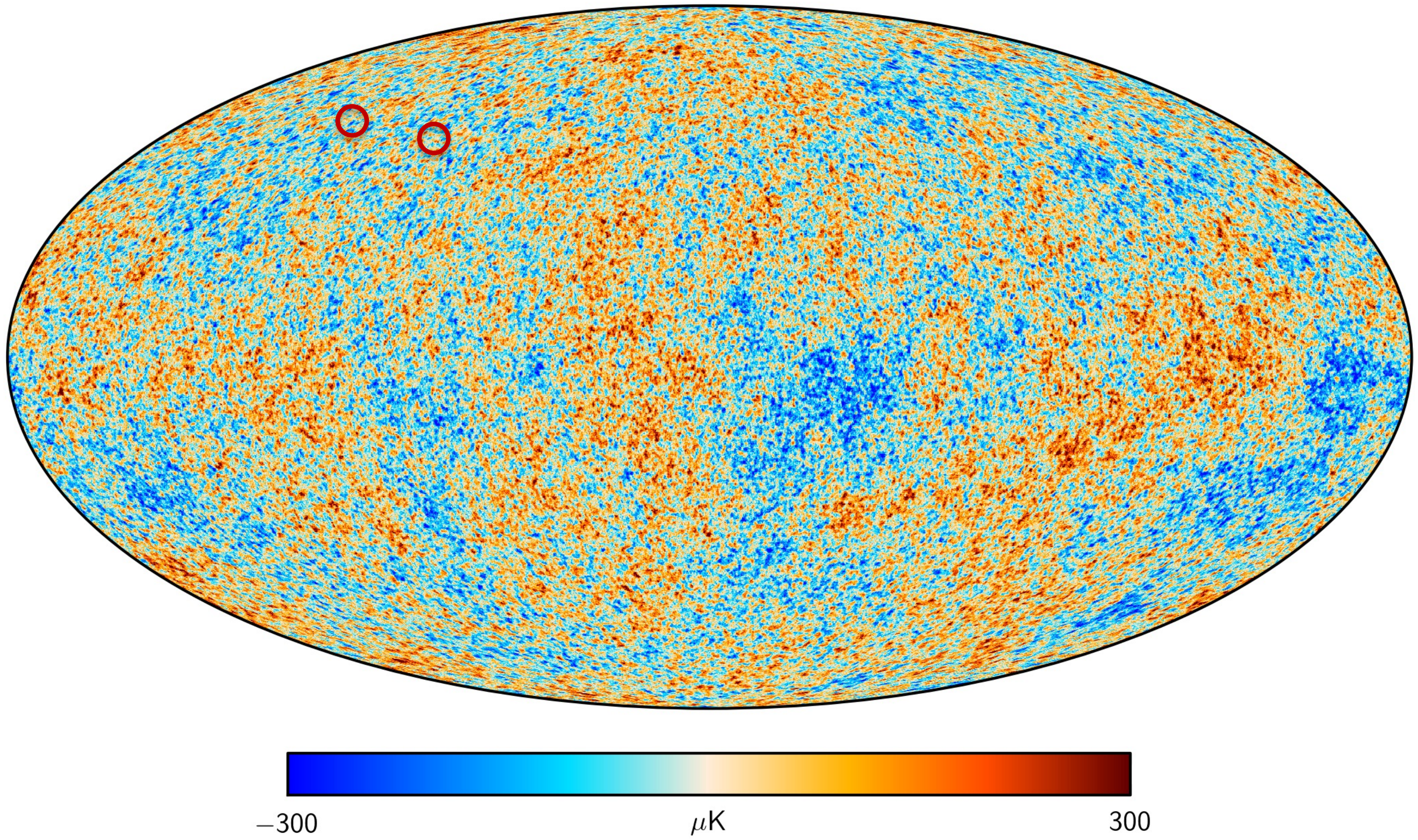
$$y \propto \langle \delta T \delta T \rangle \propto \sum_{\ell} (2\ell + 1) C_{\ell} \simeq \int \ell(\ell+1) C_{\ell} \mathrm{d} \ln \ell$$



Angular resolution of spectrometer

Good resolution reduces distortions by mixing

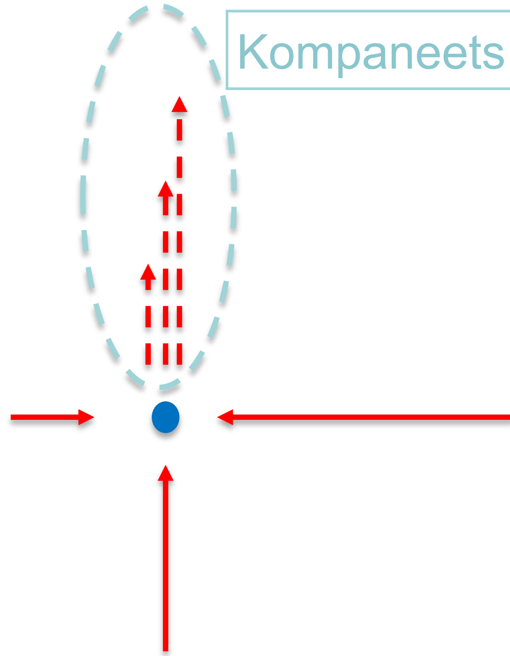




Baryon frame interpretation

Kompaneets unhappy

Mixing of Planck spectra
with different T

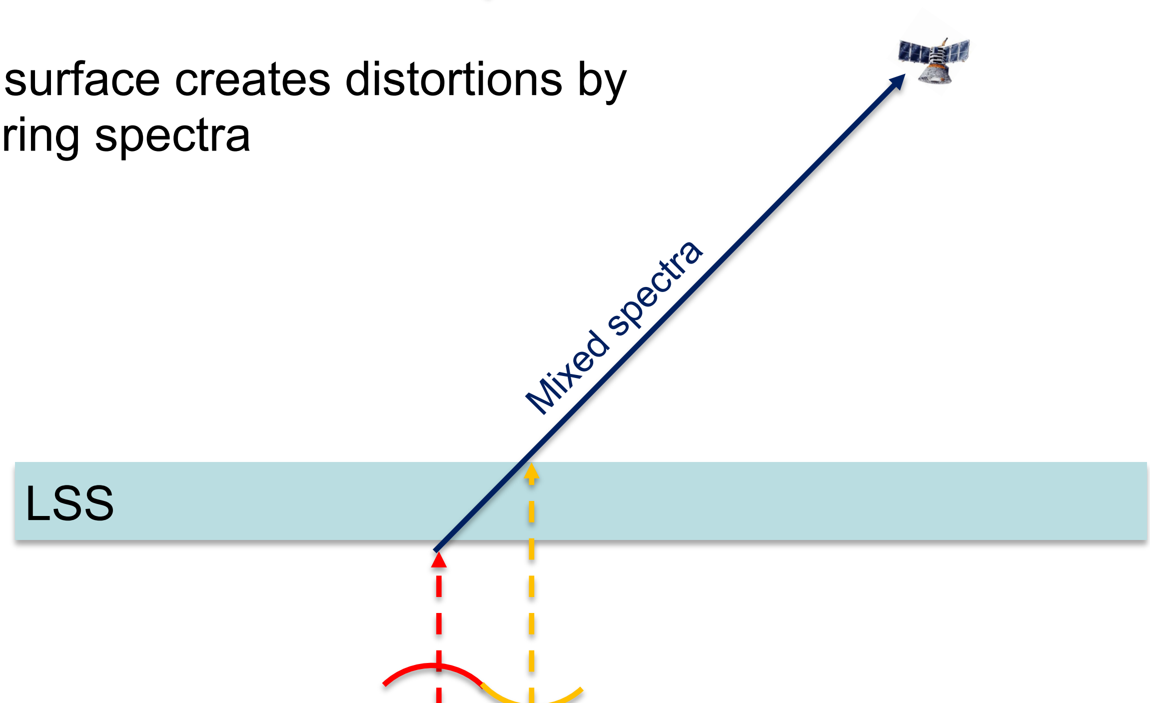


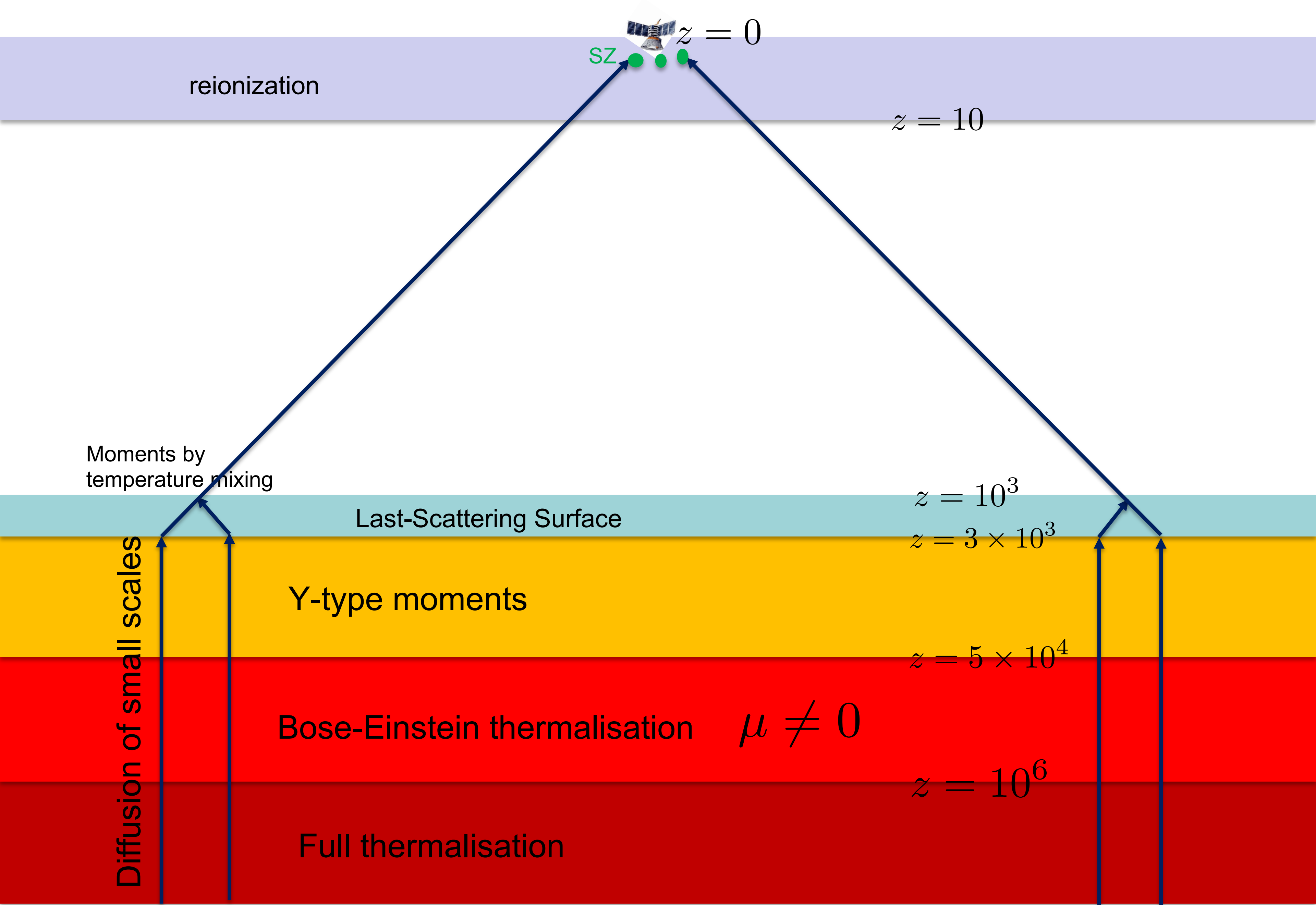
$$\frac{1}{2}(T_1^3 + T_2^3) = T^3$$

$$\frac{1}{2}(T_1^4 + T_2^4) = T^4$$

Impossible

Last scattering surface creates distortions by mixing neighboring spectra





$z = 0$
SZ
reionization
$z = 10$
Moments by temperature mixing
$z = 10^3$
Last-Scattering Surface
$z = 3 \times 10^3$
Y-type moments
$z = 5 \times 10^4$
Bose-Einstein thermalisation
$\mu \neq 0$
$z = 10^6$
Full thermalisation
Diffusion of small scales

Statistics with distortions

Quadratic physics

$$\delta\rho_\gamma\delta\rho_\gamma$$

Compressed into

$$\mu$$

1) Average over all sky : info on spectrum

$$\langle\delta\rho_\gamma\delta\rho_\gamma\rangle \rightarrow \bar{\mu} = \langle\mu\rangle \approx \int \frac{k^2 \mathrm{d}k}{2\pi^2} P_i(k) \left(\delta\rho_\gamma\right)^2$$

2) Cross-correlation : info on bispectrum

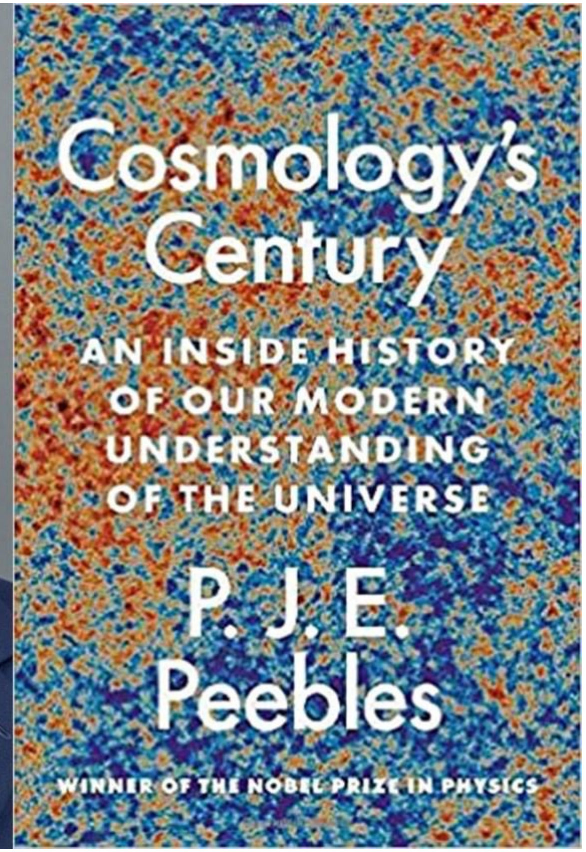
$$\langle\delta\rho_\gamma\delta\rho_\gamma\delta\rho_\gamma\rangle \rightarrow \langle\mu\delta T\rangle$$

Short scales hidden in mu.
Correlator for large scales.

Measures

$$\langle\zeta\zeta\zeta\rangle \propto f_{\mathrm{NL}} P_L P_S$$

See talk by M. Remazeilles



at the time of writing, there is no significant evidence of a departure from a thermal spectrum measured over an impressively broad range of frequencies.

Thanks

Pessimism in cosmology

1948 : Alpher & Gamow : *There should be a microwave background,*
but we are not gonna detect it (CIB should hide it)

1965 : CMB detected by Penzias & Wilson
We need to establish blackbody (atmosphere is a pain)
and to detect anisotropies (initial fluctuations).

1990 Blackbody.

1992 CMB anisotropies

2003 Better done for anisotropies

2015 More than better done for anisotropies

Today : We need a more than better done for CMB energy spectrum.
Is beyond black-body spectrum reachable ?
In average ?
With angular resolution?
In intensity and in polarization ?