Moment expansion and the challenge of polarized dust SED complexity for B mode detections

Léo Vacher - Jonathan Aumont - Ludovic Montier - François Boulanger - Susanna Azzoni -Mathieu Remazeilles

LiteBIRD and the *B*-modes quest

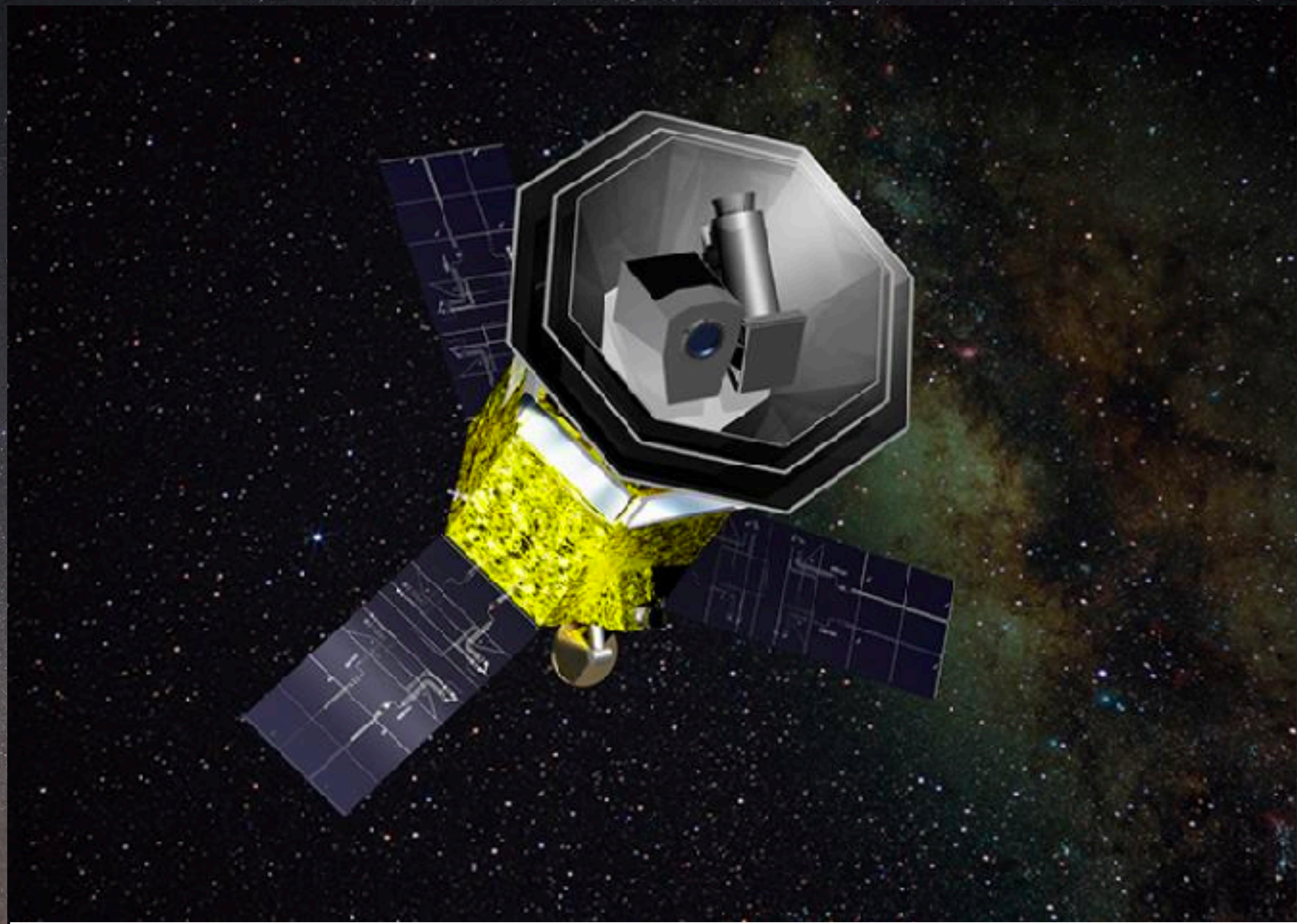
- JAXA project. Phase A CNES. ESA,NASA,CSA involved

- ***Lite*** *(Light) satellite for the studies of* ***B****-mode polarization and* ***I****nflation from cosmic background* ***R****adiation* ***D****etection*

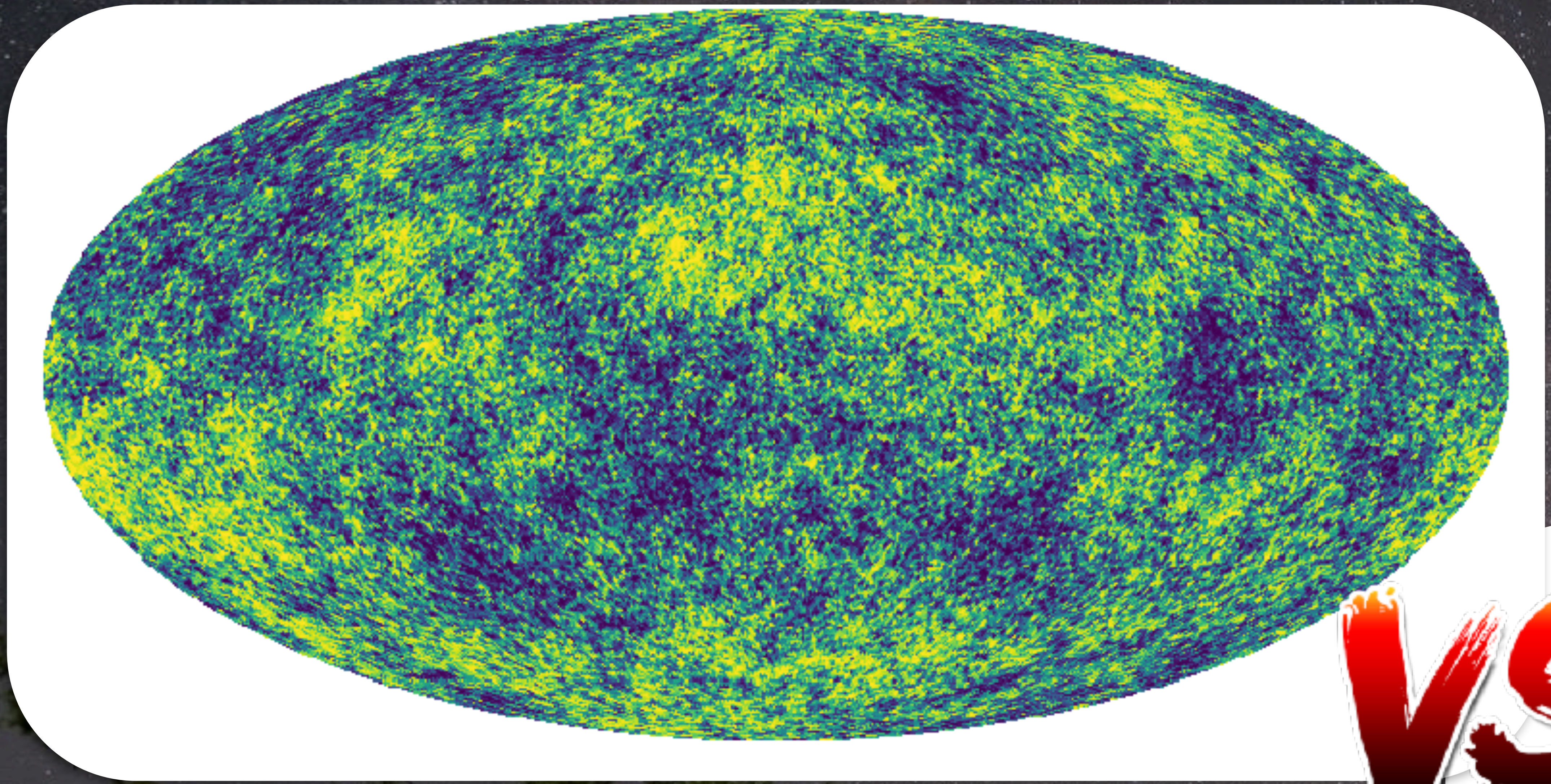
- Build to reach $\delta r = 1 \times 10^{-3}$

- 3 telescopes LFT, MFT, HFT

- Expected in 2029 at L2 for more than 3 years of observation

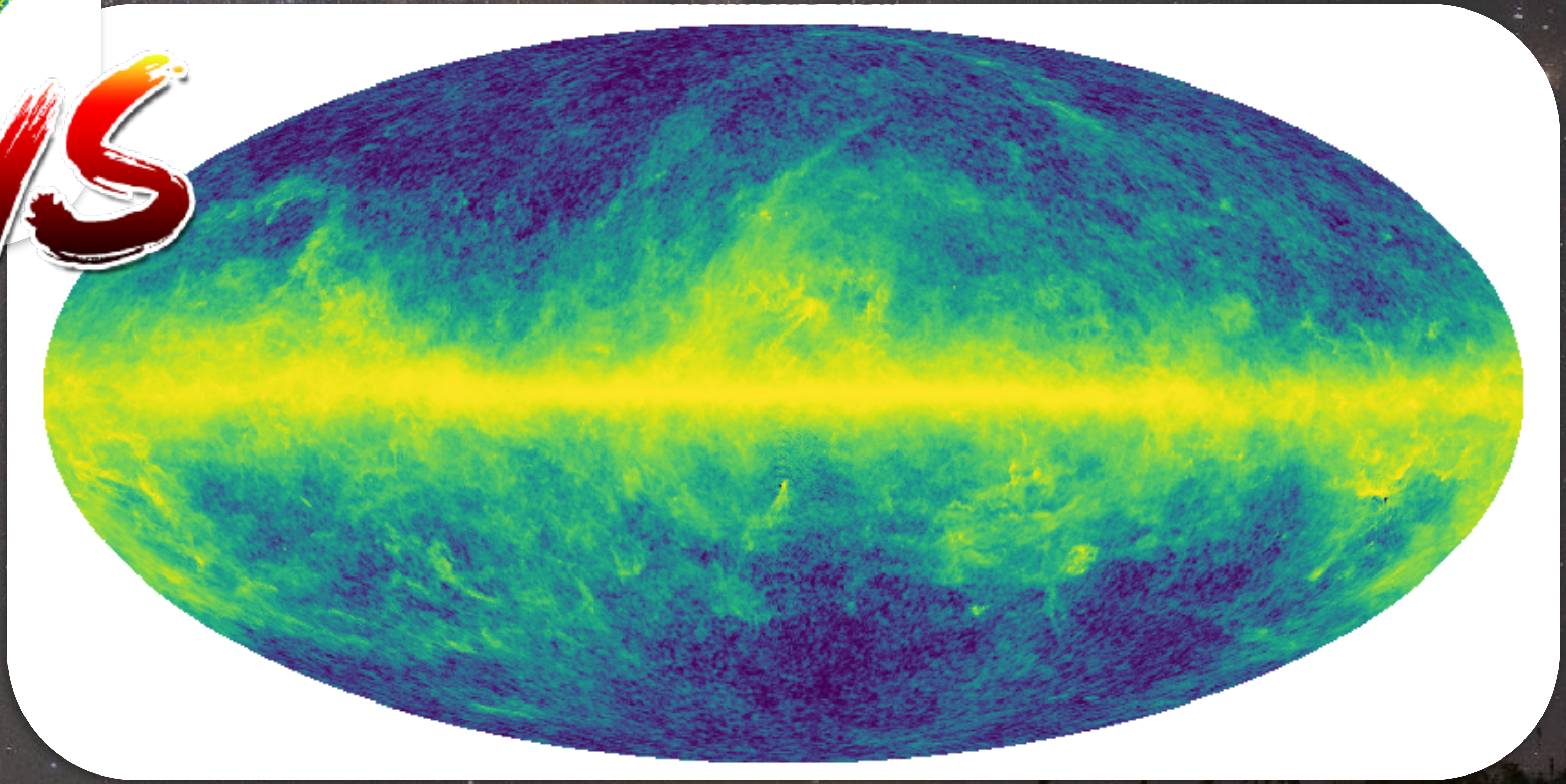


Foregrounds



Astrophysical sources emitting mainly in CMB's wavelength interval :

- ➢ Dust thermal emission
- ➢ Synchrotron
- ➢ Free-Free/ Brehmstrahlung



I. Foregrounds : problematic

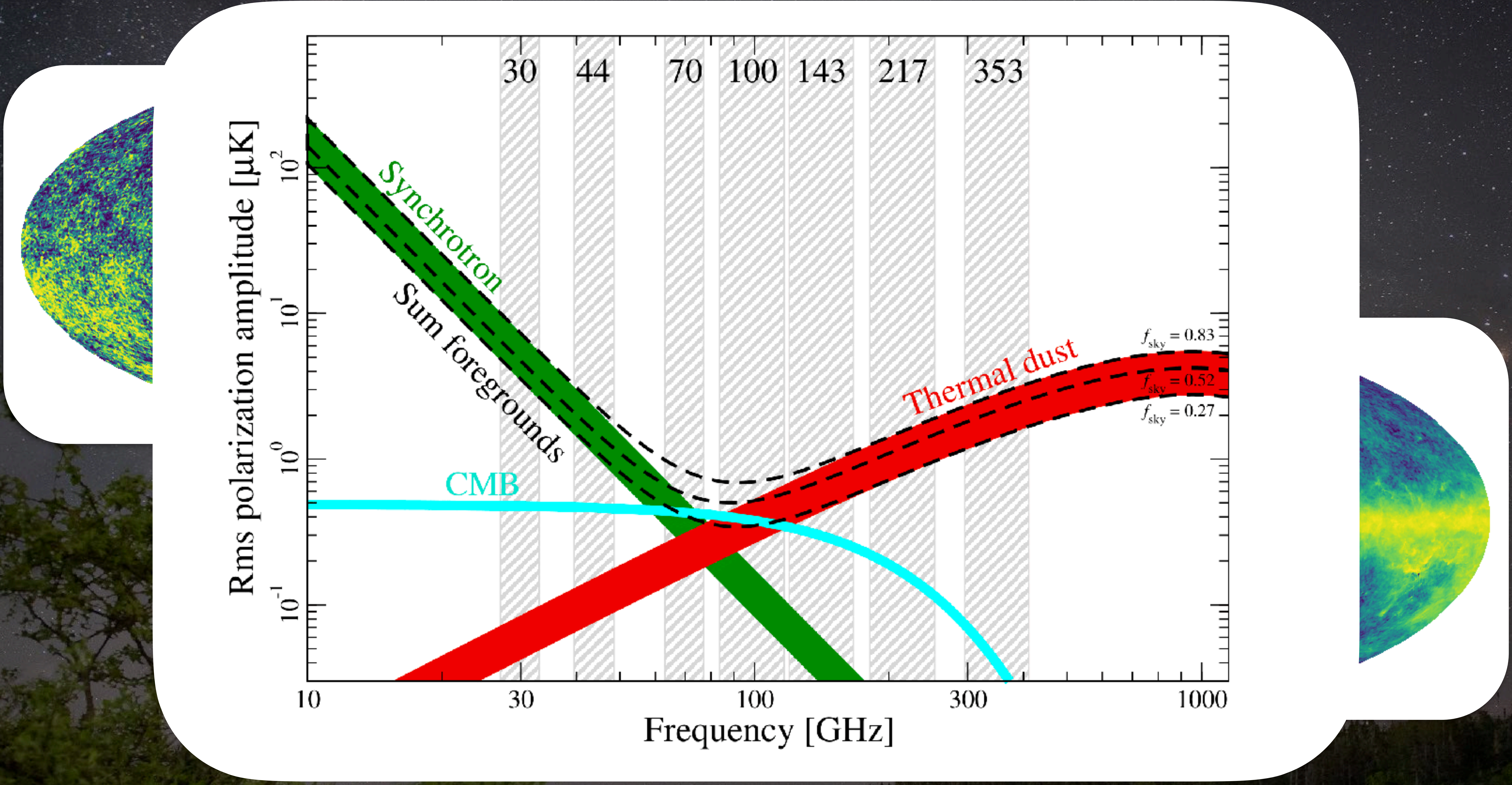
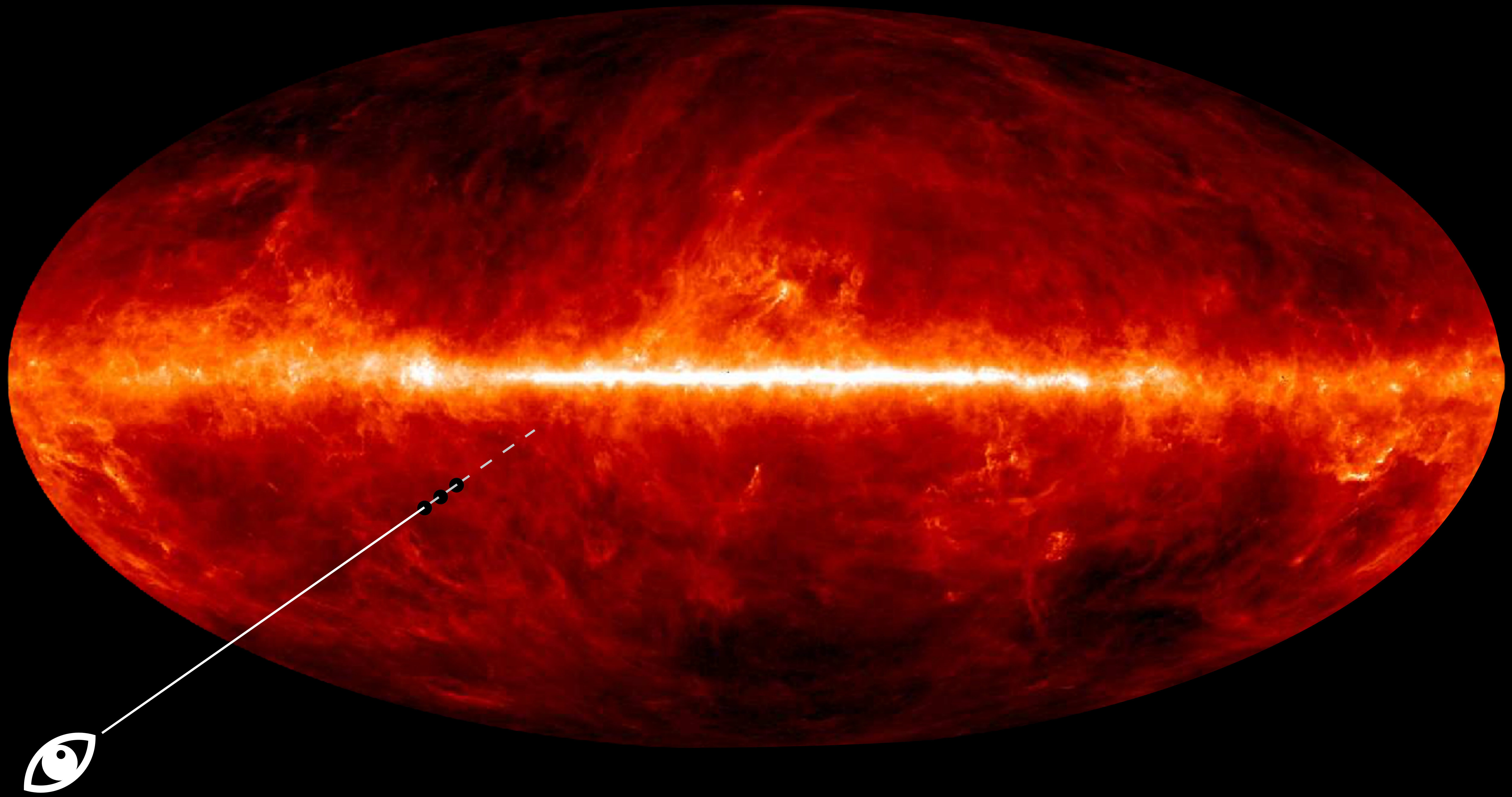


Photo-credit : Planck results 2018

Dust — **Averaging SEDs**

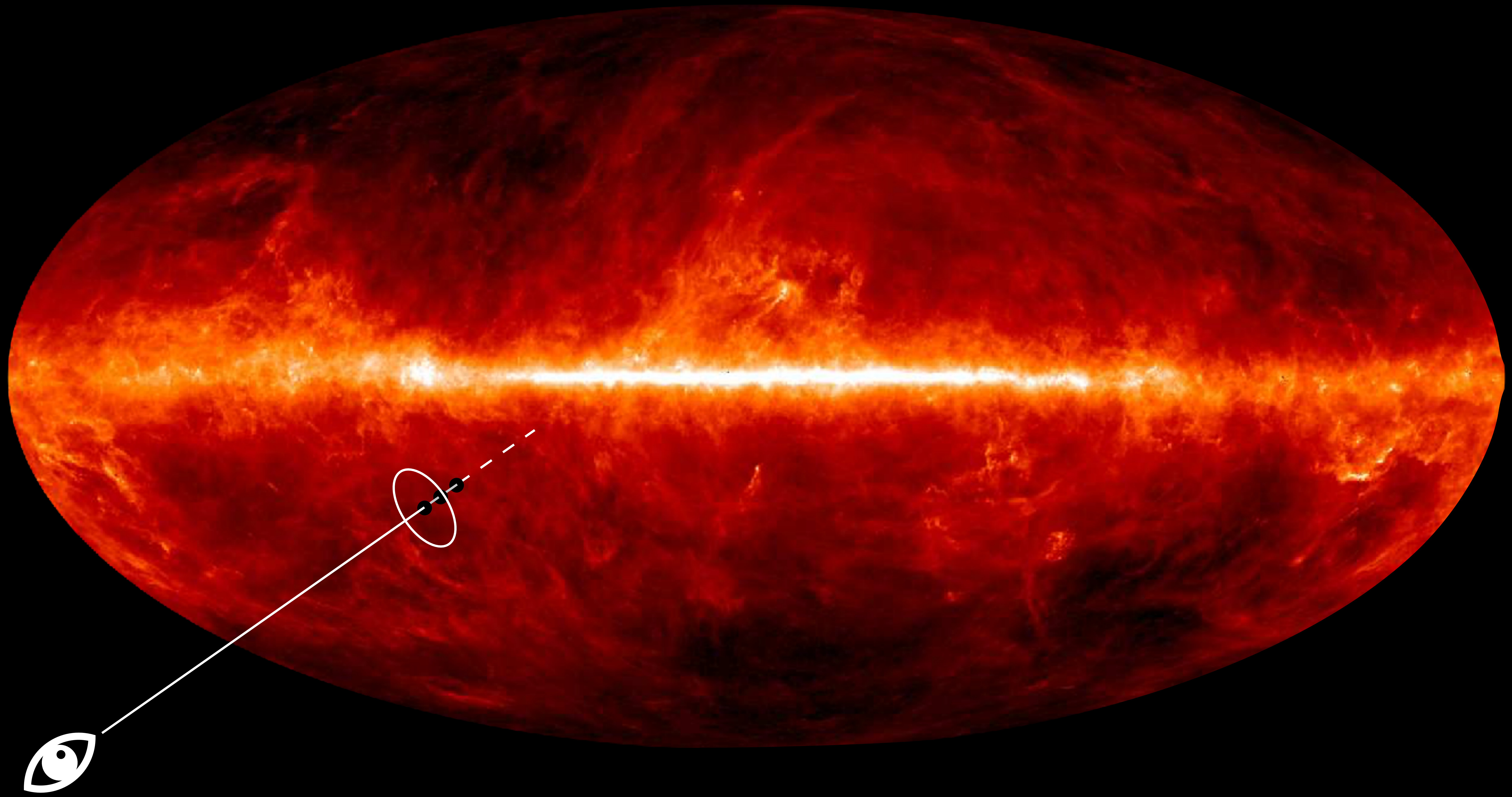


[Planck 2018 IV]

Modified black-body SED in *every* volume element

★ Line-of-sight average (*always there*!)

Dust — **Averaging SEDs**



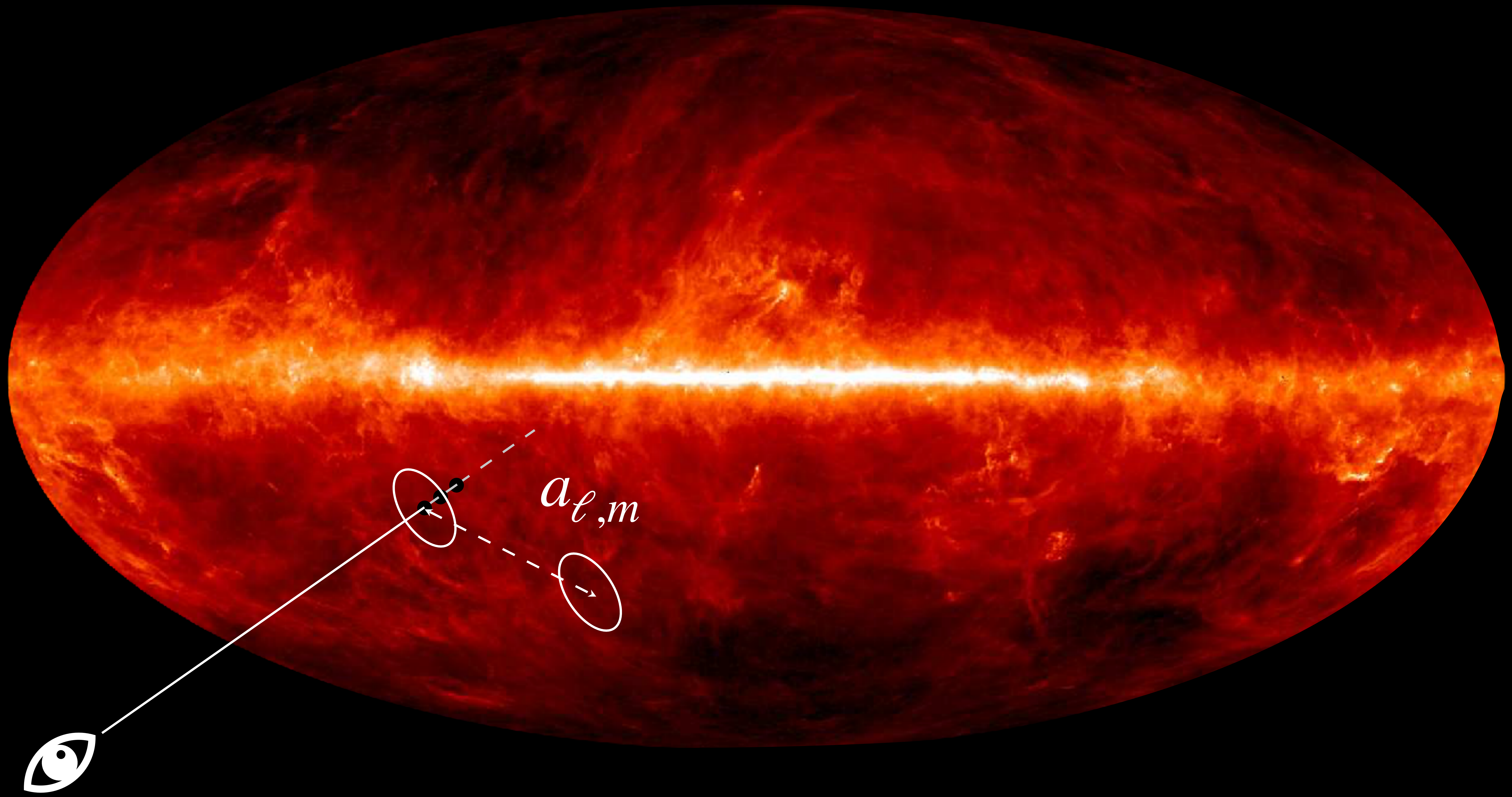
[Planck 2018 IV]

Modified black-body SED in *every* volume element

★ Line-of-sight average (*always there*!)

★ Experimental beam and frequency average

Dust — **Averaging SEDs**



[Planck 2018 IV]

Modified black-body SED in *every* volume element

- ★ Line-of-sight average (*always there*!)
- ★ Experimental beam and frequency average
- ★ Map operations average (e.g., spherical harmonic expansion)

The modified black body

Canonical model : Empirical + Non-linear

Spectral index

Modified Black-Body

dust map

$$I_{\mathrm{D}}(\nu,\vec{n}) = \left(\frac{\nu}{\nu_0}\right)^{\beta_0} \frac{B_\nu(T_0)}{B_{\nu_0}(T_0)} A(\vec{n}) = \frac{I_\nu(\beta_0,T_0)}{I_{\nu_0}(\beta_0,T_0)} A(\vec{n}).$$

Black body function

The moment expansion in pixel space

Taylor inspired expansion around the MBB in β (T fixed at T_0) :

$$I_{\mathrm{D}}(\nu,\vec{n}) = \frac{I_\nu(\beta_0,T_0)}{I_{\nu_0}(\beta_0,T_0)}\left[A(\vec{n}) + \omega_1(\vec{n})\,\ln\left(\frac{\nu}{\nu_0}\right) + \frac{1}{2}\omega_2(\vec{n})\,\ln^2\left(\frac{\nu}{\nu_0}\right) + \frac{1}{6}\omega_3(\vec{n})\,\ln^3\left(\frac{\nu}{\nu_0}\right) + \dots\right].$$

MBB (order 0) + order 1 + order 2 + order 3 + …

[Chluba et al., 2017]

The moment expansion in pixel space

Taylor inspired expansion around the MBB in β and T :

« $\Omega(\nu, T)$ »

$$I_{\mathrm{D}}(\nu,\vec{n}) = \frac{I_\nu(\beta_0,T_0)}{I_{\nu_0}(\beta_0,T_0)}\left(1+\omega_1^\beta ln\left(\frac{\nu}{\nu_0}\right)+\frac{h\omega_1^T}{kT_0^2}\left(\frac{\nu e^{\frac{h\nu}{kT_0}}}{e^{\frac{h\nu}{kT_0}}-1}-\frac{\nu_0 e^{\frac{h\nu_0}{kT_0}}}{e^{\frac{h\nu_0}{kT_0}}-1}\right)+\ldots\right.$$

MBB (order 0) + order 1 beta + order 1 T + ...

[Chluba et al., 2017]

The moment expansion in harmonic space

β only (*Mangilli et al 2021*)

$$\mathcal{D}_\ell(\nu_i \times \nu_j) = \frac{I_{\nu_i}\left(\beta_0\left(\ell\right), T_0(\ell)\right) I_{\nu_j}\left(\beta_0\left(\ell\right), T_0(\ell)\right)}{I^2_{\nu_0}\left(\beta_0\left(\ell\right), T_0(\ell)\right)} \times \Bigg\{$$

$$0^{\text{th}} \text{ order } \Big\{ \quad \mathcal{D}_\ell^{A\times A}$$

$$1^{\text{st}} \text{ order } \begin{cases} +\left[\ln\left(\frac{\nu_i}{\nu_0}\right) + \ln\left(\frac{\nu_j}{\nu_0}\right)\right] \mathcal{D}_\ell^{A\times\omega_1^\beta} \\ +\left[\ln\left(\frac{\nu_i}{\nu_0}\right)\ln\left(\frac{\nu_j}{\nu_0}\right)\right] \mathcal{D}_\ell^{\omega_1^\beta\times\omega_1^\beta} \end{cases}$$

$$2^{\text{nd}} \text{ order } \begin{cases} +\frac{1}{2}\left[\ln^2\left(\frac{\nu_i}{\nu_0}\right) + \ln^2\left(\frac{\nu_j}{\nu_0}\right)\right] \mathcal{D}_\ell^{A\times\omega_2^\beta} \\ +\frac{1}{2}\left[\ln\left(\frac{\nu_i}{\nu_0}\right)\ln^2\left(\frac{\nu_j}{\nu_0}\right) + \ln\left(\frac{\nu_j}{\nu_0}\right)\ln^2\left(\frac{\nu_i}{\nu_0}\right)\right] \mathcal{D}_\ell^{\omega_1^\beta\times\omega_2^\beta} \\ +\frac{1}{4}\left[\ln^2\left(\frac{\nu_i}{\nu_0}\right)\ln^2\left(\frac{\nu_j}{\nu_0}\right)\right] \mathcal{D}_\ell^{\omega_2^\beta\times\omega_2^\beta} \end{cases}$$

$$+ \ldots \quad \Bigg\}.$$

β and *T*

$$\mathcal{D}_\ell(\nu_i \times \nu_j) = \frac{I_{\nu_i}(\beta_0(\ell), T_0(\ell)) I_{\nu_j}(\beta_0(\ell), T_0(\ell))}{I_{\nu_0}(\beta_0(\ell), T_0(\ell))^2} \cdot \Bigg\{$$

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The moment expansion in harmonic space

$$\mathcal{D}_\ell = \frac{\ell(\ell+1)}{2\pi}\mathcal{C}_\ell$$

$$\mathcal{D}_\ell(\nu_i \times \nu_j) = \mathcal{D}_\ell(Map(\nu_i), Map(\nu_j))$$

β only (Mangilli et al

$$\mathcal{D}_\ell(\nu_i \times \nu_j) = \frac{I_{\nu_i}(\beta_0(\ell), T_0(\ell))\, I_{\nu_j}(\beta_0(\ell), T_0}{I^2_{\nu_0}(\beta_0(\ell), T_0(\ell))}$$

0th order $\left\{ \mathcal{D}_\ell^{A\times A} \right.$

1st order $\left\{ \begin{array}{l} +\left[\ln\left(\frac{\nu_i}{\nu_0}\right) + \ln\left(\frac{\nu_j}{\nu_0}\right)\right]\mathcal{D}_\ell^{A\times\omega_1^\beta} \\ +\left[\ln\left(\frac{\nu_i}{\nu_0}\right)\ln\left(\frac{\nu_j}{\nu_0}\right)\right]\mathcal{D}_\ell^{\omega_1^\beta\times\omega_1^\beta} \end{array}\right.$

2nd order $\left\{ \begin{array}{l} +\frac{1}{2}\left[\ln^2\left(\frac{\nu_i}{\nu_0}\right) + \ln^2\left(\frac{\nu_j}{\nu_0}\right)\right]\mathcal{D}_\ell^{A\times\omega_2^\beta} \\ +\frac{1}{2}\left[\ln\left(\frac{\nu_i}{\nu_0}\right)\ln^2\left(\frac{\nu_j}{\nu_0}\right) + \ln\left(\frac{\nu_j}{\nu_0}\right)\ln^2\left(\frac{\nu_i}{\nu_0}\right)\right]\mathcal{D}_\ell^{\omega_1^\beta\times\omega_2^\beta} \\ +\frac{1}{4}\left[\ln^2\left(\frac{\nu_i}{\nu_0}\right)\ln^2\left(\frac{\nu_j}{\nu_0}\right)\right]\mathcal{D}_\ell^{\omega_2^\beta\times\omega_2^\beta} \end{array}\right.$

$+\ldots \}.$

β and T

$$\mathcal{D}_\ell(\nu_i \times \nu_j) = \frac{I_{\nu_i}(\beta_0(\ell), T_0(\ell))I_{\nu_j}(\beta_0(\ell), T_0(\ell))}{I_{\nu_0}(\beta_0(\ell), T_0(\ell))^2} \cdot \{$$

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The moment expansion in harmonic space

β only *(Mangilli et al 2021)*

0^{th} order

1^{st} order

2^{nd} order

$$+\frac{1}{2}\left[\ln\left(\frac{\nu_i}{\nu_0}\right)\ln^2\left(\frac{\nu_j}{\nu_0}\right)+\ln\left(\frac{\nu_j}{\nu_0}\right)\ln^2\left(\frac{\nu_i}{\nu_0}\right)\right]\mathcal{D}_\ell^{\omega_1^\beta\times\omega_2^\beta}$$

$$+\frac{1}{4}\left[\ln^2\left(\frac{\nu_i}{\nu_0}\right)\ln^2\left(\frac{\nu_j}{\nu_0}\right)\right]\mathcal{D}_\ell^{\omega_2^\beta\times\omega_2^\beta}$$

$$+\dots\Big\}.$$

β and T

1^{st} order

$$+\mathcal{D}_\ell^{A\times\omega_1^T}\left(\Omega_i+\Omega_j-2\Omega_0\right)$$

$$+\mathcal{D}_\ell^{\omega_1^\beta\times\omega_1^T}\left(\ln\left(\frac{\nu_j}{\nu_0}\right)(\Omega_i-\Omega_0)+\ln\left(\frac{\nu_i}{\nu_0}\right)(\Omega_j-\Omega_0)\right)$$

$$+\mathcal{D}_\ell^{\omega_1^T\times\omega_1^T}\left(\Omega_i-\Omega_0\right)\left(\Omega_j-\Omega_0\right)+\dots\Big\}$$

- ★ New approach that allow to describe, at the angular power spectrum level, the dust SED distortions due to line-of-sight, beam and map operations averages
- ★ Naturally describes, in a model-independent way, the effect of spatial variations of the dust SED, additional dust components and in a more general way of dust frequency decorrelation!

Roadmap

Dust-type
CMB
Instrumental effects
Mask

Simulate LiteBIRD maps → Extract $\mathscr{C}_\ell$ → Fit SED and catch r

Frequency cross
spectra Namaster
(Purify B)

Moment expansion
Mpfit and emcee

LiteBIRD simulations with various dust components

dust models :

- ⋆ d0 : $\beta = 1.54$, $T = 20$ K
- ⋆ "d1T " : $\beta(\vec{n})$, 22 K
- ⋆ d1 : $\beta(\vec{n})$, $T(\vec{n})$

CMB :

- ⋆ With lensing "c" or without
- ⋆ $r_{\rm sim} = 0$ and $r_{\rm sim} = 0.01$

LiteBIRD simulations with various dust components

LiteBIRD :

- *LiteBIRD noise (IMO)*
- 9 highest frequencies (100-402 GHz)

Mask:

Planck Mask + cutoff + apodisation

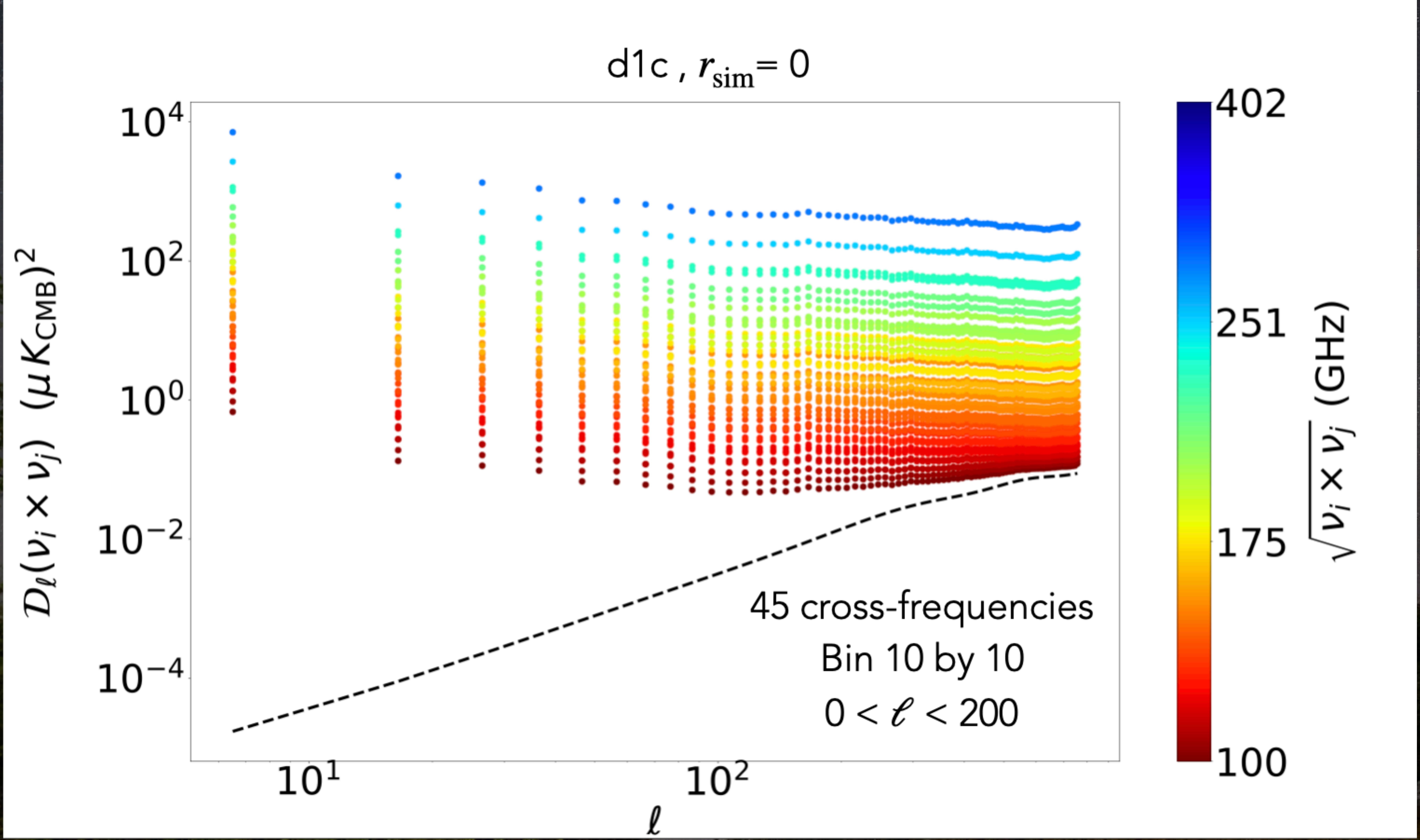
Such that $f_{\rm sky} = 0.7$

Map :

Nside = 256

500 simulations of each type

Estimation of the B-mode cross-frequency spectra (Namaster)



Best fit implementation

χ^2 minimisation using `mpfit` or `emcee`

$$\mathcal{D}_\ell^{\rm model}(\nu_i \times \nu_j) = \mathcal{D}_\ell^{\rm dust}\left(\beta(\ell), T_0(\ell), \mathcal{D}_\ell^{ab}(\nu_i \times \nu_j)\right) + \kappa \times \mathcal{D}_\ell^{\rm lensing} + r \times \mathcal{D}_\ell^{\rm tensor}$$

- ★ Moment expansion pushed at various order from 0 (MBB) to 3
- ★ $\beta(\ell)$ free at order zero and corrected through iterative process
- ★ $T_0(\ell)$ free at order zero then fixed at best fit value

Best fit implementation

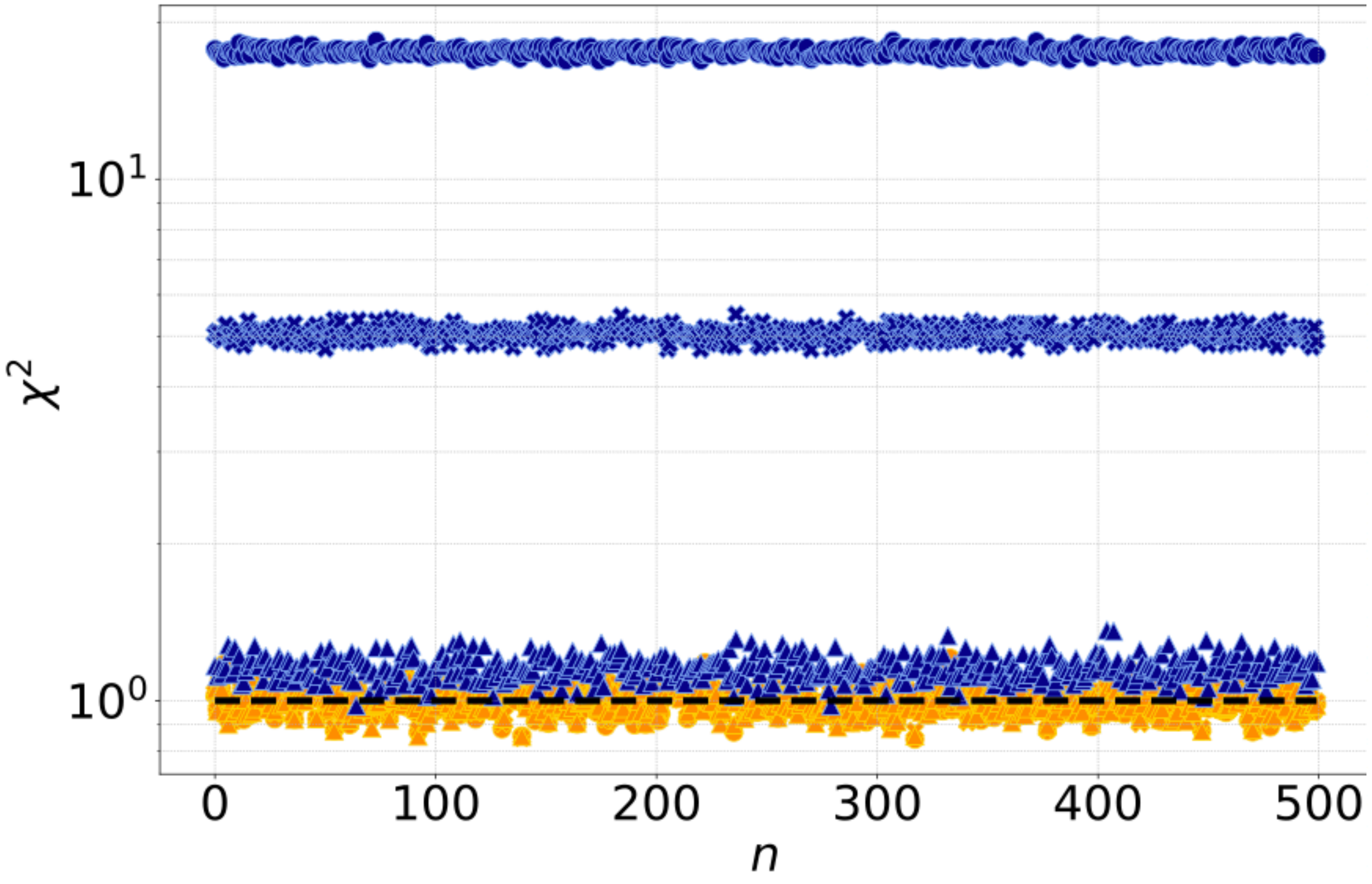
χ^2 minimisation using `mpfit` or `emcee`

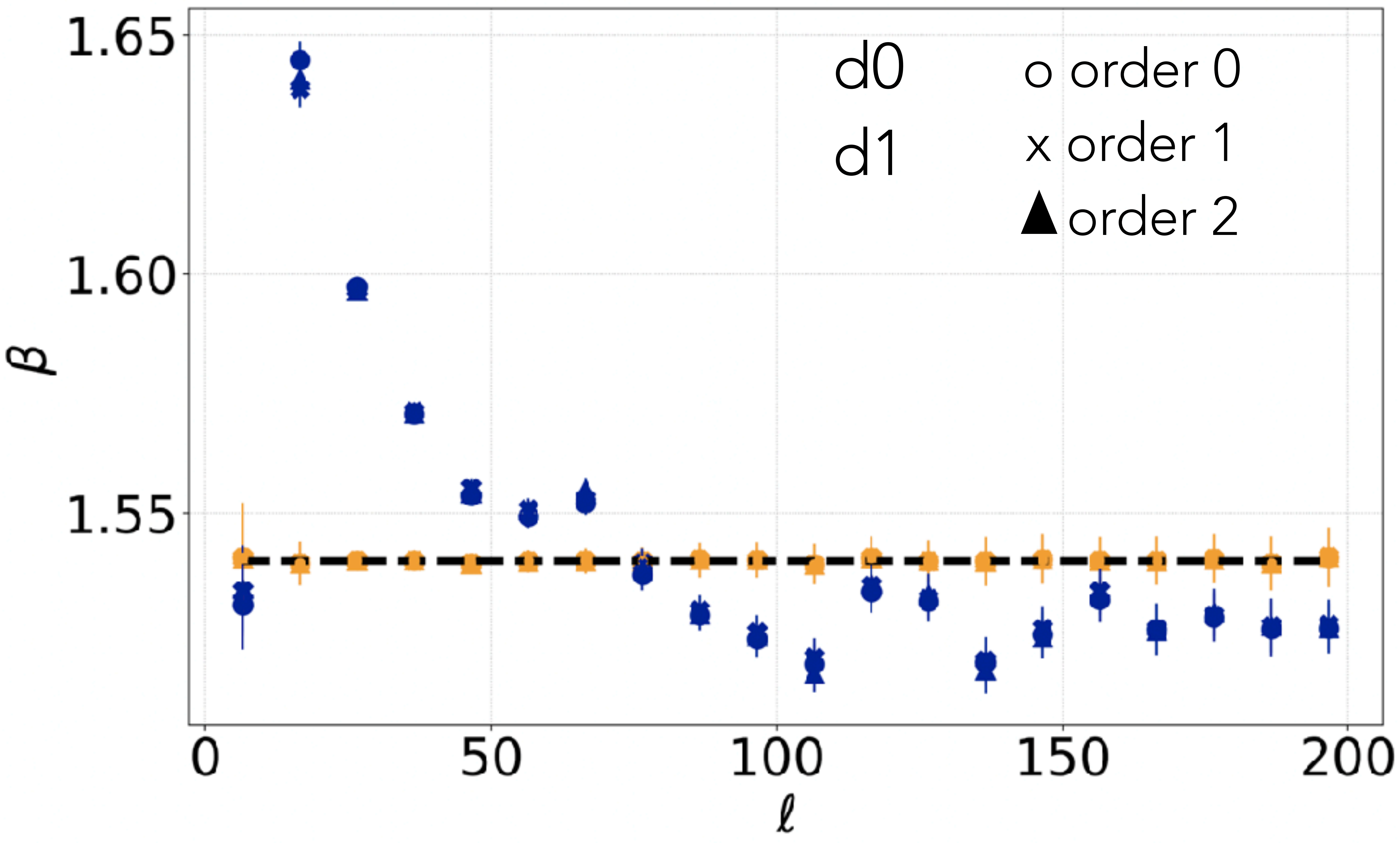
$$\mathscr{D}_\ell^{\text{model}}(\nu_i \times \nu_j) = \mathscr{D}_\ell^{\text{dust}}\left(\beta(\ell), T_0(\ell), \mathscr{D}_\ell^{ab}(\nu_i \times \nu_j)\right) + \kappa \times \mathscr{D}_\ell^{\text{lensing}} + r \times \mathscr{D}_\ell^{\text{tensor}}$$

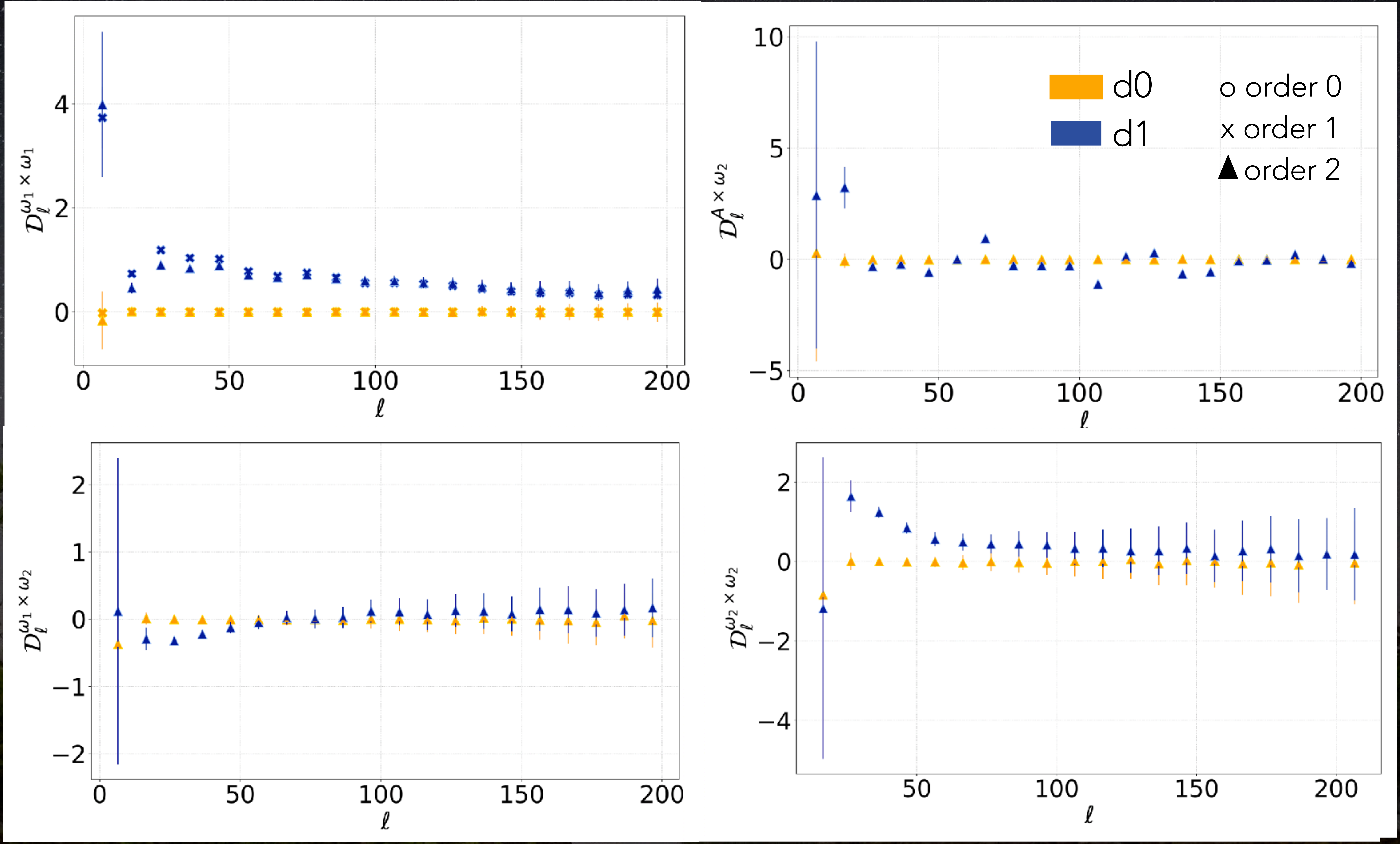
- ★ κ always fixed : 0 for d or 1 for dc
- ★ r fixed or free

astrophysique & planétologie

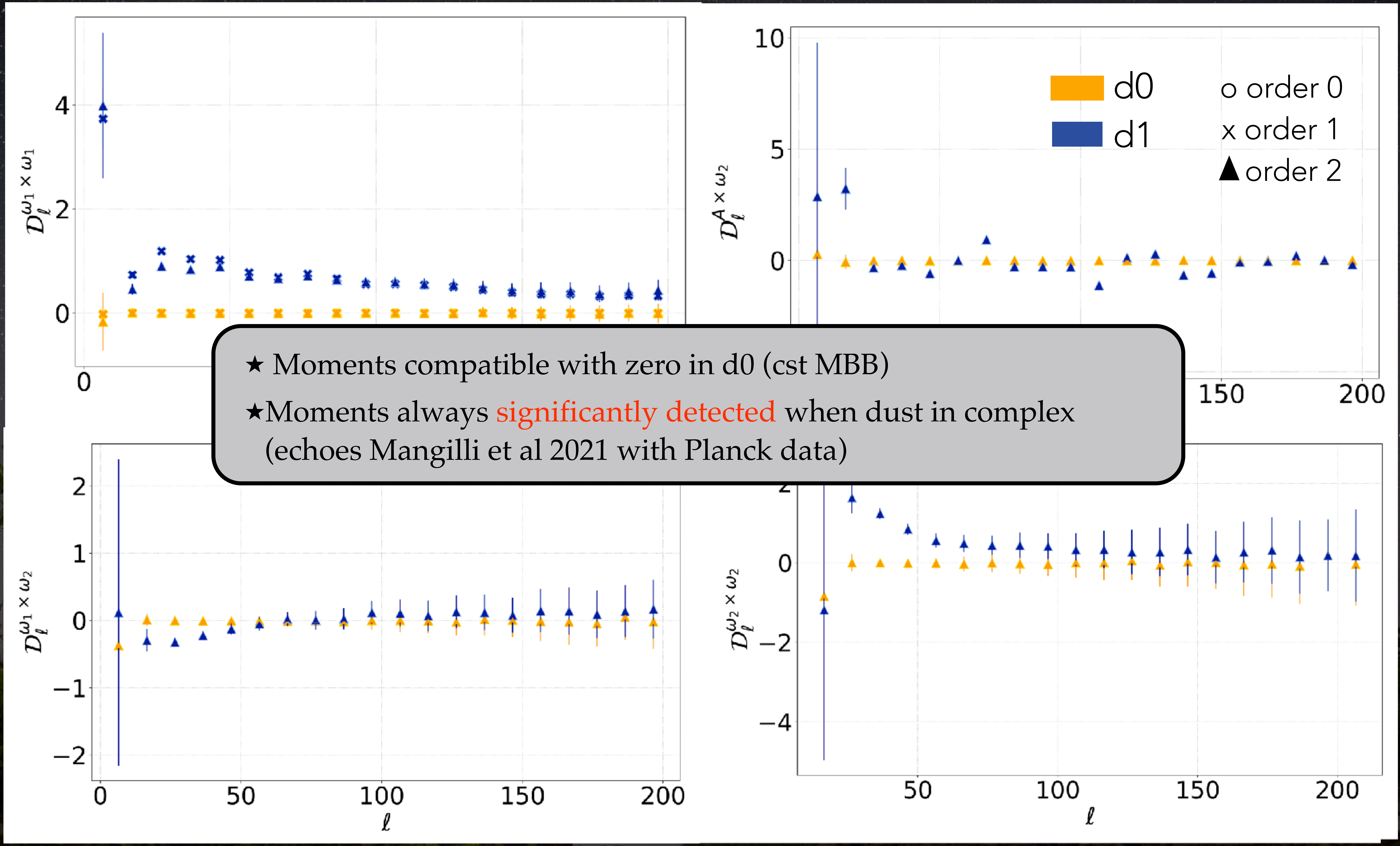
Results I : dust only (κ=0,r=0 fixed)







$\mathcal{D}_\ell^{\omega_1\times\omega_1}$
$\mathcal{D}_\ell^{A\times\omega_2}$
$\mathcal{D}_\ell^{\omega_1\times\omega_2}$
$\mathcal{D}_\ell^{\omega_2\times\omega_2}$
ℓ
d0
d1
o order 0
x order 1
▲ order 2



★ Moments compatible with zero in d0 (cst MBB)

★Moments always significantly detected when dust in complex (echoes Mangilli et al 2021 with Planck data)

Results II : dust + cmb (κ =1, *r* free, $r_{\rm sim} = 0$)

Order 0 (MBB) Order 1 beta Order 1 beta ad T Order 2 beta

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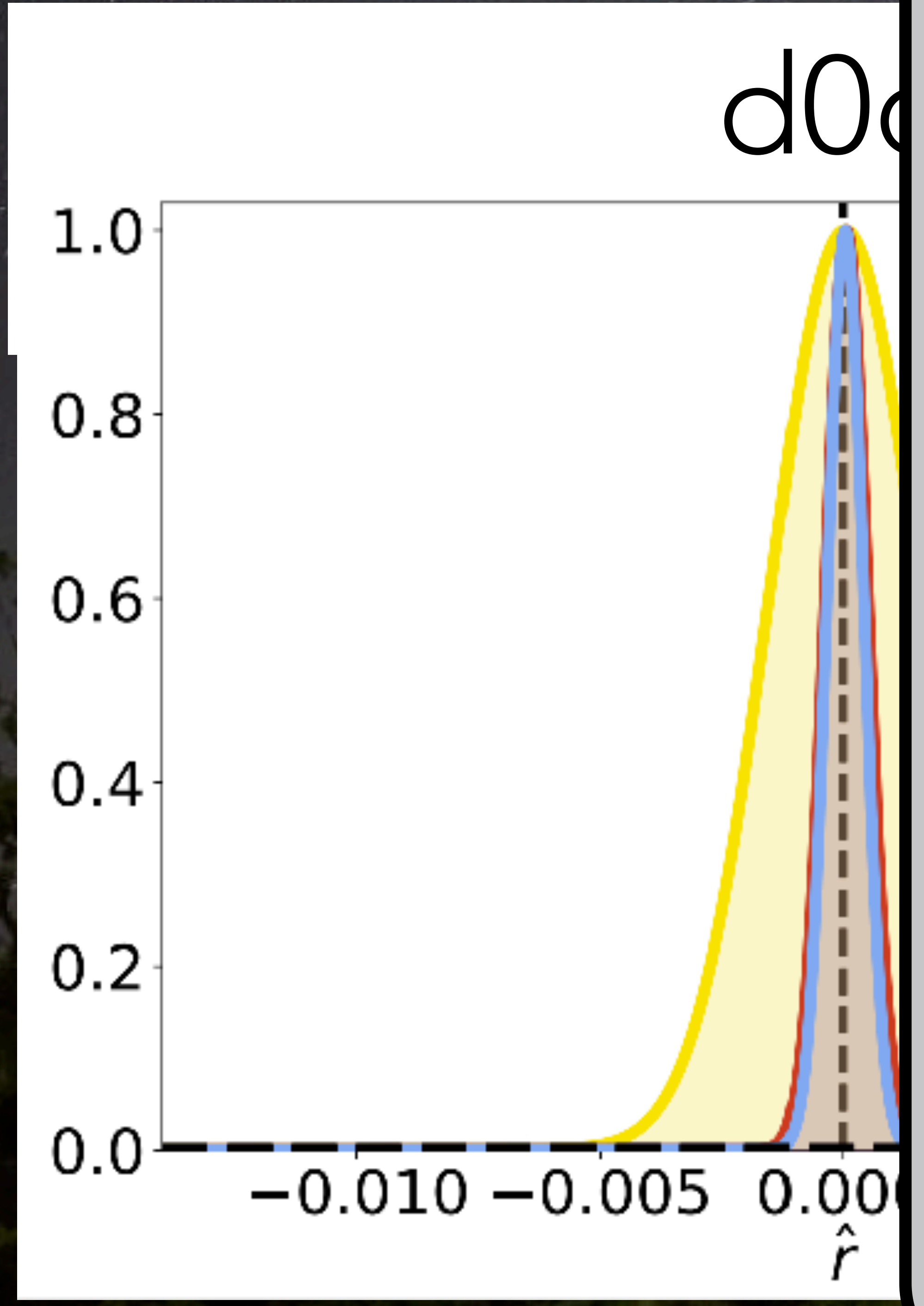


★ **Results robust with $r_{\text{sim}} \neq 0 \rightarrow$ moment expansion can be trust to detect *r***

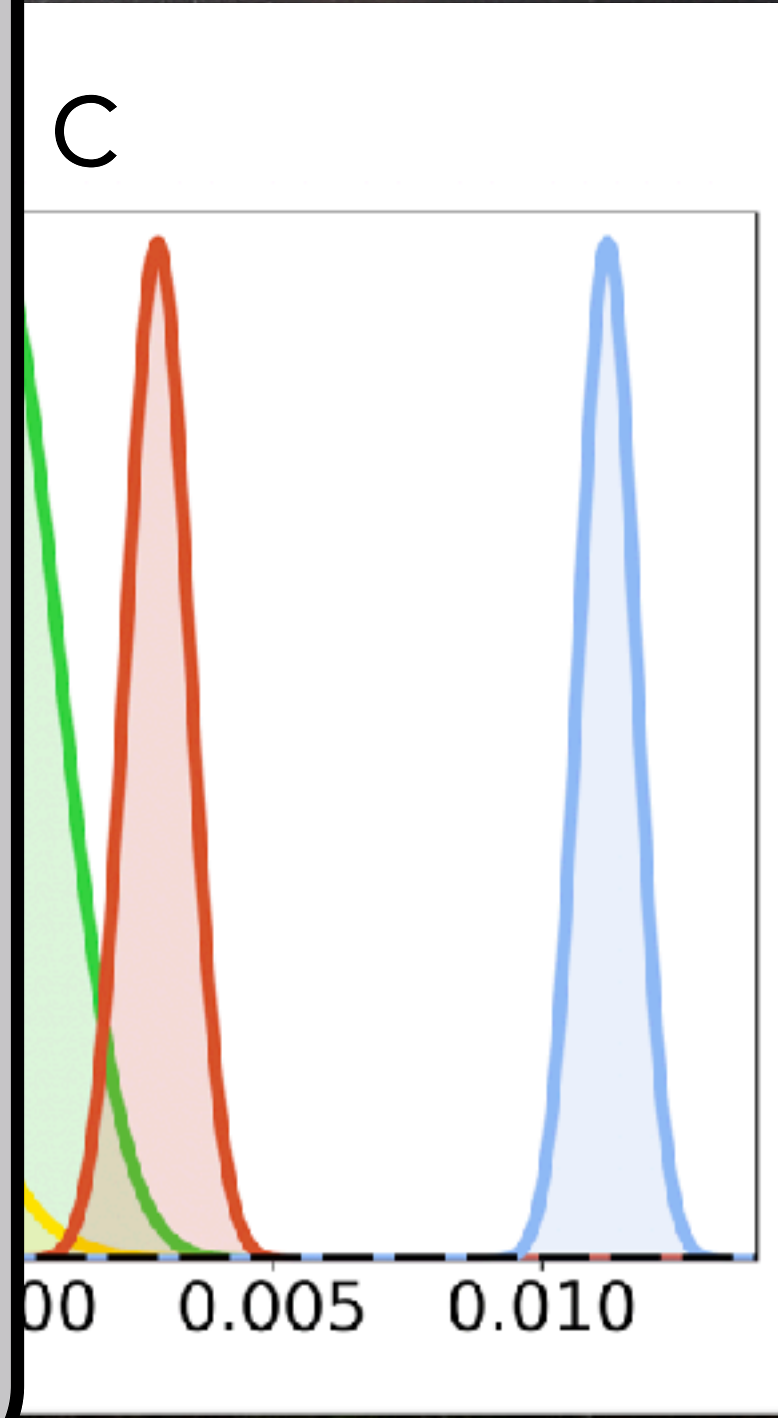
★ d0 MBB no bias + moment expansions no bias

★ d1T MBB biased + moment order 1 and 2 no bias

★ d1c : - Order 0 and 1 in beta with dispersion around noise (few 10^{-4})

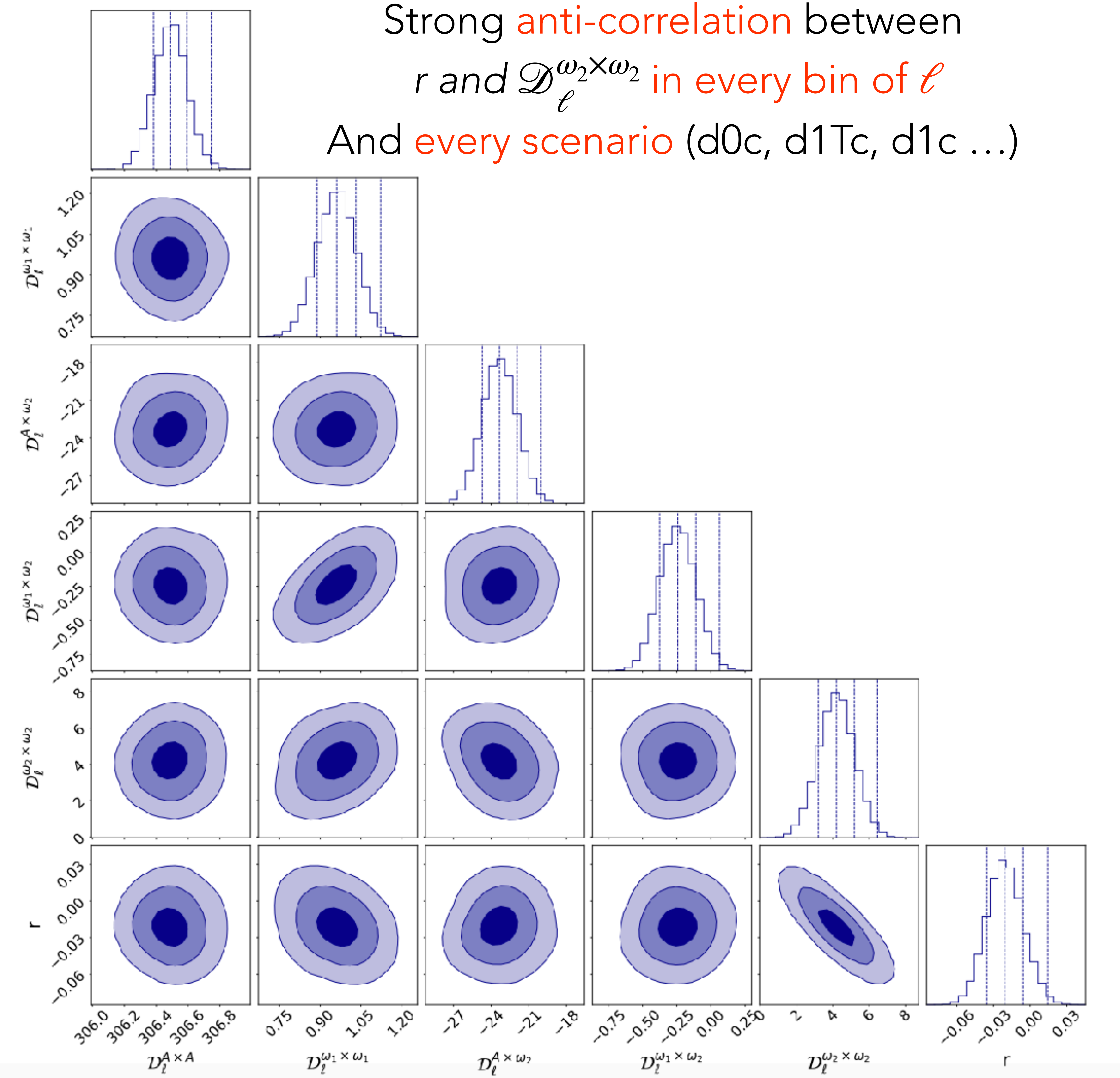
- Order 1 beta-T : no biais, dispersion around (1.1×10^{-3})
- Order 2 double the dispersion + negative bias at order 2

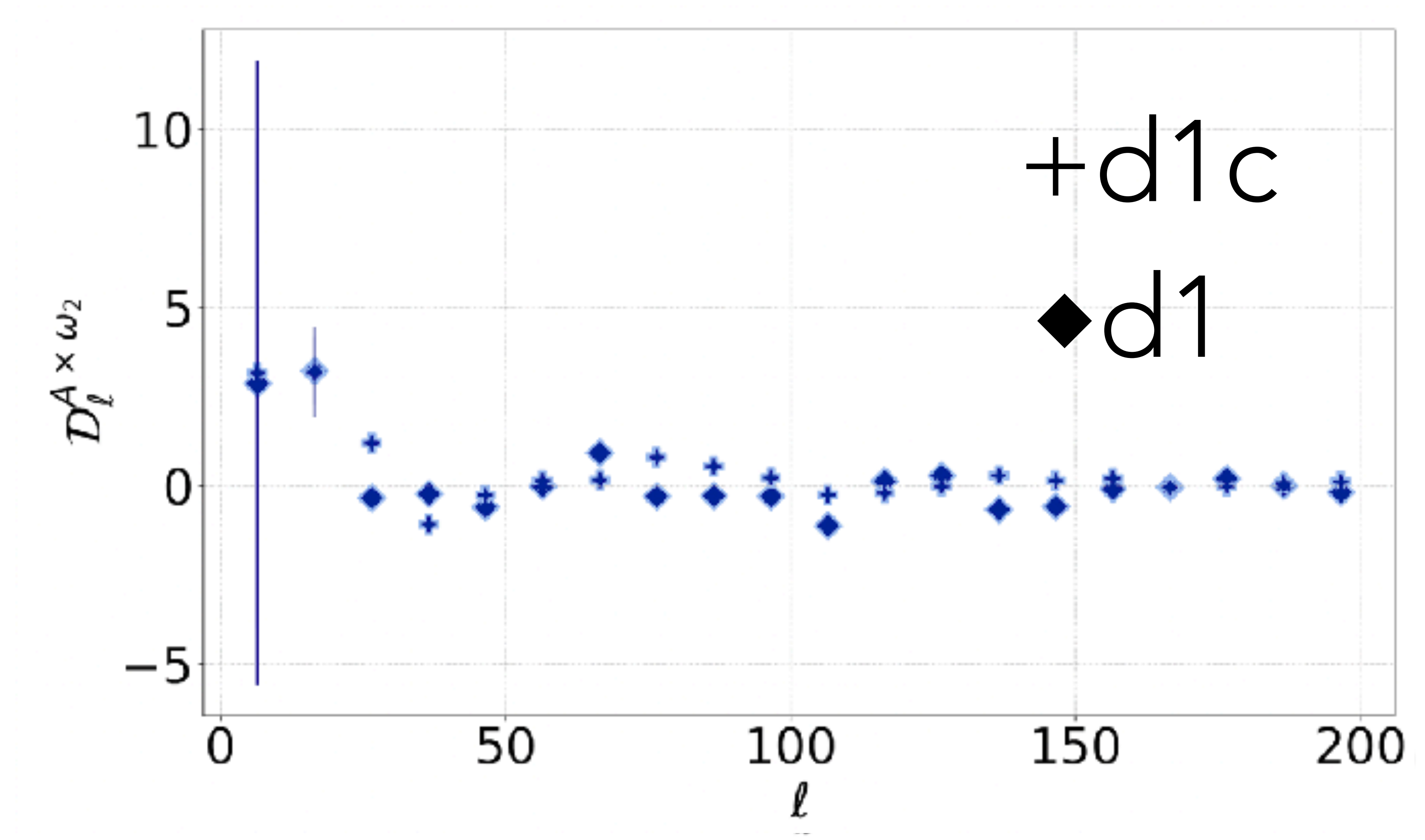
… Why ?

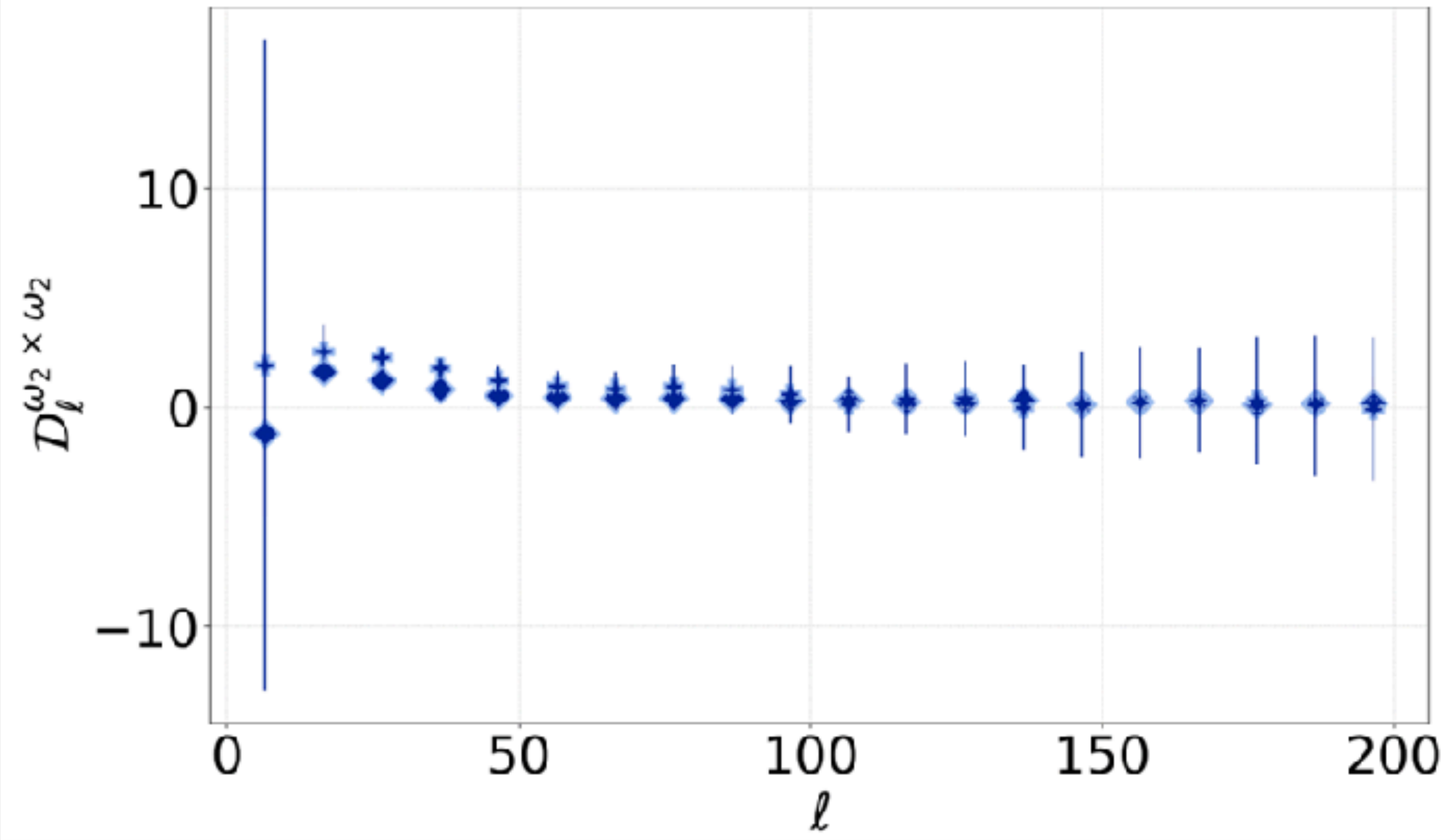


Strong anti-correlation between r and $\mathcal{D}_\ell^{\omega_2\times\omega_2}$ in every bin of ℓ

And every scenario (d0c, d1Tc, d1c …)







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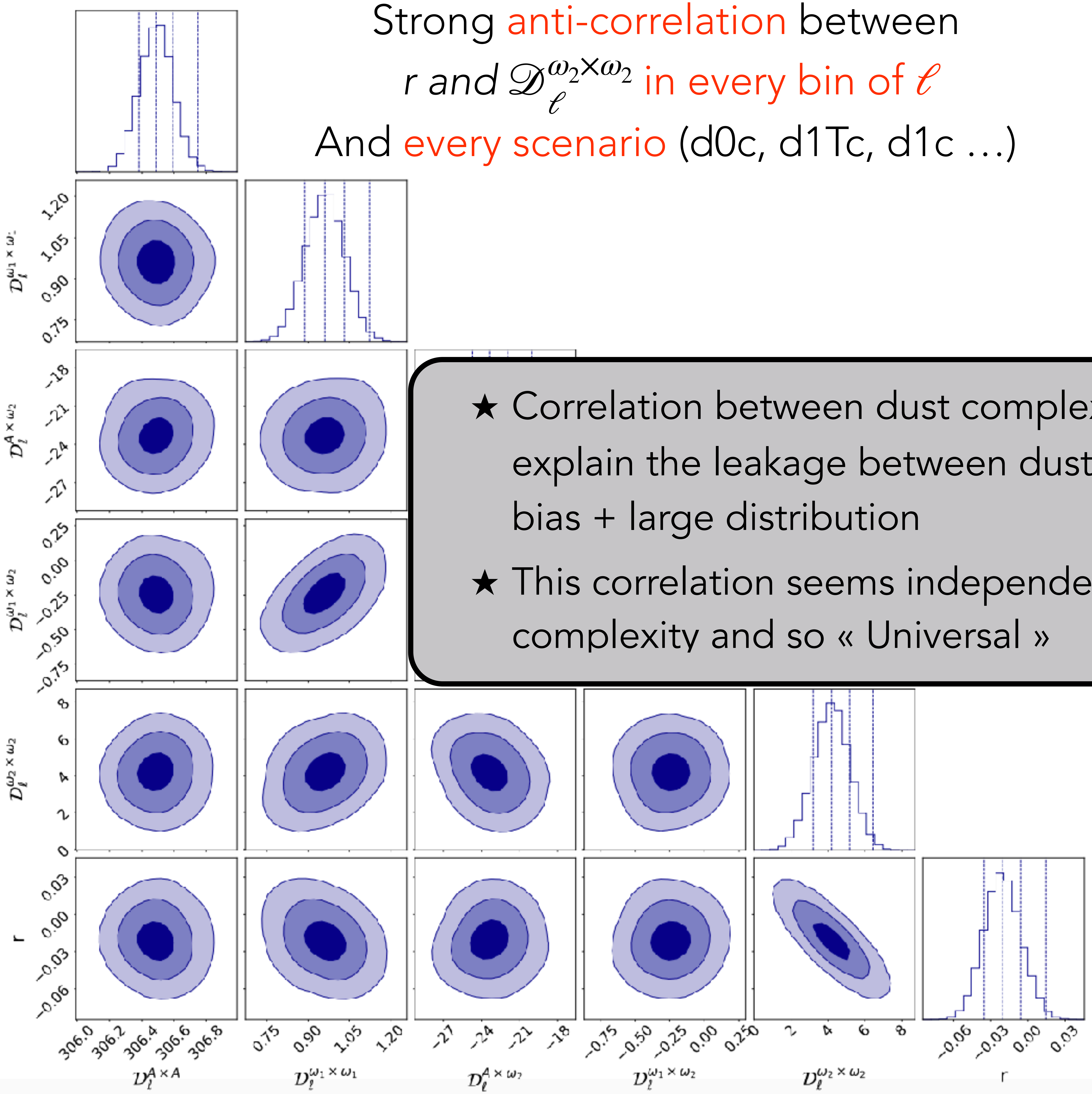
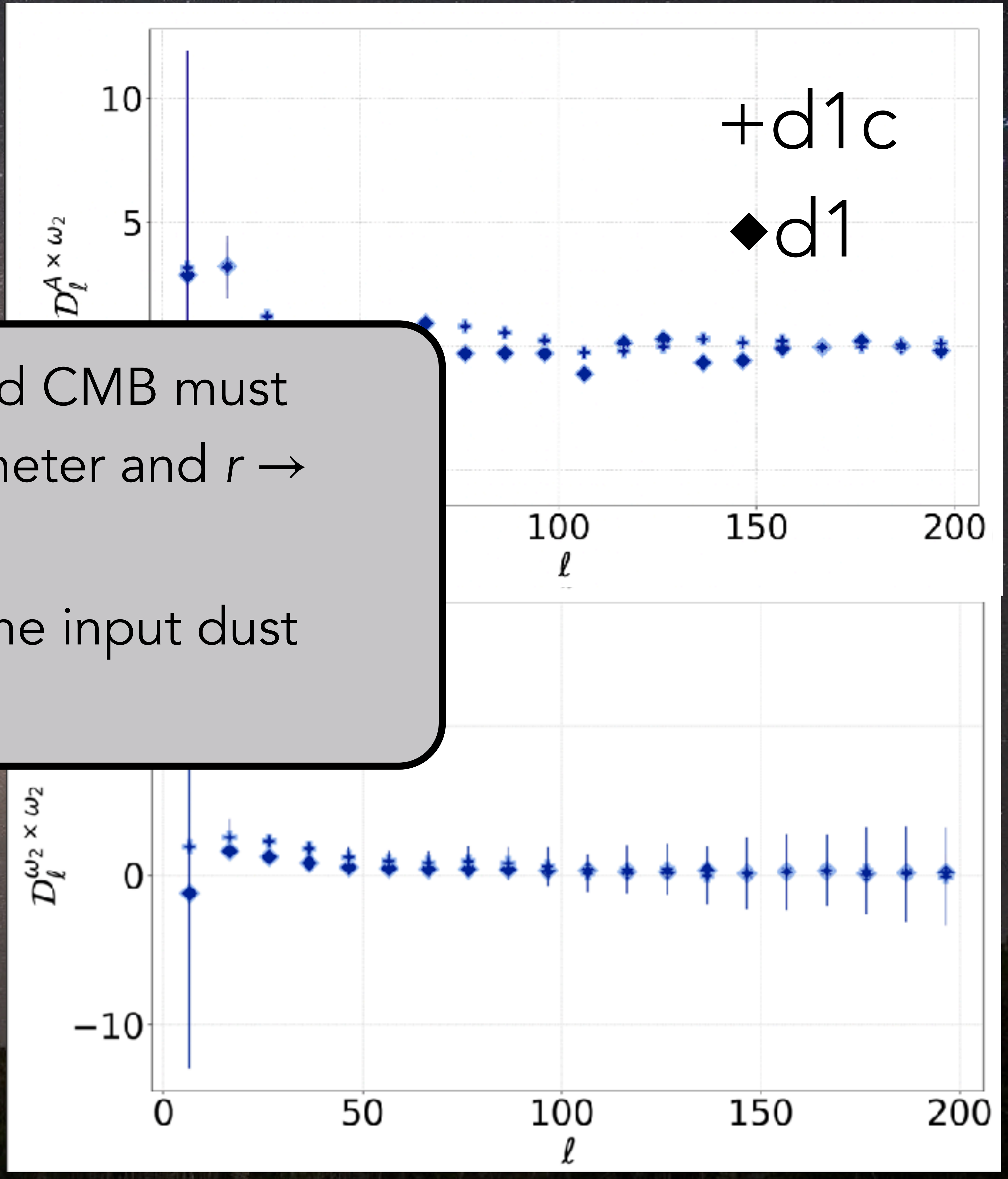




★ Correlation between dust complexity and CMB must explain the leakage between dust parameter and r → bias + large distribution

★ This correlation seems independent of the input dust complexity and so « Universal »

What's next :

- ★ Update the expansion around *T*
- ★ Finalise our first paper ! (*Vacher et al* to come)
- ★ Introduce a formalism more friendly with polarisation
- ★ Consider also synchrotron and all frequency bands
- ★ Application to other instruments (SO (Azzoni et al 2021), QUBIC ongoing)
- ★ Take into account for frequency variations of SED parameters + non gaussian dust emission
- ★ Couple moments with other methods …
  (already explored in pixel space (*Remazeilles et al 2017*))

Take away :

- ★ $\beta(\ell)$ and $T(\ell)$ can be fitted together with a MBB without biais/error-bar explosion
- ★ Temperature could be a critical parameter for LB
- ★ Using moment expansion at order 1, one can reduce/absorb the bias on r
- ★ d1 contains order 2 terms in power spectra level …
- ★ Degeneracies between dust complexity and tensor to scalar ratio ! Prevent to take into account for the additional complexity properly

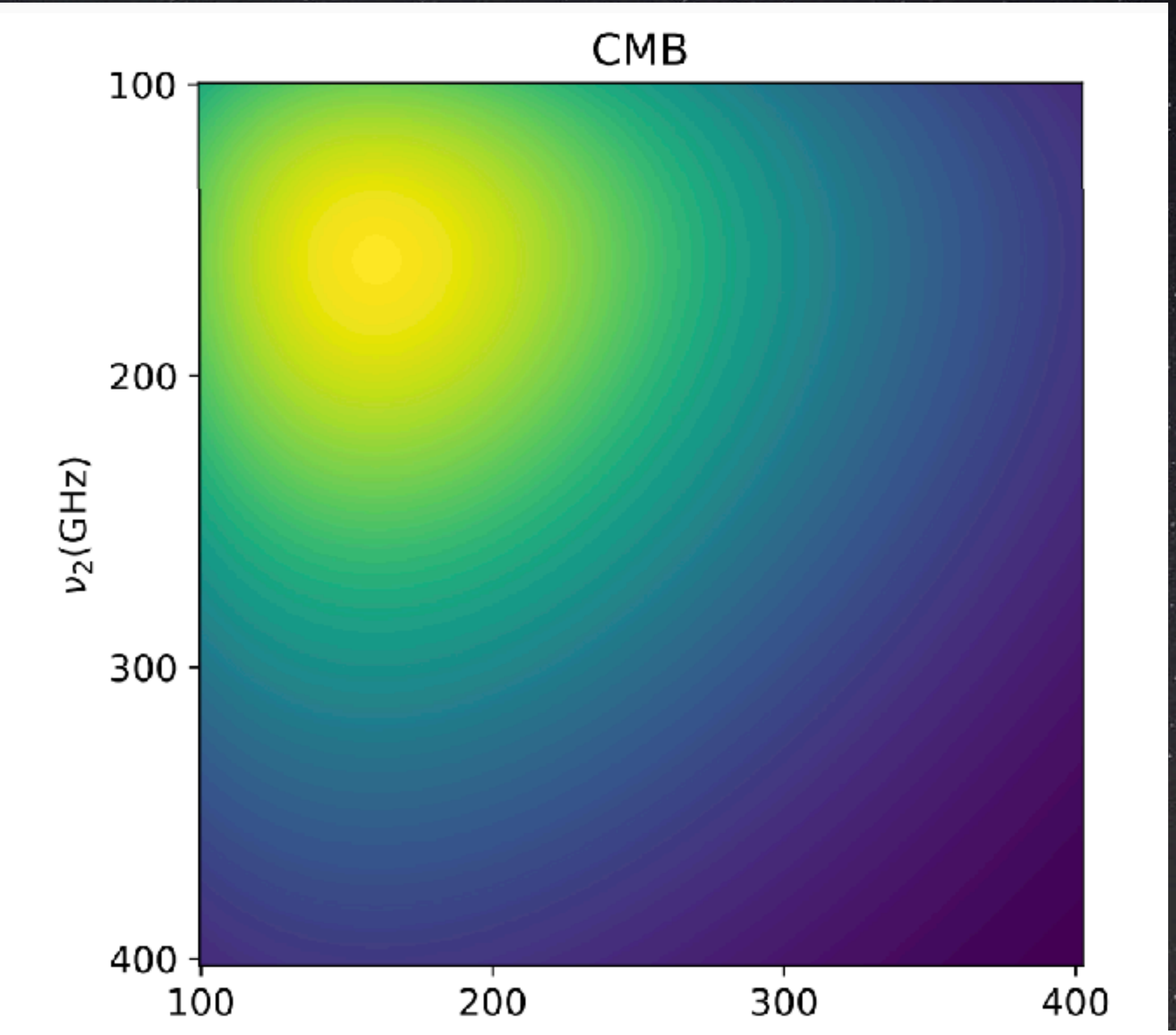
Could be problematic for component separation

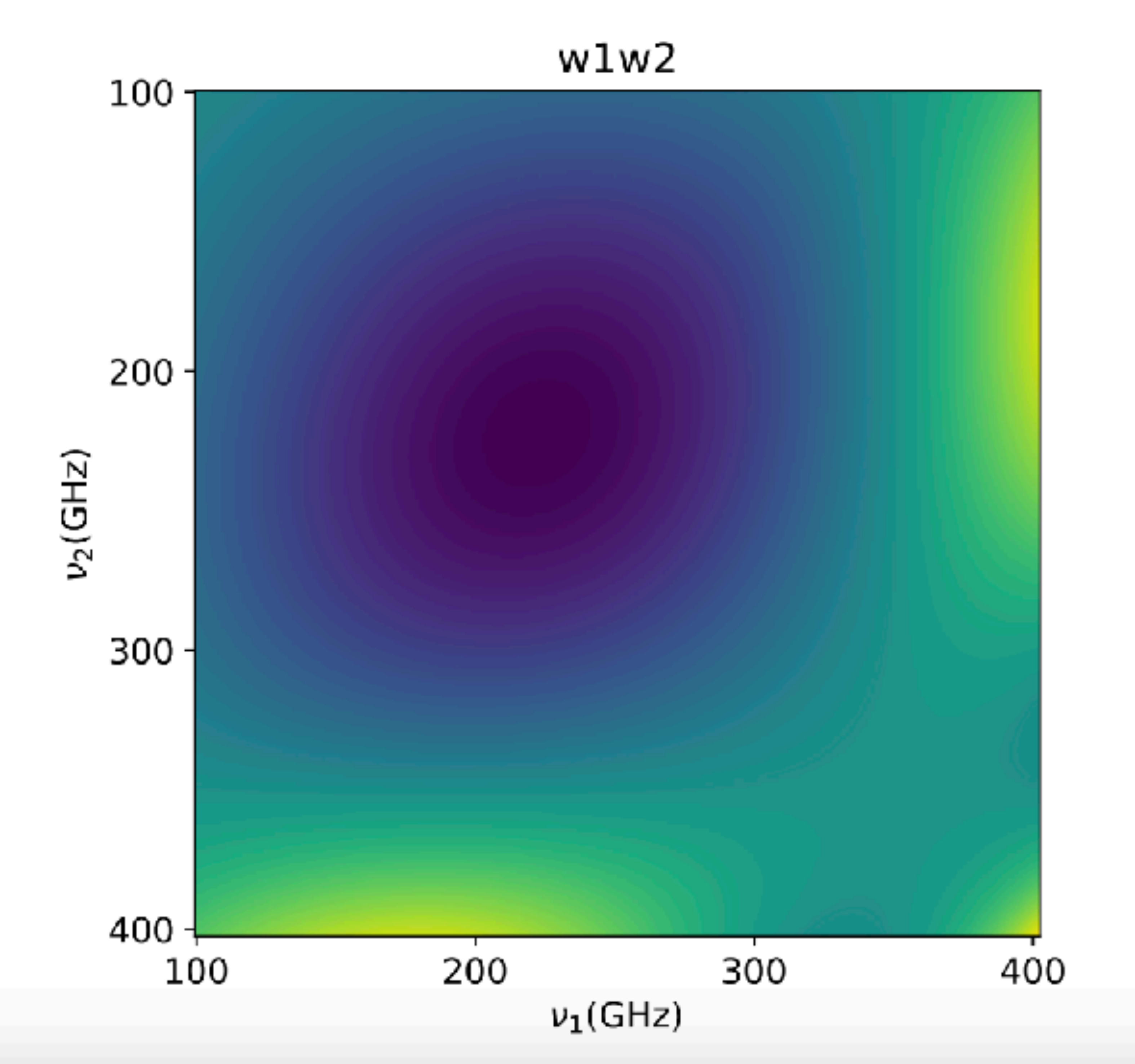
Thanks for listening !

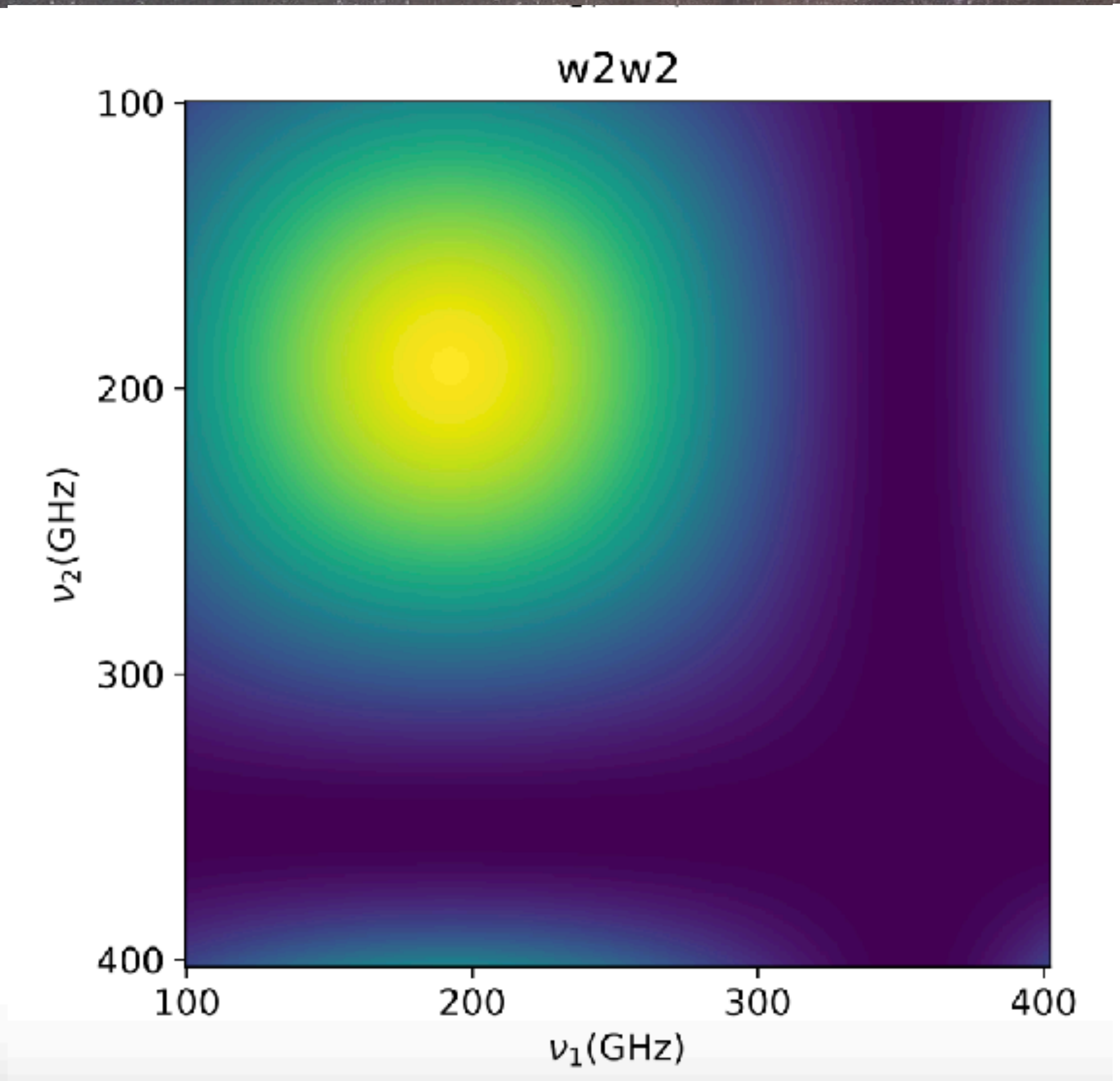
Any questions ?

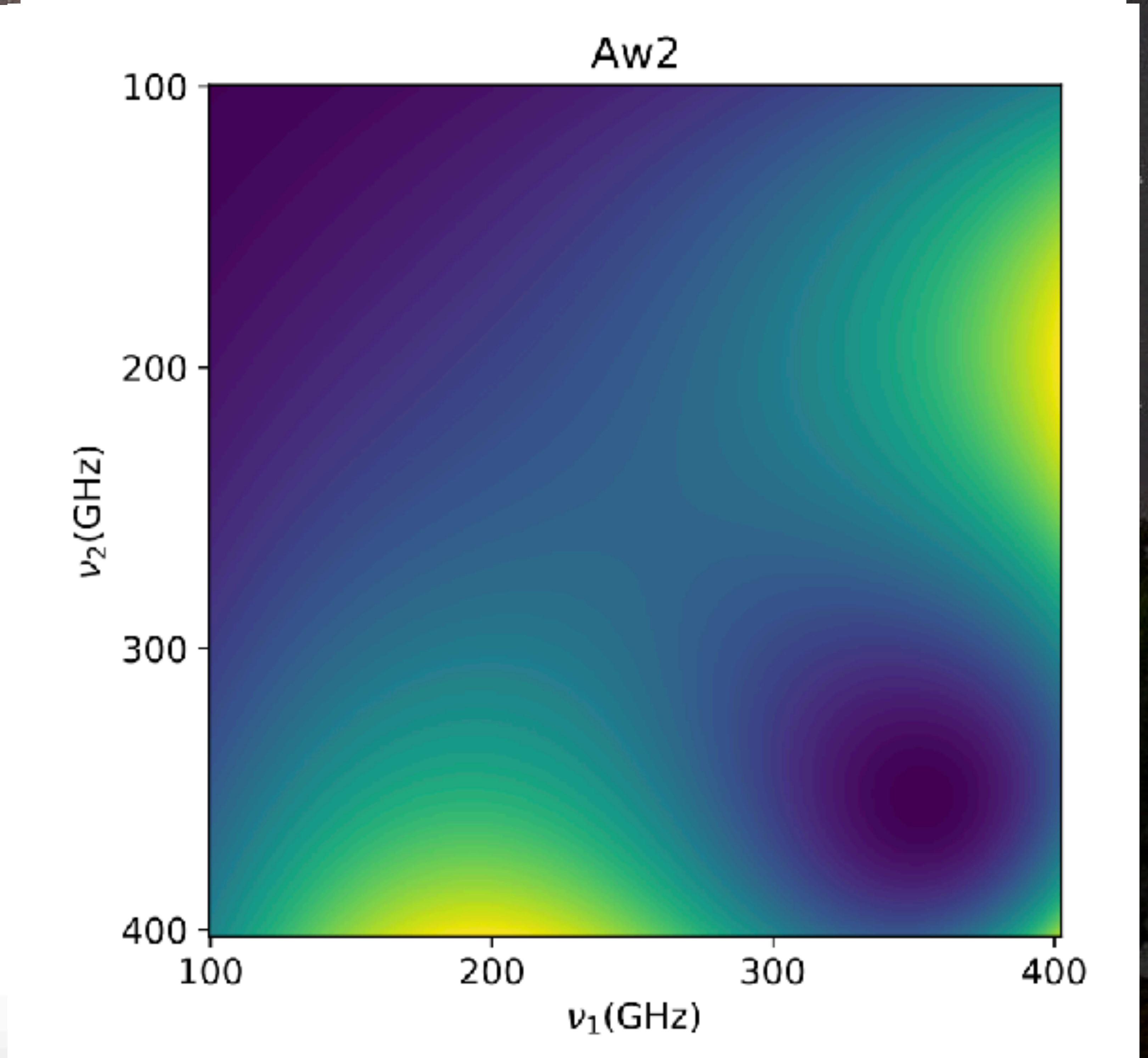
Correlations of order 1 coefficients with *r*

Correlations of order 2 coefficients with *r*

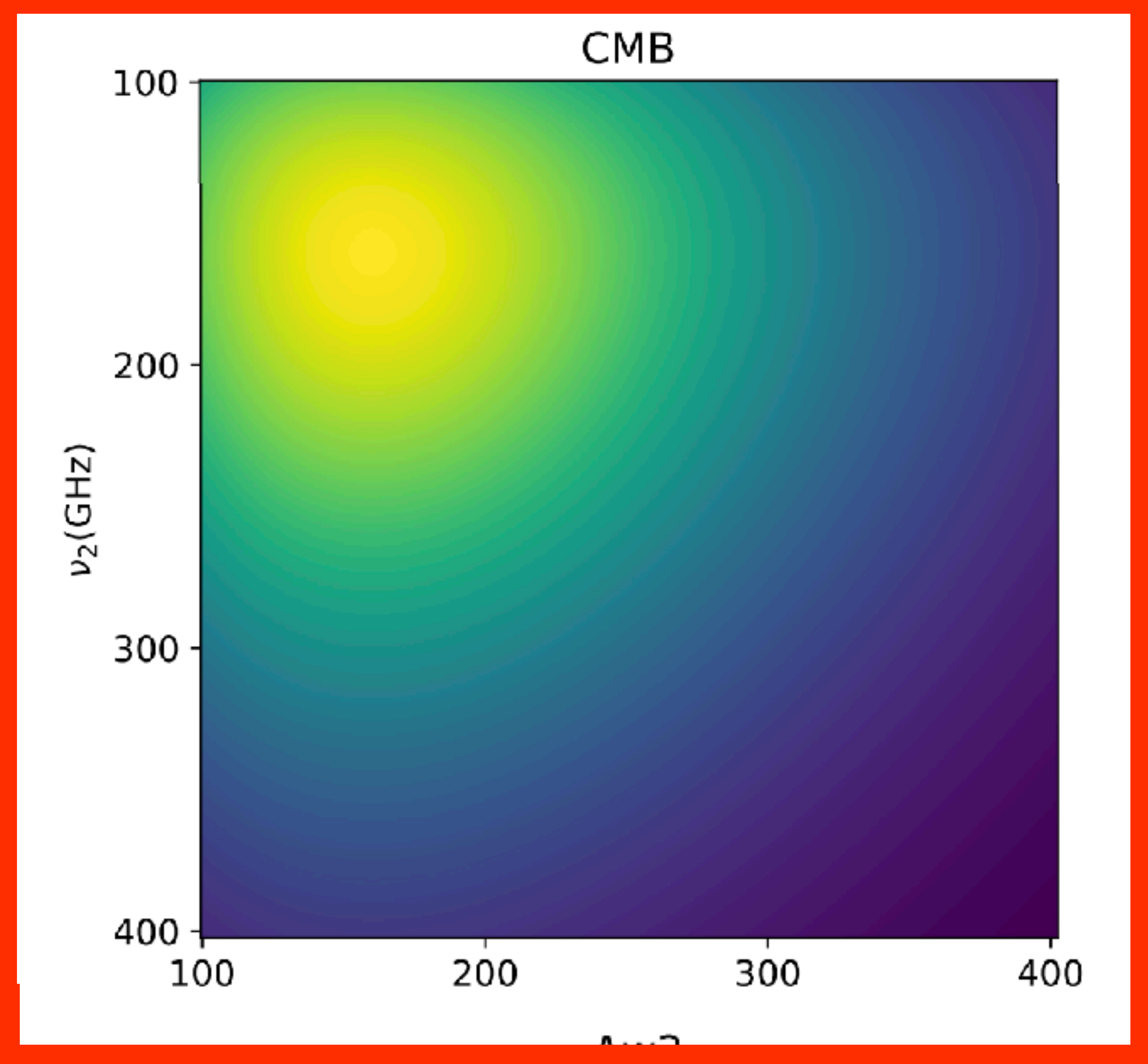


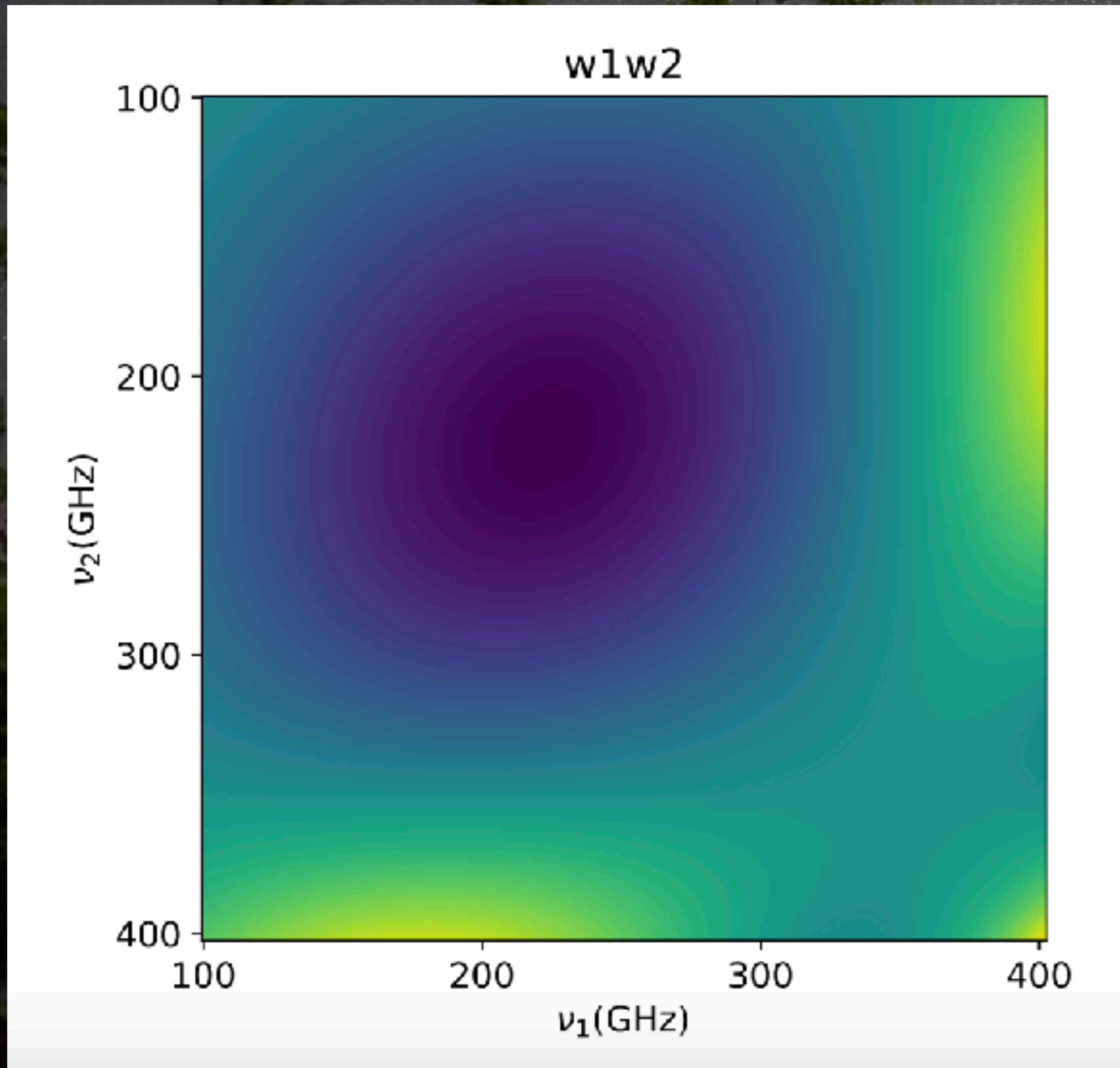


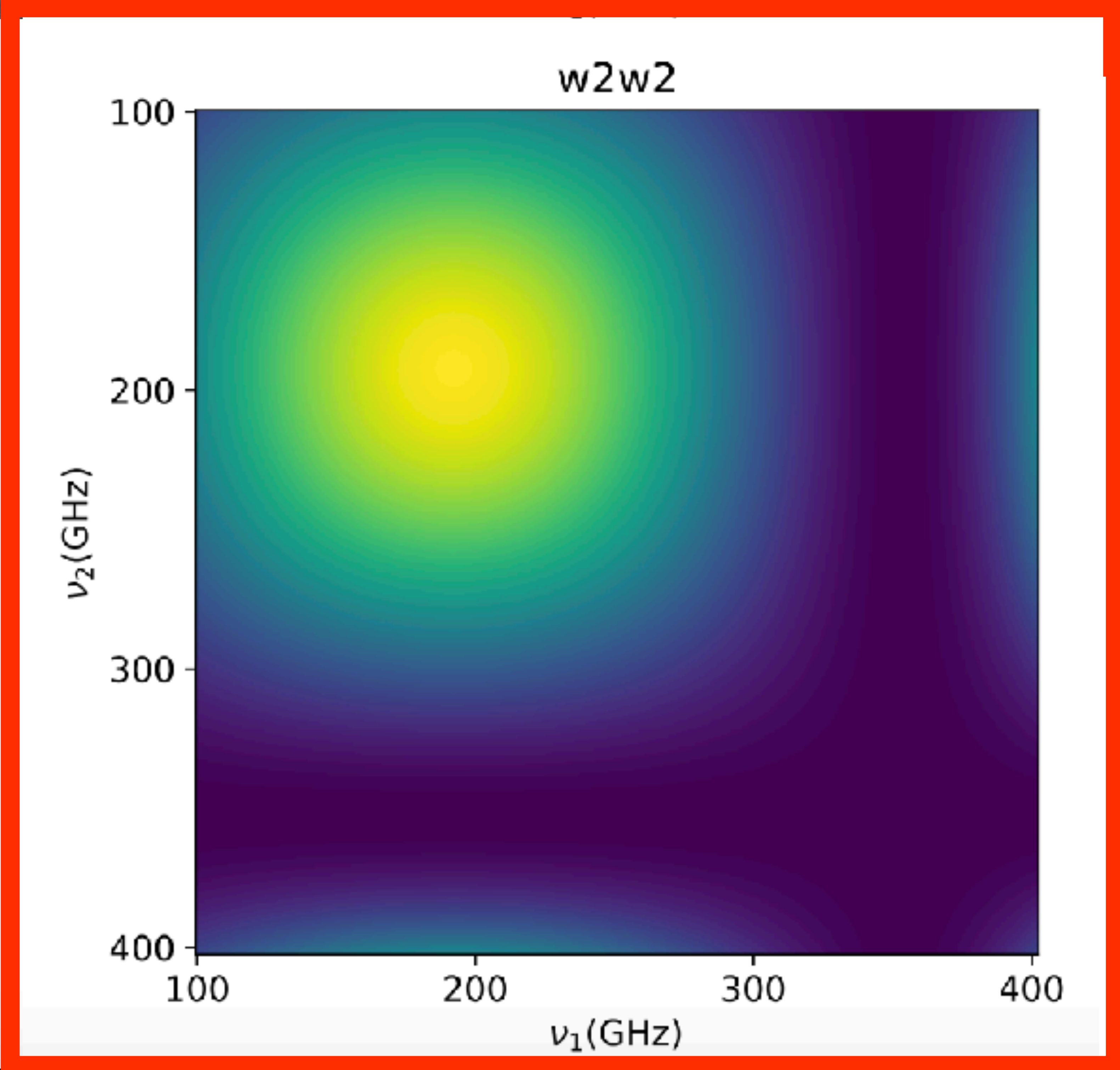


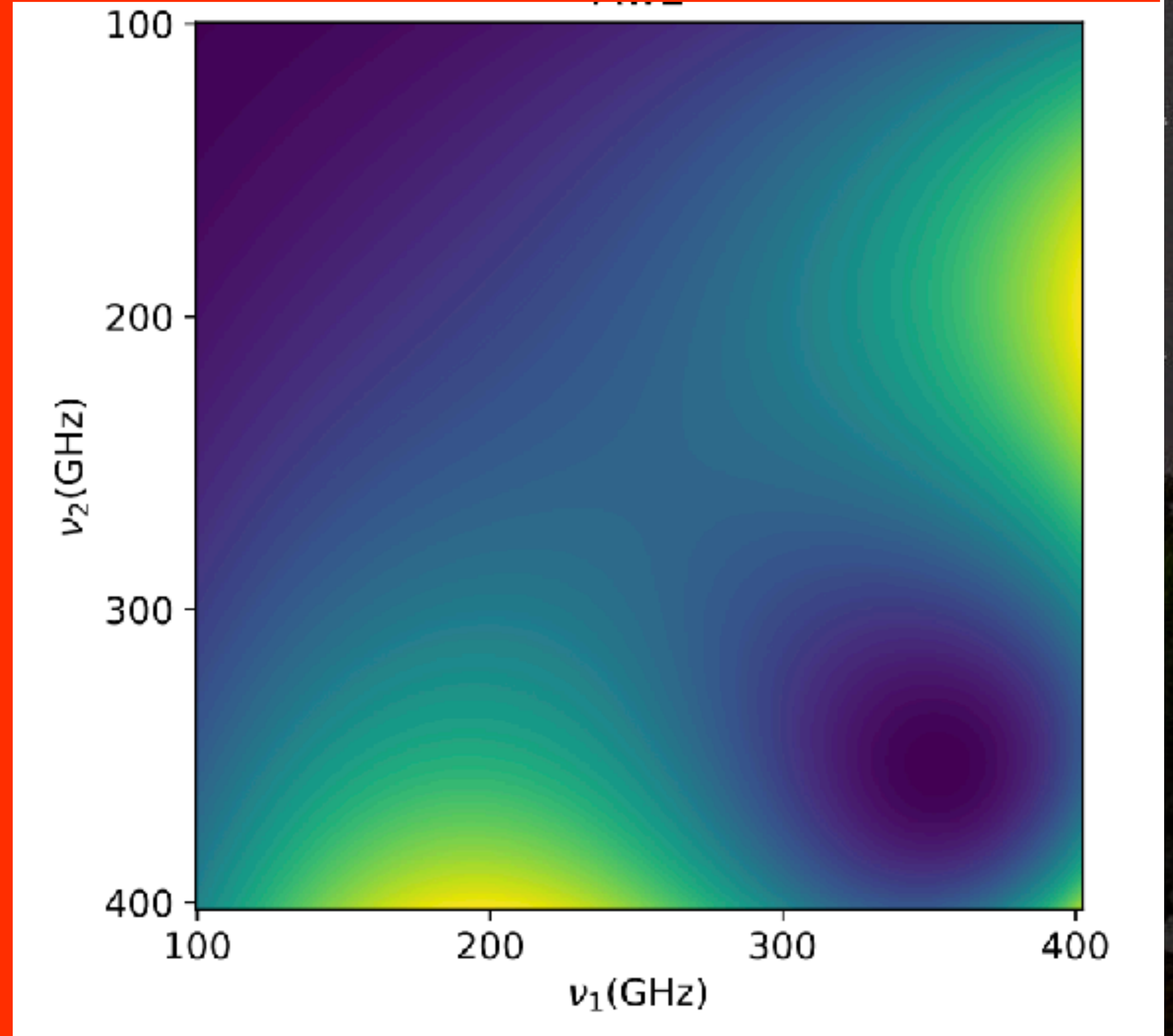


Correlations of order 2 coefficients with r









Correlations of order 2 terms with *r*

★ With LB freq. all order 2 terms are significantly correlated with the CMB

★ Correlation seems « universal » but exact value depends in a complex fashion on the instrumental parameters (sensitivity, frequency range ...)