Non-local form factors in $b \to s\ell\ell$

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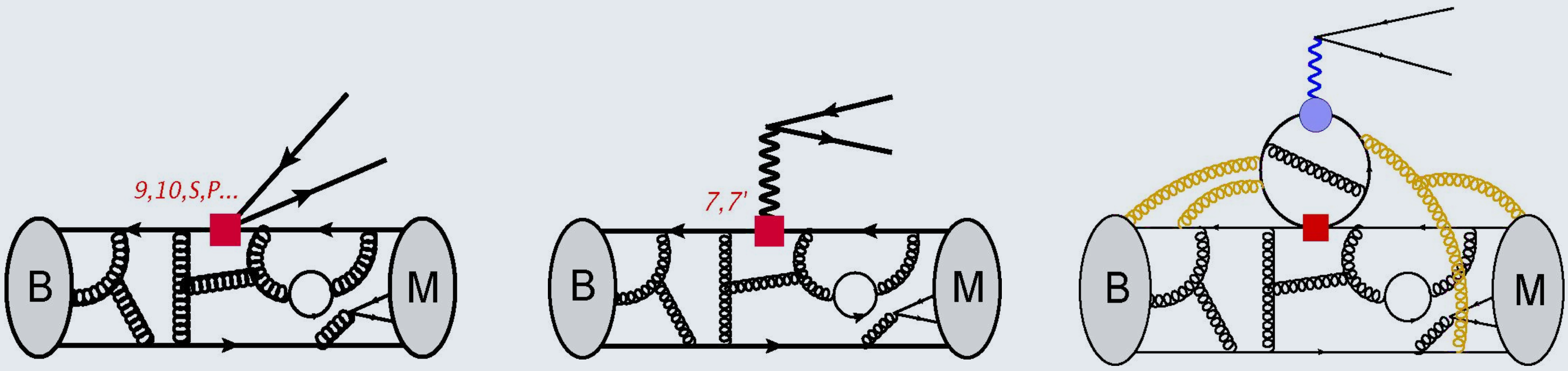

$$\mathcal{A}_\lambda^{L,R}(B \to M_\lambda \ell\ell) = \mathcal{N}_\lambda \left\{ (C_9 \mp C_{10}) \mathcal{F}_\lambda(q^2) + \frac{2 m_b M_B}{q^2} \left[C_7 \mathcal{F}_\lambda^T(q^2) - 16\pi^2 \frac{M_B}{m_b} \mathcal{H}_\lambda(q^2) \right] \right\}$$

Non-local form-factors

$$\mathcal{H}_\lambda(q^2) = i\mathcal{P}_\mu^\lambda \int d^4x\, e^{iq\cdot x} \langle \bar{M}_\lambda(k) | T\{\mathcal{J}^\mu_{\mathrm{em}}(x), \mathcal{C}_i \mathcal{O}_i(0)\} | \bar{B}(q+k) \rangle$$

→ Main contributions: O_1^c, O_2^c the so-called **“charm-loops”**

A few remarks

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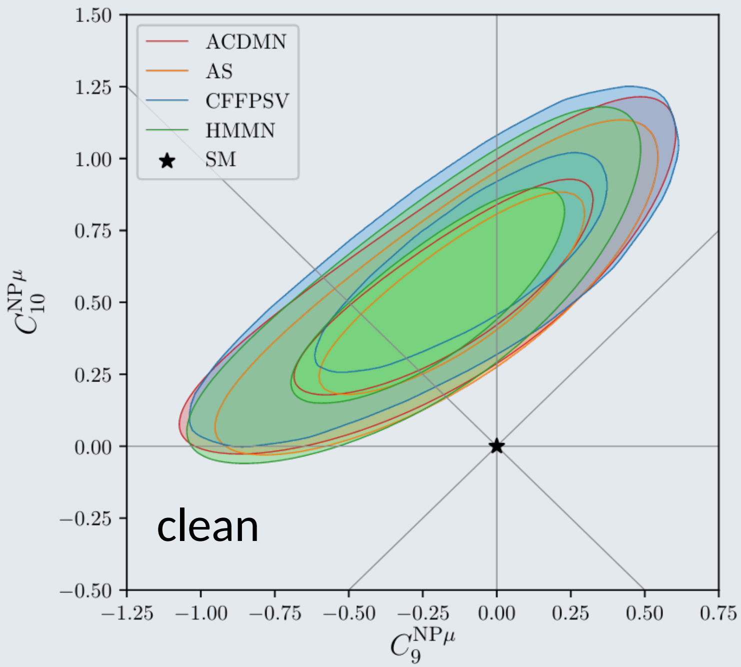
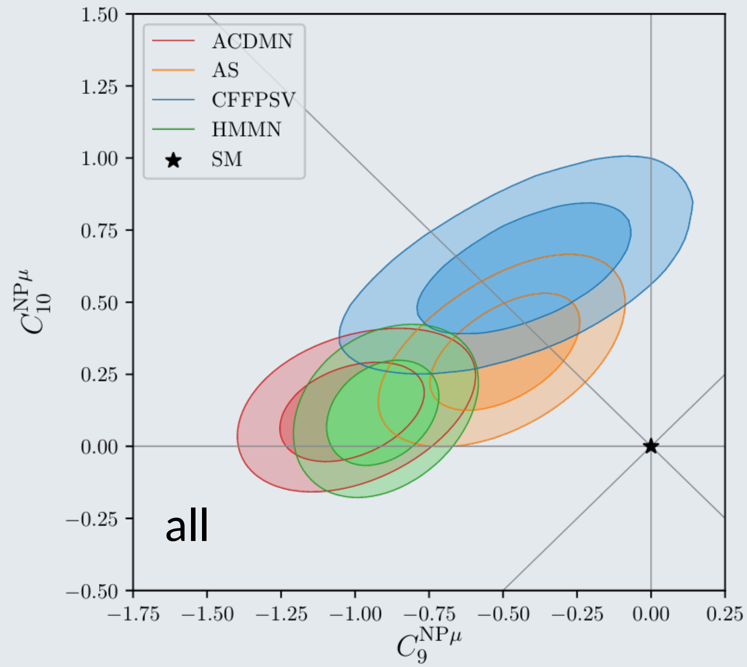
3. **Agreement** between "clean" and "not-so-clean" observables
   Charm-loops effects cannot be very large!





[Capdevila, Fedele, Neshatpour, Stangl, 2021 slides here]

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   → Significance of the C_9 vs. C_{10} fit rises from **~4σ** to **~8σ**!
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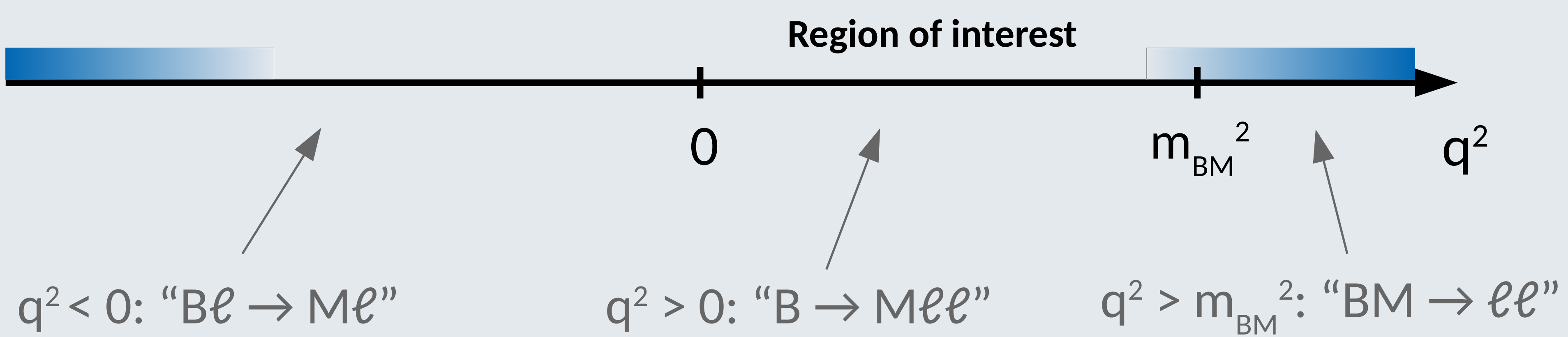
5. Theory **puzzles in $b \to s\bar{c}c$** [Lyon, Zwicky, 2014]

   We need to be careful…

Constraints on H_λ

1. Two types of **OPE** can be used for H_λ:
   - **Local** OPE $|q|^2 \gtrsim m_b^2$ [Grinstein, Piryol 2004][Beylich, Buchalla, Feldmann 2011]

     → We will discuss it later



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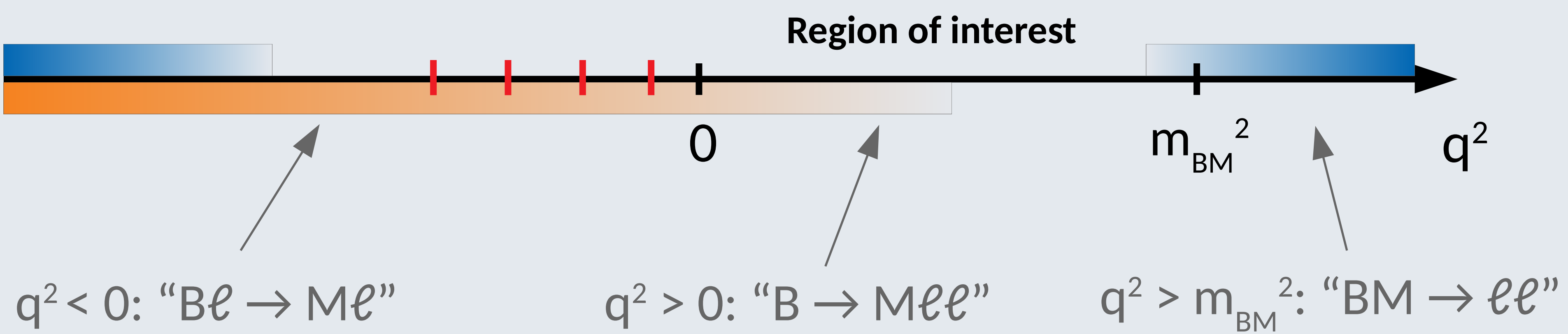
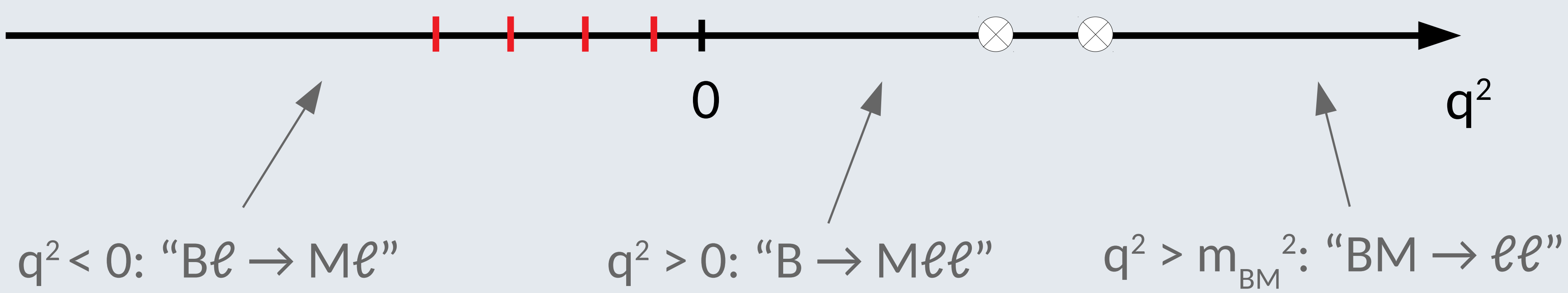
   - **Light Cone** OPE $q^2 \ll 4m_c^2$ [Khodjamirian, Mannel, Pivovarov, Wang 2010]

     → theory points at $q^2 < 0$ [Gubernari, van Dyk, Virto 2020]



Constraints on H_λ

2. Charmonium resonances:
   - H_λ presents poles at $q^2 = m_{J/\psi}^2$ and $m_{\psi(2S)}^2$
   - The residues are constrained by $B \to M\psi$



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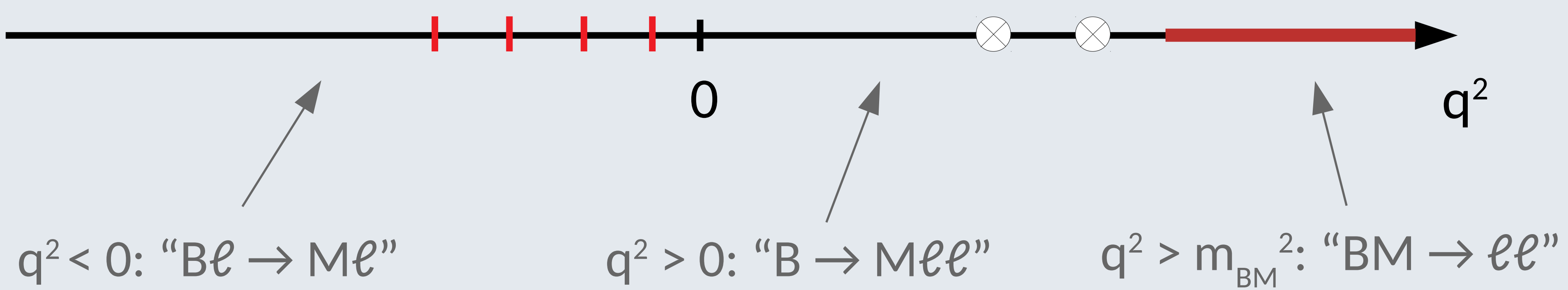
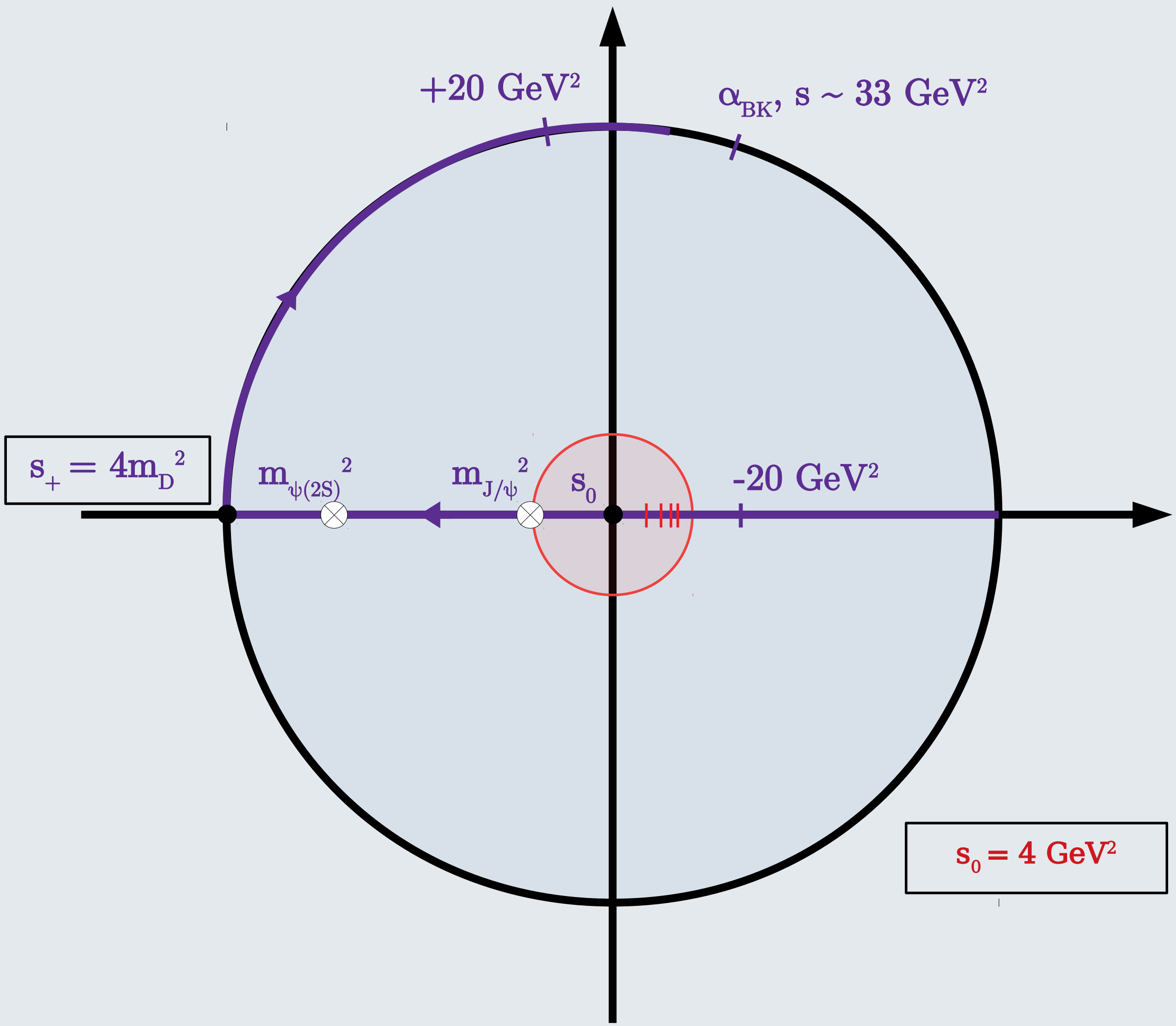
3. H_λ has a branch cut for $q^2 > 4m_D{}^2$



Parametrization of H_λ

- **z-mapping**

$$z(s) \equiv \frac{\sqrt{s_+ - s} - \sqrt{s_+ - s_0}}{\sqrt{s_+ - s} + \sqrt{s_+ - s_0}}$$



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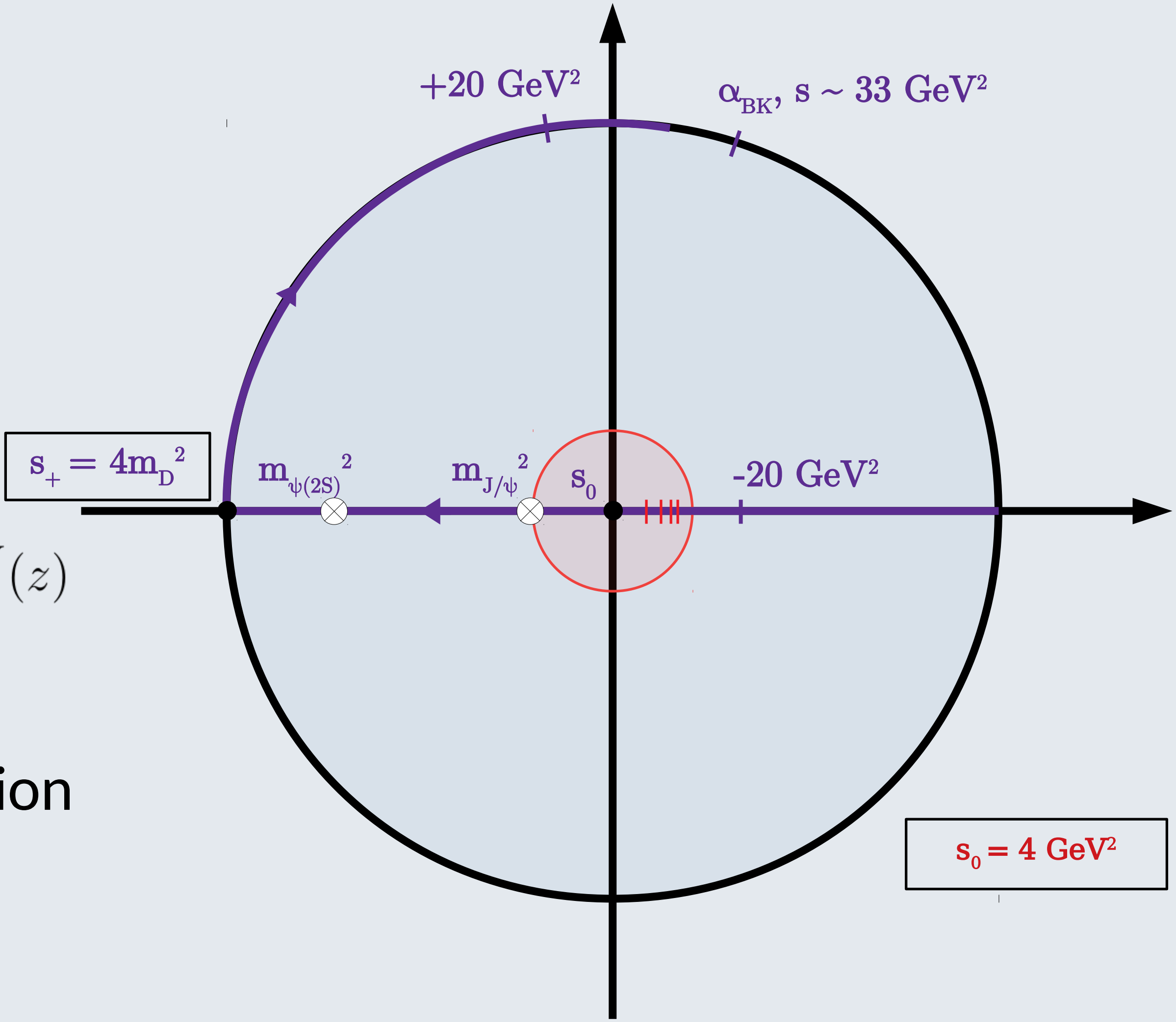
- **Analyticity**

$$\hat{\mathcal{H}}_\lambda^{B\to V}(z) \equiv \phi_\lambda^{B\to V}(z)\,\mathcal{P}(z)\,\mathcal{H}_\lambda^{B\to V}(z)$$

→ $\mathcal{P}(z)$ captures the poles

→ $\Phi(z)$ is a useful normalization



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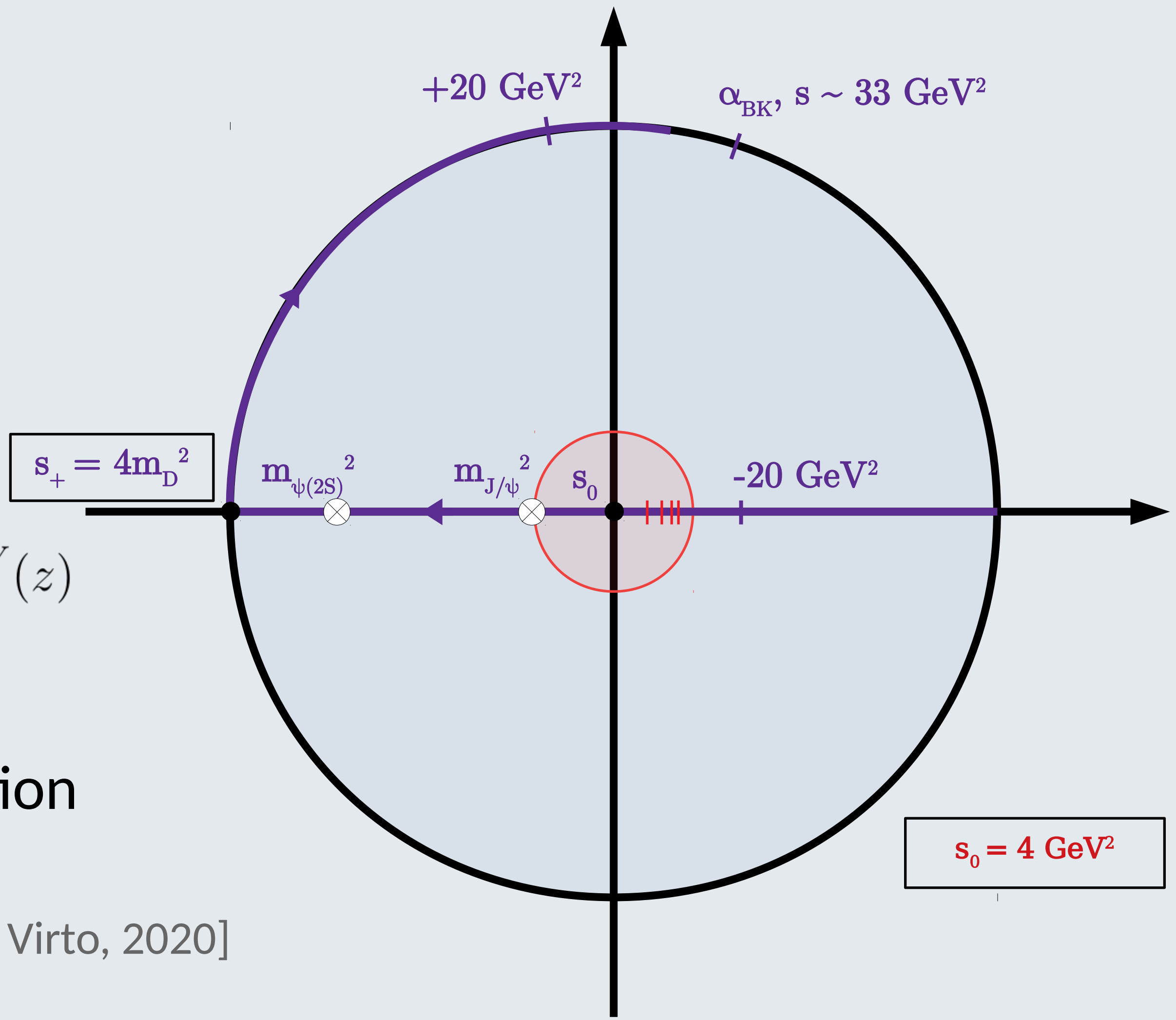
- **z-expansion** [Gubernari, van Dyk, Virto, 2020]

$$\hat{\mathcal{H}}_\lambda^{B\to M}(z) = \sum_{n=0}^{\infty} a_{\lambda,n}^{B\to M} p_n^{B\to M}(z)$$



$p_n(z)$: polynomial basis, e.g. z^n

[Bobeth, Chrzaszcz, van Dyk, Virto 2017]

Dispersive bound

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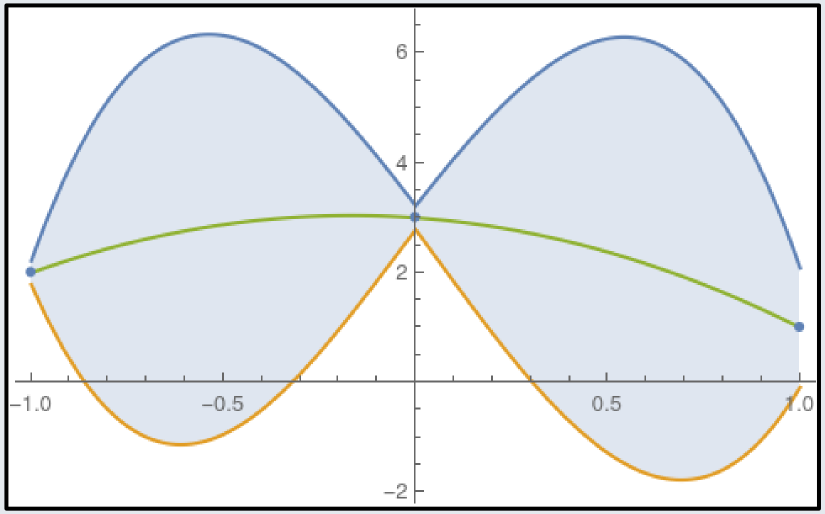
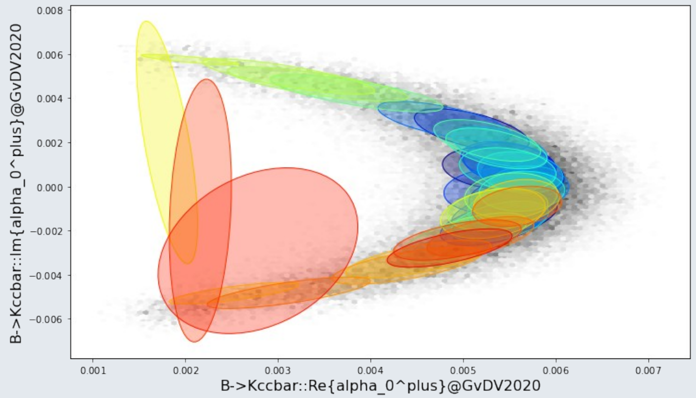
- If $p_n = z^n$, the **convergence is fast** ($z_{J/\psi} \sim -0.2$)
- **Dispersive bound** (from the local OPE)

$$\sum_{n=0}^{\infty} \left\{ 2\left|a_{0,n}^{B\to K}\right|^2 + \sum_{\lambda=\perp,\|,0} \left[2\left|a_{\lambda,n}^{B\to K^*}\right|^2 + \left|a_{\lambda,n}^{B_s\to\phi}\right|^2 \right] \right\} < 1$$

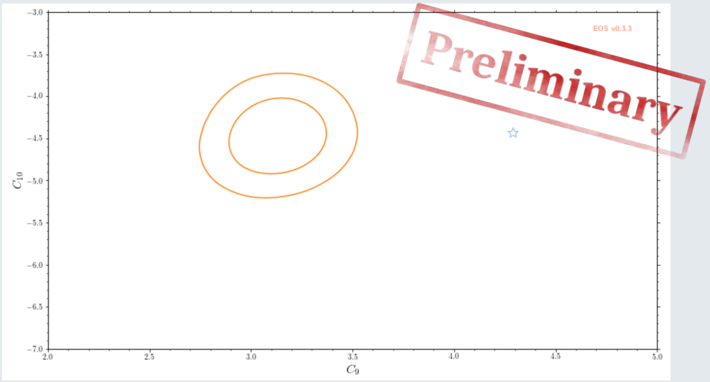


Global fit to $b \to s\ell\ell$

- The fit is performed in two steps…
  - Fit to the **local** and **non-local** form factors (described in the back-up)



  - Usual fit to the WET **Wilson coefficients**



- … using **EOS**:

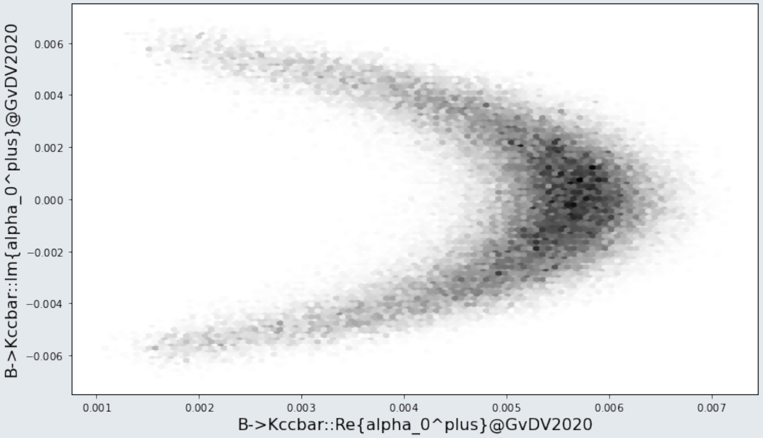
EOS is a software for a variety of applications in flavour physics. It is written in C++, but provides an interface to Python.

https://eos.github.io/

Back-up

Fit to local and non-local FFs

- **Local** and **non-local** form factors are fitted together to account for correlations (due to theory points at $q^2 < 0$)
- The posteriors **are not Gaussian distributed**...



- ...we described them as Gaussian mixture densities using **pypmc** [https://github.com/pypmc/pypmc]