

Extraction of $B_s \rightarrow D_s^{(*)} l \nu$ form factors from lattice QCD

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- Phenomenological context
- Our strategy
- Set-up and results

[B. B., P. H. Cahue, J. Heitger, S. La Cesa, J. Neuendorf, S. Zafeiropoulos, arXiv:2110.10061]

Phenomenological contexts

Lepton Flavour Universality tests are now extensively performed to reveal possible tensions with Standard Model expectations.

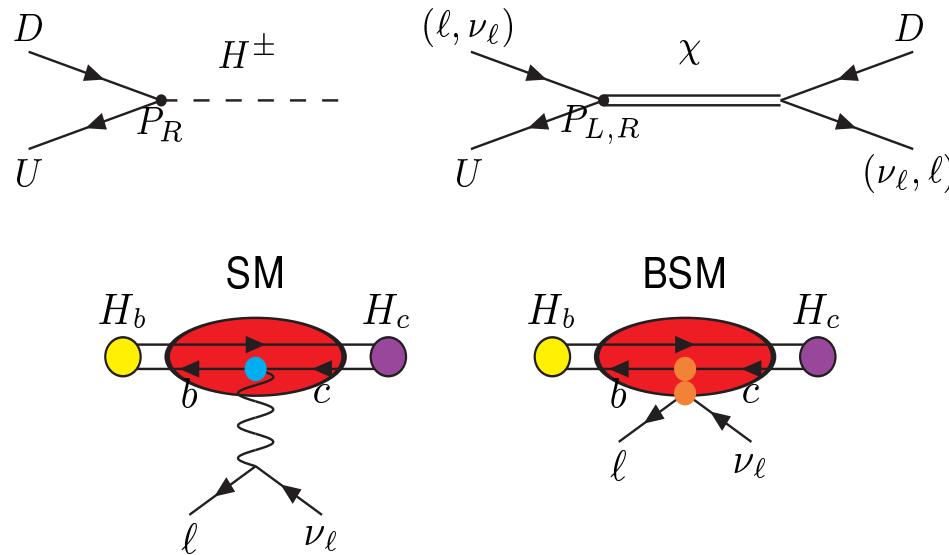
FCCC $b \rightarrow c$: $R_{D^{(*)}} \equiv \frac{\Gamma(B \rightarrow D^{(*)}\tau\nu_\tau)}{\Gamma(B \rightarrow D^{(*)}\ell\nu_\ell)}$, $\ell = e, \mu$

Discrepancy of $\sim 2\sigma$ and $\sim 3\sigma$ for R_D and R_{D^*} with SM expectations

Spectator quark $u, d \rightarrow c$: $R_{J/\psi}$: $\sim 1\sigma$ discrepancy

$u, d \rightarrow s$: $R_{D_s^{(*)}}$: ??

BSM contributions?



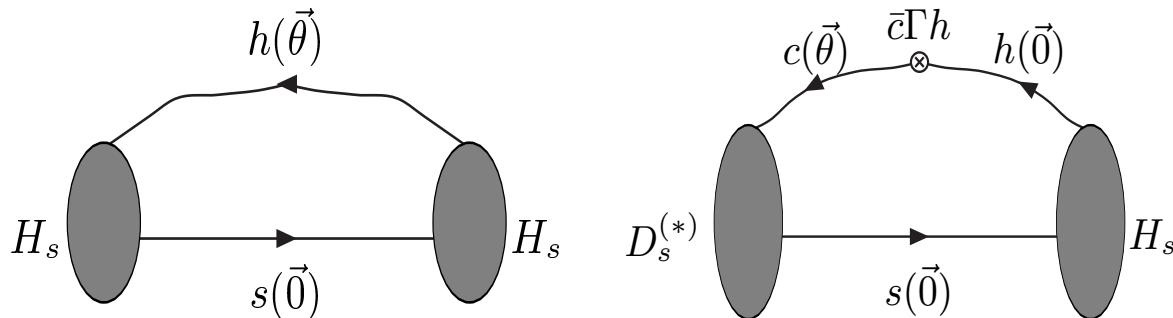
Form factors of $B_c \rightarrow J/\psi l\nu_l$, $B_s \rightarrow D_s^{(*)} l\nu_l$. Lattice QCD

Our strategy

$$\langle D_s(p_{D_s})|V^\mu|B_s(p_{B_s})\rangle = A(p_{B_s}, p_{D_s}) \mathcal{G}(w) + B(p_{B_s}, p_{D_s}) \mathcal{f}_0(w), w = \frac{p_{B_s} \cdot p_{D_s}}{m_{B_s} m_{D_s}}$$

$$\langle D_s^*(\epsilon^{(\lambda)}|A^i|B_s)\rangle(w=1) \propto h_{A_1}(w=1)$$

Computation of 2-pt and 3-pt correlation functions on the lattice.



$$C_{JJ}^{(2)}(\vec{q}, t) = \sum_{\vec{x}} e^{-i\vec{q}\cdot\vec{x}} \langle \Omega | O_{(J)}(\vec{x}, t) O_{(J)}^\dagger(0) | \Omega \rangle = \sum_n \frac{\mathcal{Z}_{J,n}^2 e^{-E_{J,n}(\vec{q}) t}}{2E_{J,n}(\vec{q})}$$

$$\xrightarrow{(E_{J,1}(\vec{q}) - E_{J,0}(\vec{q}))t \gg 1} \frac{\mathcal{Z}_{J,0}^2 e^{-E_{J,0}(\vec{q}) t}}{2E_{J,0}(\vec{q})}$$

$$\mathcal{Z}_{J,n} = \langle \Omega | O_{(J)} | n, J \rangle, \quad \langle n, J, P | m, J', P' \rangle = 2E_{J,n}(\vec{P}) \delta_{mn} \delta_{PP'} \delta_{JJ'}$$

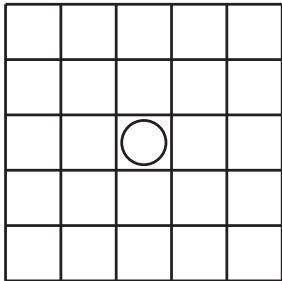
$$C_{J_1, J_2, O_\Gamma, \vec{q}, \vec{q}'}^{(3)}(t_x, t_y) = \sum_{\vec{x}, \vec{y}} e^{-i[(\vec{q}-\vec{q}')\cdot\vec{x} + \vec{q}'\cdot\vec{y}]} \langle \Omega | O_{(J_2)}(\vec{y}, t_y) O_\Gamma(\vec{x}, t_x) O_{(J_1)}^\dagger(0) | \Omega \rangle$$

$$= \sum_{n,m} \frac{\mathcal{Z}_{J_1,n} \mathcal{Z}_{J_2,m} e^{-E_{J_2,m}(\vec{q}') (t_y - t_x)} e^{-E_{J_1,n}(\vec{q}) t_x}}{2E_{J_1,n}(\vec{q}) 2E_{J_2,m}(\vec{q}')} \times \langle m, J_2, q' | \hat{O}_\Gamma | n, J_1, q \rangle$$

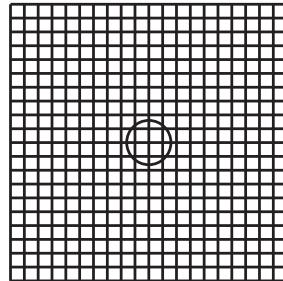
$$\xrightarrow{t_x, t_y - t_x \gg a} \frac{\mathcal{Z}_{J_1,0} \mathcal{Z}_{J_2,0}}{2E_{J_1,0}(\vec{q}) 2E_{J_2,0}(\vec{q}')} e^{-E_{J_1,0}(\vec{q}) t_x} e^{-E_{J_2,0}(\vec{q}') (t_y - t_x)} \times \langle 0, J_2, \vec{q}' | \hat{O}_\Gamma | 0, J_1, \vec{q} \rangle$$

Heavy quark on the lattice

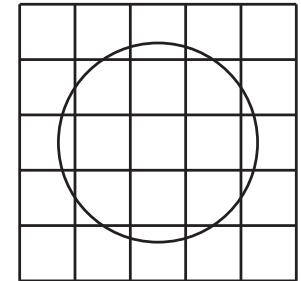
Systematics coming from potentially large discretisation effects ($\Lambda_{\text{Compt}} \sim 1/m_Q$).



Cut-off Effects



cut-off effects



cut-off effects

Our approach: Step Scaling in masses to extrapolate results at m_b , using Heavy Quark Symmetry [B. B. *et al*, '09, M. Atoui *et al*, '13]

$$A(m_{B_s}) = A(m_{D_s}) \times \frac{A(m_{H_{1s}})}{A(m_{D_s})} \times \frac{A(m_{H_{2s}})}{A(m_{H_{1s}})} \times \cdots \times \frac{A(m_{B_s})}{A(m_{H_{Ks}})}$$

$$A(m_{H_s}) = \mathcal{G}^{H_s \rightarrow D_s}(w=1), h_{A_1}^{H_s \rightarrow D_s^*}(w=1) \quad \mathcal{G}^{D_s \rightarrow D_s}(w=1) = 1$$

$$\sigma_i = \frac{A(m_{H_{(i+1)s}})}{A(m_{H_{(i)s}})} \quad \frac{m_{H_{(i+1)s}}}{m_{H_{(i)s}}} = \left(\frac{m_{B_s}}{m_{D_s}} \right)^{1/(K+1)} \equiv \lambda$$

Set-up and results

Lattice set-up: $\mathcal{O}(a)$ improved Wilson-Clover, $N_f = 2$

CLS
based

lattice	β	$L^3 \times T$	$a[\text{fm}]$	$m_\pi[\text{MeV}]$	Lm_π
A5	5.3	$32^3 \times 64$	0.075	330	4
B6		$48^3 \times 96$		280	5.2
E5	5.3	$32^3 \times 64$	0.065	440	4.7
F6		$48^3 \times 96$		310	5
F7		$48^3 \times 96$		270	4.3
G8		$64^3 \times 128$		190	4.1
N6	5.5	$48^3 \times 96$	0.048	340	4
O7		$64^3 \times 128$		270	4.2

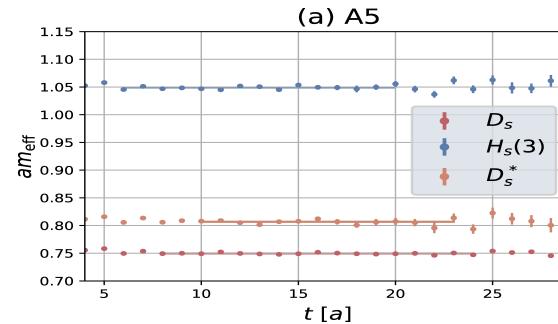
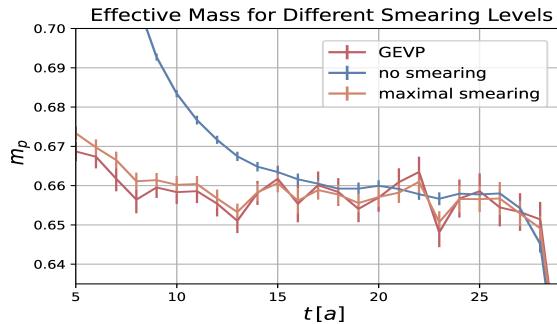
Use strange and charm quark masses determined at each ensemble from m_K^2/f_K^2
 [P. Fritzsch *et al*, '12] and m_{D_s} [J. Heitger *et al*, '13; R. Balasubramanian and B. B., '20].

Heavy quark masses tunes such that $m_{H_{(i)s}} = \lambda^i m_{D_s} \quad m_{B_s} = \lambda^6 m_{D_s}$

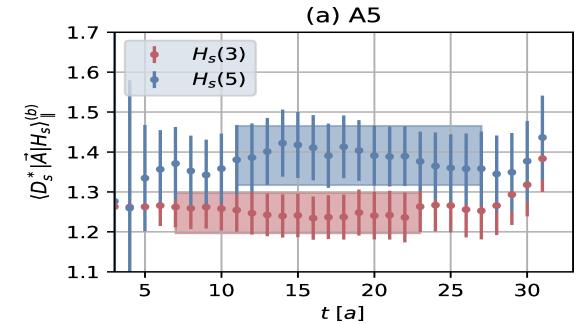
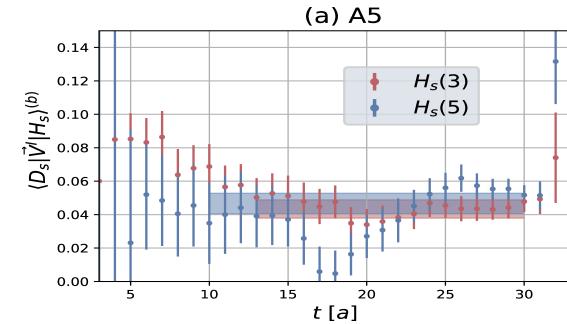
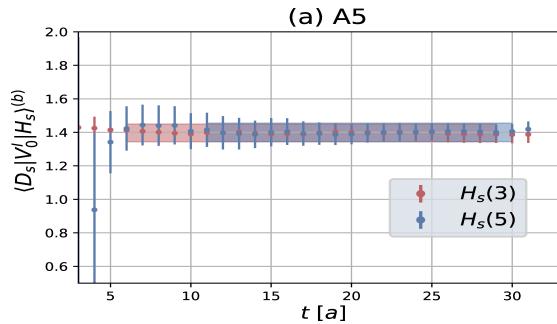
Different smearing levels $O^S(\vec{x}, \vec{y}, t) = \bar{q}_1(\vec{x}, t)\Phi^S(\vec{x} - \vec{y}, t)\Gamma q_2(\vec{y}, t)$

Find optimised linear combination of interpolating fields to improve the overlap with ground states

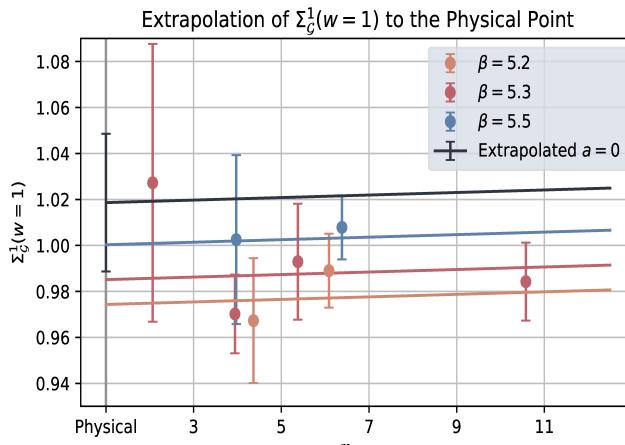
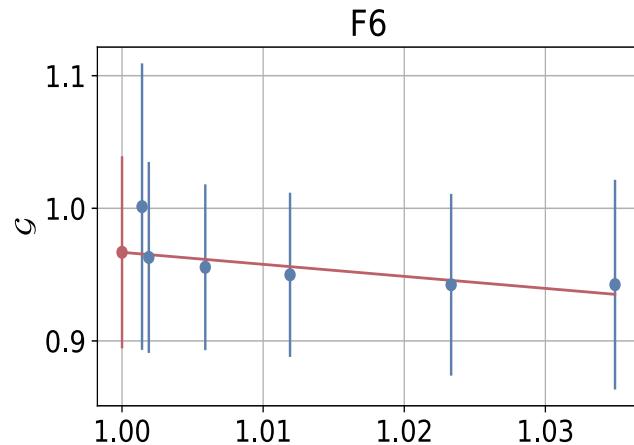
Our way to isolate ground states is reliable.



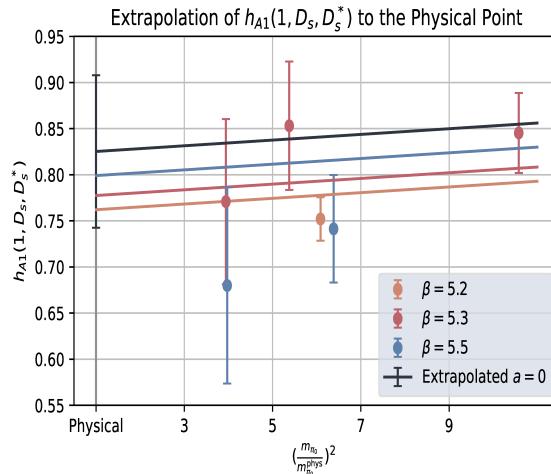
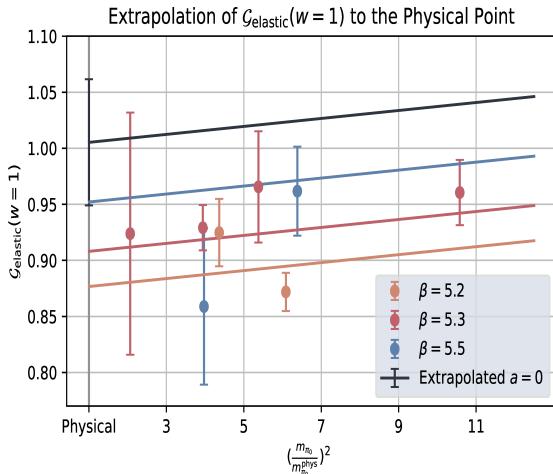
Plateaus for $\langle D_s | V^0 | H_s \rangle$ are OK, plateaus for $\langle D_s | V^i | H_s \rangle$ and $\langle D_s^* | \vec{A} | H_s \rangle_{||}$ are of lower quality.



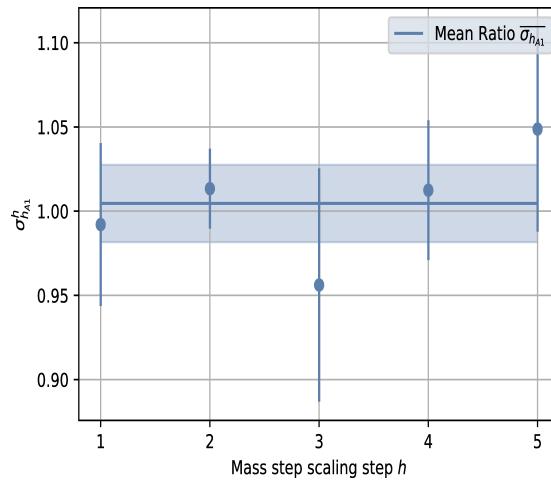
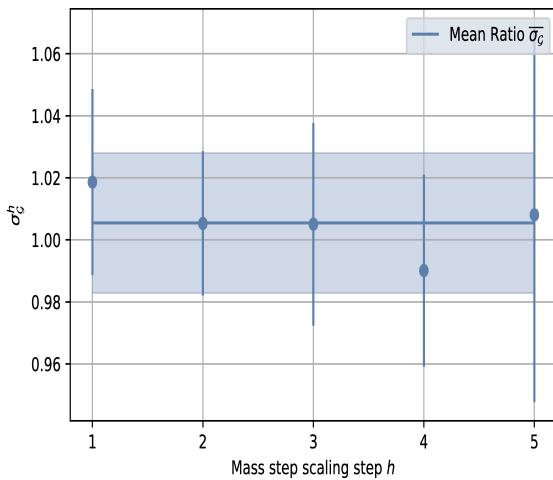
Contrary to $h_{A_1}(1)$, $\mathcal{G}(1)$ not directly accessible: extrapolation in $(w - 1)$. Extrapolation to the physical point of the Step Scaling in masses $\Sigma_{\mathcal{G}(1)}$ and $\Sigma_{h_{A_1}(1)}$



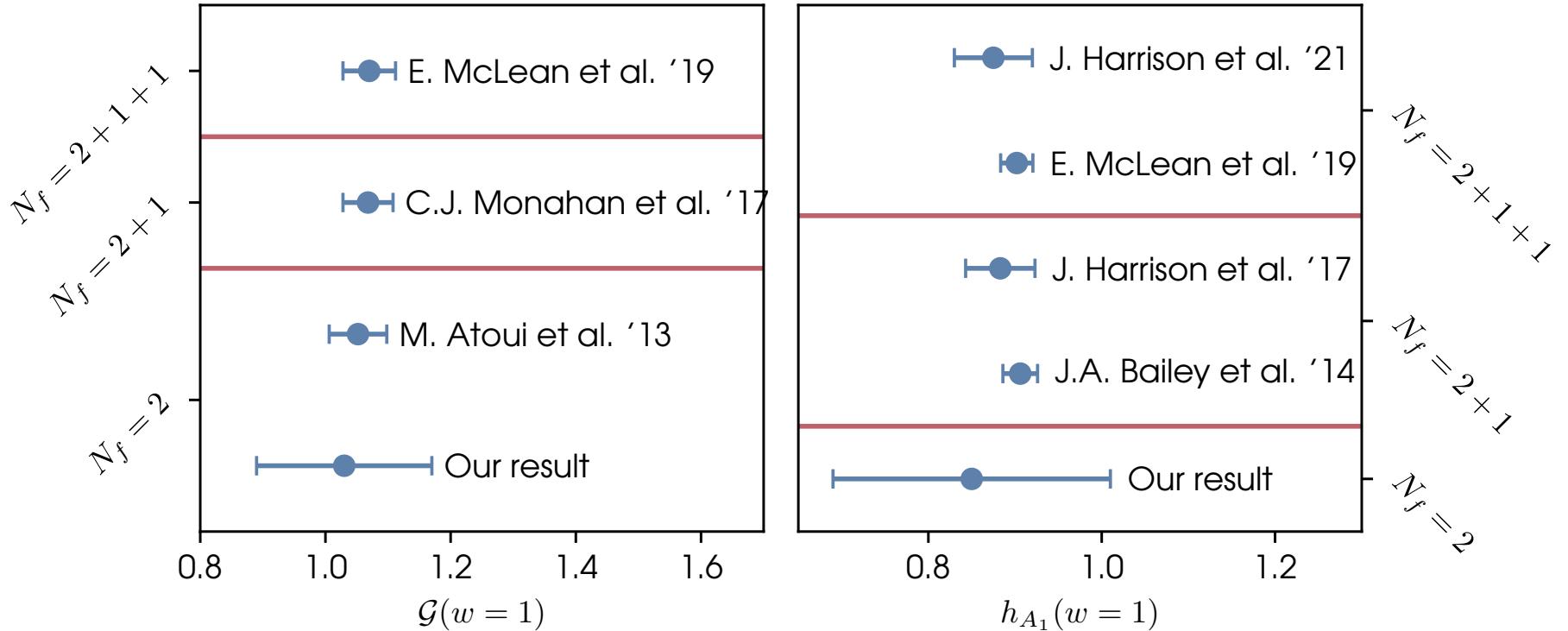
Elastic point: $\mathcal{G}(1)$ compatible with 1, h_{A_1} clearly below 1 (no normalisation from HQS)



Unfortunately the m_{H_s} dependence of $\sigma_{\mathcal{G}(1)}$ and $\sigma_{h_{A_1}(1)}$ is hardly seen.



$$G^{B_s \rightarrow D_s}(1) = \bar{\sigma}_{\mathcal{G}}^6 = 1.03(14) \quad h_{A_1}^{B_s \rightarrow D_s^*}(1) = h_{A_1}^{\text{elastic}}(1) \times \bar{\sigma}_{h_{A_1}(1)}^6 = 0.85(16)$$



The elastic point governs $\mathcal{G}^{B_s \rightarrow D_s}(1)$ and $h^{B_s \rightarrow D_s^*}(1)$.

Satisfying control on cut-off effects and contamination from excited states.

Linear extrapolation in $(w - 1)$ is enough to extract $\mathcal{G}(1)$.

Large statistical uncertainty: (too?) conservative choice of source-sink separation > 2 fm in 3-pt correlators.

The most smeared interpolating field is ultra-dominant in the operator basis: possible gain in statistics.

Outlook: computation on ensembles with $2 + 1$ dynamical quarks at the physical point.

$100^3 \times 200$ lattices: 500 Mch required in terms of computer time. Alternative strategy to simulate the b quark? Port codes on GPUs?