Relation among Lepton Flavour Violations

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Outline

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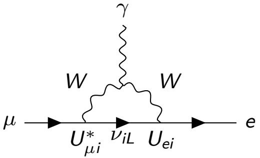
Charged Lepton Flavour Violation

Leptons:

$$\ell_\alpha = \begin{pmatrix} \nu_{\alpha L} \\ \alpha_L \end{pmatrix}, \alpha_R \qquad \text{with } \alpha = e, \mu, \tau \tag{1}$$

The Standard Model with $m_{\nu_\alpha} = 0$ has exact $U(1)_{B/3-L_\alpha}$

...but Neutrino masses break all three symmetries.
Charged Lepton Flavour Violation (CLFV)≡ short-range interaction between the charged leptons that violates LF.



CLFV assuming Dirac neutrino masses is small
$Br(\mu \to e\gamma) \simeq G_F^2 m_\nu^4 \sim 10^{-50}$

Smoking gun signal of New Physics

Experimental searches

The experimental sensitivities on LFV processes are impressive and will improve

$$Br(\mu \to e\gamma) < 4.2 \times 10^{-13} \to 6 \times 10^{-14}$$ A. M. Baldini et al., arXiv:1801.04688

$$Br(\mu \to \bar{e}ee) < 1.0 \times 10^{-12} \to 10^{-16}$$ A. Blondel et al., arXiv:1301.6113

$$Br(\mu A \to eA) < 7 \times 10^{-13} \to 10^{-16} - 10^{-18}$$ R. M. Carey et al., FERMILAB-PROPOSAL-0973

$$Br(\tau \to \bar{\ell}\ell\ell) < 2 \times 10^{-8} \to 10^{-9}$$ E. Kou et al., arXiv:1808.10567

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The possibility of very intense muon beam account for better sensitivities:

$$Br(\tau \to \ell)^2 \simeq Br_{future}(\mu \to e) \tag{2}$$

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Suppose $\mu \leftrightarrow \tau$ and $\tau \leftrightarrow e$ happen.

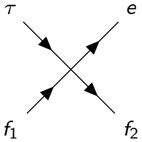
$\mu \to e$ can be radiatively generated out of $\mu \to \tau \times \tau \to e$

Can $\mu \to e$ teach us something about $\tau \leftrightarrow \ell$ LFV?

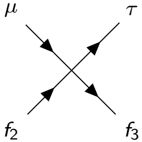
Outline

Effective Field Theory (EFT)

Assume that New Physics responsible for LFV is heavy $\Lambda \gtrsim 4$ TeV



$\sim 1/\Lambda^2$



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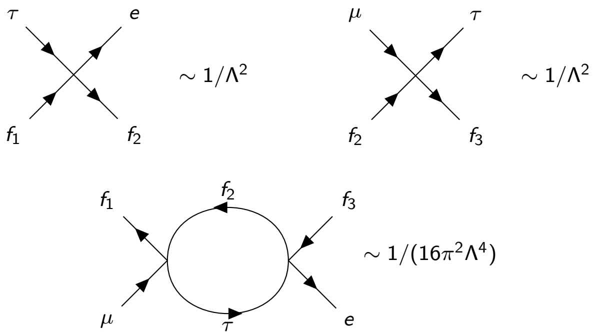
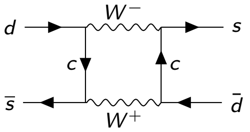


In the EFT the effect appears at dimension eight: $\mu \to e$ observables are sensitive to some $1/\Lambda^4$ contributions MA, S. Davidson, JHEP 08 2021, 2 (2021)

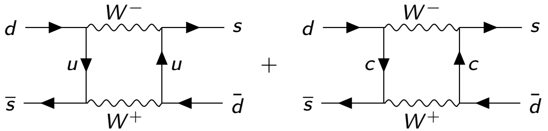
Dimension eight might be a subleading contribution of $\mu \to \tau \times \tau \to e$.

Boxes match onto dimension six



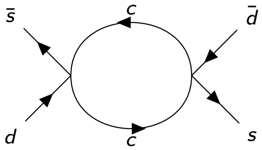
$\sim G_F^2 M_W^2 \sim 1/M_W^2$

In SM this cancels against the up diagram (GIM)



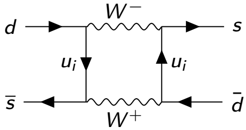
$\sim G_F^2 m_c^2$

We are left with the "fish" diagram



Including the top, GIM applies due to CKM unitarity

$$\sum_{i=u,c,t}$$



$$\sim (V_{cs}^* V_{cd})^2 G_F^2 m_c^2 + (V_{ts}^* V_{td})^2 G_F^2 m_t^2$$

The dimension six $G_F^2 m_t^2$ is suppressed by small mixing with the third generation.

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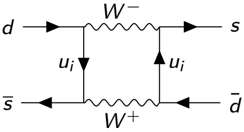

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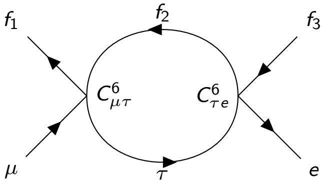
The dimension six $G_F^2 m_t^2$ is suppressed by small mixing with the third generation.

Dimension six $\mu \to \tau \times \tau \to e$ contributions are model dependent.

Our calculations provide model independent relation between LFVs.

The EFT calculation

Divergent diagrams with two dimension six $\mu \to \tau \times \tau \to e$ insertion renormalize dimension eight $\mu \to e$ operators



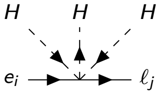
$$C^8_{e\mu}(E) \sim \frac{C^6_{\tau\mu} C^6_{e\tau}}{16\pi^2} \log\left(\frac{\Lambda}{E_{exp}}\right) \qquad E_{exp} \equiv \text{exp. scale}$$

- We only calculate contributions that are (estimated to be) within exp. sensitivities.
- We compute the $(dim6)^2$ one-loop running in the Standard Model EFT ($E > M_W$)
- SMEFT operator list is reduced using the Equations of Motion (EoM)
  EoM get corrections from dimension six operators: need to take it into account when considering EFT up to $1/\Lambda^4$ order.

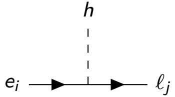
The EFT calculation

The dimension eight operators generated in the $\Lambda \to M_W$ running are matched onto low energy contact interactions at $E = M_W$ (EFT with W, Z, h, t integrated out).

Tree-level matching contributions from two dimension six operator insertion when $\langle H \rangle = v = 174$ GeV:

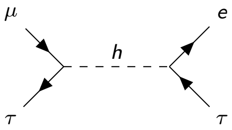


$= \frac{C^{ji}_{eH}}{\Lambda^2}(\bar{\ell}_j H e_i)(H^\dagger H)$



$\propto C^{ji}_{eH}\frac{v^2}{\Lambda^2}$

Match onto scalar and tensor $\mu e \tau\tau$ operators

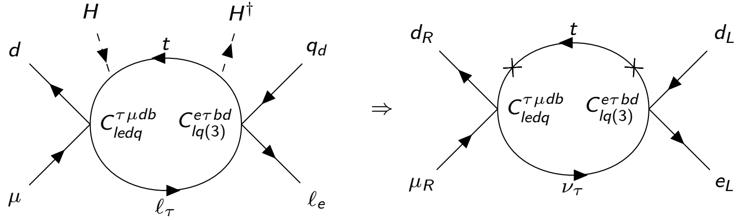


Low energy operators run from M_W to $E_{exp} \longrightarrow$ Experiment constrain $C^6_{\tau\mu} C^6_{e\tau}$.

We neglect $(dim6)^2$ running below M_W.

Outline

Take $\mathcal{O}_{ledq} = (\bar{\ell}_\tau \mu)(\bar{d} q_b)$, $\mathcal{O}_{lq(3)} = (\bar{\ell}_e \gamma^\alpha \tau^a \ell_\tau)(\bar{q}_b \gamma_\alpha \tau^a q_d)$ at Λ.



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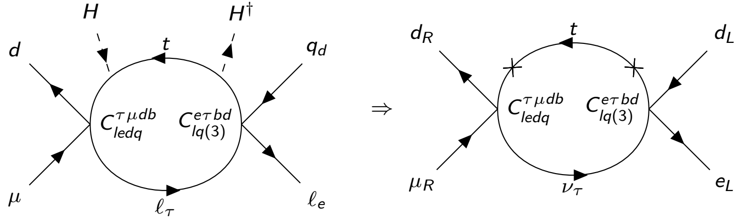

Induces a scalar contact interaction at M_W

$$\simeq \frac{m_t^2}{\Lambda^2} \frac{C_{ledq}^{\tau\mu db} C_{lq(3)}^{e\tau bd}}{4\pi^2\Lambda^2} \log\left(\frac{\Lambda}{m_W}\right) (\bar{e} P_R \mu)(\bar{d} P_R d) \tag{3}$$

that contributes to $\mu A \to eA$.

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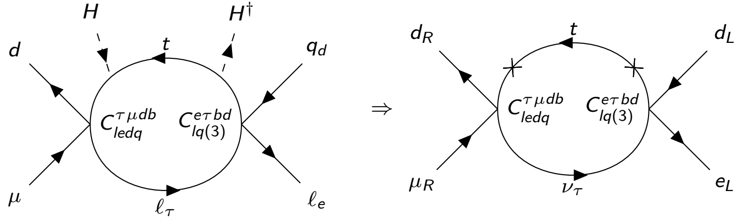

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that contributes to $\mu A \to eA$. The upcoming sensitivity is $Br(\mu A \to eA) \sim 10^{-16} - 10^{-18}$.

Similar to WIMP scattering, the interaction per nucleon add coherently in Spin Independent conversion (enhanced by the atomic mass number of the target)

Scalar quark operators have large matching coefficients with nucleon operators

$\mu \to e$ vs $\tau \to \ell$

Take New Physics at 4 TeV, future $\mu \to e$ conversion gives the following (sensitivity) constrain

$$\left(\frac{v}{\Lambda}\right)^4 \left| C_{ledq}^{\tau\mu db} C_{lq(3)}^{e\tau bd} \right| \lesssim 1.04 \times 10^{-9} \tag{4}$$

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The operators contribute to LFV B_d^0 decays

$$Br(B_d^0 \to \tau\mu) < 1.4 \times 10^{-5} \to \left(\frac{v}{\Lambda}\right)^2 \left| C_{ledq}^{\tau\mu db} \right| < 3.8 \times 10^{-4} \tag{5}$$

$$Br(B_d^0 \to \tau e) < 2.8 \times 10^{-5} \to \left(\frac{v}{\Lambda}\right)^2 \left| C_{lq(3)}^{e\tau bd} \right| < 2 \times 10^{-3} \tag{6}$$

The direct searches put a looser bound on the coefficients product.

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Suppose that (with an order magnitude better sensitivity $Br(B_d^0 \to \tau\mu) \simeq 10^{-6}$) we observe $\tau \leftrightarrow \mu$ but not $\mu \to e$, then $\tau \to e$ is constrained:

$$\left(\frac{v}{\Lambda}\right)^2 \left| C_{lq(3)}^{e\tau bd} \right| < 8.67 \times 10^{-6} \tag{7}$$

Outline

Summary

- $\mu \to e$ experimental improvement is expected in the near future
- Sensitivity on $\mu \to \tau \times \tau \to e$ can compete with direct $\tau \to \ell$ LFV searches
- Our calculation give model independent relation among the three lepton flavour violations
- We calculate a subset of unknown RGEs for dimension eight operators in SMEFT

THANK YOU!

BACK-UP SLIDES

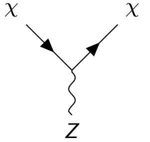
Equation of Motions

Two dimension six operators $\mathcal{O}_1^{(6)}$, $\mathcal{O}_2^{(6)}$ are on-shell equivalent if they lead to the same $S-$matrix elements

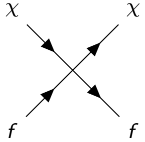
This happens if $\mathcal{O}_1^{(6)} - \mathcal{O}_2^{(6)} = Q\frac{\delta S}{\delta \phi} \equiv \mathcal{O}_{EOM}$ where Q is some operator, S is the action and ϕ is some field.

$\mathcal{O}_{EOM}$ vanishes when the Equation of Motions are satisfied and have zero $S-$ matrix elements:

$(\bar{\chi}\gamma^\mu\chi)(\partial^\nu Z_{\nu\mu} - m_Z^2 Z_\mu)$



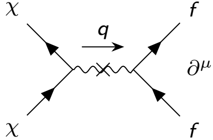
$\overset{equivalent}{\sim} \sum_f g_Z^f (\bar{\chi}\gamma^\mu\chi)(\bar{f}\gamma_\mu f)$



because

$$(\bar{\chi}\gamma^\mu\chi)(\partial^\nu Z_{\mu\nu} - m_Z^2 Z_\mu - \sum_f g_Z^f(\bar{f}\gamma_\mu f)) \equiv \mathcal{O}_{EOM} \tag{8}$$

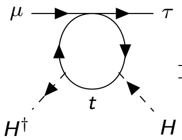
Diagrammatically, if we attach a f current to the Z, the propagator cancel with the vertex



$\partial^\mu Z_{\mu\nu} - m_Z^2 Z_\mu \sim (\Box_{\mu\nu} - m^2 g_{\mu\nu})Z^\nu$ and $\frac{i}{\Box_{\mu\nu} - m_Z^2 g_{\mu\nu}}$ is the Z propagator

Equations of Motion

EOM are corrected by dimension six contributions. If I have a divergent diagram that generate $\mathcal{O}_2^{(6)}$ that is on-shell equivalent to $\mathcal{O}_1^{(6)}$ (that sits in the basis), I renormalize dimension eight operator applying the EOM.
For example:

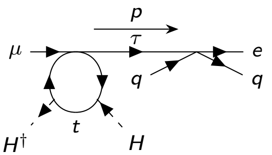


$\supset \frac{iA}{\epsilon\Lambda^2}(\not{D}\bar{\tau}\mu)(H^\dagger H)$

The EOM is $-i\not{D}\bar{\tau} - y_\tau \bar{\ell} H + \frac{C_{qe}^{e\tau qq}}{\Lambda^2}(\bar{q}\gamma_\alpha q)\bar{e}\gamma^\alpha + \ldots = 0$. We get a divergence at dimension eight

$$\frac{A C_{qe}^{e\tau qq}}{\epsilon\Lambda^4}(\bar{e}\gamma^\alpha \mu)(\bar{q}\gamma_\alpha q)(H^\dagger H) \tag{9}$$

Diagrammatically:



Vertex cancel the propagator $\not{p} \times \frac{i}{\not{p}}$

Other interesting diagrams