Natural Feeble Interactions via Fuzzy Branes

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Introduction & Motivations

Different motivations to have suppressed interaction couplings in BSM physics:

- hierarchy of Yukawa couplings in flavor physics,
- origin of neutrino masses,
- dark matter particles (WIMP's, FIMP's, ...),
- axions,
- gravity,
- ...

If small coupling technically natural ('t Hooft naturalness): Still interesting to have a fundamental origin for a small parameter (Dirac naturalness).

Talk: New way to generate suppressed couplings via an extra dimension of space & branes in the framework of UV nonlocal QFT.

Contents

1. Why UV Nonlocality?
   - Quantum Spacetime $\Rightarrow$ UV Nonlocality
   - String-Inspired Toy Model of Fuzzy Particles

2. Split Fermions in an Extra Dimension of Space
   - Extra Dimensions of Space & Branes
   - Split Fermions in a Domain Wall (AS Model)
   - Split Fermions via Fuzzy Branes

Now we are going to look at...

1. Why UV Nonlocality?
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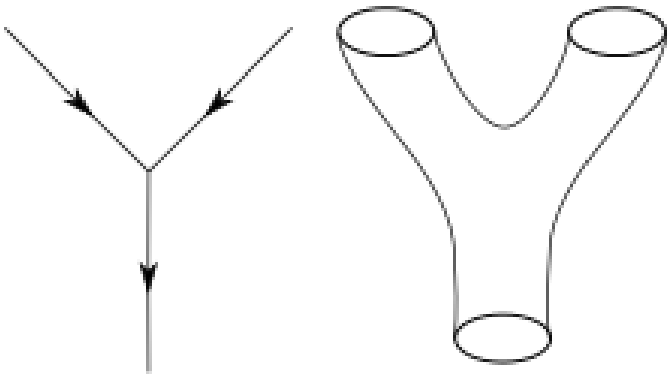
Quantum Spacetime ⇒ UV Nonlocality

- UV nonlocality ⇒ Better UV behavior than local QFT's (R. Feynman).
- Quantum gravity (string theory, loop quantum gravity (LQG), ...)
  & Quantum spacetime (LQG, noncommutative geometry, ...)
  ⇒ Minimal length scale in Nature ⇒ UV nonlocality.
- Strong indications that some UV nonlocal QFT of pure gravity are super-renormalizable or even finite on Minkowski spacetime (L. Modesto).
- In this talk: Prototypes of nonlocal QFT's inspired by string field theory (SFT).
  ⇒ No notion of microcausality! But macrocausality seems OK...



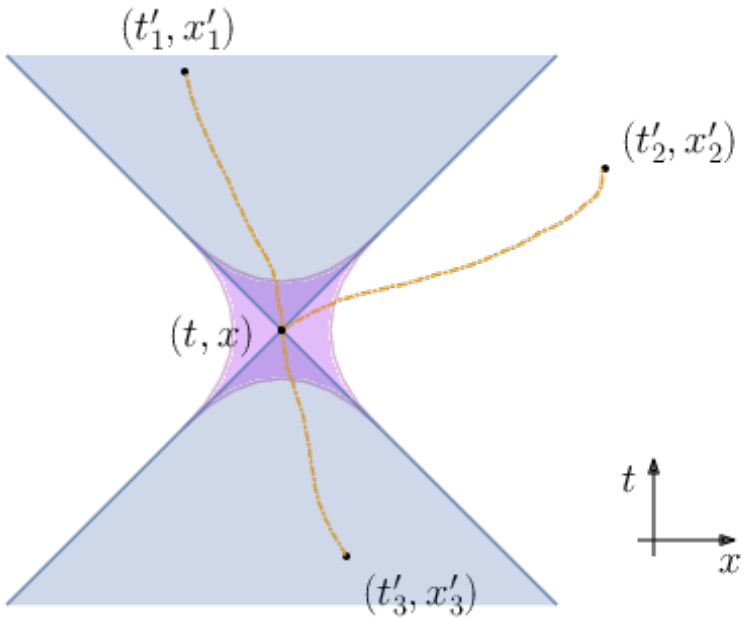


Figure: Interaction vertex & light cone: string versus pointlike particle

String-Inspired Toy Model of Fuzzy Particles

L. Buoninfante, G. Lambiase, A. Mazumdar, Nucl. Phys. B 944 (2019) 114646

Real scalar ϕ on a 4D Euclidean spacetime $\mathbb{R}^4$ of signature $(++++)$:

$$\mathcal{L} = -\frac{1}{2}\,\phi(x)\left(\Box - m^2\right)\phi(x) + \frac{\lambda}{4!}\left[e^{f(\Box)}\phi(x)\right]^4(x)\,, \quad \Box = \partial_\mu\partial_\mu\,.$$

Delocalized scalar field via a smearing infinite derivative operator:

$$\widetilde{\phi}(x) = e^{f(\Box)}\phi(x)\,, \quad f(\Box) = \frac{\Box - m^2}{\Lambda^2}\,, \text{ nonlocal scale: } \Lambda = 1/\ell\,.$$

Nonlocality manifest by introducing the smearing heat kernel $K(x-x')$:

$$\widetilde{\phi}(x) = \int d^4x'\; K(x-x')\,\phi(x')\,, \quad K(x-x') = e^{f(\Box)}\,\delta(x-x') \propto \exp\left[-\left(\frac{|x-x'|}{2\ell}\right)^2\right]\,.$$

$\Rightarrow$ SFT-like form factor. Move nonlocality: *interaction* $\rightarrow$ *kinetic* terms:

$$\mathcal{L} = -\frac{1}{2}\,\widetilde{\phi}(x)\,e^{-2f(\Box)}\left(\Box - m^2\right)\widetilde{\phi}(x) + \frac{\lambda}{4!}\,\widetilde{\phi}^4(x) \quad \Rightarrow \quad \Pi(p^2) = \frac{-ie^{-2f(p^2)}}{p^2 + m^2}\,.$$

$\Rightarrow$ No ghost at tree level!

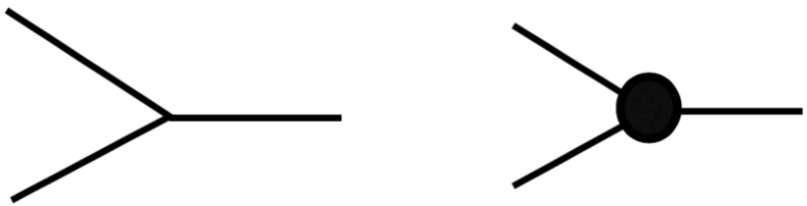


Figure: Local versus nonlocal vertex

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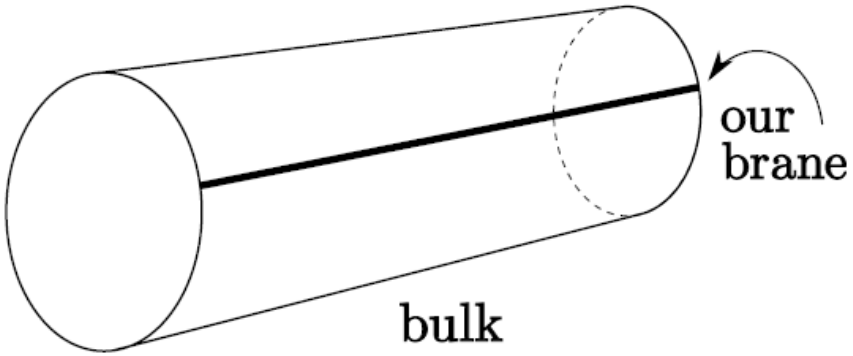
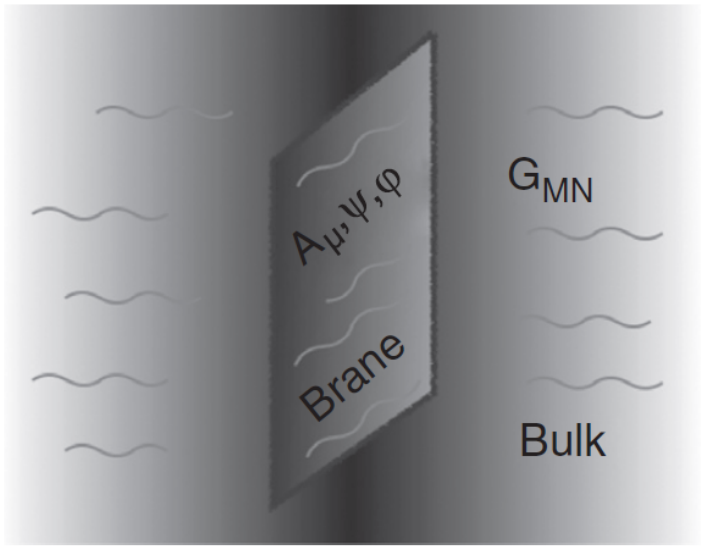
Extra Dimensions of Space & Branes (1/2) – Bottom-Up Approach

Local EFT's with extra dimensions of space & δ-like branes.

- δ-like brane = Hypersurface where fields and/or interactions can be localized.
- Bulk = Whole spacetime outside the branes.





5D action with multibranes:

$$S_{5D} = \int d^4x\, dy \left[\mathcal{L}_{bulk} + \sum_{i=1}^{N} \delta(y - y_i)\, \mathcal{L}^{(i)}_{brane} \right] .$$

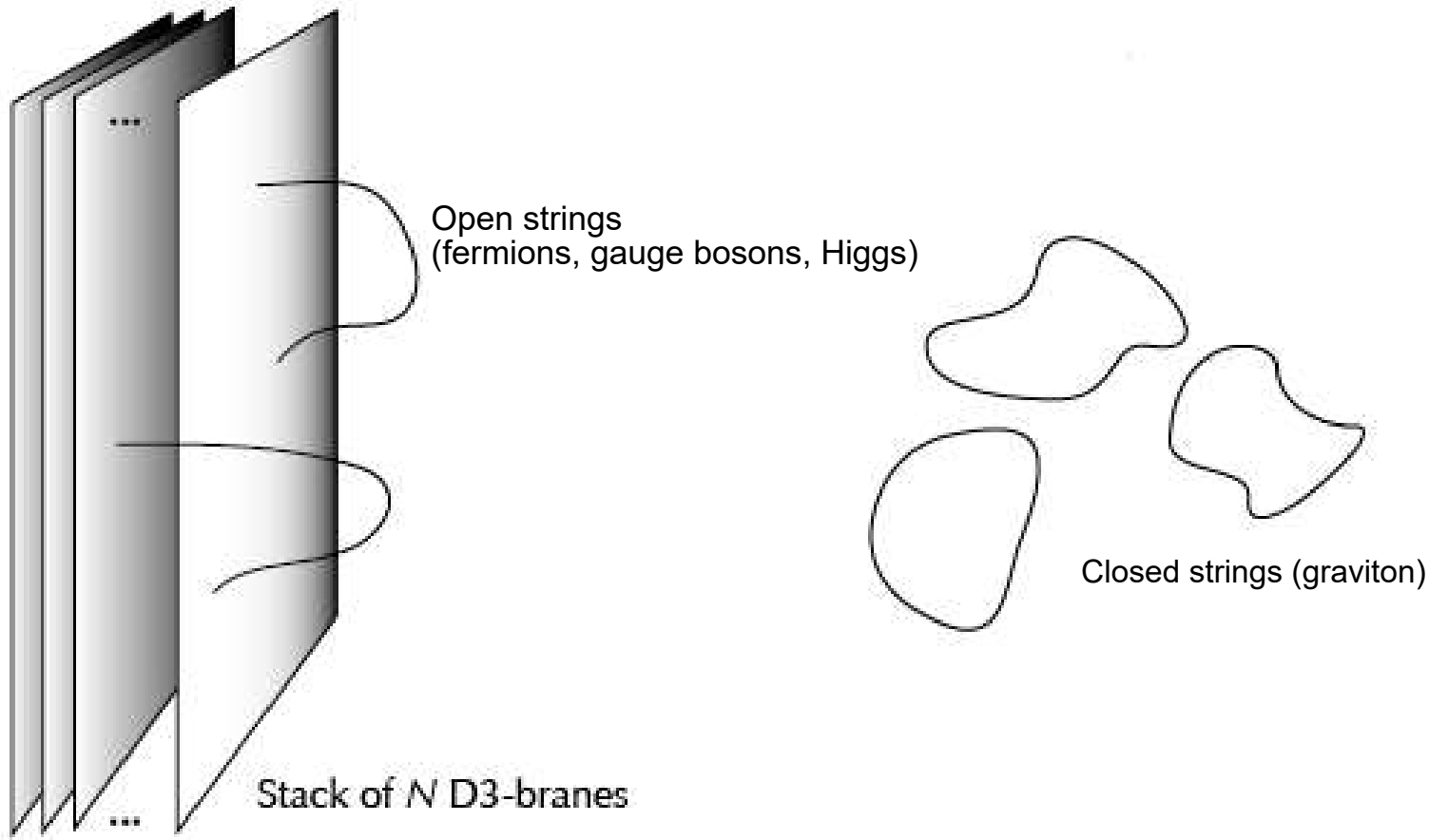
Kaluza-Klein (KK) dimensional reduction (5D $\rightarrow$ 4D).
$\Rightarrow$ 4D KK-modes ϕ_n & bulk wave functions f_n:

$$\Phi(x, y) = \sum_{n}^{\infty} f_n(y)\, \phi_n(x) .$$

Extra Dimensions of Space & Branes (2/2) – Top-Down Approach

String theory (heterotic strings & superstrings):

- 6 extra dimensions of space, D-branes, O-planes, ...
- UV nonlocality.

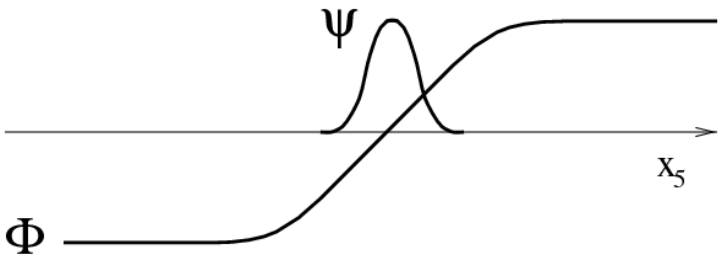


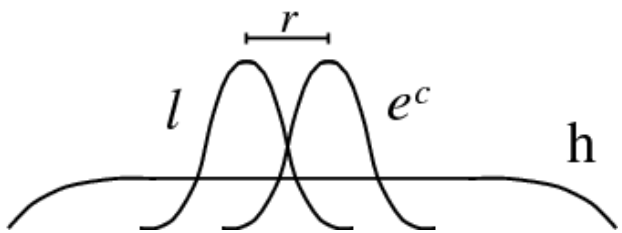
Split Fermions in a Domain Wall (AS Model)

N. Arkani-Hamed, M. Schmaltz, Phys. Rev. D 61 (2000) 033005

AS model:

- Domain wall from a bulk scalar field Φ in a flat extra dimension of space.
- Bulk Yukawa coupling $\mathcal{O}(1)$ between Φ & a bulk fermion field Ψ.
  $\Rightarrow$ 1 zero mode: 4D chiral fermion trapped in the domain wall with a Gaussian bulk wave function.
- 2 bulk fermions $\Psi^{(1)}$ & $\Psi^{(2)}$ with $\neq$ bulk masses $\mathcal{O}(1)$.
  $\Rightarrow$ 4D chiral zero modes: left-handed $\psi_L^{(1)}$ & right-handed $\psi_R^{(2)}$.
  $\Rightarrow$ Gaussian wave functions peaked at $\neq$ positions along the extra dimension.
- Bulk Higgs field: 4D zero mode h with a flat bulk wave function.
  $+$ Bulk Yukawa couplings $\mathcal{O}(1)$ to $\Psi^{(1)}$ & $\Psi^{(2)}$.
  $\Rightarrow$ 4D Yukawa couplings between the zero modes h, $\psi_L^{(1)}$ & $\psi_R^{(2)}$ naturally $\ll 1$.

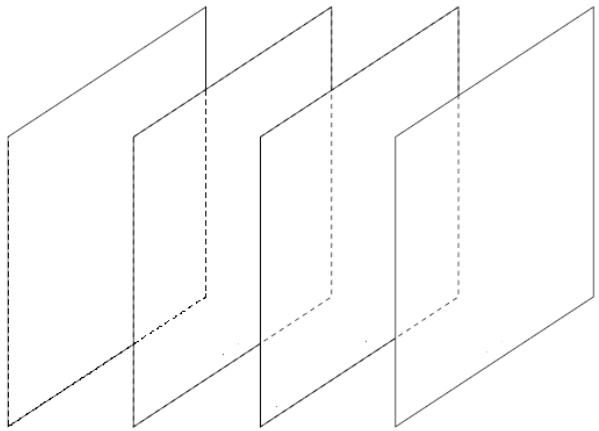




Split Fermions via Fuzzy Branes (1/3)

F. Nortier, Working paper, DOI: 10.5281/zenodo.4898023

Goal: Reproduce AS model of split fermions: domain wall $\rightarrow$ multiple fuzzy branes.



Model:

- 5D Euclidean spacetime $(+++++)$.
- 1 flat extra dimension of space: interval of length $\rho \Rightarrow M_{KK} = 1/\rho$ (KK-scale).
- 4D Weyl fermions $\psi_L^{(1)}$ & $\psi_R^{(2)}$ on 2 different δ-like branes (away from the boundaries).
- 1 bulk Higgs field H.
- Only 2 nonlocal scales (to simplify): $\Lambda_H = 1/\ell_H$ & $\Lambda_\Psi = 1/\ell_\Psi$.
  + mild hierarchy: $\Lambda_H \rightarrow \infty \gg \Lambda_\Psi \gg M_{KK}$.
  $\Rightarrow$ 5D EFT: local bulk Higgs field $(\ell_H \rightarrow 0)$ + delocalized brane-fermions $\psi_L^{(1)}$ & $\psi_R^{(2)}$.
- Yukawa interaction between H, $\psi_L^{(1)}$ & $\psi_R^{(2)}$.

F. Nortier, Working paper, DOI: 10.5281/zenodo.4898023

Delocalized brane-fermion fields:

$$\Psi_{L/R}^{(1/2)}(x,y) = e^{\Box_5/\Lambda_\Psi^2}\,\psi_{L/R}^{(1/2)}(x)\,\delta(y-y_{1/2})\,, \quad \Box_5 = \partial_\mu\partial_\mu + \partial_y^2\,,$$
$$= e^{\Box_4/\Lambda_\Psi^2}\,\psi_{L/R}^{(1/2)}(x)\,g(y-y_{1/2})\,, \quad \Box_4 = \partial_\mu\partial_\mu\,.$$

Transverse smearing function (Gaussian):

$$g(y-y_{1/2}) = \frac{1}{\sqrt{4\pi}\,\ell_\Psi}\exp\left[-\left(\frac{y-y_{1/2}}{2\ell_\Psi}\right)^2\right]\,.$$

5D action:

$$S_{5D} = \int d^4x \int_0^\rho dy \left[\mathcal{L}_H - \sum_{i=1}^{2}\delta(y-y_i)\,\psi_{L/R}^{(i)\dagger}\left(i\not{\partial}\right)\psi_{L/R}^{(i)} + Y\,\Psi_L^{(1)\dagger}\,H\,\Psi_R^{(2)} + \text{H.c.}\right]\,.$$

KK decomposition of the 5D Higgs field with a VEV:

$$H(x,y) = \frac{v}{\sqrt{\rho}} + \sum_{n=0}^{+\infty} f_n(y)\,h_n(x)\,, \quad f_0(y) = \sqrt{\frac{1}{\rho}}\,.$$

Split Fermions via Fuzzy Branes (3/3)

F. Nortier, Working paper, DOI: 10.5281/zenodo.4898023

- Effective 4D Yukawa coupling with the zero mode Higgs boson:

$$y_0 = Y \int_0^\rho dy\; g(y - y_1)\, f_0(y)\, g(y - y_2) = \frac{Y}{\sqrt{8\pi\rho}\,\ell_\Psi} \exp\left(\frac{-r^2}{8\ell_\Psi^2}\right) .$$

  Natural 5D Yukawa coupling $Y \sim \Lambda_\Psi^{3/2}$, and interbrane distance $r = |y_2 - y_1| \gg \ell_\Psi$:

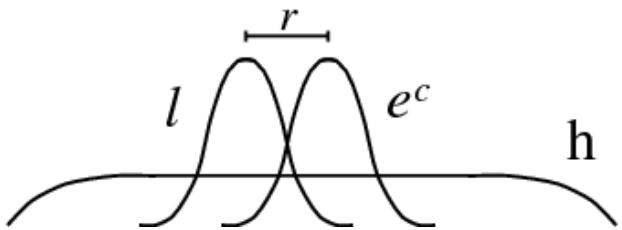
$$y_0 \sim \sqrt{\frac{\ell_\Psi}{8\pi\rho}} \exp\left(\frac{-r^2}{8\ell_\Psi^2}\right) \ll 1\,!$$

  $\Rightarrow$ Nonlocal length scale ℓ_Ψ plays the same role as the domain wall width in AS model.

- Local limit ($\ell_\Psi \to 0$) of the 5D Yukawa term with $y_1 \neq y_2$:

$$Y\, \Psi_L^{(1)\dagger} H\, \Psi_R^{(2)} \quad \to \quad Y\, \psi_L^{(1)\dagger} H\, \psi_R^{(2)}\, \delta(y - y_1)\, \delta(y - y_2) = 0\,.$$

  $\Rightarrow$ Fields on different δ-like branes cannot interact in a local 5D EFT!



Conclusion – Summary, Potential Problems & Outlook

Summary on UV nonlocality:

- Motivated to include gravity in a UV complete quantum theory of Nature.
- Has a smearing effect on interaction vertices & poinlike sources.
- Extra dimensions + branes $\Rightarrow$ Fuzzy branes.
  $\Rightarrow$ Brane-fields can interact with naturally suppressed effective 4D couplings.

$\Rightarrow$ New model building issues for both energy & intensity frontiers!

Potential problems:

- UV nonlocal QFT's usually defined on Euclidean spacetime (Wick rotation not defined: essential singularity at ∞ on the complex plane):
  $\Rightarrow$ Artificial analytic continuation (S-matrix: only real momenta of asymptotic states).
  $\Rightarrow$ Lorentzian starting point? UV change of signature? (ex: LQG)
- Simplest nonlocal theories $\Rightarrow$ Ghosts may reappear in dressed propagators...
  But: Recent formulation where this problem seems to be absent (L. Modesto).

Outlook:

- Building a consistent nonlocal UV completion of the SM.
  $\Rightarrow$ gauge hierarchy problem, asymptotic safety, stability of Higgs mass & potential, ...
- What about interplay with quantum spacetime constructions?
  ex: stochastic spacetimes, κ-Minkowski spacetime, ...
- Interplay with noncommutative QFT's?
  $\Rightarrow$ Other UV nonlocal features (Lorentz symmetry breaking, UV/IR mixing, ...).
  $\Rightarrow$ Constraining gauge groups: only $U(N)$ in θ-Poincaré noncommutative QFT's.
  New results on κ-Minkowski spacetime (J.-C. Wallet).
  $\Rightarrow$ $U(N)$ gauge theories consistent only in 5D $\Rightarrow$ extra dimension of space!

Last words...

Thank you for your attention!