Spatial resolution in the Belle II Silicon Vertex Detector

GdR-Inf 2021 Annual Workshop

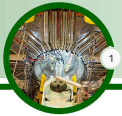
Robin Leboucher

Lucas Martel

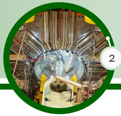
November 16, 2021

https://commons.wikimedia.org/wiki/File:KEK_Belle_II_Detector_(2).jpg

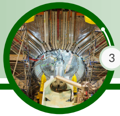
Contents



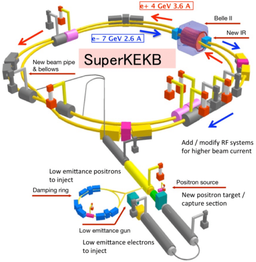
Circle Image Source: `https://commons.wikimedia.org/wiki/File:KEK_Belle_II_Detector_(2).jpg` (modified)



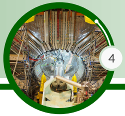
The Belle II Experiment



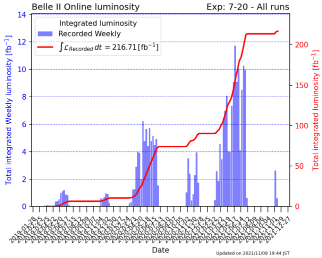
- International collaboration based in Japan.
- Data taking since **2019**.
- Comprised of asymmetric e^+e^- **SuperKEKB** collider (10.58 GeV) and **Belle II** detector.
- Main target : discovery of **BSM** physics.

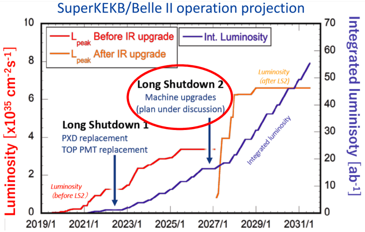


The Belle II Experiment

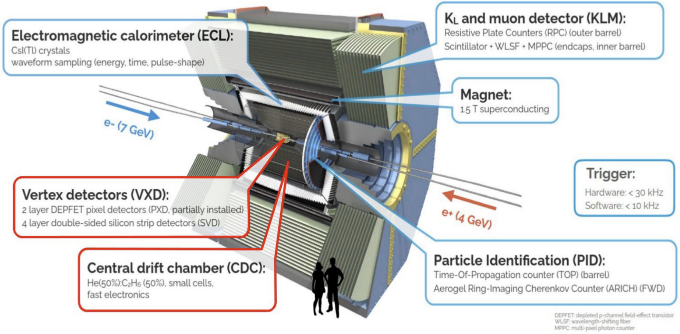


- Data taking since **2019**.
- Current Luminosity:
  - **Highest instantaneous luminosity** in the world $2.4 \times 10^{34}\ cm^{-2}s^{-1}$
  - Already collected $219\ fb^{-1}$
- Luminosity target:
  - Peak luminosity $\sim 6 \times 10^{35}\ cm^{-2}s^{-1}$
  - Collect $50\ ab^{-1}$
- Thanks to nano-beam scheme, high current





Belle II Detector



T. Abe et al. *Belle II Technical Design Report*. 2010, E Kou et al. "The Belle II Physics Book". In: *Progress of Theoretical and Experimental Physics* 2019.12 (Dec. 2019). DOI: 10.1093/ptep/ptz106

SVD roles:

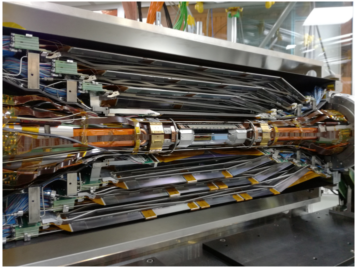
- Extrapolate tracks to PXD
  - essential for reconstruction of decay vertices
  - PXD region of interest for data reduction
- Stand-alone tracking for low momentum tracks
- PID with dE/dx

Silicon Vertex Detector

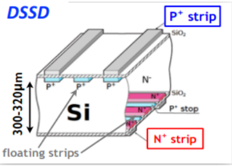
SVD:

4 layers (SVD) of double-sided Si strip sensors (DSSD):

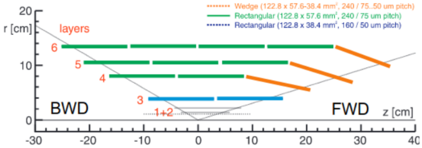
- Side u/P along $r\phi$-direction
- Side v/N along z-direction



Double Sided Strip Detector DSSD



3 shapes of DSSD used in ladders

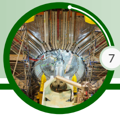


Digital estimated resolution counting floating strips:

$$\sigma_{digit} = \frac{Pitch}{2\sqrt{12}} \quad (1)$$

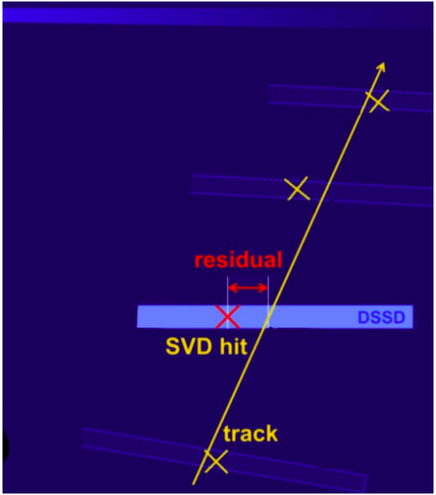
Layer (Side)	3 (u/P)	456 (u/P)	3 (v/N)	456 (v/N)
Strip Pitch (μm)	50	75	160	240
Digital Resolution (μm)	7	10	23	34

Section 2



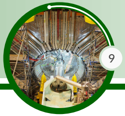
Spatial Resolution
Motivations

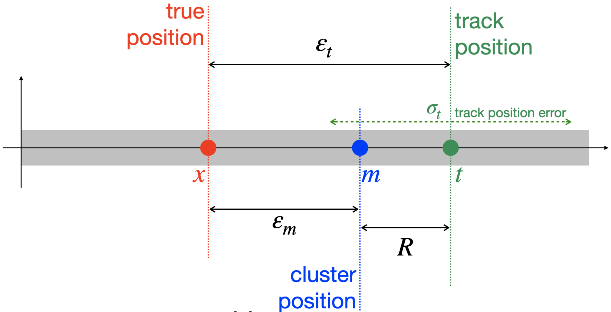
- Excellent Spatial resolution is mandatory for SVD reconstruction, crucial input for tracking:
  - Improve quality of reconstructed tracks and vertices
  - Correctly propagate uncertainty on hit's position to track parameters



Spatial Resolution

Ingredients





True Position	x		
Cluster Position	m	$\rightarrow$	Center of Gravity
Unbiased Track Position	t	$\rightarrow$	Track fitting
Track position error	σ_t		

Residuals:

Residual	$R = m - t$
Track Residual	$\varepsilon_t = t - x$
True Residual	$\varepsilon_m = m - x$

Spatial Resolution σ_{cl}:

$$\sigma_{cl}^2 := Var[\varepsilon_m] \quad (2)$$

$$:= E[(m-x)^2] - E[(m-x)]^2 \quad (3)$$

Nonetheless the true position x is only available in Monte-Carlo samples.

Section 3

Event by Event Method

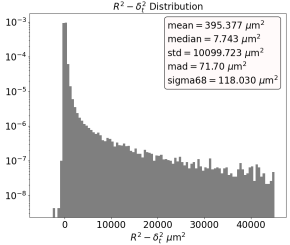
Event by event method:

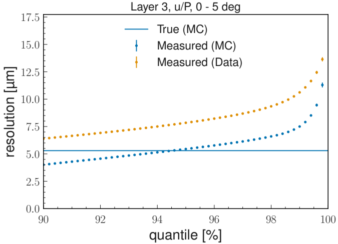
Subtract in quadrature the effect of the error on track extrapolation on residual:

$$\sigma_{cl}^2 = \langle R^2 - \sigma_t^2 \rangle_{trunc} \quad (4)$$

- Discrepancy between true and measured resolution on simulation
- Solved by optimising the quantile truncation on $R^2 - \sigma_t^2$ following:

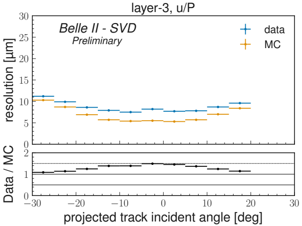
$$\mathrm{FOM} = \frac{(\sigma_{true} - \sigma_{cl})^2}{(\Delta\sigma_{cl})^2} \quad (5)$$

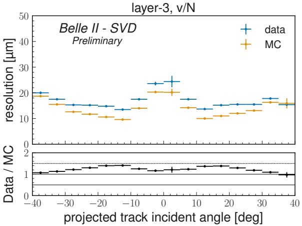


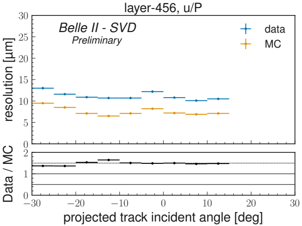


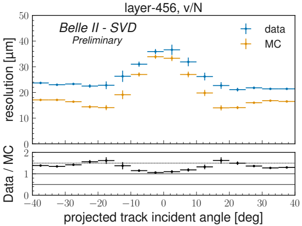
Event by Event Method

Data and Simulation results

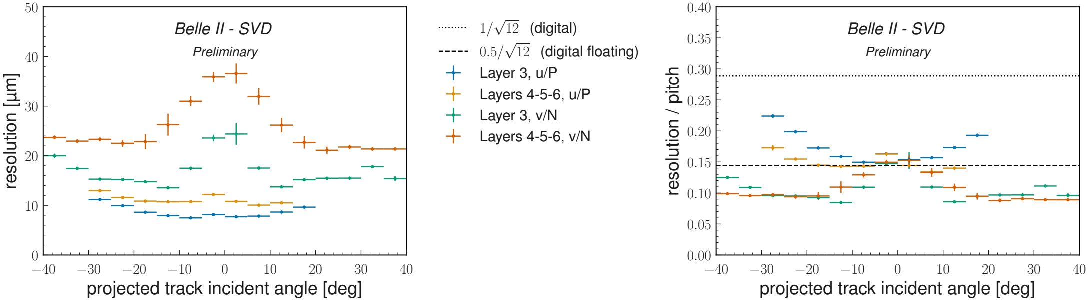








Event by Event Method
Data results



- v/N Side reaches the digital resolution for perpendicular tracks (low cluster size). Smaller resolution for higher angle and cluster size.
- Maybe still some room for improvements on u/P side

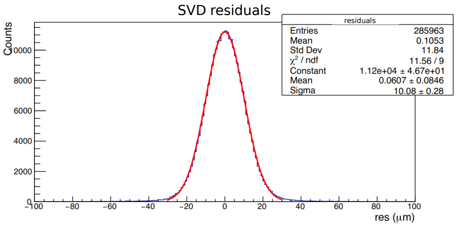
Global Method

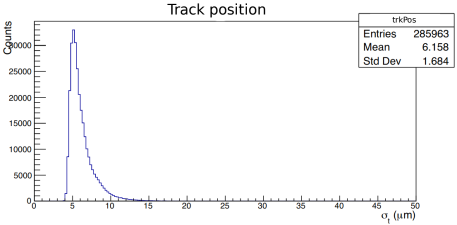
Global Method:

Based on Mean Absolute Deviation (MAD) as best estimator : no optimization on MC needed.

$$\sigma_{cl}^2 = \langle R^2 - \sigma_t^2 \rangle \simeq mad(R)^2 - median(\sigma_t)^2 - mad(\sigma_t)^2$$

- $mad(y) = 1.4826 \times median(\mid y - median(y) \mid)$
- mad and median more robust against outliers than standard deviation and mean.

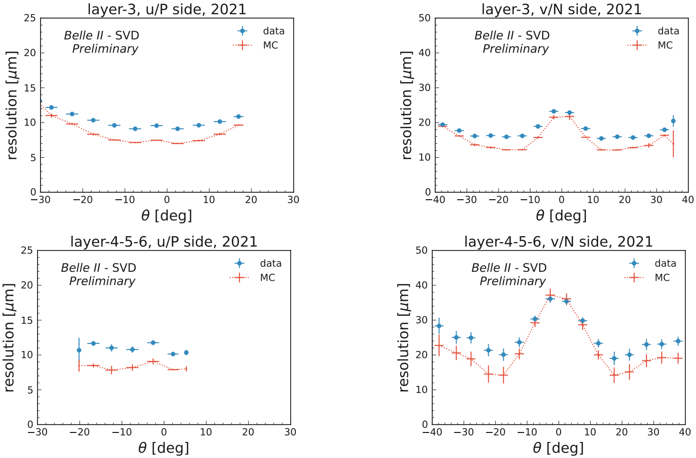




Global Method

Data and simulation results

- Results in agreement with event by event method: data/MC agreement, angular distribution..
- The Global Method allows to estimate the spatial resolution in a robust way.
- Small discrepancies between data and simulation.



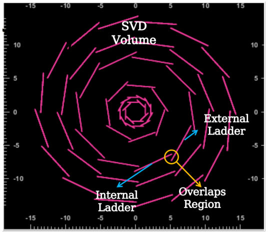
Overlapping Method

Overlapping Method:

- Select tracks in fiducial area with two hits on the same layer and on consecutive ladders
- Compare residuals computed for the pair of overlapping ladder, double residuals:

$$\Delta R = R_{int} - R_{ext} \qquad (6)$$

- Apply geometrical correction due to non-parallel sensors
- Resolution is the σ_{68} width of a Student-T distribution fit

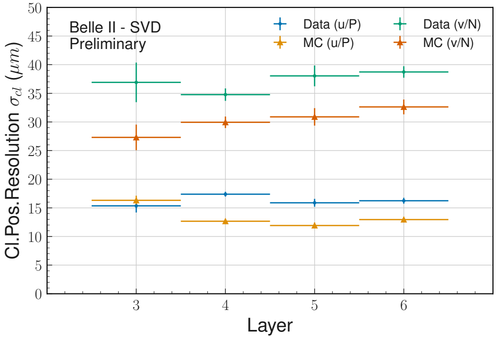


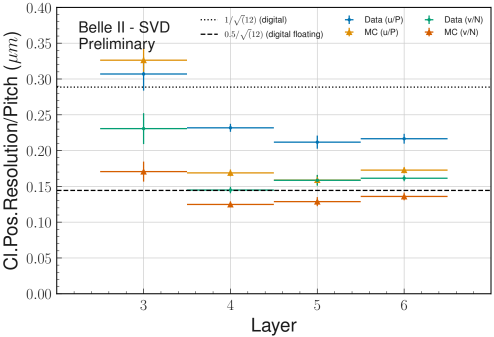
- Marginally sensitive to Coulomb scattering
- Decouple contribution of tracking uncertainty $\rightarrow$ assuming the error on the track extrapolation cancel out in the double difference!
- Most critical side for resolutions in u/P direction, due to the reduced overlapping region: limited angle range and statistics.

CMS Tracker Collaboration. "Stand-alone Cosmic Muon Reconstruction Before Installation of the CMS Silicon Strip Tracker". In: *Journal of Instrumentation* 4.05 (May 2009). DOI: 10.1088/1748-0221/4/05/P05004

Overlaps

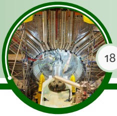
Data results





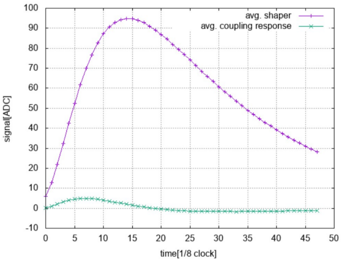
- Outermost v/N side layers are in agreement with expected digital resolution
- Measured resolutions with overlaps approach are larger than expected/other methods, even on simulation

Section 4



Charge coupling between strips

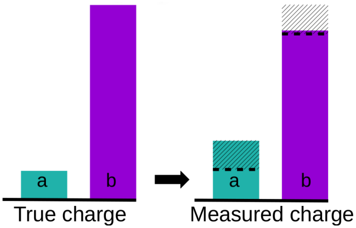
- A signal is injected on a given strip, the waveform of the adjacent channel is also checked.
- A small signal is observed on the adjacent channel with a lower time ($\simeq 27ns$)



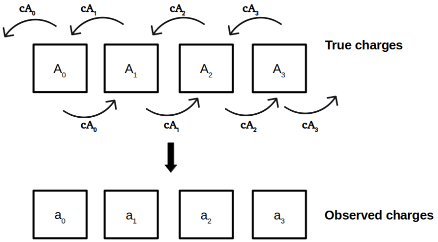
Expected effect on strip charges:

$$charge_{obs}(a) = charge_{true}(a) + c \times charge_{true}(b)$$

Strip charge is used to compute position : impact on resolution !



Correction by unfolding

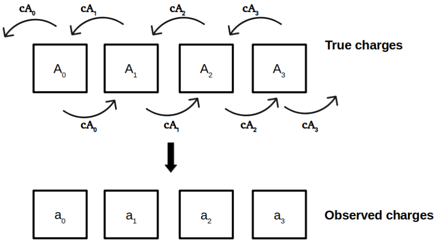


The observed charges follow:

$$a_i = (1 - 2c)A_i + c(A_{i-1} + A_{i+1})$$

From the observations on data, c = 0.1

Correction by unfolding



The observed charges follow:

$$a_i = (1 - 2\mathbf{c})A_i + \mathbf{c}(A_{i-1} + A_{i+1})$$

From the observations on data, c = 0.1

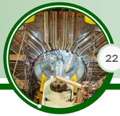
To correct this effect, we link the *observed* charges to the *true* charges :

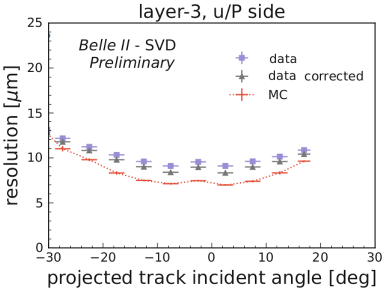
$$\begin{pmatrix} 1-2c & c & 0 & 0 \\ c & 1-2c & c & 0 \\ 0 & c & 1-2c & c \\ 0 & 0 & c & 1-2c \end{pmatrix} \begin{pmatrix} A_1 \\ A_2 \\ A_3 \\ A_4 \end{pmatrix} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \end{pmatrix}$$

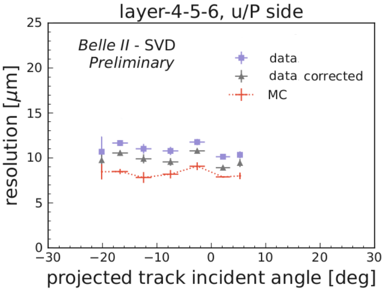
Then **unfold** the charges by inversion :

$$\begin{pmatrix} A_1 \\ A_2 \\ A_3 \\ A_4 \end{pmatrix} = M^{-1} \begin{pmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \end{pmatrix}$$

Impact on resolution

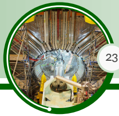






- This correction only yields improvements for u/P side.
- The unfolding method improves data/MC agreement : allows to breach $\simeq$ **20 - 30** % of the gap.
- Correction implemented in the Belle II analysis software

Summary



- Different method developed to estimate spatial resolution :

Event By Event	Global	Overlaps
Good estimation of spatial resolution	Good estimation of spatial resolution	Marginally sensitive to Coulomb scattering
Data/MC agreement	No optimization needed	
Optimization on MC needed	Small Data/MC discrepancies	Estimated resolution higher than expected

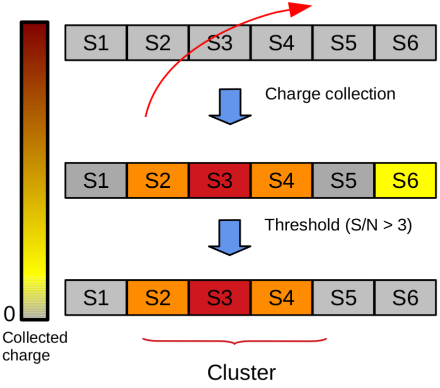
- Development of the unfolding method: correction of data that improves the spatial resolution measured on u/P side.

Thanks for your attention

Backups

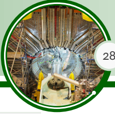
Cluster reconstruction

- For each cluster, computation of:
  - Charge S_{cl}
  - Position x_{cl}
  - Time t_{cl}
- For a cluster made of n strips
- $S_{cl} = \Sigma S_i$
- $x_{cl} = \Sigma x_i S_i \,/\, \Sigma s_i$ (Center of gravity)
- $t_{cl} = \Sigma t_i S_i \,/\, \Sigma s_i$ (Center of gravity)



Estimation of cluster position resolution

- We define the quantity σ^2_{cl} as : $\sigma_{cl}^2 = < res^2 - \sigma_t^2 > = < res^2 > - < \sigma_t^2 >$
  $= \text{mean(res)}^2 - \text{mean}(\sigma_t)^2 - \text{std.dev}(\sigma_t)^2$
- Since the distributions are not perfectly gaussian use instead :
  - mean(x) ≈ median(x)
  - Std.dev(x) ≈ mad(x) ≡ 1.4826*median(|x - median(x)|)
- Then $\sigma^2_{cl} = \text{mad(res)}^2 - \text{median}(\sigma_t)^2 - \text{mad}(\sigma_t)^2$



Errors estimation in Event by event method:

1. **Statistical uncertainties**
   Taking variance of resolution squared as variance of sample mean

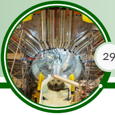
$$\Delta\sigma_{Cl} = \frac{1}{2\sigma_{Cl}}\sqrt{\frac{\langle (R^2 - {\delta_t}^2)^2\rangle - \sigma_{Cl}^4}{N}}$$

2. **Systematic uncertainties**
   Adding in quadrature:
   - Variation in resolution measurement with and without selection on residual
   - Variation in resolution measurement with quantile truncation at ∓0.2% (step) between optimal quantile

Errors estimation in Global method:

1. **Statistical uncertainties**
   - For median $\frac{mad}{\sqrt{N}}$
   - *For mad* $\sqrt{2}\frac{mad}{\sqrt{N-1}}$

2. **Systemac uncertainties:**
   Difference with another robust estimator that should give the same result for Gaussian distributions
   - For median $|median - midhinge|$ (average of the first and third quartiles)
   - For mad $|mad - \sigma_{68}|$



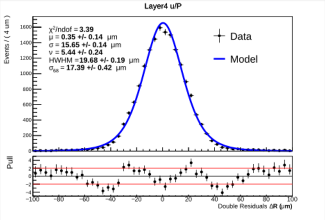
Method for estimate resolution with overlapping:

1. Apply geometical correction factor on double residuals

2. Fit ***DeltaRes*** with a student's t-distribution:

$$T(\mathrm{X},\nu,\mu,\sigma) = \frac{exp\left(\Gamma\left(\frac{\nu+1}{2}\right) - \Gamma\left(\frac{\nu}{2}\right)\right)}{\sigma\sqrt{\pi\nu}}\left(1+\frac{(X-\mu)^2}{\sigma^2\nu}\right)^{-\frac{\nu+1}{2}}$$

normalisation parameter N
number of degree of freedom ν
mean μ
variance σ^2



3. The resolution is the σ_{68} of the fitted student's t-model T

$$\sigma_{cl} = \sigma_{68}(T(\mathrm{X},\nu,\mu,\sigma)) = \frac{\chi_{84}(T(\mathrm{X},\nu,\mu,\sigma)) - \chi_{16}(T(\mathrm{X},\nu,\mu,\sigma))}{2}$$

True Resoltion in Monte-Carlo:

$$\sigma_{68}(m - x)$$

Cluster position m
True position x

Method for estimate resolution uncertainties:

1. Vary fitted parameters (N, μ, ν, σ) within the fit uncertainties (± Fit errors)

2. Compute student's T-model with new parameters

3. Taking σ_{68} resolution of this new model

4. Take as resolution uncertainty for each layer half the maximal variation of the recomputed σ_{68}:

$$\frac{max(\sigma_{68}) - min(\sigma_{68})}{2}$$

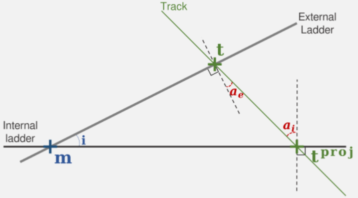
Geometrical correction:

Project parallel to the tracks, the external residual on internal ladder.

u/P Side:

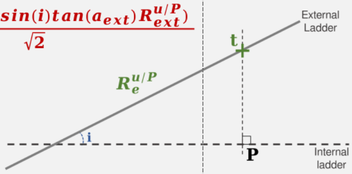
$$\Delta R = \frac{R_{int} - R_{ext} * C}{\sqrt{1 + C^2}}$$

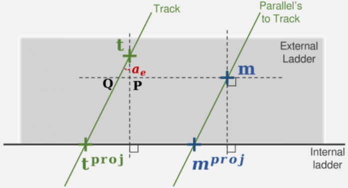
with $$C = \frac{\cos a_{ext}}{\cos a_{int}}$$



v/N Side:

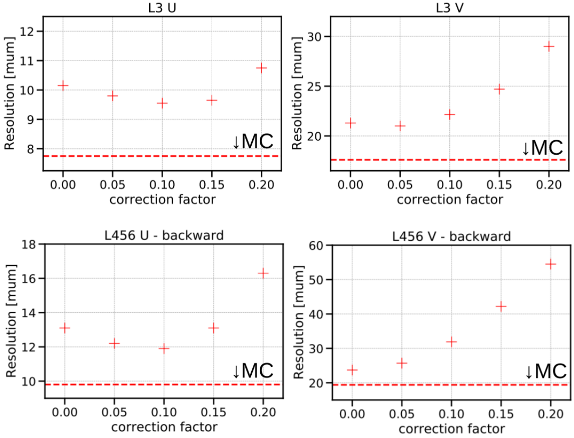
$$\Delta R = \frac{R_{int} - (R_{ext} + \sin(i)\tan(a_{ext}) R^{u/P}_{ext})}{\sqrt{2}}$$





Cluster unfolding method – estimation of best correction factor

- The resolution is computed for each type of sensor for different values of **c** ranging from **0** to **0.2**
- The unfolding method does not improve the resolution on V-side sensors
- However, **U-side** sensors do benefit from the method, with an optimal gain with **c = 0.1** for all type of sensors



Cluster unfolding method – estimation of best correction factor

- The resolution is computed for each type of sensor for different values of **c** ranging from **0** to **0.2**
- The unfolding method does not improve the resolution on V-side sensors
- However, **U-side** sensors do benefit from the method, with an optimal gain with **c = 0.1** for all type of sensors

Sensors - U side	c = 0 (original)	c = 0.05	c = 0.1 (corrected)	c = 0.15	c = 0.20	MC (goal)
L3.1	9.6	9.3	9.1	9.3	10.7	7.2
L3.2	10.7	10.3	10	10	10.8	8.3
L456 backward	13.1	12.2	11.9	13.1	16.3	9.8
L456 origami	12.6	11.8	11.5	12.6	15.2	9.1
l456 slanted	11.6	10.9	10.7	12	14.8	8.9

Sensors - V side	c = 0	c = 0.05	c = 0.1	c = 0.15	c = 0.20	MC
L3.1	25.1	24.5	24.8	25.6	27.8	21.1
L3.2	17.5	17.5	19.5	23.8	30.2	14.1
L456 backward	23.7	25.7	31.9	42.2	54.5	19.4
L456 origami	26.5	28.5	33.4	40.3	46.2	22.5
l456 slanted	29.3	29	31.2	37.3	49.6	23.4