

Status of the TDCPV measurements of the $B \rightarrow K_s \pi^+ \pi^- \gamma$ and $B \rightarrow K_s \pi^0 \gamma$ decays at the Belle II experiment

Tristan Fillinger

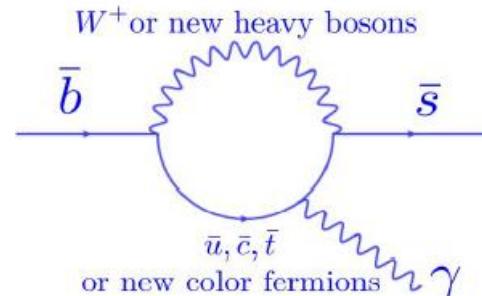
16/11/21

GDR – InF annual workshop



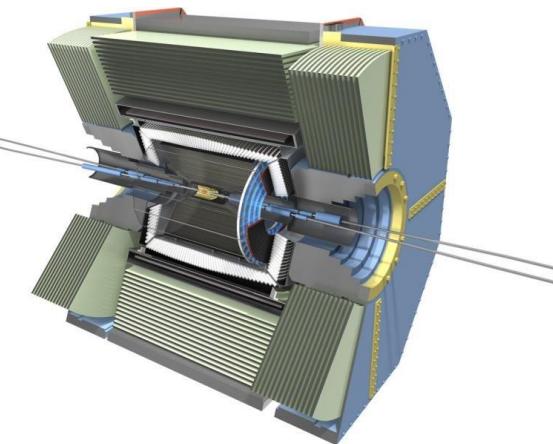
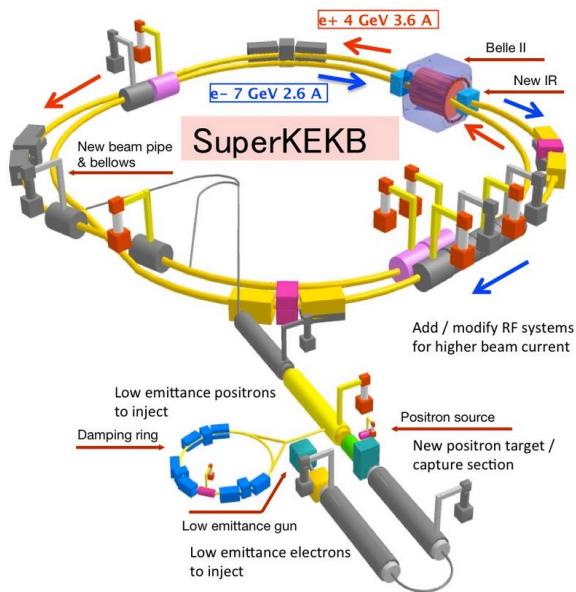
$b \rightarrow s\gamma$ physics

- Goal : Discover New Physics (beyond Standard Model) with $b \rightarrow s\gamma$ process



- Described by Wilson coefficient C_7 and C'_7
- In standard model, photon almost fully polarized
- New physics: Different coupling which enhance right-handed photon contributions (MSSM, LRSM, SUSY SU(5) GUT...)
 - Atwood *et al.*, Mixing-Induced CP Asymmetries in Radiative B Decays in and beyond the Standard Model, Phys. Rev. Lett. 79, 185
 - E. Kou *et al.*, Photon polarization in the $b \rightarrow s$ processes in the Left-Right Symmetric Model, JHEP 12 (2013) 102 [1305.3173]
 - N. Haba *et al.*, Search for new physics via photon polarization of $b \rightarrow s$, JHEP 03 (2015) 160 [1501.00668]
- Two methods to determine photon polarization:
 - Angular distribution measurement
 - Time-dependent CP asymmetry measurement (with the Belle II experiment)

SuperKEKB collider and Belle II



- Started in 2019
- Electron (7 GeV) - Positron (4 GeV) collider
- B, charm and τ factory

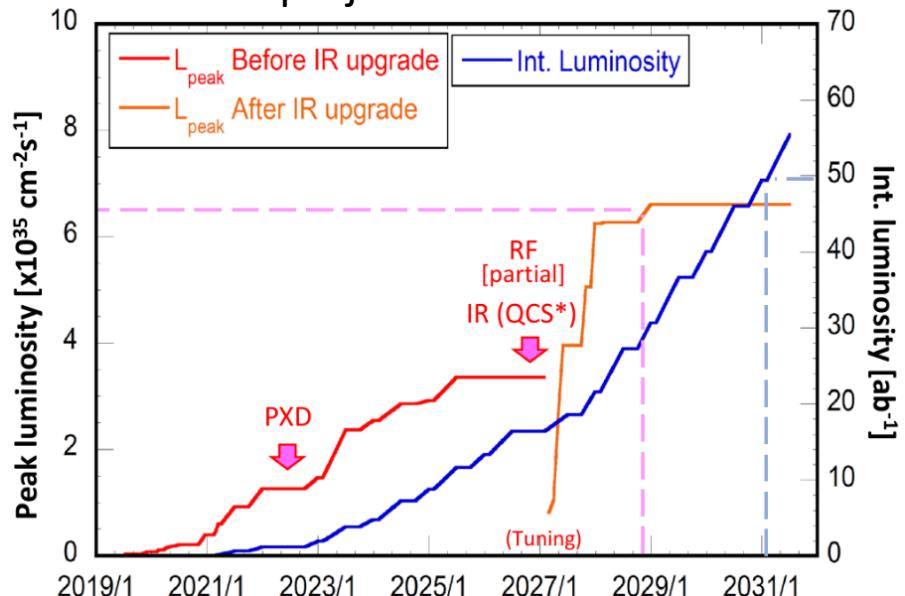
Today

- Peak luminosity: $3.1 \cdot 10^{34} \text{ cm}^{-2} \text{ s}^{-1}$
- Int. luminosity: $\sim 220 \text{ fb}^{-1}$ of data collected

Goal

- Peak luminosity: $6 \cdot 10^{35} \text{ cm}^{-2} \text{ s}^{-1}$
- Int. luminosity: 50 ab^{-1}
- Belle Int. luminosity: 1 ab^{-1}

Current projection

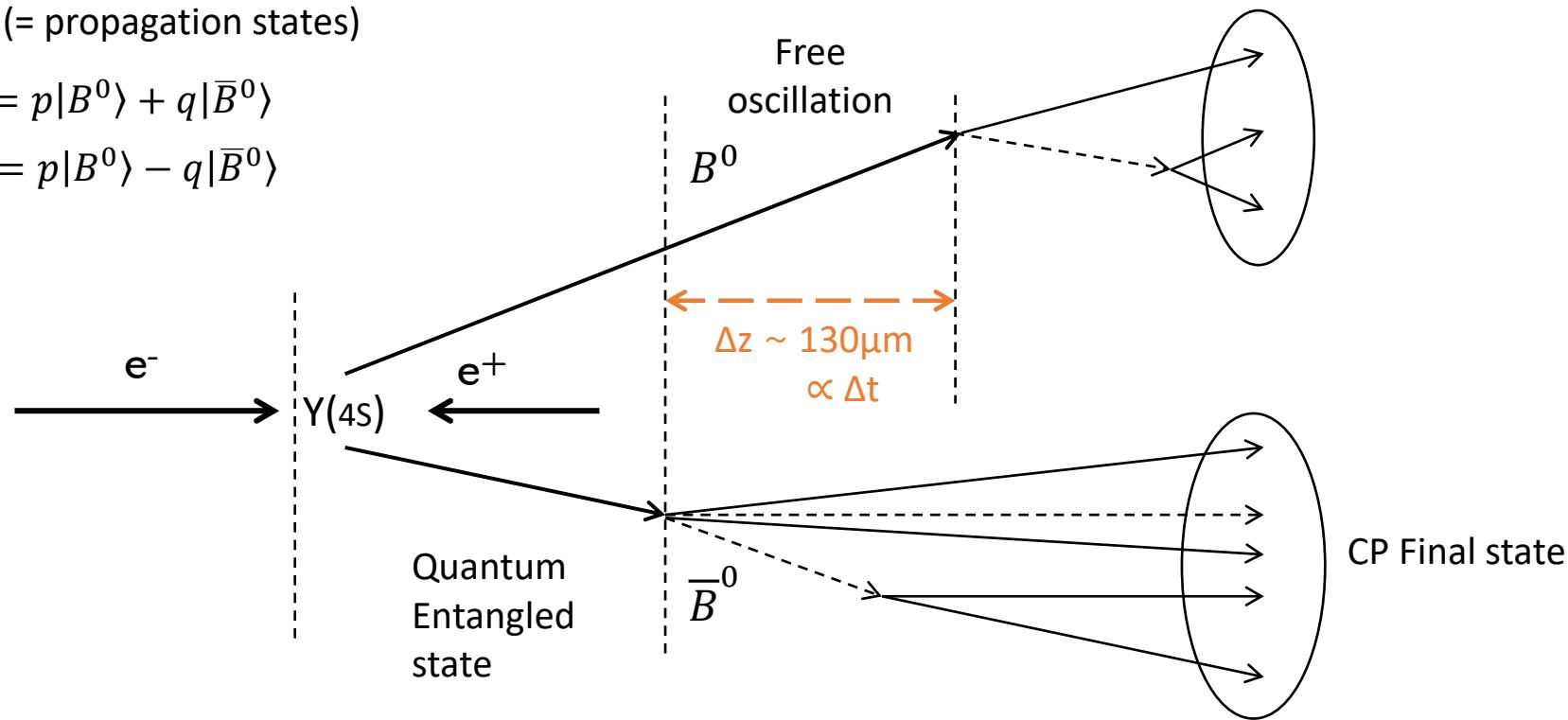


Time dependent CP violation (TDCPV)

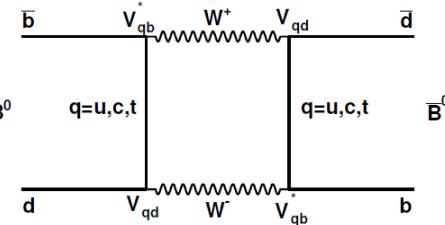
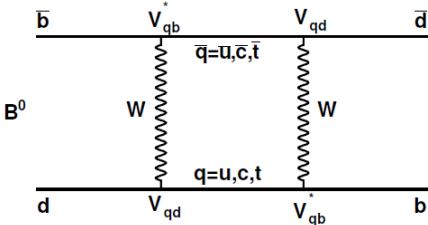
Mass states (= propagation states)

$$|B_L\rangle = p|B^0\rangle + q|\bar{B}^0\rangle$$

$$|B_H\rangle = p|B^0\rangle - q|\bar{B}^0\rangle$$



Feynman Diagram describing the oscillations

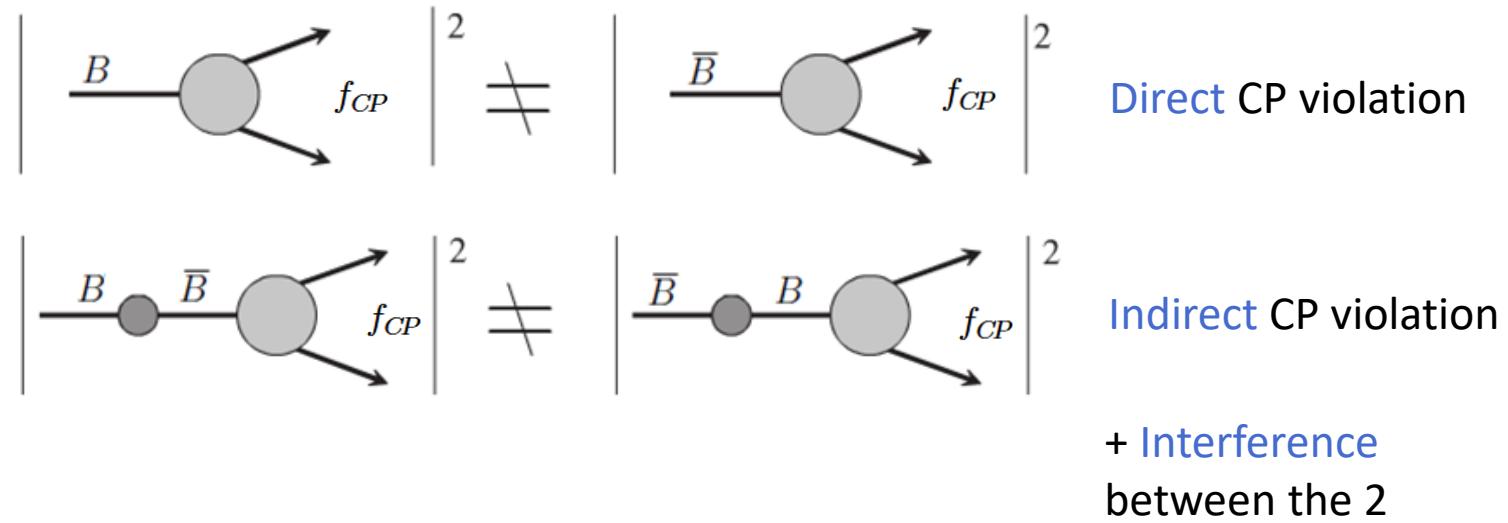


Asymmetry

$$\mathcal{A}_{CP}(\Delta t) = \frac{\Gamma(B_{tag=B^0}(\Delta t) \rightarrow f_{CP}) - \Gamma(B_{tag=\bar{B}^0}(\Delta t) \rightarrow f_{CP})}{\Gamma(B_{tag=B^0}(\Delta t) \rightarrow f_{CP}) + \Gamma(B_{tag=\bar{B}^0}(\Delta t) \rightarrow f_{CP})}$$

Time dependent CP violation (TDCPV)

- **Source of the CP violation:**



- **B meson decay rate:**

$$\frac{\Gamma(B_{\text{tag}=B^0}(\Delta t) \rightarrow f_{CP}) - \Gamma(B_{\text{tag}=\bar{B}^0}(\Delta t) \rightarrow f_{CP})}{\Gamma(B_{\text{tag}=B^0}(\Delta t) \rightarrow f_{CP}) + \Gamma(B_{\text{tag}=\bar{B}^0}(\Delta t) \rightarrow f_{CP})} \cong S \sin(\Delta m \Delta t) - C \cos(\Delta m \Delta t)$$

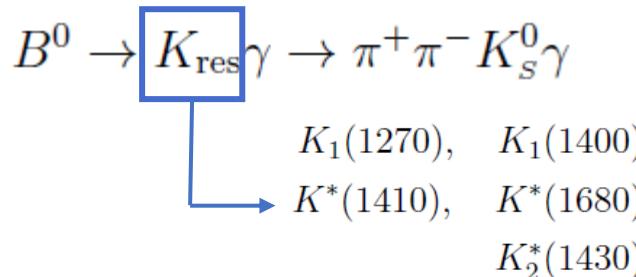
↓ ↓
CP violation coefficient

- In the SM: S and $C = 0$
- S value constrains C_7 and C'_7

TDCPV with Dalitz separation

- Channels chosen for the TDCPV analysis:** $B^0 \rightarrow K_s^0 \pi^0 \gamma$ and $B^0 \rightarrow K_s^0 \pi^+ \pi^- \gamma$

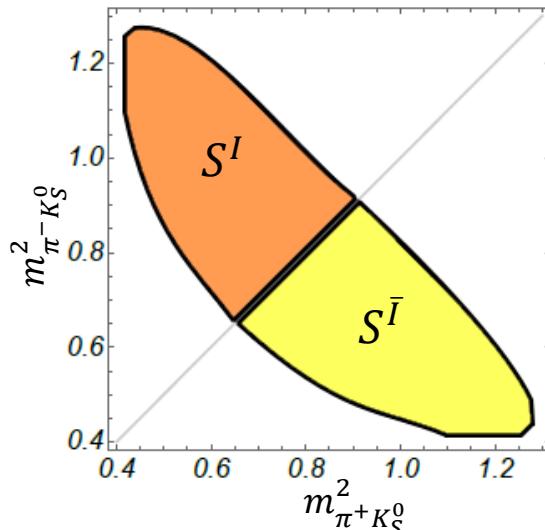
$B^0 \rightarrow (K^*(892)^0 \rightarrow \pi^0 K_S^0) \gamma$ CP final state → Measure **S** and **C** directly



$B^0 \rightarrow K_{\text{res}} \gamma \rightarrow (\rho^0 K_S^0) \gamma \rightarrow K_S^0 (\pi^+ \pi^-) \gamma,$ CP final stat
 $B^0 \rightarrow K_{\text{res}} \gamma \rightarrow (K^* \pi^-) \gamma \rightarrow (K_S^0 \pi^+) \pi^- \gamma,$ Non-CP final stat
 $B^0 \rightarrow K_{\text{res}} \gamma \rightarrow ((K\pi)_0^+ \pi^-) \gamma \rightarrow (K_S^0 \pi^+) \pi^- \gamma,$ Non-CP final stat

Dilution factor: $\mathcal{D} = \frac{S_{\pi^+ \pi^- K_S^0 \gamma}}{S_{\rho^0 K_S^0 \gamma}}$

Obtained with $B^+ \rightarrow K^+ \pi^+ \pi^- \gamma$ analysis



→ Measure **S** and **C** directly considering the dilution factor

→ Use new method from Simon AKAR *et al.* which separate the Dalitz plane to gain more information on C_7 and C'_7

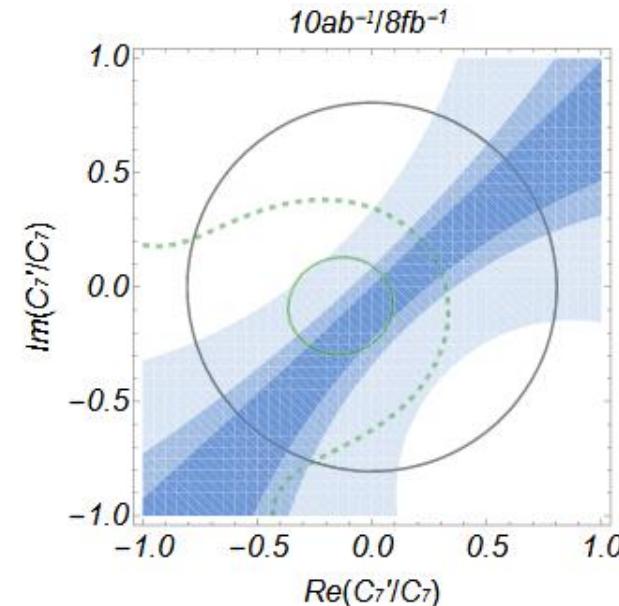
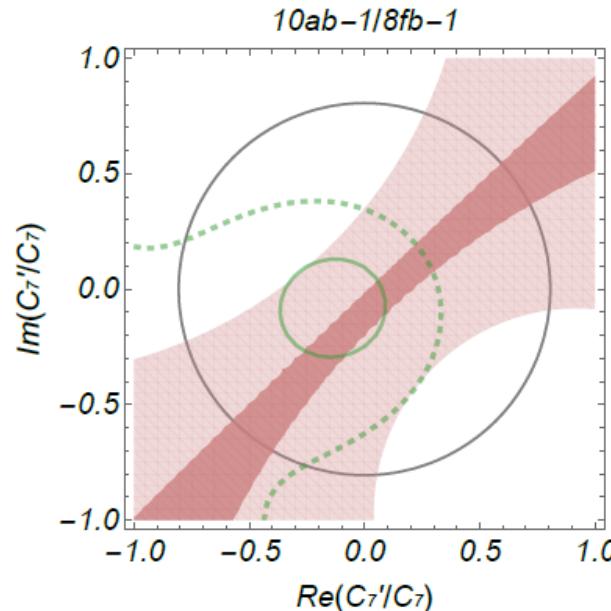
[Akar *et al.*, JHEP 09 \(2019\) 034](#)

New observables: $S_{\pi^+ \pi^- K_S^0 \gamma}^+ = S^I + S^{\bar{I}}$ $S_{\pi^+ \pi^- K_S^0 \gamma}^- = S^I - S^{\bar{I}}$

TDCPV with Dalitz separation

Constrains (3σ) on imaginary and real part of C_7 and C'_7 using:

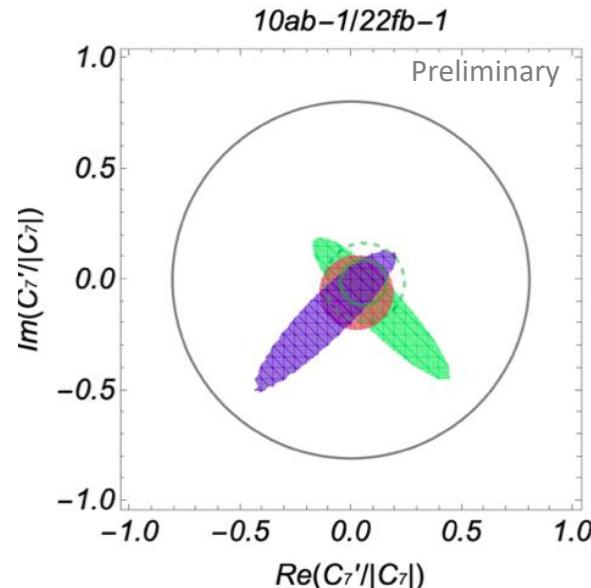
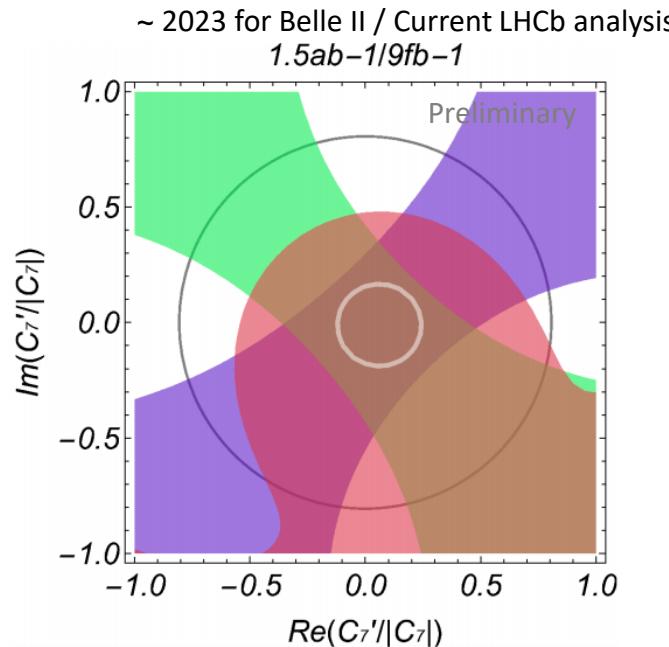
[Akar et al., JHEP 09 \(2019\) 034](#)



- Red: $S_{\pi^0 K_S^0 \gamma}$
Eur. Phys. J. C77 (2017) 895 [1612.07233]
- Dark red: $S_{\pi^0 K_S^0 \gamma}$ with expected precision
at Belle II
- Green: Angular coefficient of $B \rightarrow K^* ee$ at $q^2 \rightarrow 0$ (LHCb)
Dashed line: JHEP 04 (2015) 064 [1501.03038]
Full: Expected precision at LHCb Run II
- Grey: BR measurement constraint
Eur. Phys. J. C 77 (2017) 895
Phys. Rev. Lett. 114 (2015) 221801

TDCPV with Dalitz separation

Constrains on imaginary and real part of C_7 and C'_7 using:



[Akar et al., JHEP 09 \(2019\) 034](#)

Plot updated by E. Kou in a private conversation

- Combination of $S_{\pi^+\pi^-K_S^0\gamma}^+$, $S_{\pi^+\pi^-K_S^0\gamma}^-$ with different hadronic parameters (linked to the **dilution factor**, obtained with $B^+ \rightarrow K^+\pi^+\pi^-\gamma$ analysis)
 - > Imaginary and real part of C_7 and C'_7 much more constrained
 - > Competitive with the LHCb measurement

Belle II analysis plan

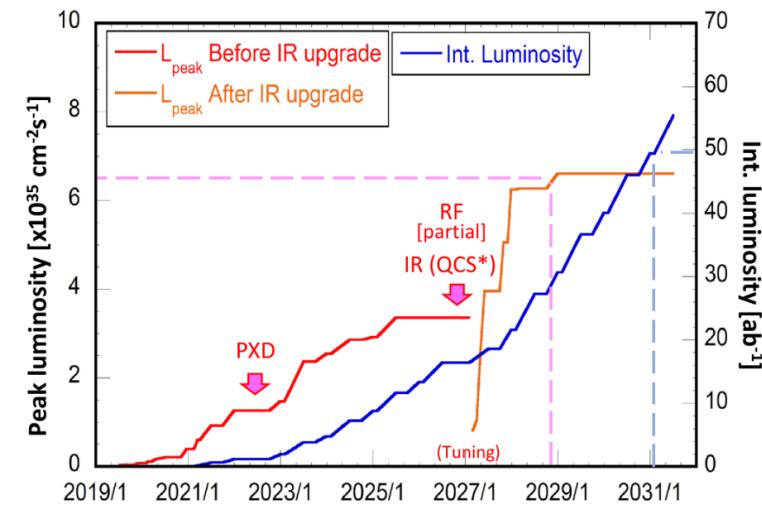
- **Already measured:**

- [BaBar PRD78 \(2008\)](#): $S_{\pi^0 K_S^0 \gamma} = -0.78 \pm 0.59 \pm 0.09$, full stat ($467\text{e}6 B\bar{B}$)
- [Belle PRD74 \(2006\)](#): $S_{\pi^0 K_S^0 \gamma} = -0.10 \pm 0.31 \pm 0.07$, partial stat ($535\text{e}6 B\bar{B}$)
- [BaBar PRD93 \(2015\)](#): $S_{\pi^+ \pi^- K_S^0 \gamma} = 0.14 \pm 0.25 \pm 0.03$, full stat ($471\text{e}6 B\bar{B}$)
- [Belle PRL101 \(2008\)](#): $S_{\pi^+ \pi^- K_S^0 \gamma} = 0.11 \pm 0.33 \pm 0.07$, full stat ($657\text{e}6 B\bar{B}$)

Statistically limited

- $\text{BR}(K_S^0 \pi^+ \pi^- \gamma) = 1 \times 10^{-5}$
- $\text{BR}(K_S^0 \pi^0 \gamma) = 7 \times 10^{-6}$

Need a large dataset

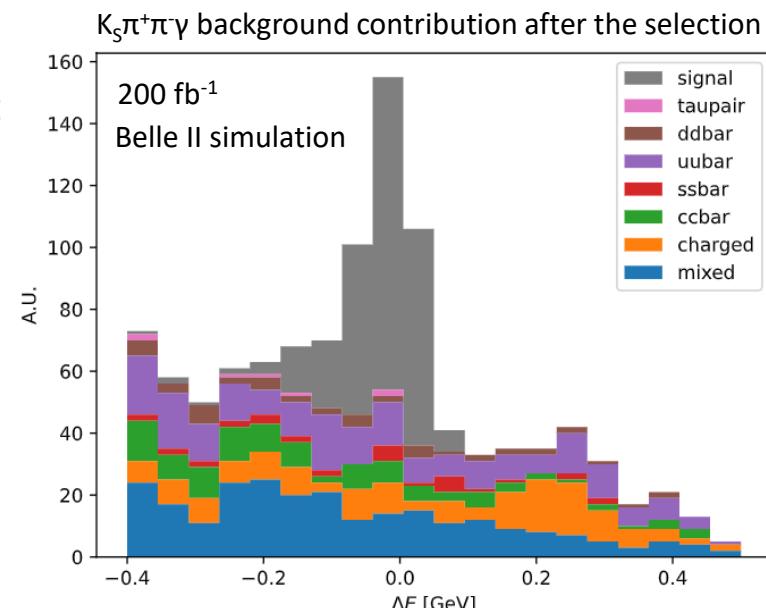


- **Short term plan:** Rediscovery and branching fraction measurement for both channels

Analysis strategy

- Reconstruction and preselection
 - Reconstruct final states particles and K_s^0 , then K_{res} resonance and finally the B meson with a vertex fit
- Selection
 - Apply particle identification requirements on π^\pm and π^0 veto on γ
 - Optimize the selection with $\frac{S}{\sqrt{S+B}}$ FOM with a few variables:
 - Multivariate classifier: remove dominant background $e^+e^- \rightarrow q\bar{q}$ ($c\bar{c}, s\bar{s}, u\bar{u}, d\bar{d}$)
 - $m_{K_s^0\pi^0}$: reduce other K_{res} (& soft cut for $m_{K\pi\pi}$)
 - K_s^0 decay length significance: good K_s^0 selection
 - M_{bc} : reduce continuum and $B\bar{B}$ background
 - Best candidate selection selecting the best B^0 vertex fit

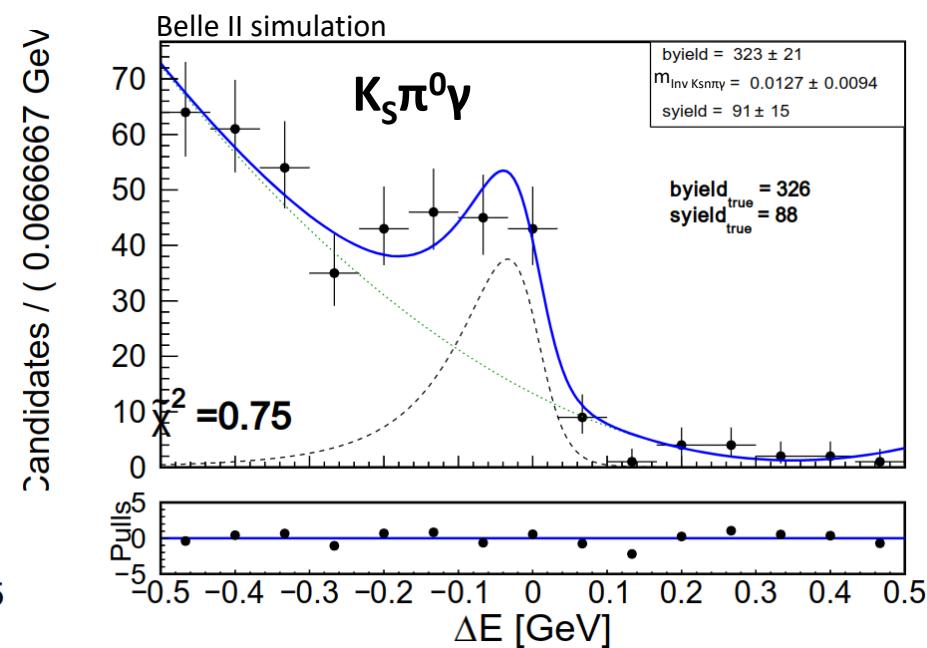
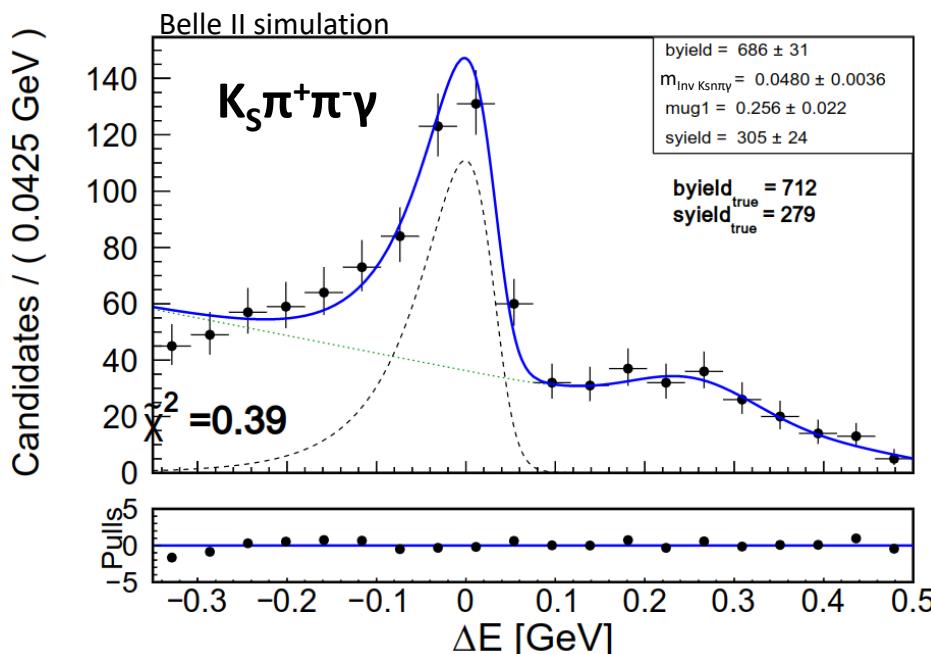
> $\sim 7.5\%$ and $\sim 8.8\%$ efficiency for $K_s\pi^+\pi^-\gamma$ and $K_s\pi^0\gamma$ with ~ 12.3 and ~ 6.0 FOM respectively
- ΔE fit to extract the signal yield



ΔE fit

Global fit on the ΔE distribution of signal+background corresponding to 200 fb^{-1} :

- Johnson for the signal and Chebyshev for the background
- Shape parameters fixed from the background and signal only fit
- Yields and means left free**



- Fitted **yields** and μ compatible with true values
- Expected yields in final Data sample:
 - $K_S\pi^+\pi^-\gamma$: 305 ± 24
 - $K_S\pi^0\gamma$: 91 ± 15

Conclusion

- $B \rightarrow K_s \pi^+ \pi^- \gamma$ and $B \rightarrow K_s \pi^0 \gamma$ promising decays to discover NP
- **Short term:** Rediscovery and branching fraction measurement
 - Analysis close to unblinding
 - Target winter conferences
- **Long term:** Full TDCP analysis to measure S^+ and S^-

Thank you for your attention

Tristan Fillinger

16/11/21

GDR – InF annual workshop



Backup

Tristan Fillinger

16/11/21

GDR – InF annual workshop



Belle II detector

Electromagnetic calorimeter (ECL):

CsI(Tl) crystals

waveform sampling (energy, time, pulse-shape)

$e^- (7 \text{ GeV})$

Vertex detectors (VXD):

2 layer DEPFET pixel detectors (PXD, partially installed)

4 layer double-sided silicon strip detectors (SVD)

Central drift chamber (CDC):

He(50%):C₂H₆ (50%), small cells,

fast electronics

K_L and muon detector (KLM):

Resistive Plate Counters (RPC) (outer barrel)

Scintillator + WLSF + MPPC (endcaps, inner barrel)

Magnet:

1.5 T superconducting

Trigger:

Hardware: < 30 kHz

Software: < 10 kHz

Particle Identification (PID):

Time-Of-Propagation counter (TOP) (barrel)

Aerogel Ring-Imaging Cherenkov Counter (ARICH) (FWD)

DEPFET: depleted p-channel field-effect transistor

WLSF: wavelength-shifting fiber

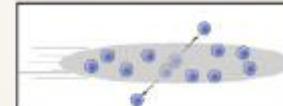
MPPC: multi-pixel photon counter



Background types

Machine background

Touschek scattering: single Coulomb scattering event between two particles of the same bunch, that are lost.



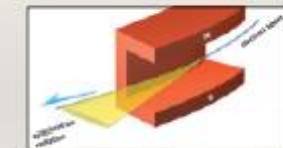
$$R_{Tou} \propto \frac{1}{\sigma E^3 n_b} I_{beam}^2$$

Beam-gas scattering: Coulomb elastic scattering or bremsstrahlung with residual gas atoms.



$$R_{bg} \propto IP$$

Synchrotron Radiation (SR): photon emission from beam particles when subject to acceleration.



$$W_{SR} \propto \frac{E^4}{\rho^2}$$

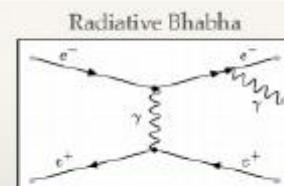
Injection background: injected bunch performing betatron oscillation around the stored bunch, resulting in particle losses especially in the interaction region.



$$R_I \propto R_{inj}$$

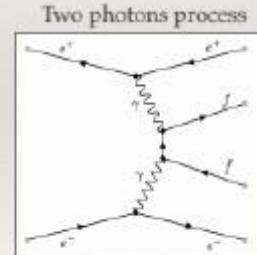
Luminosity background

Radiative Bhabha: neutron production from emitted photons (shields used for mitigation); off-energy primary particles lost in final focus magnets.



$$R_{RB} \propto L$$

Two photons process: low momentum electron-positron pairs that can generate multiple hit in the VerteX Detector.



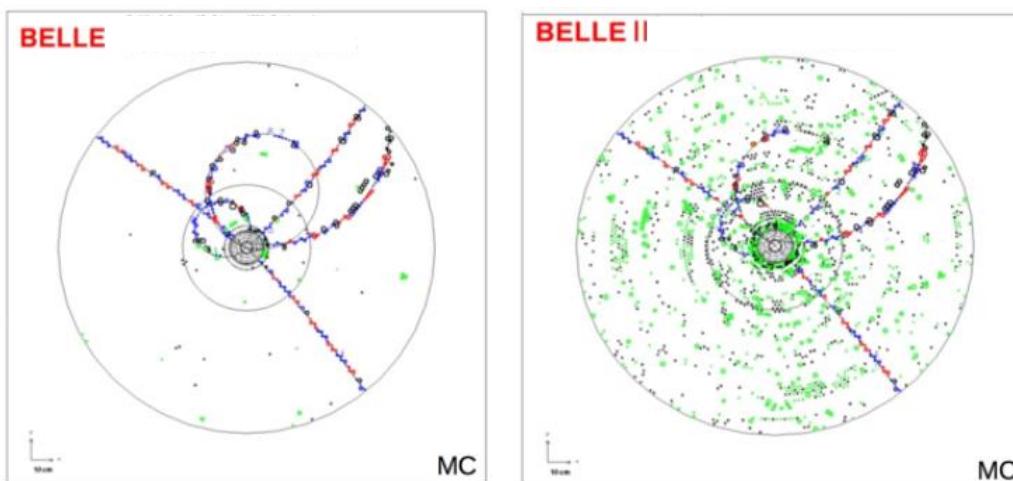
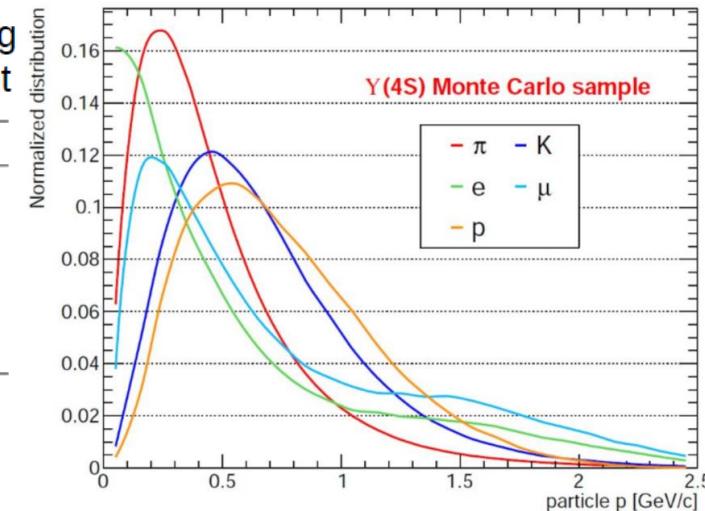
$$R_{RB} \propto L$$

Challenge of tracking at Belle II

- Average track multiplicity:
 - **11 physics tracks.**
- **Similar momentum ranges and distributions.**
- **Low momentum tracks**
 > multiple scattering, curling tracks.

Particle types visible in Tracking Detectors of typical Y(4S) event

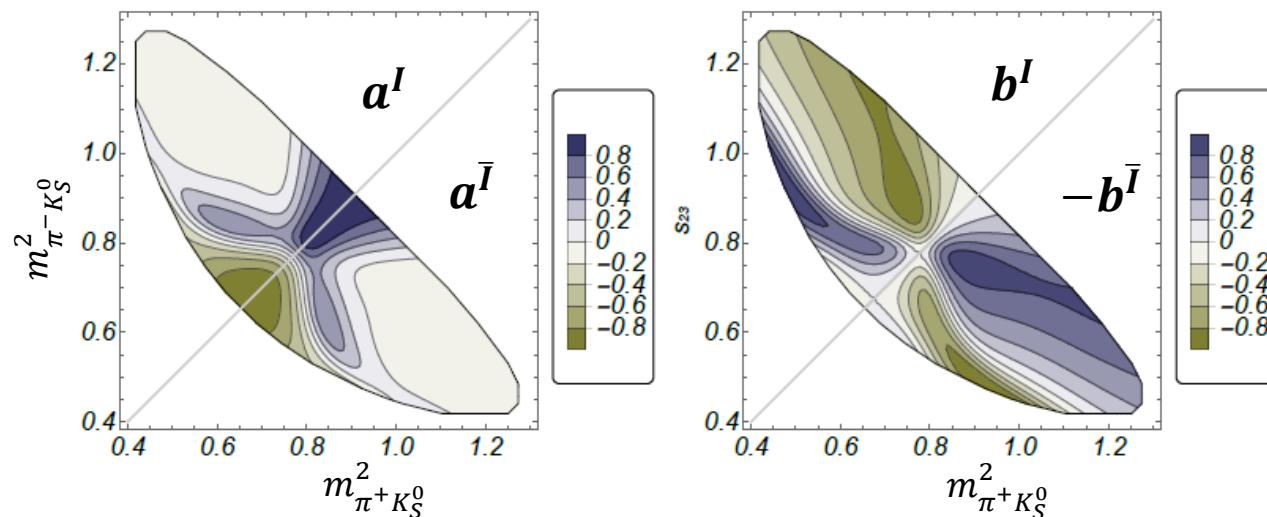
Particle type	Average fraction
π^\pm	72.8%
K^\pm	14.9%
e^\pm	5.8%
μ^\pm	4.7%
p^\pm	1.8%



- Sizeable **beam-induced background.**
- Occupancy **dominated by background**

Hadronic parameters

- Real a^I and imaginary b^I part of the hadronic contribution (hadronic parameters)
- Depend on dilution factor
- Can be obtained with $B^+ \rightarrow K^+\pi^+\pi^-\gamma$ amplitude analysis



$$S_{\pi^+\pi^-K_S^0\gamma}^+ \left(\frac{C'_{17}}{C_7}, a^I \right) = S^I + S^{\bar{I}}$$

$$S_{\pi^+\pi^-K_S^0\gamma}^- \left(\frac{C'_{17}}{C_7}, b^I \right) = S^I - S^{\bar{I}}$$

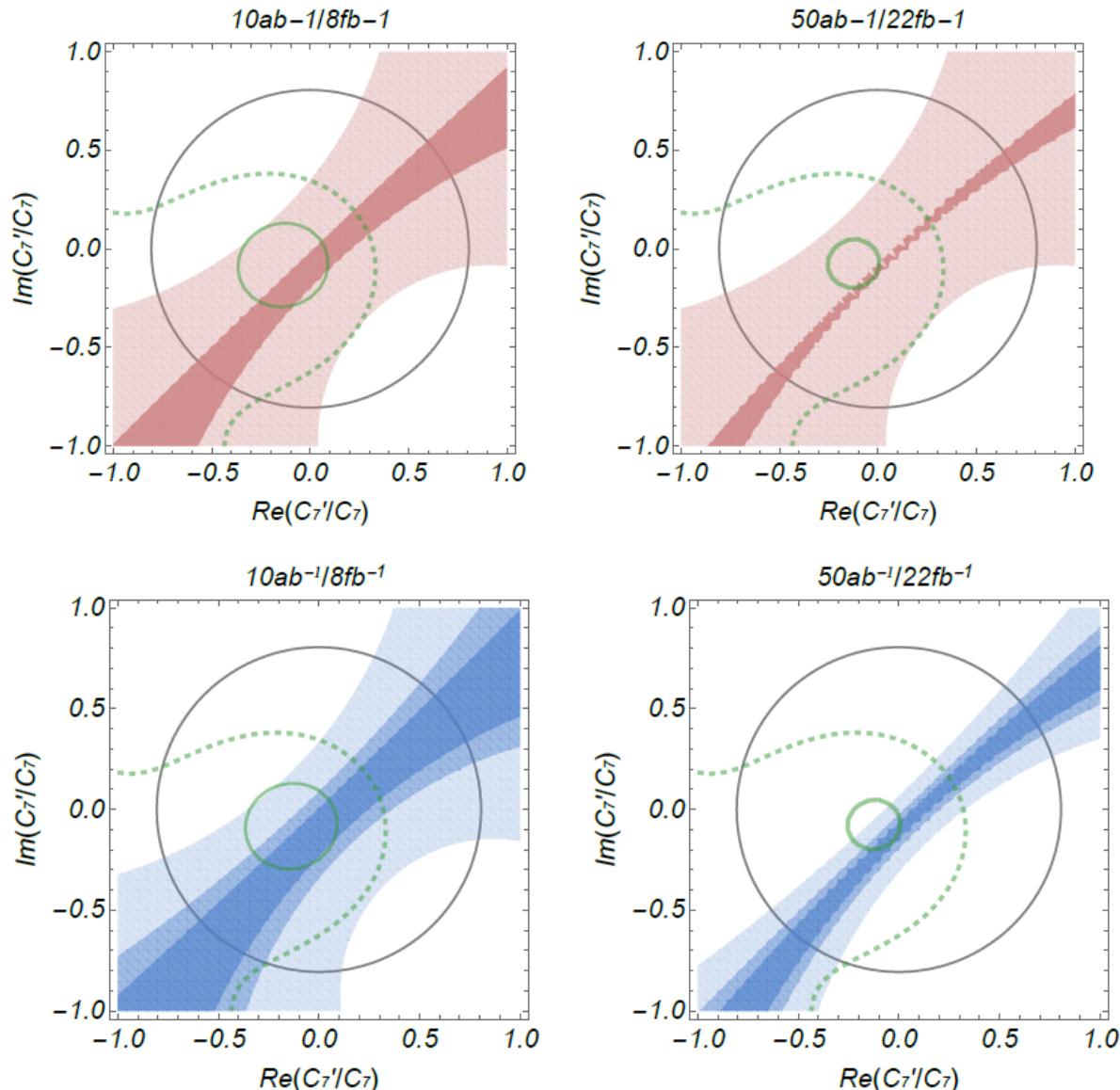
Goal of the study

Constrain on imaginary and real part of C_7 and C'_7 using:

- Red: $S_{\pi^0 K_S^0 \gamma}$
Eur. Phys. J. C77 (2017) 895 [1612.07233]
- Green: Angular coefficient of
 $B \rightarrow K^* ee$ at $q^2 \rightarrow 0$ (LHCb)
JHEP 04 (2015) 064 [1501.03038]
- Blue: $S_{\pi^+ \pi^- K_S^0 \gamma} \left(\frac{C'_7}{C_7}, \mathcal{D} \right)$ with
different dilution factor
values

$$\mathcal{D} = \frac{S_{\pi^+ \pi^- K_S^0 \gamma}}{S_{\rho^0 K_S^0 \gamma}}$$

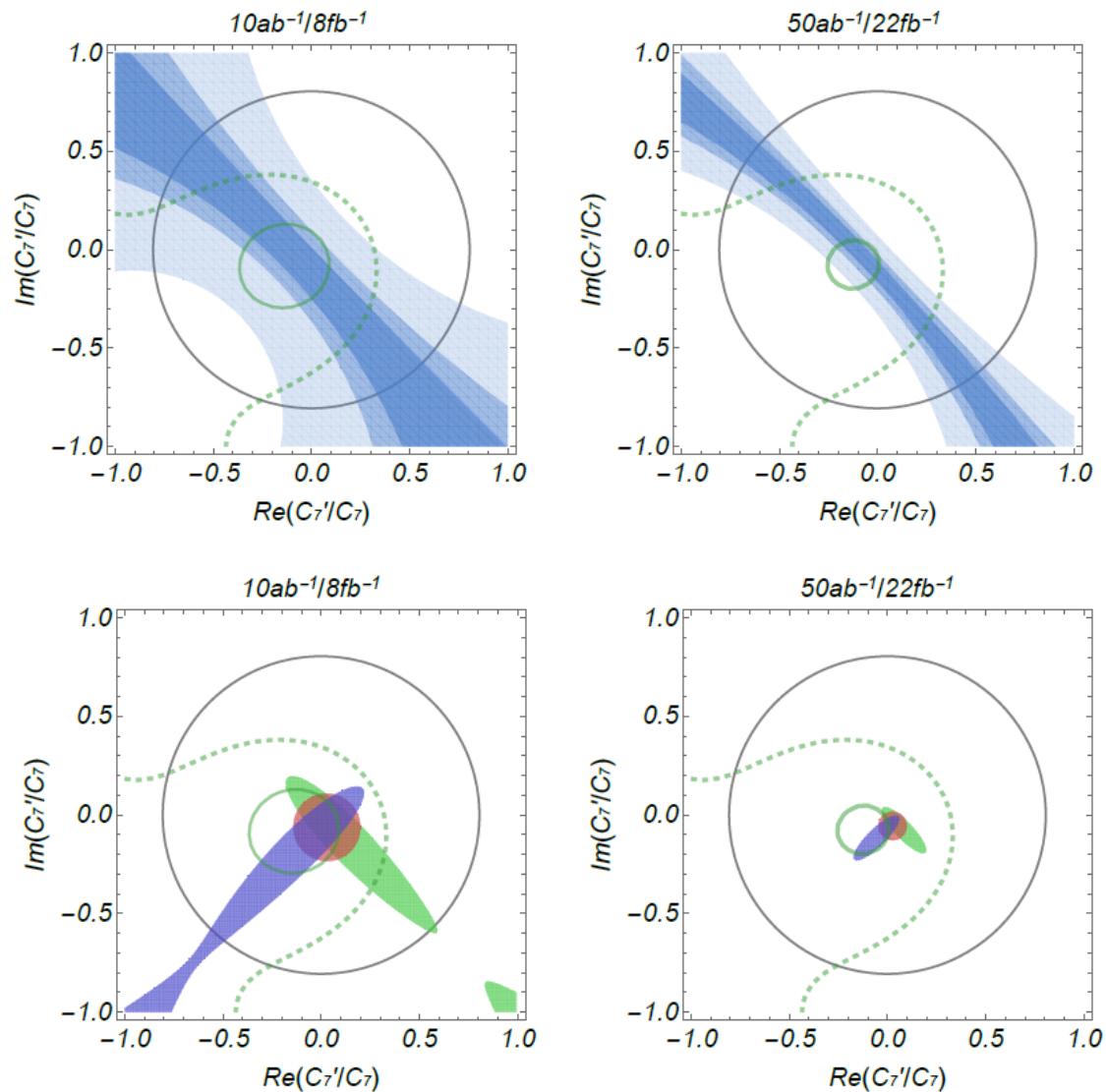
$$S_{\pi^+ \pi^- K_S^0 \gamma}^+ \left(\frac{C'_7}{C_7}, a^I \right)$$
 same area
covered



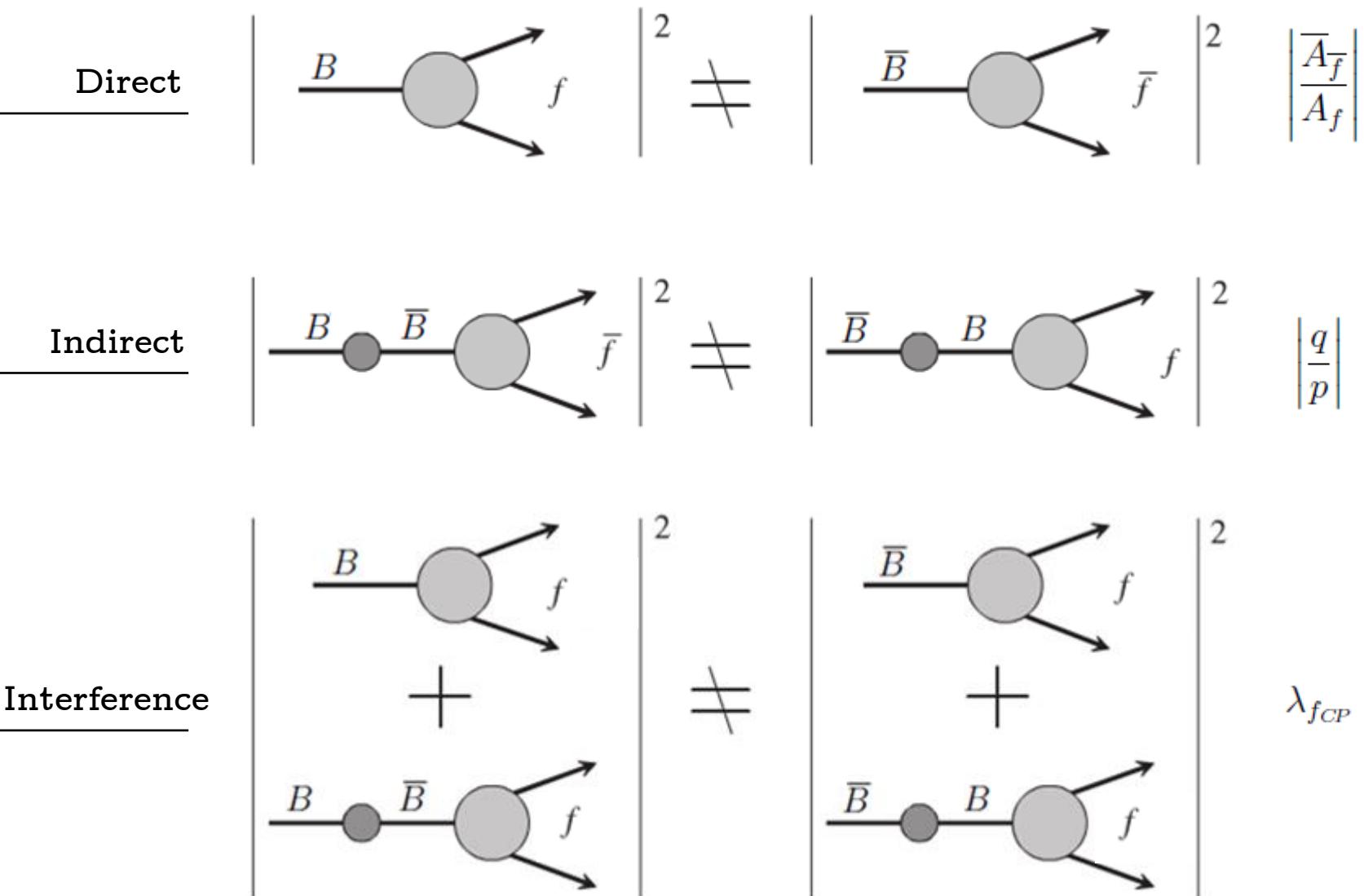
Goal of the study

Constrain on imaginary and real part of C_7 and C'_7 using:

- Blue: $S_{\pi^+\pi^-K_S^0\gamma}^-\left(\frac{C'_7}{C_7}, b^I\right)$ with different hadronic parameters (linked to the dilution factor)
- Combination of $S_{\pi^+\pi^-K_S^0\gamma}^+$, $S_{\pi^+\pi^-K_S^0\gamma}^-$ with different hadronic parameters
> Competitive with the LHCb measurement



CP violation type



CP violation type

B meson decay rate

$$f_{q_{\text{tag}}}(\Delta t) = \frac{e^{-|\Delta t|/\tau}}{4\tau} [1 + q_{\text{tag}} \underbrace{(\mathcal{S} \sin(\Delta m_d \Delta t) - \mathcal{C} \cos(\Delta m_d \Delta t))}_{\mathcal{A}_{CP}(\Delta t)}]$$

Observable containing CP violation information

$$\mathcal{S} = \frac{2\Im(\lambda_{f_{CP}})}{1 + |\lambda_{f_{CP}}|^2} \neq 0 \Rightarrow \text{Violation CP } \textit{indirecte} \quad \mathcal{S} = -\eta_{CP} \sin(2\beta)$$

$$\mathcal{C} = \frac{1 - |\lambda_{f_{CP}}|^2}{1 + |\lambda_{f_{CP}}|^2} \neq 0 \Rightarrow \text{Violation CP } \textit{directe} \quad \mathcal{C} = 0$$

$$\lambda_{f_{CP}} = \eta_{CP} \frac{q}{p} \frac{\overline{A}_{\bar{f}}}{A_f} \neq 1 \Rightarrow \text{Violation CP} \quad \left| \frac{\overline{A}_{\bar{f}}}{A_f} \right| \neq 1 \quad \left| \frac{q}{p} \right| \neq 1$$

Time dependent CP violation (TDCPV)

B meson decay rate:

$$f_{q_{\text{tag}}}(\Delta t) = \frac{e^{-|\Delta t|/\tau}}{4\tau} [1 + q_{\text{tag}} (\mathcal{S} \sin(\Delta m_d \Delta t) - \mathcal{C} \cos(\Delta m_d \Delta t))]$$

- \mathcal{C} : Direct CP asymmetry
- \mathcal{S} : Indirect CP asymmetry

- τ : B^0 lifetime
- q_{tag} : $\begin{cases} +1 \text{ for } B^0 \\ -1 \text{ for } \bar{B}^0 \end{cases}$
- $\Delta m_d = M_H - M_L$
- Δt : Time difference between the decay of the B

→ In the standard model : \mathcal{S} and $\mathcal{C} = 0$ in $b \rightarrow s$ radiative decays due to its V-A structure (photon almost fully polarized) but sensitive to new physics

Atwood et al: <https://doi.org/10.1103/PhysRevLett.79.185>

→ If new physics : \mathcal{S} and $\mathcal{C} \neq 0$

No deviations have been observed but the measurement is statistically limited

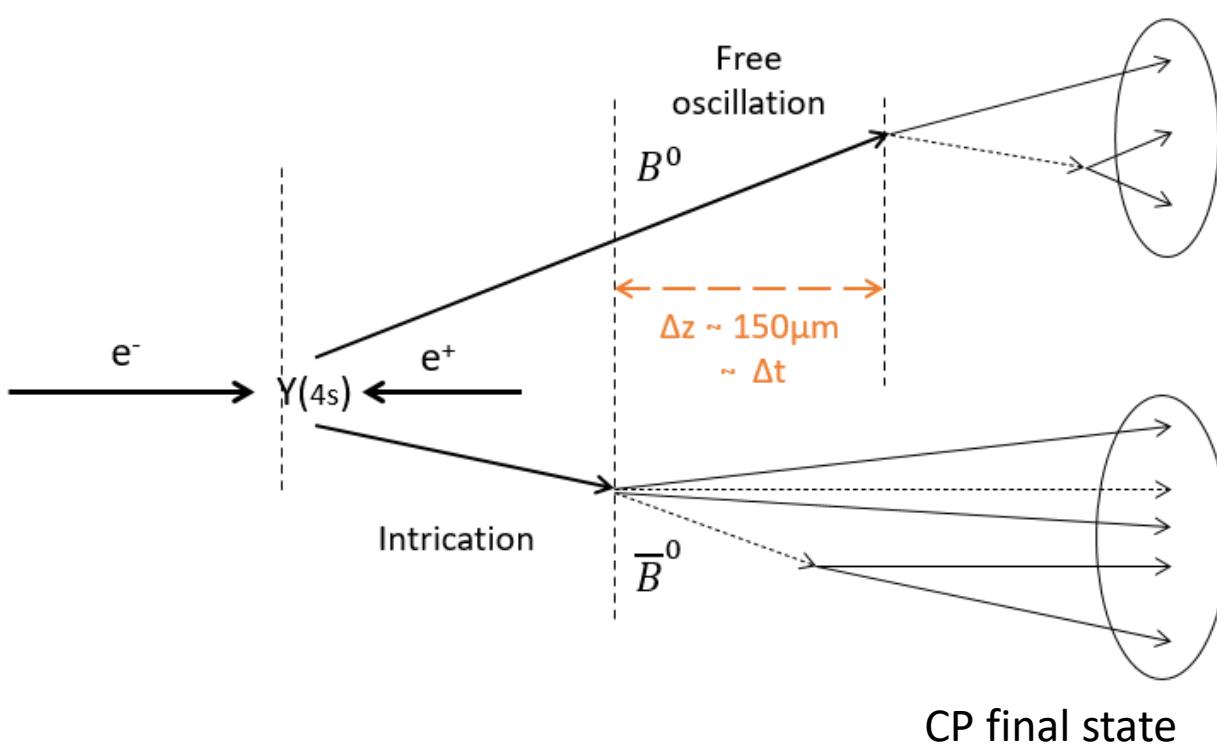
Belle: Li Jin et al: Phys.Rev.Lett.101:251601,2008

Babar: P. del Amo Sanchez et al: 10.1103/PhysRevD.93.052013

TDCP asymmetry, experimental function

$$P(\Delta t, q) = f_{sig} \frac{e^{-|\Delta t|/\tau}}{4\tau} * [1 - q \Delta w + q(1 - 2w)(A \cos \Delta m_d \Delta t + S \sin \Delta m_d \Delta t)] \otimes R_{sig} + (1 - f_{sig}) P_{bkg}$$

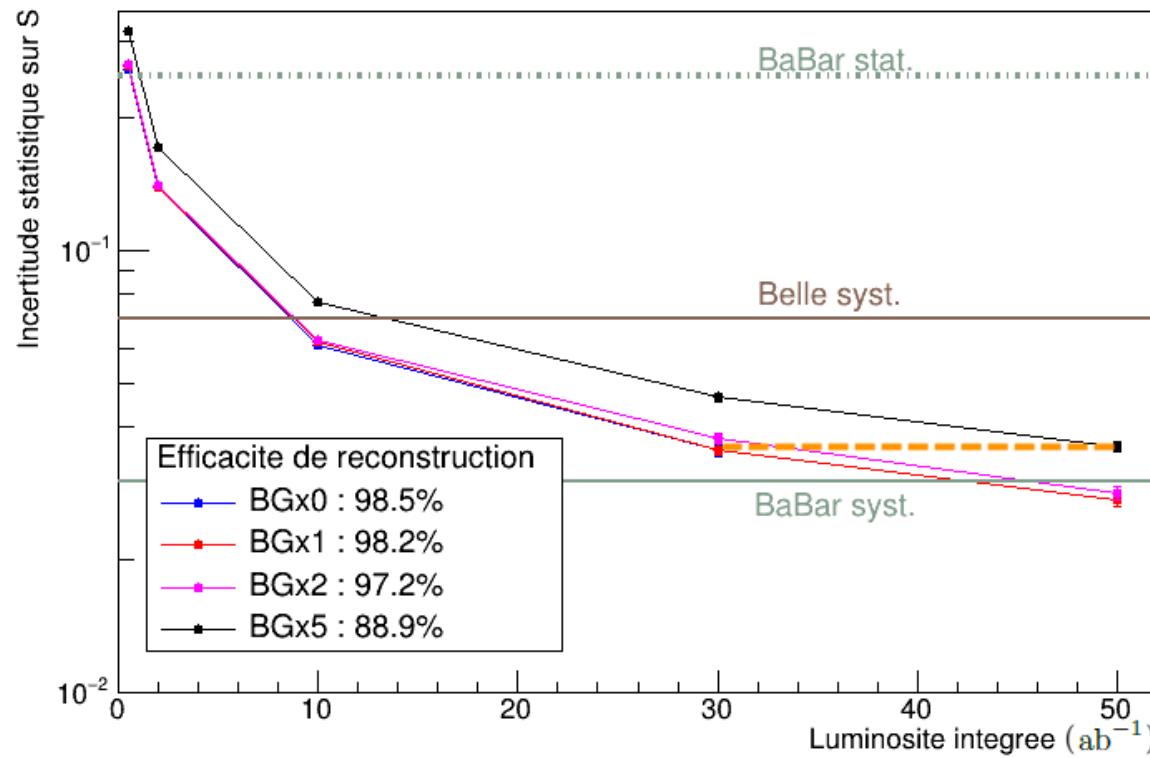
Exponential decay	Oscillation	Background
-------------------	-------------	------------



- $\tau : B^0$ lifetime
 - $q_{tag} : \begin{cases} +1 & \text{for } B^0 \\ -1 & \text{for } \bar{B}^0 \end{cases}$
 - $\Delta m_d = M_H - M_L$
 - Δt : Time difference between the decay of the B
 - w : Mistag factor
 - f_{sig} : Signal fraction
 - R_{sig} : Resolution function
 - P_{bkg} : Background pdf

Results – TDCPV ToyMC

- Simple case: $\begin{cases} B^0 \rightarrow K (\rightarrow \pi\pi) \pi\pi\gamma \\ B^0 \rightarrow \pi + X \end{cases}$ → 5 pions in the final state
- Number of event for each pseudo-experiment: $N = L\sigma(\epsilon_\pi)^5$



Dataset used

- **MC: 200 fb⁻¹ sample**
 - $B^0\bar{B}^0, B^+B^-, c\bar{c}, s\bar{s}, u\bar{u}, d\bar{d}, \tau\tau$
 - Same size as the final Data dataset
 - Used to optimize the selection, determine the fit functions and background rejection
- **MC Signal only samples:**
 - 2M evenements
 - $B^0 \rightarrow X_{sd} \gamma \rightarrow K_s^0 \rho^0 \rightarrow \pi^+\pi^- \pi^+\pi^-$
 - $B \rightarrow K^{*0} \gamma \rightarrow K_s^0 \pi^0 \rightarrow \pi^+\pi^-$
 - Used to calculate the signal efficiency
- **Data: 190 fb⁻¹**

Analysis strategy

- Reconstruction and preselection
 - Reconstruct final states particles and K_s
 - Reconstruct K_{res} resonance
 - Reconstruct B meson + vertex fit
- Apply “systematic” cuts (π^\pm identification and γ pollution from π^0)
- Apply Continuum Suppression ([CSMVA](#))
 - Remove the dominant background : $e^+e^- \rightarrow q\bar{q}$ ($c\bar{c}, s\bar{s}, u\bar{u}, d\bar{d}$)
 - Trained with a fastBDT using 30 continuum variables: Fox-Wolfram moments, CLEO cones...
- Optimize the selection based on a few variables
- Best candidate selection selecting the best B^0 vertex fit
- ΔE fit
 - Determine PDFs shapes by separating signal and background at MC level
 - Fit with free yields

Selection strategy

- Selected variables:
 - **CSMVA output:** reduce continuum
 - $m_{K_s^0\pi^0}$: reduce other K_{res} (& soft cut for $m_{K\pi\pi}$)
 - K_s^0 decay length significance: good K_s selection, further reduce continuum
 - M_{bc} : reduce continuum and $B\bar{B}$ background
- Figure of merit (FOM): $\frac{S}{\sqrt{S+B}}$
- In the signal range: $5.26 < m_{bc} < 5.3 \text{ GeV}/c$; $-0.2 < \Delta E < 0.1 \text{ GeV}$
- Selection procedure:
 1. For each variable, the FOM is measured as a function of the [variable cut values](#) and the one with the [highest FOM](#) is chosen.
 2. All found cuts are applied, then a new variable is optimized the same way
 3. Repeat until no variable left
- The order of the variables is also optimized

Selection efficiencies on MC

 $K_S\pi\pi\gamma$

Cut	cut value	FOM
$m_{K_s\pi\pi}$	$m_{K_s\pi\pi} < 1.8 \text{ GeV}/c^2$	2.46 ± 0.09
M_{bc}	$M_{bc} > 5.275 \text{ GeV}/c^2$	3.58 ± 0.13
CSMVA	$\text{CSMVA} > 0.9$	8.2 ± 0.4
$K_s^{sigDist}$	$K_s^{sigDist} > 45$	12.3 ± 0.6

Cut	sgn eff [%]	bkg retention [%]
Reconstruction + preselection	26.7 ± 0.7	–
$m_{K_s\pi\pi}$	79.4 ± 1.3	19.94 ± 0.06
M_{bc}	90.7 ± 1.0	38.60 ± 0.16
CSMVA	55.6 ± 1.9	4.90 ± 0.11
$K_s^{sigDist}$	82.8 ± 1.9	19.8 ± 0.9
Combined selection	41.7 ± 1.8	0.375 ± 0.020
Single candidate selection	85.2 ± 2.0	84.2 ± 1.1
Total efficiency	7.5 ± 0.4	–

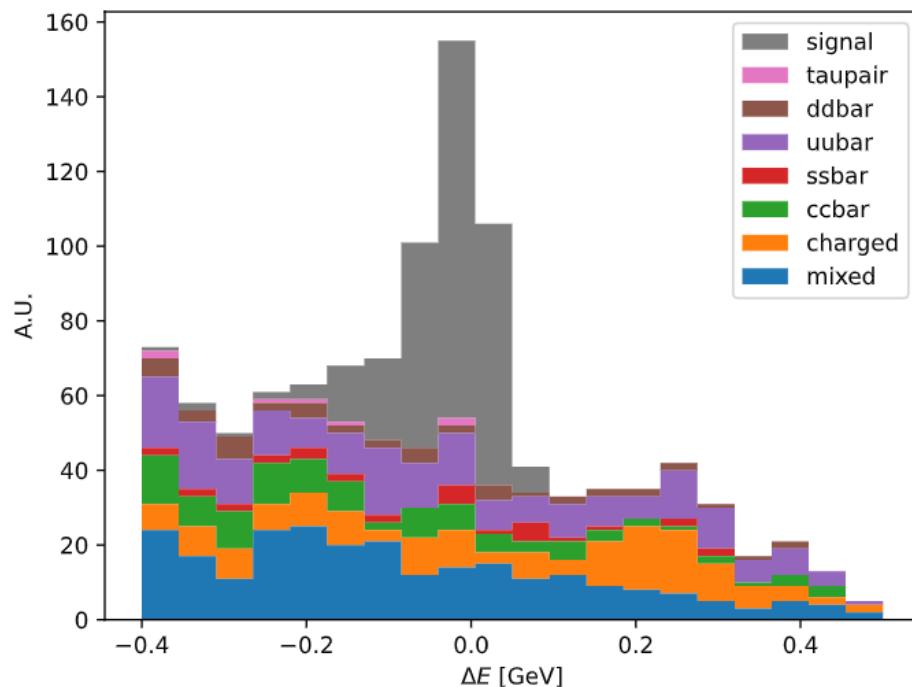
 $K_S\pi^0\gamma$

Cut	cut value	FOM
$K_s^{sigDist}$	$K_s^{sigDist} > 72$	1.14 ± 0.08
CSMVA	$\text{CSMVA} > 0.94$	4.3 ± 0.4
$m_{K_s\pi^0}$	$m_{K_s\pi^0} < 1 \text{ GeV}/c^2$	5.0 ± 0.4
M_{bc}	$M_{bc} > 5.275 \text{ GeV}/c^2$	6.0 ± 0.5

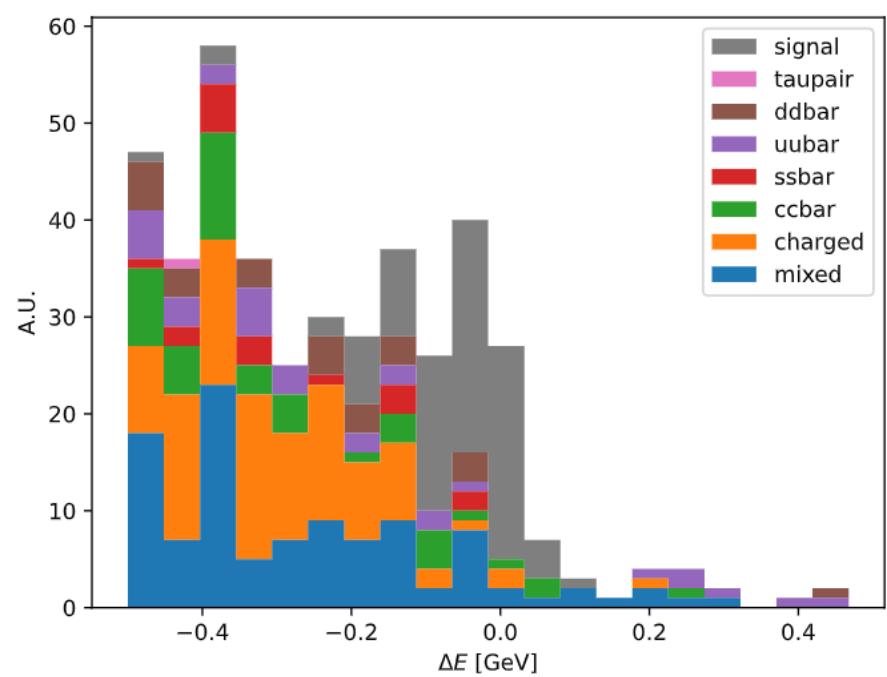
Cut	sgn eff [%]	bkg retention [%]
Reconstruction + preselection	28.865 ± 0.032	–
$K_s^{sigDist}$	77.76 ± 0.05	11.30 ± 0.06
CSMVA	54.41 ± 0.07	1.62 ± 0.07
$m_{K_s\pi^0}$	95.86 ± 0.04	61.4 ± 2.1
M_{bc}	87.10 ± 0.07	46.6 ± 2.8
Combined selection	35.33 ± 0.06	0.052 ± 0.004
Single candidate selection	86.57 ± 0.08	58.0 ± 2.1
Total efficiency	8.827 ± 0.020	–

Background contributions

$\text{K}_s \pi\pi\gamma$

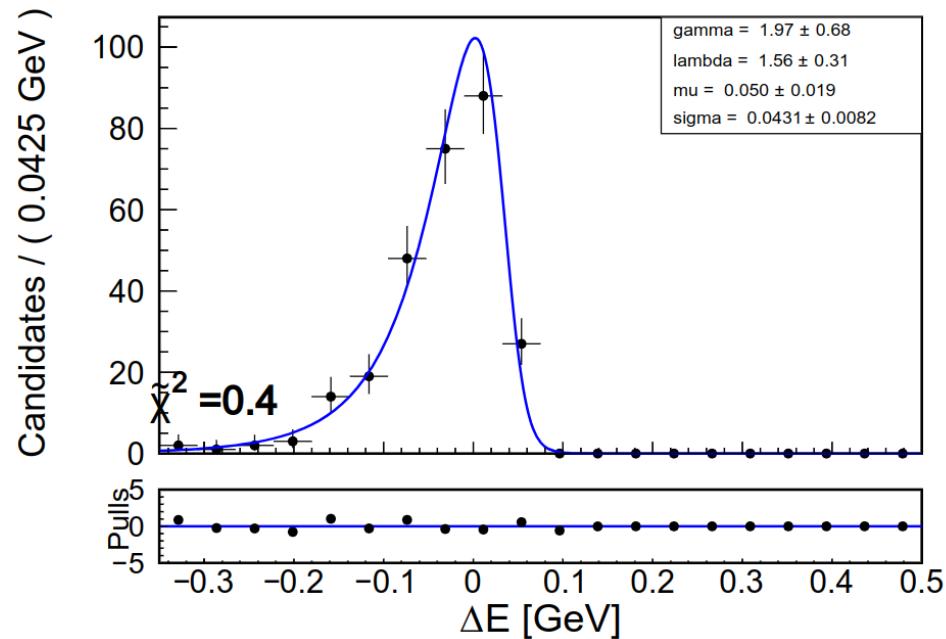
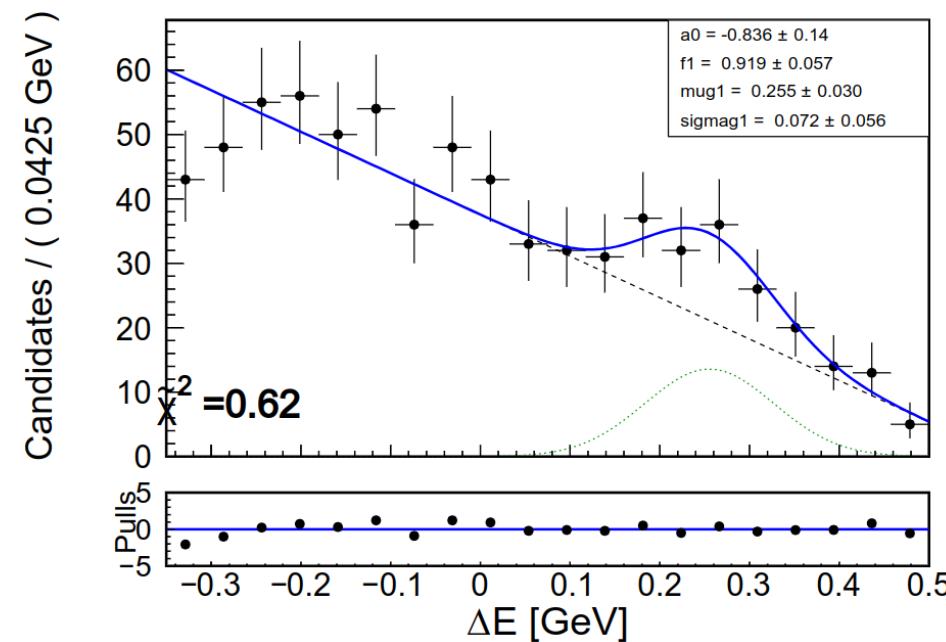


$\text{K}_s \pi^0\gamma$



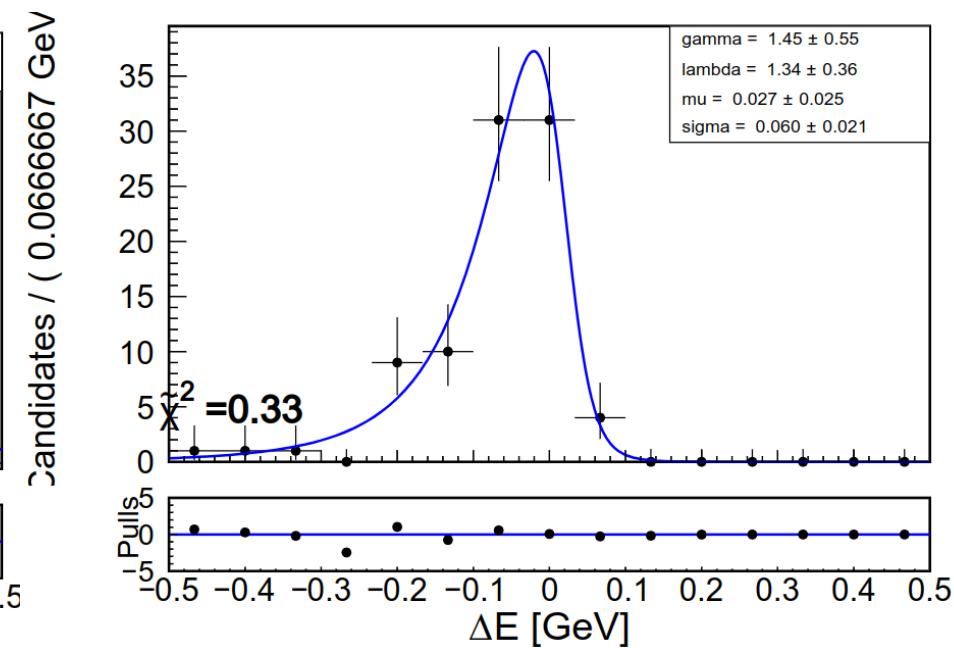
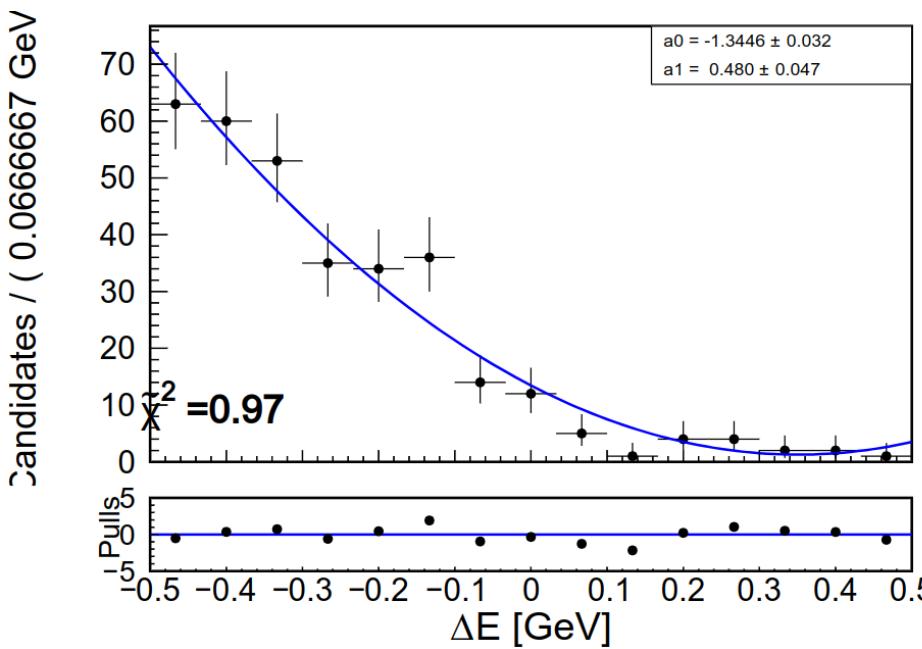
ΔE templates – $K_S \pi\pi\gamma$

- Done on the 200 fb^{-1} (to be 700 fb^{-1}) data sample:
 - In the range: $-0.35 < \Delta E < 0.5 \text{ GeV}$
 - With the 5 cuts from the selection + single candidate selection
 - Signal fit: Johnson, 4 parameters free: μ, σ central region, $\lambda \gamma$ slopes
 - Background fit: Chebychev + Gaussian, 1 + 2 parameters free



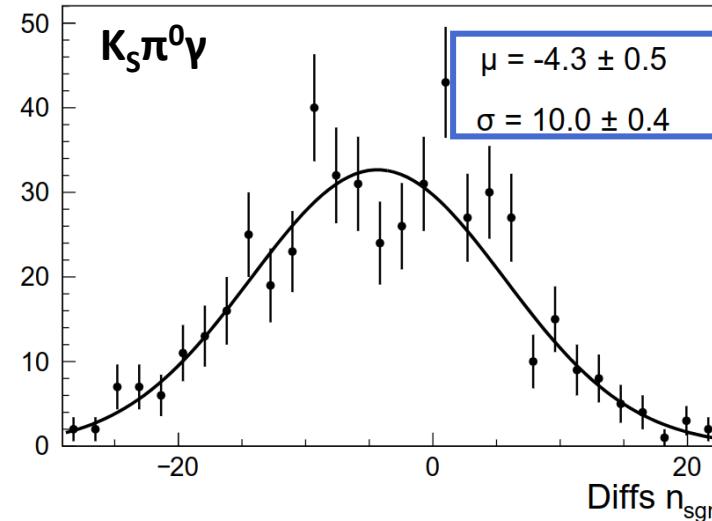
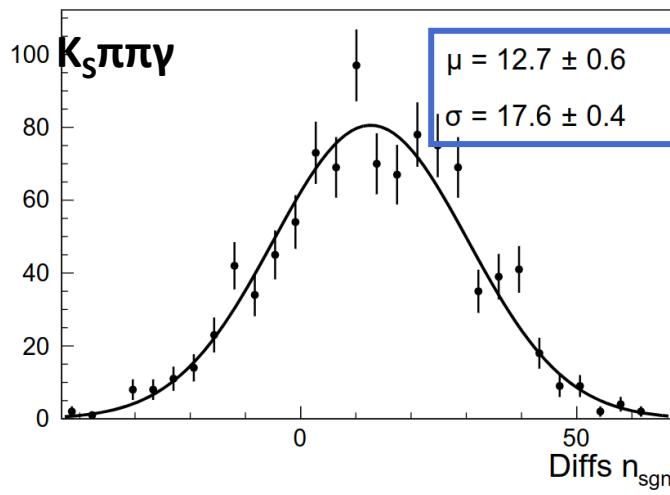
ΔE templates – $K_S \pi^0 \gamma$

- Done on the 200 fb^{-1} (to be 700 fb^{-1}) data sample:
 - In the range: $-0.4 < \Delta E < 0.5 \text{ GeV}$
 - With the 5 cuts from the selection + single candidate selection
 - Signal fit: Johnson, 4 parameters free: μ, σ central region, $\lambda \gamma$ slopes
 - Background fit: Chebychev, 2 parameters free



Rigorous Toys

Principle: On the 200 fb^{-1} data sample: bootstrap $500 \times 200 \text{ fb}^{-1}$ dataset and fit



$$\text{Diff}_i = n_{\text{sgn } i} - n_{\text{true sgn}}$$

True yields
 $K_S \pi \pi \gamma : 279$
 $K_S \pi^0 \gamma : 88$

- Minor bias and overestimation of the uncertainty
- Small enough to be treated as a systematic ($\sim 4.7\%$ and $\sim 4.8\%$ for $K_S \pi \pi \gamma$ and $K_S \pi^0 \gamma$)