# Measurement of the CKM Angle $\gamma$ with $B^0 \to DK^+\pi^-$, $D \to K^0_{\rm S}h^+h^-$ (Double Dalitz) at LHCb

Yuya Shimizu

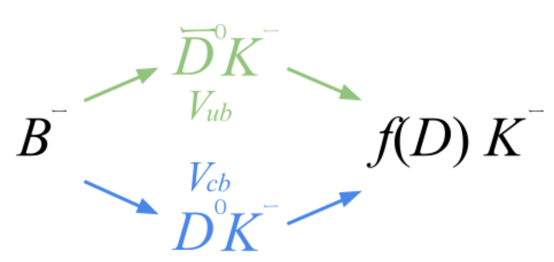
Université Paris-Saclay, France

GDR-InF annual meeting 17.11.2021

# Outline

- Measurement of $\gamma$
- Use of 3-body decay of $D$, BPGGSZ method
- $\gamma$ with $B^0$ Dalitz and $D$ Dalitz, Double Dalitz method
- Ongoing work at the LHCb

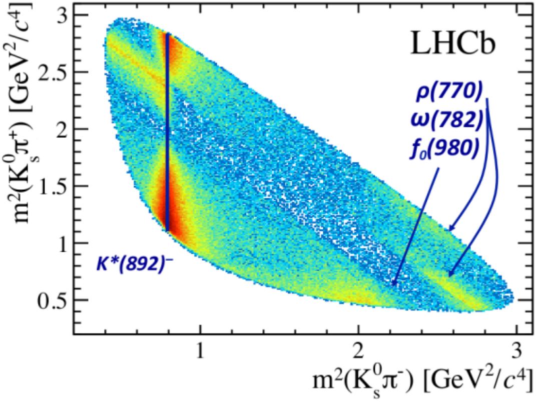
**Interference between $b \to u$ and $b \to c$**




- A typical channel to measure $\gamma$ is $B^{\pm} \to DK^{\pm}$
- The decay can go via either $D^0$ or $\overline{D}^0$
- Interference when the final state $f$ is accessible from both $D^0$ and $\overline{D}^0$
- One such example is $D \to K^0_S \pi^+ \pi^-$

# Self-conjugated 3-body decay of $D$ (BPGGSZ method)

- 3-body decay of $D$ can be used [1] [2]
- For example, $B^{\pm} \to DK^{\pm}$, $D \to K^0_{\mathrm{S}}\pi^+\pi^-$
- The sensitivity to $\gamma$ is enhanced thanks to the resonances in $D \to K^0_{\mathrm{S}}\pi^+\pi^-$
  - Cabibbo suppressed $D \to K^0_{\mathrm{S}}\rho$
  - Doubly Cabibbo suppressed $D \to K^{*+}\pi^-$



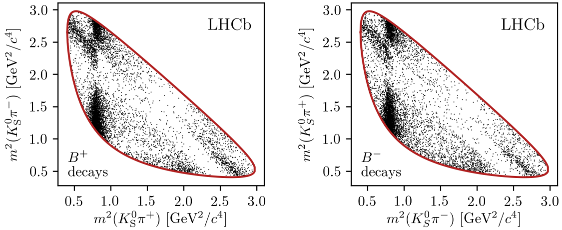
- The amplitude of $B^{\pm} \to DK^{\pm}$, $D \to K_{\mathrm{S}}^{0}\pi^{+}\pi^{-}$ can be written as

$$A \propto \overline{A}_D + r_B e^{i(\delta_B+\gamma)} A_D$$

- $r_B$: ratio of the suppressed to the favoured decays
- $\delta_B$: strong phase difference between the suppressed and favoured decays
- $\delta_D$: strong phase difference between $D^0$ $(\overline{D}^0) \to f$

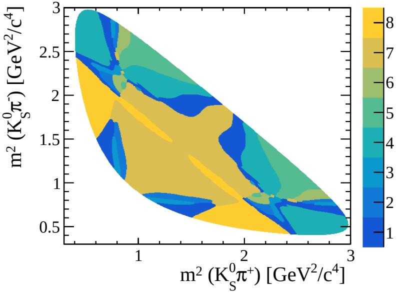
## Partial width as a function of the position on the Dalitz plane

$$d\Gamma_{B^{\pm}\to DK^{\pm}}(x) = \overline{A}_D^2 + r_B A_D^2 + 2\overline{A}_D A_D[r_B \cos(\delta_B \pm \gamma)\cos\delta_D + r_B \sin(\delta_B \pm \gamma)\sin\delta_D]$$




$B^+ \to DK^+$ $B^- \to DK^-$

# Binned $D$ Dalitz



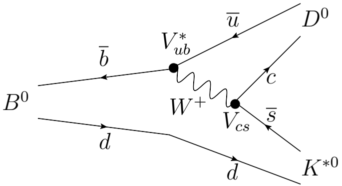

- Binned $D$ Dalitz method is model-independent
- Binning scheme is chosen to maximise sensitivity to $\gamma$ [3]
- Run 1+2 analysis has measured $\gamma = (68.7^{+5.2}_{-5.1})^\circ$, which is the most precise measurement of $\gamma$ from a single analysis [4]

## Number of events in bin *i*

$$N^{+}_{\pm i} = h_{B+}[K_{\mp i} + (x_+^2 + y_+^2)K_{\pm i} + 2\sqrt{K_i K_{-i}}(x_+ c_{\pm i} - y_+ s_{\pm i})]$$

$$N^{-}_{\pm i} = h_{B-}[K_{\pm i} + (x_-^2 + y_-^2)K_{\mp i} + 2\sqrt{K_i K_{-i}}(x_- c_{\pm i} - y_- s_{\pm i})]$$

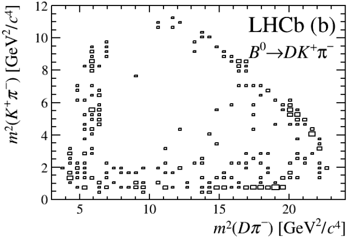
- $x_\pm = r_B \cos(\delta_B \pm \gamma)$
- $h_{B\pm}$: normalisation factor,
- $K_{+(-)i}$: fraction of $D^0$ ($\overline{D}^0$) $\to f$ in bin $i$, estimated using $B^\pm \to D\pi^\pm$ control mode
- $y_\pm = r_B \sin(\delta_B \pm \gamma)$
- $c_{\pm i}, s_{\pm i}$: sine and cosine of the strong phase difference between $D^0$ ($\overline{D}^0$) $\to f$, taken from CLEO and BESIII

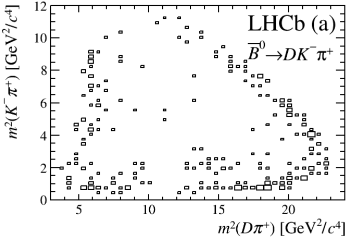



- The branching fraction is small ($\sim 5 \times 10^{-5}$)
- However, $r_B \sim 0.3$ which provides larger interference than $B^{\pm} \to DK^{\pm}$ ($\sim 0.1$)
- Model-independent BPGGSZ analysis has been done for Run 1 [5]
- Run 1+2 analysis is ongoing

# Measuring $\gamma$ with $B^0 \to DK^+\pi^-$

- We can include the entire phase space of $B^0 \to DK^+\pi^-$
- Having different resonances can give additional sensitivity to $\gamma$
  - $B^0 \to DK_0^*(1430)^0$
  - $B^0 \to DK_2^*(1430)^0$
  - $B^0 \to D_2^*(2460)^- K^+$
- No need to take into account the coherence factor as in $B^0 \to DK^{*0}$
- Simultaneously use $B$ Dalitz and $D$ Dalitz $\to$ Double Dalitz [6] [7]




$B$ Dalitz plot [8]

# Double Dalitz method

- The amplitude of $B^0 \to DK^+\pi^-$, $D \to K^0_{\mathrm{S}}\pi^+\pi^-$ can be written as

$$A \propto \overline{A}_B \overline{A}_D + e^{i\gamma} A_B A_D$$

- $A_B(\overline{A}_B)$: amplitude of $B^0 \to D^0(\overline{D}^0)K^+\pi^-$
- $A_D(\overline{A}_D)$: amplitude of $D^0(\overline{D}^0) \to f$

**Partial width as a function of the position on the Double Dalitz plane**

$$\begin{aligned} d\Gamma_{B^0\to DK^+\pi^-}(x) =& \overline{A}_B^2 \overline{A}_D^2 + A_B^2 A_D^2 + 2\overline{A}_B \overline{A}_D A_B A_D \\ & [(\cos\delta_B \cos\delta_D - \sin\delta_B \sin\delta_D)\cos\gamma - (\cos\delta_B \sin\delta_D - \sin\delta_B \cos\delta_D)\sin\gamma] \end{aligned}$$

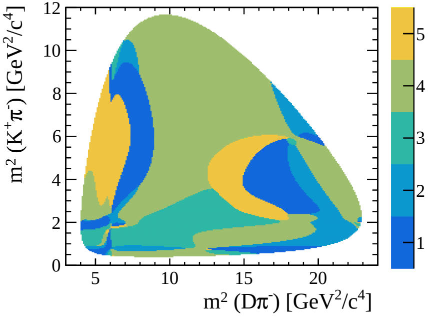
# Double Dalitz observables

**Number of events in each bin for $B^0 \to DK^+\pi^-$ (for $\overline{B}^0$ $\gamma \to -\gamma$)**

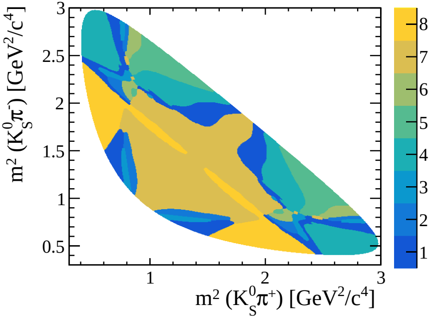
$$N_{\alpha i} = h\left\{\bar{\kappa}_\alpha K_{+i} + \kappa_\alpha K_{-i} + 2\sqrt{\kappa_\alpha K_{+i}\bar{\kappa}_\alpha K_{-i}}\left[(\chi_\alpha c_i - \sigma_\alpha s_i)\cos\gamma - (\chi_\alpha s_i + \sigma_\alpha c_i)\sin\gamma\right]\right\}$$

- $\alpha$ for $B$ Dalitz bin, $i$ for $D$ Dalitz bin
- $\kappa_\alpha$ ($\bar{\kappa}_\alpha$) fraction of $B^0 \to D^0(\overline{D}^0)K^+\pi^-$ in each bin $\alpha$
- $\chi_\alpha$, $\sigma_\alpha$ cosine and sine of strong phase difference between $B^0 \to D^0(\overline{D}^0)K^+\pi^-$ in each bin $\alpha$
- $K_{+(-)i}$ fraction of $D^0$ ($\overline{D}^0$) $\to f$ in bin $i$
- $c_i$, $s_i$ cosine and sine of strong phase difference between $D^0$ ($\overline{D}^0$) $\to f$
- $h$ overall normalisation factor
- $\kappa_\alpha, \bar{\kappa}_\alpha, \chi_\alpha, \sigma_\alpha$ **are shared across all decay modes and float in the fit**
  - additional decay modes improve precision to these parameters

- The binning scheme is based on $B^0$ and $D$ Dalitz planes from [7], [3]
- A single three–body $B$ ($D$) decay results in $2 \times 5$ ($2 \times 8$) bins.
- A Double Dalitz decay results in $2 \times 5 \times 16 = 160$ bins
- For 160 observables we have 23 free parameters
- $B^0 \to (D \to K_S^0 K^+ K^-) K^+ \pi^-$ can also be used with a suitable $D$ Dalitz binning



B Dalitz Plane [7]



D Dalitz Plane [3]

# Additional Decays

- We could add other decays in addition to 3-body $D$ final states
- $D \to K^+\pi^-$
  - the favoured control mode with low sensitivity to $\gamma$
  - but a high statistics provides sensitivity to the $B$ phase space parameters
  - it adds 10 observables:

$$N_\alpha = h\left\{\bar{\kappa}_\alpha + r_D^2\kappa_\alpha + 2\sqrt{\kappa_\alpha\bar{\kappa}_\alpha}\left[(\chi_\alpha\cos(\gamma-\delta_D) - \sigma_\alpha\sin(\gamma-\delta_D)\right]\right\}$$

- $D \to K^-\pi^+$
  - less sensitive compared to $B^+ \to DK^+$ because $r_B$ is larger
  - also have to manage the $B_s^0 \to D^{(*)}K^-\pi^+$ background
- $D \to K^+K^-, \pi^+\pi^-$
- $D \to \pi^+\pi^-\pi^+\pi^-, K^+\pi^-\pi^+\pi^-, K^-\pi^+\pi^+\pi^-$
  - for $K3\pi$ modes binning can be used [9]
- In principle, we could add more (e.g. $D \to h^+h^-\pi^0$)

# Extraction of $\gamma$

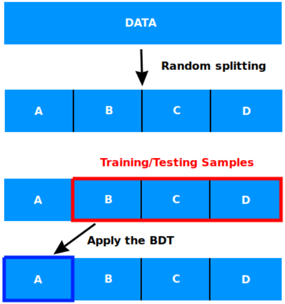
- First we need to remove background
  - Trigger and stripping requirement
  - Boosted Decision Tree to remove combinatorial background
  - Particle identification requirement particularly against $B^0 \to D\pi^+\pi^-$
  - and so on
- We fit the $B^0$ invariant mass globally rather than fit for each of 160 bins
- To get $N_{\alpha,i}$, we need to subtract the number of background
- For partially reconstructed background or mis-ID background we can use `Laura++` and ongoing $B^0 \to D^* K^+\pi^-$, $B_s^0 \to D^* K^-\pi^+$ analysis to get the distribution in Dalitz space

# Boosted Decision Tree

- Signal: truth-matched MC 11, 12, 15, 16, 17, 18 (proportional to the luminosity)
- Background: Run 1 and Run 2 data, $m_{B^0} > 5.5$ GeV
- The samples are treated separately for Run 1 and Run 2
- We split the $D$ decay modes into categories of topology rather than training a BDT for each mode
- BDTs for each of the following categories and for each Run:
  1. KsHH LL with $D \to K^0_S\pi^+\pi^-$ LL
  2. KsHH DD with $D \to K^0_S\pi^+\pi^-$ DD
  3. HH with $D \to K^+K^-$
  4. HHHH with $D \to \pi^+\pi^-\pi^+\pi^-$
     - Last two are for additional decay modes
- We checked using different BDTs for each $D$ final state does not improve performance

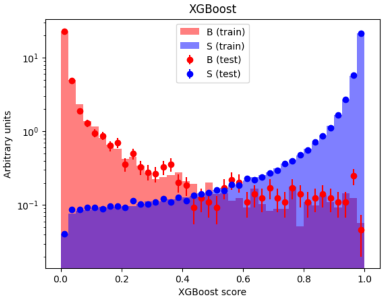
# Boosted Decision Tree

- BDT with `XGBoost` to suppress combinatorials.
- The k-fold cross BDT method with $k = 4$ is exploited.
- It allows us to use events in the fitting region for training a BDT.
- We apply PID cuts on the companion particles before the training.

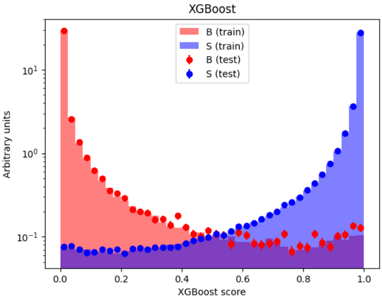


# Training/Testing sample comparison

- Good agreement between the training and testing samples
- No significant overtraining
- Good separation between signal and background



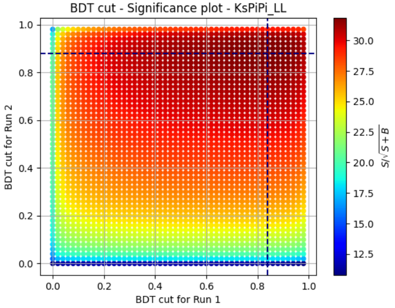

$D \to K^0_{\mathrm{S}}\pi^+\pi^-$ DD Run1

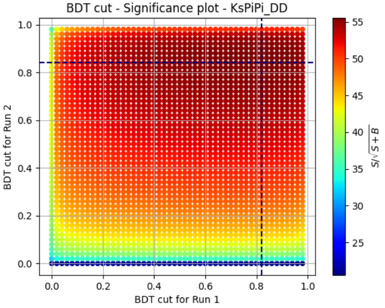



$D \to K^0_{\mathrm{S}}\pi^+\pi^-$ DD Run 2

# Optimising the BDT cut

- We optimise the BDT cut by maximising the figure of merit $S/\sqrt{S+B}$, where $S$ and $B$ are the sum of signal and background around $B^0$ mass for both Runs
- The initial values of $S$ and $B$ are extracted from a simplified $B^0$ mass fit
- The FOM is then evaluated at each working point from the efficiency
- We set the cut at 0.85 for both Runs



# Summary

- Double Dalitz method with $B^0 \to DK^+\pi^-$, $D \to K^0_\mathrm{S} h^+ h^-$ is a promising way to measure $\gamma$
- Including decays as $D \to h^+h^-$ or $D \to h^+h^-h^+h^-$ further improves sensitivity to $\gamma$
- The analysis at the LHCb is still at an early stage
- We aim to achieve $\sigma(\gamma) \sim 5^\circ$ with Run 1+2 data

[1] A. Soffer A. Giri, Y. Grossman and J. Zupan.
Determining $\gamma$ using $B^{\pm} \to DK^{\pm}$ with multibody $D$ decays.
*Phys. Rev. D*, 68:054018, 2003.

[2] A. Bonder.
Proceedings of BINP special analysis meeting on Dalitz analysis.
24-26 Sep. 2002, unpublished.

[3] J. Libby et al.
Model-independent determination of the strong-phase difference between $D^0$ and $\bar{D}^0 \to K^0_{S,L} h^+ h^-$ $(h = \pi, K)$ and its impact on the measurement of the CKM angle $\gamma/\phi_3$.
*Phys. Rev. D*, 82:112006, 2010.

[4] R. Aaij et al.
Measurement of the CKM angle $\gamma$ in $B^{\pm} \to DK^{\pm}$ and $B^{\pm} \to D\pi^{\pm}$ decays with $D \to K^0_{\mathrm{S}} h^+ h^-$.
*JHEP*, 02:169, arXiv:2010.08483, 2021.

[5] R. Aaij et al.
Model-independent measurement of the CKM angle $\gamma$ using $B^0 \to DK^{*0}$ decays with $D \to K^0_{\mathrm{S}} \pi^+ \pi^-$ and $K^0_{\mathrm{S}} K^+ K^-$.
*JHEP*, 10:131, arXiv:1604.01525, 2016.

[6] T. Gershon and A. Poluektov.
Double Dalitz Plot Analysis of the Decay $B^0 \to DK^+\pi^-$, $D \to K^0_S \pi^+ \pi^-$.
*Phys. Rev. D*, 81:014025, 2010.

[7] D. Craik, T. Gershon, and A. Poluektov.
Optimising sensitivity to $\gamma$ with $B^0 \to DK^+\pi^-$, $D \to K^0_{\mathrm{S}} \pi^+ \pi^-$ double Dalitz plot analysis.
*Phys. Rev. D*, 97(5):056002, 2018.

[8] R. Aaij et al.
Constraints on the unitarity triangle angle $\gamma$ from Dalitz plot analysis of $B^0 \to DK^+\pi^-$ decays.
*Phys. Rev. D*, 93(11):112018, 2016.
[Erratum: Phys.Rev.D 94, 079902 (2016)].

[9] T. Evans, J. Libby, S. Malde, and G. Wilkinson.
Improved sensitivity to the CKM phase $\gamma$ through binning phase space in $B^- \to DK^-$, $D \to K^+\pi^-\pi^-\pi^+$ decays.
*Phys. Lett. B*, 802:135188, 2020.

[10] R. Aaij et al.
Measurement of the CKM angle $\gamma$ in $B^\pm \to DK^\pm$ and $B^\pm \to D\pi^\pm$ decays with $D \to K_\mathrm{S} h^+ h^-$.
2020.
submitted to JHEP.

[11] M. Ablikim et al.
Model-independent determination of the relative strong-phase difference between $D^0$ and $\bar{D}^0 \to K^0_{S,L}\pi^+\pi^-$ and its impact on the measurement of the CKM angle $\gamma/\phi_3$.
*Phys. Rev. D*, 101(11):112002, 2020.

[12] M. Ablikim et al.
Improved model-independent determination of the strong-phase difference between $D^0$ and $\bar{D}^0 \to K^0_{\mathrm{S,L}}K^+K^-$ decays.
*Phys. Rev. D*, 102(5):052008, 2020.

# BACK UP

# Decay Summary

Expected number of events in each bin ($\alpha$ for $B$ bin, $i$ for $D$ bin) for $B^0 \to DK^+\pi^-$ (to get $\overline{B}^0$ then $\gamma \to -\gamma$)

$$N_{\alpha i} = h \left\{ \bar{\kappa}_\alpha K_{+i} + \kappa_\alpha K_{-i} + 2\kappa_D \sqrt{\kappa_\alpha K_{+i} \bar{\kappa}_\alpha K_{-i}} \left[ (\chi_\alpha c_i - \sigma_\alpha s_i) \cos\gamma - (\chi_\alpha s_i + \sigma_\alpha c_i \sin\gamma \right] \right\}$$

| Decay | Parameters | Observables |
|---|---|---|
| $D \to K_s^0 \pi^+\pi^-$ | $K_{\pm i}$ from $B^+ \to Dh^+$ BPGGSZ[10] <br> $c_i$, $s_i$ from CLEO+BES-III[11], $\kappa_D = 1$ | 160 |
| $D \to K_s^0 K^+ K^-$ | $K_{\pm i}$ from $B^+ \to Dh^+$ BPGGSZ[10] <br> $c_i$, $s_i$ from CLEO+BES-III[12], $\kappa_D = 1$ | 40 |
| $D \to K^+\pi^-$ | $K_{+i} = 1$, $K_{-i} = r_D^2$, $c_i, s_i = \cos, \sin(-\delta_D)$, $\kappa_D = 1$ | 10 |
| $D \to K^-\pi^+$ | $K_{+i} = r_D^2$, $K_{-i} = 1$, $c_i, s_i = \cos, \sin(\delta_D)$, $\kappa_D = 1$ | 10 |
| $D \to h^+h^-$ | $K_{+i} = 1$, $K_{-i} = 1$, $c_i = 1$, $s_i = 0$, $\kappa_D = 1$ | 10 |
| $D \to K^+\pi^-\pi^0$ | $K_{+i} = 1$, $K_{-i} = r_D^2$, $c_i, s_i = \cos, \sin(-\delta_D)$, $\kappa_D$ | 10 |
| $D \to K^-\pi^+\pi^0$ | $K_{+i} = r_D^2$, $\kappa_D$, $K_{-i} = 1$, $c_i, s_i = \cos, \sin(\delta_D)$, $\kappa_D$ | 10 |
| $D \to h^+h^-\pi^0$ | $K_{+i} = 1$, $K_{-i} = 1$, $c_i = 1$, $s_i = 0$, $\kappa_D = (2F^+ - 1)$ | 10 |
| $D \to K^-\pi^+\pi^+\pi^-$ | $K_{\pm i}$ from $B^+ \to Dh^+$ BPGGSZ <br> $c_i$, $s_i$ from CLEO+BES-III, $\kappa_D = 1$ | 80 |
| $D \to \pi^+\pi^-\pi^+\pi^-$ | $K_{+i} = 1$, $K_{-i} = 1$, $c_i = 1$, $s_i = 0$, $\kappa_D = (2F^+ - 1)$ | 10 |