

Measuring η - η' mixing with $B_{(s)}^0 \rightarrow J/\psi \eta^{(\prime)}$ decays at LHCb

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Available physics at LHCb

Channel	Expt. branching ratio	Discussion
$\eta \rightarrow 2\gamma$	39.41(20)%	chiral anomaly, $\eta - \eta'$ mixing
$\eta \rightarrow 3\pi^0$	32.68(23)%	$m_u - m_d$
$\eta \rightarrow \pi^0\gamma\gamma$	$2.56(22) \times 10^{-4}$	χ PT at $\mathcal{O}(p^6)$, leptophobic B boson, light Higgs scalars
$\eta \rightarrow \pi^0\pi^0\gamma\gamma$	$< 1.2 \times 10^{-3}$	χ PT, axion-like particles (ALPs)
$\eta \rightarrow 4\gamma$	$< 2.8 \times 10^{-4}$	$< 10^{-11}$ [52]
$\eta \rightarrow \pi^+\pi^-\pi^0$	22.92(28)%	$m_u - m_d$, P/CP violation, light Higgs scalars
$\eta \rightarrow \pi^+\pi^-\gamma$	4.22(8)%	chiral anomaly, theory input for singly-virtual TFF and $(g - 2)_\mu$, P/CP violation
$\eta \rightarrow \pi^+\pi^-\gamma\gamma$	$< 2.1 \times 10^{-3}$	χ PT, ALPs
$\eta \rightarrow e^+e^-\gamma$	$6.9(4) \times 10^{-3}$	theory input for $(g - 2)_\mu$, dark photon, protophobic X boson
$\eta \rightarrow \mu^+\mu^-\gamma$	$3.1(4) \times 10^{-4}$	theory input for $(g - 2)_\mu$, dark photon
$\eta \rightarrow e^+e^-$	$< 7 \times 10^{-7}$	theory input for $(g - 2)_\mu$, BSM weak decays
$\eta \rightarrow \mu^+\mu^-$	$5.8(8) \times 10^{-6}$	theory input for $(g - 2)_\mu$, BSM weak decays, P/CP violation
$\eta \rightarrow \pi^0\pi^0\ell^+\ell^-$		C/CP violation, ALPs
$\eta \rightarrow \pi^+\pi^-e^+e^-$	$2.68(11) \times 10^{-4}$	theory input for doubly-virtual TFF and $(g - 2)_\mu$, P/CP violation, ALPs
$\eta \rightarrow \pi^+\pi^-\mu^+\mu^-$	$< 3.6 \times 10^{-4}$	theory input for doubly-virtual TFF and $(g - 2)_\mu$, P/CP violation, ALPs

Channel	Expt. branching ratio	Discussion
$\eta' \rightarrow \eta\pi^+\pi^-$	42.6(7)%	large- N_c χ PT, light Higgs scalars
$\eta' \rightarrow \pi^+\pi^-\gamma$	28.9(5)%	chiral anomaly, theory input for singly-virtual TFF and $(g - 2)_\mu$, P/CP violation
$\eta' \rightarrow \eta\pi^0\pi^0$	22.8(8)%	large- N_c χ PT
$\eta' \rightarrow \omega\gamma$	2.489(76)% [55]	theory input for singly-virtual TFF and $(g - 2)_\mu$
$\eta' \rightarrow \omega e^+e^-$	$2.0(4) \times 10^{-4}$	theory input for doubly-virtual TFF and $(g - 2)_\mu$
$\eta' \rightarrow 2\gamma$	2.331(37)% [55]	chiral anomaly, $\eta - \eta'$ mixing
$\eta' \rightarrow 3\pi^0$	2.54(18)% (*)	$m_u - m_d$
$\eta' \rightarrow \mu^+\mu^-\gamma$	$1.09(27) \times 10^{-4}$	theory input for $(g - 2)_\mu$, dark photon
$\eta' \rightarrow e^+e^-\gamma$	$4.73(30) \times 10^{-4}$	theory input for $(g - 2)_\mu$, dark photon
$\eta' \rightarrow \pi^+\pi^-\mu^+\mu^-$	$< 2.9 \times 10^{-5}$	theory input for doubly-virtual TFF and $(g - 2)_\mu$, P/CP violation, dark photon, ALPs
$\eta' \rightarrow \pi^+\pi^-e^+e^-$	$2.4^{(+1.3)}_{(-1.0)} \times 10^{-3}$	theory input for doubly-virtual TFF and $(g - 2)_\mu$, P/CP violation, dark photon, ALPs
$\eta' \rightarrow \pi^0\pi^0\ell^+\ell^-$		C/CP violation, ALPs

LHCb can access high \mathcal{B} modes via:

- $D^+_{(s)} \rightarrow h^+ \eta(')$ - [JHEP 06 (2021) 019] & [Phys. Lett. B764 (2016) 233]
- $B^0_{(s)} \rightarrow J/\psi \eta(')$ [JHEP 01 (2015) 024] - showing Run-2 update today

η - η' Mixing

- The physical η and η' states can be described in terms of isospin singlets ($|\eta_s\rangle$, $|\eta_q\rangle$),

$$|\eta_q\rangle = \frac{1}{\sqrt{2}}(|u\bar{u}\rangle + |d\bar{d}\rangle), \quad |\eta_s\rangle = |s\bar{s}\rangle$$

- Taking account for a glueball state $|gg\rangle$, we have a description with two angles (ϕ_P and ϕ_G)

$$|\eta\rangle = \cos \phi_P |\eta_q\rangle - \sin \phi_P |\eta_s\rangle, \quad |\eta'\rangle = \cos \phi_G (\sin \phi_P |\eta_q\rangle + \cos \phi_P |\eta_s\rangle) + \sin \phi_G |gg\rangle$$

- The branching ratios can be found by projecting to the $|d\bar{d}\rangle$ or $|s\bar{s}\rangle$ states:

$$\mathcal{B}(B^0_q \rightarrow J/\psi \eta^{(\prime)}) = \frac{\tau_{B^0_q} M_{B^0_q} (\Phi_q^{\eta^{(\prime)}})^3}{64\pi} |V_{cq} V_{cb}^*|^2 |F(B^0_q \rightarrow J/\psi q\bar{q})|^2 \left| \langle q\bar{q} | \eta^{(\prime)} \rangle \right|^2$$

- where $\Phi_{(s)}^{\eta^{(\prime)}}$ are the phase-space factors and $F(B^0_q \rightarrow J/\psi q\bar{q})$ are the form factors.

Ways to measure η - η' mixing: $R_{\eta^{(\prime)}}$

- A way to measure ϕ_P is to measure $R_{\eta^{(\prime)}}$

$$R_{\eta^{(\prime)}} = \frac{\mathcal{B}(B^0 \rightarrow J/\psi \eta^{(\prime)})}{\mathcal{B}(B_s^0 \rightarrow J/\psi \eta^{(\prime)})}$$

- Assuming SU(3) symmetry $F(B^0 \rightarrow J/\psi d\bar{d}) = F(B_s^0 \rightarrow J/\psi s\bar{s})$ and $\tan^2 \theta_c = |V_{cd}/V_{cs}|^2$

$$R_{\eta'} = \frac{\tau_{B^0}}{\tau_{B_s^0}} \frac{m_{B^0}}{m_{B_s^0}} \left(\frac{\Phi_d^{\eta'}}{\Phi_s^{\eta'}} \right)^3 \times \frac{\tan^2 \theta_c}{2} \times \tan^2 \phi_P, \quad R_\eta = \frac{\tau_{B^0}}{\tau_{B_s^0}} \frac{m_{B^0}}{m_{B_s^0}} \left(\frac{\Phi_d^\eta}{\Phi_s^\eta} \right)^3 \times \frac{\tan^2 \theta_c}{2} \times \cot^2 \phi_P$$

- Although this is not sensitive to ϕ_G .
- $\phi_P = (46.3 \pm 2.3)^\circ$ LHCb Run-1 value [JHEP 01 (2015) 024]
- PDG 2020 mass formulae: $\phi_P = 43.4^\circ$
- χ -PT: $\phi_P = (39.3 \pm 1.0)^\circ$
- Lattice QCD: $\phi_P = (38.8 \pm 4.6)^\circ$ - both averages from [Gan, Kubis, Passemar and Tulin (2020)]

Ways to measure η - η' mixing: $R_{(s)}$

- A way to measure both ϕ_P and ϕ_G is to measure $R_{(s)}$

$$R_{(s)} = \frac{\mathcal{B}(B^0(s) \rightarrow J/\psi\eta')}{\mathcal{B}(B^0(s) \rightarrow J/\psi\eta)}$$

$$R = \left(\frac{\Phi_d^{\eta'}}{\Phi_d^\eta} \right)^3 \cos^2 \phi_G \tan^2 \phi_P, \quad R_s = \left(\frac{\Phi_s^{\eta'}}{\Phi_s^\eta} \right)^3 \cos^2 \phi_G \cot^2 \phi_P$$

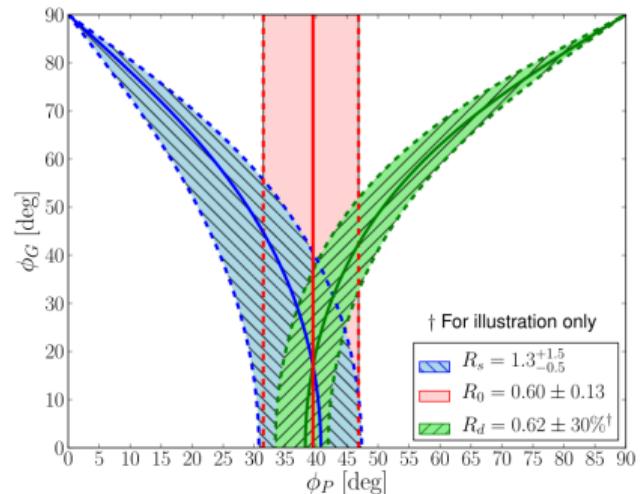
- LHCb Run-1: $\phi_P = (43.5^{+1.4}_{-2.8})^\circ$, $\phi_G = (0 \pm 24.6)^\circ$ [[JHEP 01 \(2015\) 024](#)]
- PDG 2020 mass formulae: $\phi_P = 43.4^\circ$
- χ -PT: $\phi_P = (39.3 \pm 1.0)^\circ$
- Lattice QCD: $\phi_P = (38.8 \pm 4.6)^\circ$ - both averages from [[Gan, Kubis, Passemar and Tulin \(2020\)](#)]

Ways to measure η - η' mixing: R_0

- A way to measure ϕ_P is to measure R_0

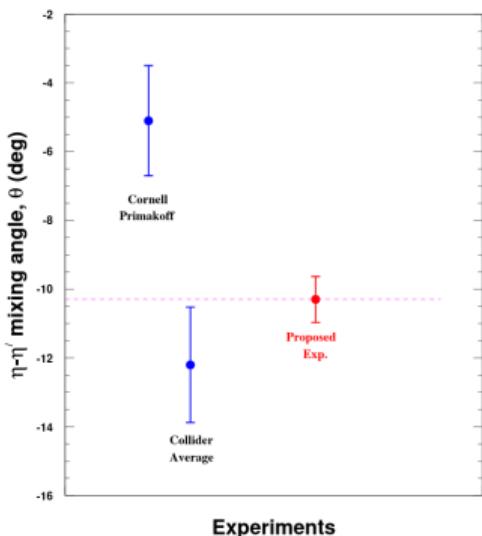
$$R_0 = \frac{\mathcal{B}(B^0 \rightarrow J/\psi\eta)}{\mathcal{B}(B^0 \rightarrow J/\psi\pi^0)} = \left(\frac{\Phi_d^\eta}{\Phi_d^{\pi^0}} \right)^3 \times \cos^2 \phi_P$$

- This is suggested in [Fleischer, Knegjens, Ricciardi (2011)]
- This gave $\phi_P = (40^{+7}_{-8})^\circ$ from PDG \mathcal{B} of the time
- We are working on this as well, but I am focusing on $R_{(s)}$ and $R_{\eta'}$ today
- Measurements together can be used to create a $\phi_G - \phi_P$ phase space



Tension in experiments

- There is some tension for η - η' mixing between Cornell experiment using Primakoff effect ($\gamma\gamma^{(*)} \rightarrow \gamma\eta$, where $\gamma^{(*)}$ is a virtual photon in Coulomb field of a nuclei)



ϕ_P is angle relative to the “ideal-mixing” angle

$$\theta_P = \phi_P - \arctan \sqrt{2}$$

- LHCb Run-1: $\theta_P = (-11.2^{+1.4}_{-2.8})^\circ$ and $\theta_P = (-8.4 \pm 2.3)^\circ$
- PDG: $\theta_P = -11.3^\circ$
- χ -PT: $\theta_P = (-15.4 \pm 1.0)^\circ$
- Lattice QCD: $\theta_P = (-15.9 \pm 4.6)^\circ$
- An update with LHCb Run-2 data value may help resolve this

Plot from JLAB η proposals
(2009)

Analysis Strategy

- We want to measure both the absolute \mathcal{B} and $\eta\text{-}\eta'$ mixing of $B^0(s) \rightarrow J/\psi\eta^{(\prime)}$

Channel	\mathcal{B}
$\eta \rightarrow \gamma\gamma$	$(39.41 \pm 0.20)\%$
$\eta \rightarrow \pi^+\pi^-\pi^0$	$(22.92 \pm 0.28)\%$
$\eta \rightarrow \pi^+\pi^-\gamma$	$(4.22 \pm 0.08)\%$

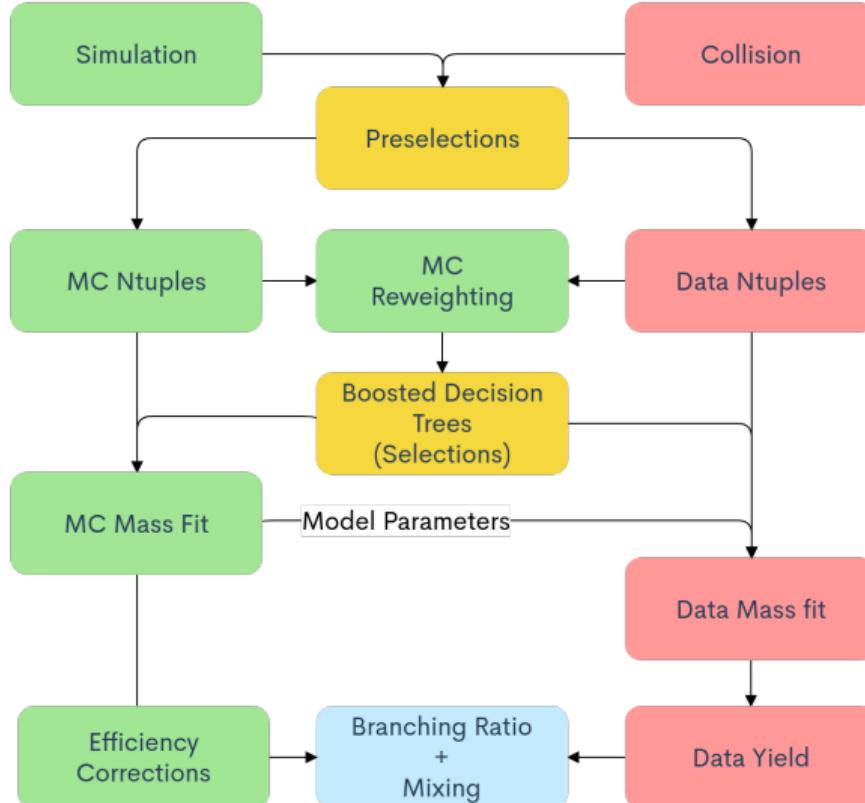
Channel	\mathcal{B}
$\eta' \rightarrow \gamma\gamma$	$(2.307 \pm 0.033)\%$
$\eta' \rightarrow \pi^+\pi^-\eta$	$(42.5 \pm 0.5)\%$
$\eta' \rightarrow \rho^0[\pi^+\pi^-]\gamma$	$(29.5 \pm 0.4)\%$

- Normalisation: $\mathcal{B}(B^0 \rightarrow J/\psi\rho^0[\pi^+\pi^-]) = (2.55_{-0.16}^{+0.18}) \times 10^{-5}$ [PDG 2020]
- Using pp collision data (taken 2016-18) corresponding to $\int \mathcal{L} = 5.6 \text{ fb}^{-1}$ at 13 TeV.

$$\mathcal{B}(B^0(s) \rightarrow J/\psi\eta^{(\prime)}) = \frac{N_{B^0(s) \rightarrow J/\psi\eta^{(\prime)}}^s}{N_{B^0 \rightarrow J/\psi\rho^0}^s} \times \frac{\varepsilon_{B^0 \rightarrow J/\psi\rho^0}^{MC}}{\varepsilon_{B^0(s) \rightarrow J/\psi\eta^{(\prime)}}^{MC}} \times \frac{\mathcal{B}(B^0 \rightarrow J/\psi\rho^0)}{\mathcal{B}(\eta^{(\prime)} \rightarrow X)} \left(\frac{f_s}{f_d} \right)^{-1}$$

- $f_s/f_d(13 \text{ TeV}) = 0.2539 \pm 0.0079$ [Phys. Rev. D 104, 032005]

Analysis Steps

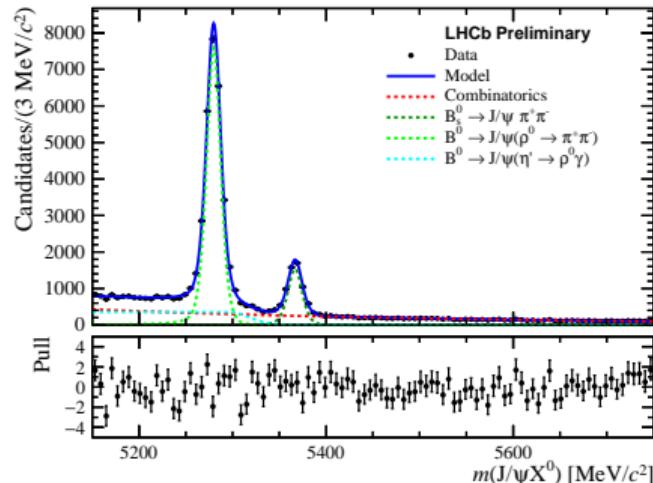


Selection Summary

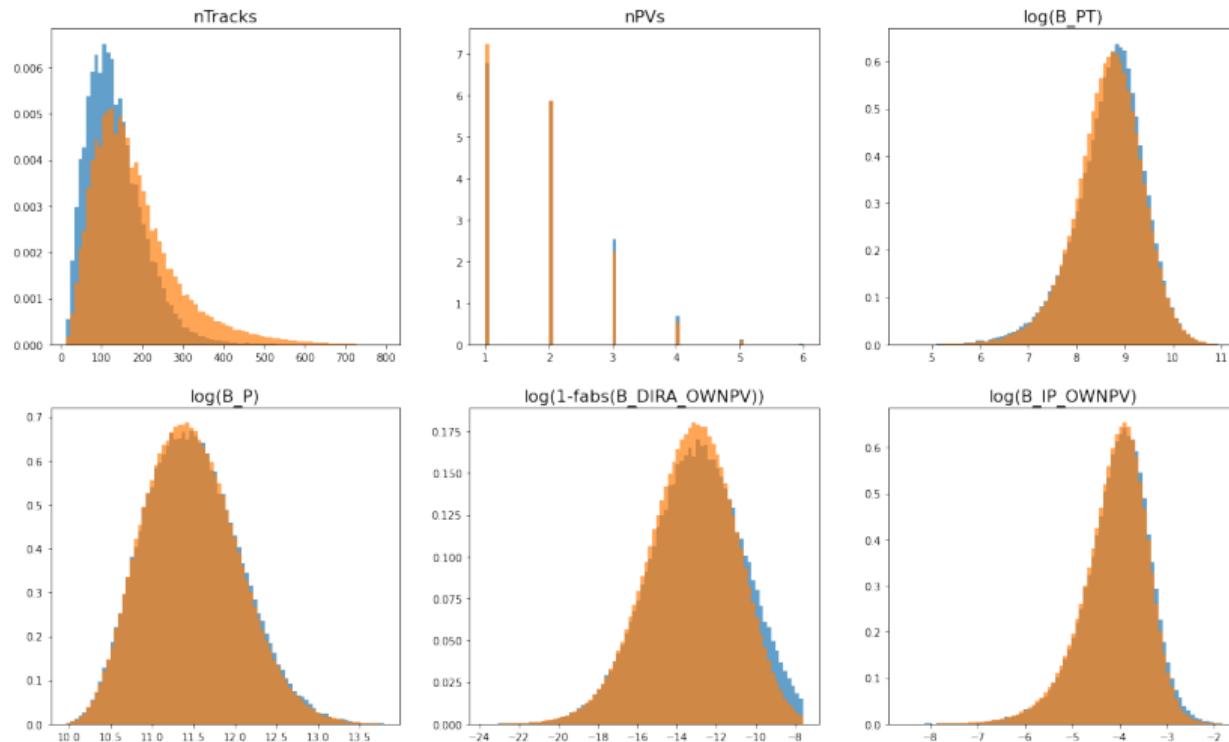
- Triggers (Muon and dimuon)
- Stripping: FullDSTDiMuonJpsi2MuMuDetachedLine (selections for $J/\psi \rightarrow \mu^+ \mu^-$)
- Vertex cuts:
 - $\log(\chi_{\text{vtx}}^2/\text{ndf}) < 4$, DTF $\chi^2/\text{ndf} < 5$, $\chi_{\text{IP}}^2(\mu) > 4$, $\chi_{\text{IP}}^2(\pi) > 9$
- p_T cuts:
 - For all modes ($B_{(s)}^0 \rightarrow J/\psi X^0$): $p_T^{X^0} > 2000$ MeV
 - $B_{(s)}^0 \rightarrow J/\psi \eta' [\pi^+ \pi^- \eta]$: $p_T^\eta > 1200$ MeV, $p_T^\gamma > 500$ MeV
 - $B_{(s)}^0 \rightarrow J/\psi \eta [\pi^+ \pi^- \pi^0]$: $p_T^{\pi^0} > 1000$ MeV, $p_T^\gamma > 200$ MeV
 - $B_{(s)}^0 \rightarrow J/\psi \eta^{(')} [\pi^+ \pi^- \gamma]$: $p_T^\gamma > 500$ MeV
- PID of charged particles:
 - π^\pm : $\mathcal{P}_\pi * (1 - \mathcal{P}_K) > 0.2$
- Mass vetoes: $J/\psi K^+ \pi^-$, $J/\psi K^+$, and $J/\psi \pi^+ \pi^-$ (for $\eta^{(')}$ modes)
- 3-D optimised selections (more on this later):
 - η and η' mass windows
 - photon CL
 - an MVA: Boosted Decision Trees

Gradient Boosted Reweighting

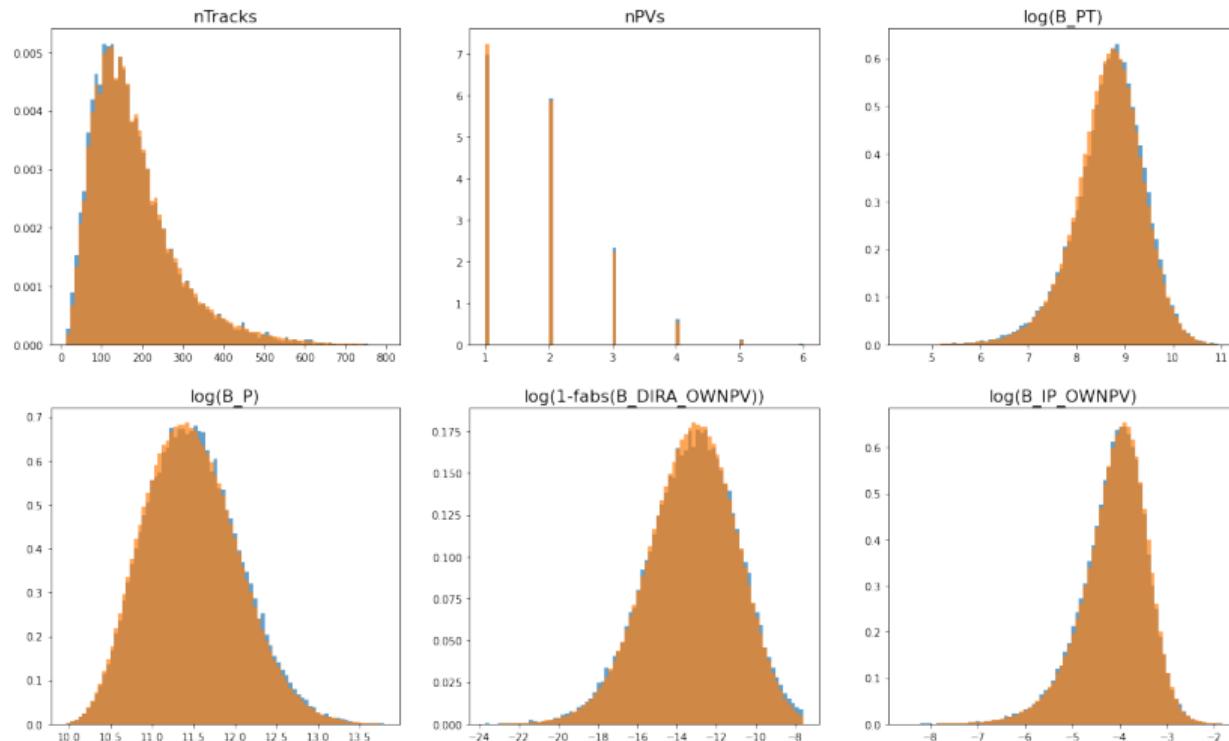
- A reweighting procedure using Boosted Decision Trees [[Rogozhnikov \(2016\)](#)]
- Data-driven corrections of MC samples
- Using sWeighted $J/\psi\pi^+\pi^-$ data compared to MC to train the reweighter
- Trained using common variables across the modes to align ($B p_T$, p , IP and DIRA with nTracks and nPVs)
- This is done before training the BDT in order to improve the alignment of the input variables



$B^0 \rightarrow J/\psi \rho^0$ real data vs. MC before reweighting

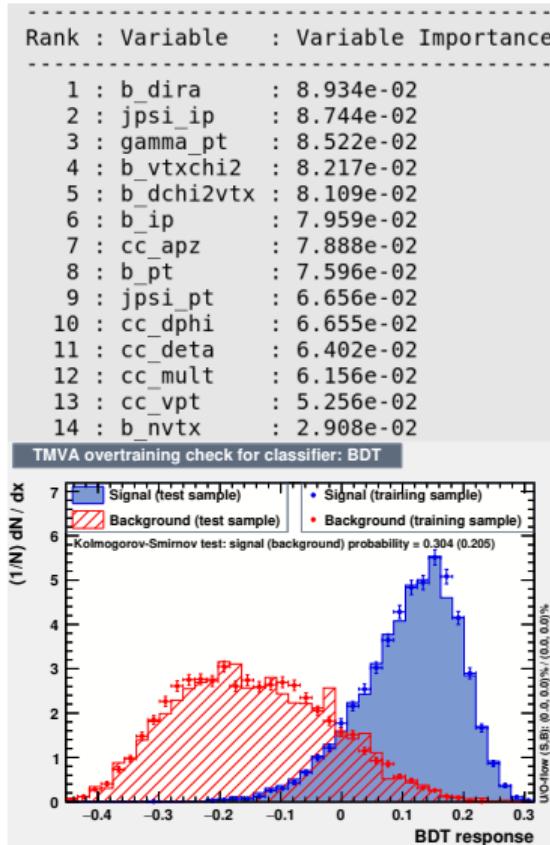


$B^0 \rightarrow J/\psi \rho^0$ real data vs. MC after reweighting

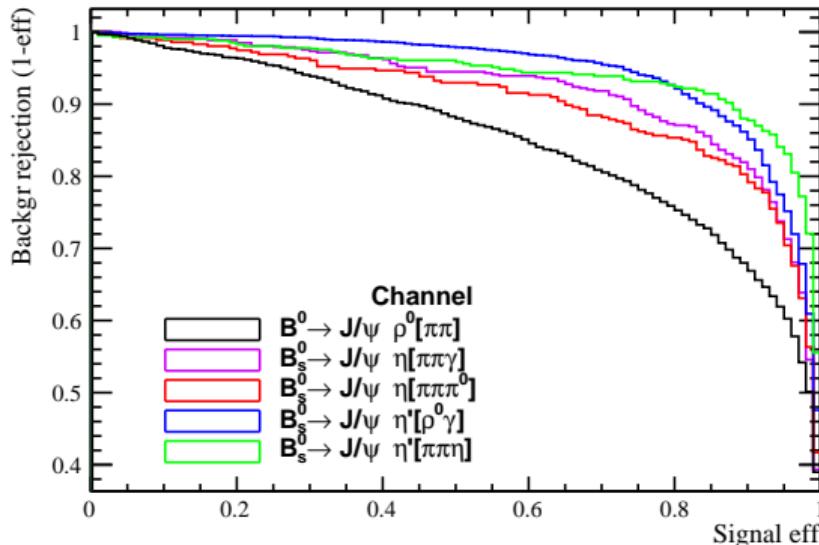


Boosted Decision Trees

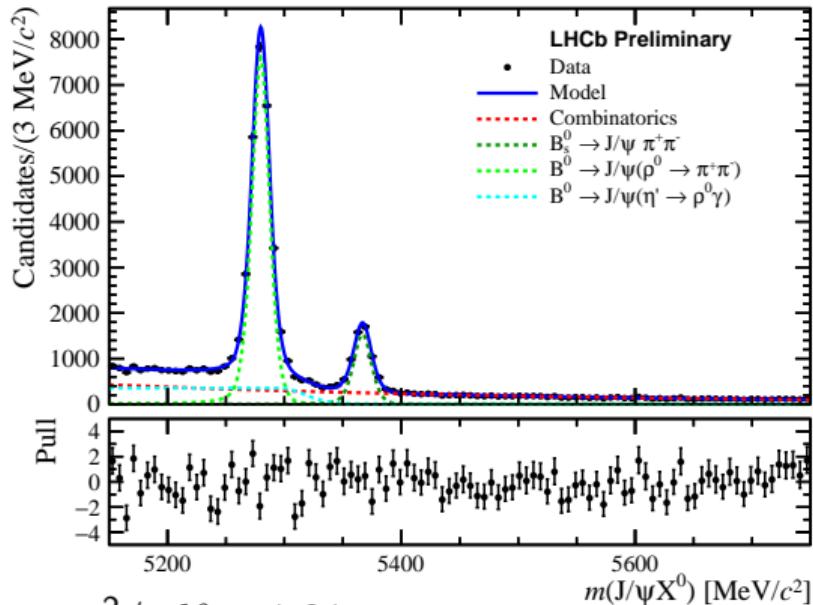
← 13/14 inputs chosen for signal-background discrimination



Signal category: MC samples
Background: J/ψ mass side bands in data



Normalisation channel $B^0 \rightarrow J/\psi \rho^0$ mass fit - Run-2



- $\chi^2/\text{ndf} = 1.31$
- $N(B^0) = (2.611 \pm 0.021) \times 10^4$
- $N(B_s^0) = (5.30 \pm 0.09) \times 10^3$

Fit models:

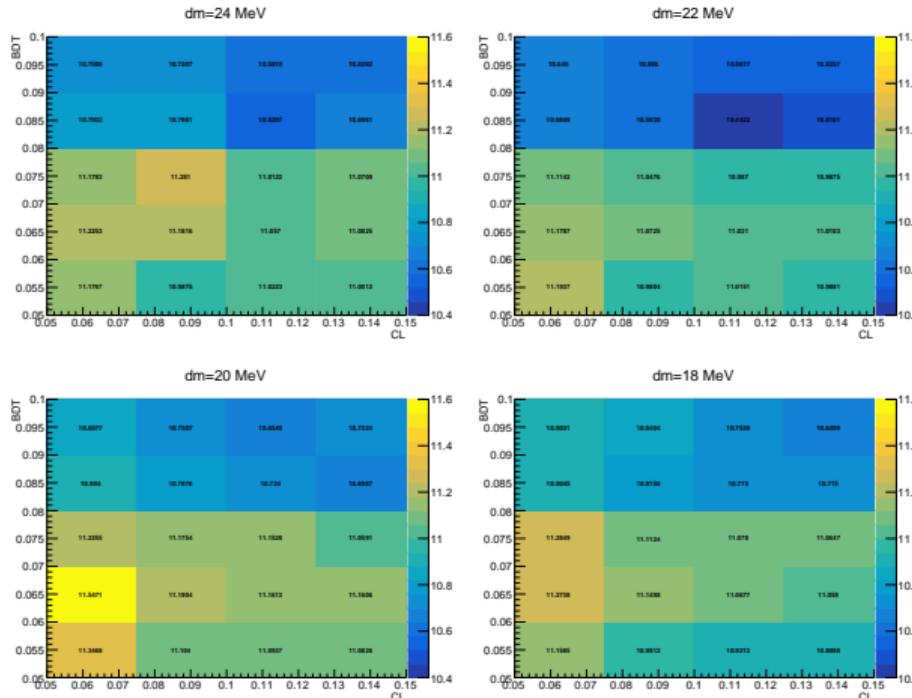
- Signal & physics backgrounds: PDF double-sided Hypatia function with two tails (fixed from MC fits)
- Combinatorics: a free exponential
- Physics backgrounds are fixed on the $J/\psi \rho^0$ yield and the ratios of ε^{MC} and \mathcal{B}

BDT selection decision

- $B^0 \rightarrow J/\psi \rho^0$ has high purity so 1-D optimisation of the BDT for maximised signal significance was sufficient
- Other modes use 3-D optimisation

3-D optimisation: a toy study

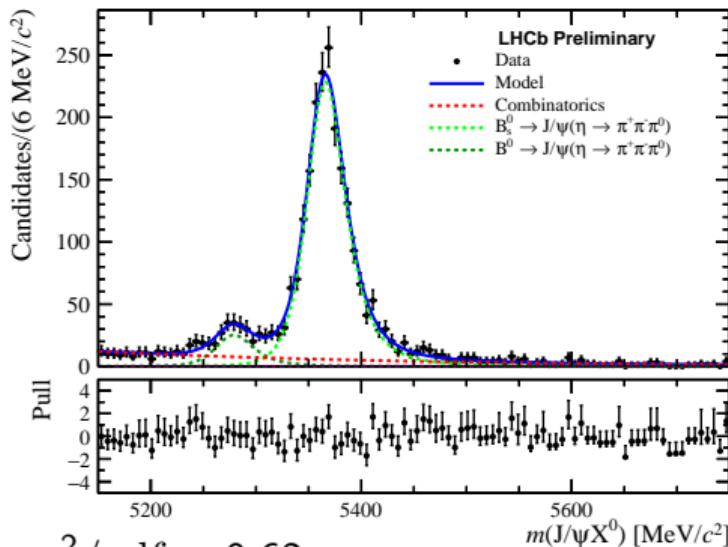
- 3-D optimisation of the BDT + $\eta^{(')}$ mass window (dm) + photon CL
- FOM: The significance of the Cabibbo-suppressed B^0 peak



- Fits MC samples
- Uses $J/\psi\rho^0$ to initially estimate yields
- Generates toy MC and refits to give significance - repeated 100 times per combined cut
- The combined cut with the maximum mean significance is then used
- This is to improve reliability and compensates for statistical fluctuations

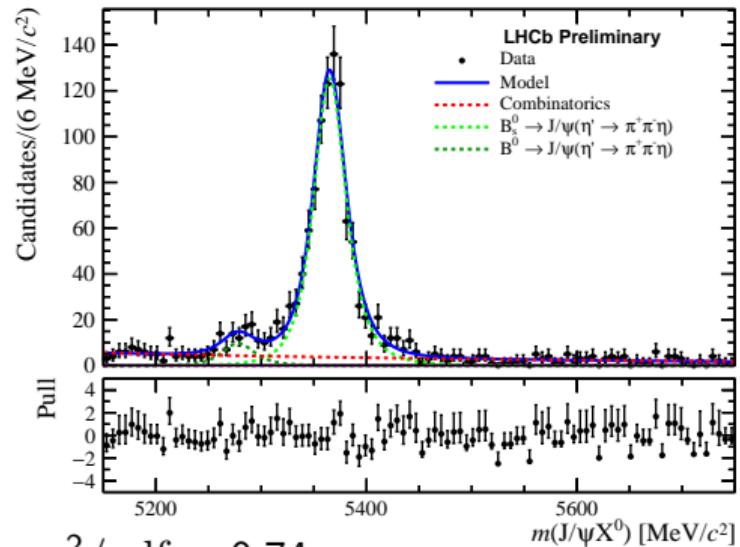
$B^0_{(s)} \rightarrow J/\psi \eta^{(\prime)}$ diphoton mass fits - Run-2

$$\eta \rightarrow \pi^+ \pi^- \pi^0$$



- $\chi^2/\text{ndf} = 0.62$
- $N(B^0) = 218 \pm 22$
- $N(B^0_s) = 1979 \pm 49$

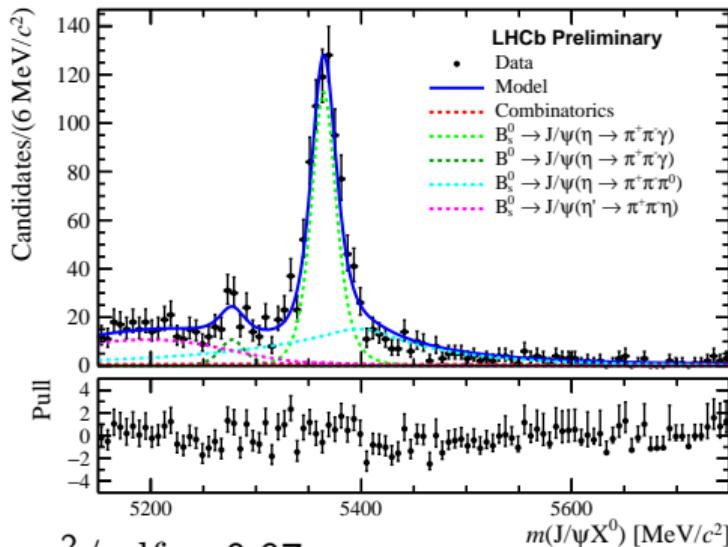
$$\eta' \rightarrow \pi^+ \pi^- \eta$$



- $\chi^2/\text{ndf} = 0.74$
- $N(B^0) = 72 \pm 14$
- $N(B^0_s) = 936 \pm 34$

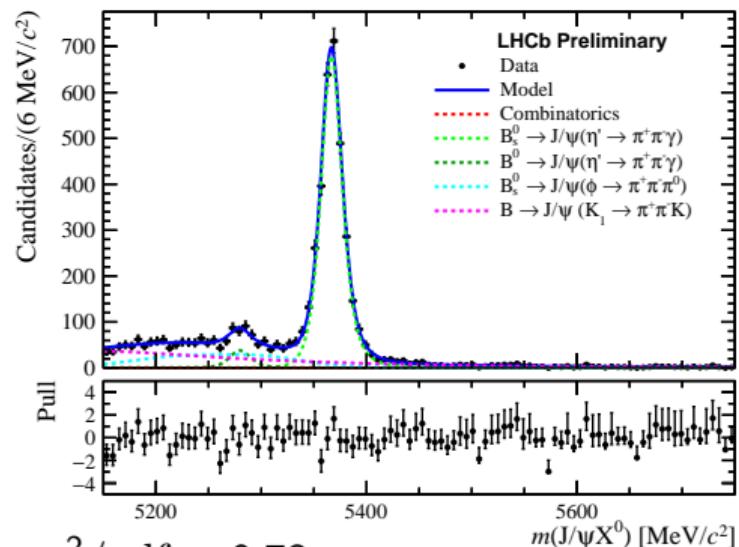
$B^0_{(s)} \rightarrow J/\psi \eta^{(\prime)}$ single photon mass fits - Run-2

$$\eta \rightarrow \pi^+ \pi^- \gamma$$



- $\chi^2/\text{ndf} = 0.97$
- $N(B^0) = 59 \pm 14$
- $N(B_s^0) = 622 \pm 19$

$$\eta' \rightarrow (\rho^0 \rightarrow \pi^+ \pi^-) \gamma$$



- $\chi^2/\text{ndf} = 0.73$
- $N(B^0) = 176 \pm 28$
- $N(B_s^0) = 3164 \pm 60$

Statistics Comparison

Great improvement on statistics since last LHCb study

Channel	$N(B^0)$ Run-1	$N(B_s^0)$ Run-1	$N(B^0)$ Run-2	$N(B_s^0)$ Run-2
$\eta \rightarrow \pi^+ \pi^- \pi^0$	34 ± 11	524 ± 27	218 ± 22	1979 ± 49
$\eta \rightarrow \pi^+ \pi^- \gamma$			59 ± 14	622 ± 19
$\eta' \rightarrow \pi^+ \pi^- \eta$	26.8 ± 7.5	333 ± 20	72 ± 14	936 ± 34
$\eta' \rightarrow (\rho^0 \rightarrow \pi^+ \pi^-) \gamma$	71 ± 22	988 ± 45	176 ± 28	3164 ± 60

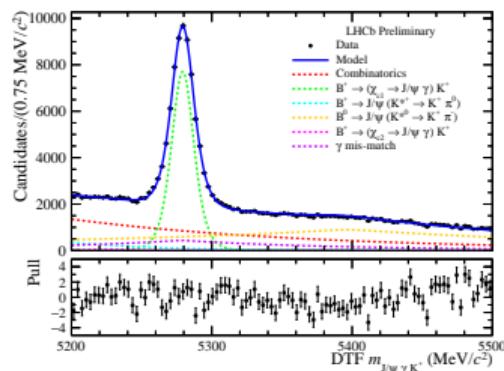
Efficiency Corrections - Photon Reconstruction

- Dedicated performance analysis
- Follows a similar analysis strategy as \mathcal{B} measurements

$$B^+ \rightarrow (\chi_{c1} \rightarrow J/\psi \gamma) K^+ \text{ normalised to } B^+ \rightarrow J/\psi K^+$$

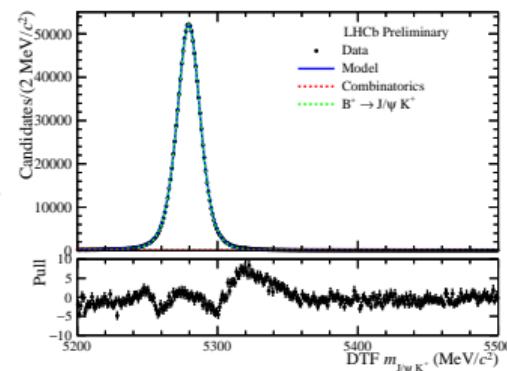
2018 mass fits

$\leftarrow \chi_{c1} K^+$ and $J/\psi K^+ \rightarrow$



$$\eta_\gamma^{cor} = \frac{N_{J/\psi\gamma K^+}^s}{N_{J/\psi K^+}^s} \times \frac{\epsilon_{J/\psi K^+}^{MC}}{\epsilon_{J/\psi\gamma K^+}^{MC}} \times \frac{\mathcal{B}_{J/\psi K^+}}{\mathcal{B}_{J/\psi\gamma K^+}}$$

This is also done with
 $B^+ \rightarrow J/\psi(K^{*+} \rightarrow K^+\pi^0)$ for π^0 reco.



- Corrections are produced for each data-taking year at LHCb
- These are applied as a multiplicative factor for the MC efficiencies ($\epsilon_\gamma^{MC} \rightarrow \epsilon_\gamma^{data}$)
- Largest source of uncertainty from \mathcal{B} ($B^+ \rightarrow \chi_{c1} K^+$) & \mathcal{B} ($B^+ \rightarrow J/\psi K^{*+}$) - Belle-II?

Resulting uncertainties on the η - η' mixing angle ϕ_P

$$R_{\eta^{(\prime)}} = \frac{\mathcal{B}(B^0 \rightarrow J/\psi \eta^{(\prime)})}{\mathcal{B}(B_s^0 \rightarrow J/\psi \eta^{(\prime)})}, \quad R_{(s)} = \frac{\mathcal{B}(B^0_{(s)} \rightarrow J/\psi \eta')}{\mathcal{B}(B^0_{(s)} \rightarrow J/\psi \eta)}$$

\mathcal{B} Ratio	Statistics	Total
$R_\eta (\gamma)$	10.0%	10.1%
$R_\eta (\gamma\gamma)$	4.4%	4.4%
$R_{\eta'} (\gamma)$	7.1%	7.2%
$R_{\eta'} (\gamma\gamma)$	8.7%	8.8%
$R (\gamma)$	9.2%	9.2%
$R (\gamma\gamma)$	6.7%	6.7%
$R_s (\gamma)$	1.2%	1.4%
$R_s (\gamma\gamma)$	1.3%	1.4%

- Errors greatly reduced due to $R \propto \tan^2 \phi_P \Rightarrow \delta(\phi_P) \propto \delta(R)/([2R + 2]\sqrt{R})$
- Expected improvement over averaging across all modes and calculation methods
- Needs systematic studies
- Current combined uncertainties of $\sim 1\%$ on ϕ_P
- LHCb Run-1 uncertainties: $\sim 5\%$

Resulting uncertainties on the absolute branching ratios

$\mathcal{B}(B_s^0 \rightarrow J/\psi \eta')$:

Channel	Statistics	Total
$\eta \rightarrow \pi^+ \pi^- \pi^0$	2.6%	10.3%
$\eta \rightarrow \pi^+ \pi^- \gamma$	3.2%	11.1%
$\eta' \rightarrow \pi^+ \pi^- \eta$	3.7%	10.6%
$\eta' \rightarrow (\rho^0 \rightarrow \pi^+ \pi^-) \gamma$	2.1%	10.8%

$\mathcal{B}(B^0 \rightarrow J/\psi \eta')$:

Channel	Statistics	Total
$\eta \rightarrow \pi^+ \pi^- \pi^0$	9.8%	13.7%
$\eta \rightarrow \pi^+ \pi^- \gamma$	24.5%	26.5%
$\eta' \rightarrow \pi^+ \pi^- \eta$	19.2%	21.4%
$\eta' \rightarrow (\rho^0 \rightarrow \pi^+ \pi^-) \gamma$	15.4%	18.4%

Channel	LHCb Run-1	PDG2020
η	18.9%	17.5%
η'	15.4%	12.1%

- Limited by uncertainty on external inputs:
 - $\sigma(\mathcal{B}(B^0 \rightarrow J/\psi \rho^0)) = 6.7\%$
 - $\sigma(f_s/f_d) = 3.1\%$
 - $\sigma(\eta_{\gamma(\gamma)}^{cor}) = 7.5\%(6.5\%)$

Channel	LHCb Run-1	PDG2020
η	38.6%	21.3%
η'	31.8%	31.6%

- Limited by statistical uncertainties:
 - Run-3 data would help

Overview and Outlook

Overview

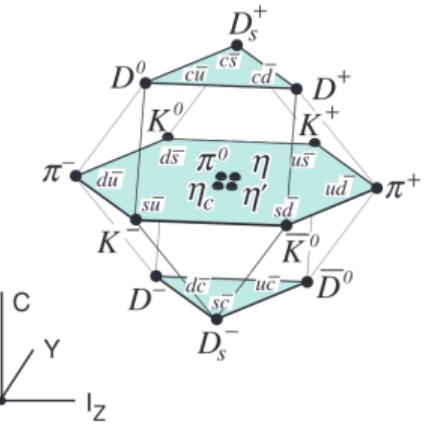
- We can improve the precision of LHCb's Run-1 η - η' mixing measurement by a factor of 5.
- We are currently able to measure the branching fractions with uncertainties expected to be lower than the PDG average (esp. for $\mathcal{B}[B^0 \rightarrow J/\psi \eta^{(\prime)}]$).
- Caveat: This is before further systematical uncertainties have been analysed — though it is expected to be small compared to external factors (\mathcal{B} and η_{γ}^{cor}).

Outlook for LHCb

- With these channels in Run-3 \rightarrow more stats \rightarrow possible CPV measurement (ϕ_s).
 - Using measurements of $\mathcal{B}(B^0 \rightarrow J/\psi \eta^{(\prime)})$ we can test $SU(3)_F$ flavour symmetry and provide hadronic corrections to control penguin parameters. [Fleischer, Knegjens, Ricciardi (2011)]
- CPV searches with $D^+_{(s)} \rightarrow h^+ \eta^{(\prime)}$ - ongoing Run-2 analysis

Back-Up

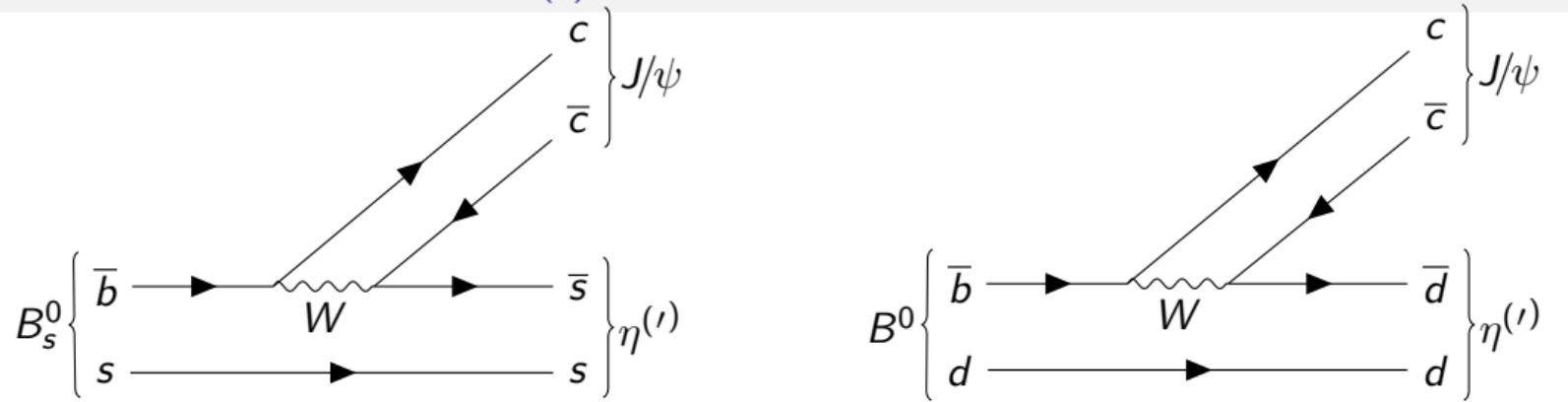
η and η' physics



[PDG 2020]

- η and η' provide opportunities for the testing of many Standard Model as well as Beyond Standard Model physics
- There is particular interest testing fundamental effects of QCD
- $SU(3)_V$ flavour symmetry breaking causes η and η' being admixtures of η_1 and η_8
 - $SU(3)_V$ pseudoscalar octet ($\pi^0, \pi^\pm, K^\pm, K^0, \bar{K}^0$ and η_8)
 - $U(1)_A$ pseudoscalar singlet (η_1)
- low-energy QCD symmetries in kaon and pion sector are well studied. η and η' are behind.

Dominant diagram of $B_{(s)}^0 \rightarrow J/\psi \eta^{(\prime)}$



$$\mathcal{B}(B_{(s)}^0 \rightarrow J/\psi \eta^{(\prime)}) = \frac{\tau_{B_{(s)}^0} p_q^{*\eta^{(\prime)}}}{8\pi M_{B_{(s)}^0}^2} |V_{cq} V_{cb}^*|^2 \left(p_q^{*\eta^{(\prime)}} \right)^2 |F(B_s^0 \rightarrow J/\psi q\bar{q})|^2 \left| \langle q\bar{q} | \eta^{(\prime)} \rangle \right|^2, \quad q = d, s$$

$$R_{\eta^{(\prime)}} = \frac{\mathcal{B}(B^0 \rightarrow J/\psi \eta^{(\prime)})}{\mathcal{B}(B_s^0 \rightarrow J/\psi \eta^{(\prime)})} = \frac{\tau_{B^0}}{\tau_{B_s^0}} \left(\frac{p_d^{*\eta^{(\prime)}}}{p_s^{*\eta^{(\prime)}}} \right)^3 \frac{M_{B_s^0}^2}{M_{B^0}^2} \left| \frac{V_{cd}}{V_{cs}} \right|^2 \left| \frac{F(B^0 \rightarrow J/\psi d\bar{d})}{F(B_s^0 \rightarrow J/\psi s\bar{s})} \right|^2 \left| \frac{\langle d\bar{d} | \eta^{(\prime)} \rangle}{\langle s\bar{s} | \eta^{(\prime)} \rangle} \right|^2$$

$$p_q^{*\eta^{(\prime)}} = M_{B_{(s)}^0}^2 \Phi_q^{\eta^{(\prime)}} / 2$$

η - η' Mixing - singlet and octet basis

- The physical η and η' and η_G (glueball) states can be described in terms of octet and singlet eigenstates ($|\eta_1\rangle$, $|\eta_8\rangle$) and a glueball state $|gg\rangle$
- a description can be made with mixing angles (θ_P and ϕ_G)

$$\begin{pmatrix} |\eta\rangle \\ |\eta'\rangle \\ |\eta_G\rangle \end{pmatrix} = \begin{pmatrix} \cos \theta_P & -\sin \theta_P & 0 \\ \sin \theta_P & \cos \theta_P & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos \theta_G & -\sin \theta_G \\ 0 & \sin \theta_G & \cos \theta_G \end{pmatrix} \begin{pmatrix} |\eta_1\rangle \\ |\eta_8\rangle \\ |gg\rangle \end{pmatrix}$$

- where $|\eta_8\rangle = \frac{1}{\sqrt{6}}(|u\bar{u}\rangle + |d\bar{d}\rangle - |s\bar{s}\rangle)$ and $|\eta_1\rangle = \frac{1}{\sqrt{3}}(|u\bar{u}\rangle + |d\bar{d}\rangle + |s\bar{s}\rangle)$

Available physics - full list

Channel	Expt. branching ratio	Discussion
$\eta \rightarrow 2\gamma$	39.41(20)%	chiral anomaly, η - η' mixing
$\eta \rightarrow 3\pi^0$	32.68(23)%	$m_u = m_d$
$\eta \rightarrow \pi^0\gamma\gamma$	$2.56(22) \times 10^{-4}$	χ PT at $O(p^6)$, leptophobic B boson, light Higgs scalars
$\eta \rightarrow \pi^0\pi^0\gamma\gamma$	$< 1.2 \times 10^{-3}$	χ PT, axion-like particles (ALPs)
$\eta \rightarrow 4\gamma$	$< 2.8 \times 10^{-4}$	$< 10^{-11}$ [52]
$\eta \rightarrow \pi^+\pi^-\pi^0$	22.92(28)%	$m_u = m_d$, P/CP violation, light Higgs scalars
$\eta \rightarrow \pi^+\pi^-\gamma$	4.22(8)%	chiral anomaly, theory input for singly-virtual TFF and $(g-2)_\mu$, P/CP violation
$\eta \rightarrow \pi^+\pi^-\gamma\gamma$	$< 2.1 \times 10^{-3}$	χ PT, ALPs
$\eta \rightarrow e^+e^-\gamma$	$6.9(4) \times 10^{-3}$	theory input for $(g-2)_\mu$, dark photon, protophobic X boson
$\eta \rightarrow \mu^+\mu^-\gamma$	$3.1(4) \times 10^{-4}$	theory input for $(g-2)_\mu$, dark photon
$\eta \rightarrow e^+e^-$	$< 7 \times 10^{-7}$	theory input for $(g-2)_\mu$, BSM weak decays
$\eta \rightarrow \mu^+\mu^-$	$5.8(8) \times 10^{-6}$	theory input for $(g-2)_\mu$, BSM weak decays, P/CP violation
$\eta \rightarrow \pi^0\pi^0\ell^+\ell^-$		C/CP violation, ALPs
$\eta \rightarrow \pi^+\pi^-e^+e^-$	$2.68(11) \times 10^{-4}$	theory input for doubly-virtual TFF and $(g-2)_\mu$, P/CP violation, ALPs
$\eta \rightarrow \pi^+\pi^-\mu^+\mu^-$	$< 3.6 \times 10^{-4}$	theory input for doubly-virtual TFF and $(g-2)_\mu$, P/CP violation, ALPs
$\eta \rightarrow e^+e^-e^+e^-$	$2.40(22) \times 10^{-5}$	theory input for $(g-2)_\mu$
$\eta \rightarrow e^+e^-\mu^+\mu^-$	$< 1.6 \times 10^{-4}$	theory input for $(g-2)_\mu$
$\eta \rightarrow \mu^+\mu^-\mu^+\mu^-$	$< 3.6 \times 10^{-4}$	theory input for $(g-2)_\mu$
$\eta \rightarrow \pi^+\pi^-\pi^0\gamma$	$< 5 \times 10^{-4}$	direct emission only
$\eta \rightarrow \pi^+\pi^-\nu_e$	$< 1.7 \times 10^{-4}$	second-class current
$\eta \rightarrow \pi^+\pi^-$	$< 4.4 \times 10^{-6}$ [53]	P/CP violation
$\eta \rightarrow 2\pi^0$	$< 3.5 \times 10^{-4}$	P/CP violation
$\eta \rightarrow 4\pi^0$	$< 6.9 \times 10^{-7}$	P/CP violation

Channel	Expt. branching ratio	Discussion
$\eta' \rightarrow \eta\pi^+\pi^-$	42.6(7)%	large- N_c χ PT, light Higgs scalars
$\eta' \rightarrow \pi^+\pi^-\gamma$	28.9(5)%	chiral anomaly, theory input for singly-virtual TFF and $(g-2)_\mu$, P/CP violation
$\eta' \rightarrow \eta\pi^0\pi^0$	22.8(8)%	large- N_c χ PT
$\eta' \rightarrow \omega\gamma$	2.489(76)% [53]	theory input for singly-virtual TFF and $(g-2)_\mu$
$\eta' \rightarrow \omega e^+e^-$	$2.0(4) \times 10^{-4}$	theory input for doubly-virtual TFF and $(g-2)_\mu$
$\eta' \rightarrow 2\gamma$	2.331(37)% [53]	chiral anomaly, η - η' mixing
$\eta' \rightarrow 3\pi^0$	2.54(18)% (*)	$m_u = m_d$
$\eta' \rightarrow \mu^+\mu^-\gamma$	$1.09(27) \times 10^{-4}$	theory input for $(g-2)_\mu$, dark photon
$\eta' \rightarrow e^+\pi^-\gamma$	$4.73(30) \times 10^{-4}$	theory input for $(g-2)_\mu$, dark photon
$\eta' \rightarrow \pi^+\pi^-\mu^+\mu^-$	$< 2.9 \times 10^{-5}$	theory input for doubly-virtual TFF and $(g-2)_\mu$, P/CP violation, dark photon, ALPs
$\eta' \rightarrow \pi^+\pi^-e^+e^-$	$2.4(^{+1.3}_{-1.0}) \times 10^{-3}$	theory input for doubly-virtual TFF and $(g-2)_\mu$, P/CP violation, dark photon, ALPs
$\eta' \rightarrow \pi^0\pi^0\ell^+\ell^-$		C/CP violation, ALPs
$\eta' \rightarrow \pi^+\pi^-\pi^0$	$3.61(17) \times 10^{-3}$	$m_u = m_d$, C/CP violation, light Higgs scalars
$\eta' \rightarrow 2(\pi^+\pi^-)$	$8.4(9) \times 10^{-5}$	theory input for doubly-virtual TFF and $(g-2)_\mu$
$\eta' \rightarrow \pi^+\pi^-2\pi^0$	$1.8(4) \times 10^{-4}$	
$\eta' \rightarrow 2(\pi^+\pi^-)\pi^0$	$< 1.8 \times 10^{-3}$	ALPs
$\eta' \rightarrow K^+\pi^\mp$	$< 4 \times 10^{-5}$	weak interactions
$\eta' \rightarrow \pi^+\pi^-\nu_e$	$< 2.1 \times 10^{-4}$	second-class current
$\eta' \rightarrow \pi^0\gamma\gamma$	$3.20(24) \times 10^{-3}$	vector and scalar dynamics, B boson, light Higgs scalars
$\eta' \rightarrow \eta\gamma\gamma$	$8.3(3.5) \times 10^{-5}$ [56]	vector and scalar dynamics, B boson, light Higgs scalars
$\eta' \rightarrow 4\pi^0$	$< 4.94 \times 10^{-5}$ [57]	(S-wave) P/CP violation
$\eta' \rightarrow e^+\pi^-$	$< 5.6 \times 10^{-9}$	theory input for $(g-2)_\mu$, BSM weak decays
$\eta' \rightarrow \mu^+\mu^-$		theory input for $(g-2)_\mu$, BSM weak decays
$\eta' \rightarrow \ell^+\ell^-\ell^+\ell^-$		theory input for $(g-2)_\mu$
$\eta' \rightarrow \pi^+\pi^-\pi^0\gamma$		B boson
$\eta' \rightarrow \pi^+\pi^-$	$< 1.8 \times 10^{-5}$	P/CP violation
$\eta' \rightarrow 2\pi^0$	$< 4 \times 10^{-4}$	P/CP violation

See JLAB physics review
 Gan, Kubis, Passemar and Tulin (2020)

Ways to measure η - η' mixing outside of LHCb: $\Gamma(\eta^{(\prime)} \rightarrow \gamma\gamma)$

- A way to measure θ_P is to measure $\Gamma(\eta^{(\prime)} \rightarrow \gamma\gamma)$ at e^+e^- colliders and Primakoff experiments

$$\Gamma(\eta^{(\prime)} \rightarrow \gamma\gamma) = \frac{M_{\eta^{(\prime)}}^3}{4\pi} \left| \kappa_{\eta^{(\prime)}} \right|^2$$

$$\begin{pmatrix} \kappa_\eta \\ \kappa_{\eta'} \end{pmatrix} = -i\alpha \left(\frac{1}{4\pi} \frac{1}{\sqrt{3}} ((M(\theta_P)F)^{-1})^T \begin{pmatrix} 1 \\ \sqrt{8} \end{pmatrix} + \pi \frac{t_1}{9/\sqrt{3}F_\pi} M(\theta_P) \begin{pmatrix} 7M_\pi^2 - 4M_K^2 \\ \sqrt{8}(2M_\pi^2 + M_K^2) \end{pmatrix} \right)$$

- where there are the form factors $F = \begin{pmatrix} F_{88} & F_{81} \\ F_{81} & F_{11} \end{pmatrix}$,
 mixing matrix $M(\theta_P) = \begin{pmatrix} \cos \theta_P & -\sin \theta_P \\ \sin \theta_P & \cos \theta_P \end{pmatrix}$ and a Low Energy Constant $t_1 = \frac{F_\pi^2}{m_\rho^4}$
- More detail in [JLAB \$\eta\$ proposals \(2009\)](#)

Testing $SU(3)_F$ symmetry breaking

Using LHCb's two measurements of ϕ_P

- ϕ_P from the ratios $R_{\eta^{(\prime)}}$ assumes $SU(3)_F$ symmetry.
- ϕ_P from the ratios $R_{(s)}$ does not require the same approximation.
- Controlling for systematics means the differences between the two measurements would imply breaking of $SU(3)_F$ symmetry.
- Note the LHCb Run-1 measurements with these methods were
 - $\phi_P = (46.3 \pm 2.3)^\circ$ from the ratios $R_{\eta^{(\prime)}}$
 - $\phi_P = (43.5^{+1.4}_{-2.8})^\circ$ from the ratios $R_{(s)}$

Testing $SU(3)_F$ symmetry breaking

Non-factorisable $SU(3)_F$ symmetry breaking with \mathcal{B} ($B^0 \rightarrow J/\psi K^0$)

- A method highlighted in [Fleischer, Knegjens, Ricciardi (2011)]
- This can use our new measurements of the mixing angles and \mathcal{B}

$$K_{SU(3)}^{\eta(\prime)} \equiv \frac{\tau_{B^0}}{\tau_{B_s^0}} \left[\frac{M_{B^0} \Phi_d^{K^0}}{M_{B_s^0} \Phi_s^{\eta(\prime)}} \right]^3 \left[\frac{F_1^{B^0 K^0}(M_{J/\psi}^2)}{F_1^{B_s^0 \eta(\prime)}(M_{J/\psi}^2)} \right]^2 \frac{\mathcal{B}(B_s^0 \rightarrow J/\psi \eta(\prime))}{\mathcal{B}(B^0 \rightarrow J/\psi K^0)}$$

- where $F_1(M_{J/\psi}^2)$ are the respective form factors which can be projected to $|\eta_s\rangle = |s\bar{s}\rangle$

$$F_1^{B_s^0 \eta}(M_{J/\psi}^2) = -\sin \phi_P F_1^{B^0 K^0}(M_{J/\psi}^2)$$

$$F_1^{B_s^0 \eta'}(M_{J/\psi}^2) = \cos \phi_P \cos \phi_G F_1^{B^0 K^0}(M_{J/\psi}^2)$$

- $K_{SU(3)}^{\eta(\prime)} = 1$ would mean no non-factorisable $SU(3)_F$ symmetry breaking.

Testing $SU(3)_F$ symmetry breaking

Non-factorisable $SU(3)_F$ symmetry breaking with \mathcal{B} ($B^0 \rightarrow J/\psi K^0$)

- If we take LHCb's Run-1 values of ϕ_P and ϕ_G (from $R_{(s)}$) along with the PDG2020 values for the \mathcal{B} and $B_{(s)}^0$ τ and masses

$$K_{SU(3)}^\eta = 0.92^{+0.31}_{-0.21}, \quad K_{SU(3)}^{\eta'} = 0.85^{+0.23}_{-0.08}$$

- Both values are compatible with one

Resulting uncertainties - ϕ_P - extended

$$R_{\eta^{(\prime)}} = \frac{\mathcal{B}(B^0 \rightarrow J/\psi \eta^{(\prime)})}{\mathcal{B}(B_s^0 \rightarrow J/\psi \eta^{(\prime)})}, \quad R_{(s)} = \frac{\mathcal{B}(B^0_{(s)} \rightarrow J/\psi \eta')}{\mathcal{B}(B^0_{(s)} \rightarrow J/\psi \eta)}$$

\mathcal{B} Ratio	Statistics	$\mathcal{B} (\eta^{(\prime)} \rightarrow X)$	f_s/f_d	Total
$R_\eta (\gamma)$	10.0%		1.4%	10.1%
$R_\eta (\gamma\gamma)$	4.4%		1.4%	4.4%
$R_{\eta'} (\gamma)$	7.1%		1.4%	7.2%
$R_{\eta'} (\gamma\gamma)$	8.7%		1.4%	8.8%
$R (\gamma)$	9.2%	0.7%		9.2%
$R (\gamma\gamma)$	6.7%	0.6%		6.7%
$R_s (\gamma)$	1.2%	0.8%		1.4%
$R_s (\gamma\gamma)$	1.3%	0.5%		1.4%

- Errors greatly reduced due to $R \propto \tan^2 \phi_P$
- Expected improvement over averaging across all modes and calculation methods
- Needs systematic studies
- Current combined uncertainties of $\sim 1\%$ on ϕ_P
- LHCb Run-1 uncertainties: $\sim 5\%$

Resulting uncertainties - \mathcal{B} - extended

$\mathcal{B}(B_s^0 \rightarrow J/\psi \eta^{(\prime)})$ - PDG uncertainties $\sigma(\eta) = 17.5\%$ and $\sigma(\eta') = 12.1\%$

Channel	Statistics	$\mathcal{B}(\eta^{(\prime)} \rightarrow X)$	$\mathcal{B}(B^0 \rightarrow J/\psi \rho^0)$	f_s/f_d	η_γ^{cor}	Total
$\eta \rightarrow \pi^+ \pi^- \pi^0$	2.6%	1.9%	6.7%	3.1%	6.5%	10.3%
$\eta \rightarrow \pi^+ \pi^- \gamma$	3.2%	1.2%	6.7%	3.1%	7.5%	11.1%
$\eta' \rightarrow \pi^+ \pi^- \eta$	3.7%	1.3%	6.7%	3.1%	6.5%	10.6%
$\eta' \rightarrow (\rho^0 \rightarrow \pi^+ \pi^-) \gamma$	2.1%	1.4%	6.7%	3.1%	7.5%	10.8%

$\mathcal{B}(B^0 \rightarrow J/\psi \eta^{(\prime)})$ - PDG uncertainties $\sigma(\eta) = 21.3\%$ and $\sigma(\eta') = 31.6\%$

Channel	Statistics	$\mathcal{B}(\eta^{(\prime)} \rightarrow X)$	$\mathcal{B}(B^0 \rightarrow J/\psi \rho^0)$	η_γ^{cor}	Total
$\eta \rightarrow \pi^+ \pi^- \pi^0$	9.8%	1.9%	6.7%	6.5%	13.7%
$\eta \rightarrow \pi^+ \pi^- \gamma$	24.5%	1.2%	6.7%	7.5%	26.5%
$\eta' \rightarrow \pi^+ \pi^- \eta$	19.2%	1.3%	6.7%	6.5%	21.4%
$\eta' \rightarrow (\rho^0 \rightarrow \pi^+ \pi^-) \gamma$	15.4%	1.4%	6.7%	7.5%	18.4%

Recent experimental publications - high energy frontier

- LHCb

- Run-1 η - η' mixing measurement with $B_{(s)}^0 \rightarrow J/\psi \eta^{(\prime)}$ [JHEP 01 (2015) 024]
- Search for decay $\Lambda_b^0 \rightarrow \Lambda \eta^{(\prime)}$ [JHEP 09 (2015) 006]
- Search for decay $B_s^0 \rightarrow \eta' \phi$ [JHEP 05 (2017) 158]
- CP Violation in $D_{(s)}^+ \rightarrow h^+ \eta$ [JHEP 06 (2021) 019]

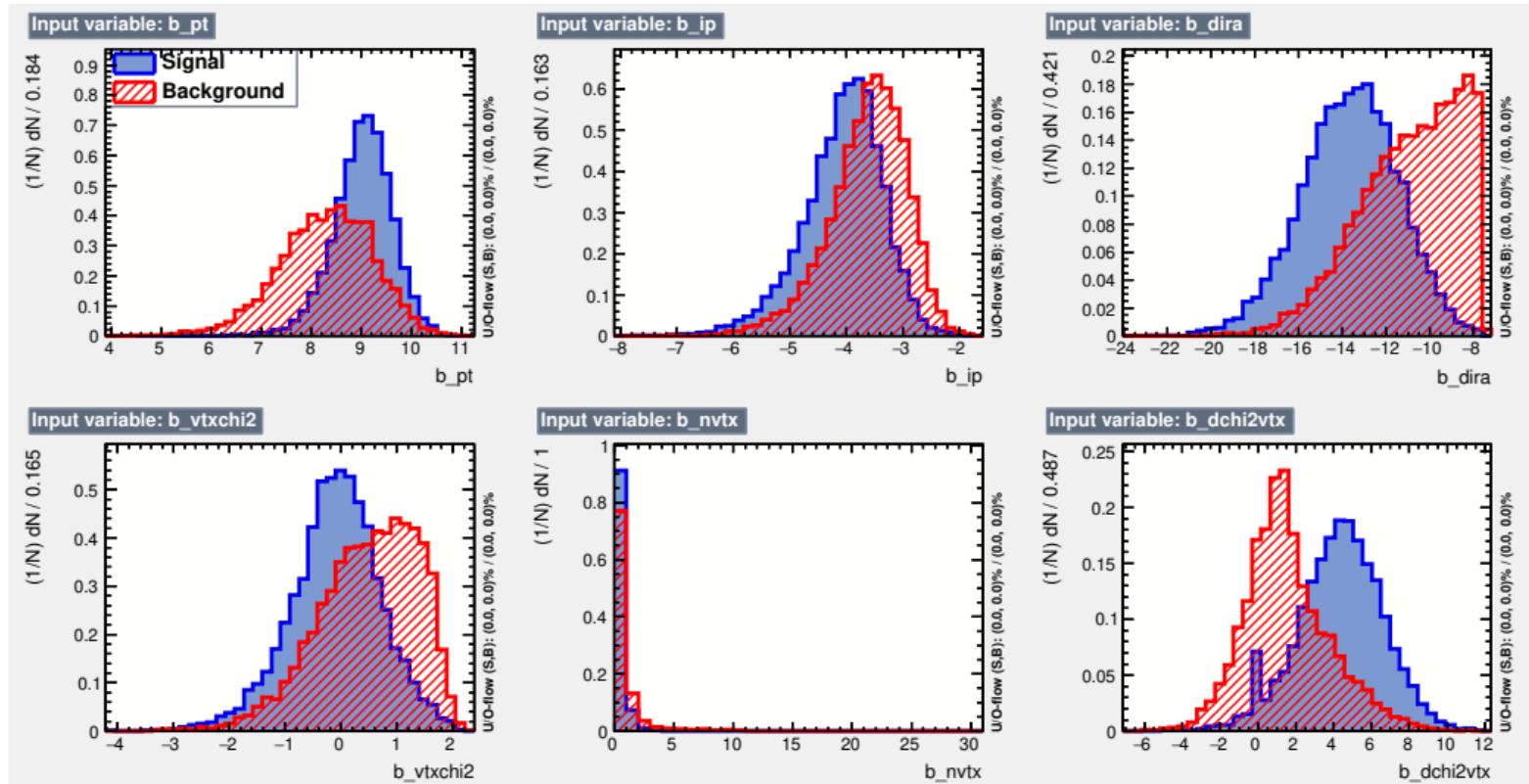
- Belle-I&II

- Search for $B_s^0 \rightarrow \eta' \eta$ [Phys.Rev.D 104 (2021) 3, L031101]
- CP Violation in decays $D^0 \rightarrow \pi^+ \pi^- \eta$, $D^0 \rightarrow K^+ K^- \eta$ and $D^0 \rightarrow \phi \eta$ [JHEP 09 (2021) 075]
- Many searches planned in [The Belle II Physics Book]

- BaBar

- Light meson spectroscopy in $\eta_c \rightarrow \eta^{(\prime)} h^+ h^-$ [Phys.Rev.D 104 (2021) 7, 072002]
- LFV in $D^0 \rightarrow X^0 e^\pm \mu^\pm$ (including $X^0 = \eta$) [Phys.Rev.D 101 (2020) 11, 112003]
- Measurement of the form factors of $\gamma^* \gamma^* \rightarrow \eta'$ [Phys.Rev.D 98 (2018) 11, 112002]

BDT inputs - $\eta' \rightarrow \rho^0 \gamma$ (1)



BDT inputs - $\eta' \rightarrow \rho^0 \gamma$ (2)

