

Axion Effective Action

Pham Ngoc Hoa Vuong
(LPSC, Grenoble)

In collaboration with Jeremie Quevillon, Christopher Smith
to appear on arXiv

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Outline of this talk

I. Building Axion Effective Action

- Anomaly & EFT coefficients
- Functional method: Integrating out chiral-fermions & derivative couplings

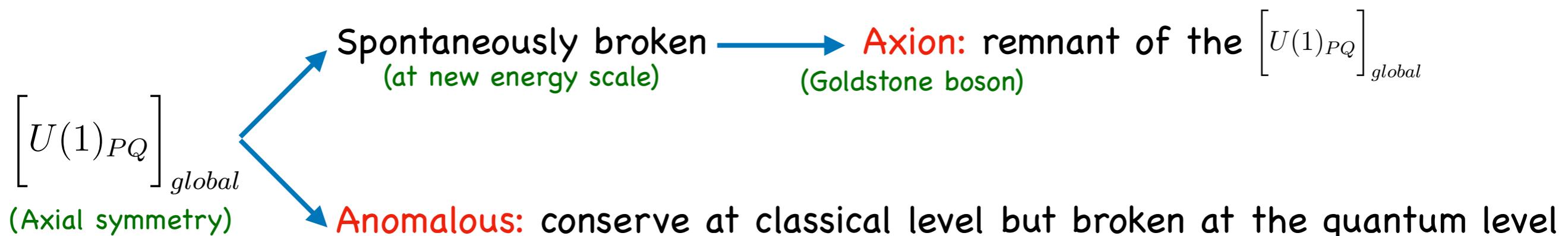
Summary

I. Building axion EFTs: Axion-gauge bosons couplings (1)

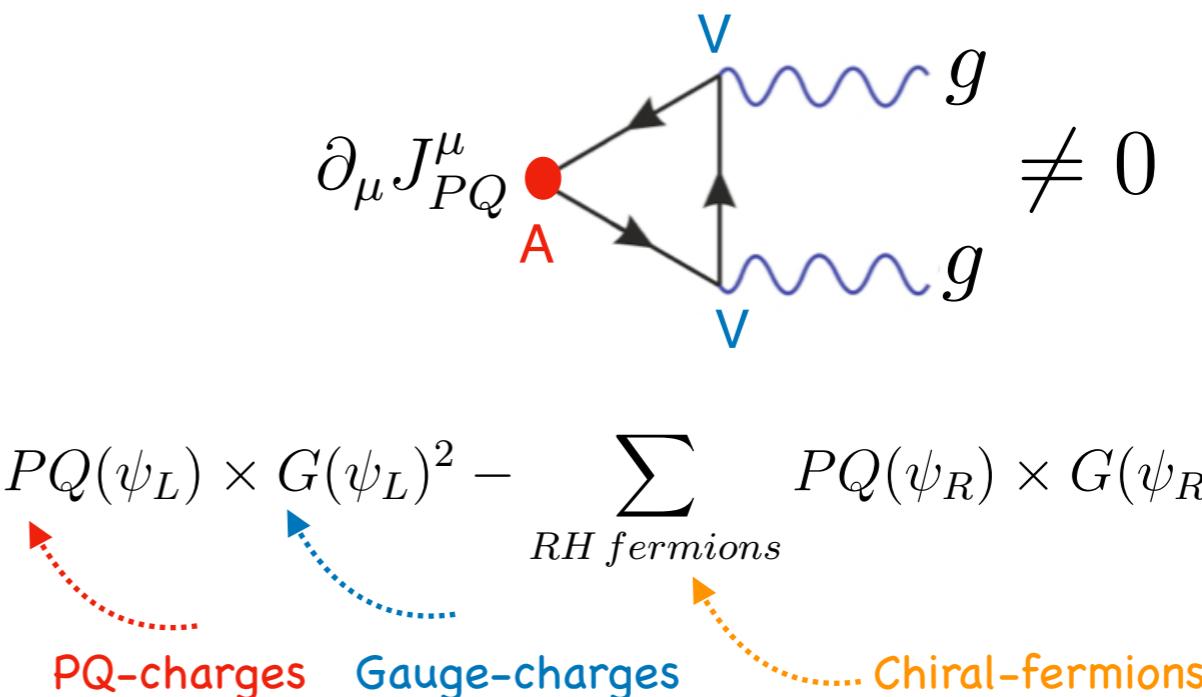
#Anomalous coefficients

- “Peccei-Quinn” paradigm:

$$\left[\text{SM symmetries} \right]_{\text{local}} \otimes \left[U(1)_{PQ} \right]_{\text{global}}$$



- Anomalous coefficient: $\mathcal{A}_{aGG}^{PQ} = \sum_{LH \text{ fermions}} PQ(\psi_L) \times G(\psi_L)^2 - \sum_{RH \text{ fermions}} PQ(\psi_R) \times G(\psi_R)^2 \neq 0$



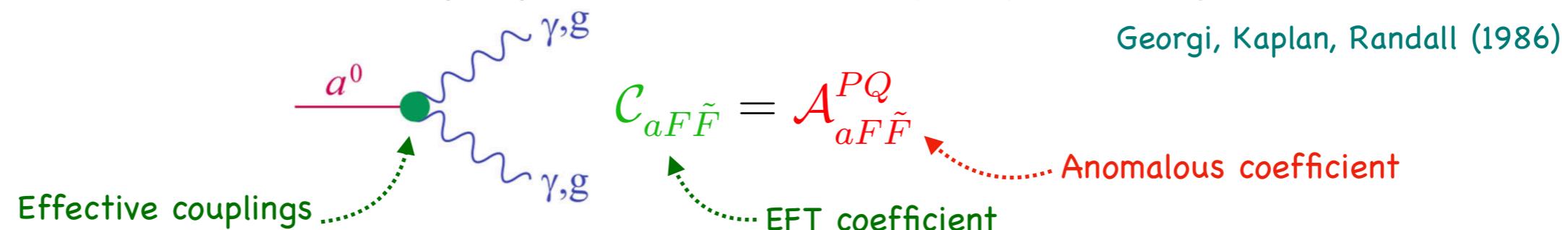
I. Building axion EFTs: Axion-gauge bosons couplings (2)

#EFT coefficients

- Axion-gauge bosons couplings play an essential role in axion phenomenology

$$\mathcal{L}_{\text{EFT}} \supset -\frac{\mathcal{C}_{aF\tilde{F}}}{16\pi^2 f_a} a F_{\mu\nu} \tilde{F}_{\mu\nu}$$

- Axion couples with massless vector gauge fields: (for example: photons, gluons)



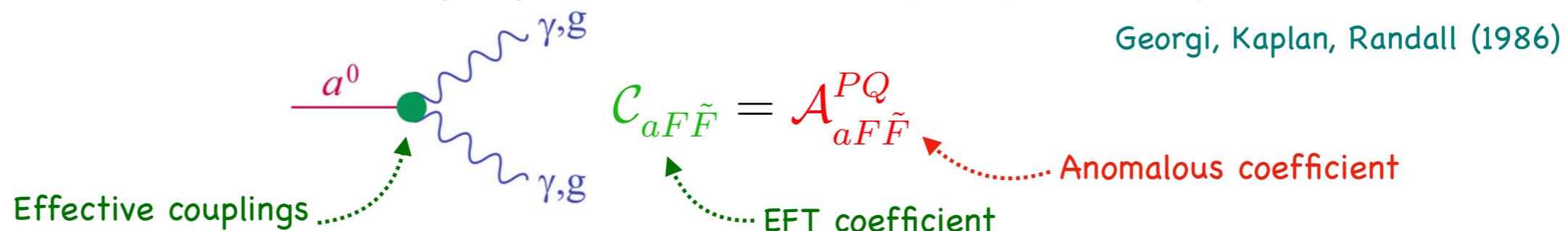
I. Building axion EFTs: Axion-gauge bosons couplings (2)

#EFT coefficients

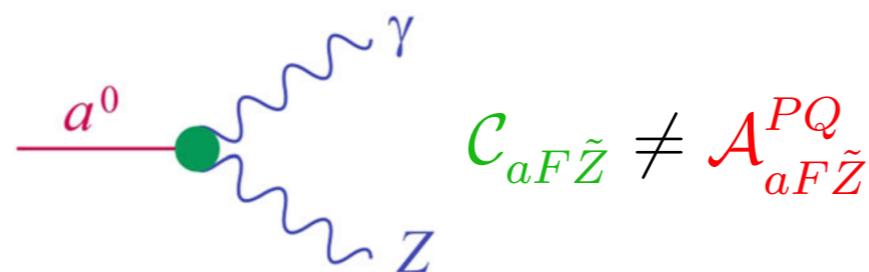
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- Axion couples with massless vector gauge fields: (for example: photons, gluons)



- Axion couples with massive chiral gauge fields: (for example: Z-boson, W-boson)



(J. Quevillon, C. Smith , arXiv:1903.12559)

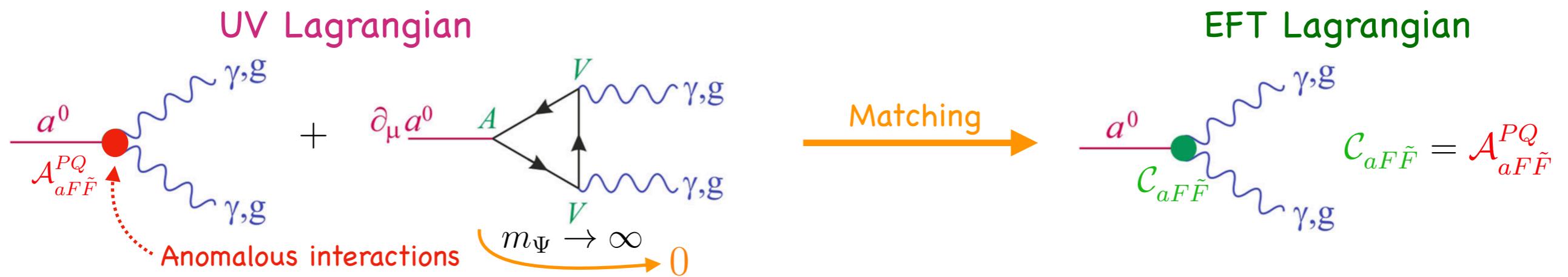
(Q. Bonnefoy, L. Di Luzio, C. Grojean, A. Paul, and A. N. Rossia , arXiv:2011.10025)

Main message: anomalous coefficients do not fully capture all Axion EFT couplings

I. Building axion EFTs: Axion-gauge bosons couplings (3)

$$\mathcal{L}_{\text{UV}} \supset -\frac{\mathcal{A}_{aF\tilde{F}}^{PQ}}{16\pi^2} \frac{a}{f_a} F\tilde{F} + \frac{(\partial_\mu a)}{f_a} \bar{\Psi} (g_V^{PQ} \gamma^\mu - g_A^{PQ} \gamma^\mu \gamma^5) \Psi + \bar{\Psi} (V_\mu \gamma^\mu - A_\mu \gamma^\mu \gamma^5 - m_\Psi) \Psi$$

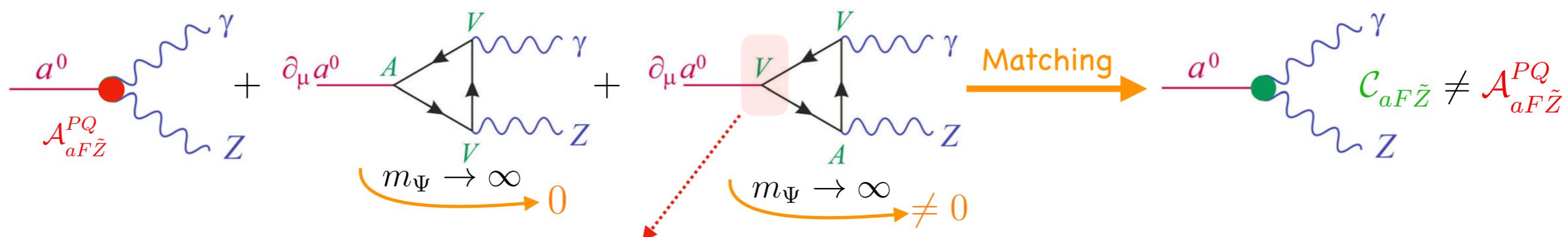
- When axion couples with massless vector gauge fields:



- When axion couples with massive chiral gauge fields:

(J. Quevillon, C. Smith , arXiv:1903.12559)

Example: $a \rightarrow Z\gamma$

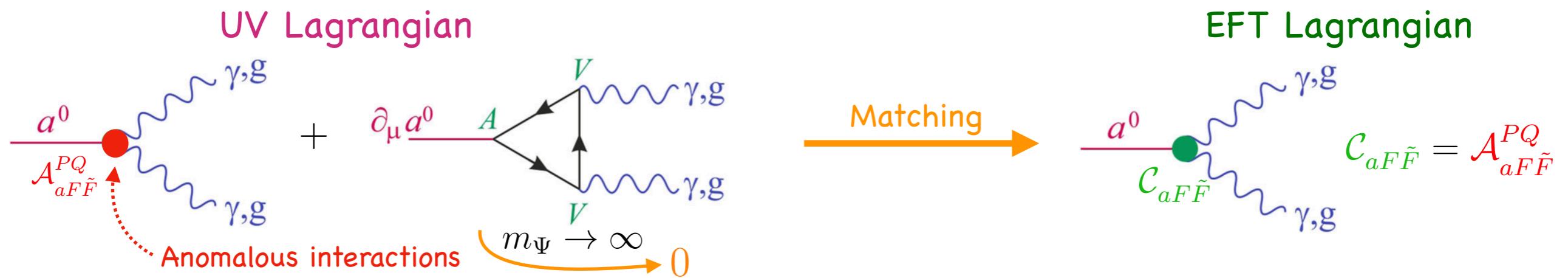


Vector current of PQ-symmetry is anomalous
(analogous with the anomalous of fermion number current)

I. Building axion EFTs: Axion-gauge bosons couplings (4)

$$\mathcal{L}_{\text{UV}} \supset -\frac{\mathcal{A}_{aF\tilde{F}}^{PQ}}{16\pi^2} \frac{a}{f_a} F\tilde{F} + \frac{(\partial_\mu a)}{f_a} \bar{\Psi} (g_V^{PQ} \gamma^\mu - g_A^{PQ} \gamma^\mu \gamma^5) \Psi + \bar{\Psi} (V_\mu \gamma^\mu - A_\mu \gamma^\mu \gamma^5 - m_\Psi) \Psi$$

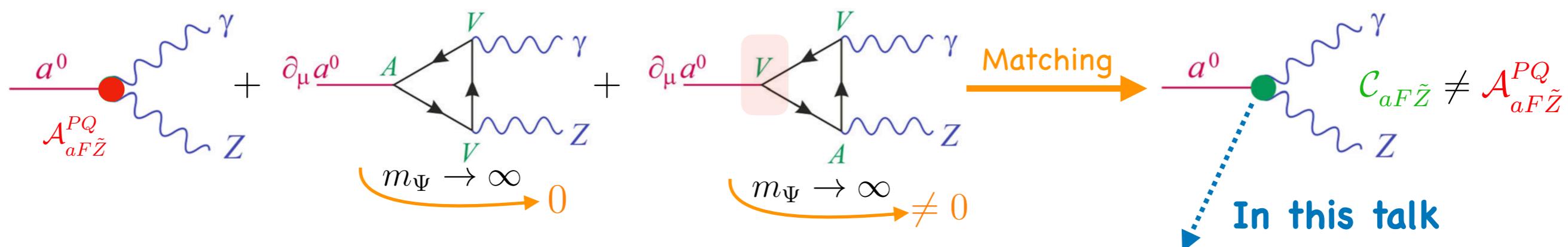
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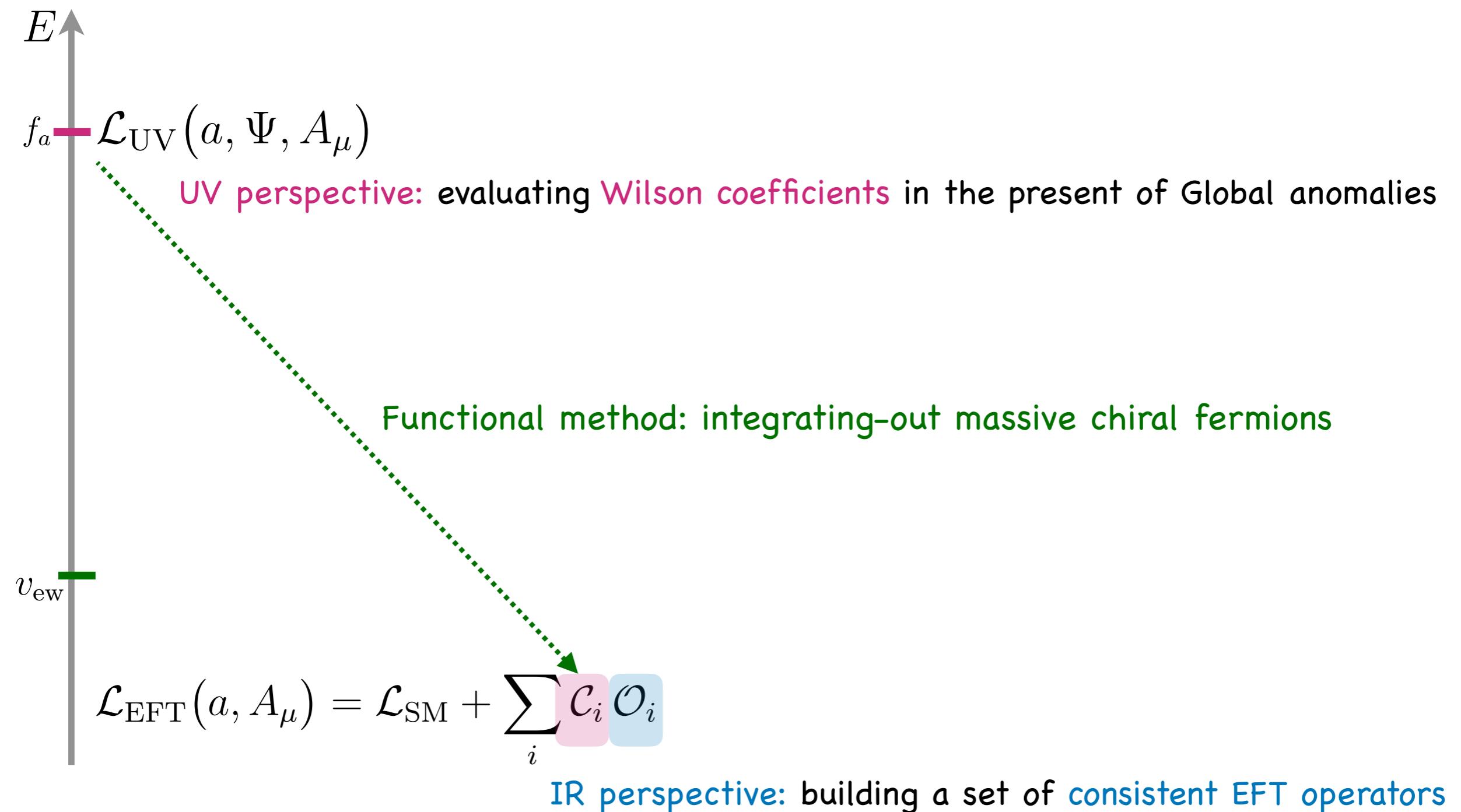
(J. Quevillon, C. Smith , arXiv:1903.12559)

Example: $a \rightarrow Z\gamma$



Reformulate this phenomena by: Matching with Path-integral approach & Building a consistent low-energy EFT for axion phenomenology

I. Building axion EFTs: UV/IR point-of-view



I. Building axion EFTs: Set up axion UV Lagrangian

- Starting point: axion UV Lagrangian

$$\mathcal{L}_{\text{UV}}^{\text{fermion}} = \bar{\Psi} (i\partial_\mu \gamma^\mu + g_V V_\mu \gamma^\mu - g_A A_\mu \gamma^\mu \gamma^5) \Psi - y_\Psi (\bar{\Psi}_L \phi_A \Psi_R + \text{h.c.})$$

- PQ-symmetry: $\Psi_L \rightarrow e^{i(g_V^{PQ} + g_A^{PQ})\theta} \Psi_L$, $\Psi_R \rightarrow e^{i(g_V^{PQ} - g_A^{PQ})\theta} \Psi_R$, $\phi_A \rightarrow e^{i(2g_A^{PQ})\theta} \phi_A$

- PQ spontaneously broken: $\phi_A \supset f_a \exp \left[i g_\phi^{PQ} \frac{a(x)}{f_a} \right]$, with $g_\phi^{PQ} = 2g_A^{PQ}$

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Fermion field-dependent reparametrisation: $\Psi_L \rightarrow e^{i(g_V^{PQ} + g_A^{PQ})a(x)} \Psi_L$, $\Psi_R \rightarrow e^{i(g_V^{PQ} - g_A^{PQ})a(x)} \Psi_R$

$$\mathcal{L}_{\text{UV}}^{\text{fermion}} = \mathcal{L}_{\text{UV}}^{\text{Anomalous}} + \bar{\Psi} \left(i\partial_\mu \gamma^\mu + g_V V_\mu \gamma^\mu - g_A A_\mu \gamma^\mu \gamma^5 - M + \frac{\partial_\mu a}{f_a} [g_V^{PQ} \gamma^\mu - g_A^{PQ} \gamma^\mu \gamma^5] \right) \Psi$$

Path-integral measure
is not invariant under the chiral transformation

$$\int \mathcal{D}\bar{\Psi} \mathcal{D}\Psi \rightarrow (\log \mathcal{J}) \int \mathcal{D}\bar{\Psi} \mathcal{D}\Psi$$

Contribute to EFT coefficients at one-loop level
=> Evaluate the one-loop effective action

$$\mathcal{L}_{\text{UV}}^{\text{Anomalous}} \supset -\frac{\mathcal{A}_{aF\tilde{F}}^{PQ}}{16\pi^2} \frac{a}{f_a} F \tilde{F}$$

I. Building axion EFTs: One-Loop Effective Action

We parameterise the shape of UV Lagrangian as follows:

$$\mathcal{L}_{\text{UV}}^{\text{fermion}}[\Psi_H, \phi] \supset \bar{\Psi}_H \left[i \partial_\mu \gamma^\mu - M + X[\phi] \right] \Psi_H$$

general coupling with background fields

Example: $X[\phi] = V_\mu[\phi]\gamma^\mu - A_\mu[\phi]\gamma^\mu\gamma^5 - W_1[\phi]i\gamma^5$

Path Integral: extract the one-loop (heavy-only) piece:

$$S_{\text{eff}}^{\text{1-loop}} = -i \text{Tr} \log \left(-\frac{\delta^2 S}{\delta \Psi_H^2} \Big|_{\Psi_H, c} \right) = -i \text{Tr} \log \left(i \partial_\mu \gamma^\mu - M + X[\phi] \right)$$

evaluating functional trace

$$\mathcal{L}_{\text{EFT}}^{\text{1loop}} = i \text{tr} \sum_{n=1}^{\infty} \frac{1}{n} \int \frac{d^4 q}{(2\pi)^4} \left[\frac{-1}{q + M} \left(-i \partial_\mu \gamma^\mu - V_\mu[\phi]\gamma^\mu + A_\mu[\phi]\gamma^\mu\gamma^5 + W_1[\phi]i\gamma^5 \right) \right]^n$$

- Expanding order by order (ex: up to n=6)
- Integrating over momentum q (use Dimensional Regularisation for divergence integrals)
- Evaluating the Dirac traces (careful with γ^5)

I. Building axion EFTs: Anomaly-related operators: The problems (1)

- Power counting: new operator structures appear

$$\mathcal{L}_{\text{EFT}}^{\text{1loop}} \supset \omega_{AAA} (\partial_\mu a) A_\nu \tilde{F}_A^{\mu\nu}$$

- Wilson coefficients:

$$\omega_{AAA} \begin{cases} \supset \int \frac{d^4 q}{(2\pi)^d} \frac{1}{q^4 - M^4} : \text{divergence integral} & \xrightarrow{\text{Dimensional regularisation}} \\ & \text{(evaluate integrals in d-dimensions)} \\ \supset \text{tr}(\cdots \gamma^5) & \xrightarrow{\text{'t' Hooft \& Veltman's scheme: might obtain wrong results}} \\ & \text{(vector component of PQ-symmetry can be anomalous !)} \end{cases}$$

I. Building axion EFTs: Anomaly-related operators: The problems (2)

- Power counting: new operator structures appear

$$\mathcal{L}_{\text{EFT}}^{\text{1loop}} \supset \omega_{AAA} (\partial_\mu a) A_\nu \tilde{F}_A^{\mu\nu}$$

- Wilson coefficients:

$$\omega_{AAA} \begin{cases} \supset \int \frac{d^4 q}{(2\pi)^d} \frac{1}{q^4 - M^4} : \text{divergence integral} \\ \supset \text{tr}(\cdots \gamma^5) \end{cases} \xrightarrow{\text{Dimensional regularisation}} \text{(evaluate integrals in d-dimensions)}$$

(t' Hooft & Veltman's scheme: might obtain wrong results
(vector component of PQ-symmetry can be anomalous !)

What if we try hard to evaluate this coefficient?

Key point: ambiguity on the location of γ^5

(t' Hooft & Veltman)

$$\begin{aligned} \text{tr} \left(\gamma_a \not{V}^i \gamma_b \not{V}^j \gamma_c \not{P} \gamma_d \not{A}^k \gamma^5 \right) \Big|_{d=4-\epsilon} \longrightarrow & \alpha_1 \text{tr} \left(\gamma_a \not{V}^i \gamma^5 \gamma_b \not{V}^j \gamma_c \not{P} \gamma_d \not{A}^k \right) \Big|_{d=4-\epsilon} + \beta_1 \text{tr} \left(\gamma_a \not{V}^i \gamma_b \not{V}^j \gamma^5 \gamma_c \not{P} \gamma_d \not{A}^k \right) \Big|_{d=4-\epsilon} \\ & + \theta_1 \text{tr} \left(\gamma_a \not{V}^i \gamma_b \not{V}^j \gamma_c \not{P} \gamma^5 \gamma_d \not{A}^k \right) \Big|_{d=4-\epsilon} + \eta_1 \text{tr} \left(\gamma_a \not{V}^i \gamma_b \not{V}^j \gamma_c \not{P} \gamma_d \not{A}^k \gamma^5 \right) \Big|_{d=4-\epsilon} \end{aligned}$$

=> ω_{AAA} (free parameters)

decide if a symmetry is broken or not

I. Building axion EFTs: Anomaly-related operators: The problems (3)

- Power counting: new operator structures appear

$$\mathcal{L}_{\text{EFT}}^{\text{1loop}} \supset \omega_{AAA} (\partial_\mu a) A_\nu \tilde{F}_A^{\mu\nu}$$

- Wilson coefficients:

$$\omega_{AAA}$$

$\supset \int \frac{d^4 q}{(2\pi)^d} \frac{1}{q^4 - M^4}$: divergence integral \rightarrow Dimensional regularisation
 (evaluate integrals in d-dimensions)

$\supset \text{tr}(\cdots \gamma^5)$ \rightarrow t' Hooft & Veltman's scheme: might obtain wrong results
 (vector component of PQ-symmetry can be anomalous !)

- EFT operators:

$$(\partial_\mu a) A_\nu \tilde{F}_A^{\mu\nu}$$

PQ-invariant
 Virtually gauge-invariant ! $(\delta_A [(\partial_\mu a) A_\nu \tilde{F}_A^{\mu\nu}] = [(\partial_\mu a) \theta_A \tilde{F}_A^{\mu\nu}])$

Problem: ambiguous Wilson coefficient but virtually gauge-invariant operator

=> No counter terms to fix the value of ω_{AAA}

I. Building axion EFTs: Anomaly-related operators: The solution (1)

- Let's gauge the PQ-symmetry: (Q. Bonnefoy, L. Di Luzio, C. Grojean, A. Paul, and A. N. Rossia , arXiv:2011.10025)

$$\mathcal{L}_{\text{EFT}}^{\text{1loop}} \supset \omega_{AAA} (\partial_\mu a) A_\nu \tilde{F}_A^{\mu\nu}$$

↓
Introduce an auxiliary gauge field: A_μ^{PQ}

$$\mathcal{L}_{\text{EFT}}^{\text{1loop}} \supset \omega_{AAA} (\partial_\mu a - A_\mu^{PQ}) A_\nu \tilde{F}_A^{\mu\nu}$$

Chern-Simon operators

PQ-invariant
Virtually gauge-invariant !

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After EW symmetry breaking: Chiral fermions & gauge fields obtain their mass

$$\mathcal{L}_{\text{UV}}^{\text{fermion}} \supset \bar{\Psi} \left(-M \frac{\pi_A(x)}{v_A} i\gamma^5 \right) \Psi \Rightarrow \text{New counter terms appear!}$$

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Enforcing gauge-invariant

$$\mathcal{L}_{\text{EFT}}^{\text{1loop}} \supset \omega_{AAA} (\partial_\mu a - A_\mu^{PQ}) A_\nu \tilde{F}_A^{\mu\nu} + \eta_{APA} \frac{\pi_A(x)}{v_A} F_{PQ}^{\mu\nu} \tilde{F}_A^{\mu\nu}$$

Goldstone-gauge bosons operators

Imposing non-trivial constrain on Wilson coefficients

I. Building axion EFTs: Anomaly-related operators: The solution (2)

- Evaluating new counter terms:

$$\mathcal{L}_{\text{EFT}}^{\text{1loop}} \supset i \text{tr} \sum_{n=1}^{\infty} \frac{1}{n} \int \frac{d^4 q}{(2\pi)^4} \left[\frac{-1}{q + M} \left(-i\partial_\mu \gamma^\mu - (V_\mu \gamma^\mu - A_\mu \gamma^\mu \gamma^5) + \pi_A(x) i\gamma^5 \right. \right.$$

$$\left. \left. - \{V_\mu^{PQ} \gamma^\mu - A_\mu^{PQ} \gamma^\mu \gamma^5\} \right) \right]^n$$

- Directly expand the master formula
- Finite integrals, unambiguous Dirac traces

$$\mathcal{L}_{\text{EFT}}^{\text{1loop}} \supset \frac{1}{4\pi^2} \pi_A(x) F_{V^{PQ}} \tilde{F}_V + \frac{1}{12\pi^2} \pi_A(x) F_{A^{PQ}} \tilde{F}_A$$

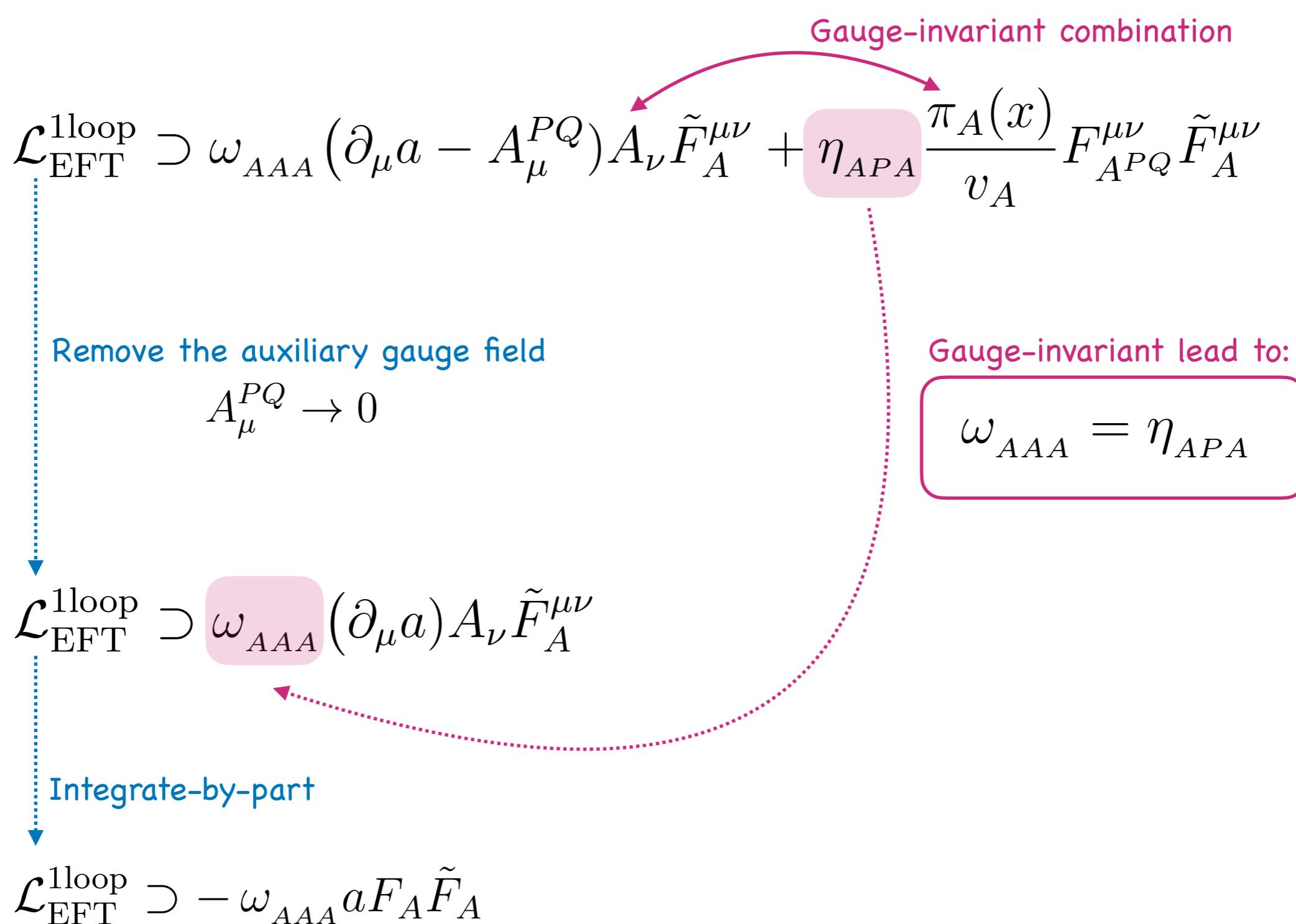
Not gauge invariant!
 $\delta_A \pi_A(x) = v\theta_A$

Loop & Dirac traces coefficients

I. Building axion EFTs: Anomaly-related operators: The solution (3)

- Wilson coefficients & Gauge-invariant combinations:

Example:



I. Building axion EFTs: Summary

- Axion bosonic EFT Lagrangian:

Anomalous structure of the theory

$$\mathcal{L}_{\text{EFT}}^{\text{axion}} \supset \mathcal{L}_{\text{UV}}^{\text{Anomalous}} + \left[\frac{1}{4\pi^2 f_a} g_V^{PQ} g_A g_V (\partial_\mu a) A_\nu \tilde{F}_V^{\mu\nu} + \frac{1}{12\pi^2 f_a} g_A^{PQ} g_A g_A (\partial_\mu a) A_\nu \tilde{F}_A^{\mu\nu} \right]$$

Non-decoupling effect after integrating-out chiral fermions

Their combination will generate the true value of EFT coefficient

- Gauge-invariant features:

Matching by Feynman diagrams

- Study Ward identities
(To do for each diagram)

Matching by Path integral

- Study gauge-invariant combination of EFT operators
(Hint a universal cooking recipe,
Universal coefficients can be computed once-and-for-all)

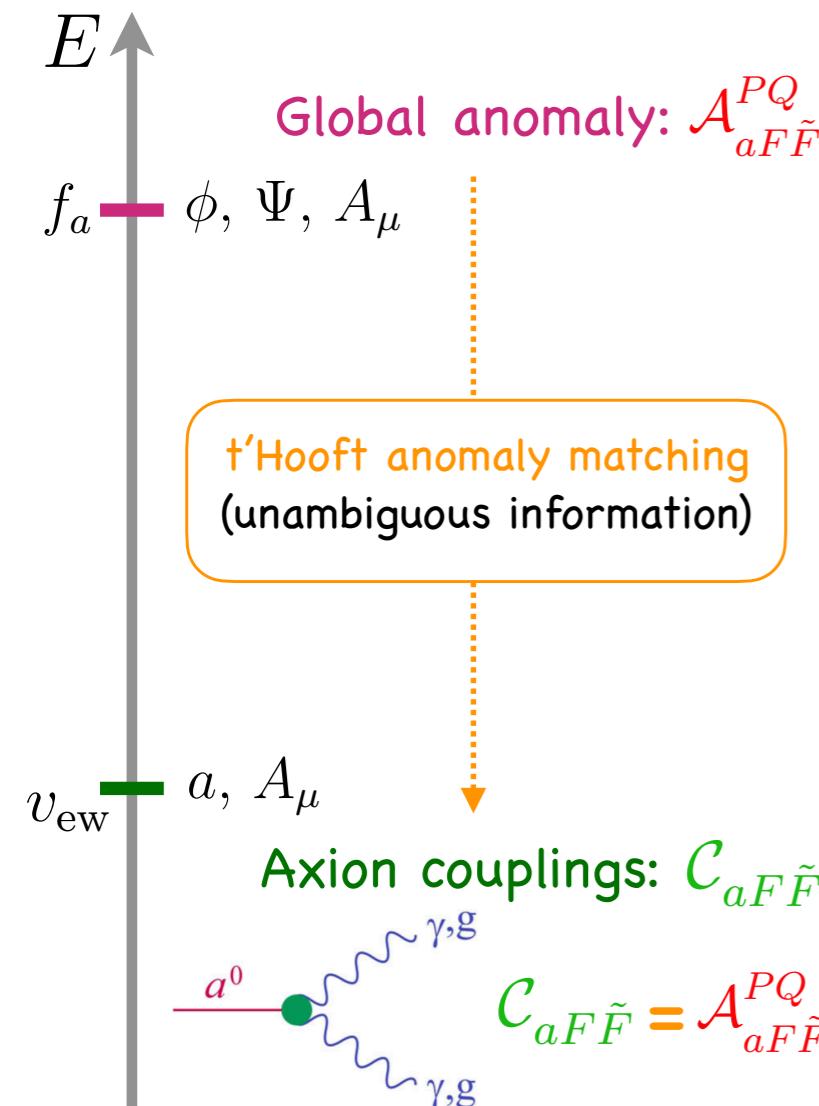
- Our approach can generalise to build EFTs for ALPs

Backup slides

I. Backup slides: Anomalous coefficient vs EFT coefficient

- Axion-gauge boson couplings play an essential role in axion phenomenology

$$\mathcal{L}_{\text{EFT}} \supset -\frac{\mathcal{C}_{aF\tilde{F}}}{16\pi^2 f_a} a F_{\mu\nu} \tilde{F}_{\mu\nu}$$



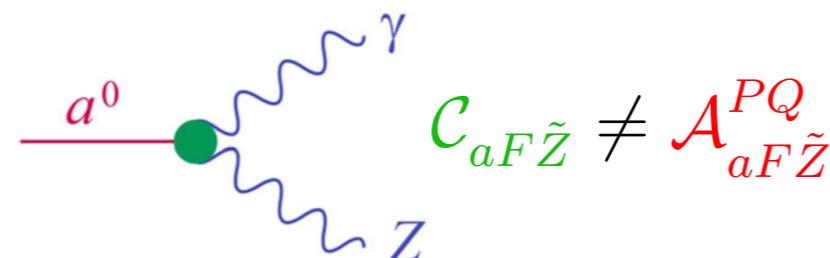
- Axion couples with massless vector gauge fields: Photons/Gluons

$$\mathcal{C}_{aF\tilde{F}} = \mathcal{A}_{aF\tilde{F}}^{PQ}$$

But, recently...

- Axion couples with massive chiral gauge fields: Z, W^\pm

In DFSZ-like axion:



(J. Quevillon, C. Smith , arXiv:1903.12559)

(Q. Bonnefoy, L. Di Luzio, C. Grojean, A. Paul, and A. N. Rossia , arXiv:2011.10025)

Main message: anomalous coefficients do not fully capture all Axion EFT couplings

I. Backup slides: Generic EFTs from the IR point-of-view

- Without knowledge of UV-complete theory, any QFT is just an EFT
- Use effective operators to parametrise new physics at higher energy scale

$$\mathcal{L}^{EFT} = \mathcal{L}_{d=4}^{SM} + \sum_{d, i} \frac{c_i}{\Lambda^{d-4}} \mathcal{O}_{d>4}^i$$

Wilson coefficients
Encapsulate effect of new physics

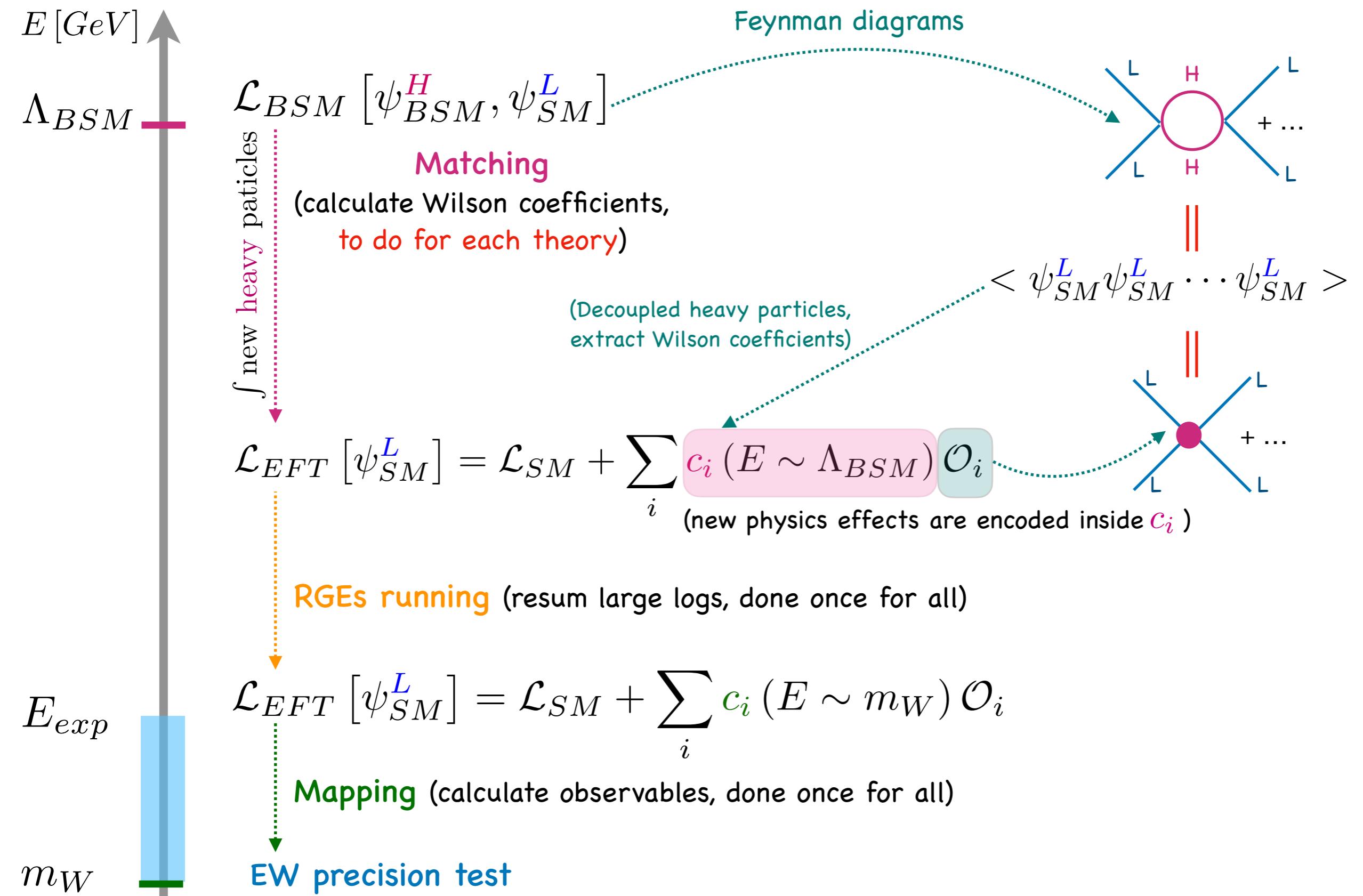
Non-renormalizable operators
Made up of **gauge-invariant** combination of SM fields

Cut-off energy scale

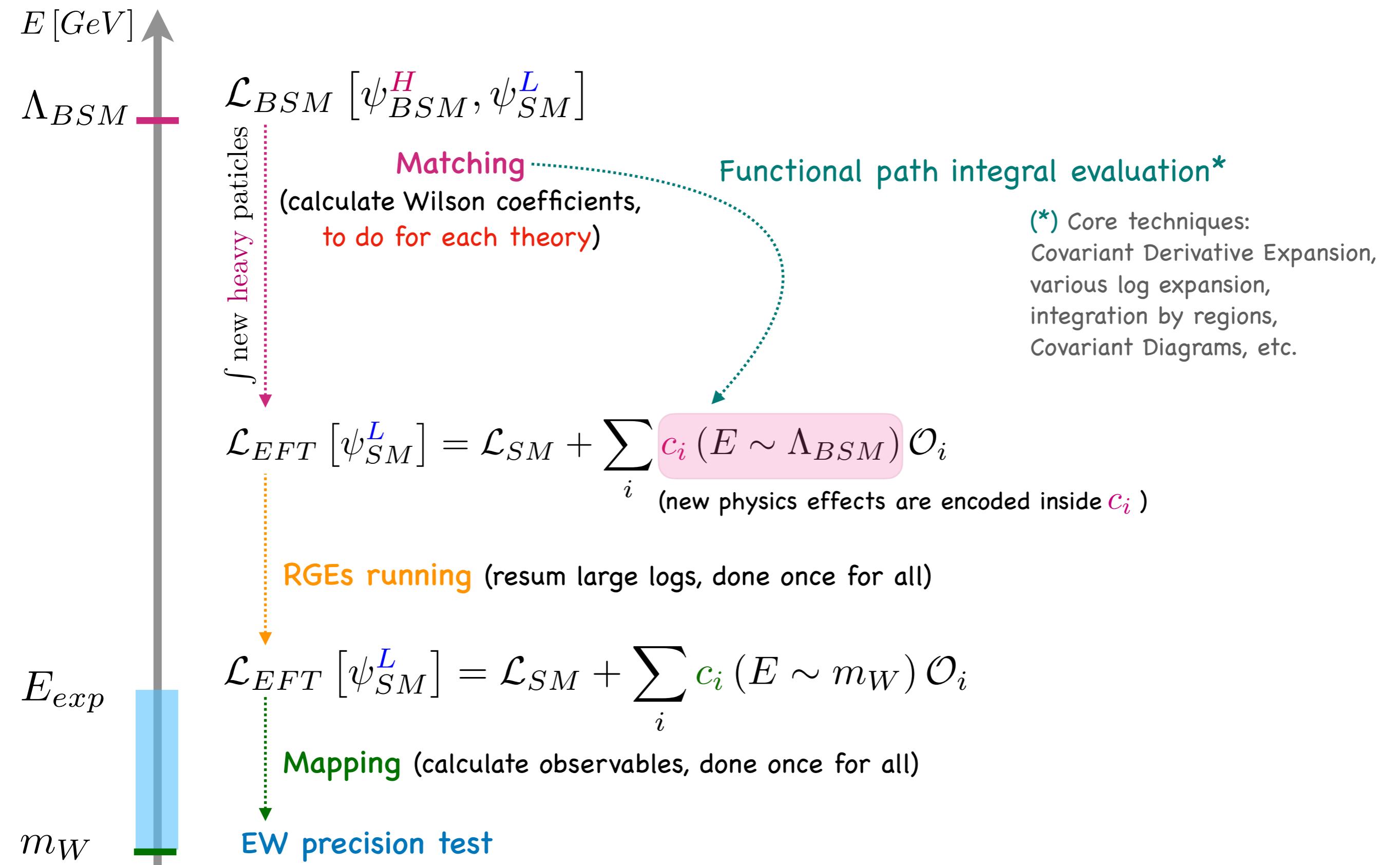
The diagram illustrates the structure of the Effective Field Theory (EFT) Lagrangian. It shows the total Lagrangian \mathcal{L}^{EFT} as the sum of the Standard Model Lagrangian $\mathcal{L}_{d=4}^{SM}$ and a series of non-renormalizable operators. The non-renormalizable operators are represented by a sum over dimensions d and indices i , where each term is proportional to a Wilson coefficient c_i divided by a cut-off energy scale Λ^{d-4} . Two boxes provide additional context: one for Wilson coefficients (encapsulating the effect of new physics) and another for non-renormalizable operators (made up of gauge-invariant combinations of SM fields).

- Once deviation with SM (in precision Electroweak, precision Higgs, precision Flavour, etc)
=> BSM theory

I. Backup slides: Generic EFTs from the UV point-of-view (1)



I. Backup slides: Generic EFTs from the UV point-of-view (2)



II. Backup slides: One-Loop Effective Action (1)

Path integral formalism: $e^{iS_{eff}[\psi_{SM}^L](\mu)} = \int \mathcal{D}\psi_{BSM}^H e^{iS[\psi_{BSM}^H, \psi_{SM}^L](\mu)}$

Find classical solution by solving EOM:

$$\frac{\delta S [\psi_{BSM}^H, \psi_{SM}^L]}{\delta \psi_{BSM}^H} \Bigg|_{\psi_{BSM}^H = \psi_{BSM,c}} = 0 \Rightarrow \psi_{BSM,c}(\psi_{SM}^L)$$

Expand action around minimum:

$$S [\psi_{BSM}^H] = S [\psi_{BSM,c} + \eta] = S [\psi_{BSM,c}] + \frac{1}{2} \frac{\delta^2 S}{\delta (\psi_{BSM}^H)^2} \Bigg|_{\psi_{BSM,c}} \eta^2 + \mathcal{O}(\eta^3)$$

Integrate over quantum fluctuation η :

$$e^{iS_{eff}[\psi_{SM}^L]} = e^{iS[\psi_{BSM,c}^H]} \left[\det \left(- \frac{\delta^2 S}{\delta (\psi_{BSM}^H)^2} \Bigg|_{\psi_{BSM,c}} \right) \right]^{-c_s}$$

c_s is spin factor ($c_s = +1/2$ for real scalar, -1 for Dirac fermion)

Re-write the determinant, $\det(A) = e^{\text{Tr log } A}$:

$$S_{eff} [\psi_{SM}^L] = S [\psi_{BSM,c}^H (\psi_{SM}^L), \psi_{SM}^L] + i c_s \text{Tr log} \left(- \frac{\delta^2 S}{\delta (\psi_{BSM}^H)^2} \Bigg|_{\psi_{BSM,c}} \right)$$

Tree-level

One-loop level

I. Backup slides: One-Loop Effective Action (1)

- We parameterise the shape of UV Lagrangian as follows:

$$\mathcal{L}_{\text{UV}}^{\text{fermion}}[\Psi_H, \phi] \supset \bar{\Psi}_H \left[P_\mu \gamma^\mu - M + X[\phi] \right] \Psi_H$$

general coupling with background fields

Example: $X[\phi] = V_\mu[\phi]\gamma^\mu - A_\mu[\phi]\gamma^\mu\gamma^5 - W_1[\phi]i\gamma^5$

Path-integral

Extract the one-loop (**heavy-only**) piece:

$$S_{\text{eff}}^{1\text{-loop}} = -i \text{Tr} \log \left(-\frac{\delta^2 S}{\delta \Psi_H^2} \Big|_{\Psi_H, c} \right) = -i \text{Tr} \log (P_\mu \gamma^\mu - M + X[\phi]) \equiv -i \text{Tr} \log \Delta_H$$

Evaluate the Trace by inserting complete set of spatial and momentum states:

$$S_{\text{eff}}^{1\text{-loop}} = -i \int d^d x \int \frac{d^d q}{(2\pi)^d} \text{tr} \log (e^{iq \cdot x} \Delta_H e^{-iq \cdot x}) = -i \int d^d x \int \frac{d^d q}{(2\pi)^d} \text{tr} \log (\underline{\Delta_H})_{P_\mu \rightarrow P_\mu - q_\mu}$$

Expansion by regions => Extract **short-distance** fluctuation which contribute to the **local** EFT operators
 (Fuentes-Martin, Portoles, Ruiz-Femenia, arXiv:1607.02142)

$$\mathcal{L}_{\text{EFT}}^{\text{1loop}} = i \text{tr} \sum_{n=1}^{\infty} \frac{1}{n} \int \frac{d^4 q}{(2\pi)^4} \left[\frac{-1}{q + M} \left(-P - V_\mu[\phi]\gamma^\mu + A_\mu[\phi]\gamma^\mu\gamma^5 + W_1[\phi]i\gamma^5 \right) \right]^n$$

One need to: expand order by order (ex: up to n=6),
 integrate over momentum q (careful to γ^5 in D-dimension), evaluate the Dirac traces

II. Backup slides: One-Loop Effective Action (Bosonic form)

We parameterise the shape of UV Lagrangian as follows:

$$\mathcal{L}_{UV} = \mathcal{L}_{SM} + \left[\Phi_{\textcolor{magenta}{H}}^\dagger F(\phi_{SM}) + h.c \right] + \Phi_{\textcolor{magenta}{H}}^\dagger [P^2 - m_{\Phi_{\textcolor{magenta}{H}}}^2 - U(\phi_{SM})] \Phi_{\textcolor{magenta}{H}}$$

Linear coupling,
contribute to tree-level

Quadratic coupling,
contribute to heavy-only 1-loop

Notations: $P_\mu = iD_\mu$ (kinetic momentum operator, hermitian)
 Φ_H (heavy fields can be bosons or fermions)

Extract the one-loop (heavy-only) piece:

$$S_{eff}^{1-loop} = ic_s \text{Tr} \log \left(- \frac{\delta^2 S}{\delta \Phi_{\textcolor{magenta}{H}}^2} \Big|_{\Phi_{\textcolor{magenta}{H}},c} \right) = ic_s \text{Tr} \log [-P^2 + m_{\Phi_{\textcolor{magenta}{H}}}^2 + U(\phi_{SM})] \equiv ic_s \text{Tr} \log \Delta_H$$

Evaluate the Trace by inserting complete set of spatial and momentum states:

$$S_{eff}^{1-loop} = ic_s \int d^d x \int \frac{d^d q}{(2\pi)^d} \text{tr} \log (e^{iq \cdot x} \Delta_H e^{-iq \cdot x}) = ic_s \int d^d x \int \frac{d^d q}{(2\pi)^d} \text{tr} \log (\Delta_H)_{P_\mu \rightarrow P_\mu - q_\mu}$$

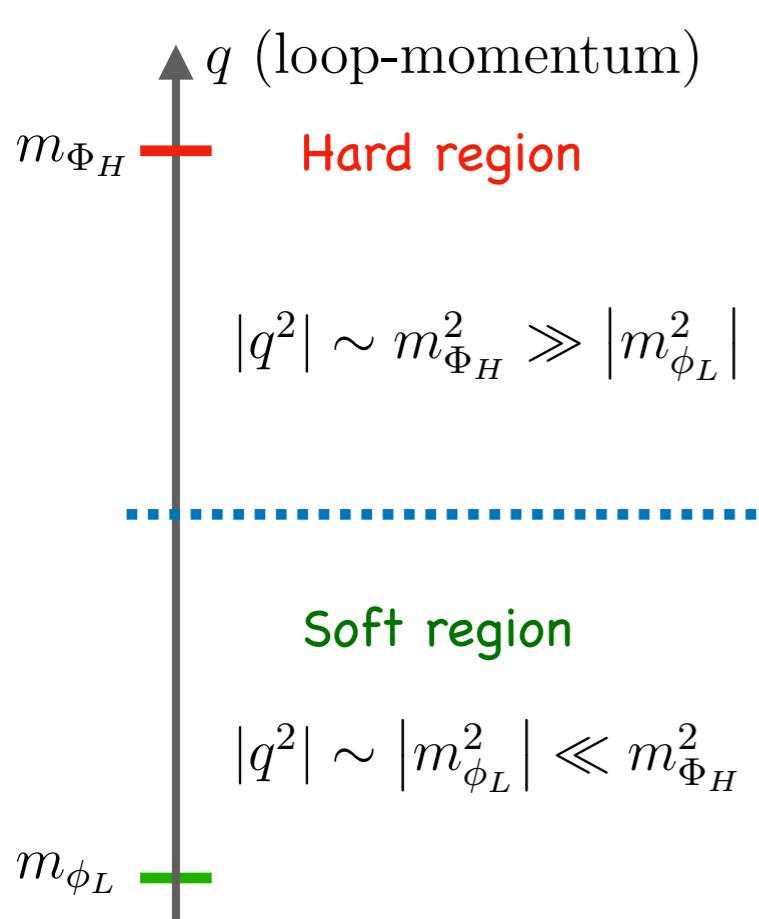
Core techniques to proceed the matching computations (quick overview):

- Expansion by regions => Extract short-distance fluctuation which contribute to the local EFT operators
(Fuentes-Martin, Portoles, Ruiz-Femenia, arXiv:1607.02142)
- Covariant Derivative Expansion => Manifestly gauge-invariant in each step of the computation
(B. Henning, X. Lu and H. Murayama, arXiv:1412.1837)
- Covariant Diagrams => Keep track of the series expansion (Z. Zhang, arXiv:1610.00710)

II. Backup slides: Expansion by regions

In Dim.Reg. with MS-bar scheme, each “log det X” can be separated into “hard” and “soft” region contributions:

$$\log \det X = \log \det X|_{\text{hard}} + \log \det X|_{\text{soft}}$$



Basis idea:

- **1PI effective action** include quantum fluctuation at **all scales**

$$\int d^d x \mathcal{L}_{EFT}^{1\text{-loop}} [\phi_{SM}] \neq S_{eff}^{1\text{-loop}} [\phi_{SM}]$$

- Extract **short-distance** fluctuations
=> **Local operators** in EFT Lagrangian

$$\int d^d x \mathcal{L}_{EFT}^{1\text{-loop}} [\phi_{SM}] = S_{eff}^{1\text{-loop}} [\phi_{SM}] \Big|_{\text{hard-region}}$$

Technically speaking:

- Taylor expand the integral in “hard” region, then integrate over the loop momenta

Making use of expansion by regions:

$$\mathcal{L}_{EFT}^{1\text{-loop}} = -ic_s \text{tr} \int \frac{d^d q}{(2\pi)^d} \sum_{i=1}^{\infty} \frac{1}{n} \left[\frac{1}{q^2 - m_{\Phi_H}^2} (-P^2 + 2q \cdot P + U[\phi_{SM}]) \right]^n$$

II. Backup slides: Covariant Diagrams

Main idea: Due to the **trace cyclicity**, any terms in the expansion can be presented diagrammatically !!!
Power counting is transparent => **classify diagrams very easy !**

Key points: Define building blocks + readout rules => Generate all possible diagrams at each order, evaluate the prefactor and get the EFT operators (able to **automatise easily**)

$$\mathcal{L}_{\text{EFT}}^{\text{1loop}} = i \text{tr} \sum_{n=1}^{\infty} \frac{1}{n} \int \frac{d^4 q}{(2\pi)^4} \left[\frac{-1}{q + M} \left(-\not{P} - V_\mu[\phi] \gamma^\mu + A_\mu[\phi] \gamma^\mu \gamma^5 + W_1[\phi] i \gamma^5 \right) \right]^n$$

Decompose the fermion propagator

$$\frac{-1}{q_\mu \gamma^\mu + m_H} = \frac{m_H}{q^2 - m_H^2} + \frac{-q_\mu \gamma^\mu}{q^2 - m_H^2}$$

Example:

Building blocks:

Fermion propagators:

W1 insertion:

bosonic part fermionic part

$$= m_H \quad ; \quad = -\gamma^\mu$$

$$= W_1 [\phi_L] \gamma^5$$

Readout rules:

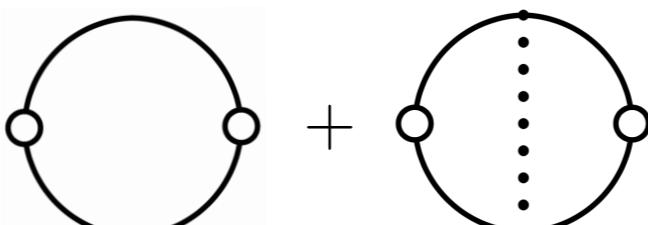
Diagram value = **prefactor** x trace (building blocks)

Prefactor: $i \frac{1}{S} \mathcal{I}[q^{2n_c}]_{ij}^{n_i n_j} \dots$

if the diagram have Z_s symmetry

Let's compute W_1^2 term:

$$(W_1)^2 =$$



$$= i \frac{1}{2} m_H^2 \mathcal{I}_i^2 \text{tr} (W_1 \gamma^5 W_1 \gamma^5) + i \frac{1}{2} \mathcal{I}[q^2]^2_i \text{tr} (W_1 \gamma^5 \gamma^\mu W_1 \gamma^5 \gamma_\mu)$$

The diagram is symmetry if we rotate 180 degree => symmetry factor = 1/2

II. Backup slides: Divergence & Regularisation

$$\mathcal{L}_{EFT}^{1-loop} = i \text{tr} \sum_{n=1}^{\infty} \frac{1}{n} \int \frac{d^d q}{(2\pi)^d} \left[\frac{-1}{q_\mu \gamma^\mu + m_H} (-\not{P} + W_0[\phi_L] + i W_1[\phi_L] \gamma^5 + V_\mu[\phi_L] \gamma^\mu + A_\mu[\phi_L] \gamma^\mu \gamma^5) \right]^n$$

Any **difficulties** in this computations ? YES, we have γ^5 in D-dimension !!!

Let's do an example and see...

$$\mathcal{O}(W_1^2) = -\frac{i}{2} m_i^2 \mathcal{I}_i^2 \text{tr} (W_1^2 \gamma^5 \gamma^5) - \frac{i}{2} \mathcal{I}[q^2]_i^2 \text{tr} (W_1^2 \gamma^5 \gamma^\mu \gamma^5 \gamma_\mu)$$

The 1-loop integral is divergence,
using Dim.Reg. to evaluate the integral

$$\mathcal{I}[q^2]_i^2 = \frac{m_i^2}{2} \left[1 - \log \frac{m_i^2}{\mu^2} + \left(\frac{2}{\epsilon} - \gamma_E + \log 4\pi \right) \right]$$

Evaluate the Dirac trace in D-dimension

Key points:

- Due to the issue of γ^5 in D-dimension, we used Breitenlohner-Maison- t'Hooft Veltman scheme (**BMHV**)
- We must **keep** the terms $\mathcal{O}(\epsilon)$ in the Dirac traces, since they will **cancel out** the divergence term $\frac{2}{\epsilon}$ of the 1-loop integrals

divergence is cancelled => extra finite term

$$\mathcal{O}(W_1^2) = i \left\{ -2 m_i^2 \mathcal{I}_i^2 + (8 + 2\epsilon) \frac{m_i^2}{2} \left[1 - \log \frac{m_i^2}{\mu^2} + \left(\frac{2}{\epsilon} - \gamma_E + \log 4\pi \right) \right] \right\} \text{tr} (W_1^2)$$

result of Dirac trace in BMHV-scheme

No need to evaluate
Dirac algebra

I. Backup slides: Evaluating Chern-Simon operators

$$\mathcal{L}_{\text{EFT}}^{\text{1loop}} = i \text{tr} \sum_{n=1}^{\infty} \frac{1}{n} \int \frac{d^4 q}{(2\pi)^4} \left[\frac{-1}{q + M} \left(-\not{P} - V_\mu[\phi] \gamma^\mu + A_\mu[\phi] \gamma^\mu \gamma^5 + W_1[\phi] i \gamma^5 \right) \right]^n$$

- Power counting: Chern-Simon operator structures

$$\mathcal{O}(PV_{PQ}AV), \mathcal{O}(PA_{PQ}AA)$$

- The coefficients are ambiguous. One should not naively evaluate these coefficients
 \Rightarrow How to have enough freedom in dim. reg. to choose which currents are conserved or not?
- In $d > 4$ dimension: $\{\gamma^\mu, \gamma^5\} = 0$ & trace cyclicity can **not** hold simultaneously
- The usual ambiguity (choice of integration variables) \longrightarrow ambiguity on the location of γ^5
 (from divergence integrals) t' Hooft & Veltman
- One can uses this ambiguity \rightarrow free parameters \rightarrow decide if a symmetry is broken or not

$$\begin{aligned} \text{tr} \left(\gamma_a \not{V}^i \gamma_b \not{V}^j \gamma_c \not{P} \gamma_d \not{A}^k \gamma^5 \right) \Big|_{d=4-\epsilon} \xrightarrow{\hspace{1cm}} & \alpha_1 \text{tr} \left(\gamma_a \not{V}^i \gamma^5 \gamma_b \not{V}^j \gamma_c \not{P} \gamma_d \not{A}^k \right) \Big|_{d=4-\epsilon} + \beta_1 \text{tr} \left(\gamma_a \not{V}^i \gamma_b \not{V}^j \gamma^5 \gamma_c \not{P} \gamma_d \not{A}^k \right) \Big|_{d=4-\epsilon} \\ & + \theta_1 \text{tr} \left(\gamma_a \not{V}^i \gamma_b \not{V}^j \gamma_c \not{P} \gamma^5 \gamma_d \not{A}^k \right) \Big|_{d=4-\epsilon} + \eta_1 \text{tr} \left(\gamma_a \not{V}^i \gamma_b \not{V}^j \gamma_c \not{P} \gamma_d \not{A}^k \gamma^5 \right) \Big|_{d=4-\epsilon} \end{aligned}$$

- Main output: $\omega_{VAV}(\bar{a}, \bar{b}), \omega_{AAA}(\bar{c}, \bar{d})$ ready to impose gauge-invariant

Backup slides: Integrate out heavy fermions

Starting point: Let's write down the UV Lagrangian for fermions

$$\mathcal{L}_{UV} [\Psi_H, \phi_L] = \mathcal{L}_0 [\phi_L] + \overline{\Psi}_H (\gamma_\mu P^\mu - m_H - X_H [\phi_L]) \Psi_H$$

general coupling with background fields

The effective action resulting from integrating out **heavy-only fermions**,

$$S_{eff}^{1-loop} = -i \operatorname{Tr} \log (\gamma_\mu P^\mu - m_H - X_H [\phi_L])$$

Two way of proceeding:

1. Squaring the quadratic operators, using the trick $\operatorname{Tr} \log(AB) = \operatorname{Tr} \log A + \operatorname{Tr} \log B$

$$S_{eff}^{1-loop} = -\frac{i}{2} \operatorname{Tr} \log (-P^2 + m_H^2 + U_{fermion}) ,$$

$$\text{where } U_{fermion} = -\frac{i}{2} \sigma^{\mu\nu} G'_{\mu\nu} + 2m_H X_H[\phi_L] + X_H^2 + [\not{P}, X_H[\phi_L]]$$

=> Then we can use the master formula in UOLEA as mentioned before

Disadvantages:

- Not straight forward to derive EFT operators due to the complicated of the background function $U_{fermion}$
- If $X_H[\phi_L]$ contains Dirac matrices, the quantity $[\not{P}, X_H[\phi_L]]|_{P_\mu \rightarrow P_\mu - q_\mu}$, will lead to non-trivial terms which are not implemented in the UOLEA before

Backup slides: Loop integrals

Definition of the master integrals:

$$\mathcal{I}[q^{2n_c}]_i^{n_i} = \frac{i}{16\pi^2} (-M_i^2)^{2+n_c-n_i} \frac{1}{2^{n_c}(n_i-1)!} \frac{\Gamma(\frac{\epsilon}{2}-2-n_c+n_i)}{\Gamma(\frac{\epsilon}{2})} \left(\frac{2}{\bar{\epsilon}} - \log \frac{M_i^2}{\mu^2} \right)$$

The value of some master integrals:

$\tilde{\mathcal{I}}[q^{2n_c}]_i^{n_i}$	$n_c = 0$	$n_c = 1$	$n_c = 2$	$n_c = 3$
$n_i = 1$	$M_i^2 \left(1 - \log \frac{M_i^2}{\mu^2} \right)$	$\frac{M_i^4}{4} \left(\frac{3}{2} - \log \frac{M_i^2}{\mu^2} \right)$	$\frac{M_i^6}{24} \left(\frac{11}{6} - \log \frac{M_i^2}{\mu^2} \right)$	$\frac{M_i^8}{192} \left(\frac{25}{12} - \log \frac{M_i^2}{\mu^2} \right)$
$n_i = 2$	$-\log \frac{M_i^2}{\mu^2}$	$\frac{M_i^2}{2} \left(1 - \log \frac{M_i^2}{\mu^2} \right)$	$\frac{M_i^4}{8} \left(\frac{3}{2} - \log \frac{M_i^2}{\mu^2} \right)$	$\frac{M_i^6}{48} \left(\frac{11}{6} - \log \frac{M_i^2}{\mu^2} \right)$
$n_i = 3$	$-\frac{1}{2M_i^2}$	$-\frac{1}{4} \log \frac{M_i^2}{\mu^2}$	$\frac{M_i^2}{8} \left(1 - \log \frac{M_i^2}{\mu^2} \right)$	$\frac{M_i^4}{32} \left(\frac{3}{2} - \log \frac{M_i^2}{\mu^2} \right)$
$n_i = 4$	$\frac{1}{6M_i^4}$	$-\frac{1}{12M_i^2}$	$-\frac{1}{24} \log \frac{M_i^2}{\mu^2}$	$\frac{M_i^2}{48} \left(1 - \log \frac{M_i^2}{\mu^2} \right)$
$n_i = 5$	$-\frac{1}{12M_i^6}$	$\frac{1}{48M_i^4}$	$-\frac{1}{96M_i^2}$	$-\frac{1}{192} \log \frac{M_i^2}{\mu^2}$
$n_i = 6$	$\frac{1}{20M_i^8}$	$-\frac{1}{120M_i^6}$	$\frac{1}{480M_i^4}$	$-\frac{1}{960M_i^2}$

Table 7. Commonly-used degenerate master integrals $\tilde{\mathcal{I}}[q^{2n_c}]_i^{n_i} \equiv \mathcal{I}[q^{2n_c}]_i^{n_i} / \frac{i}{16\pi^2}$, with $\frac{2}{\bar{\epsilon}} = \frac{2}{\epsilon} - \gamma + \log 4\pi$ dropped. All nondegenerate (including mixed heavy-light) master integrals can be reduced to degenerate master integrals by Eq. (A.2).