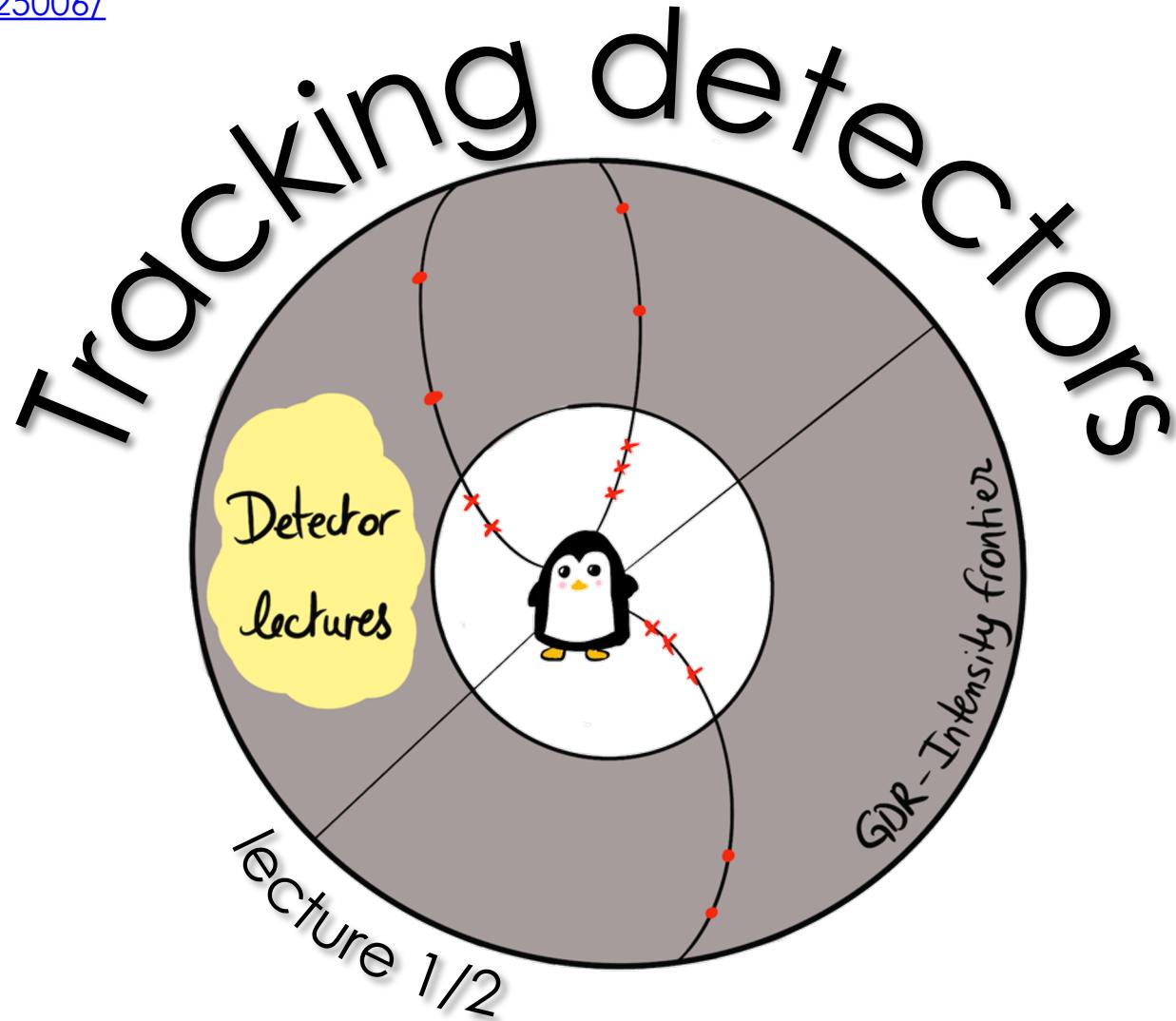


November 2021

<https://indico.in2p3.fr/event/25006/>

INTENSITY
frontier



Jerome Baudot



Beyond detectors:
lecture on Tracking algorithms by P.Billoir

Tracking: what is that about ?



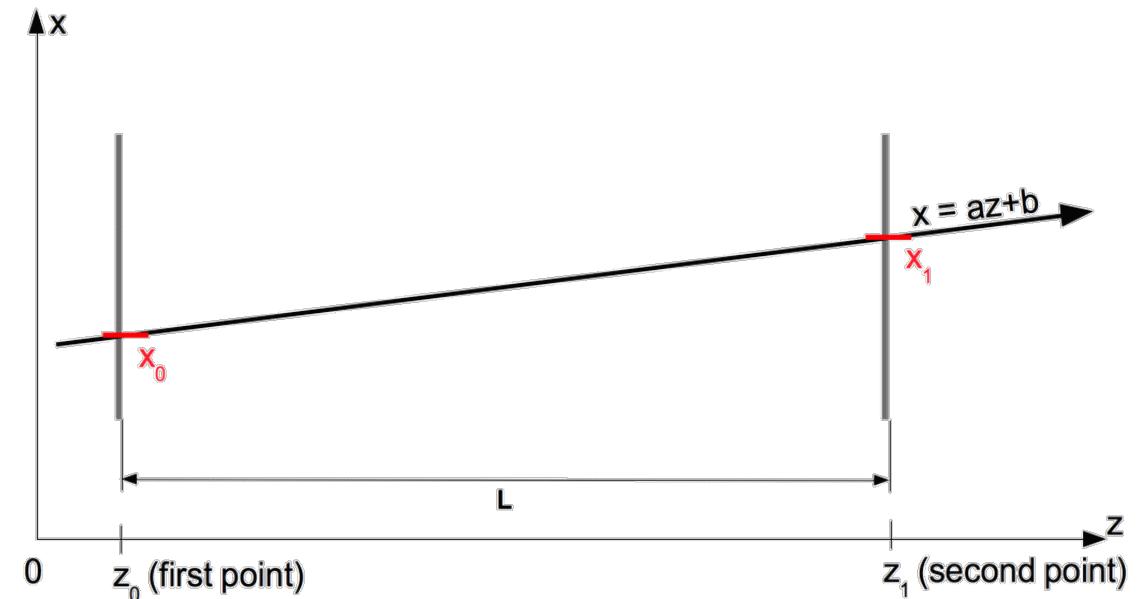
Tracking version 0.0

■ Hypothesis:

- Two sensors
 - perfect position measurement
 - Infinitely thin
- 1 straight track
 - 2 parameters (a,b)

■ Estimation of track parameters

- Assuming track model is straight
- No uncertainty !



$$a = \frac{x_1 - x_0}{z_1 - z_0}, \quad b = \frac{x_0 z_1 - x_1 z_0}{z_1 - z_0}$$

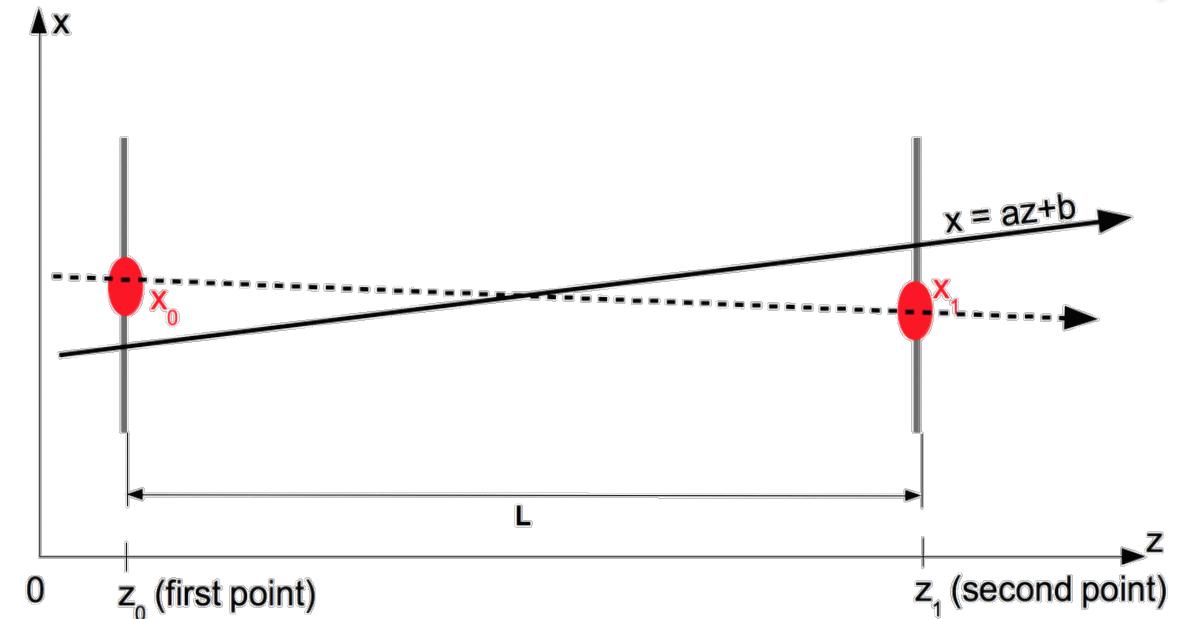
Tracking version 1.0

■ Hypothesis:

- Two sensors
 - Positions with UNCERTAINTY σ_{det}
 - Infinitely thin
- 1 straight track
 - 2 parameters (a,b)

■ Estimation of track parameters

- Assuming track model is straight
- Uncertainties from error propagation
 - Decreases will lever arm ($z_1 - z_0$)



$$a = \frac{x_1 - x_0}{z_1 - z_0}, b = \frac{x_0 z_1 - x_1 z_0}{z_1 - z_0}$$

$$\sigma_a = \frac{\sqrt{2}}{z_1 - z_0} \sigma_{\text{det}}, \sigma_b = \frac{\sqrt{z_1^2 + z_0^2}}{z_1 - z_0} \sigma_{\text{det}}$$

$$\text{cov}_{a,b} = -\frac{\sqrt{z_1 + z_0}}{z_1 - z_0} \sigma_{\text{det}}$$

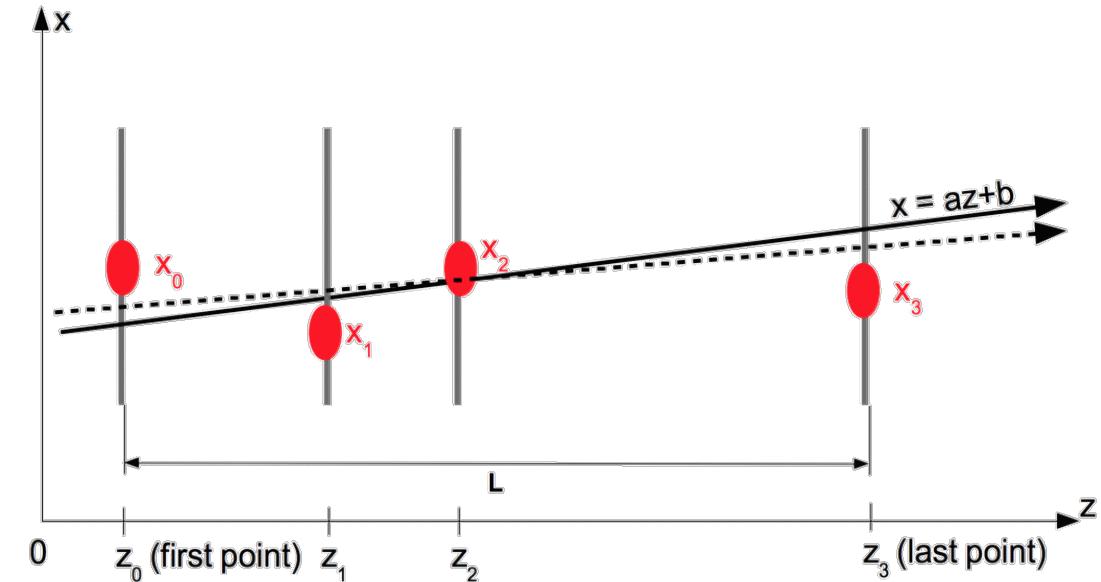
Tracking version 1.1

Hypothesis:

- More than two sensors
 - Positions with uncertainty σ_{det}
 - Infinitely thin
- 1 straight track
 - 2 parameters (a,b)

Estimation of track parameters

- Assuming track model is straight
 - Need **FITTING PROCEDURE** least square
 - Need covariance matrix (4x4) of measurements (here diagonal)
 - Uncertainties from error propagation
 - Detail depends on geometry
- => Both estimation & uncertainties improve



$$a = \frac{S_1 S_{xz} - S_x S_z}{S_1 S_{z^2} - (S_z)^2}, \quad b = \frac{S_x S_{z^2} - S_z S_{xz}}{S_1 S_{z^2} - (S_z)^2}$$

$$\sigma_a^2 = \frac{S_1}{S_1 S_{z^2} - (S_z)^2}, \quad \sigma_b^2 = \frac{S_{z^2}}{S_1 S_{z^2} - (S_z)^2}$$

$$\text{cov}_{a,b} = \frac{-S_z}{S_1 S_{z^2} - (S_z)^2}$$

$$S_{f(x,z)} = \sum_{i=\text{first point}}^{\text{last point}} \frac{f(x_i, z_i)}{\sigma_{\text{det}}^2}$$

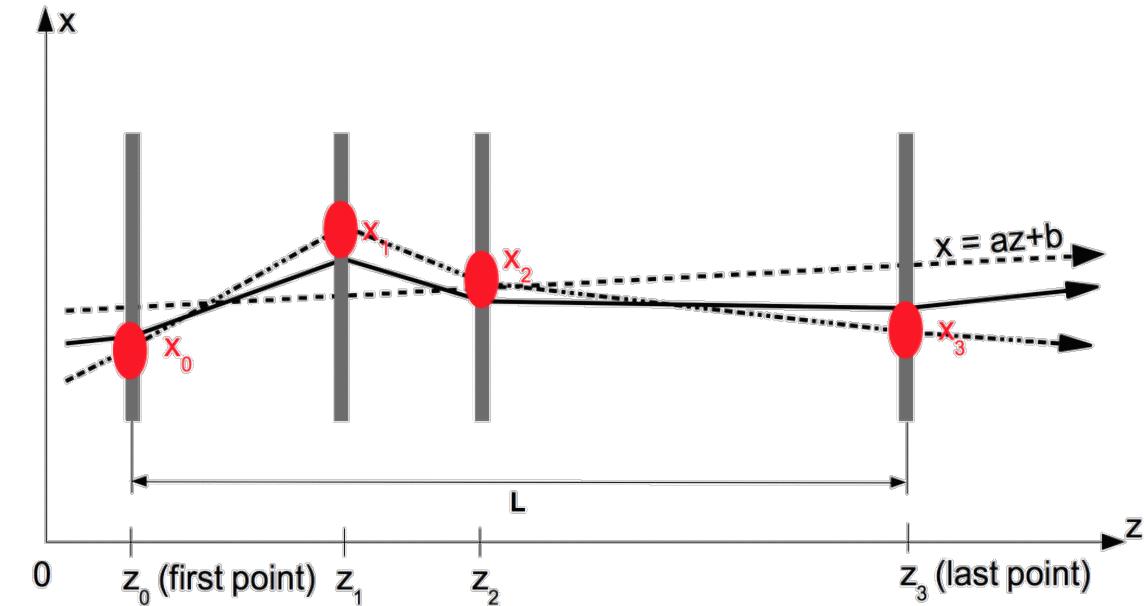
Tracking version 2.0

■ Hypothesis:

- More than two sensors
 - Positions with uncertainty σ_{det}
 - With some THICKNESS → physics effect
- 1 straight track
- 2 parameters (a,b)
- But may change along track path

■ Estimation of track parameters

- Assuming track model is straight
 - Need fitting procedure → least square
 - Global AND local estimations
 - Need covariance matrix (4x4) of measurements physics effect → **NON DIAGONAL** terms
 - Uncertainties from error propagation
- => same estimators but increased uncertainties



$$a = \frac{S_1 S_{xz} - S_x S_z}{S_1 S_{z^2} - (S_z)^2}, \quad b = \frac{S_x S_{z^2} - S_z S_{xz}}{S_1 S_{z^2} - (S_z)^2} \quad (\text{Global estimation})$$

Complex covariant matrix expression

- correlation between sensors
- Various implementations possible

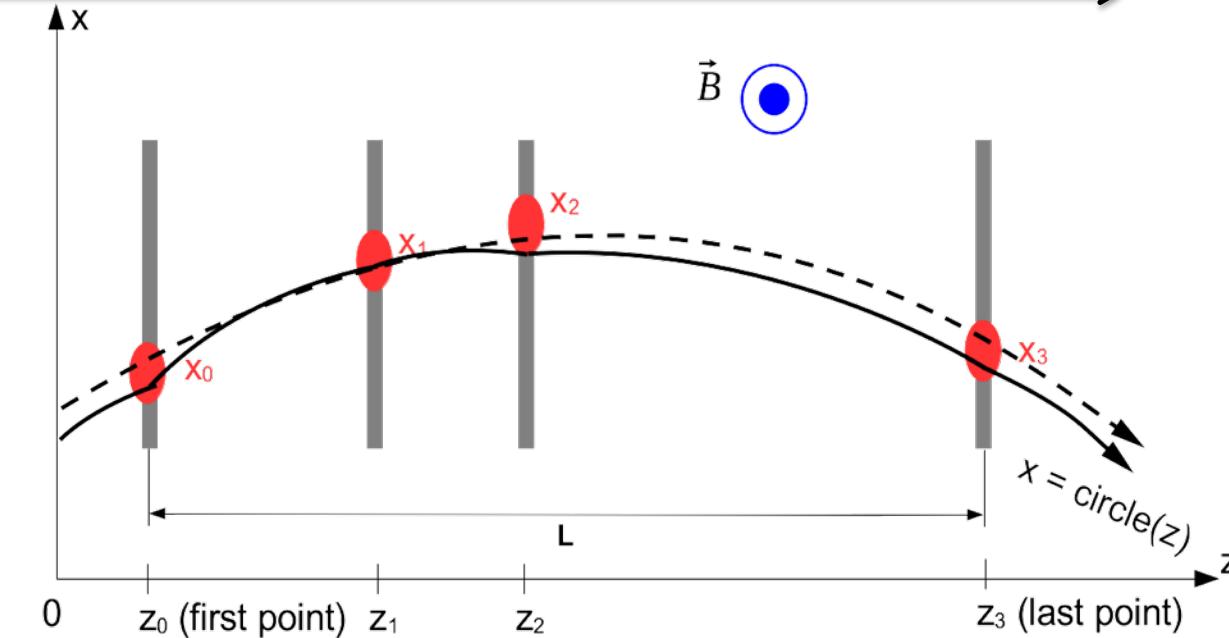
Tracking version 2.1

■ Hypothesis:

- More than two sensors
 - Positions with uncertainty σ_{det}
 - With some THICKNESS → physics effect
- No more straight track
 - Magnetic field → helicoidal model
 - 5 parameters → one is the momentum

■ Estimation of track parameters

- As before:
 - non-diagonal covariance matrix
 - Increased uncertainties from physics effect
- But FITTING more complex:
 - higher dimensions
 - **non-linearities**

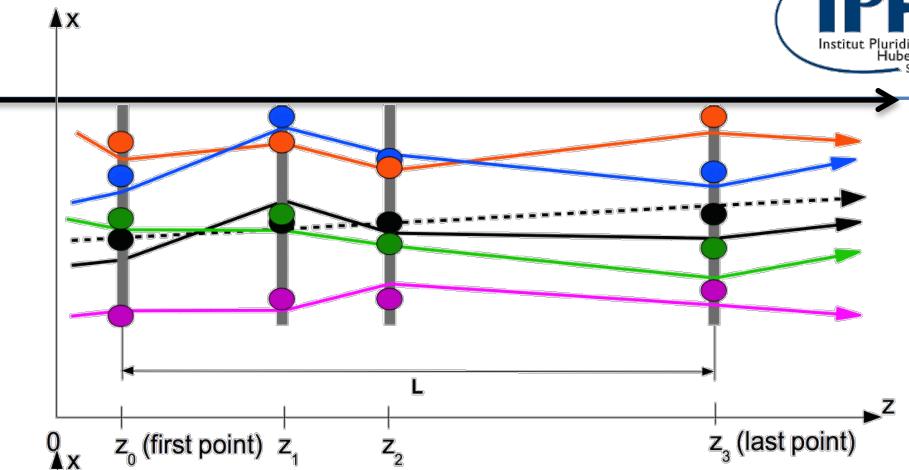


Influence of geometry from
- Overall layout
- Shape of sensing layers

Tracking RELOADED

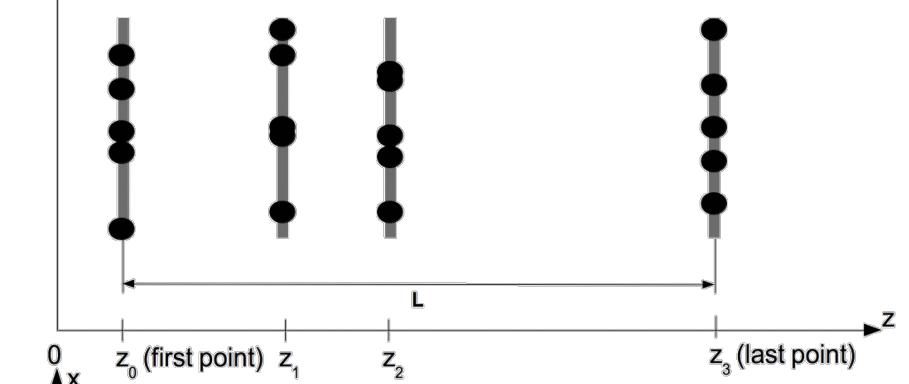
Hypothesis:

- More than two sensors
 - Positions with uncertainty σ_{det}
 - With some thickness
- **SEVERAL** straight tracks \Leftrightarrow depends on sensor timing perf.
 - Still 2 parameters (a,b)...per track!
 - But may change along track path



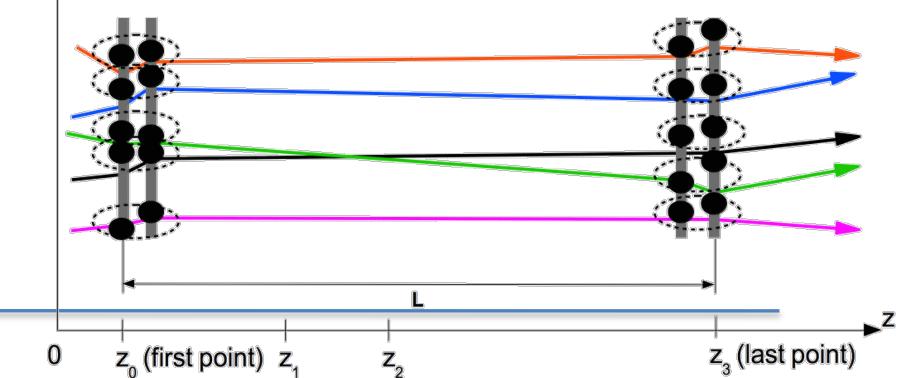
New step = FINDING

- Which hits to which tracks ?
- Strongly depends on geometry



Uncertainties on track parameters (FITTING)

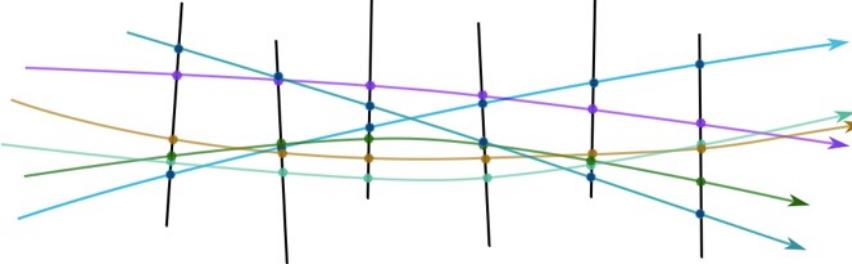
- Involve correlation
- Depend on level of wrong hit-track association



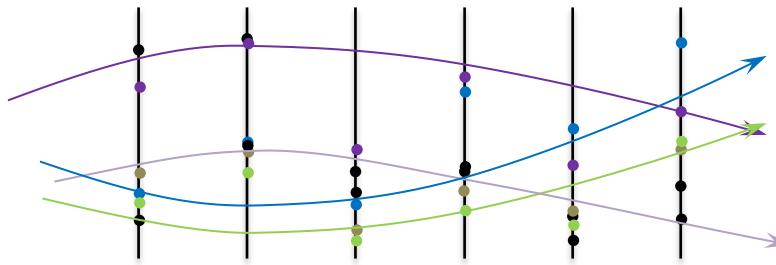
Tracking ... additional real life troubles

■ Alignment

What you mechanically constructed



What you suppose you constructed



 ALIGMENT procedure needed

■ Radiation environment

- Intensity & energy frontiers => large radiation exposure
- Total ionizing dose
 - Possible effects starting ~ 1 kGy
 - Worst conditions \sim Ggy
- Non ionizing energy loss fluence
 - Possible effects starting $\sim 10^{12} n_{eq(1MeV)}/cm^2$
 - Worst conditions $\sim 10^{17} n_{eq(1MeV)}/cm^2$



Hardening/Monitoring needed

■ Temperature

- Electronics heat-up => perf. can degrades with temp.
- Radiation tolerance depends on temperature

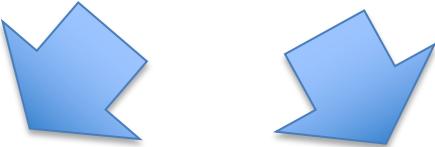


cooling => additional material

Lessons from simple considerations

■ Goals of tracking

- Identifying particles
- Measuring particle properties from track parameters



■ Detector performance

- Granularity → res. on track parameters
- Thickness → res. on track parameters
- Timing → easier finding
- Multi-layer system → fitting & finding performance

■ Algorithms

- FITTING
- FINDING

} See P.Billoir's lecture

Outline of lecture 1

- Introduction (done already)
- Figures of merit
- Some tracking systems

Figures of merit (simplistic approach)

- Physics drivers:
 - bending power & multiple scattering
- System
 - resolution on Momentum & track Origin (vertex)
- Single layer
 - resolution on Position & Time
- Finding power

Bending power

Magnetic field for momentum measurement

- Lorentz Force $\frac{d\vec{p}}{dt} = \vec{qv} \times \vec{B}$ => helix (R, λ)
for homogeneous B

(r-φ) plane circle

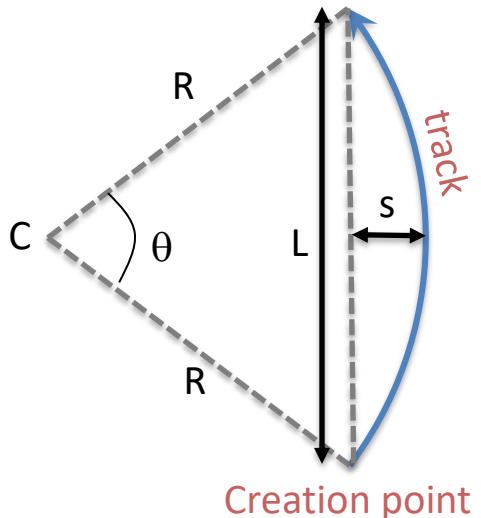
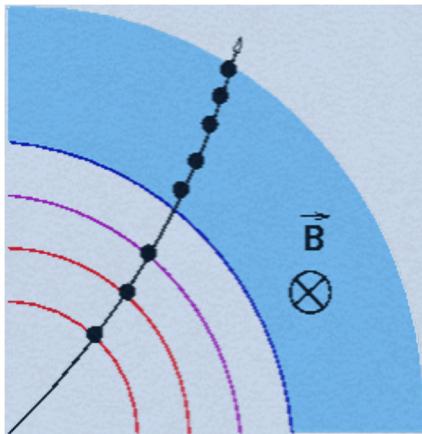
$$p_T = 0.3zBR$$

(r-z) plane Straight line

$$p_T = p \cos \lambda$$

$$\vec{p} = \vec{p}_T + \vec{p}_z$$

Collider

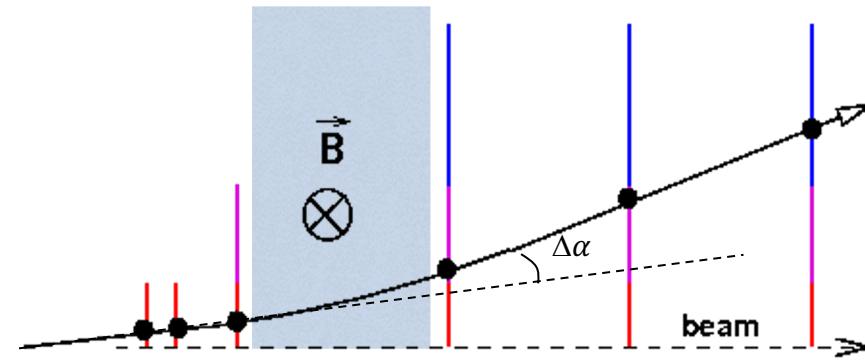


$$\frac{p_T(\text{GeV}/c)}{q} = 0.3 \cdot B(T) \cdot R(\text{m})$$

$$s \approx \frac{L^2}{8R} = 0.038 \frac{BL^2}{p_T}$$

Fixed target

$$\frac{p_T}{q} = \frac{0.3 \cdot B(T)L}{\Delta\alpha}$$



At given p_T : large BL^2 or $BL \rightarrow$ large s or $\Delta\alpha$
 \Rightarrow easier measurements

Multiple scattering

■ Reminder on the physics process

- Coulomb scattering mostly on nuclei => angular deviation
- Deviation in 3D space ($\varphi, \theta=\theta_{\text{plane}}$) (note: $\phi \in [0, 2\pi]$ uniform)
where plane = $(\vec{p}_{\text{in}}, \vec{p}_{\text{out}})$

■ Molière theory for the distribution of θ_{plane}

- central part (multiple small angles) ~ **centered gaussian** process
- Tails driven by single scatter, larger than gaussian
 - the thinner the material, the more important the tails

■ The Highland formula

- Approximates std. dev. of central part in Molière theory

$$\sigma_\theta = \frac{13.6 \text{ (MeV/c)}}{\beta p} \cdot z \cdot \sqrt{\frac{\text{thickness}}{X_0}} \cdot \left[1 + 0.038 \ln\left(\frac{\text{thickness}}{X_0}\right) \right]$$

β, p, z = particle boost, momentum, charge

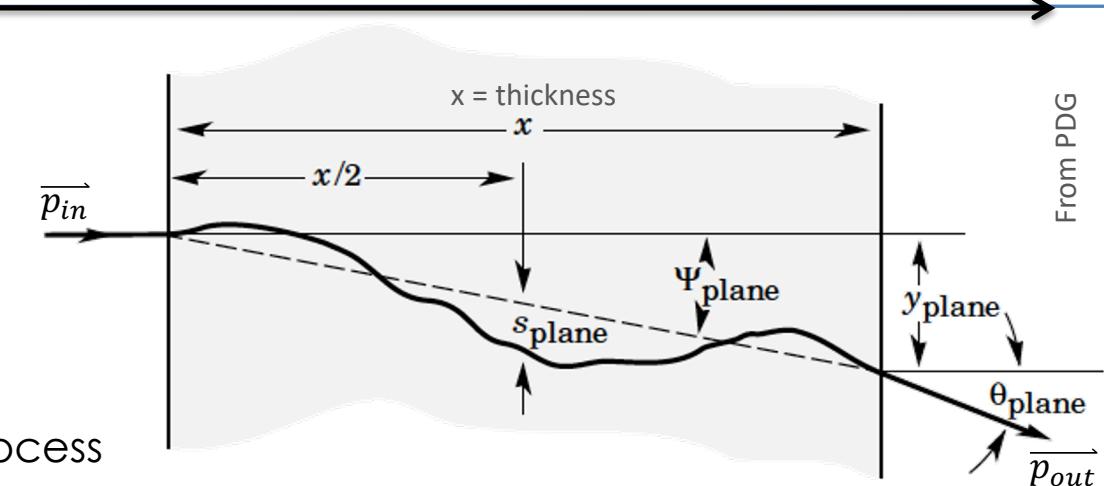
- Deviation probability increases for large angle with:

- Decreasing momentum

- Increasing $\sqrt{\text{material budget}} = \frac{\text{thickness}}{X_0}$

X_0 = radiation length

Same definition as in calorimetry... though this is accidental



Multiple scattering – some numbers

■ Various materials

material	X_0	Deviation for protons at 1 GeV		
		thickness	Budget	Shit @ 10 cm
Silicon	9.4 cm	100 μm	0.1 %	50 μm
Epoxy	30 cm	1 mm	0.3 %	90 μm
Nal	2.6 cm	1 mm	3.8%	350 μm
Gas (air-CH4)	300-700 m	10 cm	<10 ⁻³	-

- For tracking purpose, material budget/measurement point ~ % or lower

■ For layers made of various material compounds

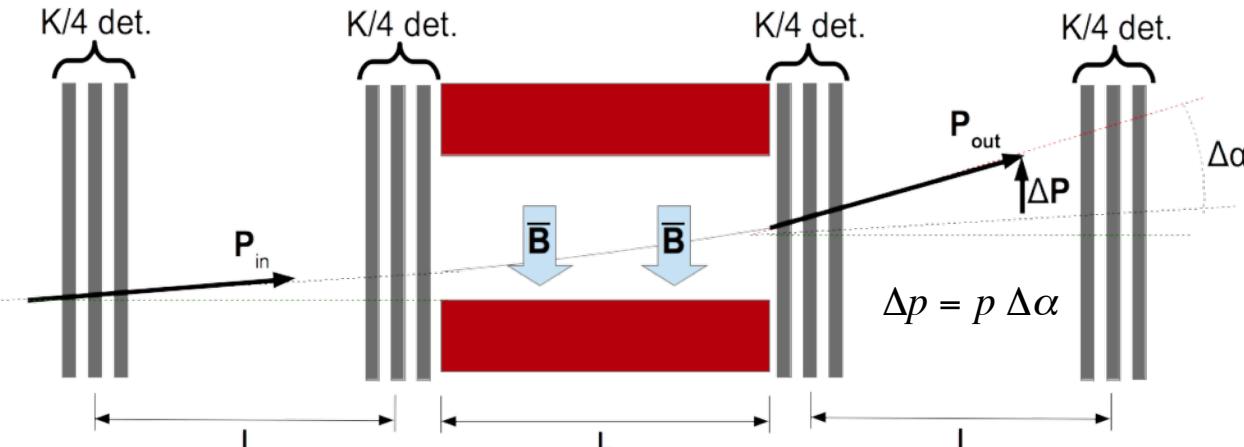
- Total thickness $T = \sum T_i$, each material (i) with $X_0(i)$
- Summing σ_i^2 undershoots the deviation
- Definition of **effective radiation length** => $X_{0,eff} = \frac{\sum_i T_i X_0(i)}{T}$

then $\sigma_{eff} = \sqrt{\frac{T}{X_{0,eff}}}$

Momentum res., fixed target geometry

Hypothesis

- K detectors, each with σ single point accuracy
- Uniform field over L from dipole
- Geometrical arrangement optimized for resolution
 - Angular determination on input/output angle: $\sigma_\alpha^2 = \frac{16 \sigma^2}{K l^2}$



Without multiple scattering

$$\frac{\sigma_p}{p} = \frac{8}{0.3q} \frac{1}{BL} \frac{\sigma}{l\sqrt{K}} p$$

Same power for L and σ
 Expected statistical behaviour \sqrt{K}
 Large lever arm l beneficial

- Note relative uncertainty proportional to p!

Multiple scattering contribution

- Bring additive term proportional to K and $\sigma_\theta = \frac{13.6 \text{ (MeV/c)}}{\beta p} \sqrt{\frac{\text{thickness}}{X_0}}$
- Factor A_K depends on layer arrangement
 => large length l without measurement detrimental!

$$\left. \frac{\sigma_p}{p} (MS) = A_N \frac{13.6 \text{ (MeV/c)}}{\beta} \sqrt{\frac{\text{thickness}}{X_0}} \right\}$$

Constant with p!

Momentum res., collider geometry

■ Hypothesis

- K detectors uniformly distributed => **compulsory to measure sagitta**
each with σ single point accuracy
- Uniform field over path length L

■ Without multiple scattering

- (Glückstern formula, Works well with large $K > 20$)

$$\frac{\sigma_{p_T}}{p_T} = \frac{\sqrt{720}}{0.3q} \frac{1}{BL^2} \frac{\sigma}{\sqrt{K+6}} p_T$$

$\left. \begin{array}{l} \text{L}^2 \text{ stronger than } \sigma \\ \text{Expected statistical behaviour } \sqrt{K} \end{array} \right\}$

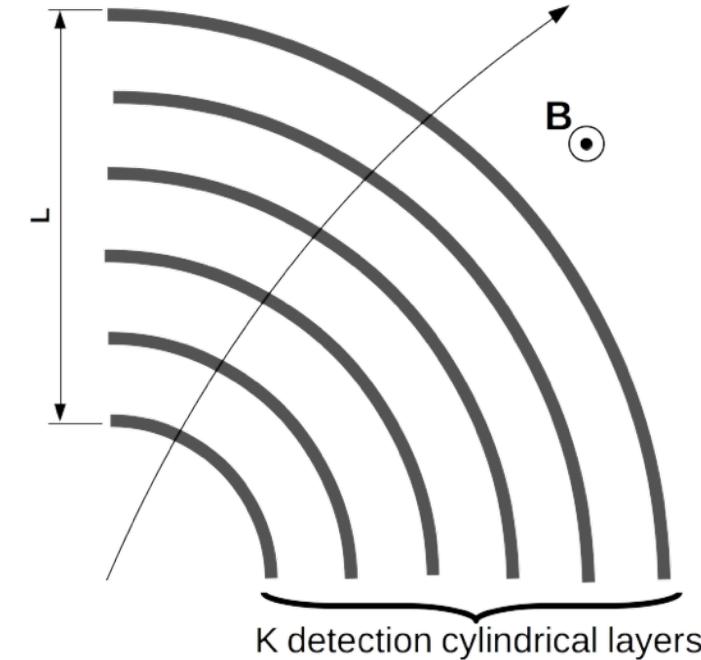
- Note relative uncertainty proportional to $p!$

■ Multiple scattering contribution

- Brings additive contribution

$$\frac{\sigma_{p_T}}{p_T} = \frac{1.43}{0.3q} \frac{1}{BL} \frac{13.6 \text{ (MeV/c)}}{\beta} \sqrt{\frac{\text{thickness}}{X_0}}$$

Constant with $p!$



Note: numerical factors will change for:

- Smaller # layers
- Non-uniform arrangement

Resolution on position vertices

- Vertex position = intersection of several tracks
=> resolution depends on #tracks, angles between tracks & track parameter resolution

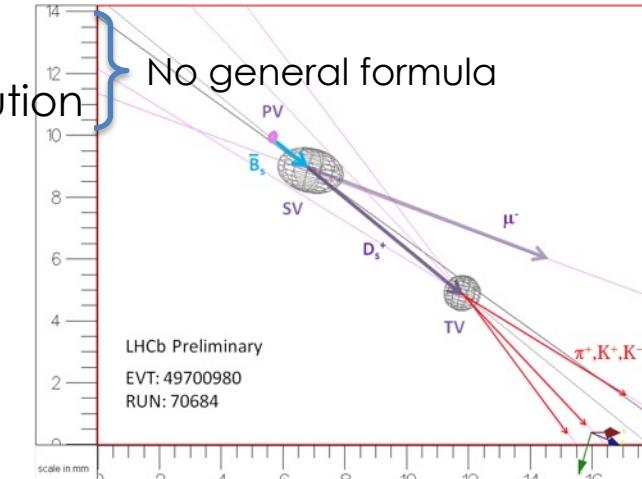
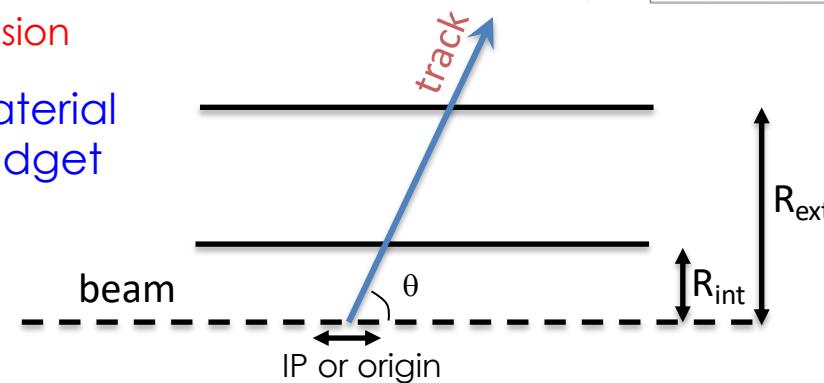
Distance of closest approach to collision (impact parameter)

- One of the parameter of the helix / similar to telescope extrapolation
- 2 crude approximations
 - close to the point of extrapolation → straight track
 - Only 2 measurement layers

$$\sigma_{IP} \propto \frac{\sqrt{R_{\text{ext}}^2 \sigma_{\text{int}}^2 + R_{\text{int}}^2 \sigma_{\text{ext}}^2}}{R_{\text{ext}} - R_{\text{int}}} + \frac{R_{\text{int}} \sigma_{\vartheta(\text{ms})}}{p \sin^{3/2}(\theta)} \quad \begin{matrix} \text{position res.} \\ \text{Distance to collision} \\ \text{Material budget} \end{matrix}$$

Lever arm

- Standard parametrization $\sigma_{IP} = a \oplus \frac{b}{p \sin^2 \theta}$
- Geometry + intrinsic resolution
- Geometry + multiple scattering



Critical addition: SIGN

- Significant help to separate primary/secondary tracks
- Defined against a direction (secondary vtx, jet)

Finding power

Iterative local algorithms

- Start from small track seed, then extrapolate track to next layers
- Power ~ probability to match correct hit
 \Rightarrow decided on distance hit to track-extrapolation (χ^2 test)

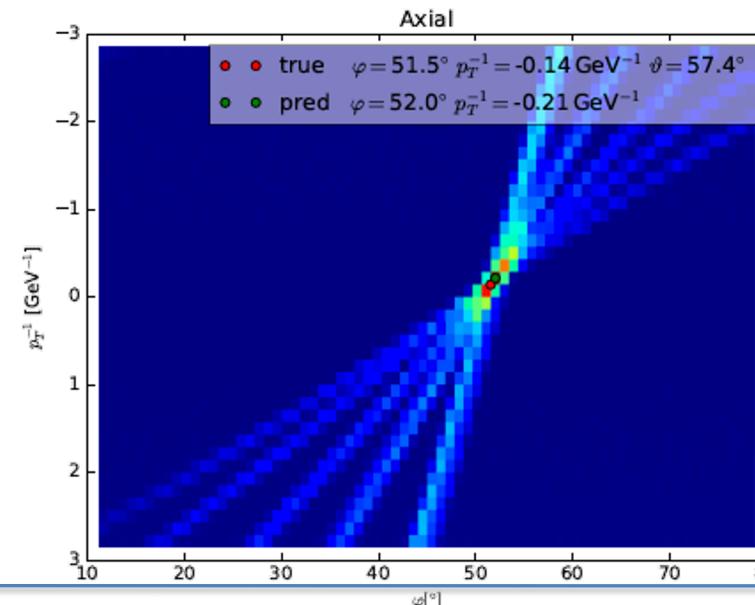
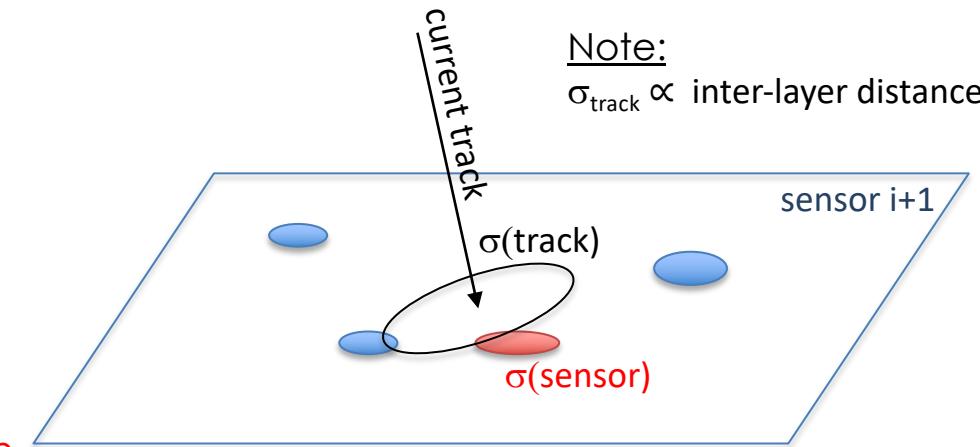
$$\text{Proba} = \frac{1}{1 + 2\pi\sigma_{eff,z} \times \sigma_{eff,\varphi} \times \rho_{bkg}}$$

- $\sigma_{eff} = \sigma(\text{sensor}) \oplus \sigma(\text{track extrapolation})$ = effective spatial resolution
- ρ_{bkg} = background hit density = bkg rate x integration time

Global algorithms

- Transform hit position \rightarrow parameter-space (=pattern-space)
 - Each hit shapes potential location of associated track
 - Large enough intersection of shapes = real track
- Granularity of pattern space = uncertainty on hit position
- Power ~ probability to intersect true-hit-shapes

$$\text{Proba} = \frac{1}{1 + 2\pi\sigma_{sensor,z} \times \sigma_{sensor,\varphi} \times \rho_{bkg}}$$



Lessons from figures of merit

■ Excellent track finder requires

- Granular sensors in space & time \leq hit rate
- Layer close to each other \leq intrinsic resolution of layer

Can a single algorithm all track search?



NO

real tracking is done with many steps

■ Excellent momentum resolution requires

- Many measurement points
- Measurement over large lever arm
- Low material budget
(depending on momentum range)

Can we get all that with a single sub-detector?



NO

(though they are similarities)



Trackers

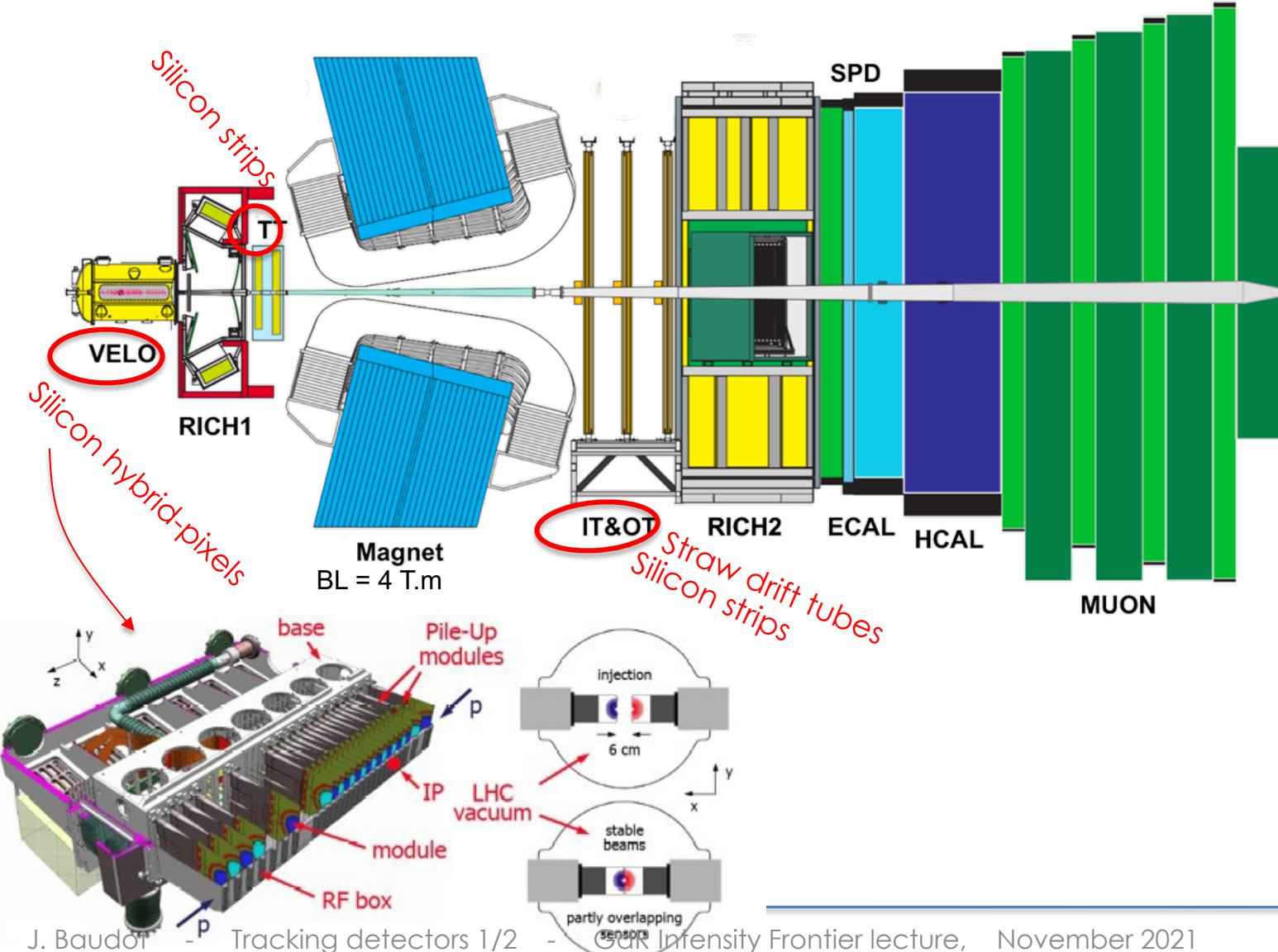
Vertex locators

Some real/future tracking systems

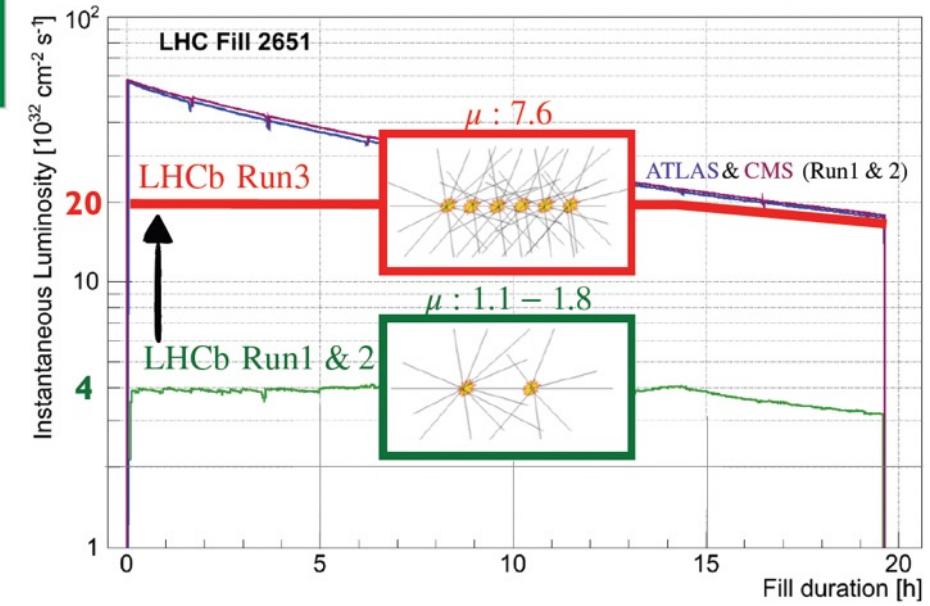


LHCb setup

- Fixed-target-like experiment at LHC p+p collisions @ 13 TeV

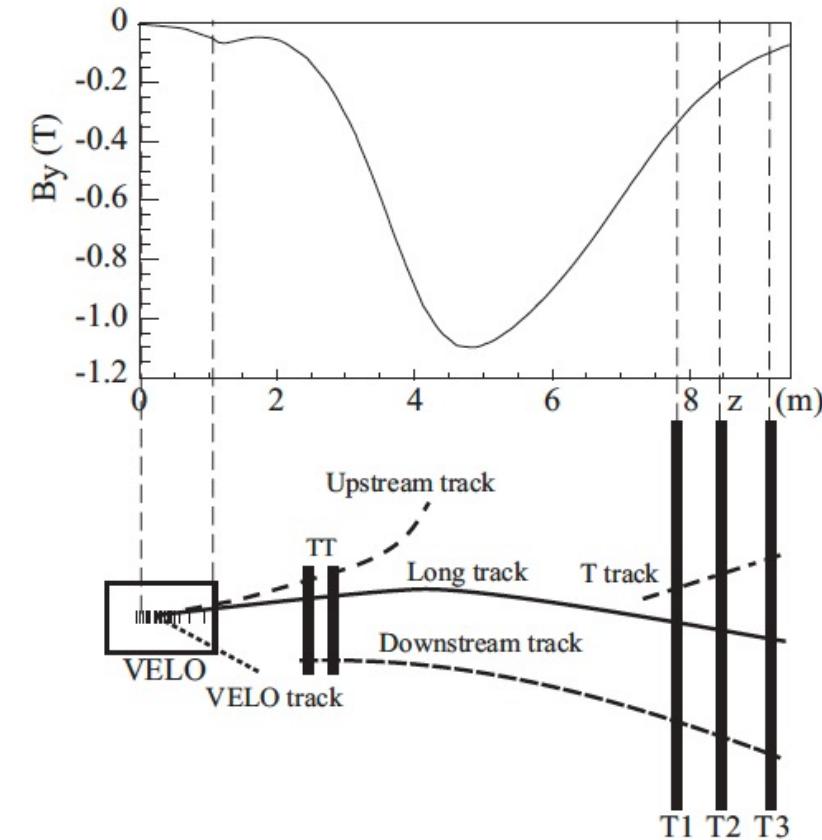
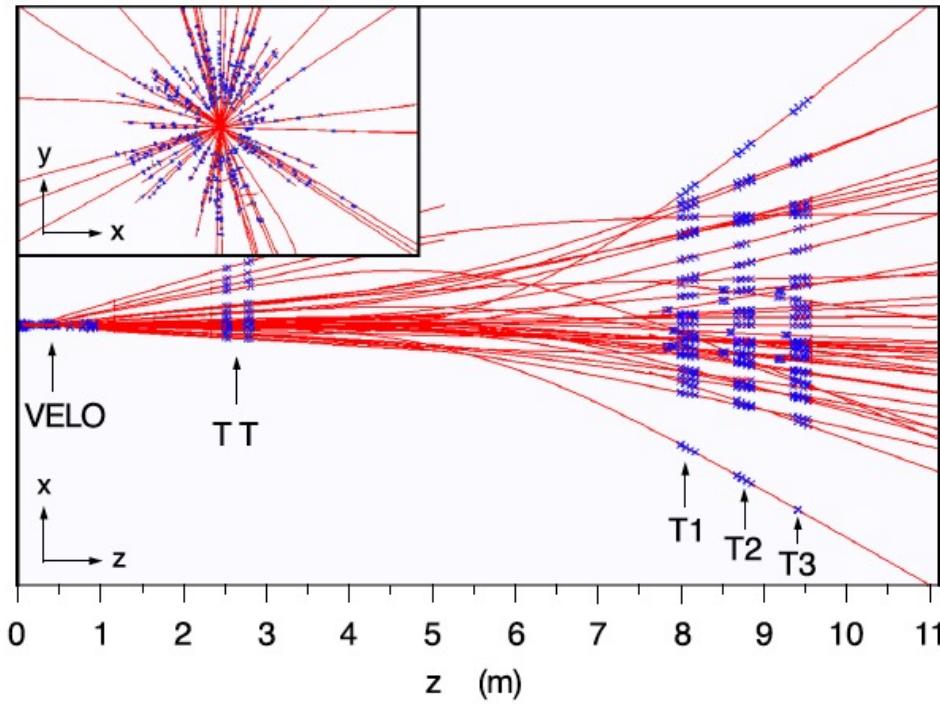


- Collision rate 40 MHz
- Trigger rates (hw & sw)
1 MHz / 100 KHz / 12 KHz



LHCb tracking

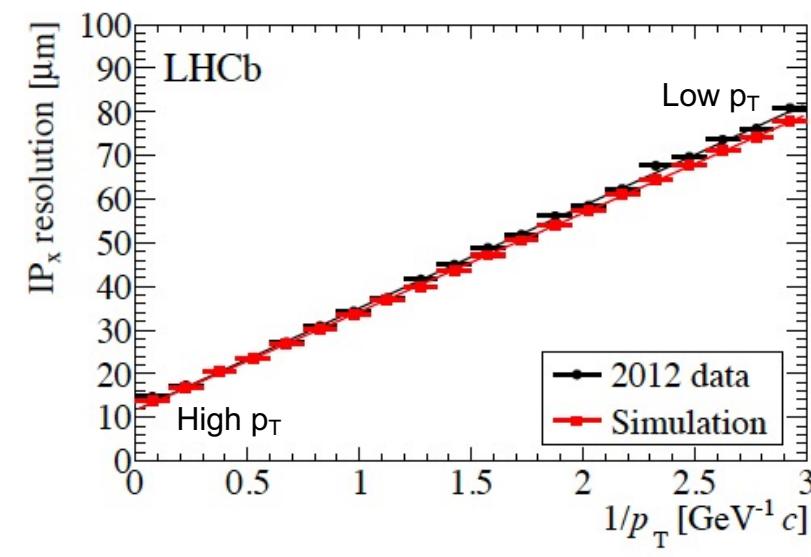
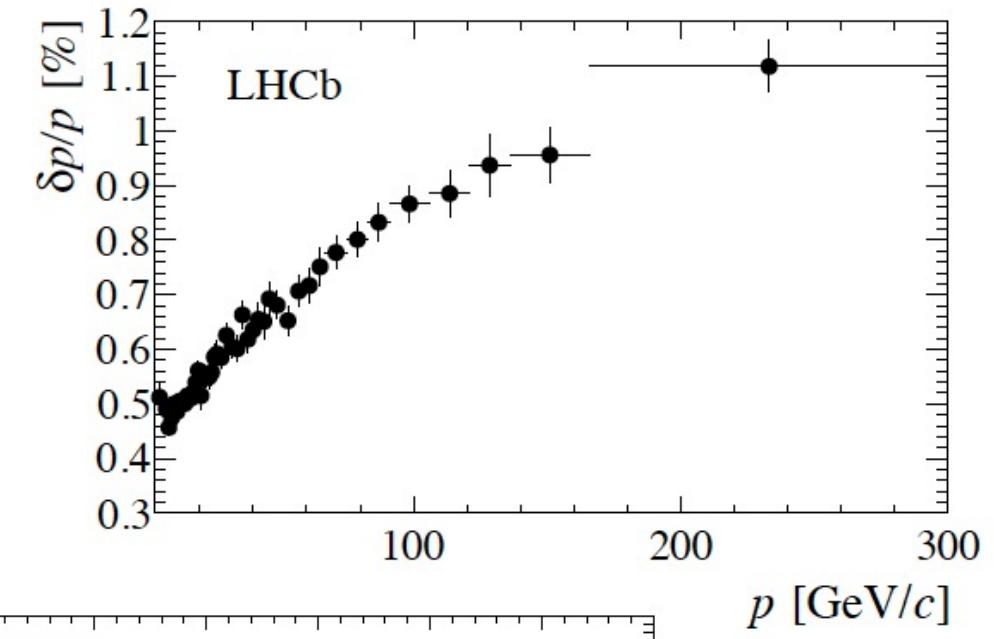
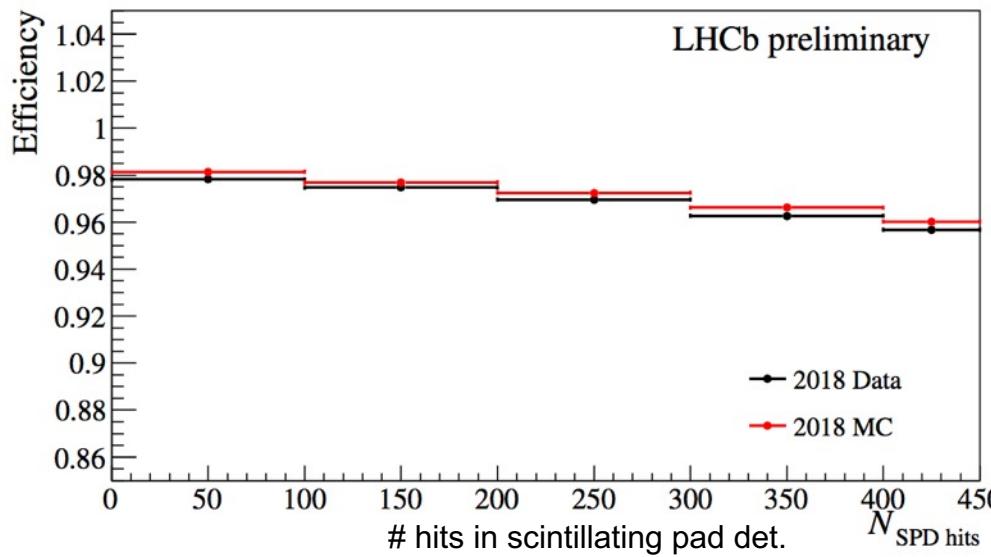
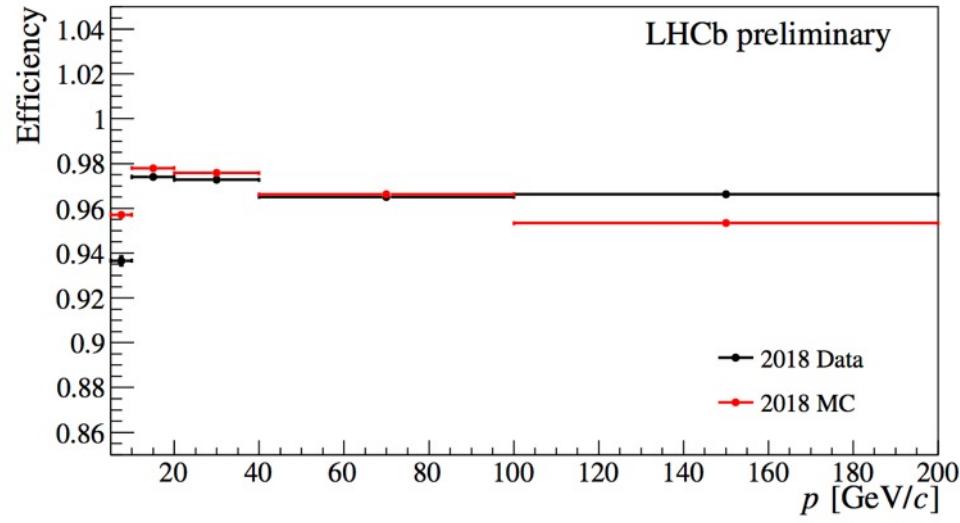
■



Occupancies:

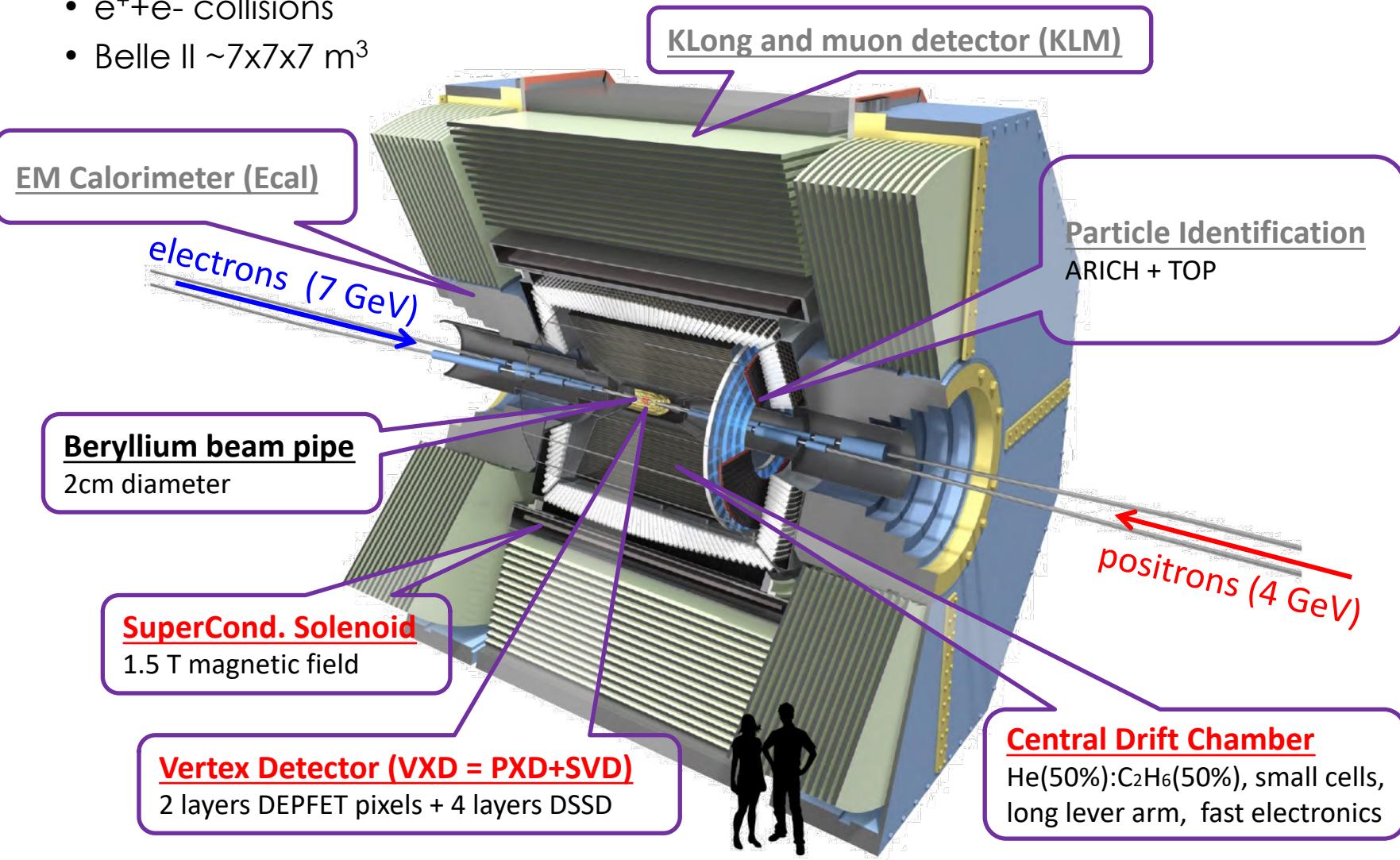
- VELO: 0.5-1 %
- TT: 0.2-1.9 %

LHCb tracking performance

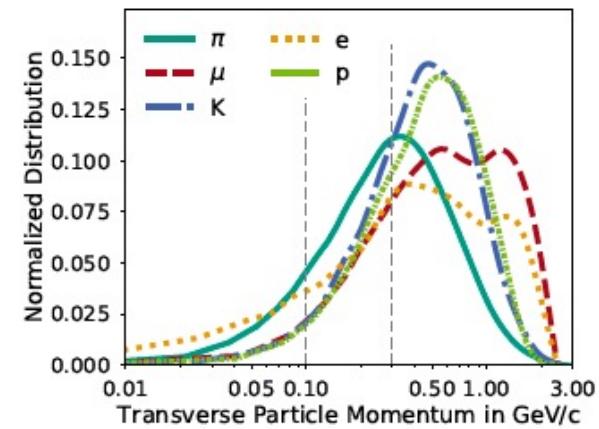


Belle II setup

- e^+e^- collisions
- Belle II $\sim 7 \times 7 \times 7 \text{ m}^3$



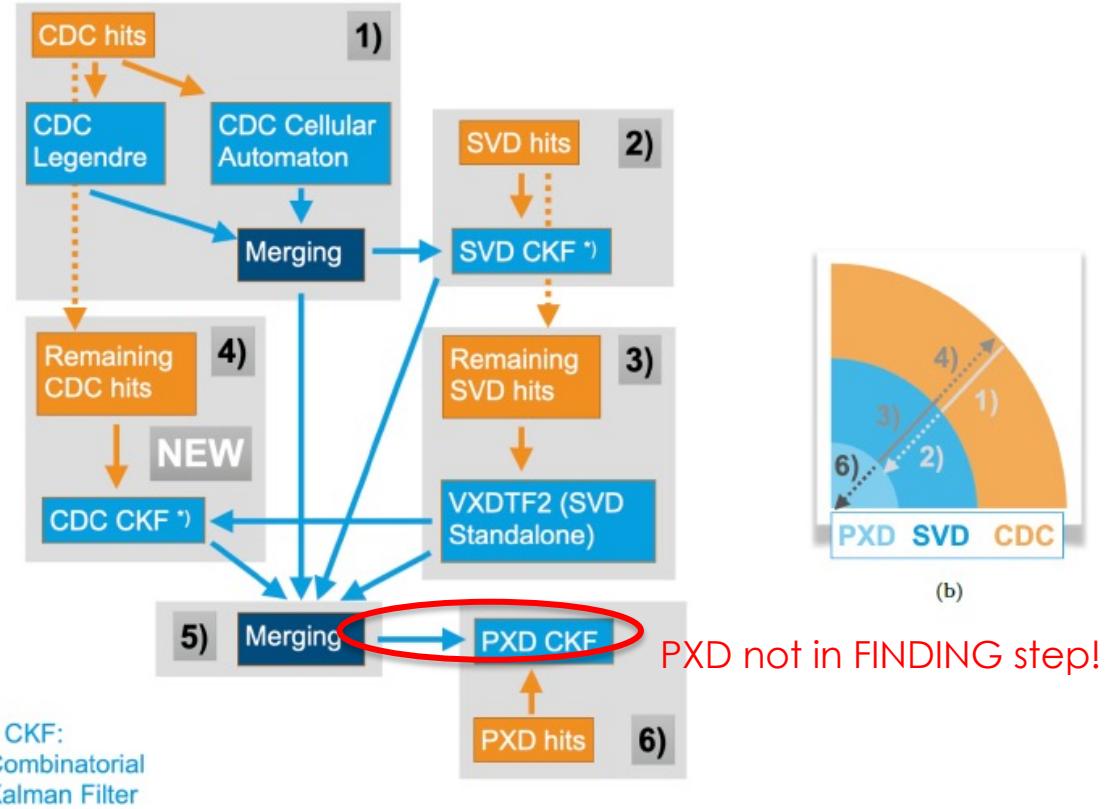
- Collision rate 250 MHz
- Trigger rates (hw & sw)
30 kHz / 6 KHz



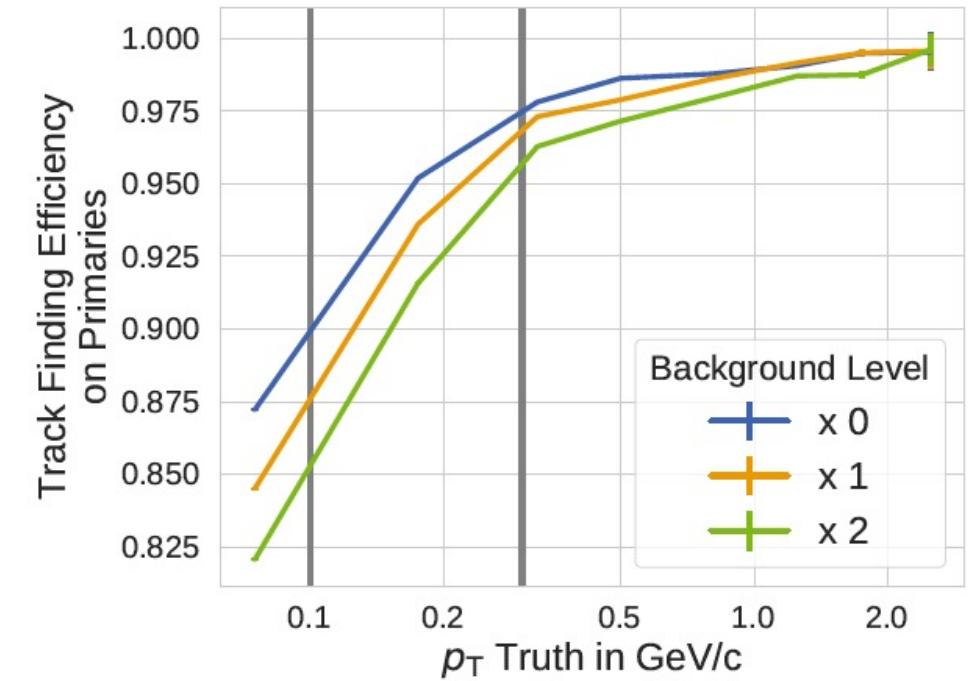
Belle II tracking

- Elementary collisions produce few tracks ($\sim 10/\text{evts}$)
- Beam induced background largely dominates occupancies. \Rightarrow at peak luminosities $> 10^{35} \text{ cm}^{-2}\cdot\text{s}^{-1}$

- Occupancies for $\text{Lumi} > 10^{35} \text{ cm}^{-2}\cdot\text{s}^{-1}$
- PXD: $\sim 1\%$, SVD: 1-3 %

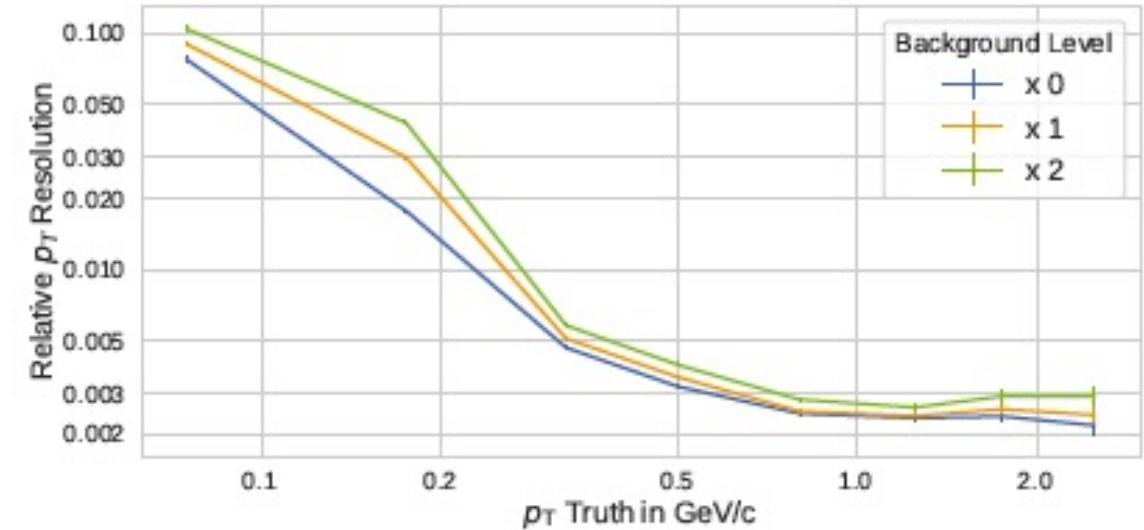
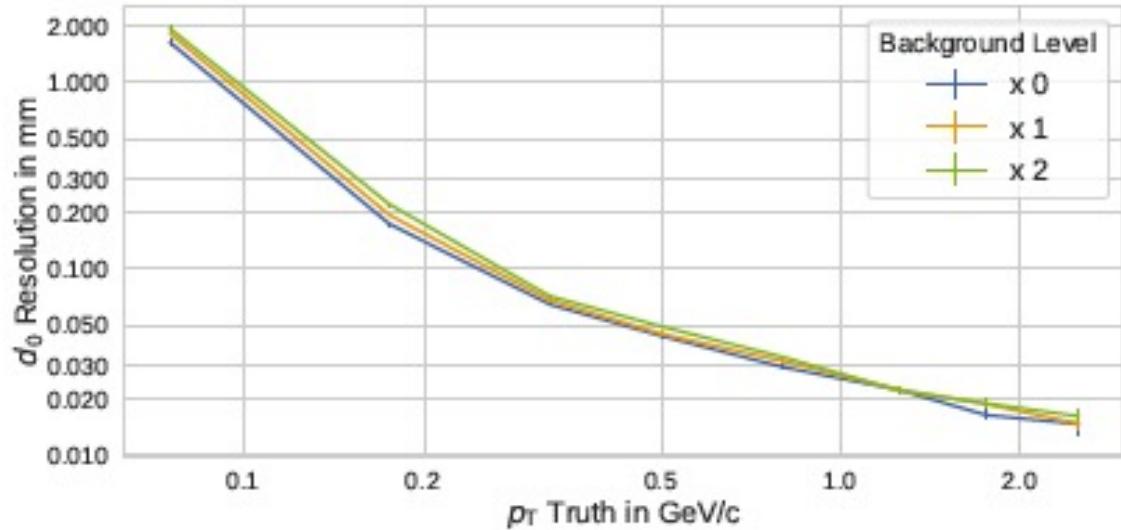


Current occupancies for $\text{Lumi} \sim \text{few } 10^{34} \text{ cm}^{-2}\cdot\text{s}^{-1}$
 $< 1\%$



Belle II tracking performance

■

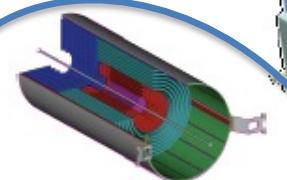


Superconducting
Coil, 4 Tesla

CALORIMETERS

ECAL 76k
scintillating
PbWO₄ crystals

HCAL Plastic
scintillator/
sand



TRACKER Pixels
Silicon Microstrips
210 m² of silicon sensors
0.6 M channels

Total weight 12500 t
Overall diameter 15 m
Overall length 21.6 m

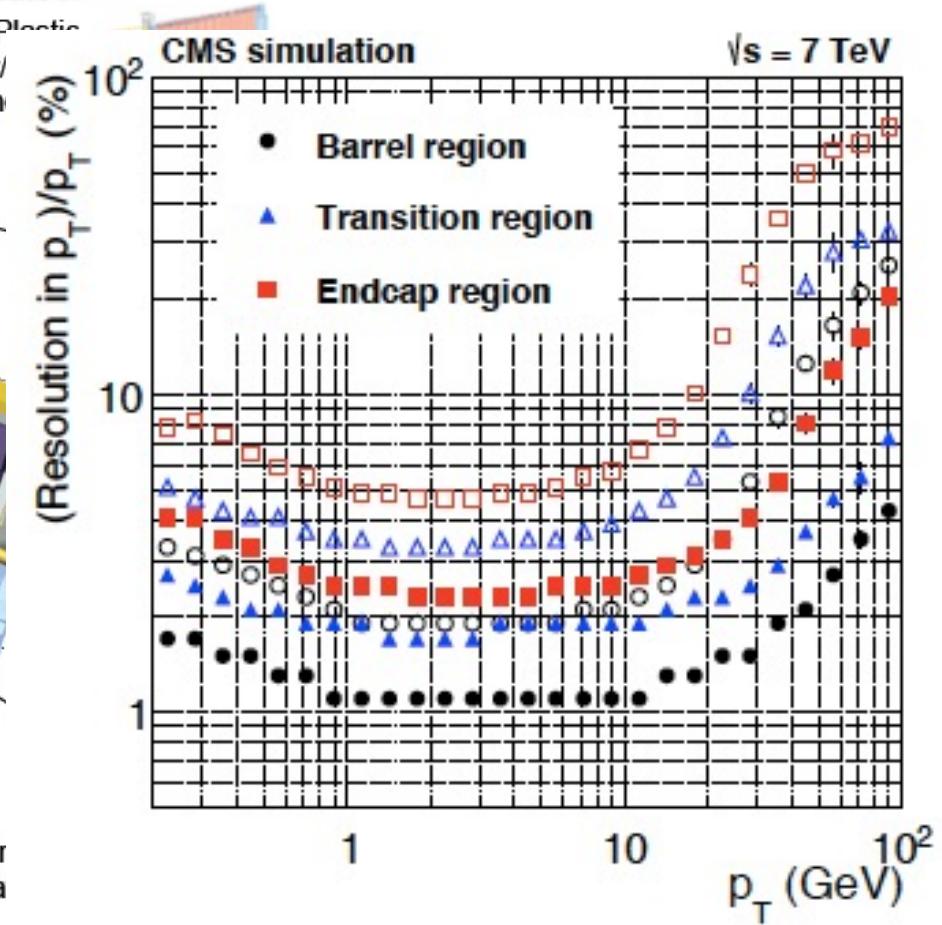
2900 scientists from
182 Institutes from
38 countries

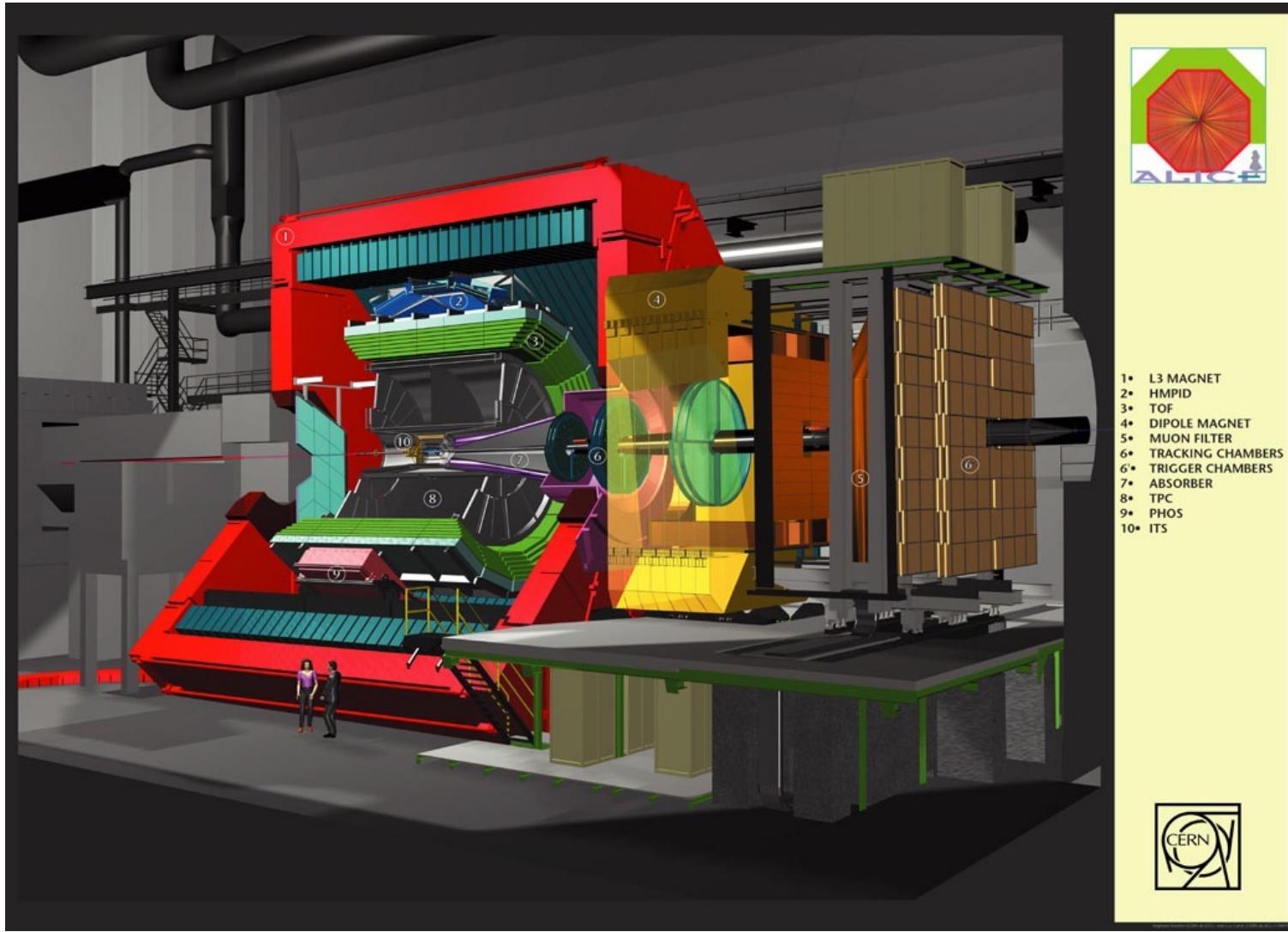
MUON BARREL
Drift Tube
Chambers (DT)

Resistive Plate
Chambers (RPC)

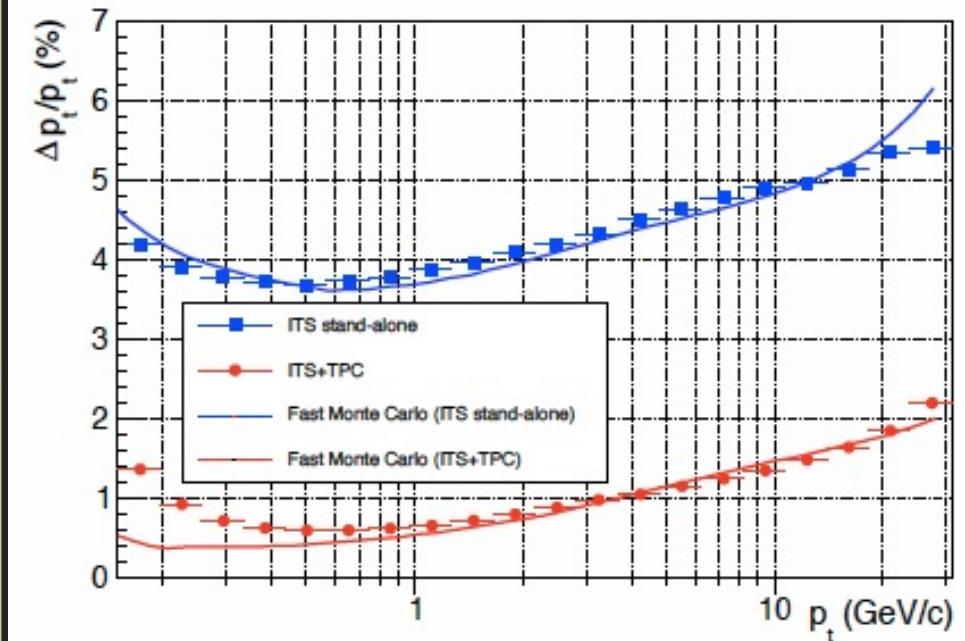
Cathode Str
Resistive Pla

- p+p collision rate 40 MHz



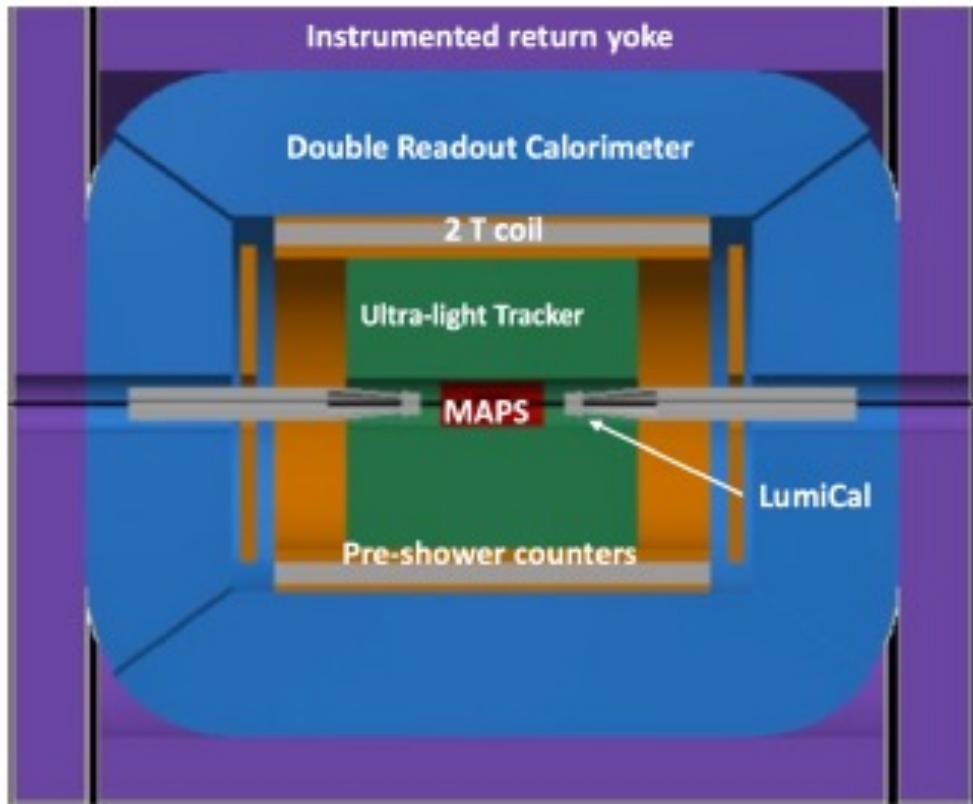


- A+A, A+p, p+p collisions



FCCee (far in the future)

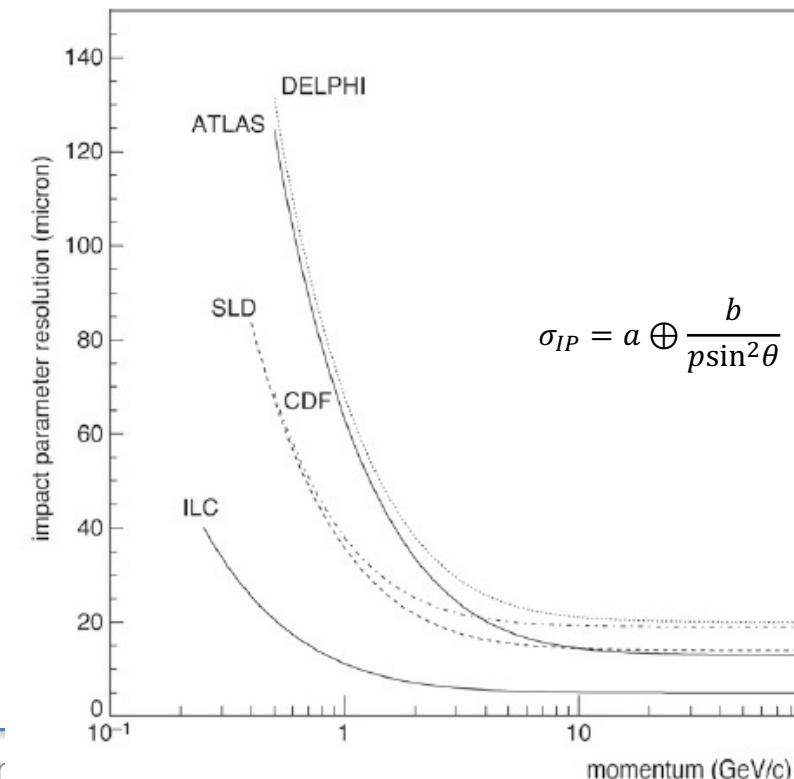
- One of the proposals
- Goals: $\sigma(d_0)/d_0 \approx 2/5/20 \mu\text{m}$ (100/10/1 GeV at 90°) and $\sigma(p_T)/p_T^2 \lesssim 2 \times 10^{-5} \text{ GeV}^{-1}$ ($p_T \gtrsim 100 \text{ GeV}$ at 90°)



13 m long x 11m high

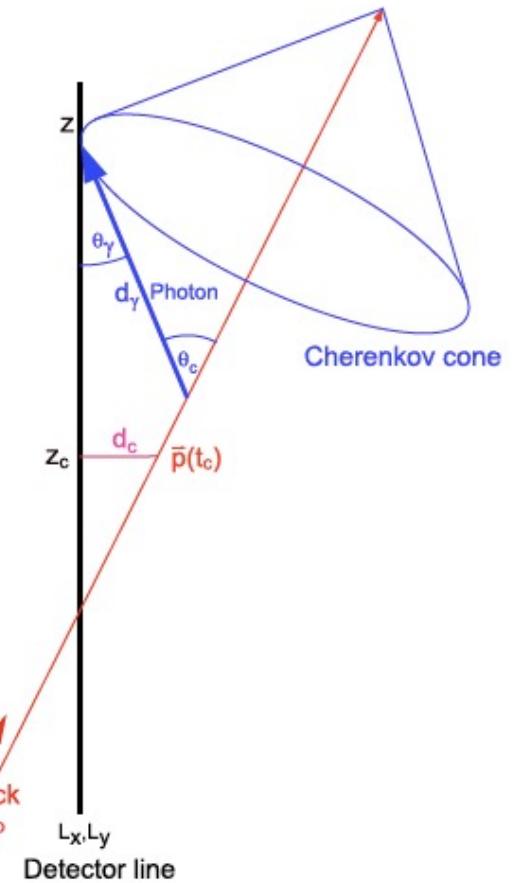
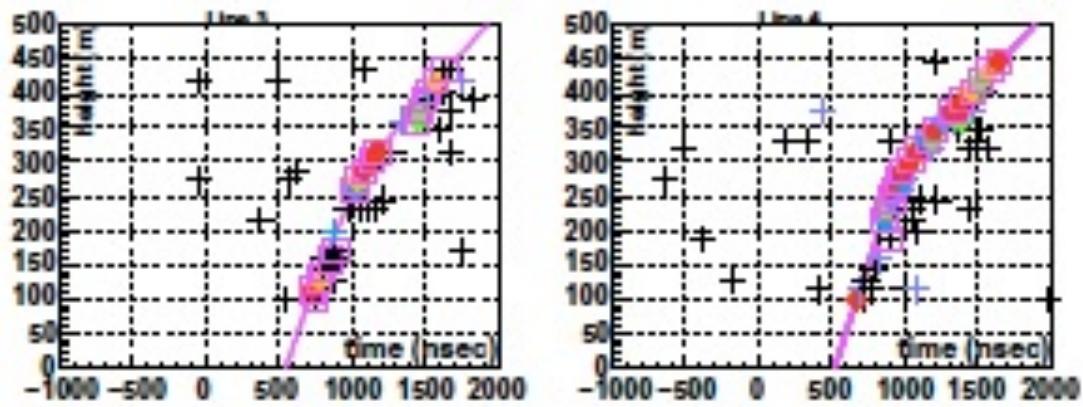
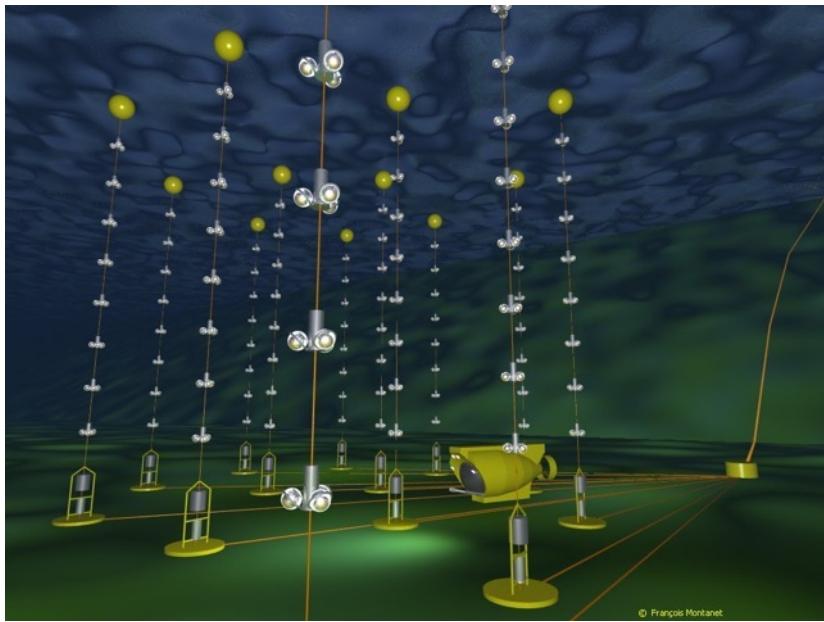
■ Technologies for tracking

- Inner layers: Monolithic CMOS pixel sensors
- Options for outer/main layers:
drift chamber, TPC, silicon sensors



5. Some tracking systems:

ANTARES



Lessons from system review

■ Various technologies

- Semi-conductor based sensors for individual, thin & inner layers
 - Mostly Silicon (sometimes diamond)
- Thin gas volumes over large area with
- Large gas volume, drift chambers or time projection chambers
- Scintillators strips, straw tubes to build large area and/or volume

■ Complementary roles of various sub-detectors

- Obvious for Tracking / Vertexing
- Also sometimes, some information need additional hints to be used properly

■ Usually detectors undergo upgrades

Join the fun !

Additional slides



Multiple scattering - geometry

$$p_{out}^2 = p_{out,z}^2 + p_{out,T}^2 \quad \left\{ \begin{array}{l} p_{out} \cos\theta \approx p_{out,z} \\ p_{out,T} = p_{out} \sin\theta \approx p_{out}\theta \end{array} \right.$$

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- In-plane description (defined by vectors \mathbf{p}_{in} , \mathbf{p}_{out})

- Corresponds to $(\phi, \theta=\theta_{\text{plane}})$ with $\mathbf{p}_{in} = \mathbf{p}_z$ and $p_{out,T}^2 = p_{out,x}^2 + p_{out,y}^2$
- $$\left\{ \begin{array}{l} p_{out} \sin\theta_x \approx p_{out} \theta_x \\ p_{out} \sin\theta_y \approx p_{out} \theta_y \end{array} \right.$$

(note : $\phi \in [0, 2\pi]$ uniform)
 $z = \text{particle charge}$

- In-space description (defined by fixed x/y axes)

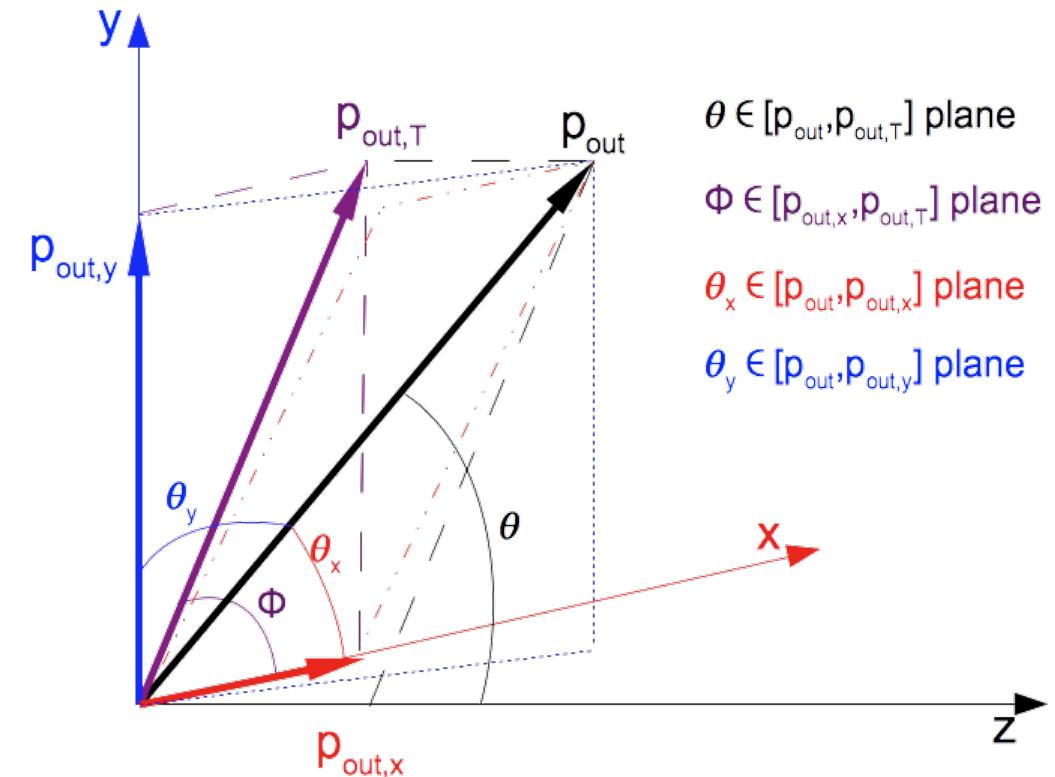
- Corresponds to (θ_x, θ_y) with

\Rightarrow

- θ_x and θ_y are independent gaussian processes

$$\theta_{\text{plane}}^2 = \theta_x^2 + \theta_y^2$$

$$\sigma_{\theta_x} = \sigma_{\theta_y} = \frac{\sigma_{\theta_{\text{plane}}}}{\sqrt{2}}$$



■ Alignment parameters

- Track model depends on additional “free” parameters, i.e. the sensor positions

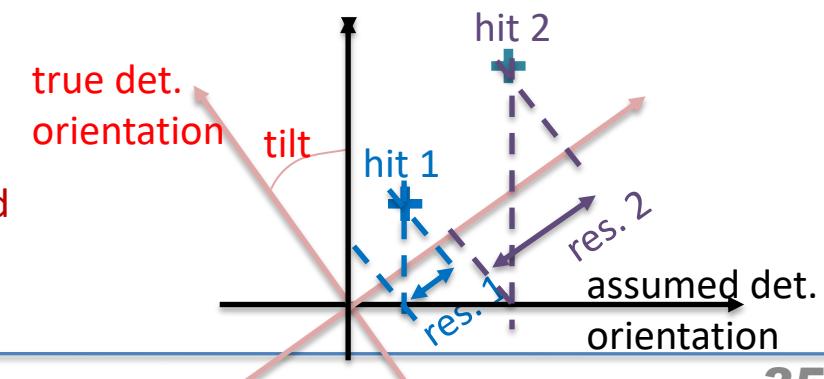
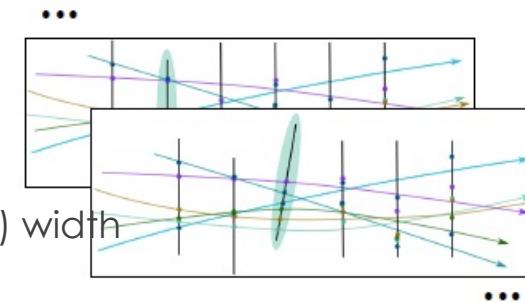
■ Methods to find the relative position of individual sensors

- Global alignment:

- Fit the new params. to minimize the overall χ^2 of a set of tracks
- Beware: many parameters could be involved (few 10^3 can easily be reached) → Millepede algo.

- Local alignment:

- Use tracks reconstructed with reference detectors
- Align other detectors by minimizing the “residual” (track-hit distance) width



In both methods (global or local alignment)

- Use a set of well known tracks and tracking-“friendly” environment to avoid bias
 - Muons (very traversing) and no magnetic field
 - Low multiplicity events

Global deformations also possible

- affect overall positions & momentum
- Corrected through observing
 - Mass peak positions
 - Systematic differences at various track angles or detector positions

