Discussion on Infinite Distance Limits

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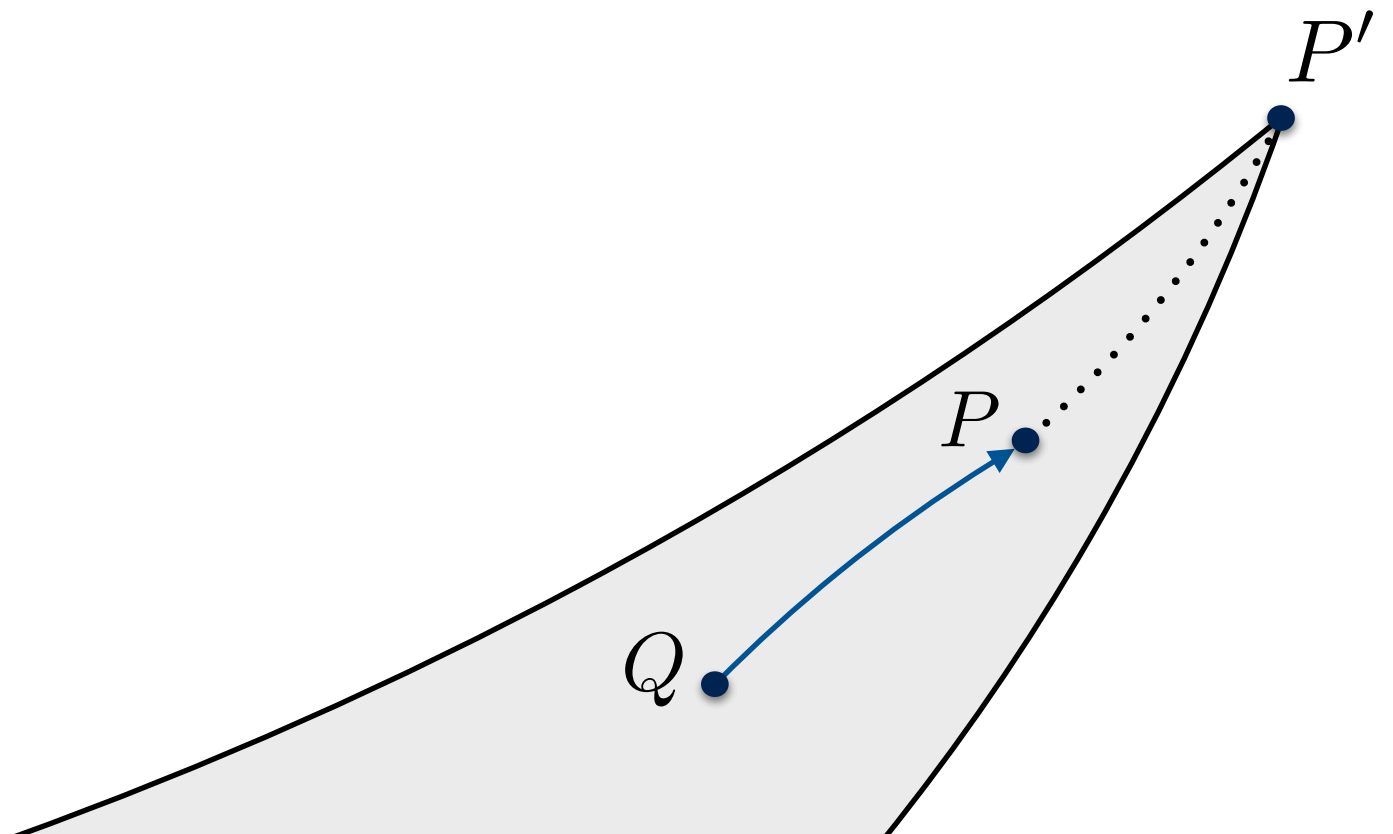
Geometry and Swampland workshop
at Banff, January 2022

Statement of the SDC

(1) $\mathcal{M}$ moduli space. For any point $P \in \mathcal{M}$ and any $C > 0$ exists another point $Q \in \mathcal{M}$ with $d(P,Q) > C$.

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(2) Approaching an infinite distance point $P' \in \partial\mathcal{M}$ with $d(P',Q) \rightarrow \infty$, there exists an infinite tower of states that becomes exponentially light. The masses of the states at P (near P') behave as

$$M_n(P) \sim M_n(Q)e^{-\lambda d(P,Q)}$$

P'

P

Q
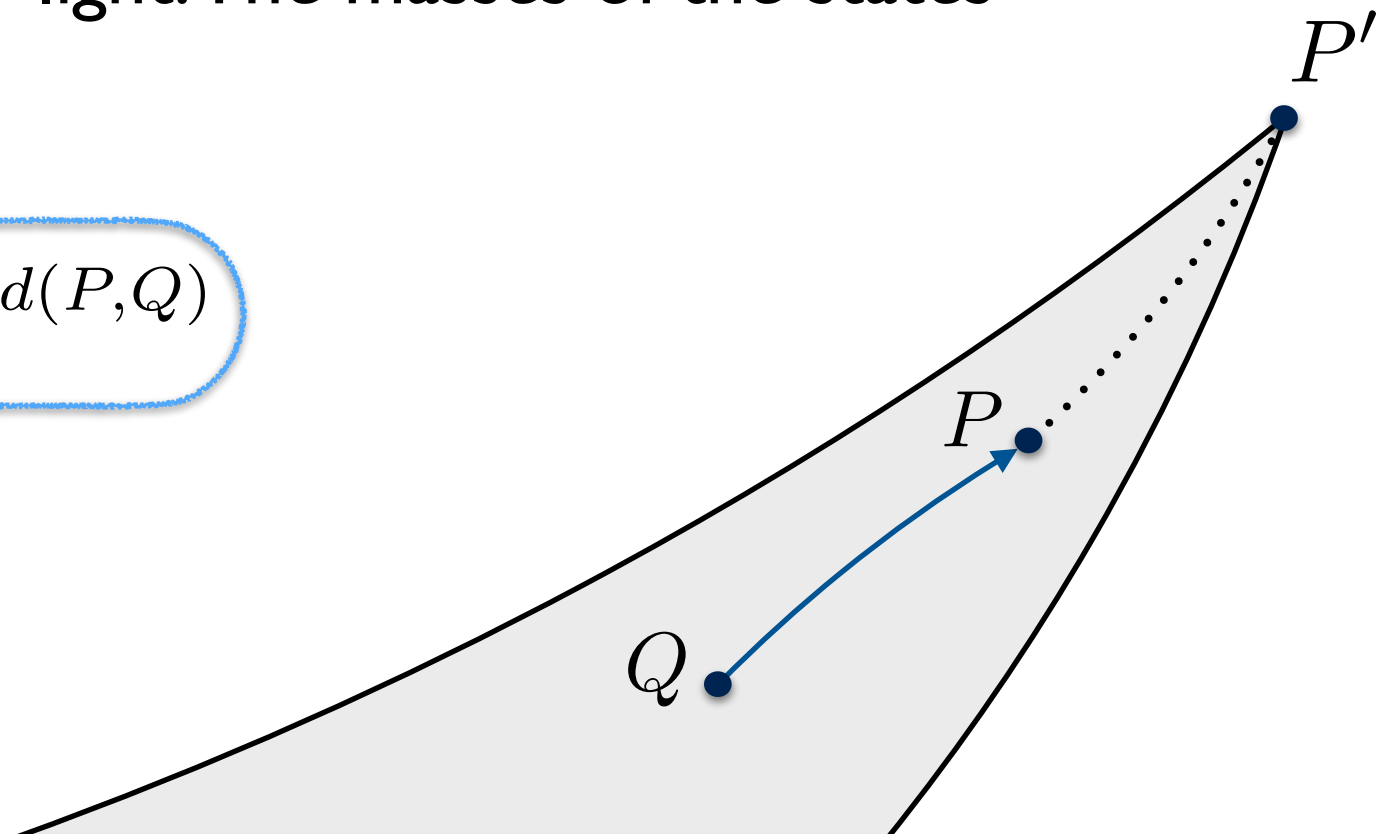
Some loose ends upfront

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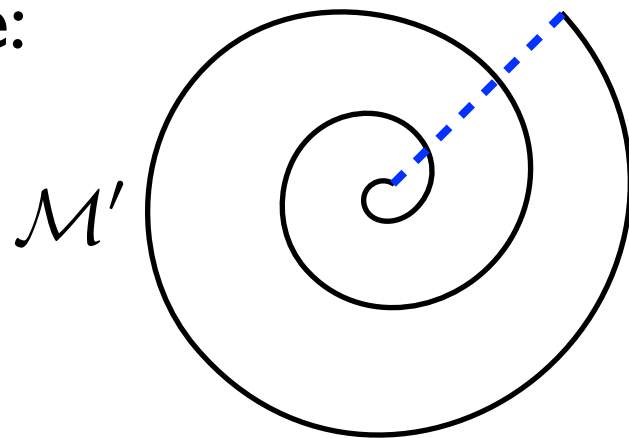
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$\mathcal{M}'$

potential with flat direction along

infinitely long spiral
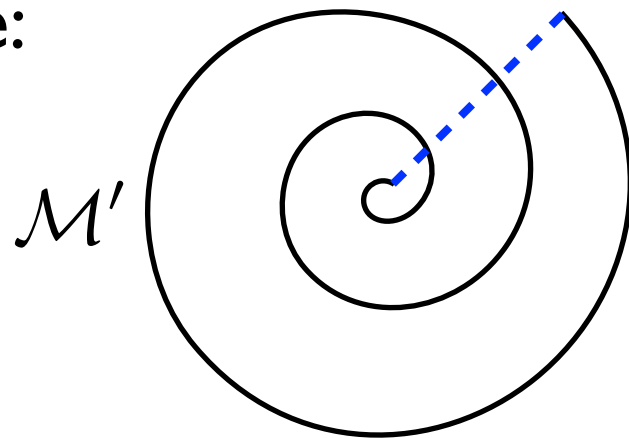
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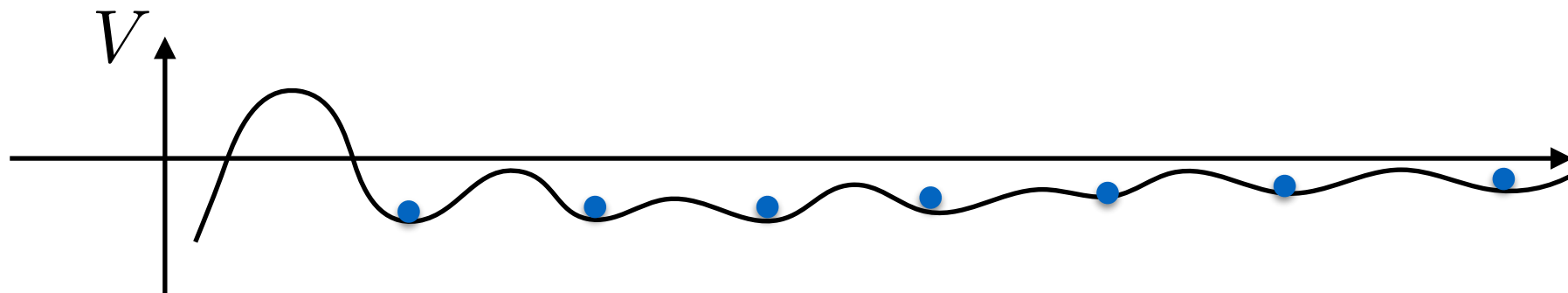
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More general parameter spaces ("discrete" paths)?

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                Emergent string conjecture   Distant axionic string conjecture

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- <u>Numerics/Pheno</u>: What is value of the constant λ ?
  When does the universal exponential behavior set in?

Many Connections with Geometry

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  study of all limits in complex structure + Kähler moduli space

  distance + asymptotic behavior ↔ monodromy + boundary splitting

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- Future perspective: connection to tame geometry
        (vast field of mathematics reaching from logic to number theory)

 $\rightarrow$ no holomorphicity... but strong finiteness properties

Status report of SDC

| Asymptotically Minkowski compactifications:			Exponential behaviour of the tower mass	Tower populated by infinitely many states	Classification of limits
More than 8 supercharges: coset spaces			🟩	🟩	🟢
8 supercharges	4d N=2 (Type II on CY3)	Vector multiplets	🟩	🟢	🟩
		Hypermultiplets	🟨	🟨	🟧
	5d/6d N=1 (M/F-theory on CY3)	Vector/tensor multiplets	🟩	🟢	🟩
		Hypermultiplets	🟨	🟨	🟧
4 supercharges: 4d N=1			🟨	🟧	🟥
No supersymmetry			🟧	🟥	🟥
Asymptotically AdS compactifications:					
Weak coupling points in d>3			🟩	🟩	🟨
Other points			🟧	🟧	🟥

Relations to other conjectures: central role in the Swampland program
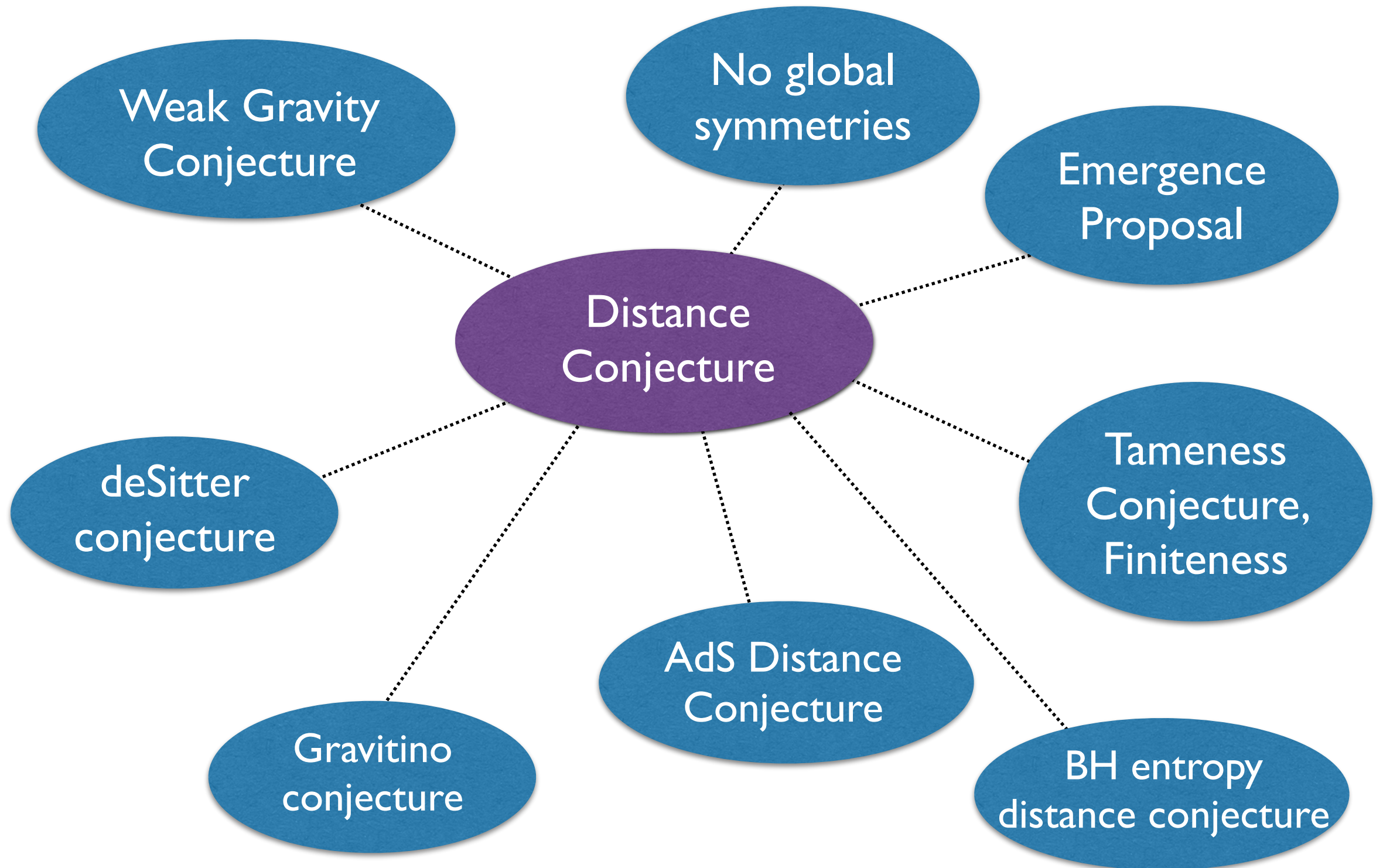
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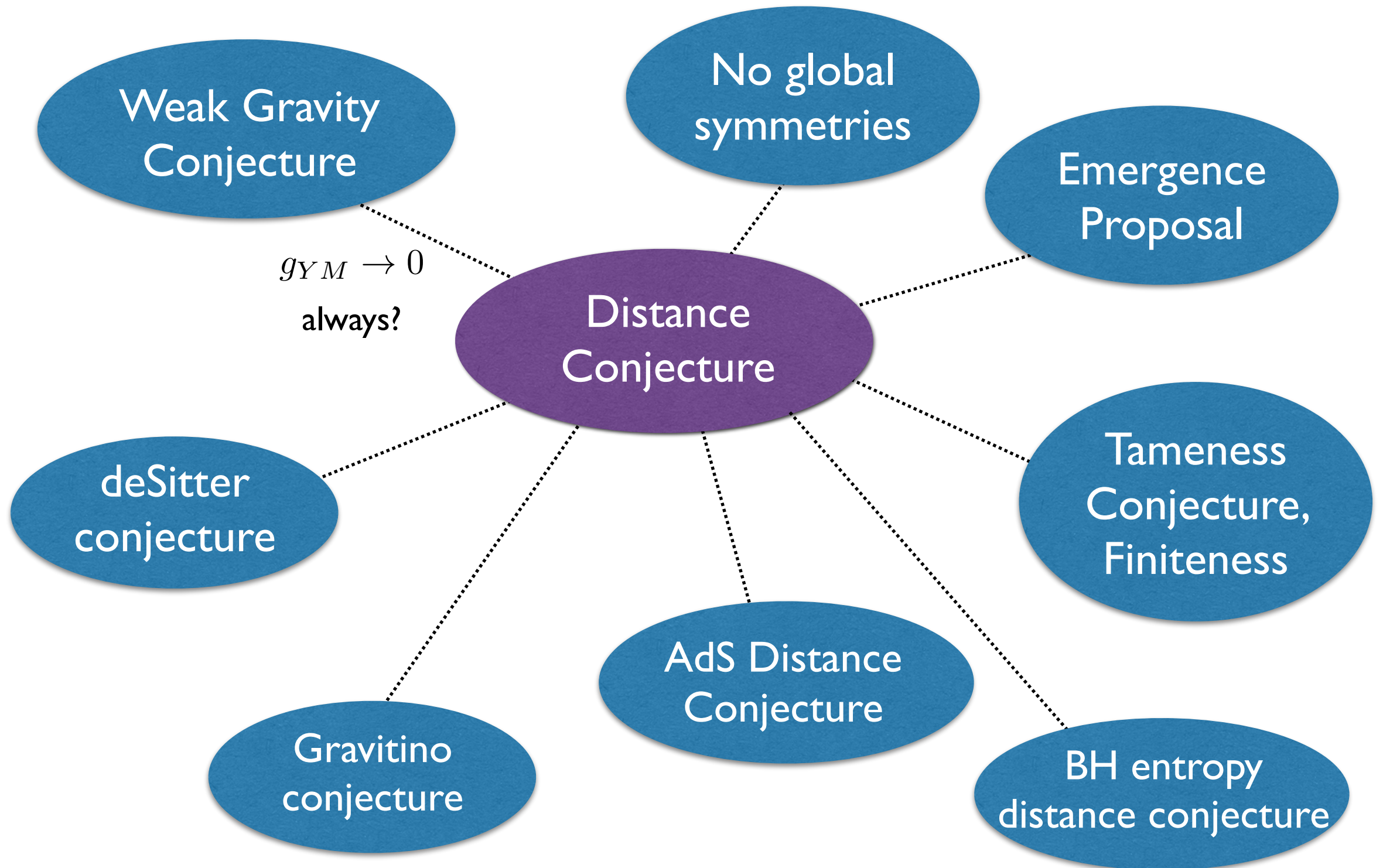
What general features do we expect at infinite distance?
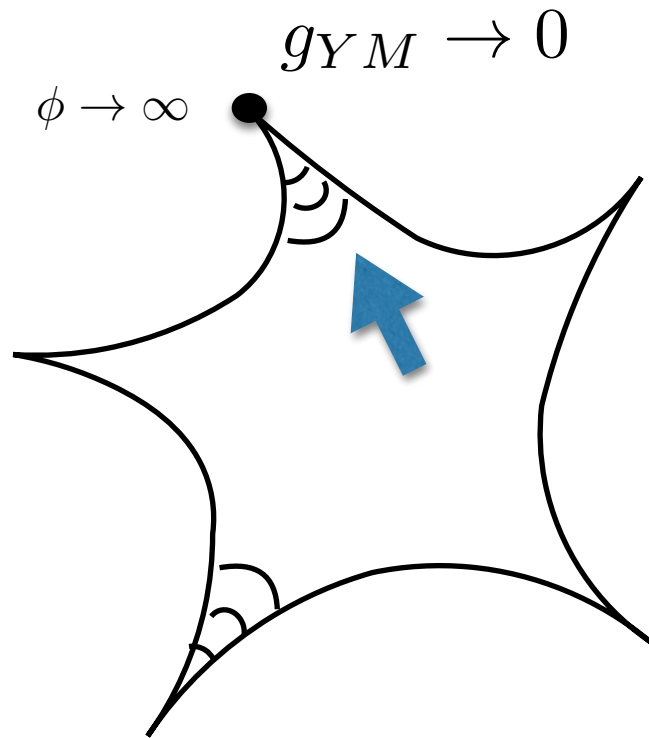
$g_{YM} \to 0$

$\phi \to \infty$

Dualities

Weakly coupled theories
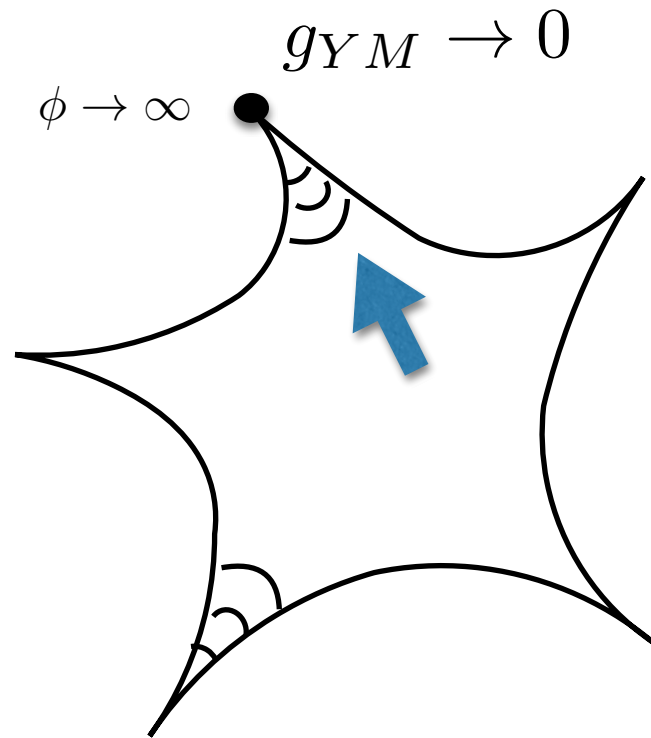
Approximate global symmetries

Runaway potentials

Einstein gravity

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Studying asymptotic limits is much more than identifying the tower!

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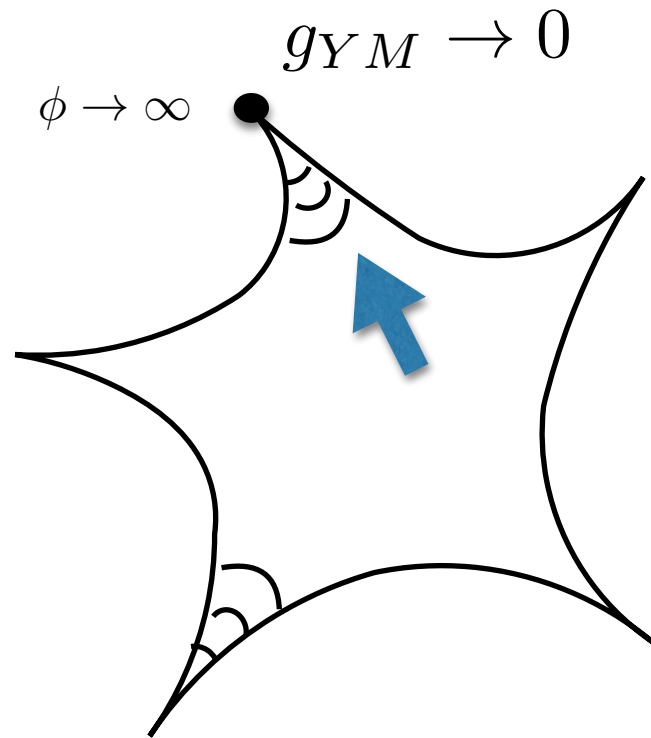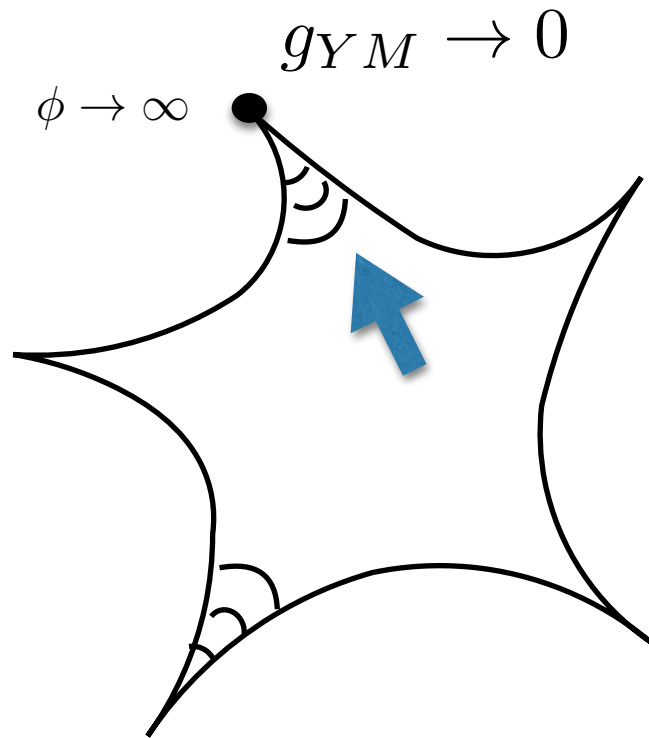Approximate global symmetries

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Runaway potentials **??**

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Where does our universe lives in the field space?

Questions

❖ What is the physical quantum gravity principle underlying the SDC? Is it only valid in Einstein gravity?

❖ Is the leading tower always made of particle or strings, or is it the lamppost effect? How many towers? Can the exponential rate be made very small?

❖ How are quantum corrections expected to affect the infinite distance limits? (obstructions, log corrections…). Can we control them to explore the interior of moduli spaces?

❖ Is there a unique geometrical structure underlying the swampland asymptotic features?

❖ What happens in the presence of a potential?

❖ Does it apply to more general configuration spaces (not just moduli spaces), even discrete ones? Why?

❖ Other questions about the status report of SDC, universal asymptotic features or our place in the field space?