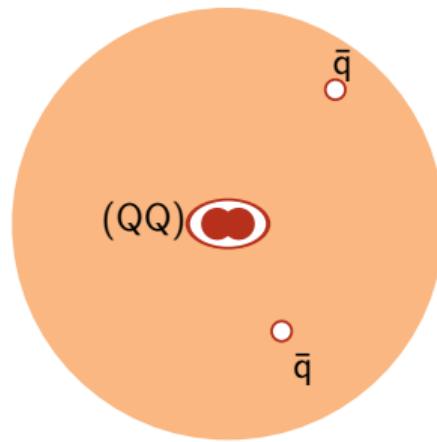


Heavy-Quark Symmetry for $QQ\bar{q}\bar{q}$ Tetraquarks

Chris Quigg

Fermilab



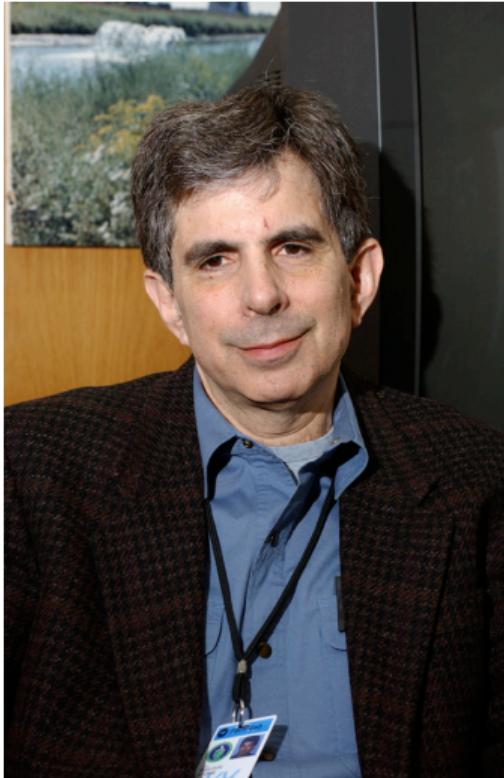
IP2∞ Lyon · *Double Charm Tetraquarks & Other Exotics* · 23 November 2021

Estia Eichten & CQ, PRL **119**, 202002 (2017) / arXiv:1707.09575

Celebrating departed colleagues ...



Misha Voloshin (1953–2020)



Sheldon Stone (1946–2021)

Heavy-quark symmetry implies stable heavy tetraquark mesons $Q_i Q_j \bar{q}_k \bar{q}_l$

In the limit of very heavy quarks Q , novel narrow doubly heavy tetraquark states (DHTQ) must exist.

HQS relates DHTQ mass to masses of a doubly heavy baryon, heavy-light baryon, and heavy-light meson.

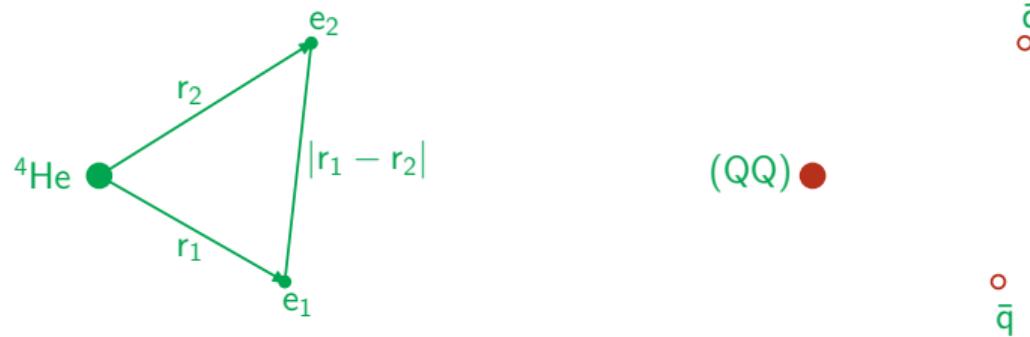
The lightest double-beauty states composed of $bb\bar{u}\bar{d}$, $bb\bar{u}\bar{s}$, and $bb\bar{d}\bar{s}$ will likely be stable against strong decays.

Heavier $bb\bar{q}_k\bar{q}_l$ states, mixed $bc\bar{q}_k\bar{q}_l$ states, and $cc\bar{q}_k\bar{q}_l$ states will likely dissociate into pairs of heavy-light mesons. Some might be seen as “double-flavor” resonances near threshold.

Away from the very-heavy Q limit, the idealization of a pointlike color- $\bar{3}$ diquark does not hold. How to improve the approximation?

When tetraquarks resemble the (textbook) helium atom . . .

Factorized system: separate dynamics for compact “nucleus,” light quarks



(Attractive, **repulsive**) one-gluon exchange for (QQ) in color- $(\bar{\mathbf{3}}, \mathbf{6})$
 $\bar{\mathbf{3}}$ *half strength of $Q\bar{Q}$ attraction in color-1*
also for string tension [Nakamura & Saito]

In heavy limit, idealize a stationary, structureless (color) charge

Stability in the heavy-quark limit

1) *Dissociation into two heavy-light mesons is kinematically forbidden.*

$$\mathcal{Q} \equiv m(Q_i Q_j \bar{q}_k \bar{q}_l) - [m(Q_i \bar{q}_k) + m(Q_j \bar{q}_l)] =$$
$$\underbrace{\Delta(q_k, q_l)}_{\text{light d.o.f.}} - \frac{1}{2} \left(\frac{2}{3} \alpha_s \right)^2 [1 + O(v^2)] \overline{M} + O(1/\overline{M}) ,$$

$\overline{M} \equiv (1/m_{Q_i} + 1/m_{Q_j})^{-1}$: reduced mass of Q_i and Q_j

$\Delta(q_k, q_l) \xrightarrow{\overline{M} \rightarrow \infty}$ independent of heavy-quark masses

For large enough \overline{M} , QQ Coulomb binding dominates, $\boxed{\mathcal{Q} < 0}$

Stability in the heavy-quark limit

2) *Decay to doubly heavy baryon and light antibaryon?*

$$(Q_i Q_j \bar{q}_k \bar{q}_l) \rightarrow (Q_i Q_j q_m) + (\bar{q}_k \bar{q}_l \bar{q}_m)$$

Core $Q_i Q_j$ is color- $\bar{\mathbf{3}}$, same as \bar{Q}_x . Up to contributions from Q motion and spin interactions,

$$m(Q_i Q_j \bar{q}_k \bar{q}_l) - m(Q_i Q_j q_m) = m(Q_x q_k q_l) - m(Q_x \bar{q}_m)$$

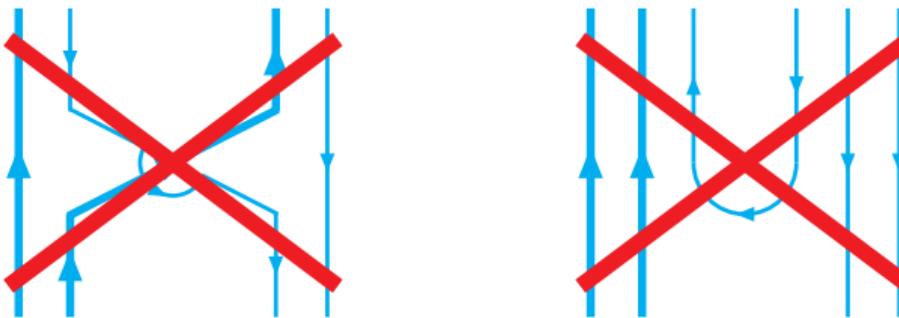
(spin configurations matter)

RHS has generic form $\Delta_0 + \Delta_1/M_{Q_x}$

Using $m(\Lambda_c) - m(D) = 416.87$ MeV and $m(\Lambda_b) - m(B) = 340.26$ MeV, we estimate $\Delta_0 \approx 330$ MeV (asymptotic mass difference).

$$\text{All } < m(\bar{p}) = 938 \text{ MeV}$$

No open strong decay channels in the heavy-quark limit!

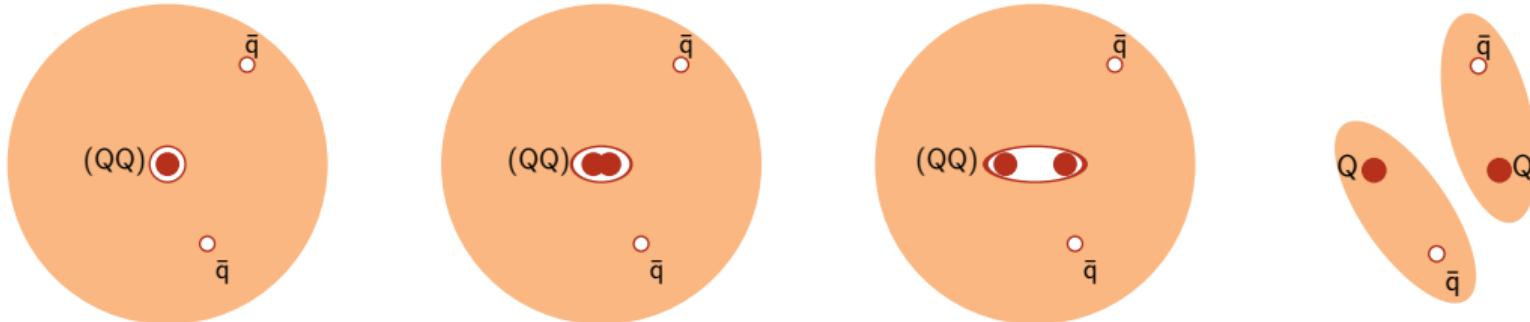


As $\bar{M} \rightarrow \infty$, stable $Q_i Q_j \bar{q}_k \bar{q}_l$ mesons must exist

Implications for the real world?

Does a tiny quasistatic diquark core make sense in our world?

At large $Q_i - Q_j$ separations, $\bar{q}_k \bar{q}_l$ cloud screens $Q_i Q_j$ interaction



Growing separation alters $\bar{\mathbf{3}}, \mathbf{6}$ mix \leadsto division into heavy-light mesons

In half-strength Cornell potential, rms core radii are small wrt tetraquark radius: $\langle r^2 \rangle^{1/2} = 0.19$ fm (bb); 0.24 fm (bc); 0.28 fm (cc). (cf. LQCD)

\therefore core + light (anti)quarks a reasonable idealization.

Mass estimates (beyond the heavy-quark limit . . .)

Use heavy-quark-symmetry relations to estimate $Q_i Q_j \bar{q}_k \bar{q}_l$ masses

$$m(\{Q_i Q_j\}\{\bar{q}_k \bar{q}_l\}) - m(\{Q_i Q_j\}q_y) = m(Q_x\{q_k q_l\}) - m(Q_x \bar{q}_y)$$

$$m(\{Q_i Q_j\}[\bar{q}_k \bar{q}_l]) - m(\{Q_i Q_j\}q_y) = m(Q_x[q_k q_l]) - m(Q_x \bar{q}_y)$$

$$m([Q_i Q_j]\{\bar{q}_k \bar{q}_l\}) - m([Q_i Q_j]q_y) = m(Q_x\{q_k q_l\}) - m(Q_x \bar{q}_y)$$

$$m([Q_i Q_j][\bar{q}_k \bar{q}_l]) - m([Q_i Q_j]q_y) = m(Q_x[q_k q_l]) - m(Q_x \bar{q}_y) .$$

+ finite-mass corrections, $\delta m = \mathcal{S} \frac{\vec{S} \cdot \vec{j}_\ell}{2M} + \frac{\mathcal{K}}{2M}$

(hyperfine + light d.o.f.)

Masses, etc., of ground-state hadrons containing heavy quarks

State	j_ℓ	Mass ($j_\ell + \frac{1}{2}$)	Mass ($j_\ell - \frac{1}{2}$)	Centroid	Spin	Splitting	\mathcal{S} [GeV 2]
$D^{(*)} (c\bar{d})$	$\frac{1}{2}$	2010.26	1869.59	1975.09	140.7		0.436
$D_s^{(*)} (c\bar{s})$	$\frac{1}{2}$	2112.1	1968.28	2076.15	143.8		0.446
$\Lambda_c (cud)_{\bar{3}}$	0	2286.46	—	—			—
$\Sigma_c (cud)_6$	1	2518.41	2453.97	2496.93	64.44		0.132
$\Xi_c (cus)_{\bar{3}}$	0	2467.87	—	—			—
$\Xi'_c (cus)_6$	1	2645.53	2577.4	2622.82	68.13		0.141
$\Omega_c (css)_6$	1	2765.9	2695.2	2742.33	70.7		0.146
$\Xi_{cc} (ccu)_{\bar{3}}$	0	3621.40	—		—		
$B^{(*)} (b\bar{d})$	$\frac{1}{2}$	5324.65	5279.32	5313.32	45.33		0.427
$B_s^{(*)} (b\bar{s})$	$\frac{1}{2}$	5415.4	5366.89	5403.3	48.5		0.459
$\Lambda_b (bud)_{\bar{3}}$	0	5619.58	—		—		
$\Sigma_b (bud)_6$	1	5832.1	5811.3	5825.2	20.8		0.131
$\Xi_b (bds)_{\bar{3}}$	0	5794.5	—		—		
$\Xi'_b (bds)_6$	1	5955.33	5935.02	5948.56	20.31		0.128
$\Omega_b (bss)_6$	1		6046.1				
$B_c (b\bar{c})$	$\frac{1}{2}$	6329	6274.9	6315.4	54		0.340

Kinetic-energy shift differs in $Q\bar{q}$ mesons and Qqq baryons . . .

Consider $\delta\mathcal{K} \equiv \mathcal{K}_{(ud)} - \mathcal{K}_d$:

$$[m((cud)_{\bar{3}}) - m(c\bar{d})] - [m((bud)_{\bar{3}}) - m(b\bar{d})]$$

$$= \delta\mathcal{K} \left(\frac{1}{2m_c} - \frac{1}{2m_b} \right) = 5.11 \text{ MeV}$$

$$\leadsto \delta\mathcal{K} = 0.0235 \text{ GeV}^2$$

$$m(\{cc\}(\bar{u}\bar{d})) - m(\{cc\}d) : \quad \frac{\delta\mathcal{K}}{4m_c} = 2.80 \text{ MeV}$$

$$m((bc)(\bar{u}\bar{d})) - m(\{bc\}d) : \quad \frac{\delta\mathcal{K}}{2(m_c + m_b)} = 1.87 \text{ MeV}$$

$$m(\{bb\}(\bar{u}\bar{d})) - m(\{bb\}d) : \quad \frac{\delta\mathcal{K}}{4m_b} = 1.24 \text{ MeV}$$

Small! (only slightly larger than isospin-breaking effects we neglect)

Estimating ground-state tetraquark masses

RHS of

$$m(Q_i Q_j \bar{q}_k \bar{q}_l) - m(Q_i Q_j q_m) = m(Q_x q_k q_l) - m(Q_x \bar{q}_m)$$

is determined from data

Ξ_{cc}^{++} $Q_i Q_j q_m$ observed; other masses from model calculations*

LHCb: $M(\Xi_{cc}^{++}) = 3621.40 \pm 0.78$ MeV

*We adopt Karliner & Rosner, *PRD* **90**, 094007 (2014)

Strong decays $(Q_i Q_j \bar{q}_k \bar{q}_l) \not\rightarrow (Q_i Q_j q_m) + (\bar{q}_k \bar{q}_l \bar{q}_m)$ \forall ground states

Must consider decays to pairs of heavy-light mesons case-by-case

Expectations for ground-state tetraquark masses, in MeV

State	J^P	$m(Q_i Q_j \bar{q}_k \bar{q}_l)$	Decay Channel	\mathcal{Q} [MeV]
$\{cc\}[\bar{u}\bar{d}]$	1^+	3978	$D^0 D^{*+}$	3875 103
$\{cc\}[\bar{q}_k \bar{s}]$	1^+	4156	$D^+ D_s^{*+}$	3977 179
$\{cc\}\{\bar{q}_k \bar{q}_l\}$	$0^+, 1^+, 2^+$	4146, 4167, 4210	$D^+ D^0, D^+ D^{*0}$	3734, 3876 412, 292, 476
$[bc][\bar{u}\bar{d}]$	0^+	7229	$B^- D^+/B^0 D^0$	7146 83
$[bc][\bar{q}_k \bar{s}]$	0^+	7406	$B_s D$	7236 170
$[bc]\{\bar{q}_k \bar{q}_l\}$	1^+	7439	$B^* D/BD^*$	7190/7290 249
$[bc][\bar{u}\bar{d}]$	1^+	7272	$B^* D/BD^*$	7190/7290 82
$[bc][\bar{q}_k \bar{s}]$	1^+	7445	DB_s^*	7282 163
$[bc]\{\bar{q}_k \bar{q}_l\}$	$0^+, 1^+, 2^+$	7461, 7472, 7493	$BD/B^* D$	7146/7190 317, 282, 349
$\{bb\}[\bar{u}\bar{d}]$	1^+	10482	$B^- \bar{B}^{*0}$	10603 -121
$\{bb\}[\bar{q}_k \bar{s}]$	1^+	10643	$\bar{B} \bar{B}_s^*/\bar{B}_s \bar{B}^*$	10695/10691 -48
$\{bb\}\{\bar{q}_k \bar{q}_l\}$	$0^+, 1^+, 2^+$	10674, 10681, 10695	$B^- B^0, B^- B^{*0}$	10559, 10603 115, 78, 136

Cf. M. Karliner & J. L. Rosner model, Phys. Rev. Lett. **119**, 202001 (2017) [arXiv:1707.07666].

They estimate deeper binding, so anticipate additional bc and cc candidates.

Real-world candidates for stable tetraquarks

$J^P = 1^+$ $\{bb\}[\bar{u}\bar{d}]$ meson, bound by 121 MeV

(77 MeV below $B^- \bar{B}^0 \gamma$)

$\mathcal{T}_{[\bar{u}\bar{d}]}^{\{bb\}}(10482)^- \rightarrow \Xi_{bc}^0 \bar{p}$, $B^- D^+ \pi^-$, and $\underbrace{B^- D^+ \ell^- \bar{\nu}}$
manifestly weak!

$J^P = 1^+$ $\{bb\}[\bar{u}\bar{s}]$ and $\{bb\}[\bar{d}\bar{s}]$ mesons, bound by 48 MeV

(3 MeV below $BB_s \gamma$)

$\mathcal{T}_{[\bar{u}\bar{s}]}^{\{bb\}}(10643)^- \rightarrow \Xi_{bc}^0 \bar{\Sigma}^-$ $\mathcal{T}_{[\bar{d}\bar{s}]}^{\{bb\}}(10643)^0 \rightarrow \Xi_{bc}^0 (\bar{\Lambda}, \bar{\Sigma}^0)$

Lattice studies also suggest stable double-beauty tetraquarks

Unstable doubly heavy tetraquarks

Resonances in “wrong-sign” (double flavor) combinations $DD, DB, BB?$

$$J^P = 1^+ \mathcal{T}_{[\bar{u}\bar{d}]}^{\{cc\}}(3978)^+, Q = +102 \text{ MeV}$$

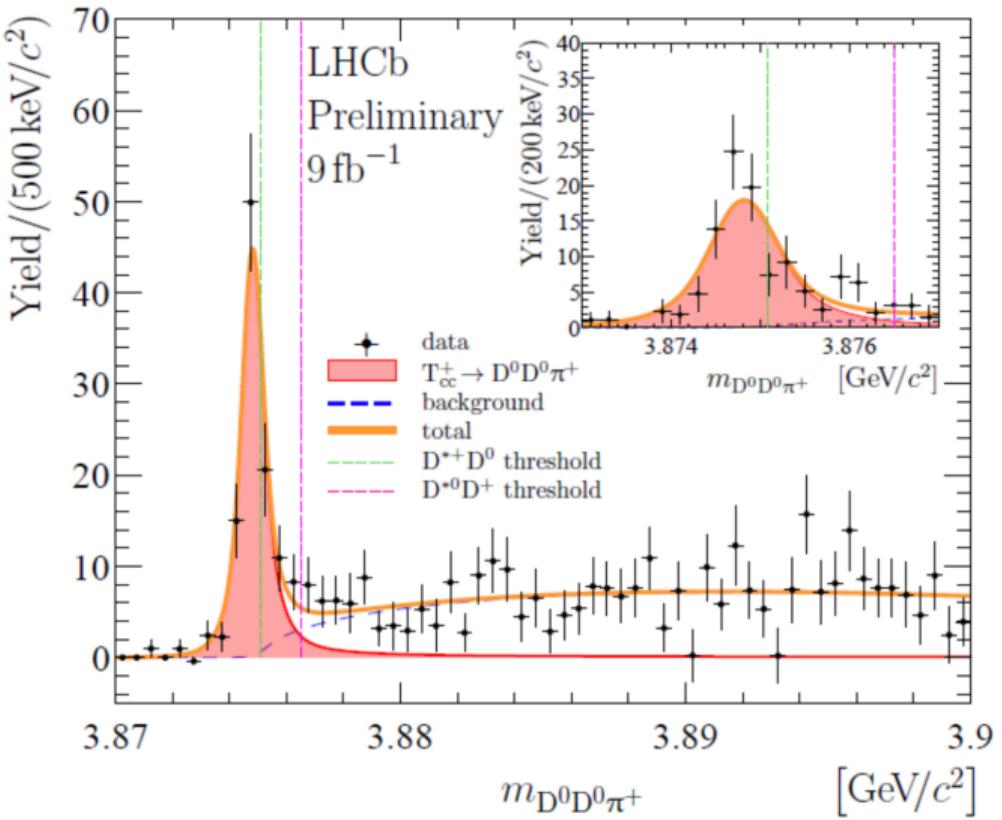
Double Charm

$$J^P = 1^+ \mathcal{T}_{[\bar{d}\bar{s}]}^{\{cc\}++}(4156) \rightarrow D^+ D_s^{*+}, Q = +179 \text{ MeV}$$

Double Charge / Double Charm

prima facie evidence for non- $q\bar{q}$ level

LHCb observes state compatible with $\mathcal{T}_{[\bar{u}\bar{d}]}^{\{cc\}+} \rightarrow D^0 D^0 \pi^+$



Reference	Year	$\delta'm$ [MeV/c 2]
J. Carlson, L. Heller and J. A. Tjon	1987	~ 0
B. Silvestre-Brac and C. Semay	1993	+19
C. Semay and B. Silvestre-Brac	1994	[−1, +13]
S. Pepin, F. Stancu, M. Genovese and J. M. Richard	1996	< 0
B. A. Gelman and S. Nussinov	2002	[−25, +35]
J. Vijande, F. Fernandez, A. Valcarce, A. and B. Silvestre-Brac	2003	−112
D. Jane and M. Rosina	2004	[−3, −1]
F. Navarra, M. Nielsen and S. H. Lee	2007	+91
J. Vijande, E. Weissman, A. Valcarce	2007	[−16, +50]
D. Ebert, R. N. Faustov, V. O. Galkin and W. Lucha	2007	+60
S. H. Lee and S. Yasui	2009	−79
Y. Yang, C. Deng, J. Ping and T. Goldman	2009	−1.8
G.-Q. Feng, X.-H. Guo and B.-S. Zou	2013	−215
Y. Ikeda, B. Charron, S. Aoki, T. Doi, T. Hatsuda, T. Inoue, N. Ishii, K. Murano, H. Nemura and K. Sasaki	2013	[−70, +124]
S.-Q. Luo, K. Chen, X. Liu, Y.-R. Liu and S.-L. Zhu	2017	+100
M. Karliner and J. Rosner	2017	$7 \pm 12 \rightarrow 1$
E. J. Eichten and C. Quigg	2017	+102
Z. G. Wang	2017	+25 ± 90
G. K. C. Cheung, C. E. Thomas, J. J. Dudek and R. G. Edwards	2017	$\lesssim 0$
W. Park, S. Noh and S. H. Lee	2018	+98
A. Francis, R. J. Hudspith, R. Lewis and K. Maltman	2018	~ 0
P. Jumankar, N. Mathur and M. Padmanath	2018	[−40, 0]
C. Deng, H. Chen and J. Ping	2018	−150
M.-Z. Liu, T.-W. Wu, V. Pavon Valderrama, J.-J. Xie and L.-S. Geng	2019	-3^{+4}_{-15}
G. Yang, J. Ping and J. Segovia	2019	−149
Y. Tan, W. Lu and J. Ping	2020	−182
Q.-F. Lü, D.-Y. Chen and Y.-B. Dong	2020	+166
E. Braaten, L.-P. He and A. Mohapatra	2020	+72
D. Gao, D. Jia, Y.-J. Sun, Z. Zhang, W.-N. Liu and Q. Mei	2020	[−250, +2]
J.-B. Cheng, S.-Y. Li, Y.-R. Liu, Z.-G. Si, T. Yao	2020	+53
S. Noh, W. Park and S. H. Lee	2021	+13
R. N. Faustov, V. O. Galkin and E. M. Savchenko	2021	+64

Unstable doubly heavy tetraquarks ^{bis}

Resonances in “wrong-sign” (double flavor) combinations $DD, DB, BB?$

$$J^P = 1^+ \mathcal{T}_{[\bar{u}\bar{d}]}^{\{bc\}}(7272)^0 \rightarrow B^*D/BD^*, Q = +82 \text{ MeV}$$

Charm + Beauty

$$J^P = 0^+ \mathcal{T}_{[\bar{u}\bar{d}]}^{[bc]}(7229)^0, Q = +83 \text{ MeV}$$

Charm + Beauty

$$1^+ \mathcal{T}_{\{\bar{q}_k\bar{q}_l\}}^{\{bb\}}(10681)^{0,-,--} \rightarrow B^-B^- \text{ etc.}, Q = +78 \text{ MeV}$$

Double Beauty

Configurations beyond $Q_i Q_j \bar{q}_k \bar{q}_l$

All quarks heavy, one-gluon exchange prevails: No stable $QQ\bar{Q}\bar{Q}$ (equal-mass) tetraquarks in very-heavy-quark limit. Support for binding of $bb\bar{q}\bar{q}$, not $bb\bar{c}\bar{c}$. Study N_c dependence, hierarchy of scales.

A. Czarnecki, B. Leng, M. B. Voloshin, "Stability of tetrons," arXiv:1708.04594, PLB **778**, 233 (2018).

Lattice-NRQCD study of $bb\bar{b}\bar{b}$: No tetraquark with mass below $\eta_b\eta_b$, $\eta_b\Upsilon$, $\Upsilon\Upsilon$ thresholds in $J^{PC} = 0^{++}, 1^{+-}, 2^{++}$ channels.

C. Hughes, E. Eichten, C. T. H. Davies, "The Search for Beauty-fully Bound Tetraquarks Using Lattice Non-Relativistic QCD," arXiv:1710.03236, PRD **97**, 054505 (2018).

Within potential models: No tetraquark composed exclusively of heavy quarks c or b .

J. M. Richard, "Fully Heavy Multiquarks," arXiv:2106.07434, Few Body Syst. **62**, no.3, 37 (2021).

How to improve HQS mass estimates

Complete the input data set by observing all doubly heavy baryons

Make a quantitative study of how the color and constituent configurations depend on $Q_i Q_j$ separation, heavy-quark masses, lighter-quark masses.
Promising vehicle: CLV long-distance Hamiltonian, which mixes $\bar{\mathbf{3}} \otimes \mathbf{3}$ and $\mathbf{6} \otimes \bar{\mathbf{6}}$ diquark–antidiquark configurations, in the heavy-quark limit.

Incorporate the insights drawn from lattice QCD, (potential-)model calculations, other approaches . . .

Correlate with what we may learn about $Q_i \bar{Q}_j \bar{q}_k q_l$ spectroscopy.

Homework for experiment

Look for additional double-flavor resonances near threshold: $\mathcal{T}_{[\bar{d}\bar{s}]}^{\{cc\}++}$.

Measure cross sections for final states containing 4 heavies: $Q_i \bar{Q}_i Q_j \bar{Q}_j$.

Discover and determine masses of doubly-heavy baryons.

Needed to implement HQS calculation of tetraquark masses.

*Intrinsic interest in these states: comparison with heavy-light mesons,
possible core excitations.*

⟨ Resolve Ξ_{cc}^+ uncertainty (SELEX/LHCb) ⟩

Find stable tetraquarks through weak decays.

Homework for theory

Develop expectations for production. A. Ali et al., "Prospects of discovering stable double-heavy tetraquarks at a Tera-Z factory," arXiv:1805.02535, Phys. Lett. B **782**, 412-420 (2018)

Refine lifetime estimates ($\lesssim 1$ ps) for stable states.(Cf. Ali's talk)

Understand how color configurations evolve with QQ (and $\bar{q}\bar{q}$) masses.

J.-M. Richard, et al., "Few-body quark dynamics for doubly-heavy baryons and tetraquarks," arXiv:1803.06155, Phys. Rev. C **97**, 035211 (2018).

Investigate stability of different body plans in the heavy-quark limit.
... up to $(Q_i Q_j)(Q_k Q_l)(Q_m Q_n)$: $B = 2$, but $Q_p Q_q Q_r$ color structure?

Heavy-quark symmetry implies stable heavy tetraquark mesons $Q_i Q_j \bar{q}_k \bar{q}_l$

In the limit of very heavy quarks Q , novel narrow doubly heavy tetraquark states must exist.

In future, heavy-quark-symmetry relations should provide increasingly reliable mass estimates for $Q_i Q_j \bar{q}_k \bar{q}_l$.

Mass estimates lead us to expect that the $J^P = 1^+$ $\{bb\}[\bar{u}\bar{d}]$, $\{bb\}[\bar{u}\bar{s}]$, and $\{bb\}[\bar{d}\bar{s}]$ states should be exceedingly narrow, decaying only through the charged-current weak interaction

Observation would herald a new form of stable matter; establish the doubly heavy color- $\bar{\mathbf{3}}$ $Q_i Q_j$ as a basic building block.

Unstable $Q_i Q_j \bar{q}_k \bar{q}_l$ tetraquarks with small Q -values will be observable as resonant pairs of heavy-light mesons DD , DB , BB .

Congratulations and thanks
to the (many) pioneers of
The Tetraquark Odyssey
and to All who have brought us
to the Opportunities before us!

Courage to those who will carry on!

Félicitations à Jean-Marc et aux Mousquetaires d'antan,
et un grand merci aux Organisateurs et à l'IP2∞!