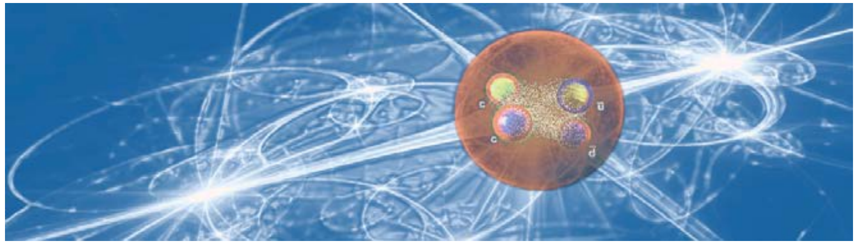
RECENT RESULTS FROM THE LATTICE ON STRONG-INTERACTION-STABLE HEAVY TETRAQUARKS



IP2I WORKSHOP ON DOUBLE CHARM TETRAQUARK AND OTHER EXOTICS

Lyon, November 22-23, 2021

KM, B. Colquhoun, A. Francis, R. Hudspith, R. Lewis

(C)FHLM: PRD102 (2020) 114506 (+earlier, in progress) [Francis, Hudspith, Lewis, Colquhoun +KM]

(J)MP: arXiv:2111.01147 [hep-lat] [Mathur, Padmanath]; PRD99 (2019) 034507 [+Junnarkar]

LMPW: arXiv:2108.10704 [hep-lat]; PRD100 (2019) 014503 [Leskovec, Meinel, Pflaumer, Wagner]

SPECIAL FEATURES OF DOUBLY HEAVY TETRAQUARK CHANNELS

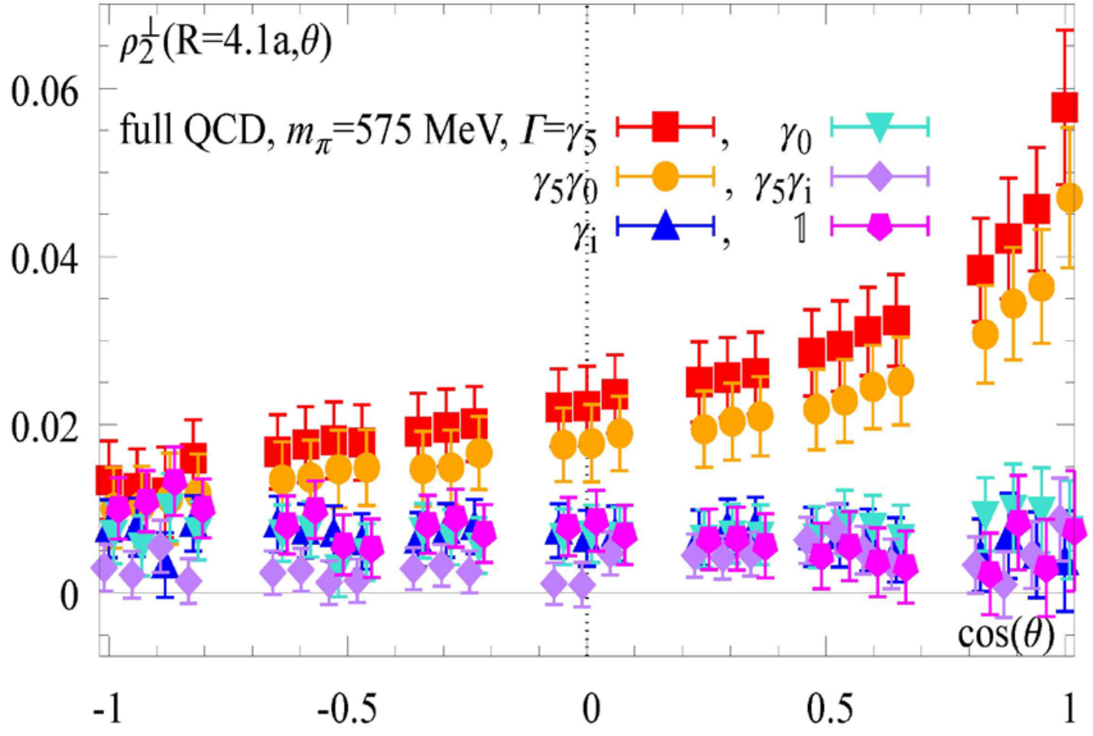
- **Attractive interactions for localized $\overline{QQ'}qq'$ configurations absent in well-separated two-heavy-meson states**
  - Color Coulomb for $\overline{QQ'}$ in 3_c ($\Rightarrow$ qq' in $\bar{3}_c$)
  - Light quark spin-dependent for qq' in "good light diquark" ($J^P=0^+$, $\bar{3}_F$, $\bar{3}_c$) configuration (as per heavy baryon splittings)
- **Attractive long-distance π-exchange in $V_H P_H$, $J^P=1^+$ channels (Manohar, Wise)**

- **Channels favorable to possible doubly heavy tetraquark binding**
  - For Q'=Q: qq' in good light diquark; QQ with $J_{QQ}=1$ (no spatial excitation), 3_c: **overall $J^P=1^+$, $\bar{3}_F$**
  - For Q≠Q': qq' in good light diquark; QQ' with $J_{QQ''}=0$ or 1 (no spatial excitation), 3_c: **overall $J^P=0^+$ or 1^+, $\bar{3}_F$**

A FEW ADDITIONAL DETAILS/OBSERVATIONS

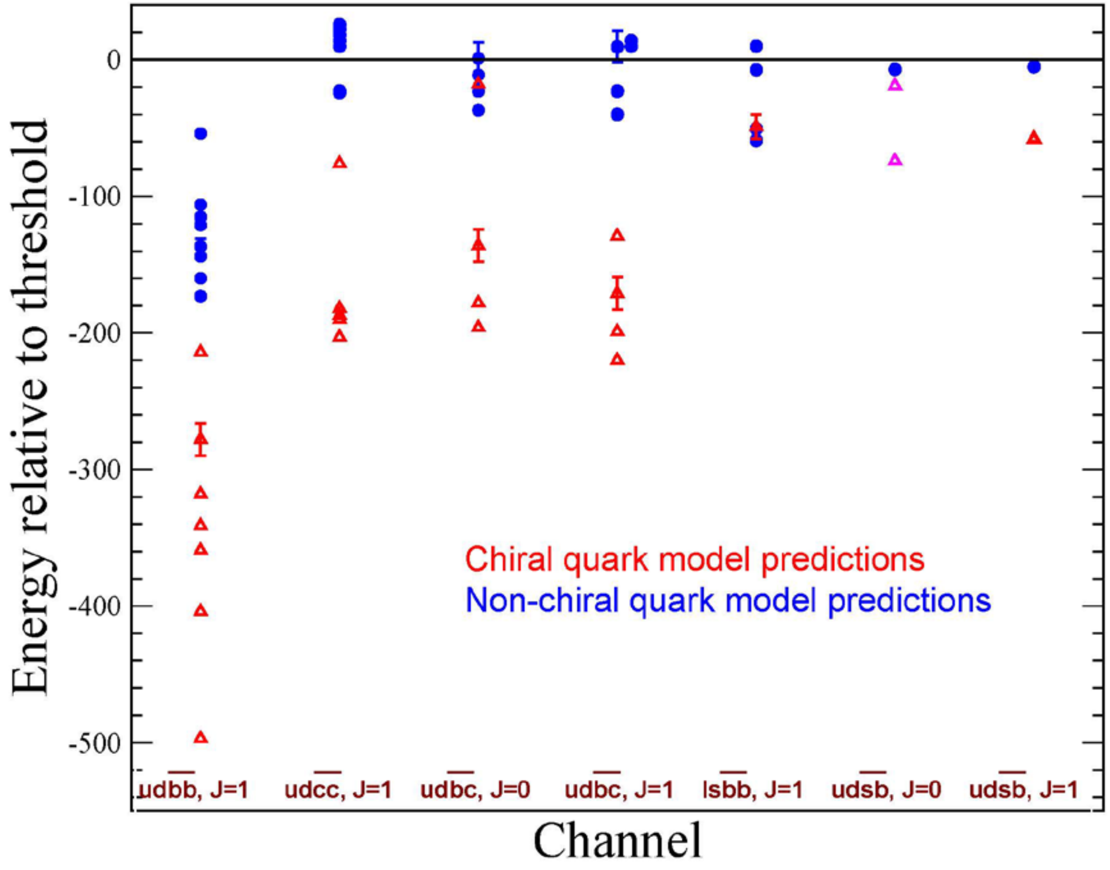
"Reality" of the good light diquark attraction: light-quark spatial correlations from the lattice

arXiv: 2106.09080



- **Colour Coulomb attraction proportional to $\overline{QQ'}$ reduced mass**
  - ➢ greatest for bb, then bc, then cc
  - ➢ bound $J^P=1^+$ doubly heavy QQ tetraquark in QCD for sufficiently heavy Q
- **Heavy baryon splittings⇒ increasing good light diquark attraction with decreasing m_q**
  - ➢ light m_π, short extrapolation to physical m_π important on lattice
  - ➢ potentially relevant for lattice if state weakly bound at physical m_π
- Σ_b-Λ_b > Σ_c-Λ_c⇒ residual $1/m_h$-suppressed heavy-light interaction repulsive

EXTENSIVE PAST INVESTIGATIONS USING QCD-INSPIRED MODELS (since mid-80s; many experts at this workshop)



- Model parameters fixed from fits to ordinary meson, baryon spectrum
- $\Rightarrow$ model qq $\bar{6}_c$, $q\bar{q}$ 8_c interactions (present in multiquark sector) entirely unconstrained
- differing model spin-flavor-color dependences $\Rightarrow$ different tetraquark results (e.g., figure)
  - chiral quark models (flavor dependent, color-independent spin-spin interactions)
  - c.f. non-chiral quark models (OGE-like color-dependent spin-spin interactions)
- **Experimental/lattice tetraquark channel results can rule out one (or both)**

DOUBLY HEAVY TETRAQUARKS ON THE LATTICE

- **Generic correlator (matrix) in a given channel**

$$C_{\mathcal{O}_1\mathcal{O}_2}(p,t) = \sum_x e^{ip\cdot x}\langle \mathcal{O}_1(x,t)\mathcal{O}_2(0,0)^\dagger\rangle\ ,$$
$$= \sum_n \langle 0|\mathcal{O}_1|n\rangle\langle n|\mathcal{O}_2|0\rangle e^{-E_n(p)t}$$

- **$\{O_k\}$: operators with channel quantum numbers**
  e.g. for Q=Q'=b, qq'=ud, $J^P=1^+$: "diquark-antidiquark", "meson-meson" ("B B*"), ...

$$D(x) = (u_a^\alpha(x))^T (C\gamma_5)^{\alpha\beta} q_b^\beta(x) \times \bar{b}_a^\kappa(x)(C\gamma_i)^{\kappa\rho}(\bar{b}_b^\rho(x))^T$$

$$M(x) = \bar{b}_a^\alpha(x)\gamma_5^{\alpha\beta} u_a^\beta(x)\ \bar{b}_b^\kappa(x)\gamma_i^{\kappa\rho} d_b^\rho(x) - \bar{b}_a^\alpha(x)\gamma_5^{\alpha\beta} d_a^\beta(x)\ \bar{b}_b^\kappa(x)\gamma_i^{\kappa\rho} u_b^\rho(x)$$

- **Zero-momentum projection, C(t) ~ exp($-M_0$ t) at large Euclidean time, t**

Generalities/practicalities

- Exponential signal decay ⟷ growth of noise at large t
- Effective mass: log[C(t)/C(t+a)] → (aM_0) at large t
- **Goal: ground-state effective mass plateaus before signal overwhelmed by noise**
- Overlaps operator-dependent, hence want operator(s) with
  - ➢ good ground-state overlap
  - ➢ reduced excited-state overlap
- **Smearing of quark fields (e.g. at source) improves ground state overlap** (our results: Coulomb gauge-fixed wall sources, local (point) or "box" sinks ⟷ C(t) from wall-local or wall-box propagators)

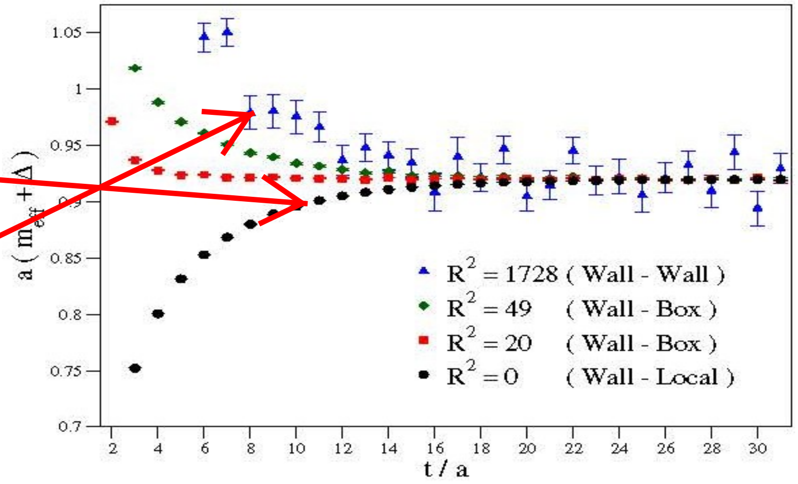
BOX SINK IMPROVEMENT

- **Wall-local propagators** (sink location (x,t)): $S_{WL}(x,t)$
  - Typically: approach ground state plateaus from below (negative excited-state contamination)
  - Late t plateaus in tetraquark channels

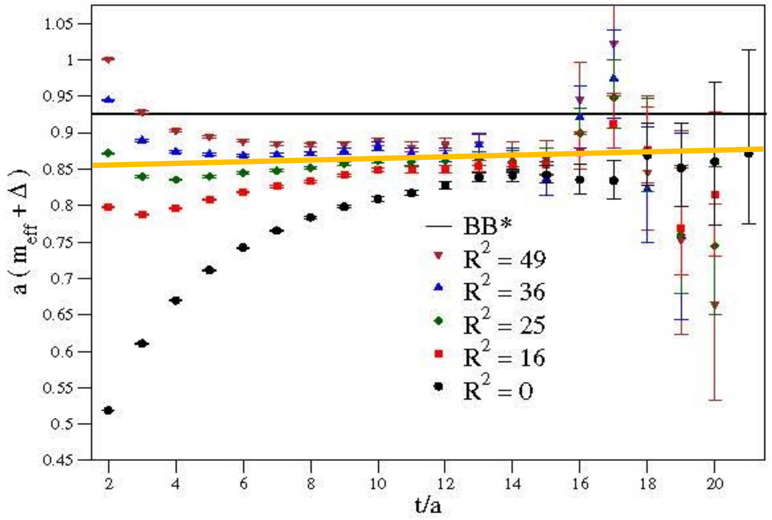
- **Wall-wall propagators**: $S_{WW}(t)=\Sigma_x S_{WL}(x,t)$
  - Advantage: ground state plateaus for diagonal correlator elements approached from above (positive excited state contamination)
  - Disadvantage: very noisy

- **Wall-box propagators**: intermediate between WL, WW: $S^R_{WB}(x,t)=\frac{1}{N}\sum_{\{r\}<R} S_{WL}(x+r,t)$. Expect
  - Intermediate behavior/reduced excited state contamination
  - Optimal improvement for R≃ground state size

- **E.g., the B_c channel**



- **E.g., the $ud\overline{bb}$ I=0, $J^P=1^+$ channel**

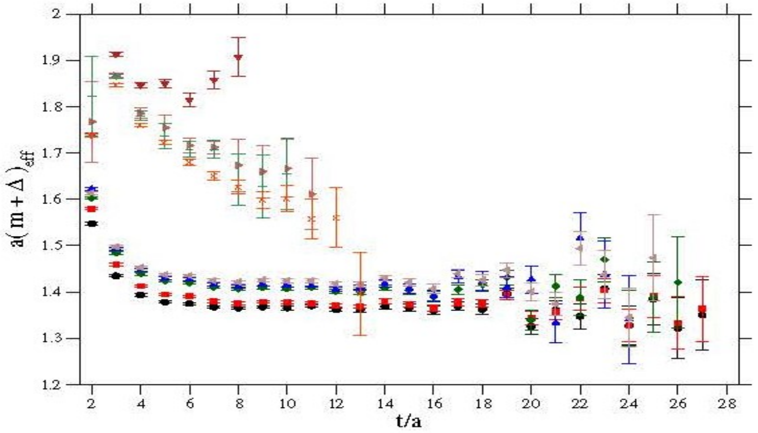


NEAR-PHYSICAL-POINT CHANNEL-SCAN FOR DEEPLY BOUND TETRAQUARK STATES

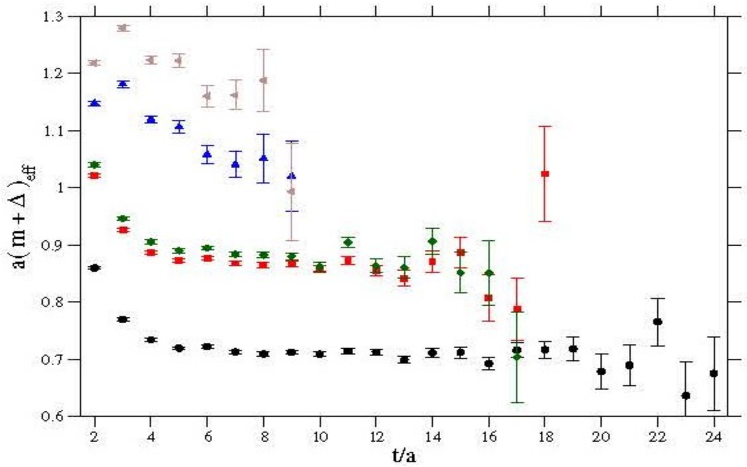
- CFHLM: PRD 102 (2020) 114506 [arXiv:2006.14294]
- Ensemble details
  - n_f=2+1, Wilson clover, Iwasaki gauge
  - NRQCD for b, relativistic heavy quark action for c
  - 48^3 x 64, 1/a=2.194(10) GeV (a~0.09 fm); larger volume extension of PACS-CS 32^3 x 64 ensembles, same parameters
  - $m_\pi = 192$ MeV, $m_\pi L = 4.2$
  - 122 configurations, 8 source locations per configuration
- **Conclusions: tetraquarks bound by more than ~10-20 MeV ruled out in all channels except for $J^P=1^+$, $\overline{3}_F$ multiplet (I=0 $ud\overline{bb}$ and I=1/2 $ls\overline{bb}$ states)**
- more on the doubly bottom states later; first the other channels

THE I=0 $ud\overline{bc}$ $J^P=0^+$ and 1^+ CHANNELS

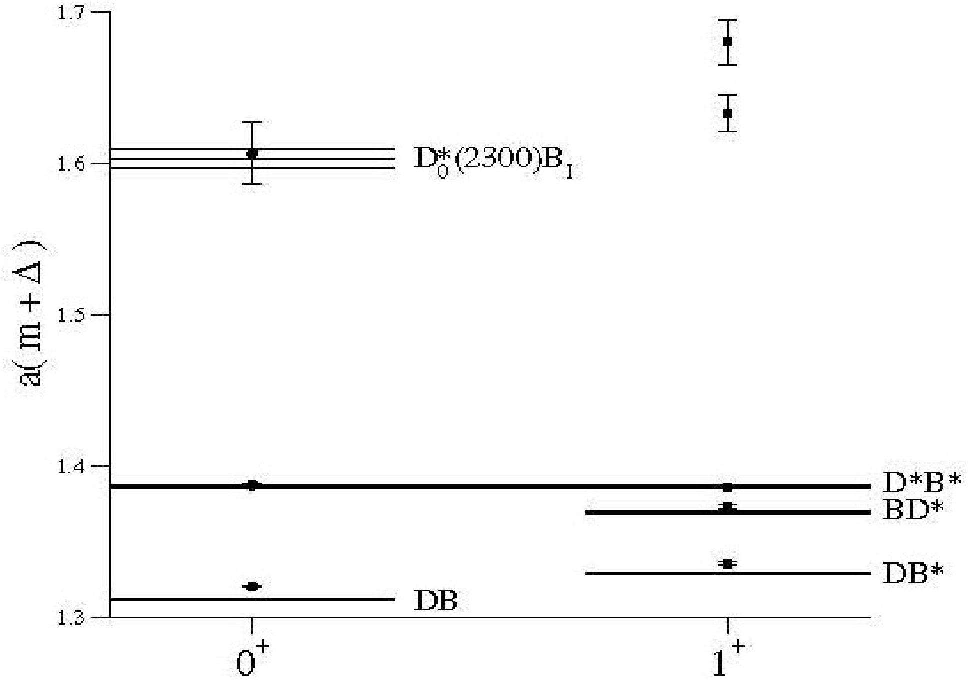
- $J^P=0^+$ effective mass



- $J^P=1^+$ effective mass



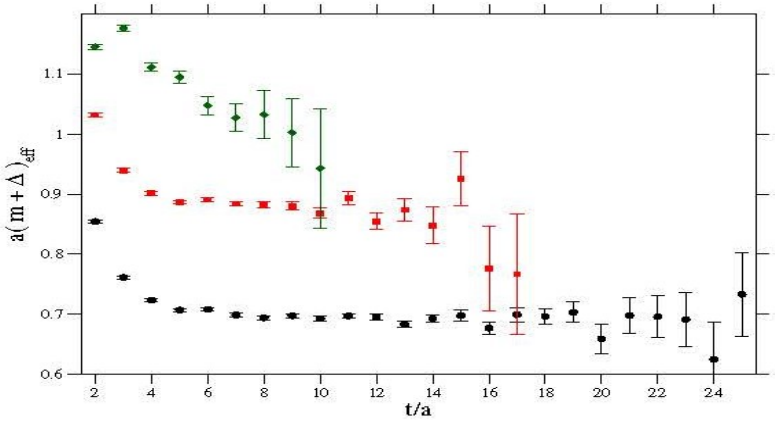
Spectra



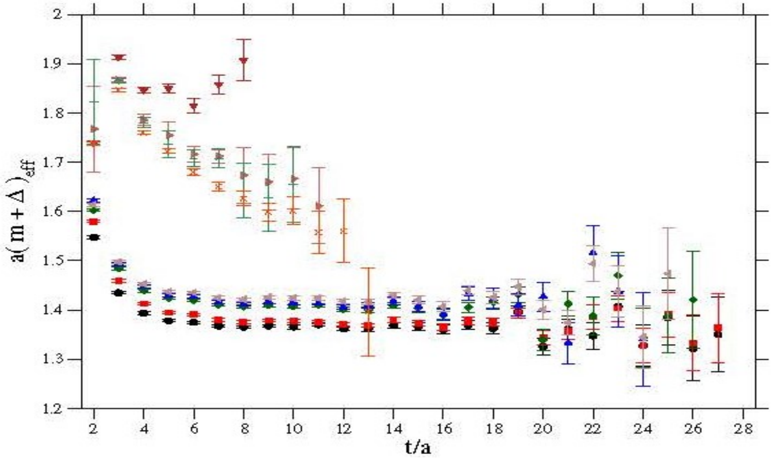
- ➢ $J^P=0^+$, 1^+ both unbound
- ➢ c.f. LMPW (preliminary): both unbound
- ➢ c.f. MP (preliminary) : $J^P=1^+$ BE $\simeq$ 20 MeV

THE I=1/2 $ls\overline{bc}$ $J^P=0^+$ and 1^+ CHANNELS

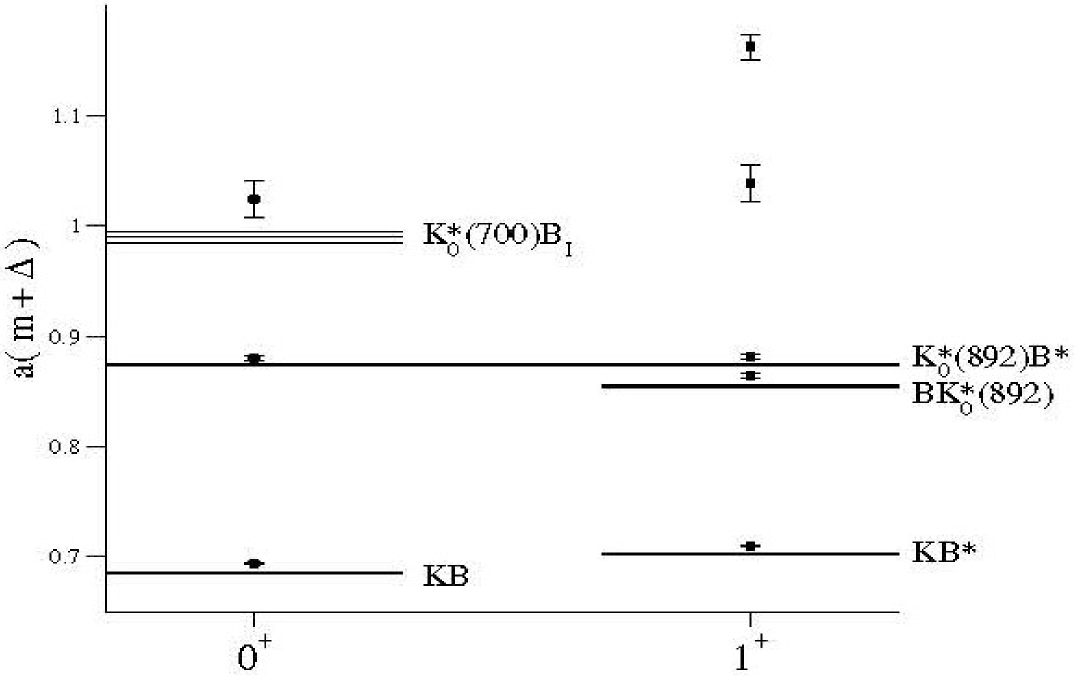
- $J^P=0^+$ effective mass



- $J^P=1^+$ effective mass



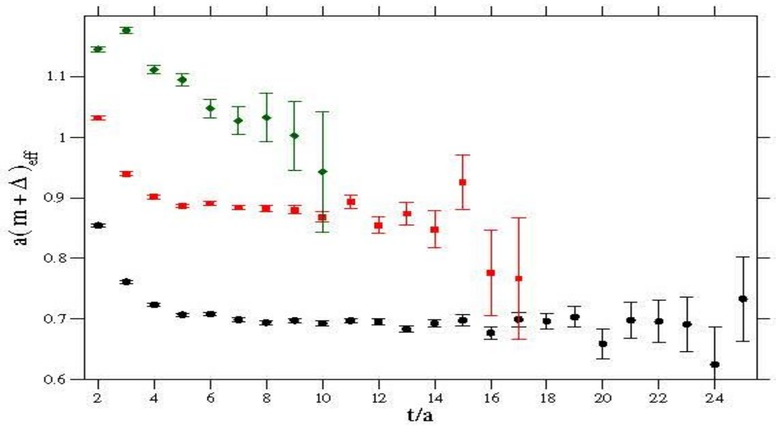
Spectra



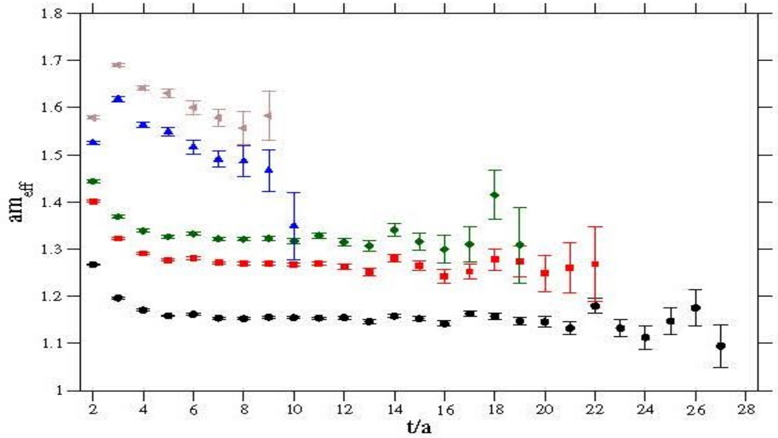
- ➢ $J^P=0^+$, 1^+ both unbound
- ➢ c.f. MP (preliminary): $J^P=1^+$ BE $\simeq 10$ MeV

THE I=0 ud$\overline{sb}$ $J^P=0^+$ and 1^+ CHANNELS

- $J^P=0^+$ effective mass



- $J^P=1^+$ effective mass



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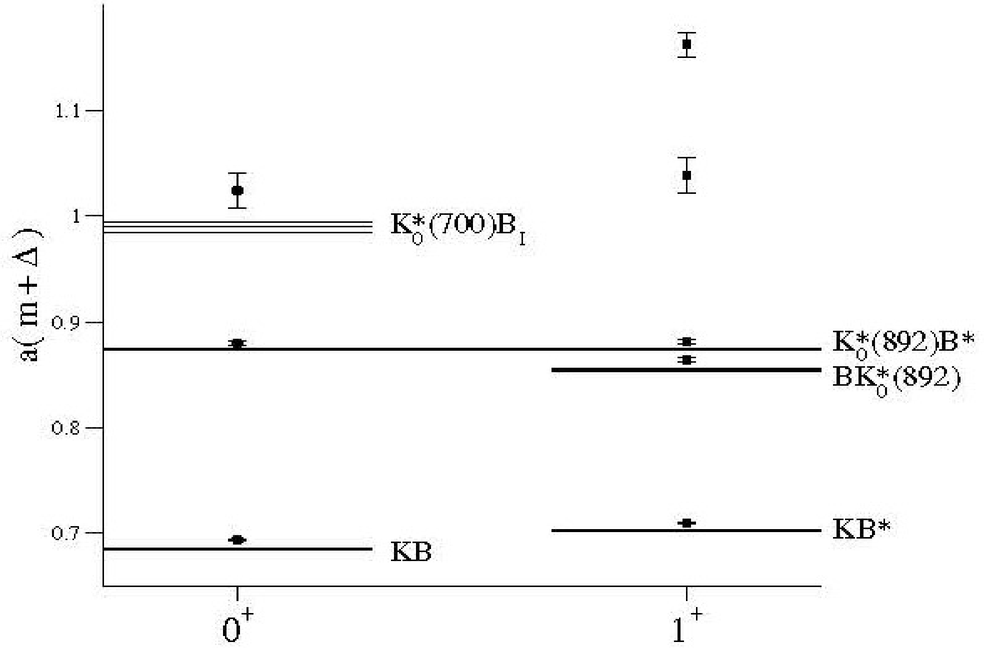
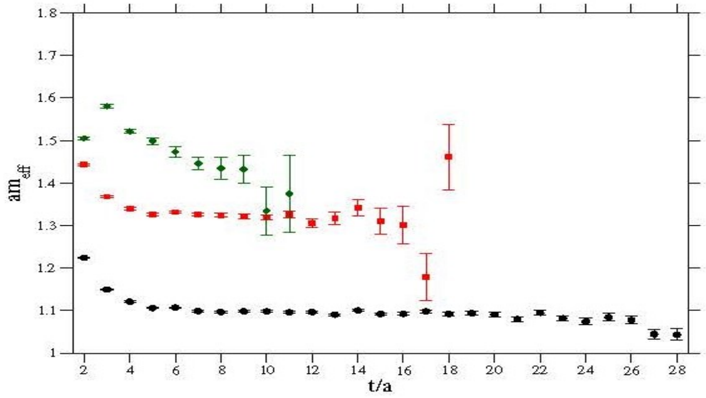


➢ $J^P=0^+$, 1^+ both unbound

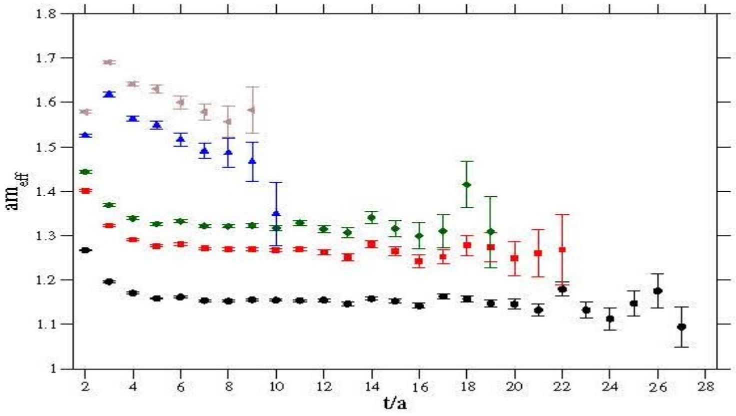
Original motivation: possible $SU(3)_F$ ud$\overline{sb}$ partner of D0's putative us$\overline{db}$ $X(5568)$

THE I=0 ud$\overline{sc}$ J^P=0^+ and 1^+ CHANNELS

- J^P=0^+ effective mass



- J^P=1^+ effective mass



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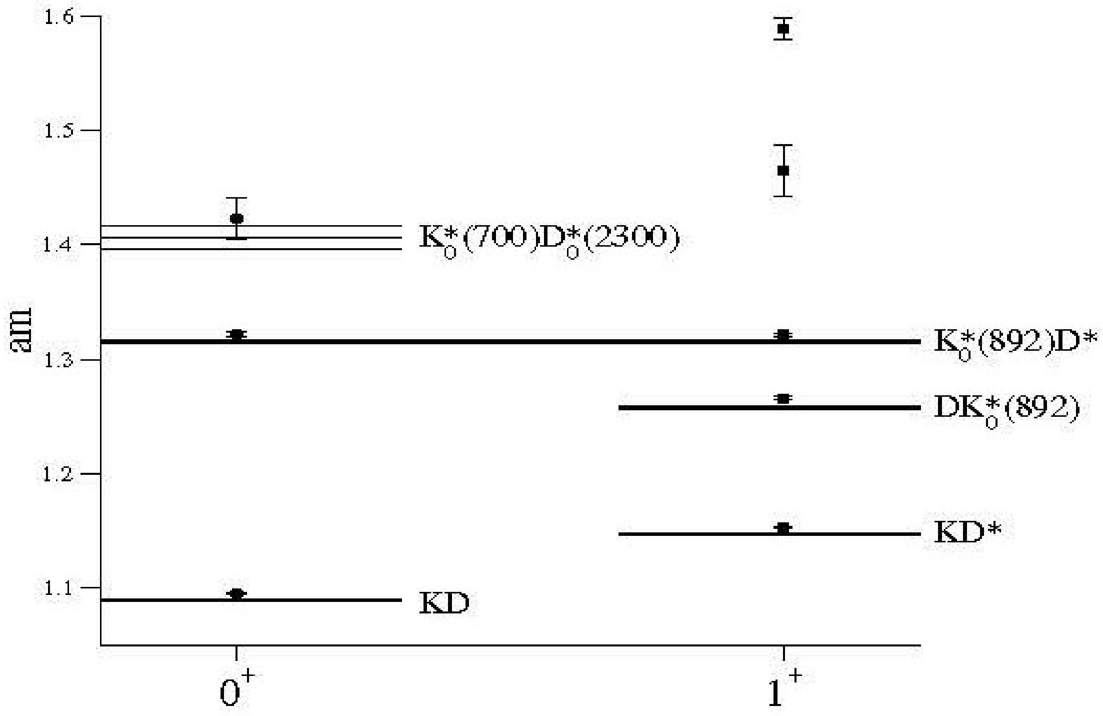
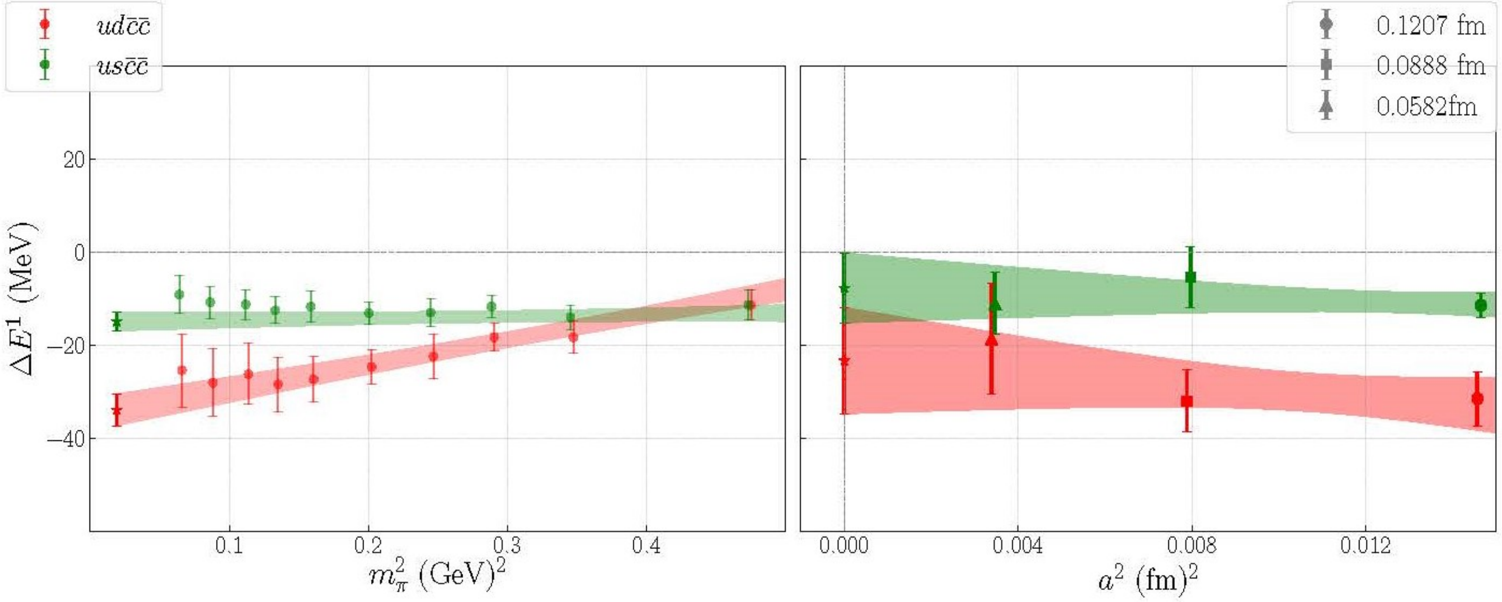


➢ J^P=0^+, 1^+ both unbound

THE $J^P=1^+$, I=0 $ud\bar{c}\bar{c}$ CHANNEL (other lattice groups)

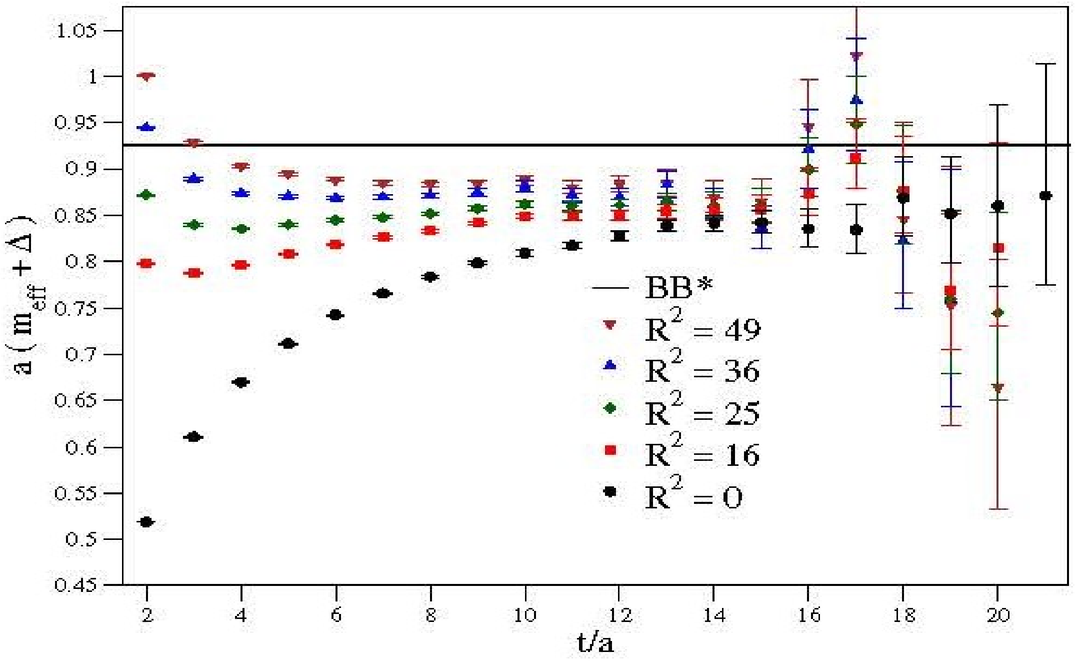
- **The LHCb experimental T_{cc} discovery (and motivation for this workshop!)**
  - very weakly bound (essentially at threshold)
  - consistent with favored I=0, $J^P=1^+$
- **HadSpec 2017 lattice [JHEP 11 (2017) 033; arXiv:1709.01417]:** no evidence for bound state or narrow resonance at m_π=391 MeV
- **JMP [PRD99 (2019) 034507]:** 23(11) MeV binding (also 8(8) MeV for I=1/2 $ls\bar{c}\bar{c}$) with caution re possible residual FV effects



THE DOUBLY BOTTOM $J^P=1^+$, $\bar{3}_F$ CHANNELS

- CFHLM: PRD 102 (2020) 114506+in progress: updates, improves wall-local study of PRL118 (2017) 142001)
  - PACS-CS n_f=2+1, Wilson clover, Iwasaki gauge, a=0.09 fm, 32^3x48 of earlier PRL, plus new larger-volume 48^3x64 ensembles with $m_\pi < 200$ MeV
  - Box-sink construction dramatically improves the ground state signal c.f. earlier PRL results
- Other groups
  - (J)MP: n_f=2+1+1 HISQ, wall-local, a-dependent minimum m_π: 153, 345, 545 MeV for a~0.12, 0.09, 0.06 fm (near physical point for coarsest)
  - LMPW: RBC/UKQCD n_f=2+1 DWF, Iwasaki gauge ensembles, 2 with physical m_π; smeared source, local sink,; 3 local operators, 2 non-local "meson-meson" for improved overlap with threshold 2-meson state

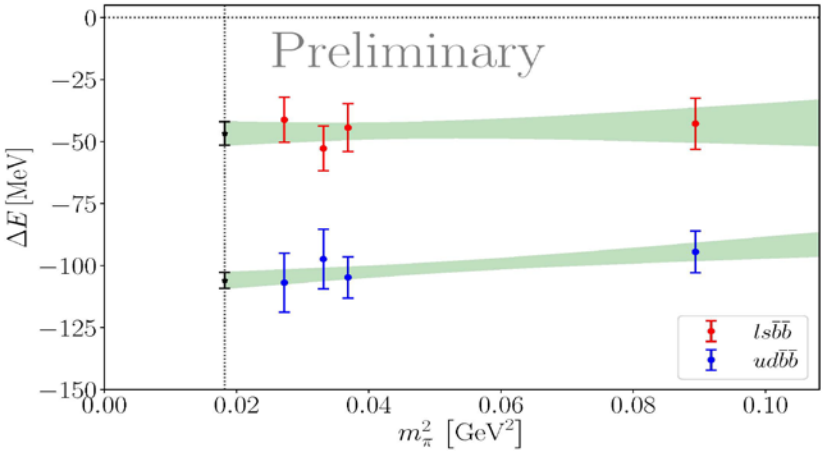
- Example of box-sink improvement for $ud\bar{b}\bar{b}$ I=0, $J^P=1^+$ channel, m_π =192 MeV



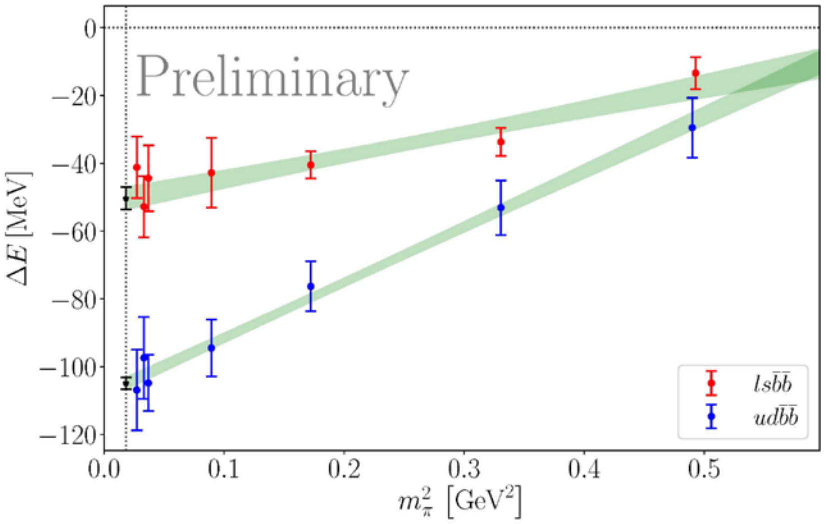
- Q: wall-local/late-plateau an issue for (J)MP?

CFHLM UPDATED $J^P=1^+$, I=0 ud$\bar{b}\bar{b}$, I=1/2 ls$\bar{b}\bar{b}$ BINDING ENERGIES

- **Fits with m_π<300 MeV**



- **Fits with m_π<702 MeV**



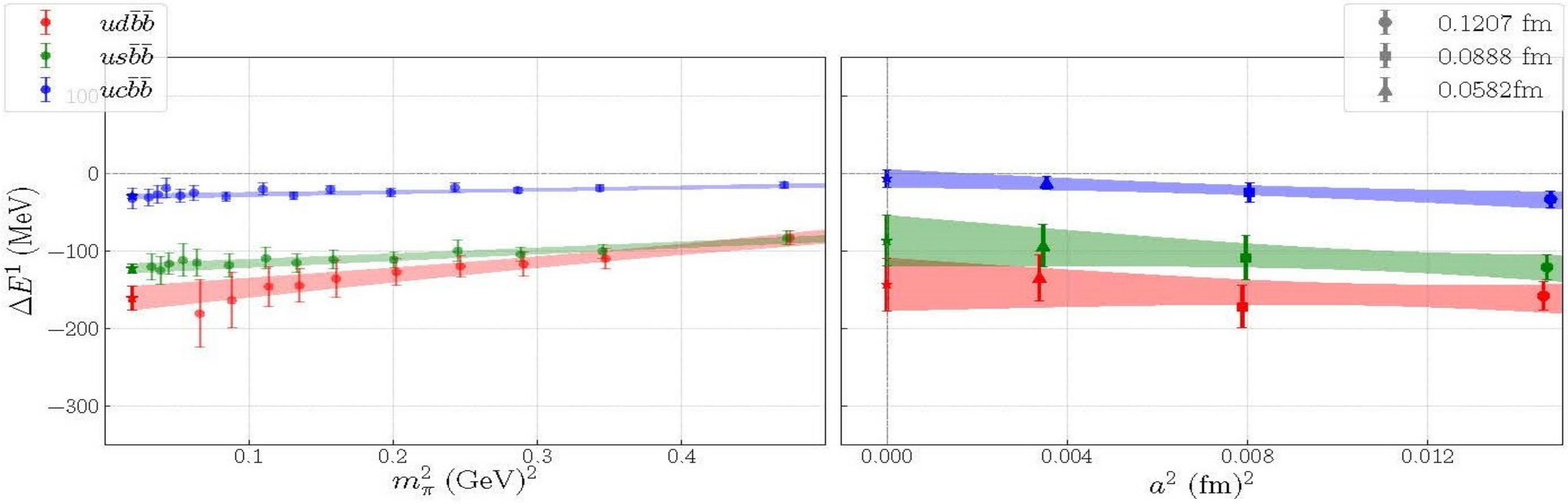
PRELIMINARY UPDATES

Errors: statistical, optimal box-sink size, R, choice only at present (i.e., preliminary)

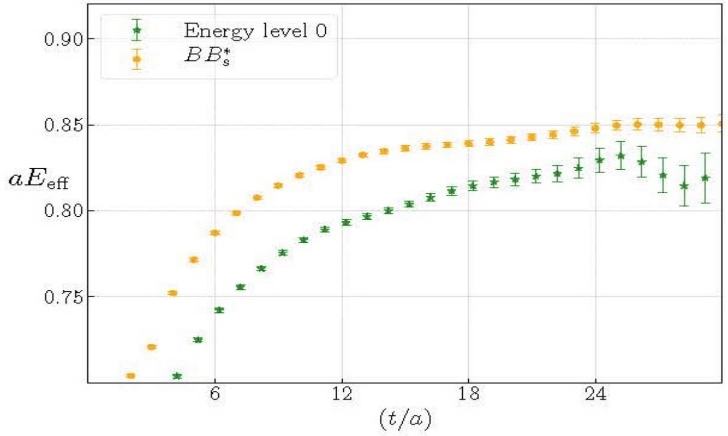
- **From m_π<300 MeV fits**
  - I=0 ud$\bar{b}\bar{b}$: 109(5) MeV
  - I=1/2 ls$\bar{b}\bar{b}$: 48(7) MeV

- **Extending fits to m_π<702 MeV**
  - I=0 ud$\bar{b}\bar{b}$: 109 → 108 MeV
  - I=1/2 ls$\bar{b}\bar{b}$: 48 → 52 MeV

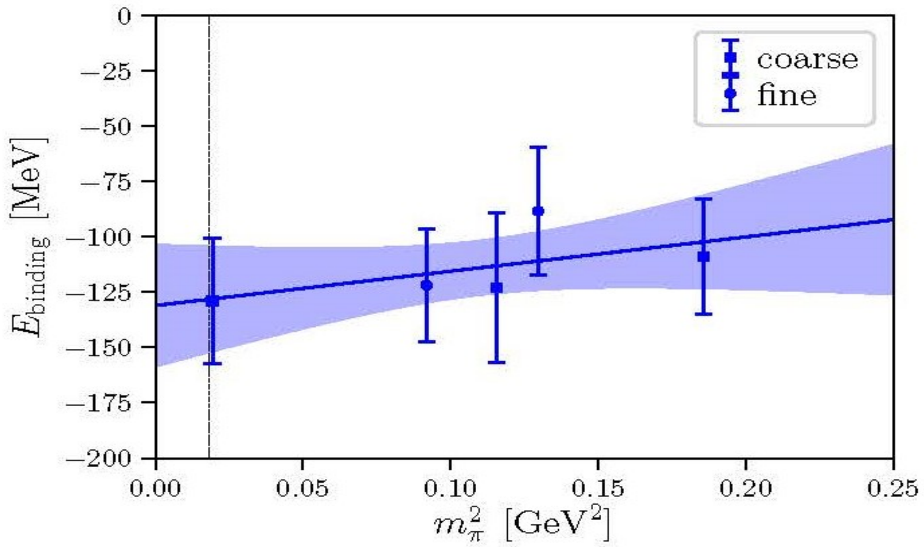
JMP $J^P=1^+$, I=0 $ud\bar{b}\bar{b}$, I=1/2 $ls\bar{b}\bar{b}$ BINDING ENERGIES



- **Binding energies**
  - I=0 $ud\bar{b}\bar{b}$: 143(34) MeV
  - I=1/2 $ls\bar{b}\bar{b}$: 87(32) MeV
- **Effective mass plateaus?** E.g. m_π=575 MeV $ls\bar{b}\bar{b}$ example



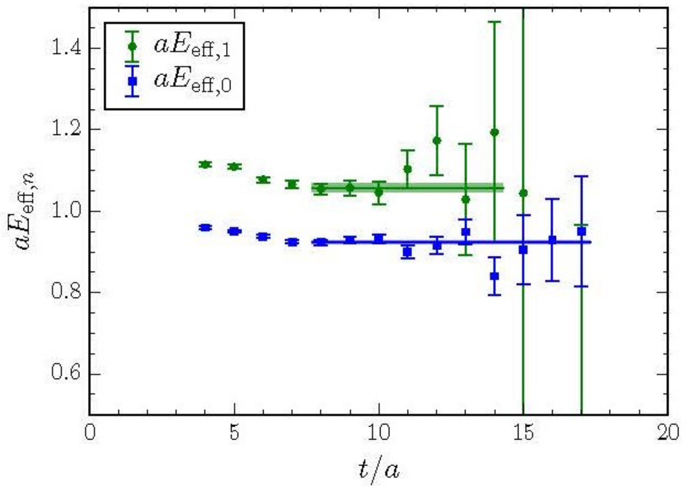
LMPW $J^P=1^+$, I=0 ud$\overline{bb}$, I=1/2 ls$\overline{bb}$ BINDING ENERGIES



I=0 ud$\overline{bb}$ physical point extrapolation

- **Binding energies**
  - I=0 ud$\overline{bb}$: 128(26) MeV (PRD100 (2019) 014503)
  - I=1/2 ls$\overline{bb}$: ~80 MeV, ~25 MeV uncertainty (preliminary Lattice 2021 report, arXiv:2108.10704)

- **I=0 ud$\overline{bb}$ m_π=340 MeV effective mass e.g.**



CONCLUSIONS/EXPECTATIONS/SPECULATIONS (1)

- **Lattice tetraquark binding energy summary**

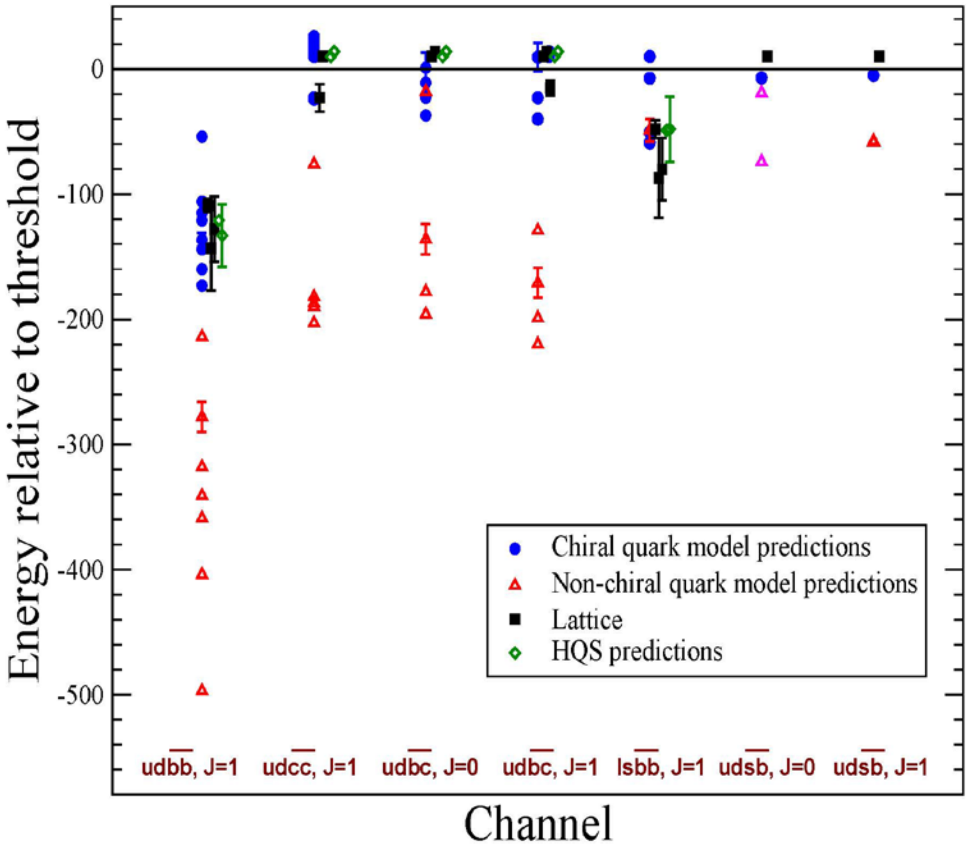
  - Clear evidence for strong-interaction-stable $J^P=1^+$, I=0 ud$\overline{bb}$ state: bound by >100 MeV, hence with only weak decays (experimental detection challenge)

  - Clear evidence for strong-interaction-stable $J^P=1^+$, I=1/2 ls$\overline{bb}$
    - **possibility of < 45 MeV binding, ~100% BF EM decay to $BB_s\gamma$** (CFHLM lattice, Eichten-Quigg/Braaten-He-Mohapatra HQS results)
    - binding > 45 MeV however also possible (JMP, preliminary LMPW): weak decays only, multiple modes, small BFs

  - No states with several x 10 MeV binding in other channels in CFHLM study

CONCLUSIONS/EXPECTATIONS/SPECULATIONS (2)

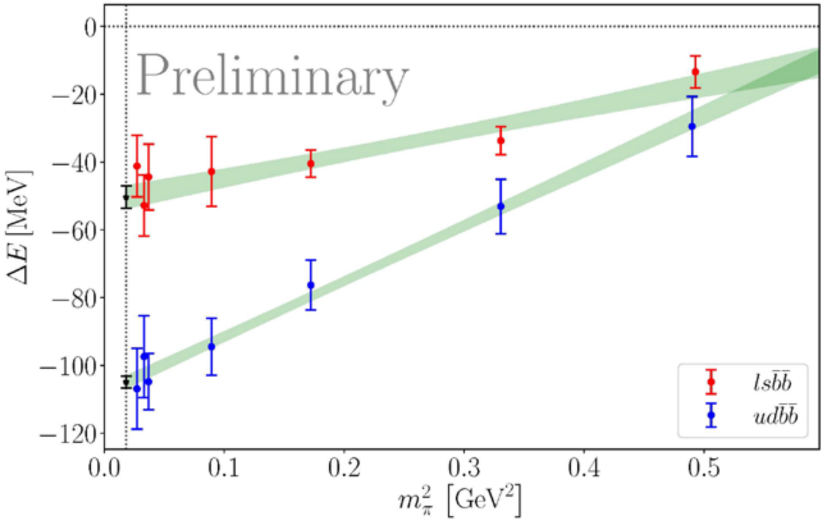
- **Lattice/LHCb T_{cc}: tests of model predictions**
    - ➢ Chiral quark models: predictions incompatible with lattice (and HQS) results/overbind T_{cc} c.f. LHCb
    - ➢ Non-chiral quark models (OGE-like spin-spin interactions): compatible with weakly bound T_{cc}, predictions in other channels in reasonable agreement with lattice (and HQS)



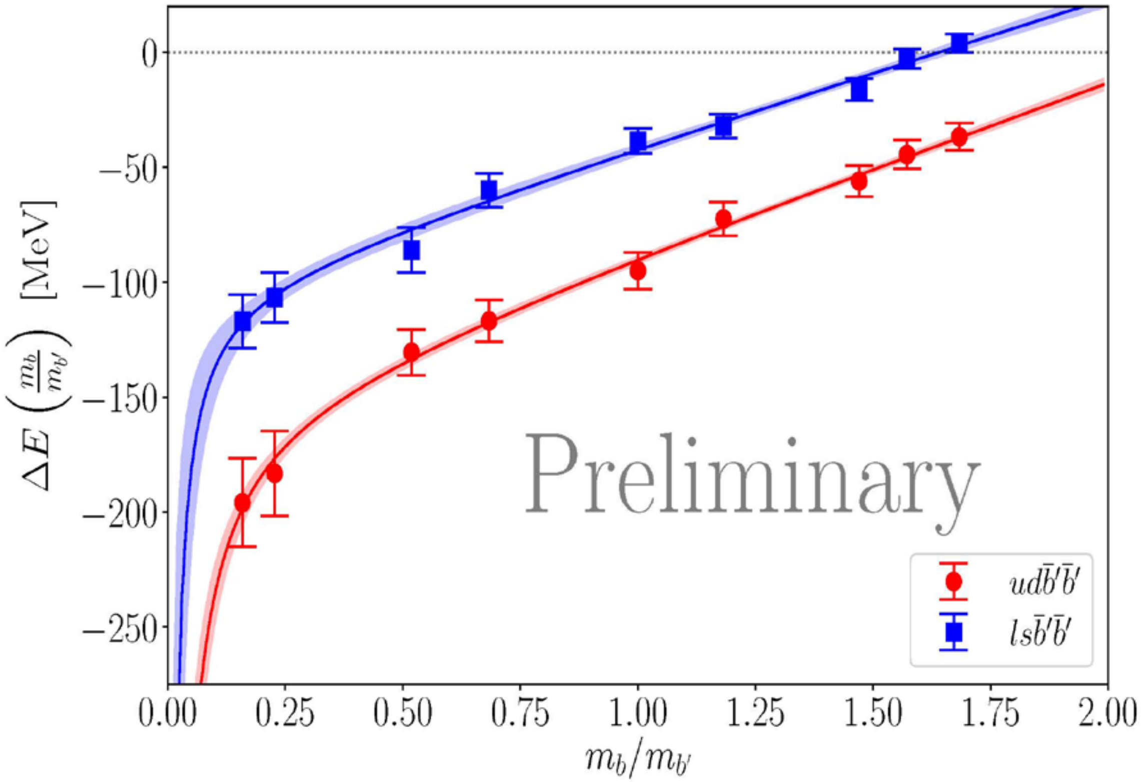
- **Conclusion: chiral quark model picture of effective low-energy interactions (in current forms) ruled out by multi-quark sector tests**

CONCLUSIONS/EXPECTATIONS/SPECULATIONS (3)

- **Key physical mechanisms at play in doubly heavy tetraquark binding**
  - Heavy-heavy color Coulomb attraction: as expected + evidence from variable b' mass study (figure)
  - Spin-dependent good-light-diquark attraction (increasing binding with decreasing light-quark mass)



- **Variable b mass study: the heavy-heavy color Coulomb (m_π=299 MeV)**



CONCLUSIONS/EXPECTATIONS/SPECULATIONS (4)

- **A possible I=0, $J^P=1^+$ ud$\overline{bc}$ state?**
    - very weak I=0, $J^P=1^+$ ud$\overline{cc}$, T_{cc} binding suggests molecular-type component, importance of long-distance π-exchange force
    - same π-exchange force operative in analogous I=0, $J^P=1^+$ ud$\overline{bc}$ channel
    - in addition, short-range attraction likely stronger for ud$\overline{bc}$ than ud$\overline{cc}$ (more attractive color Coulomb, similar good-light-diquark attraction)
    - CFHLM, LMPW, MP lattice studies all concur: no deep binding
    - suggests likelihood of I=0, $J^P=1^+$ T_{bc} analogue of T_{cc} bound by < 45 MeV, hence EM decay, likely to BDγ (unless weakly bound I=0, $J^P=0^+$ tetraquark also exists)
    - **experimental search for correlated BD feasible?**

Thanks for your attention

in memory of Gabriel Karl and Simon Eidelman

BACKUP SLIDES (1)

EXPECTATIONS FROM HEAVY BARYON SPLITTINGS

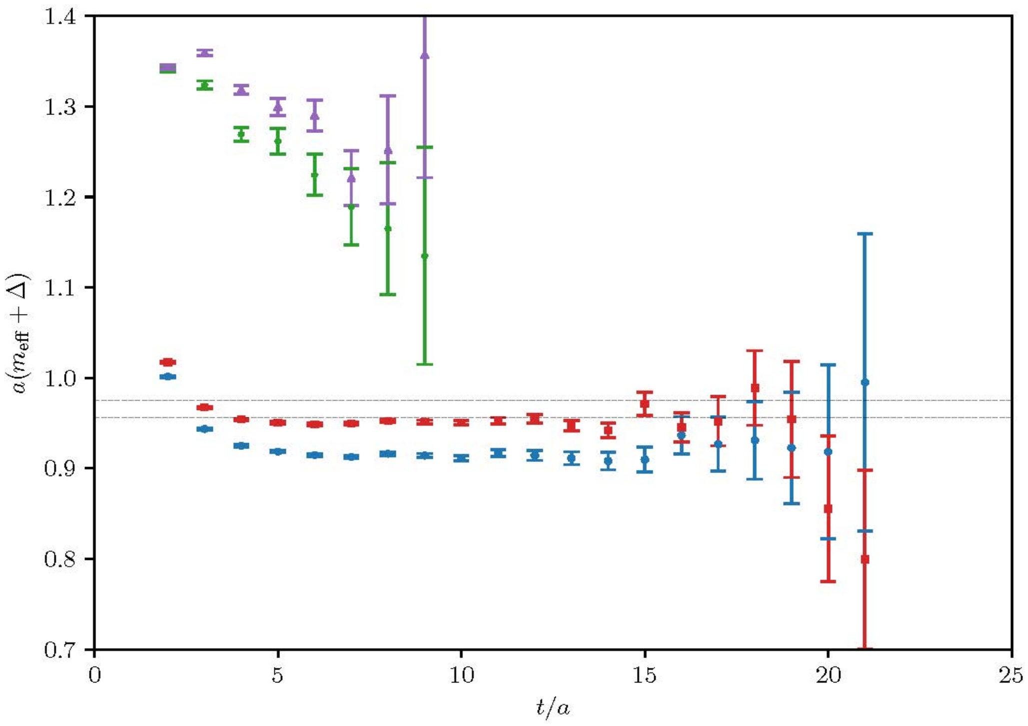
- **Observed splittings**
  - $\Sigma_b - \Lambda_b = 194$ MeV
  - $\Sigma_c - \Lambda_c = 167$ MeV
  - $\Xi_b' - \Xi_b = 142$ MeV
  - $\Xi_c' - \Xi_c = 109$ MeV

- good ud, ℓs diquark attraction relative to corresponding spin averages: ~145, 105 MeV ⇒ *increased* attraction with *decreased* m_q
- h=c vs. h=b: residual ($\propto 1/m_h$) light-heavy interactions eat into good diquark attraction with decreasing m_h

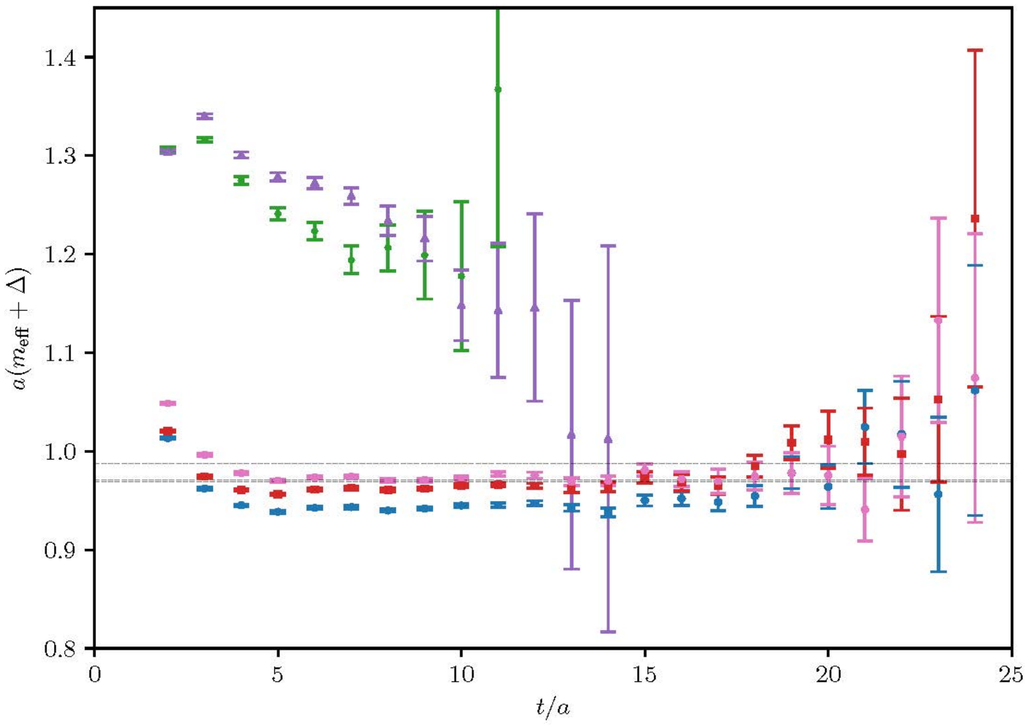
BACKUP SLIDES (2a)

- **CFHLM effective mass plot examples**
  - **PACS-CS E5 ensemble (m_π=299 MeV)**

I=0, $J^P=1^+$ $ud\overline{bb}$



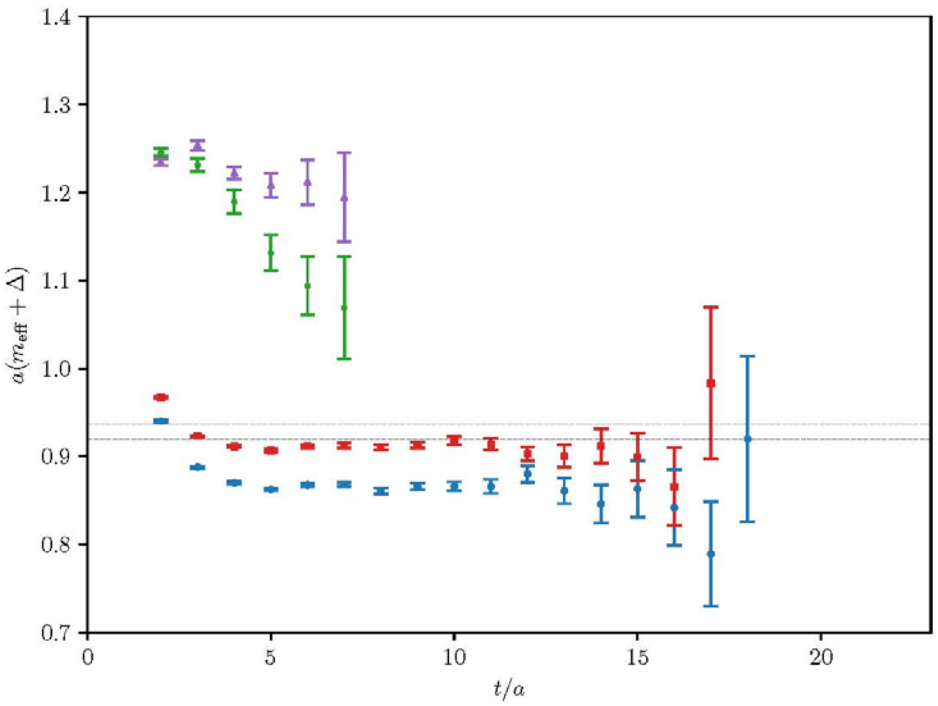
I=0, $J^P=1^+$ $ls\overline{bb}$



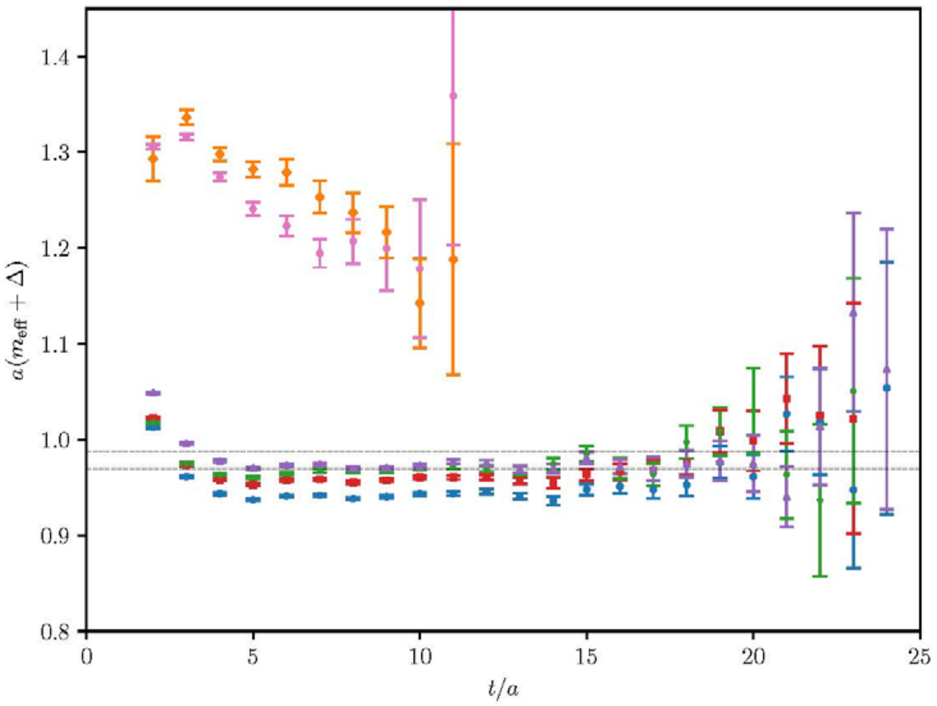
BACKUP SLIDES (2b)

- **CFHLM effective mass plot examples**
  - ➢ **new 48^3 x 64 κ_l=0.13779 ensemble (m_π=165 MeV)**

I=0, $J^P=1^+$ $ud\overline{bb}$



I=0, $J^P=1^+$ $ls\overline{bb}$



BACKUP SLIDES (3)

- **LMPW 2019, I=0, $J^P=1^+$ ud$\overline{bb}$: possible residual fit window dependence?**
  - ➢ m_π=340 MeV multi-exponential fits,, variable windows, variable # of exponentials