Introduction
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$M^+ \neq m^+$
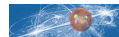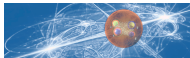0

MMmm
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BO
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$M = m$
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Conclusion
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Doubly Heavy Tetraquarks
Lessons from Atomic Physics

Jean-Marc Richard

Institut de Physique Des 2 Infinis de Lyon
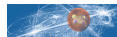Université de Lyon–IN2P3-CNRS, France

IP2I, Tcc Workshop, November 23, 2021

Table of contents

Based on recent or old work with J. Vijande, A. Valcarce, Cafer Ay, Hyam Rubinstein, S. Zouzou, C. Gignoux, B. Silvestre-Brac, M. Genovese, Fl. Stancu, J.-L. Ballot, ...

$QQ\bar{q}\bar{q}$ Cold case

- History and prehistory $QQ\bar{q}\bar{q}$

- When did it start?

- First quark model: a CERN preprint CERN-TH-3101 of 1981 (Ader, R., Taxil) published in Phys.Rev.D 25 (1982) 2370

- Confirmed and improved by L. Heller et al., M. Rosina et al., N. Barnea et al., Valcarce et al., etc.

- Other approaches: molecular (Manohar et al., G. Karl et al., N. Törnqvist, S.-L. Zhu et al., dots), QCD sum rules (M. Nielsen et al., ..., lattice (C. Michael, A.M. Green, G. Bali, P. Bicudo, ...)

- Before 1981?

- Work on mass dependence of $M^+ M^+ m^- m^-$ (Adamowski et al. 1971-72, refined by Fröhlich, Graf, Seifert, R., 1993)
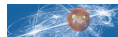
BINDING ENERGY OF THE BIEXCITONS

J. Adamowski and S. Bednarek

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and

M. Suffczynski

Institute of Physics, Polish Academy of Sciences, Warsaw

BINDING ENERGY OF FOUR-PARTICLE COMPLEXES IN SEMICONDUCTORS

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Received 17 July 1972

Proof of Stability of the Hydrogen Molecule

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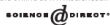
J. Fröhlich, G.-M. Graf, and M. Seifert

Theoretical Physics, Eidgenössische Technische Hochschule Zürich-Hönggerberg, Zürich, Switzerland

(Received 24 May 1993)

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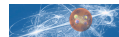
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Physics Reports 413 (2005) 1–90

www.elsevier.com/locate/physrep

Stability of few-charge systems in quantum mechanics

E.A.G. Armour[a], J.-M. Richard[b,*], K. Varga[c,d]

Introduction
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$M^+ \neq m^+$
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MMmm
○

BO
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$M = m$
○○○○

Conclusion
○

Symmetry breaking

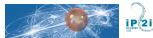- Symmetry breaking lowers the ground state

$$H = H_0 + H_1 = -\frac{\mathrm{d}^2}{\mathrm{d}x^2} + x^2 + \lambda\, x \qquad \Longrightarrow \qquad E(\lambda) = 1 - \frac{\lambda^2}{4} < E(0)\,,$$

- More generally

$$H = H_0(\text{even}) + H_1(\text{odd}) \qquad \Longrightarrow \qquad E(H) < E(H_0)$$

for parity, charge conjugation, permutation, etc.

- But competition 4-body vs. threshold,
  and often, the threshold benefits more!

Breaking particle identity

- Assume $\mu\mu\bar{\mu}\bar{\mu}$ is stable for some flavor independent interaction V
- Break particle identity in both sectors
- $Mm\bar{M}\bar{m}$ becomes unstable if $M/m \nearrow$
- QED
  - $\mu^+\mu^+\mu^-\mu^-$ stable
  - $M^+m^+M^-m^-$ unstable for $M/m \gtrsim 2.2$ (Bressanini, Varga)
- Quark model with central forces
  - $q'q'\bar{q}'\bar{q}'$ unstable
  - $Qq\bar{Q}\bar{q}$ more and more unstable, vs. $Q\bar{Q} + q\bar{q}$
- However, some metastability can be envisaged, see
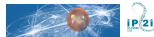 - hydrogen-antihydrogen
  - *XYZ*
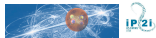
Breaking charge conjugation

- $(M^+ M^+ m^- m^-)$ vs. $(\mu^+ \mu^+ \mu^- \mu^-)$ with $2\,\mu^{-1} = M^{-1} + m^{-1}$
- Same threshold
- The decomposition $H = H_{\text{even}} + H_{\text{odd}}$

$$\frac{\boldsymbol{p}_1^2}{2M} + \frac{\boldsymbol{p}_2^2}{2M} + \frac{\boldsymbol{p}_3^2}{2m} + \frac{\boldsymbol{p}_4^2}{2m} + V =$$
$$\left[\sum \frac{\boldsymbol{p}_i^2}{2\mu} + V \right] + \left(\frac{1}{4M} - \frac{1}{4m} \right) [\boldsymbol{p}_1^2 + \boldsymbol{p}_2^2 - \boldsymbol{p}_3^2 - \boldsymbol{p}_4^2]$$

- Implies $E(H) < E(H_{\text{even}})$
- This explains why H_2 is more stable than Ps_2.
- Same reasoning holds for $QQ\bar{q}\bar{q}$ in a central interaction:
    - It starts unstable for $M = m$
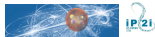   - It becomes stable if M/m large enough

The Born-Oppenheimer limit

- Eichen-Quigg rule, following Lipkin, Nussinov, . . .
- Based on heavy-diquark–heavy-antiquark symmetry
- Analogies of $Q\bar{q}$, QQq, Qqq and $QQ\bar{q}\bar{q}$

$$(QQ\bar{q}\bar{q}) \overset{?}{=} (QQq) + (Qqq) - (Q\bar{q})$$

- Exact at $M/m \to \infty$
- Works rather well for finite M/m
- Can be understood in the Born-Oppenheimer approach

The Born-Oppenheimer limit

- Already for QQq baryons, the Born-Oppenheimer method is very efficient. See, e.g. Fleck & R. (1989)
- $QQ\bar{q}\bar{q}$ for large M dominated by the $\bar{3}3$ color configuration, and one can compare the BO potentials
- BO approach: exact at $R = 0$, but V_{BO} similar for ($QQ\bar{q}\bar{q}$) and (QQq) [shifted here by $(Qqq) - (Q\bar{q})$]
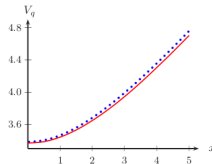
- In atomic physics differences between H_2^+ BO potential vs. H_2
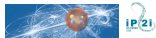
The equal-mass case

- Interesting history
- 1945: Wheeler predicted the existence of $Ps_2 = e^+ e^+ e^- e^-$ as stable except for internal annihilation
- 1946 Øre (at Yale) concluded that it is unlikely
- 1947 Hylleraas and Øre demonstrated the stability
- Refined in more advanced variational calculations
- Erroneous criticism (Sharma, Phys. Rev. 171 (1986) 36)

> than the value obtained by Ore. There are four main reasons why our value differs so much from the value calculated by Ore. First, the Hamiltonian of the system was not transformed properly in Ref. 6 in order to eliminate the kinetic energy of the center of mass.
> Second, the kinetic energy of the electron

- The wf of Øre et al. is translation invariant!
- 2007 indirect experimental evidence for Ps_2
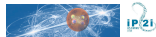
The equal-mass case

- Conflicting results also for $QQ\bar{Q}\bar{Q}$
- Simple QED → quarks extrapolation

$QQ\bar{Q}\bar{Q}$ STATES:
MASSES, PRODUCTION, AND DECAYS

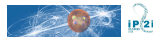Marek Karliner[a], Shmuel Nussinov[b], and Jonathan L. Rosner[c]

existence of "dipositronium" thus implies that an analog di-quarkonium state exists

- Even for the simple NR, color-additive model

$$H = \sum_i \frac{\boldsymbol{p}_i^2}{2\,m} - \frac{3}{16} \sum_{i<j} \tilde{\lambda}_i . \tilde{\lambda}_j \, v(r_{ij}) \,,$$

where v is the quarkonium potential, results differ!!!
- Vary et al., e.g., got binding
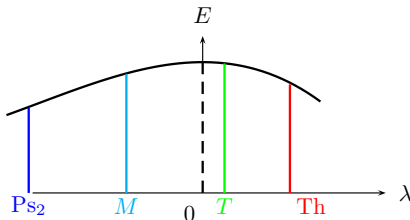- Most 4-body calculations do not get binding!
- Why?

Ps$_2$ vs. tetraquark

Meson-meson, atom-atom, Ps$_2$, tetraquark of frozen color given by

$$H = \sum \boldsymbol{p}_i^2/(2m) + \sum g_{ij} v(r_{ij}) . \quad \sum g_{ij} = 2 \quad v \text{ attrac.}$$
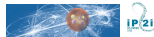
After suitable renumbering:

$$H = \sum \frac{\boldsymbol{p}_i^2}{2m} + \left(\frac{1}{3} - \lambda\right)[v_{12} + v_{23}] + \left(\frac{1}{3} + \frac{\lambda}{2}\right)[v_{13} + v_{14} + v_{23} + v_{24}] .$$

- Atomic physics Ps$_2$ vs. Threshold
- Quark model with frozen color $T = (\bar{3}, 3)$ or $M = (6, \bar{6})$

Tetraquarks penalized by the non-Abelian algebra!!!

About the stability of Ps$_2$

- 1945: Wheeler suggested Ps$_2$, but acknowledged he cannot prove its existence. Same conclusion by Øre in 1946.

- $$\Psi = \exp\left[-a(r_{12}^2 + r_{34}^2) - b \sum_{i \leq 2 \, j \geq 3} r_{ij}^2\right]$$

  gives $E \simeq -0.367$, OK for $(e^+ e^+ e^-) + e^-$, not for $(e^+ e^-)^2$??
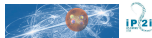
- Modern quantum chemistry: correlated Gaussians
  $$\Psi = \sum_i^N \gamma_i \left[\exp(-a_i \, r_{12}^2 - b_i \, r_{34}^2 - c_i \, r_{13}^2 - d_i \, r_{14}^2 - e_i \, r_{23}^2 - f_i \, r_{24}^2) + (a_i \leftrightarrow b_i, c_i \leftrightarrow d_i, e_i \leftrightarrow f_i \dots\right]$$
  symmetries broken and restored. With $N \geq 4$, stability.

- 1947: H.& O used (tour de force)

  $$\Psi = \exp\left[-\alpha(r_{13} + r_{24}) - \beta(r_{14} + r_{23})\right] + \alpha \leftrightarrow \beta$$

  which gives stability ($E = -0.504$, ex. -0.516)

Introduction
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$M^+ \neq m^+$
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MMmm
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BO
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$M = m$
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Conclusion
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Outlook

- Interesting analogies and differences between
  - 4 unit charges in atomic physics
  - tetraquarks in the quark model
- In both cases, the 4-body problem is delicate
- Better understanding of the role of symmetry breaking
- There is a new effect (chromo-electric) atop the more advertised chromomagnetic effect of Jaffe and others . . .
- Thanks to the LHCb collaboration for discovering at CERN this state that was predicted 40 years ago at CERN