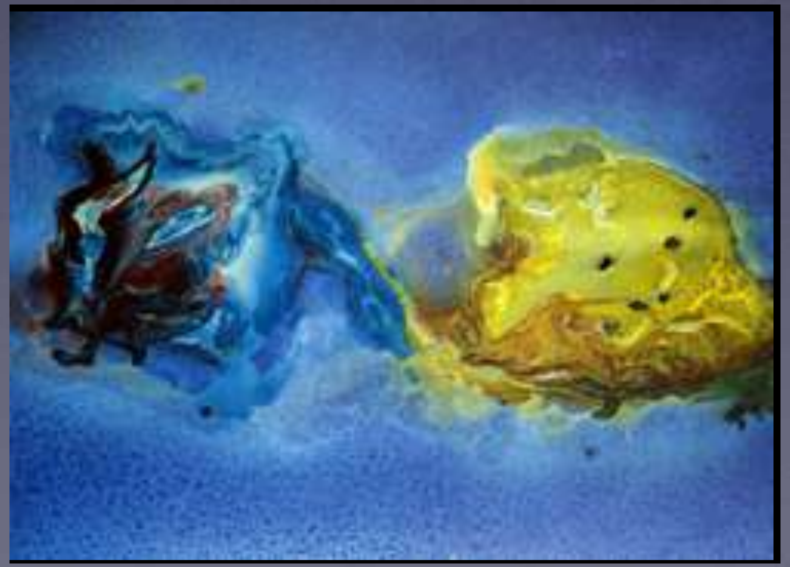
QCD i.e. Effective Field Theories and Lattice for the X Y Z Frontier

Dedicated to Gabriel Karl and atomic physics



PHYSIK DEPARTMENT TUM
T30F

NORA BRAMBILLA

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- I will show how we can address X Y Z states on the basis of an EFT called BOEFT and some lattice input i.e. directly in QCD

Plan of the talk

Quarkonium: multiscale system -> hierarchy of scales/hierarchy of NREFTs based on factorization which makes apparent symmetries hidden in QCD and increase model independent predictivity

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- BOEFT for tetraquarks, pentaquarks, doubly heavy baryons

Material for discussion/references

Heavy quarkonium: progress, puzzles, and opportunities
N. Brambilla (Munich, Tech. U.) *et al.*. Oct 2010. 181 pp.
Published in **Eur.Phys.J. C71 (2011) 1534**
e-Print: **arXiv:1010.5827 [hep-ph]**

QCD and Strongly Coupled Gauge Theories: Challenges and Perspectives
N. Brambilla (Munich, Tech. U.) *et al.*. Apr 2014. 241 pp.
Published in **Eur.Phys.J. C74 (2014) no.10, 2981**
e-Print: **arXiv:1404.3723**

chapter on exotics

N. Brambilla, S. Eidelman, C. Hanhart, A. Nefediev, C. P. Shen, C. E. Thomas, A. Vairo and C. Z. Yuan
The XYZ states: experimental and theoretical status and perspectives
Phys.Rept. 873 (2020) 1-154 • e-Print: 1907.07583 [hep-ex]

Effective field theories for heavy quarkonium
Nora Brambilla, Antonio Pineda, Joan Soto, Antonio Vairo
Rev.Mod.Phys. 77 (2005) 1423
e-Print: **hep-ph/0410047**

Born-Oppenheimer approximation in an effective field theory language
Nora Brambilla , Gastão Krein, Jaume Tarrús Castellà, Antonio Vairo
Phys.Rev. D97 (2018) no.1, 016016
e-Print: **arXiv:1707.09647**

Quarkonium Hybrids with Nonrelativistic Effective Field Theories
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Phys.Rev. D92 (2015) no.11, 114019
e-Print: **arXiv:1510.04299**

QCD spin effects in the heavy hybrid potentials and spectra Nora Brambilla, Wai Kin Lai, J. Segovia, J. Tarrus *Phys.Rev.D* 101 (2020) 5, 054040 • e-Print: 1908.11699

Spin structure of heavy-quark hybrids
N. Brambilla, Wai KIn Lai, J. Segovia, J. Tarrus A. Vairo
Phys.Rev.D 99 (2019) 1, 014017, e-Print: 1805.07713 [hep-ph]

Long range properties of 1S bottomonium states
N. Brambilla, G. Krein, J.. Tarrus, A. Vairo
Phys.Rev.D 93 (2016) 5, 054002 • e-Print: 1510.05895

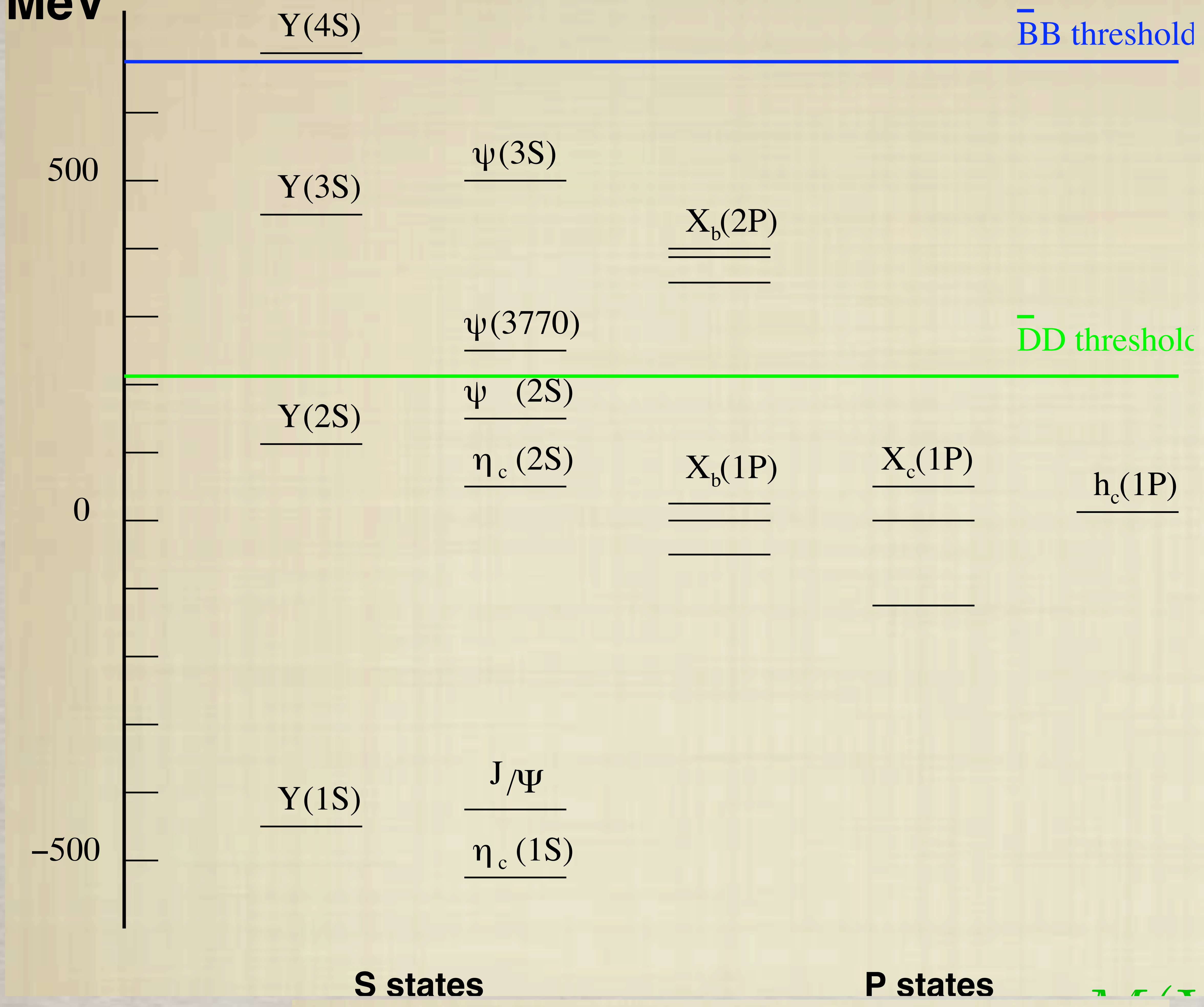
Oncala and Soto
Heavy hybrids: spectrum, decay and mixing
Phys.Rev.D 96 (2017) 1, 014004 •

Nonrelativistic effective field theory for heavy exotic hadrons
J. Soto, J. Tarrus Published in: *Phys.Rev.D* 102 (2020) 1, 014012 •

Systems with two heavy quarks: physical scales and physical significance

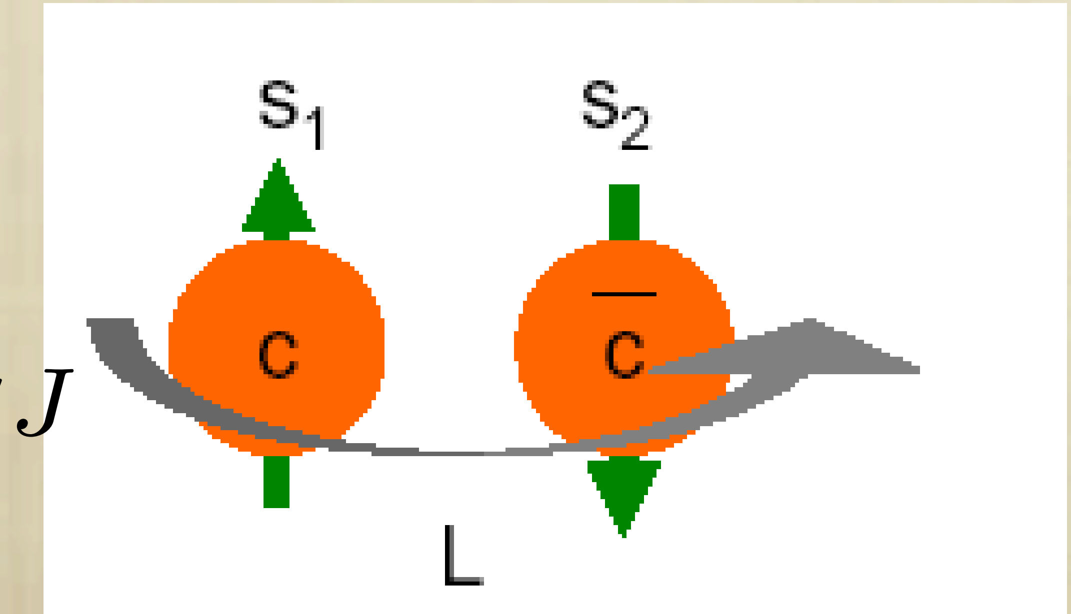
consider QQbar (quarkonium) but things are similar for QQ, QQQ etc

Quarkonium scales



Normalized with respect to $\chi_b(1P)$ and $\chi_c(1P)$

$^{2S+1}L_J$

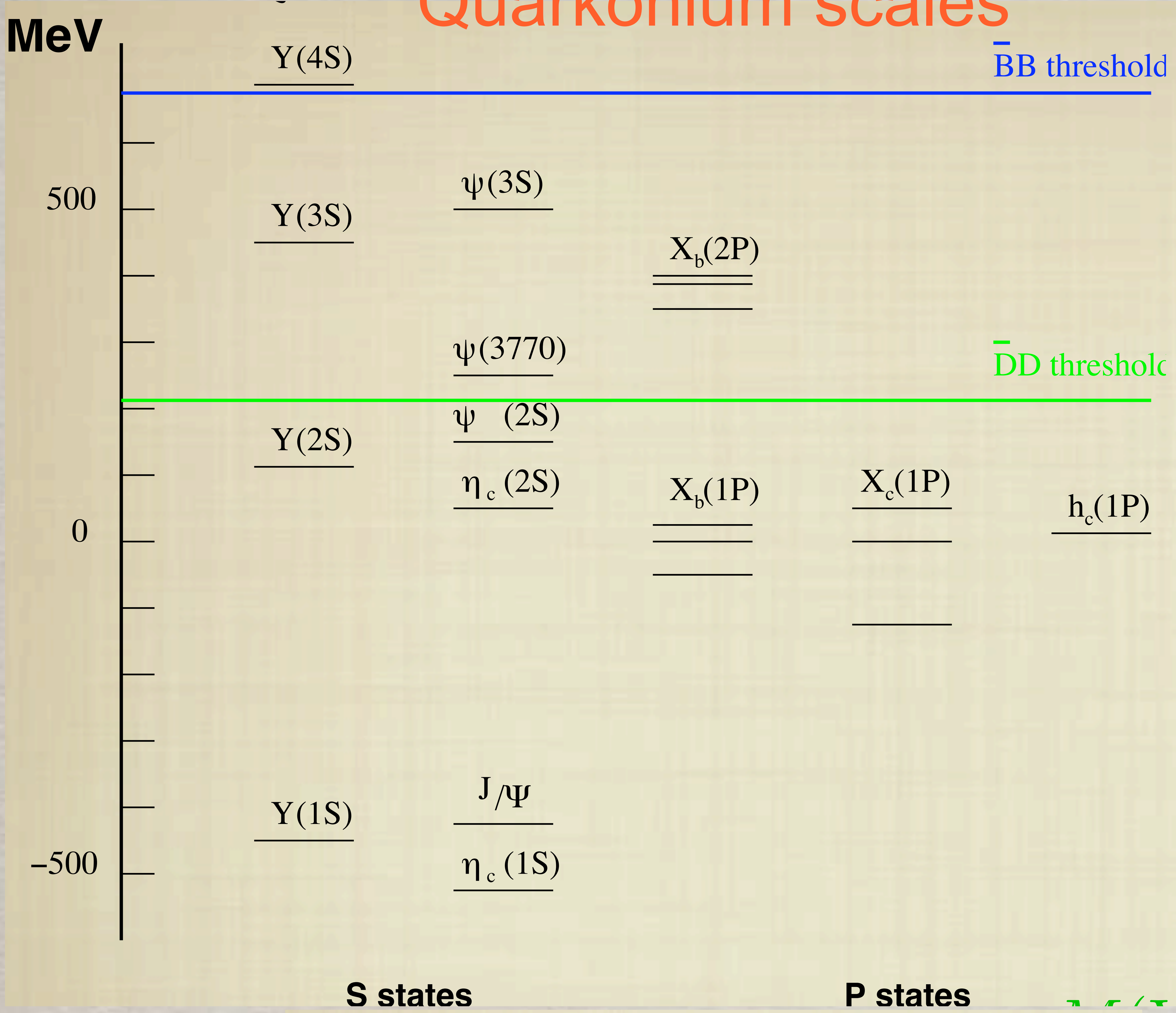


THE MASS SCALE IS PERTURBATIVE

$$m_Q \gg \Lambda_{\rm QCD}$$

$$m_b \simeq 5\,\text{GeV}; m_c \simeq 1.5\,\text{GeV}$$

Quarkonium scales



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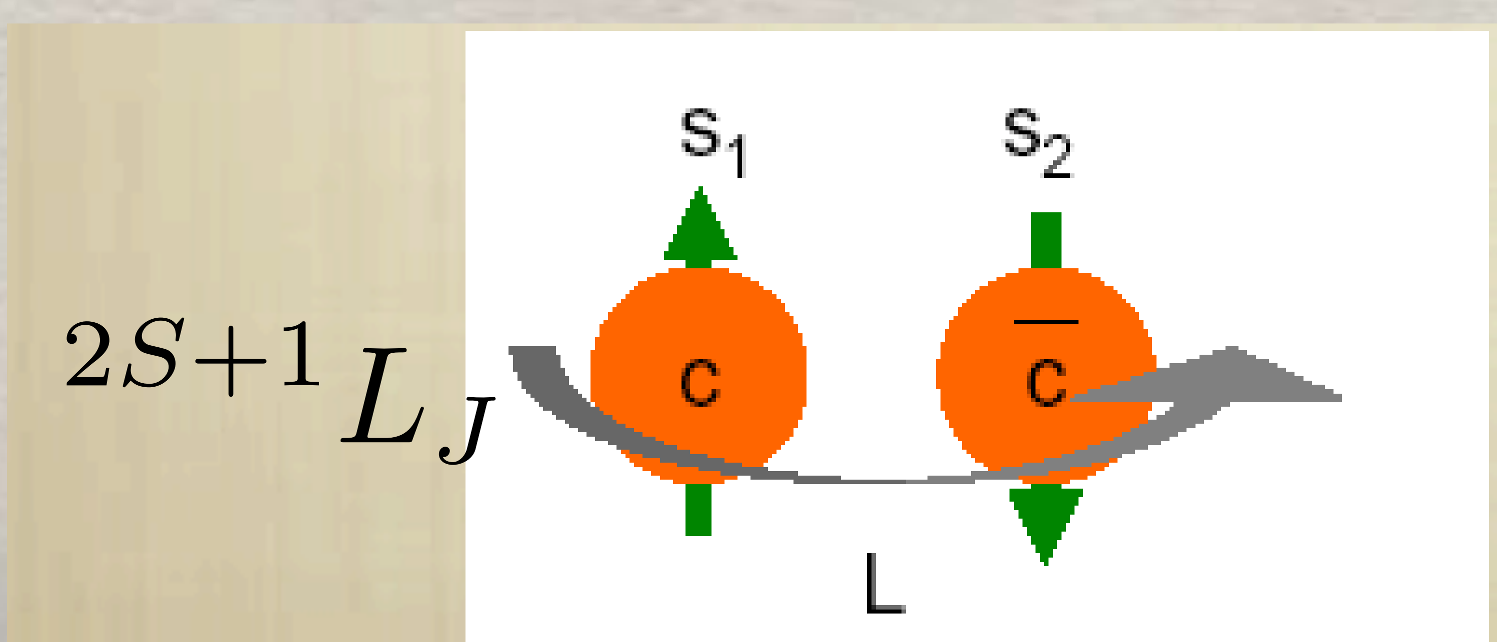

The system is nonrelativistic(NR)

$$\Delta E \sim mv^2, \Delta_{fs} E \sim mv^4$$

$$v_b^2 \sim 0.1, v_c^2 \sim 0.3$$

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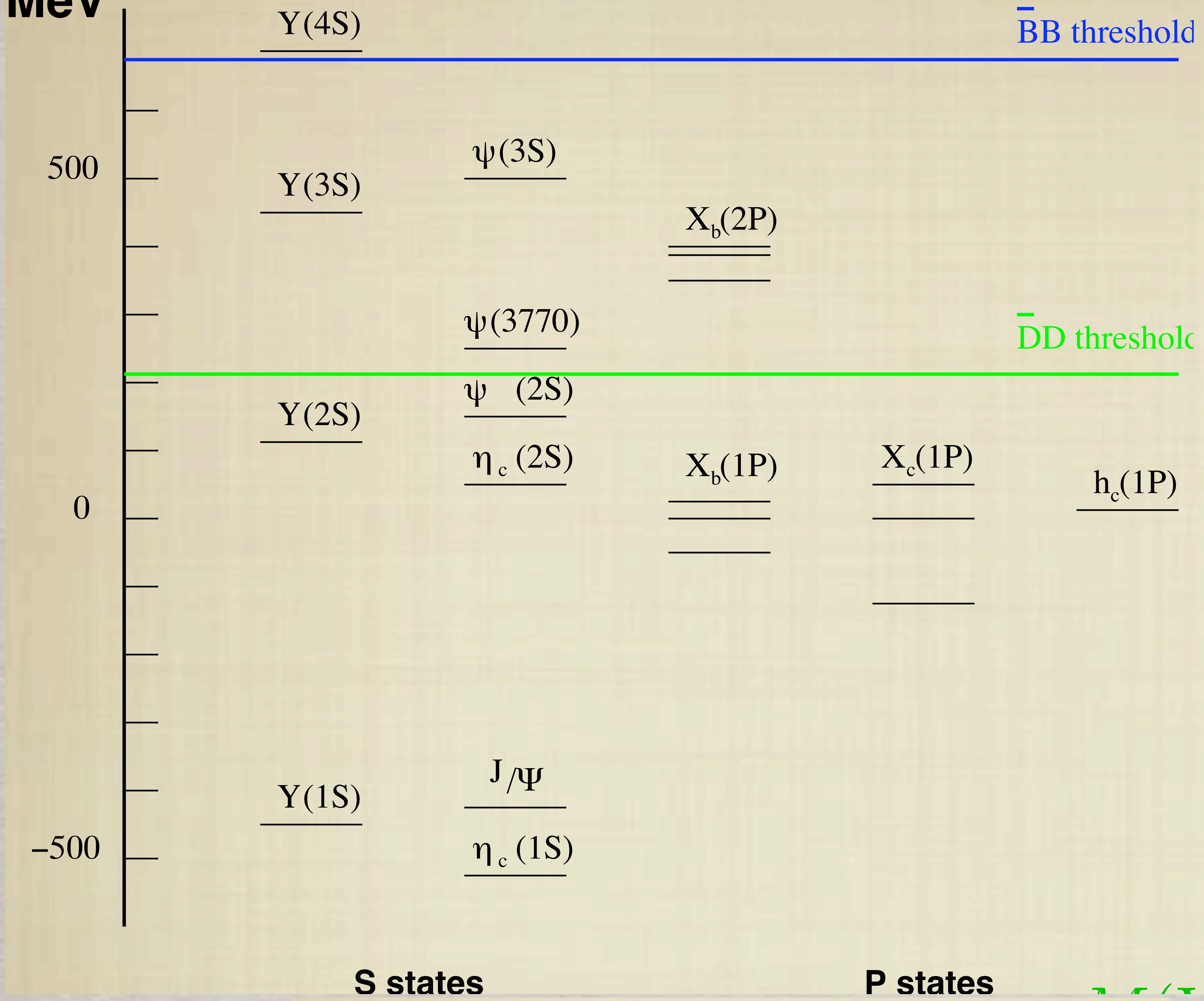

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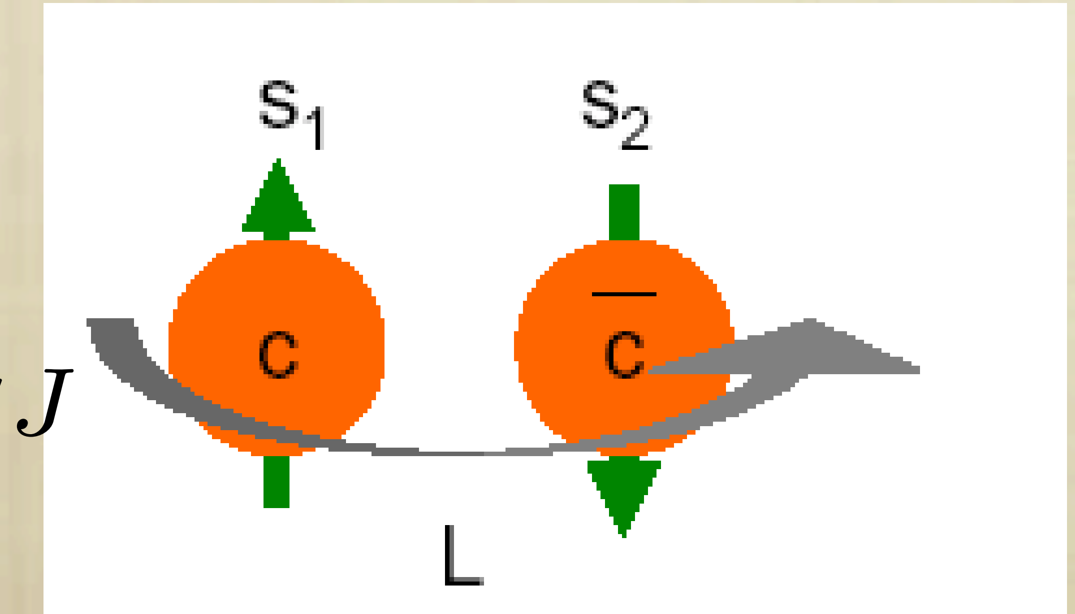

NR bound states have at least 3 scales

$$m \gg mv \gg mv^2 \quad v \ll 1$$

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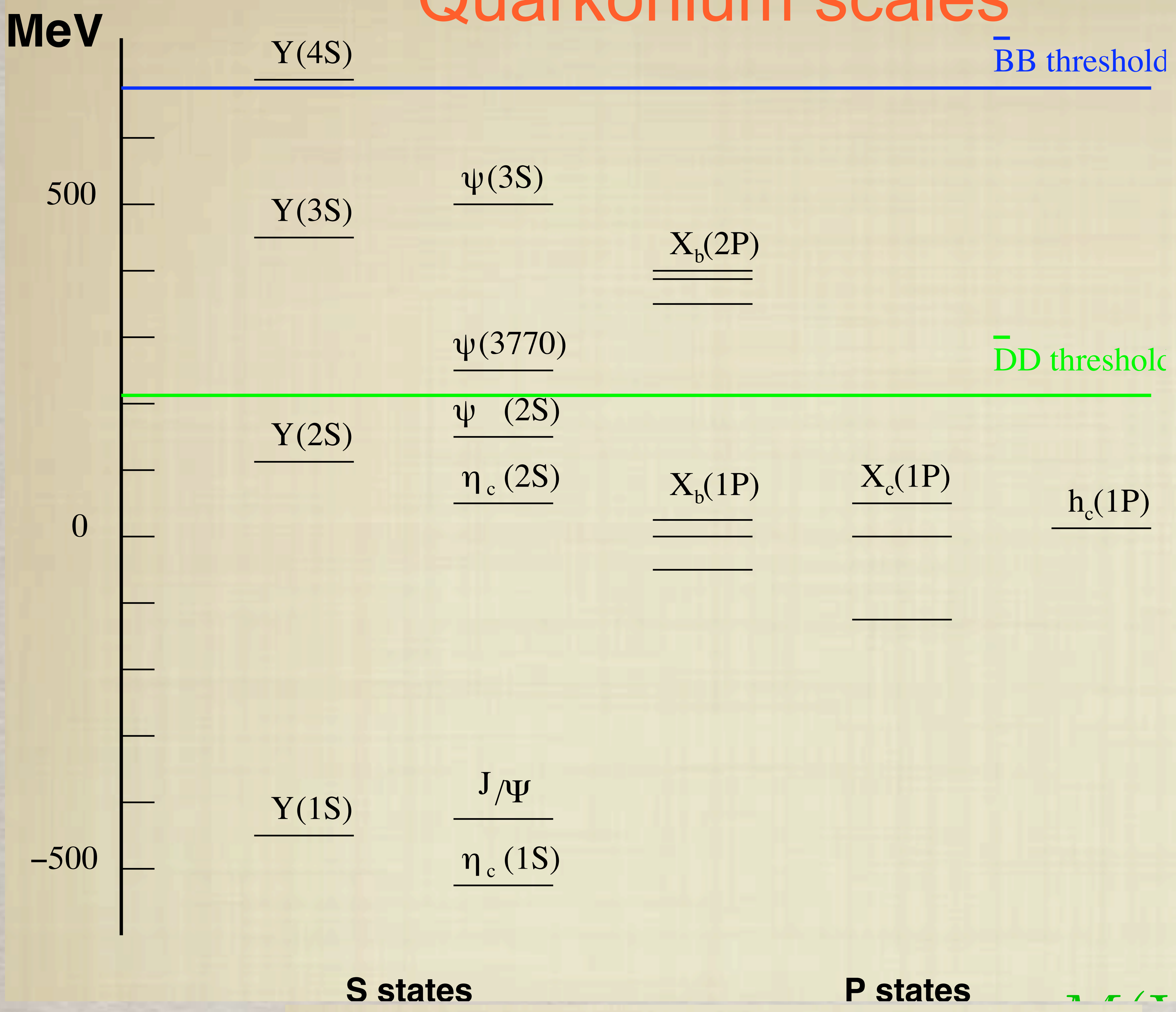

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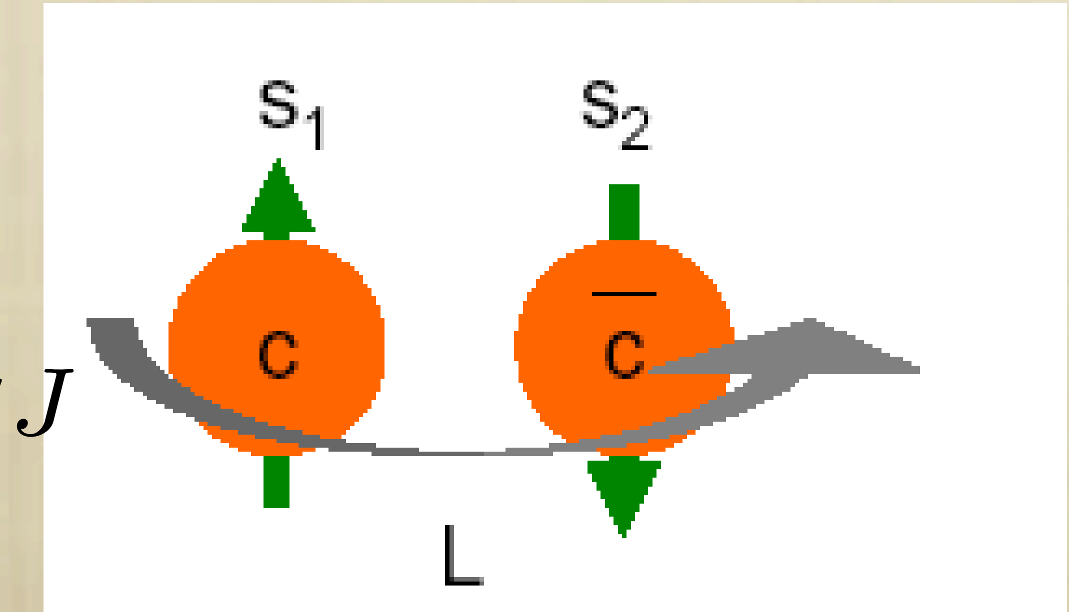

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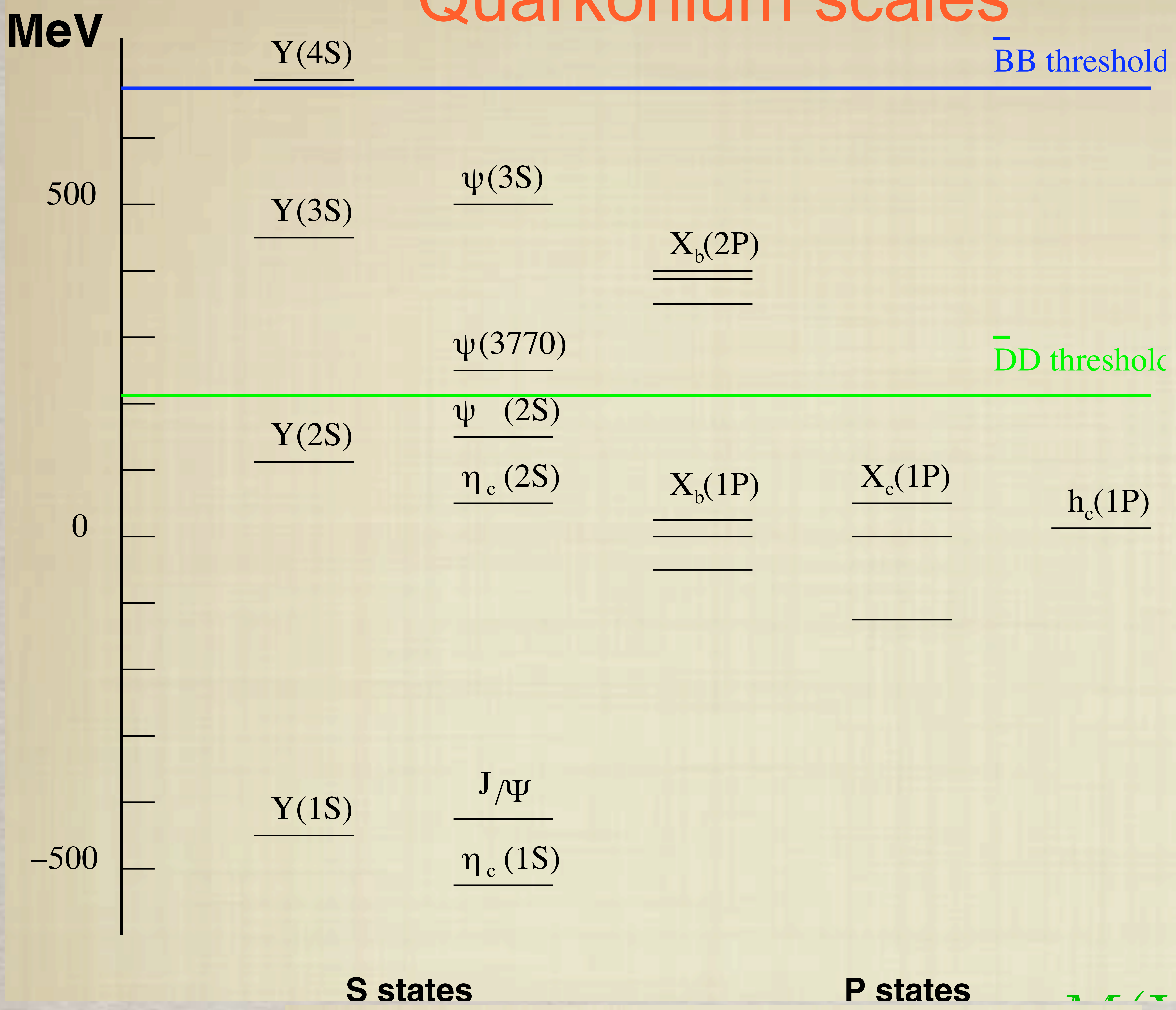

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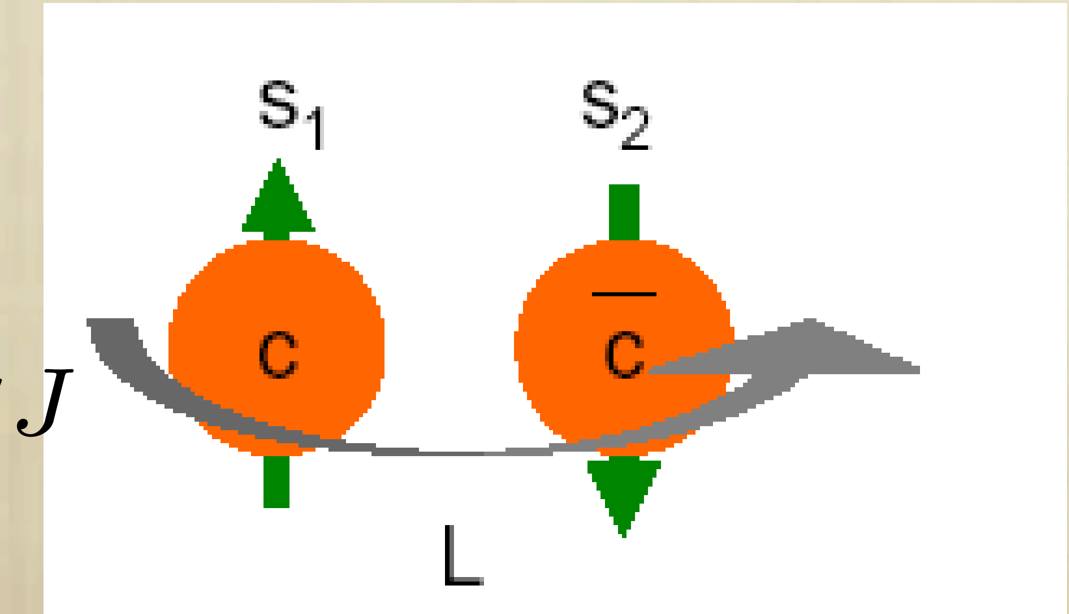

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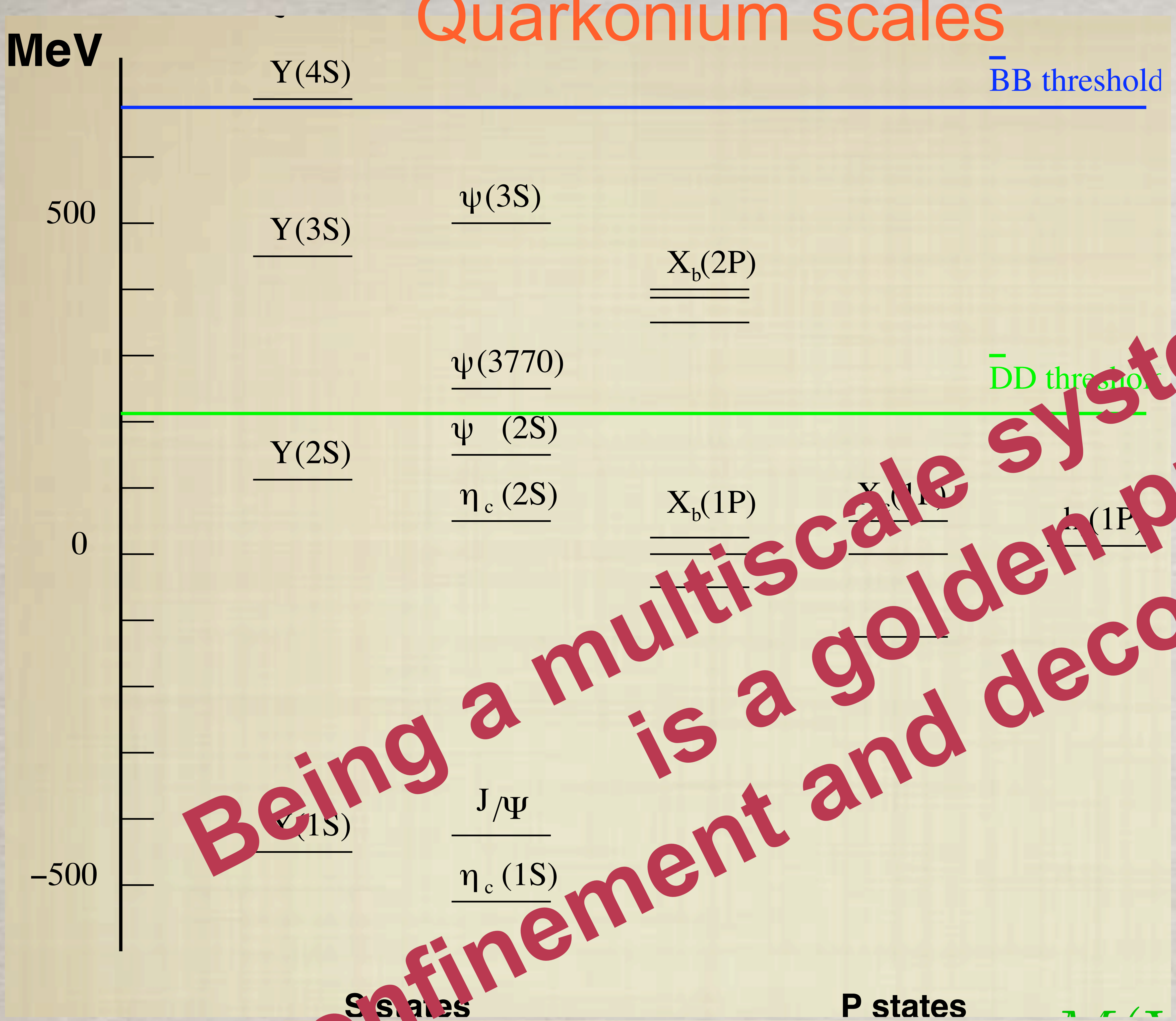

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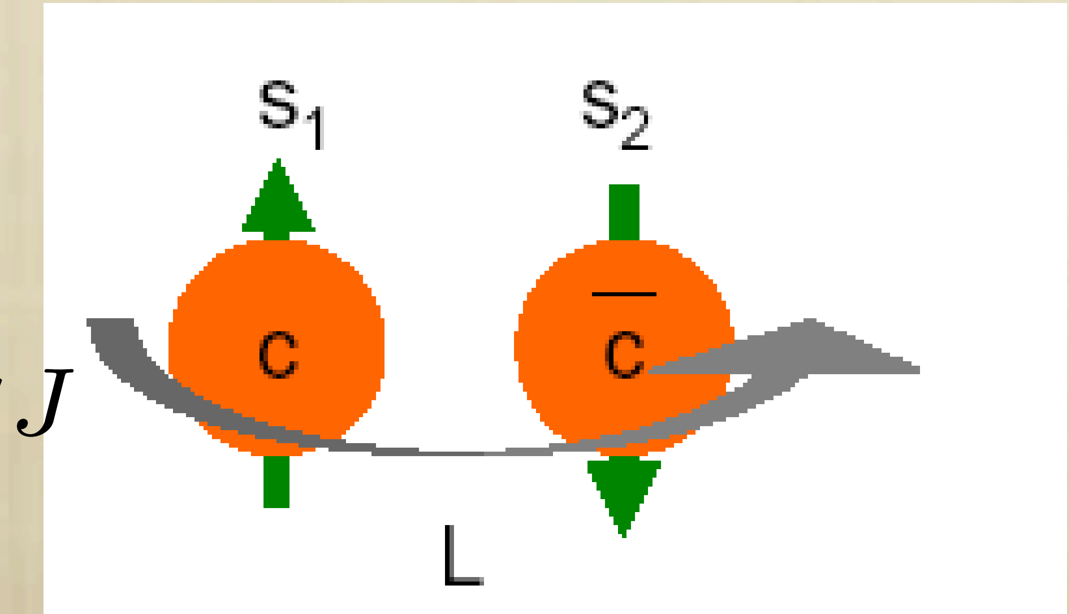

Being a multiscale system quarkonium is a golden probe of confinement and deconfinement physics

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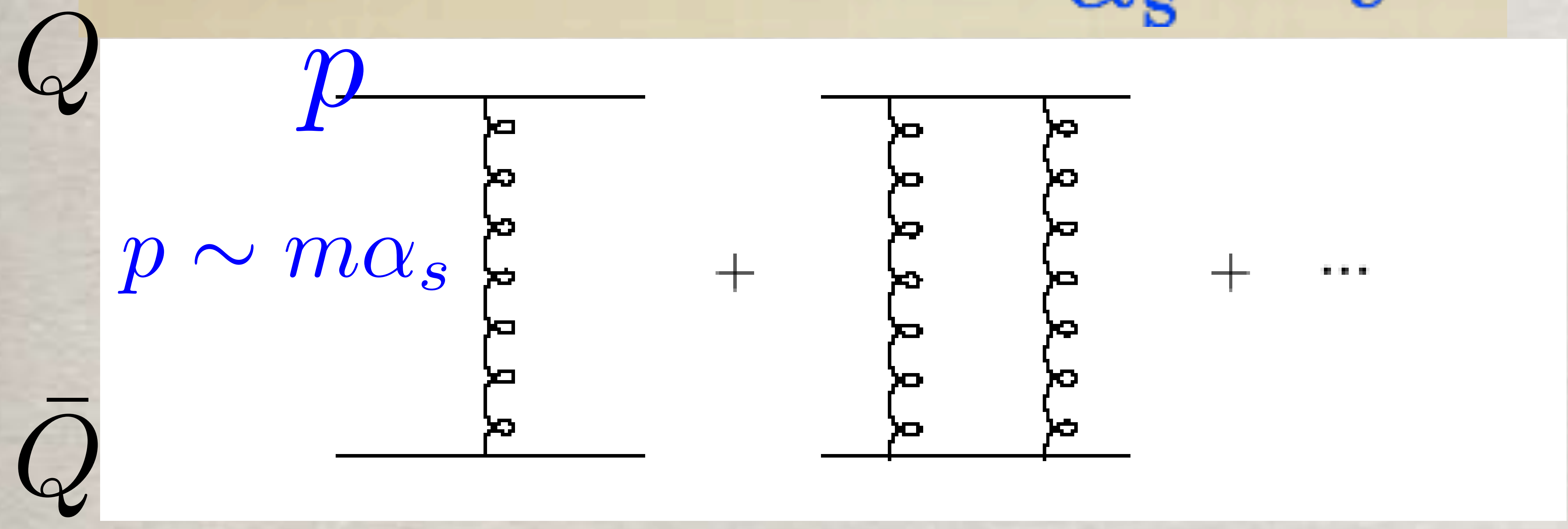

QCD THEORY OF QUARKONIUM: A VERY CHALLENGING PROBLEM

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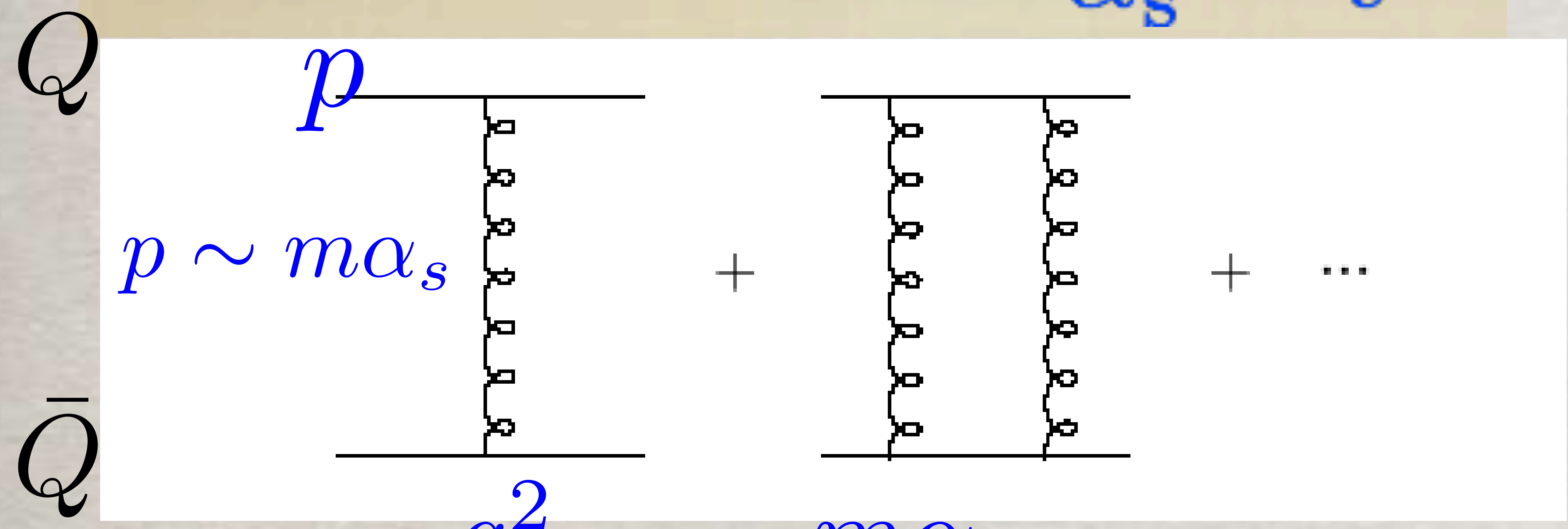

$$\frac{g^2}{p^2}(1 + \frac{m\alpha_s}{p})$$

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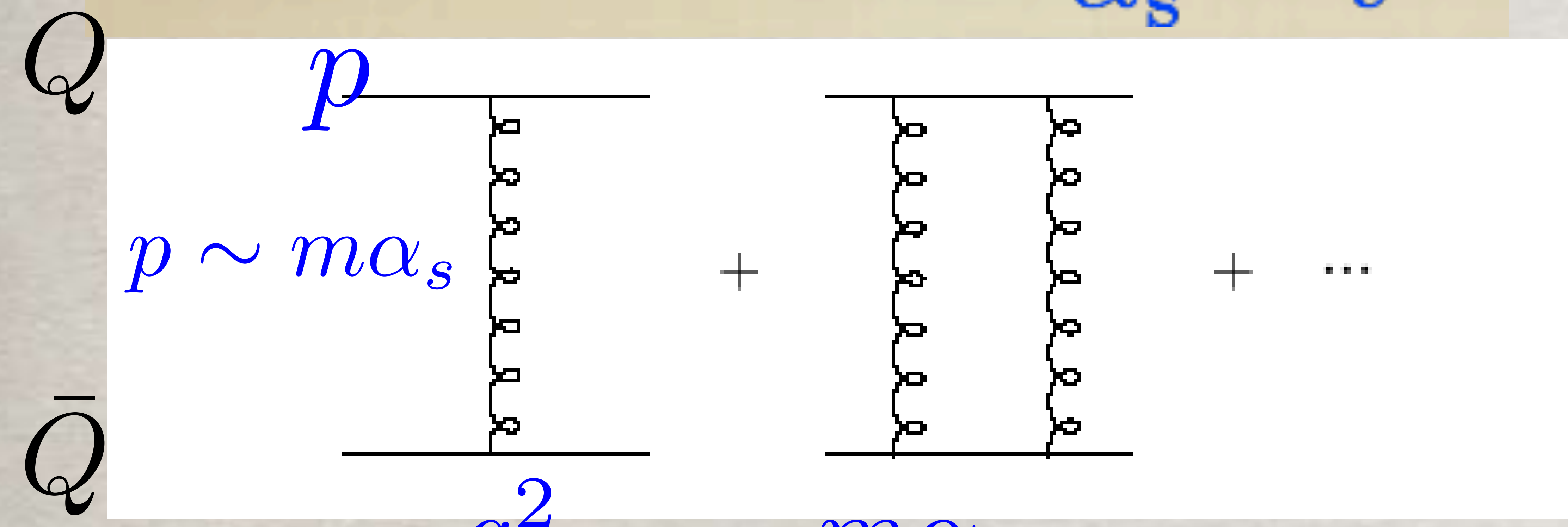

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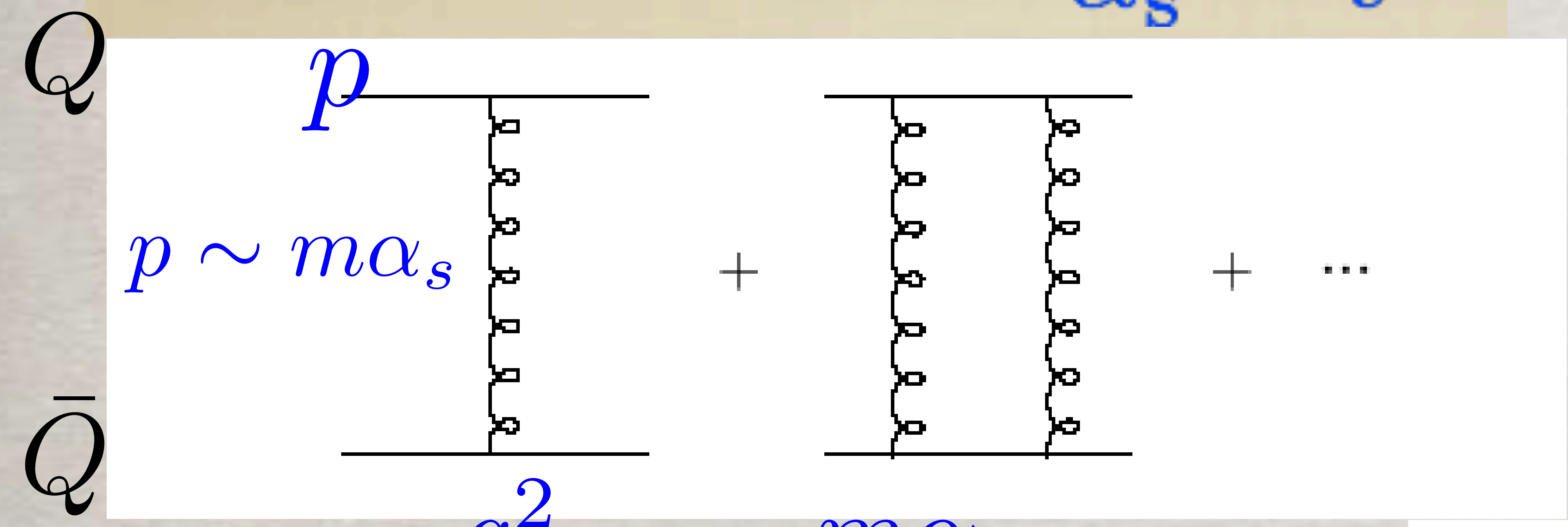

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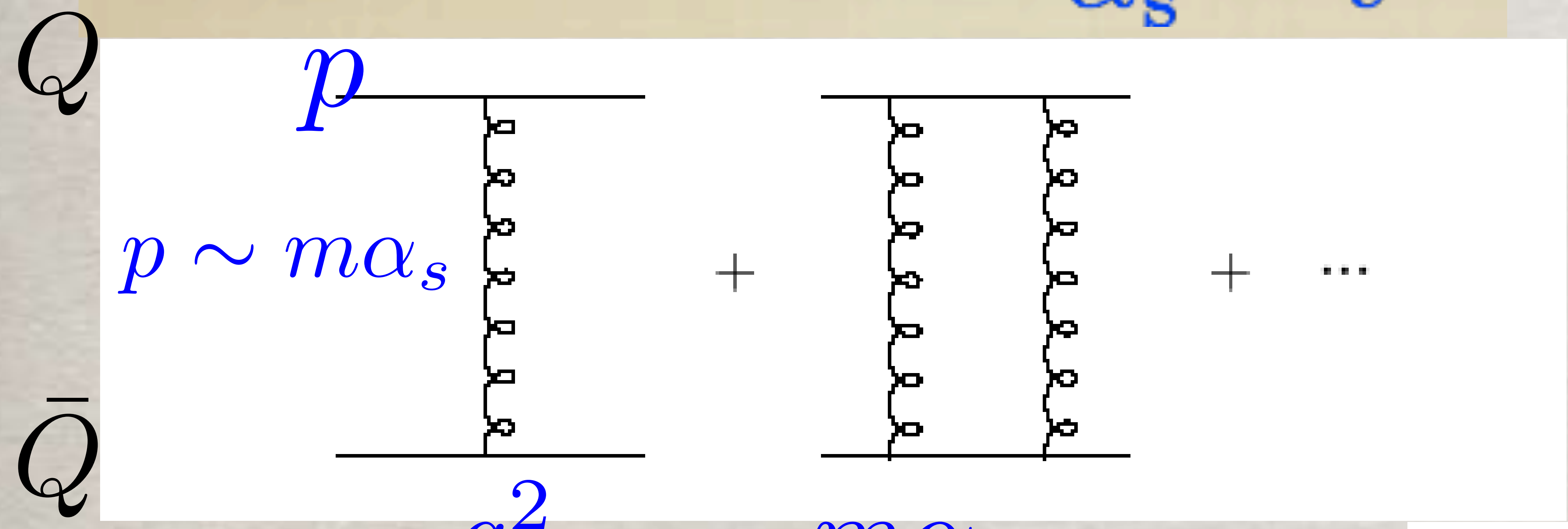

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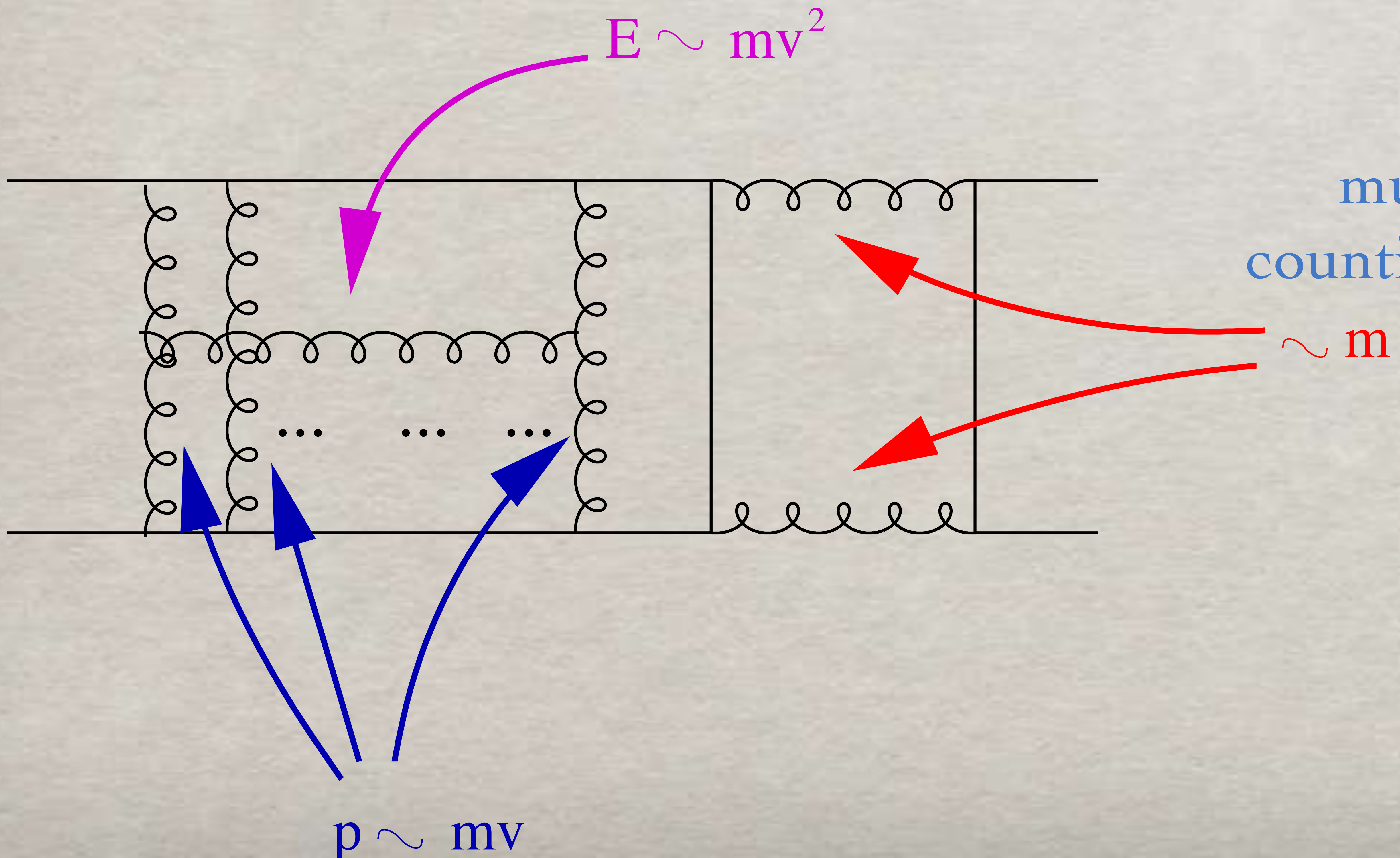

multiscale diagrams have a complicate power counting and contribute to all orders in the coupling

Difficult also for the lattice!

$$L^{-1} \ll \lambda \ll \Lambda \ll a^{-1}$$

QCD Theory of Quarkonium: a very challenging problem

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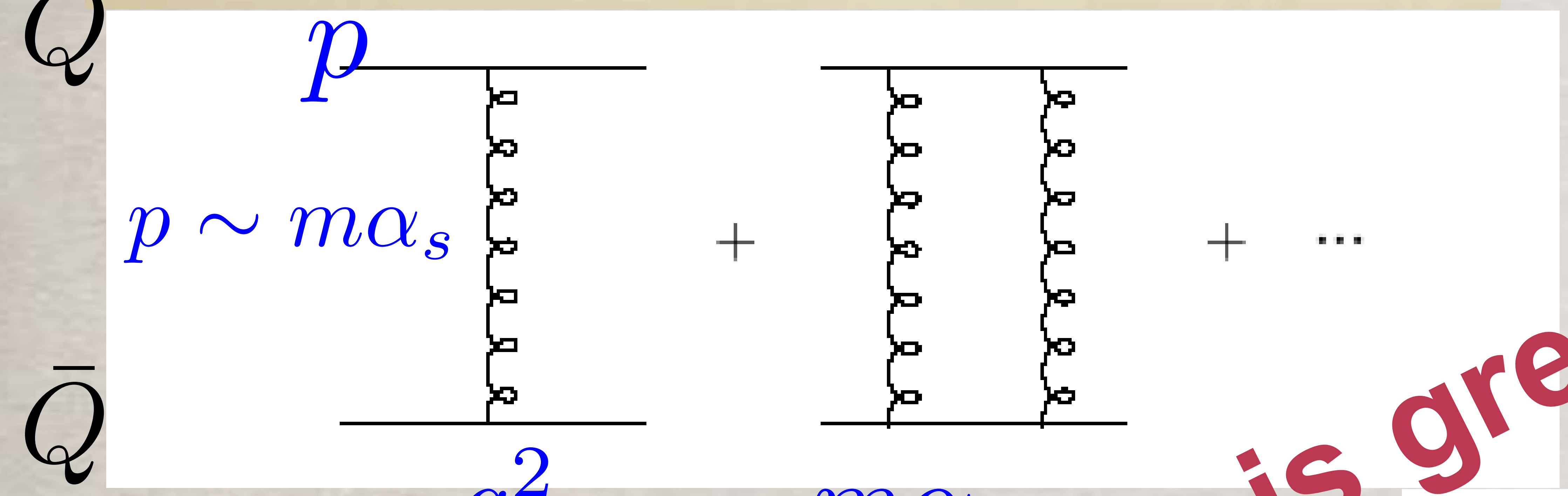

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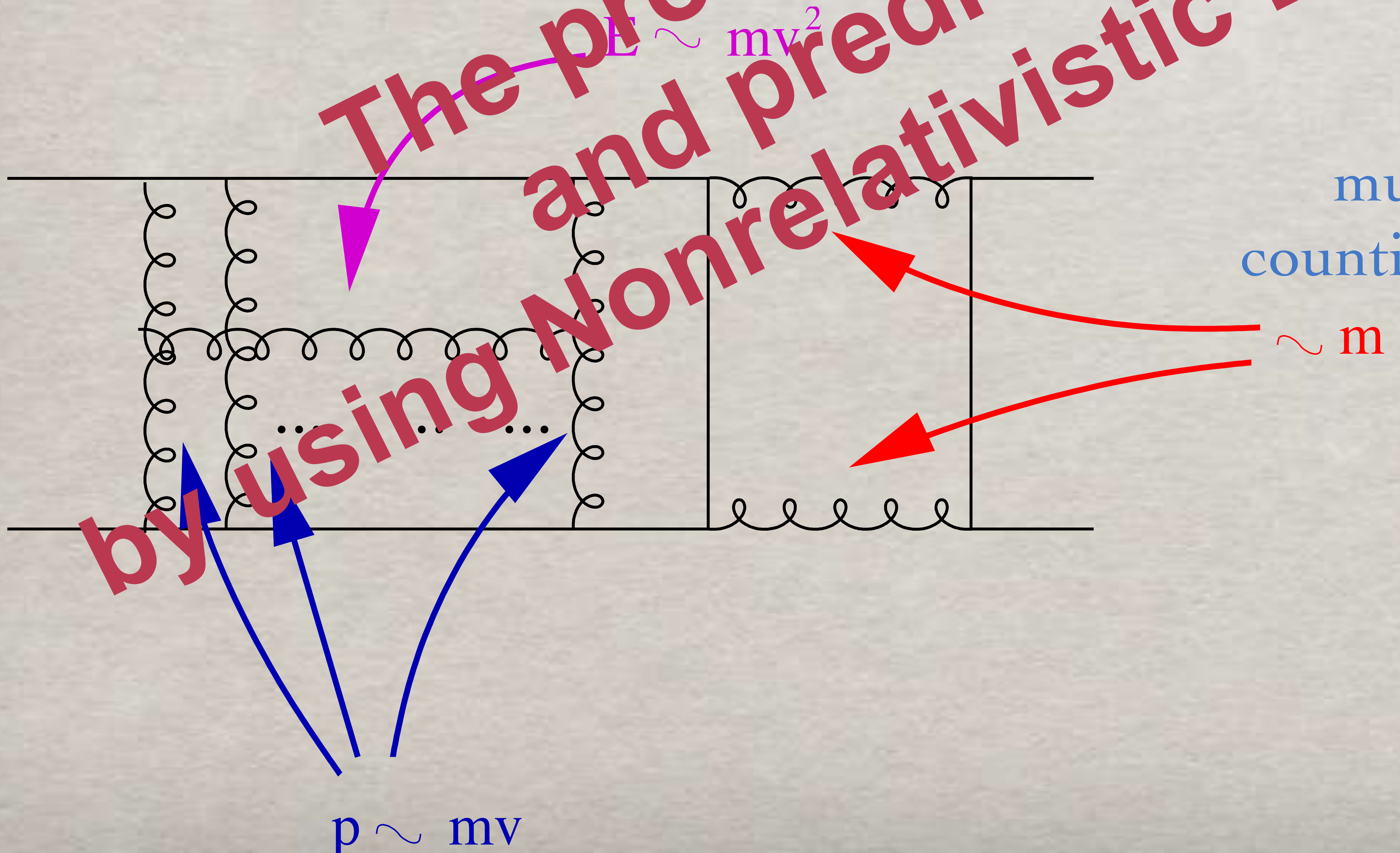
The problem is greatly simplified and predictivity is achieved by using Nonrelativistic Effective Field Theories



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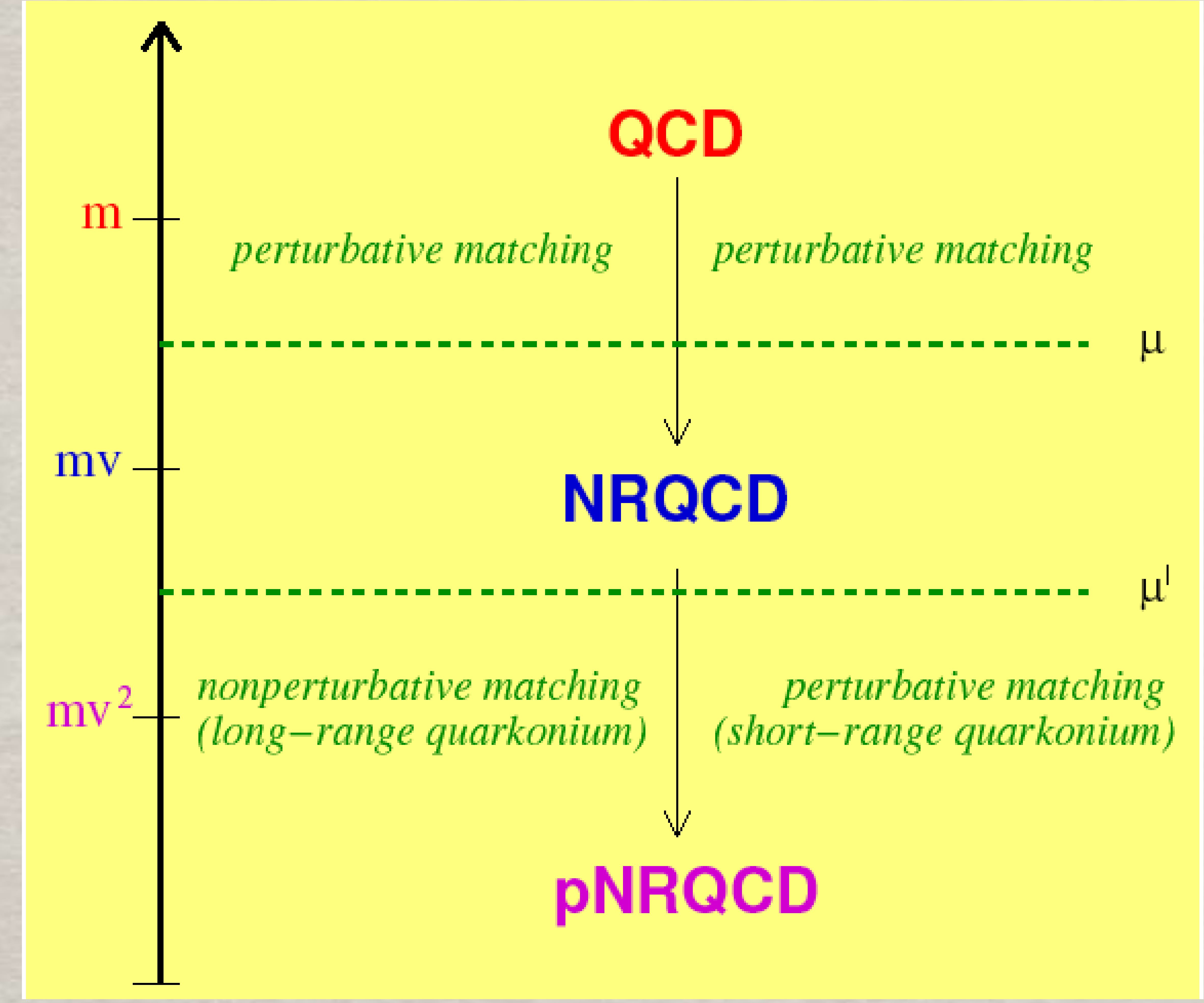
NREFTs for quarkonium



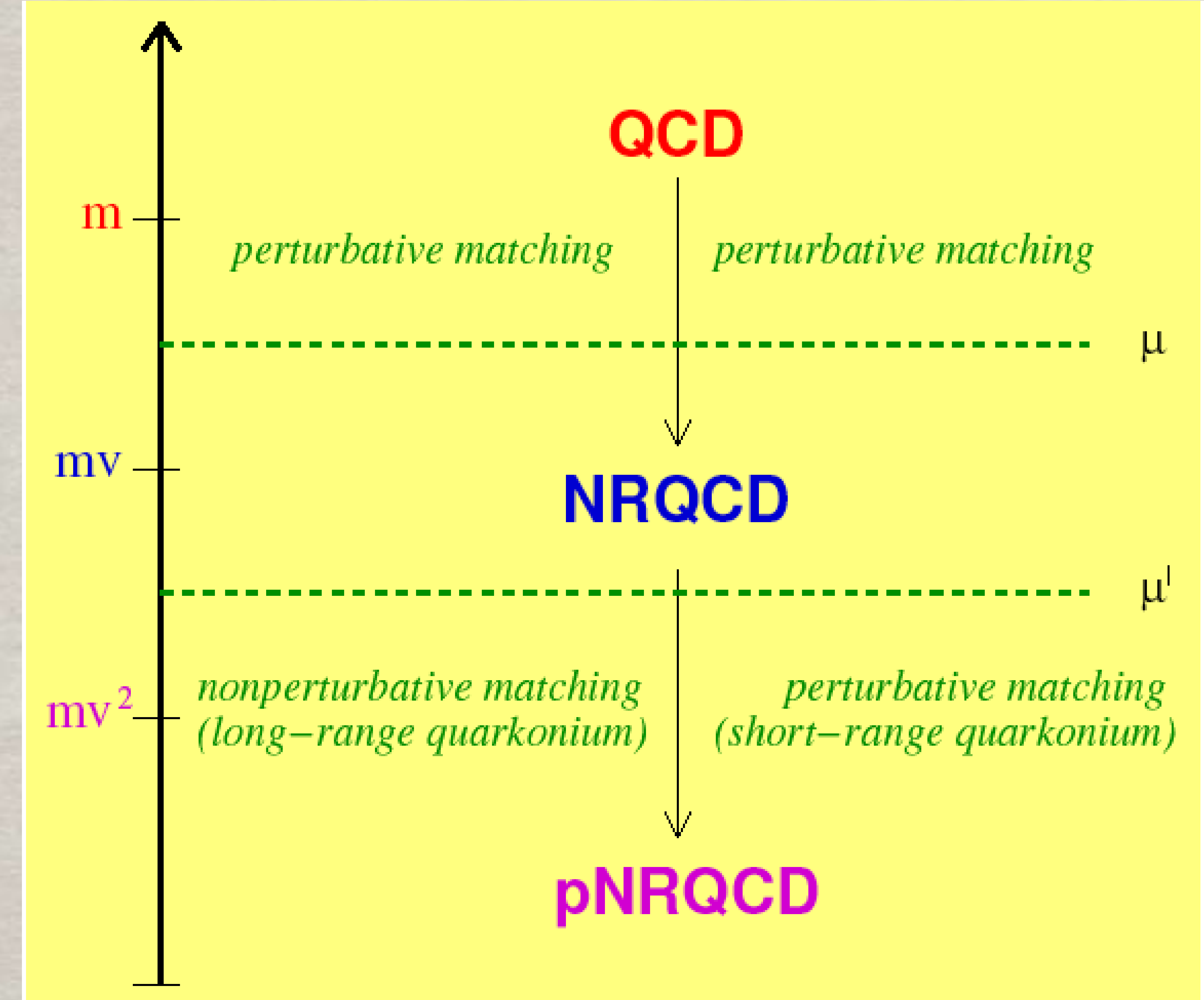
Color degrees of freedom
3X3bar=1+8
singlet and octet QQbar

Hard

Soft
(relative
momentum)

Ultrasoft
(binding energy)

NREFTs for quarkonium



$$\mathcal{L}_{\mathrm{EFT}} = \sum_n c_n(E_\Lambda/\mu) \frac{O_n(\mu,\lambda)}{E_\Lambda}$$

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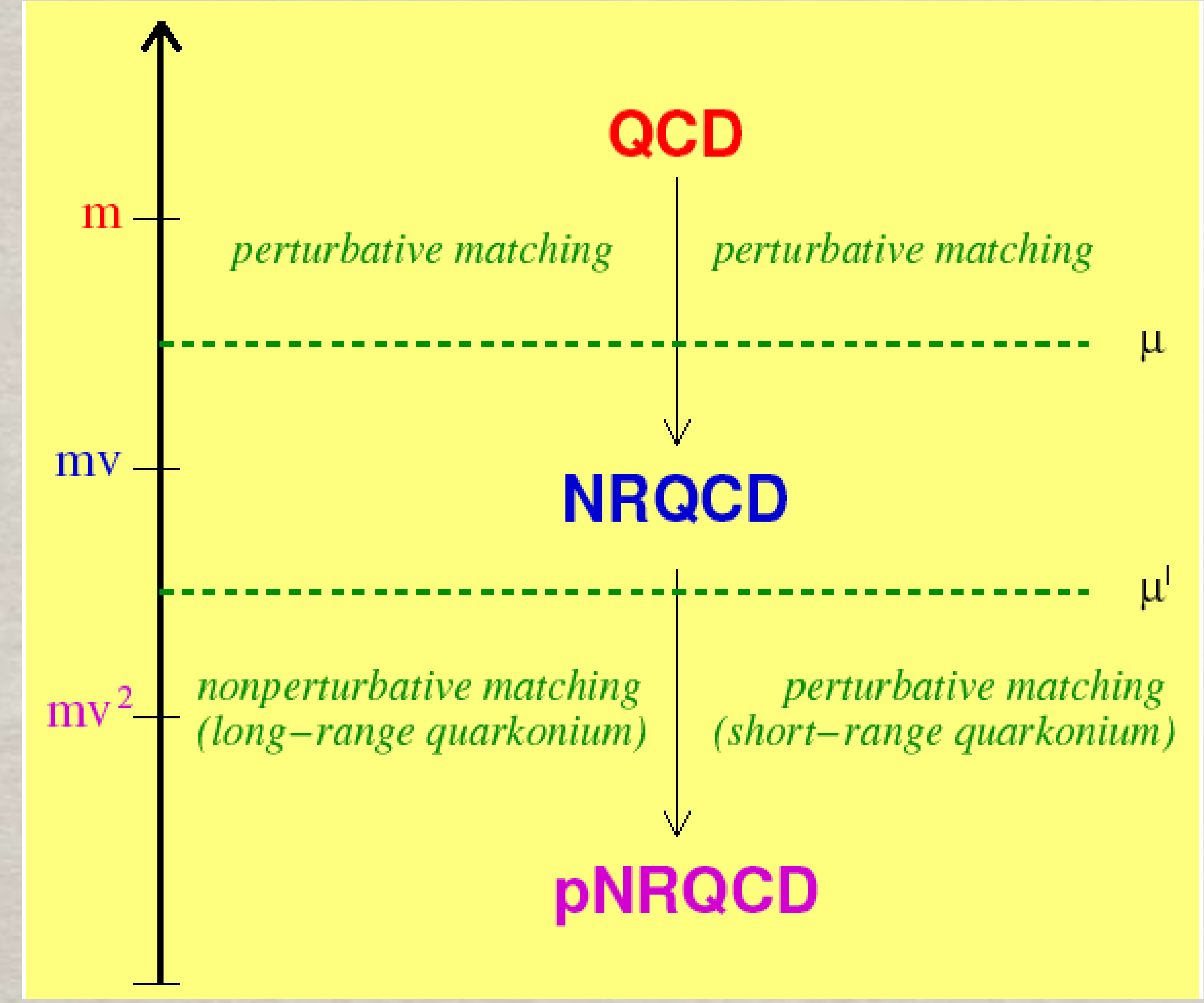

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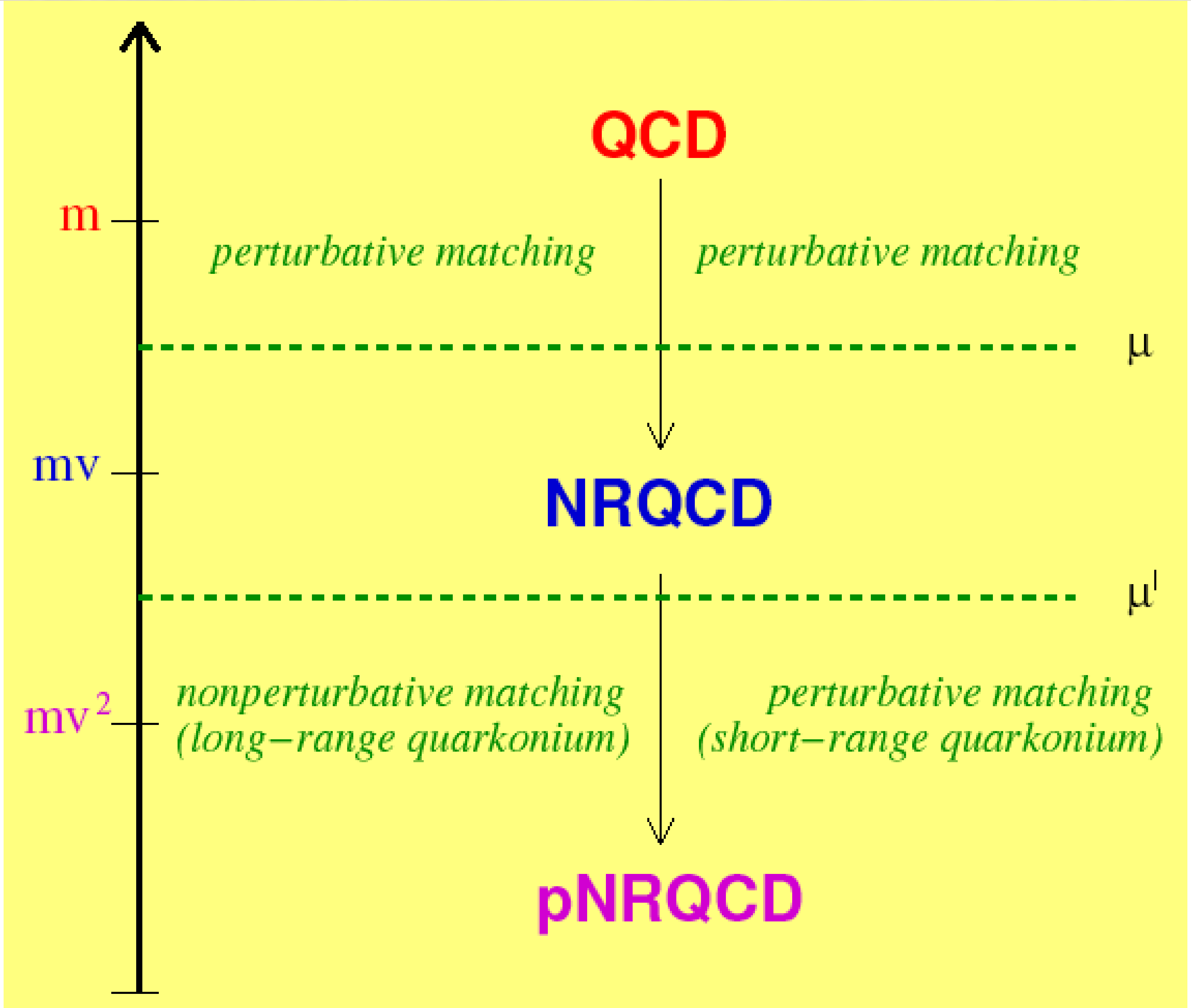

$$\frac{E_\lambda}{E_\Lambda} = \frac{mv}{m}$$

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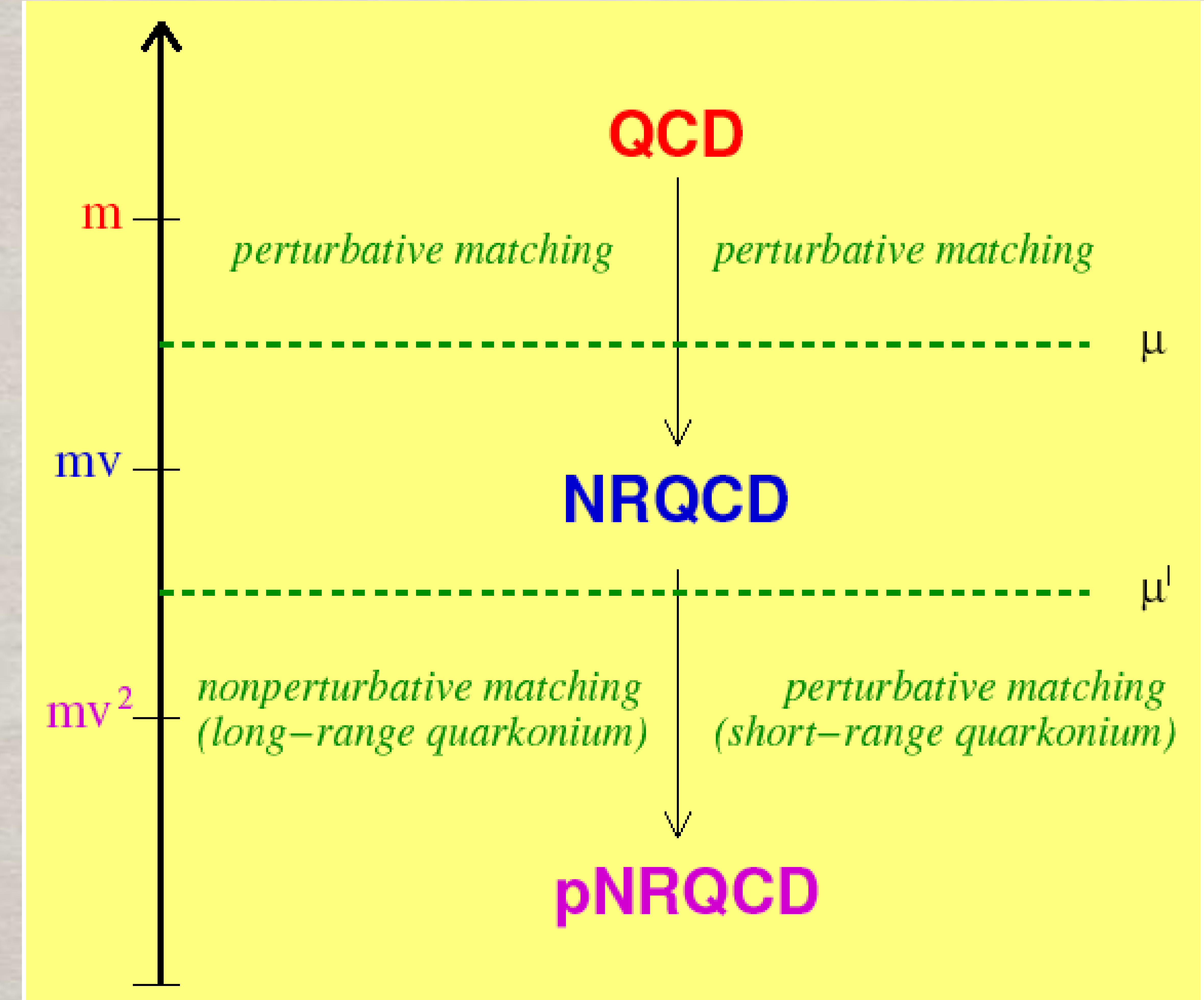

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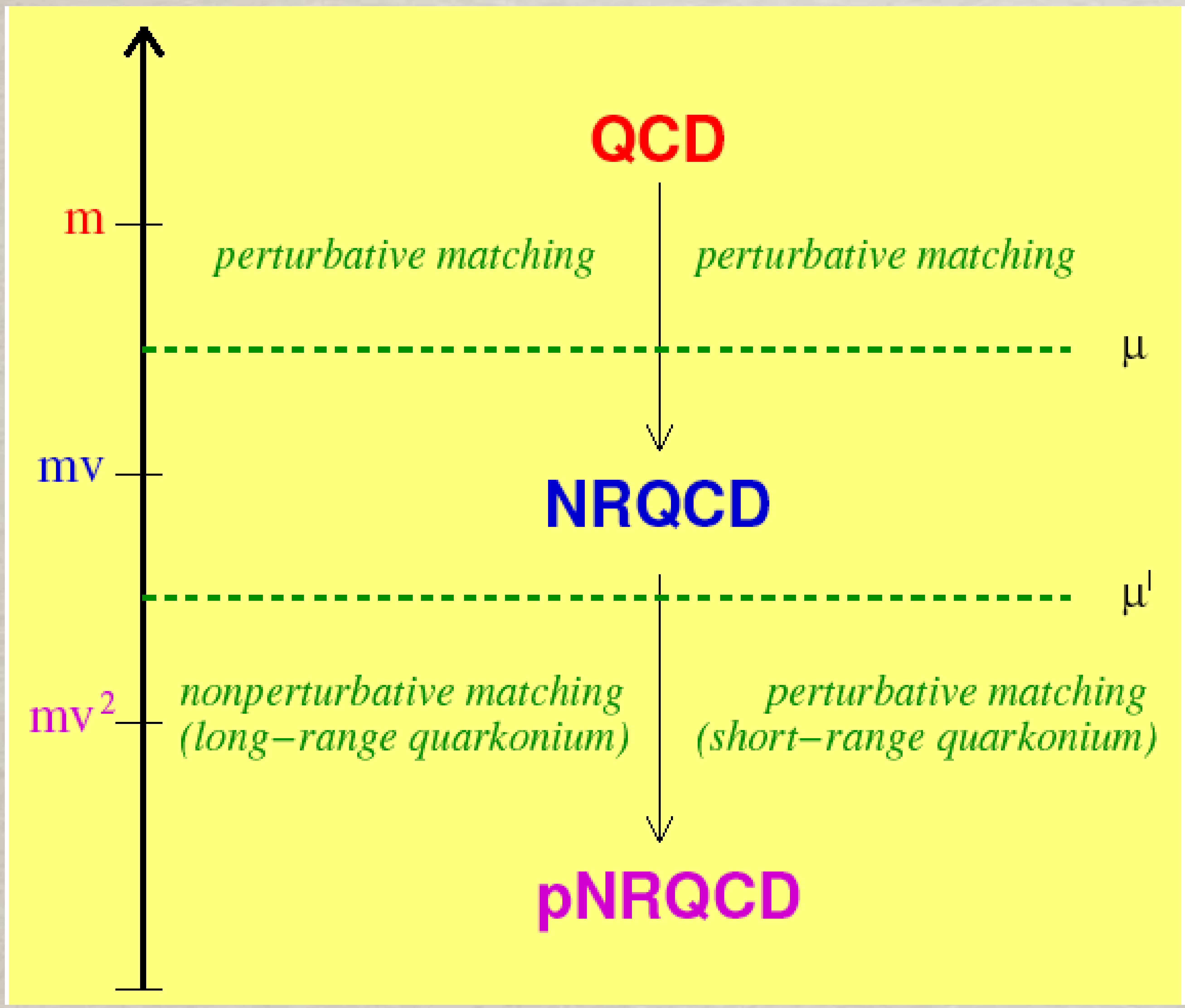
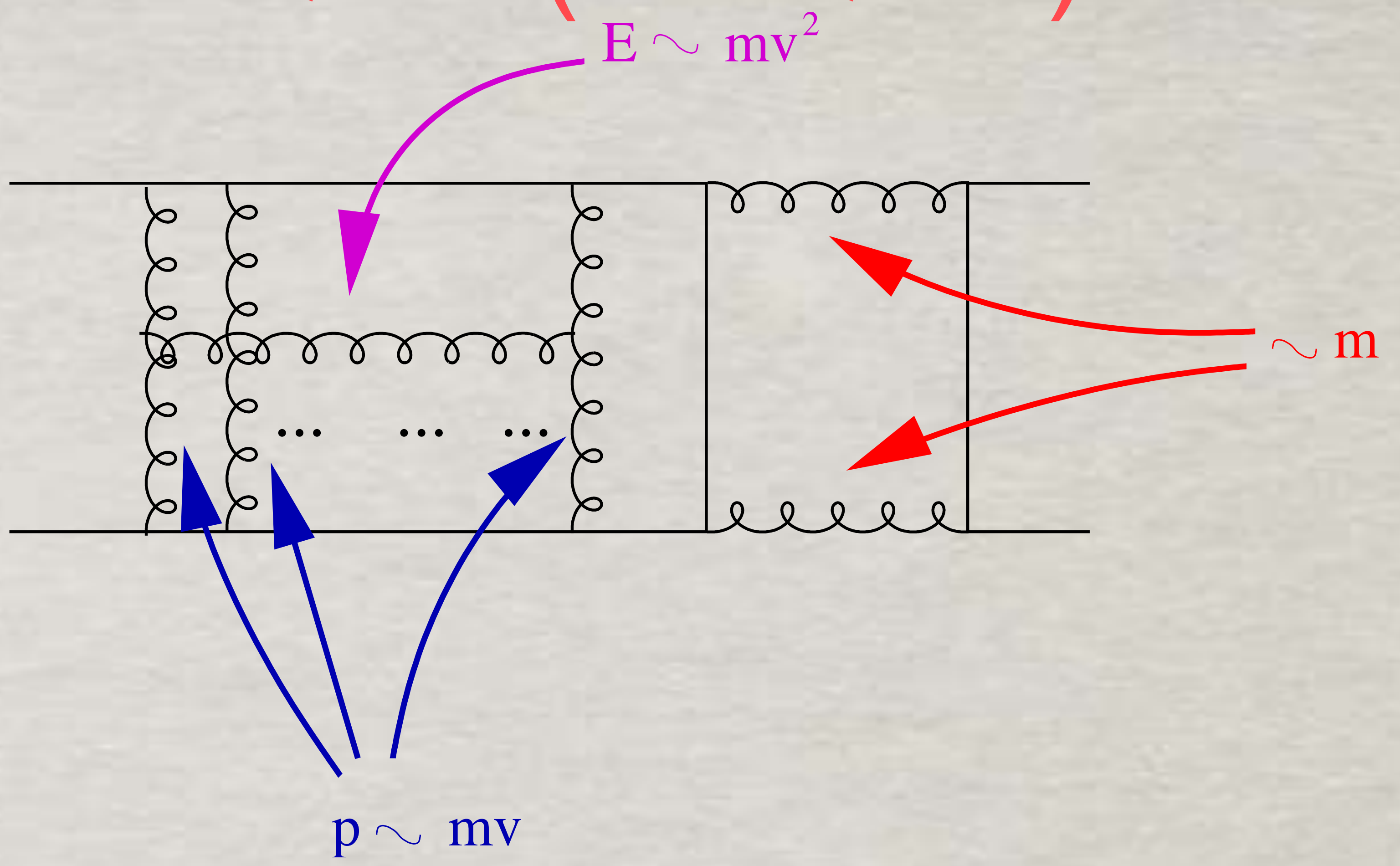
Quarkonium with NREFTs: Non Relativistic QCD (NRQCD)





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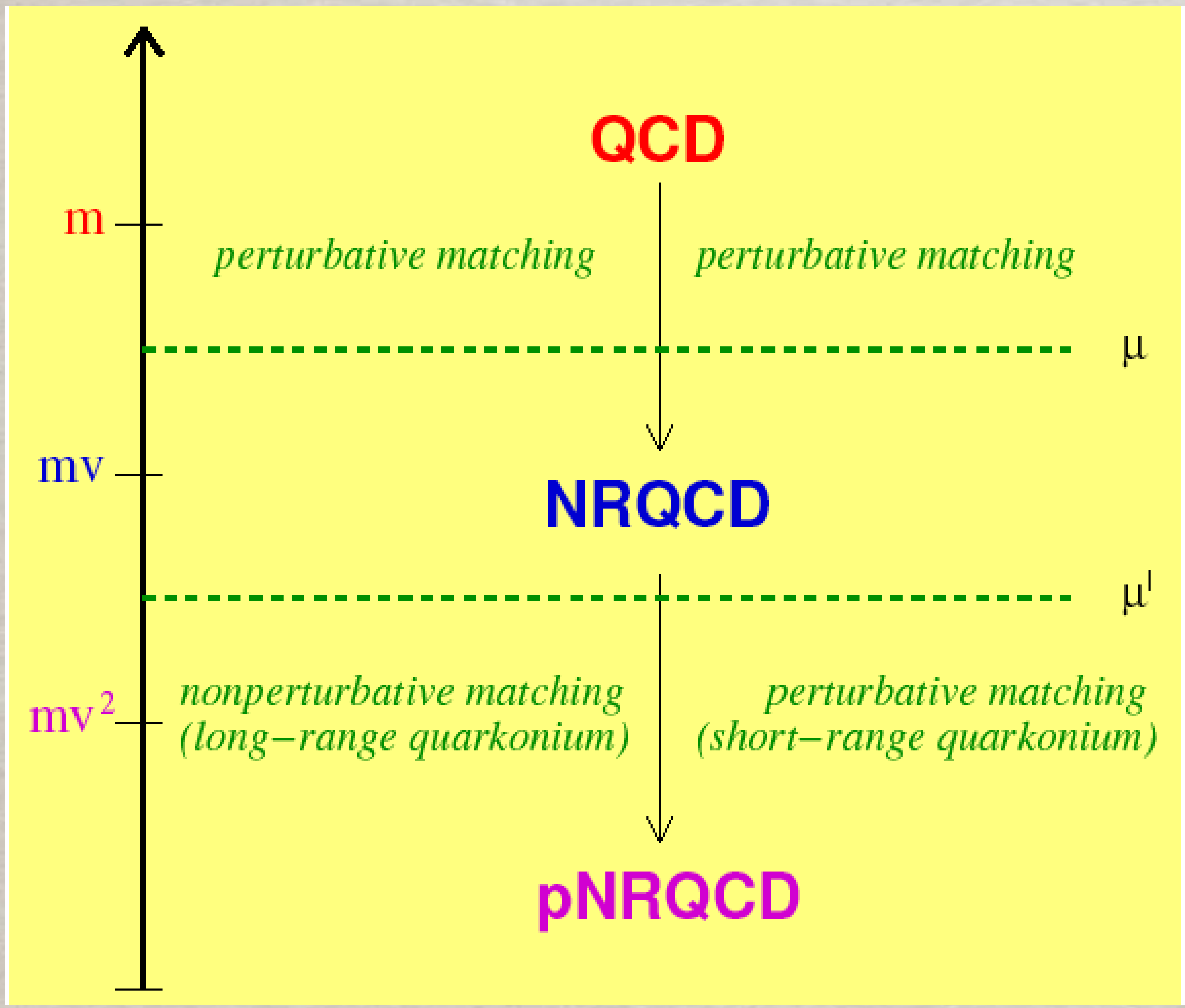

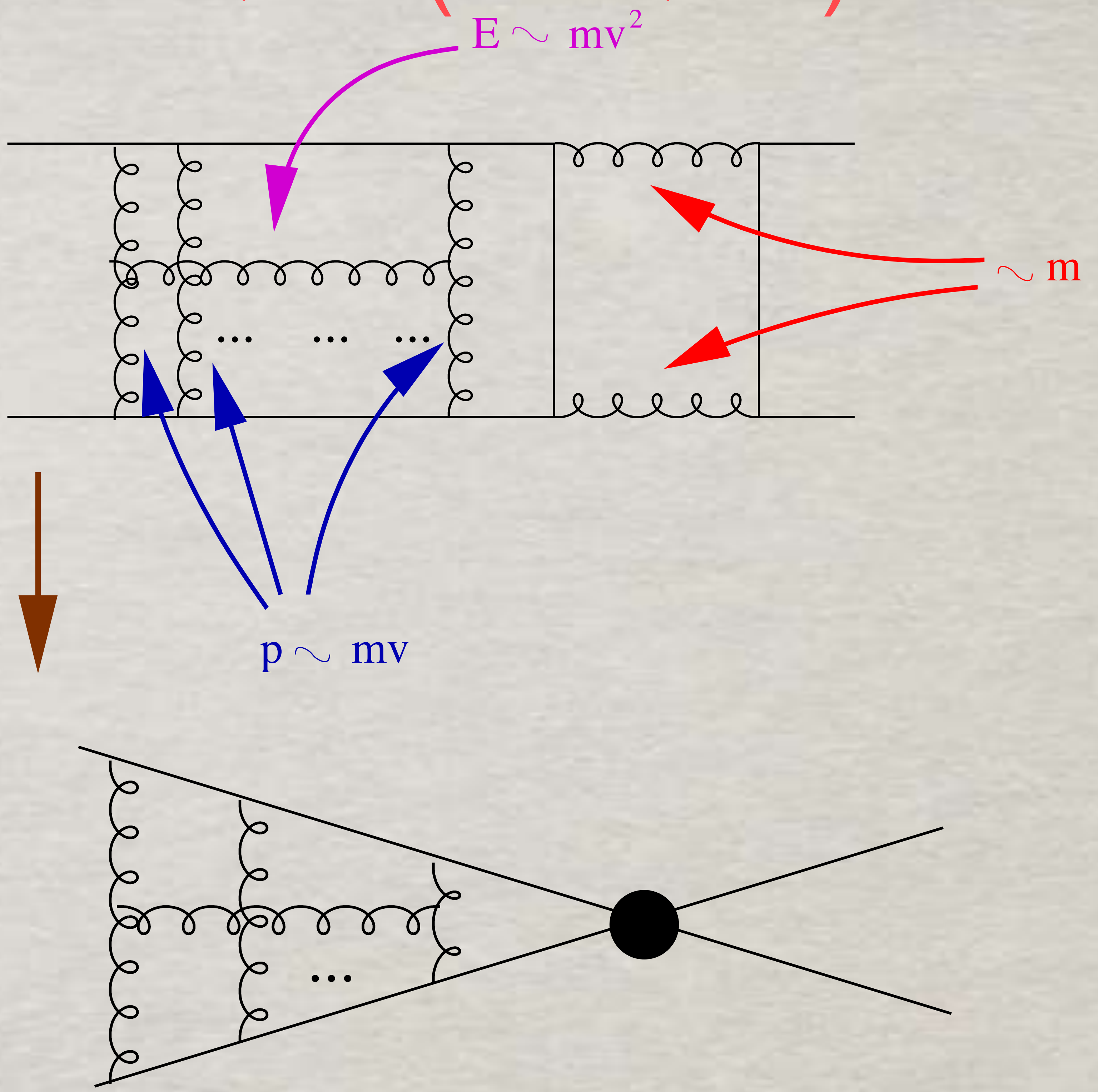

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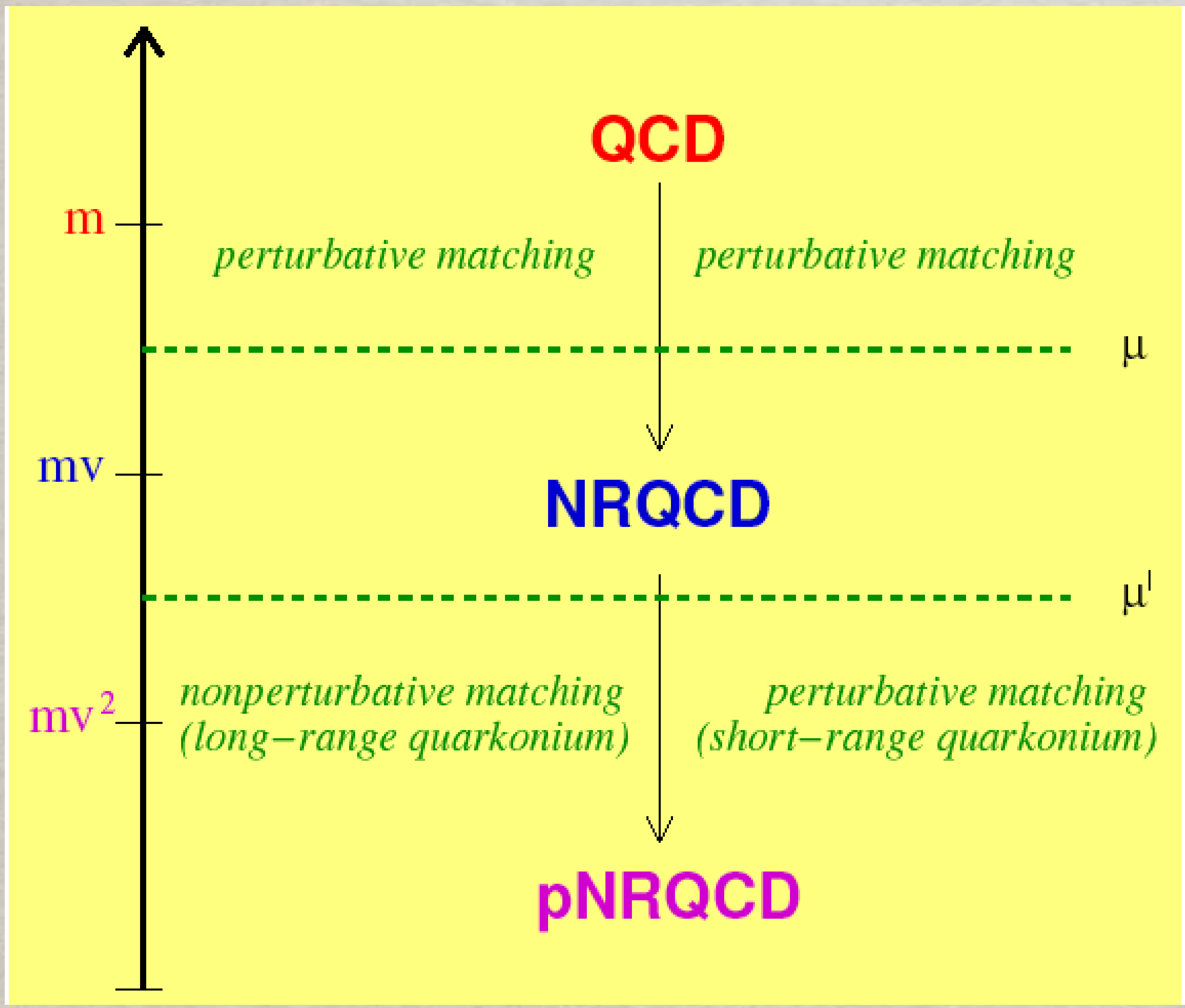
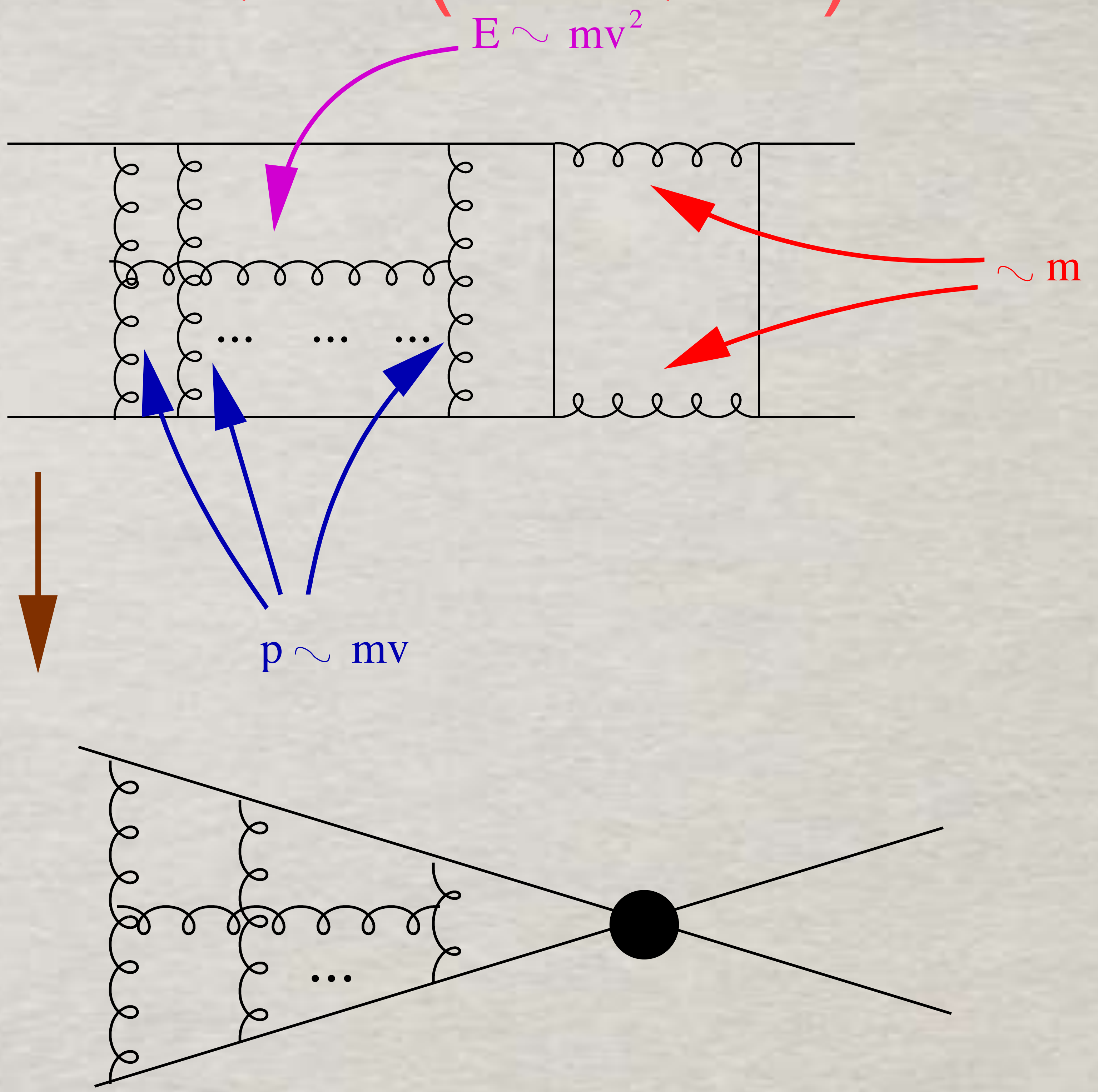
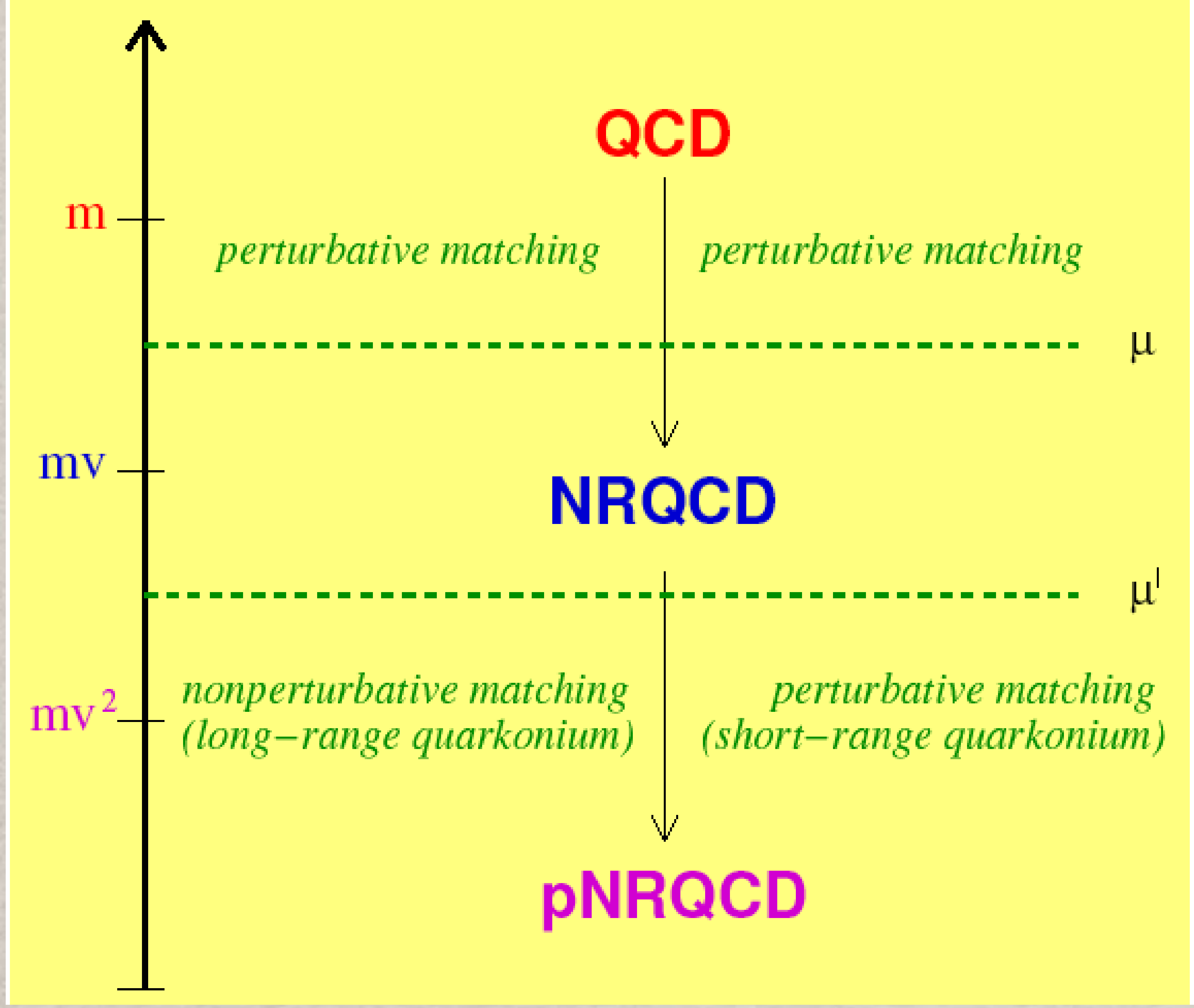
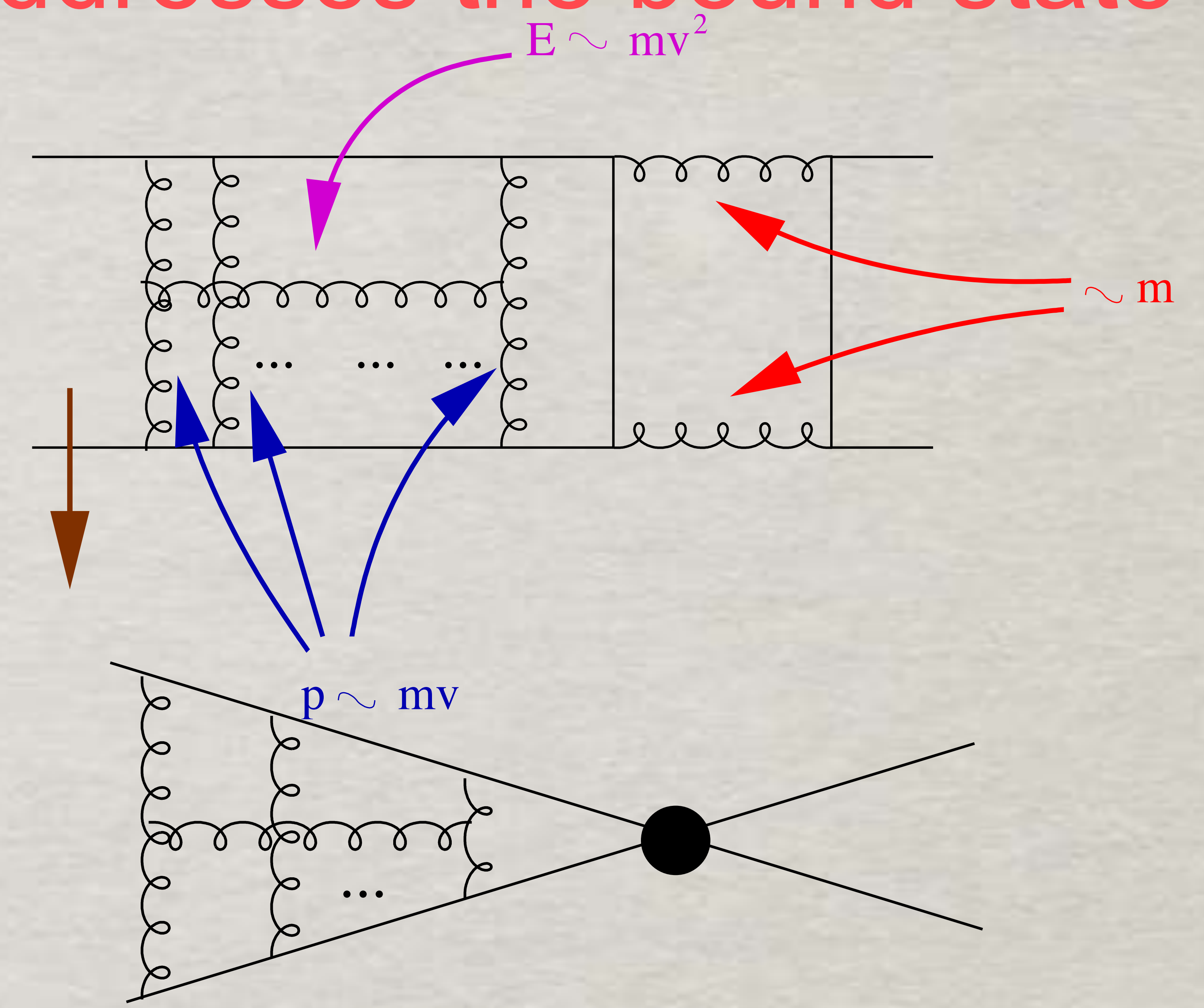




$$\mathcal{L}_{\rm NRQCD} = \sum_n c(\alpha_{\rm s}(m/\mu)) \times \frac{O_n(\mu,\lambda)}{m^n}$$

potential NonRelativistic QCD (pNRQCD): addresses the bound state formation





(with perturbative matching)

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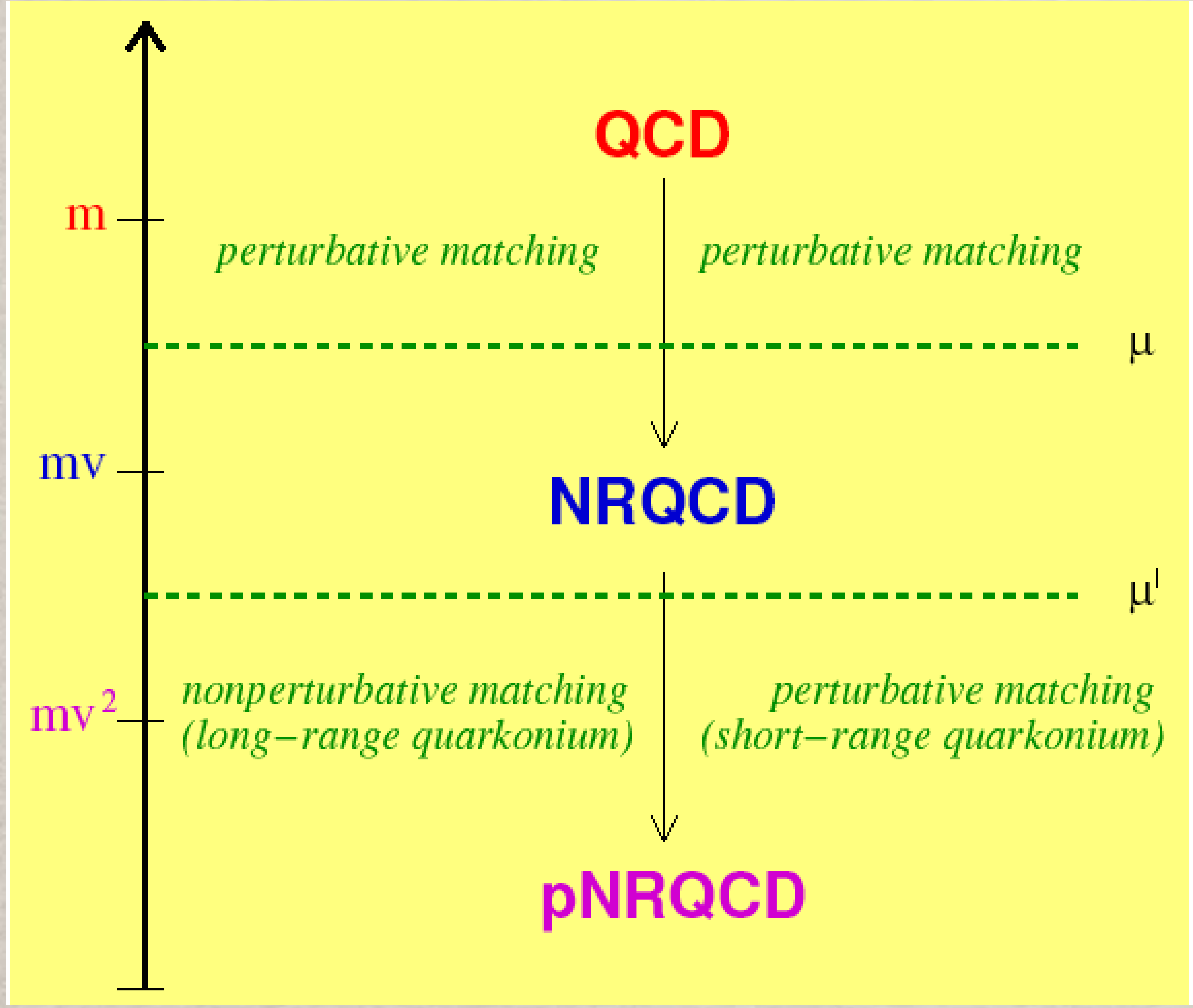
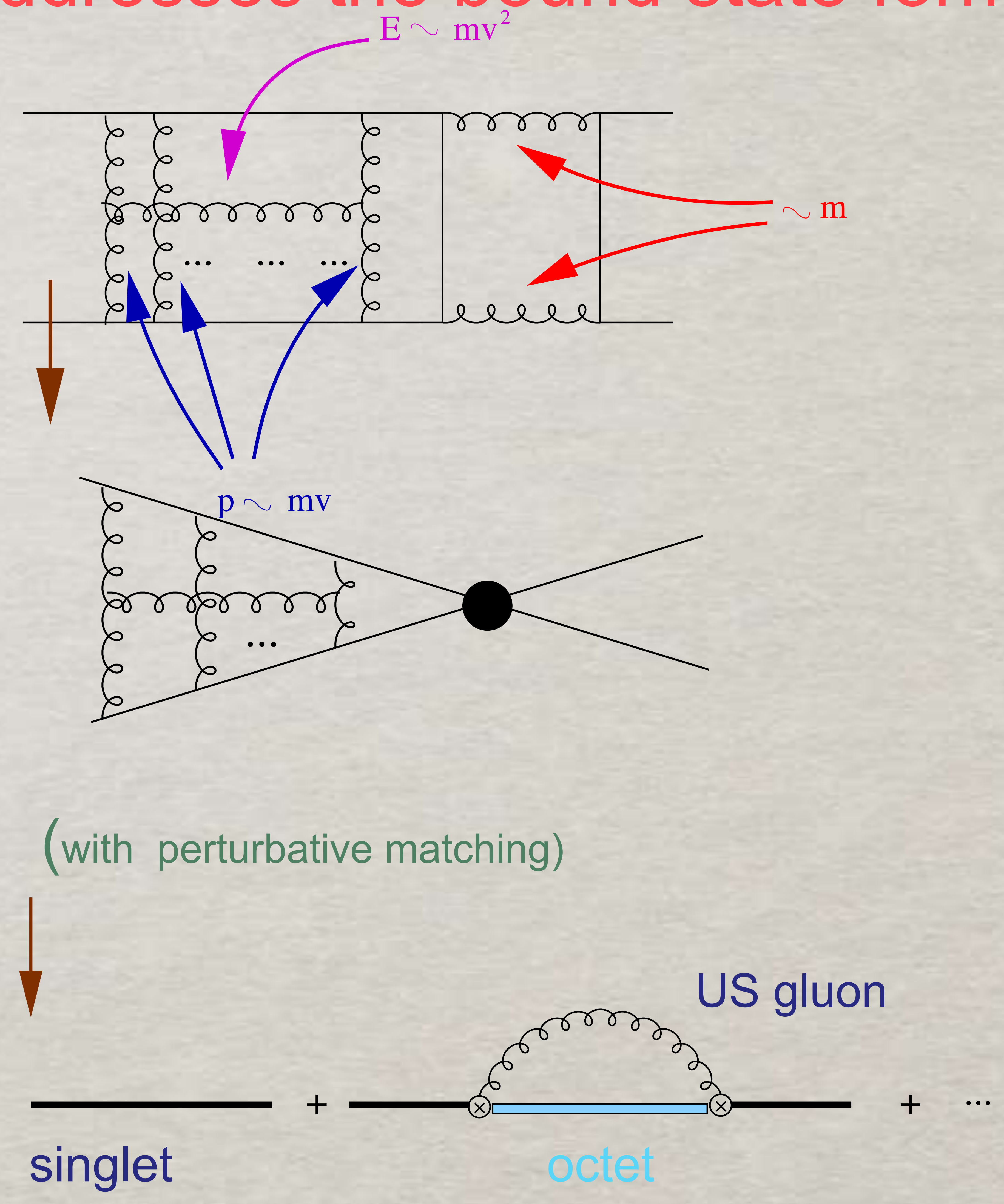
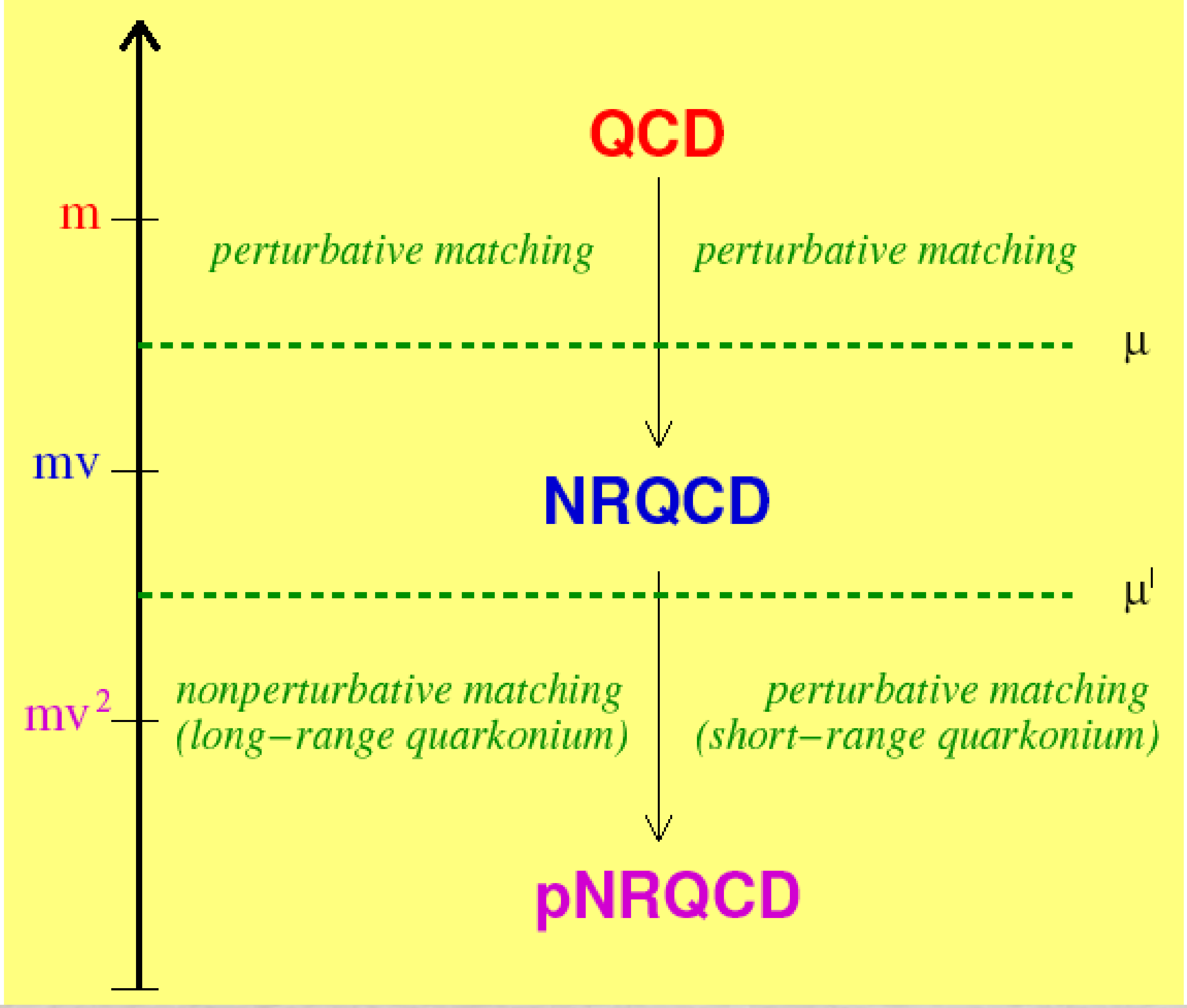
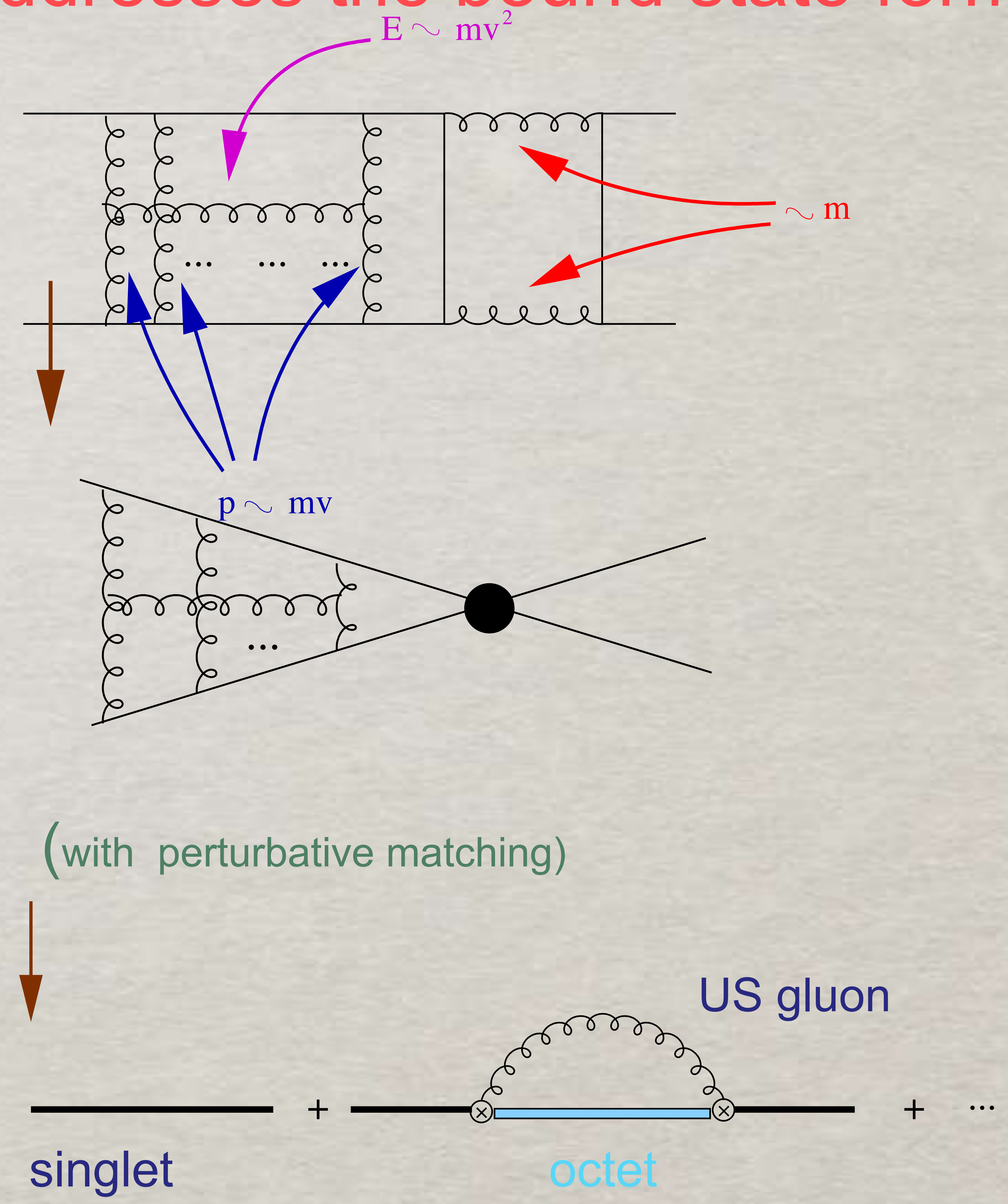




potential NonRelativistic QCD (pNRQCD): addresses the bound state formation





$$\mathcal{L}_{\rm pNRQCD} = \sum_k \sum_n \frac{1}{m^k} c_k(\alpha_s(m/\mu)) \times V(r\mu', r\mu) \times O_n(\mu', \lambda)\, r^n$$

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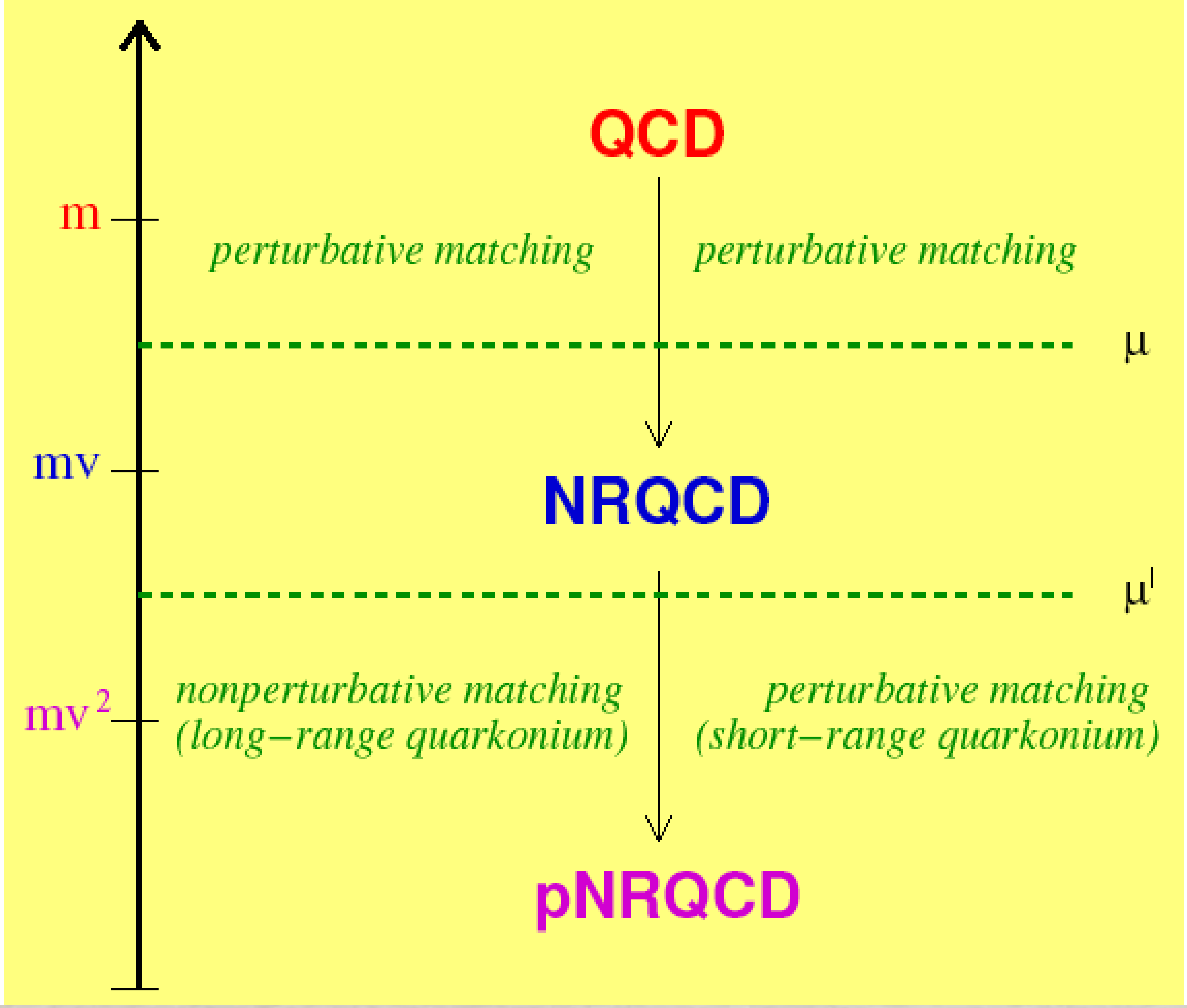

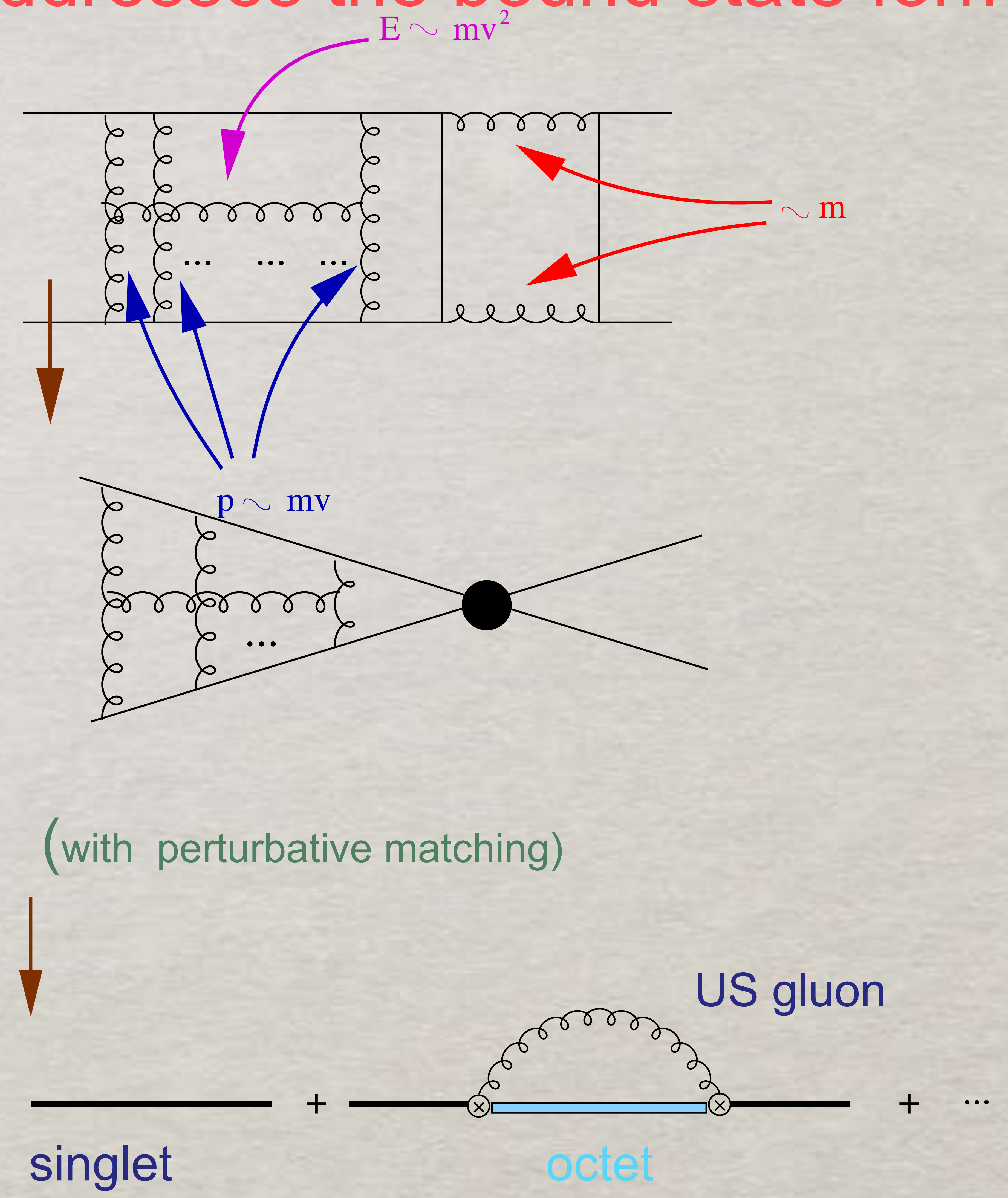

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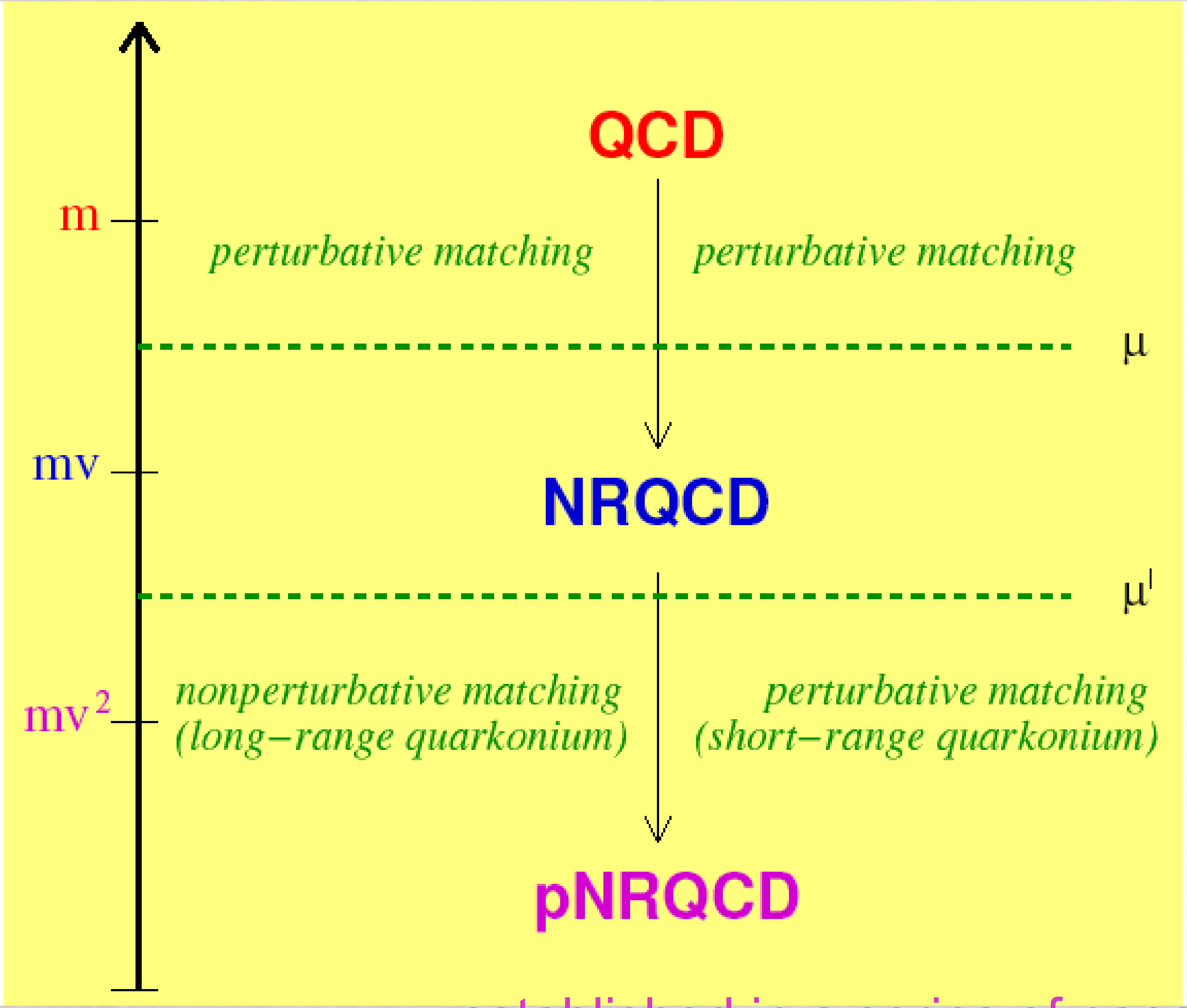
Quarkonium with EFT



Caswell, Lepage 86,
Lepage, Thacker 88
Bodwin, Braaten, Lepage 95......

Pineda, Soto 97, N.B. et al, 99,00,
Luke Manohar 97, Luke Savage 98,
Beneke Smirnov 98, Labelle 98
Labelle 98, Grinstein Rothstein 98
Kniehl, Penin 99, Griesshammer 00,
Manohar Stewart 00, Luke et al 00,
Hoang et al 01, 03->

established in a series of papers:
Pineda, Soto 97, N.B., Pineda, Soto, Vairo 99
N.B. Vairo, et al. 00–021
N.B., Pineda, Soto, Vairo Review of Modern Physis 77(2005) 1423

pNRQCD addresses the bound state dynamics

$$\mathcal{L}_{\rm pNREFT} = \int d^3r \phi^\dagger (i\partial_0 - \frac{\mathbf{p}^2}{m} - V)\phi + \Delta\mathcal{L}$$

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$$\mathcal{L}_{\text{pNREFT}} = \int d^3r \phi^\dagger (i\partial_0 - \frac{\mathbf{p}^2}{m} - V)\phi + \Delta\mathcal{L}$$

- It is obtained by integrating out hard and soft gluons with p or E scaling like m, mv.
- The d.o.f. are $Q\bar{Q}$ pairs (sometimes cast in color singlet S and color octet O) and ultrasoft modes (e.g. light quarks, low-energy gluons):
  $\phi = S$
- The Lagrangian is organized as an expansion in $1/m$ and r.
- The form of $\Delta\mathcal{L}$ and of the ultrasoft modes depends on the low energy dynamics.

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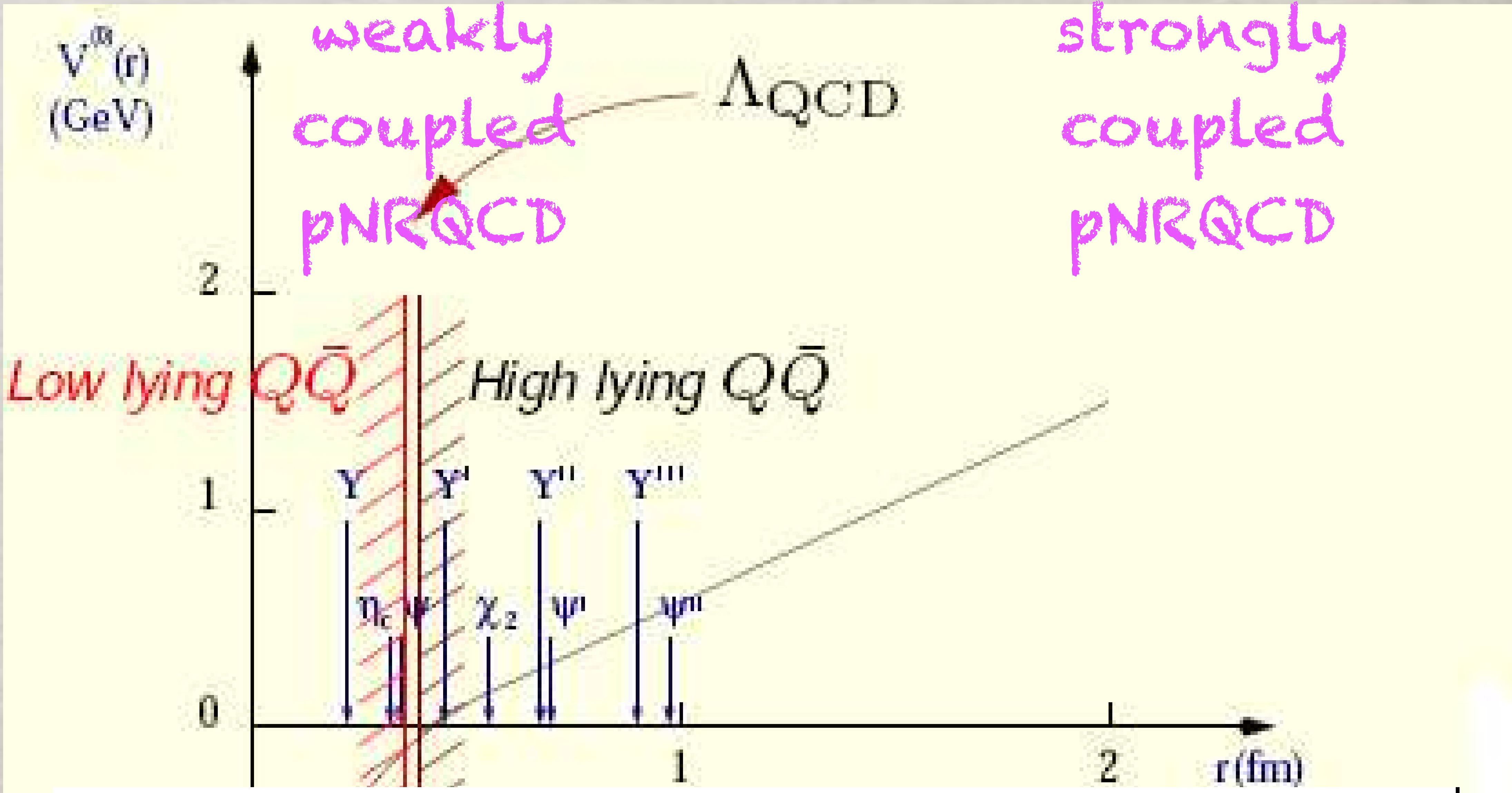
in QCD another scale is relevant Λ_{QCD}



A potential picture arises at the level of pNRQCD:
- the potential is perturbative if $mv \gg \Lambda_{\text{QCD}}$
- the potential is non-perturbative if $mv \sim \Lambda_{\text{QCD}}$

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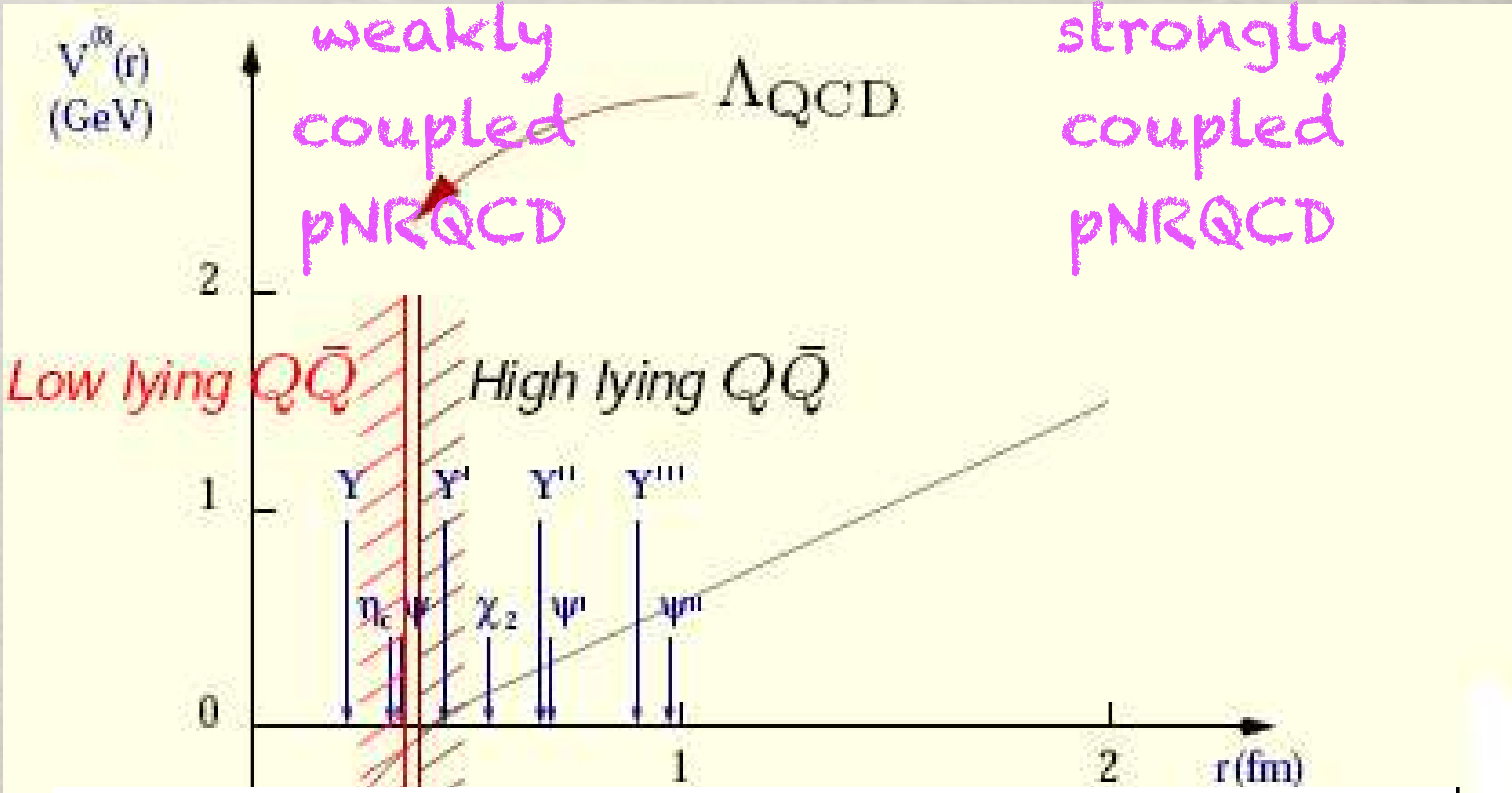
in QCD another scale is relevant $\Lambda_{\rm QCD}$

- The leading picture is Schoedinger eq.,the potentials appear once all scales above the energy have been been integrated out
- non potential effects appear as correction to the leading picture and are nonperturbative
- Any prediction of pNRQCD is a prediction of QCD at the given order of expansion
- Effects at the nonperturbative scale are carried by gauge invariant purely glue dependent correlators to be calculated on the lattice or in QCD vacuum models



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Weakly coupled pNRQCD

- Pineda Soto NP PS 64 (1998) 428
  Brambilla Pineda Soto Vairo NPB 566 (2000) 275

- If $mv \gg \Lambda_{\rm QCD}$, the matching is perturbative

  Non-analytic behaviour in $r \to$ matching coefficients V

The gauge fields are multipole expanded:
$A(R,r,t) = A(R,t) + \mathbf{r}\cdot\boldsymbol{\nabla}A(R,t) + \ldots$

$\mathbf{R}$ = center of mass
$\mathbf{r}$ = $Q\bar{Q}$ distance

$$\mathcal{L}^{\rm pNRQCD} = \int d^3r\,{\rm Tr}\,\{S^\dagger(i\partial_0 - \frac{\mathbf{p}^2}{m} - V_S + \cdots)S + O^\dagger(iD_0 - \frac{\mathbf{p}^2}{m} - V_O + \cdots)O +$$

LO in r

$$+V_A(S^\dagger\mathbf{r}\cdot g\mathbf{E}O + O^\dagger\mathbf{r}\cdot g\mathbf{E}S) + \frac{V_B}{2}(O^\dagger\mathbf{r}\cdot g\mathbf{E}O + O^\dagger O\mathbf{r}\cdot g\mathbf{E})\} + \ldots$$

NLO in r

$$-\frac{1}{4}F^a_{\mu\nu}F^{\mu\nu\,a} + \sum_{i=1}^{n_f}\bar{q}_i\,i\not{D}q_i$$

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The matching coefficients are the Coulomb potential

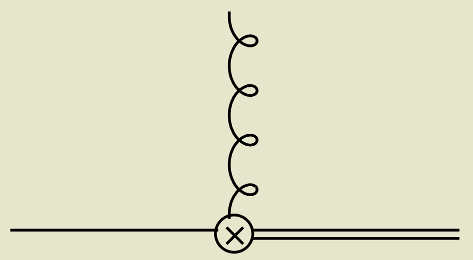
$$V_S(r) = -C_F\frac{\alpha_s}{r} + \ldots, \qquad V_o(r) = \frac{1}{2N}\frac{\alpha_s}{r} + \ldots,$$

$$V_A = 1 + \mathcal{O}(\alpha_s^2),\ V_B = 1 + \mathcal{O}(\alpha_s^2).$$

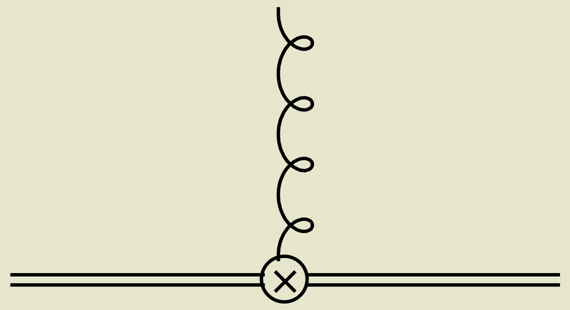
Feynman rules

$$= \theta(t)\, e^{-it(\mathbf{p}^2/m + V)}$$

$$= \theta(t)\, e^{-it(\mathbf{p}^2/m + V_o)}\left(e^{-i\int dt\, A^{\rm adj}}\right)$$



$= {\rm O}^\dagger \mathbf{r}\cdot g\mathbf{E}\,{\rm S}$

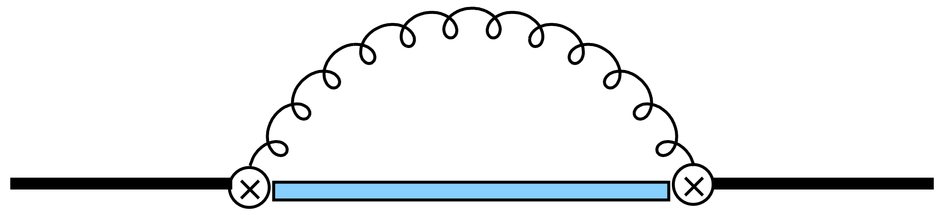


$= {\rm O}^\dagger\{\mathbf{r}\cdot g\mathbf{E},\, {\rm O}\}$

Energies at order m alpha^5 (NNNLO)

$m\alpha_s^5 \ln\alpha_s$ Brambilla Pineda Soto Vairo 99, Kniehl Penin 99
$m\alpha_s^5$ Kniehl Penin Smirnov Steinhauser 02 NNLL Pineda 02

NNNLL Peset Pineda et al 2018,2019, Kiyo Sumino 2014, Beneke, Kiyo Schuler 05,08

$$E_n = 2m + \langle n|\frac{p^2}{m} + V_s|n\rangle + \langle n|$$  $$|n\rangle$$

$\sim e^{i\Lambda_{\rm QCD} t}$

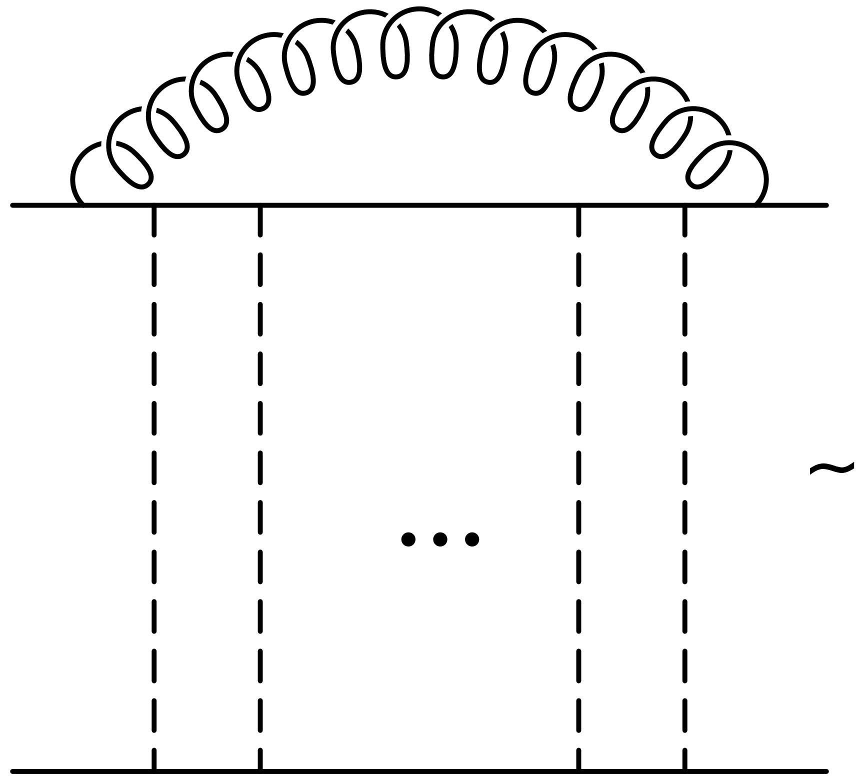
$$E_n = \langle n|H_s(\mu)|n\rangle - i\frac{g^2}{3N_c}\int_0^\infty dt\, \langle n|\mathbf{r}e^{it(E_n^{(0)}-H_o)}\mathbf{r}|n\rangle\, \langle \mathbf{E}(t)\,\mathbf{E}(0)\rangle(\mu)$$

$E_n^{(0)} - H_o \gg \Lambda_{\rm QCD} \Rightarrow \langle \mathbf{E}(t)\mathbf{E}(0)\rangle(\mu) \to \langle \mathbf{E}^2(0)\rangle$

local condensates as predicted in a paper by Misha Voloshin in 1979

$E_n^{(0)} - H_o \sim \Lambda_{\rm QCD} \Rightarrow$ no expansion possible, non-local condensates, analogous to the Lamb shift in QED

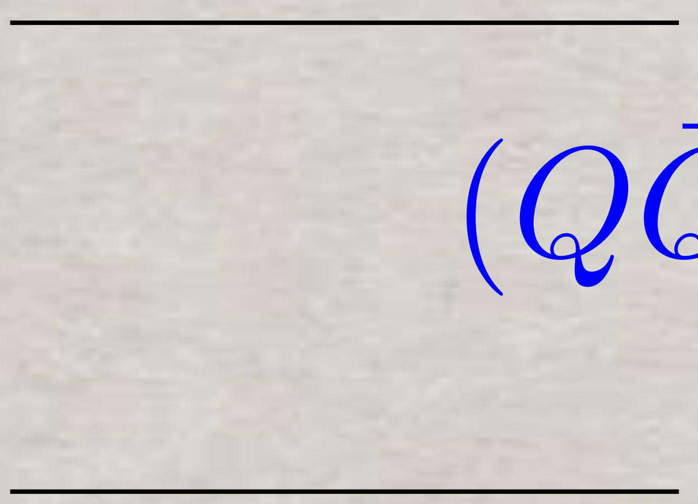
—>used to extract precise (NNNLO) determination of m_c and m_b

Σ  $\sim \alpha_s^5 \ln \alpha_s$ (Lamb-shift)

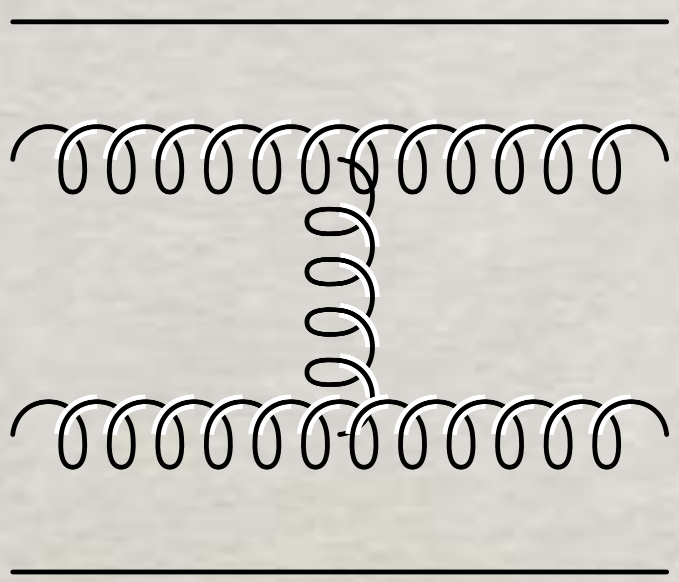
Applications of weakly coupled pNRQCD include:
ttbar production, quarkonia spectra, decays, E1 and M1 transitions, QQq and QQQ energies, thermal masses and potentials

Strongly coupled pNRQCD: Hitting the scale $\Lambda_{\rm QCD}$ $r \sim \Lambda_{QCD}^{-1}$

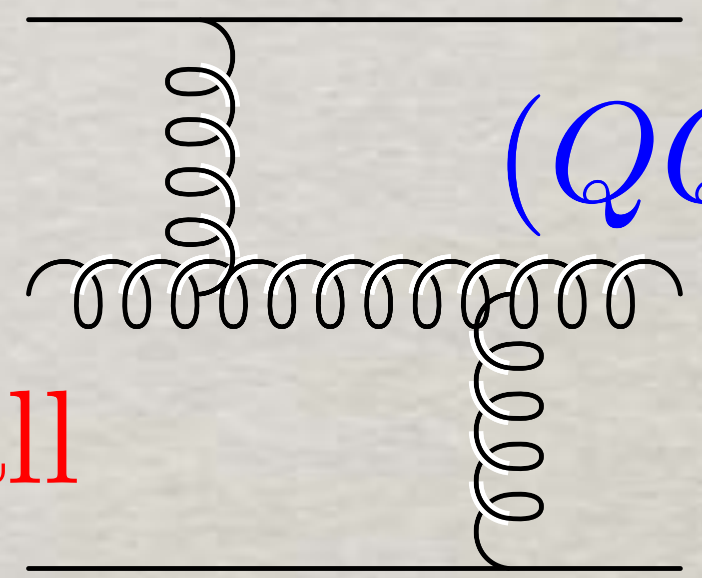
The degrees of freedoms now are



$(Q\bar{Q})_1$



$(Q\bar{Q})_1 + \mathrm{Glueball}$



$(Q\bar{Q})_8 G$

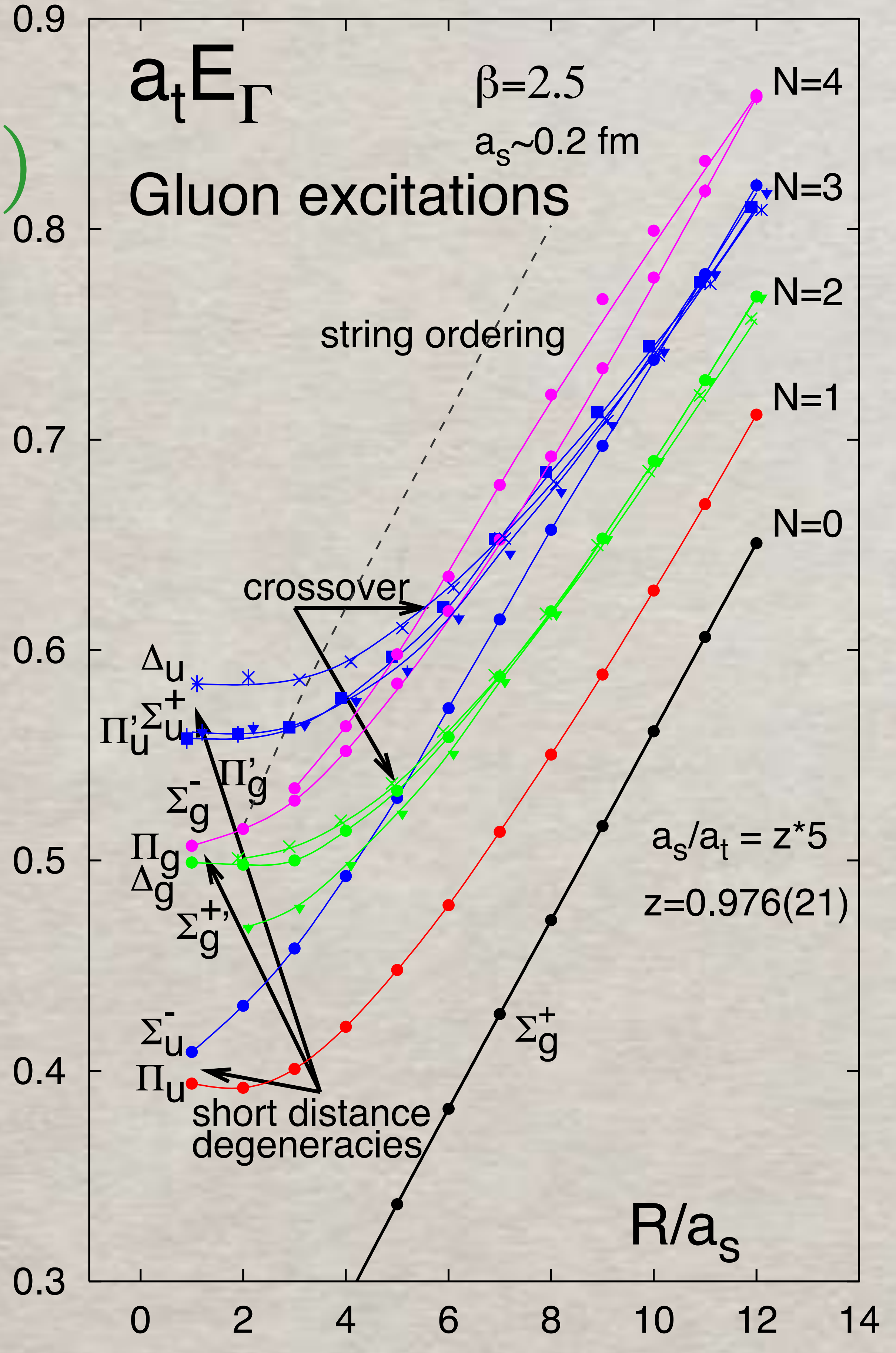
hybrid

with gluons at the scale $\Lambda_{\rm QCD}$ —>nonperturbative problem, use lattice

Strongly coupled pNRQCD: Hitting the scale Λ_{QCD} $r \sim \Lambda_{QCD}^{-1}$

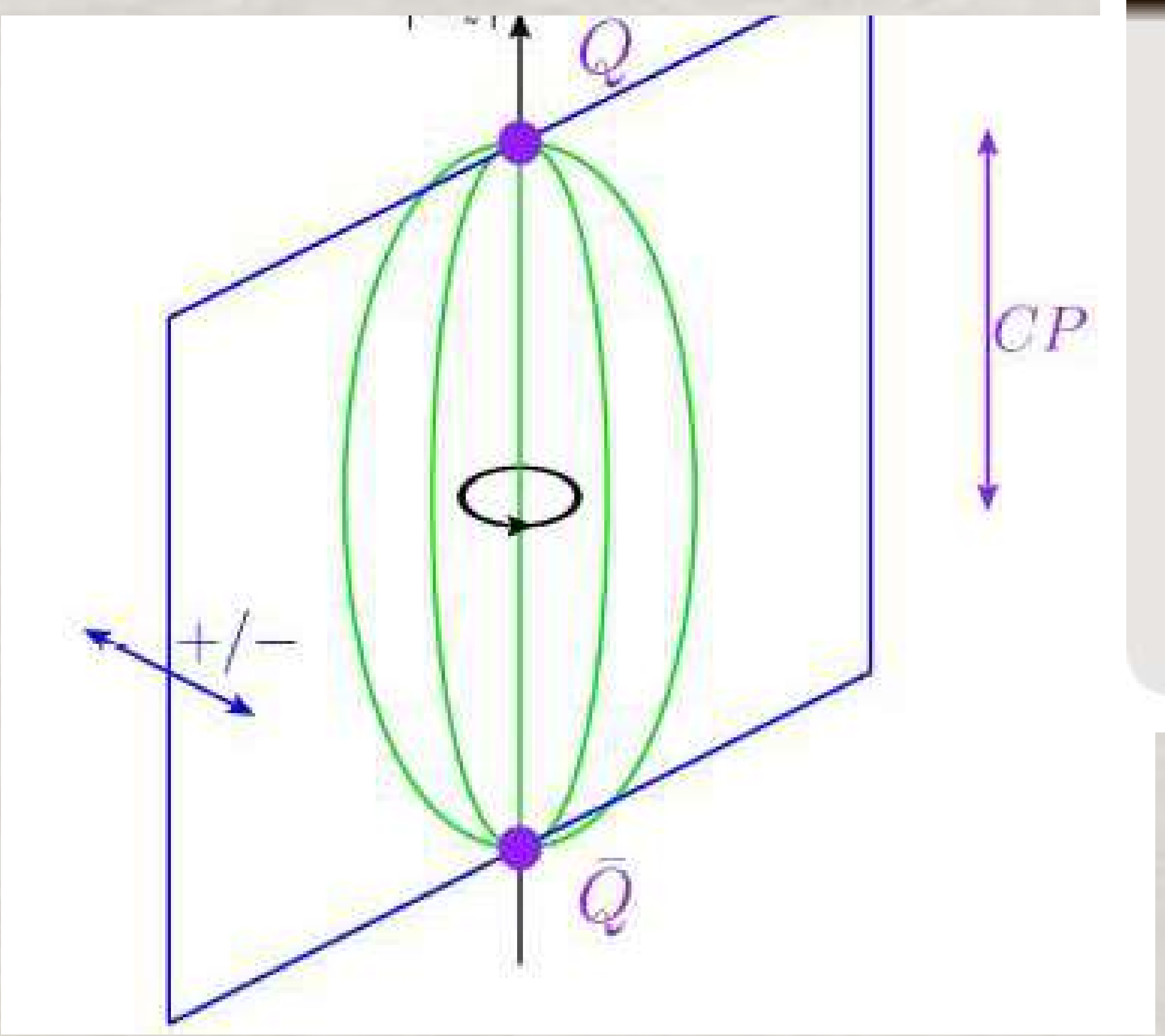
Spectrum of NRQCD static energies E^0_n from Lattice

$E_n^{(0)}$



$r = |\mathbf{x}_1 - \mathbf{x}_2|$

Λ



Irreducible representations of $D_{\infty h}$ Λ_η^σ

- $\boldsymbol{K}$: angular momentum of light d.o.f.
  $\lambda = \hat{\boldsymbol{r}} \cdot \boldsymbol{K} = 0, \pm 1, \pm 2, \pm 3, \ldots$
  $\Lambda = |\lambda| = 0, 1, 2, 3, \ldots \ (\Sigma, \Pi, \Delta, \Phi, \ldots)$
- Eigenvalue of CP: $\eta = +1\,(g),\ -1\,(u)$
- σ: eigenvalue of relfection about a plane containin

K is the angular momentum of the light degrees of freedom;same symmetry as the diatomic molecule

NRQCD

$$\mathcal{H}^{(0)}|\underline{n}; \mathbf{x}_1, \mathbf{x}_2\rangle^{(0)} = E_n^{(0)}(\mathbf{x}_1, \mathbf{x}_2)|\underline{n}; \mathbf{x}_1, \mathbf{x}_2\rangle^{(0)}$$

$$|\underline{n}; \mathbf{x}_1, \mathbf{x}_2\rangle^{(0)} = \psi^\dagger(\mathbf{x}_1)\chi(\mathbf{x}_2)|n; \mathbf{x}_1, \mathbf{x}_2\rangle^{(0)}$$

$\Lambda_\eta^\sigma \; -- > n$

NRQCD states

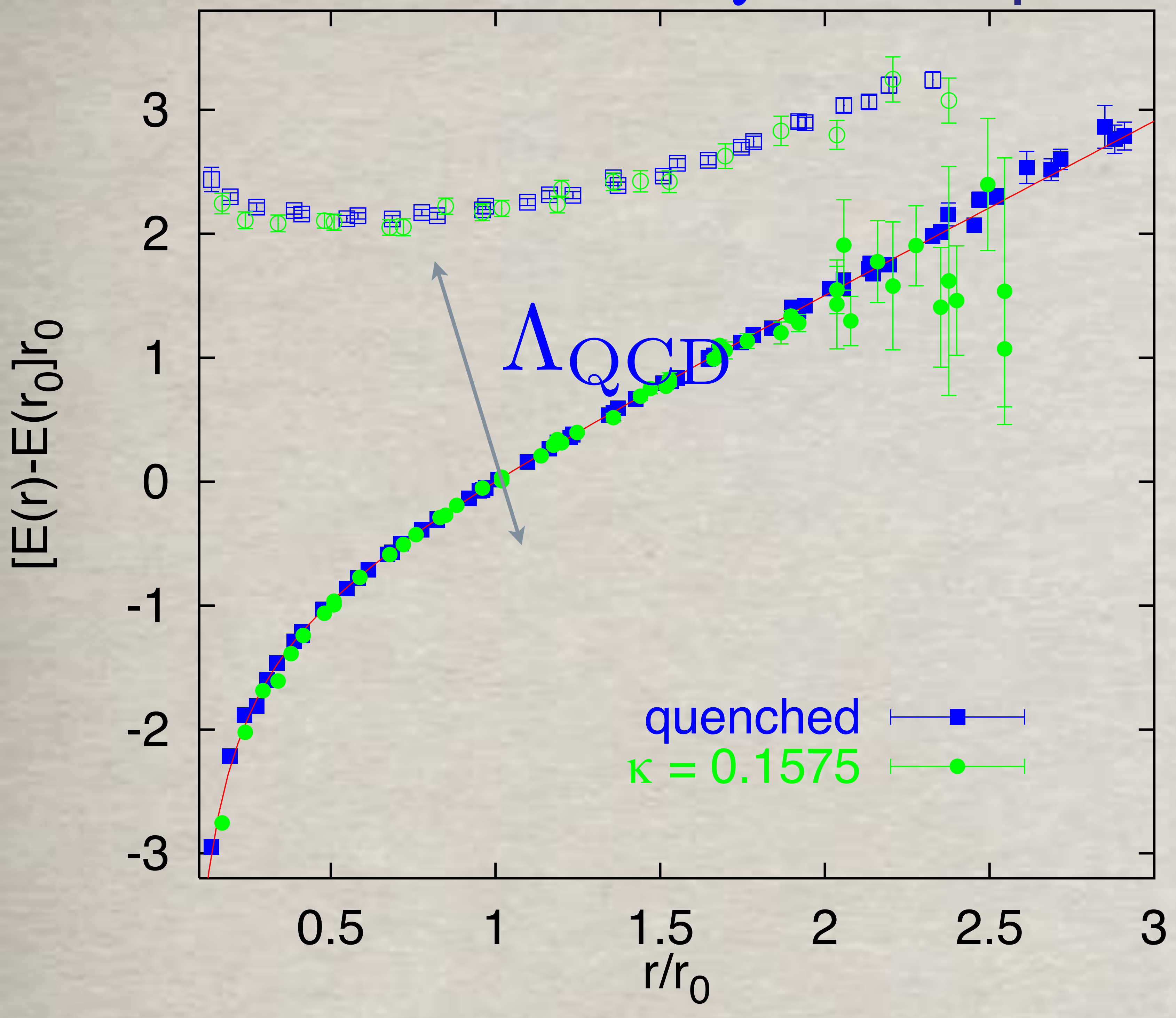
$|\underline{0}; \mathbf{x}_1 \mathbf{x}_2\rangle - >$ $|(Q\bar{Q})_1\rangle \rightarrow$ Quarkonium Singlet

pNRQCD states

$|\underline{n} > 0; \mathbf{x}_1 \mathbf{x}_2\rangle - >$ $|(Q\bar{Q})g^{(n)}\rangle \rightarrow$ Higher Gluonic Excitations

Juge Kuti Mornigstar 98-06

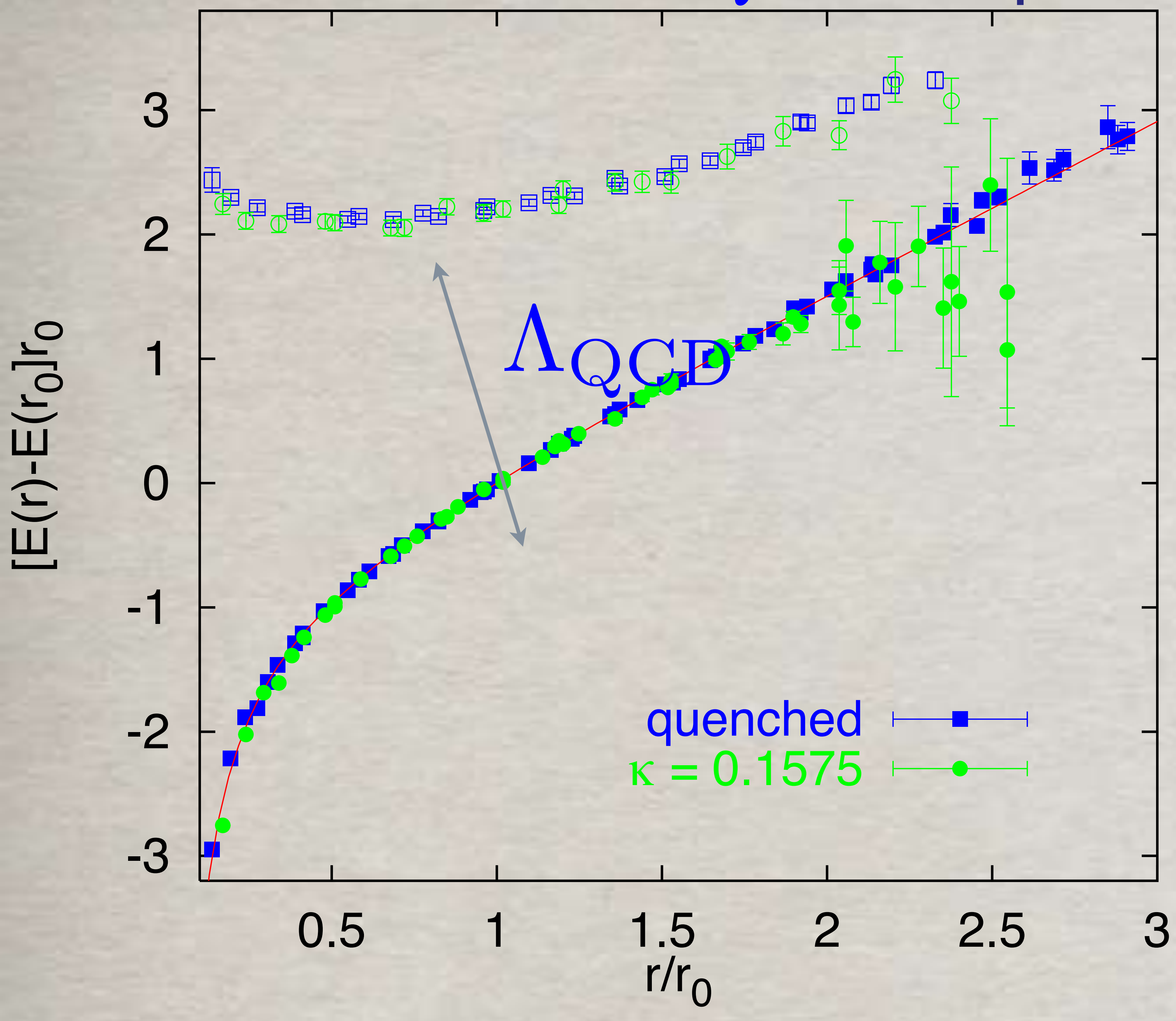
Bali et al. 98 $mv \sim \Lambda_{QCD}$



- pNRQCD and the potentials come from integrating out all scales up to mv^2
- gluonic excitations develop a gap $\Lambda_{\rm QCD}$ and are integrated out

Brambilla Pineda Soto Vairo 00

Bali et al. 98 $mv \sim \Lambda_{QCD}$



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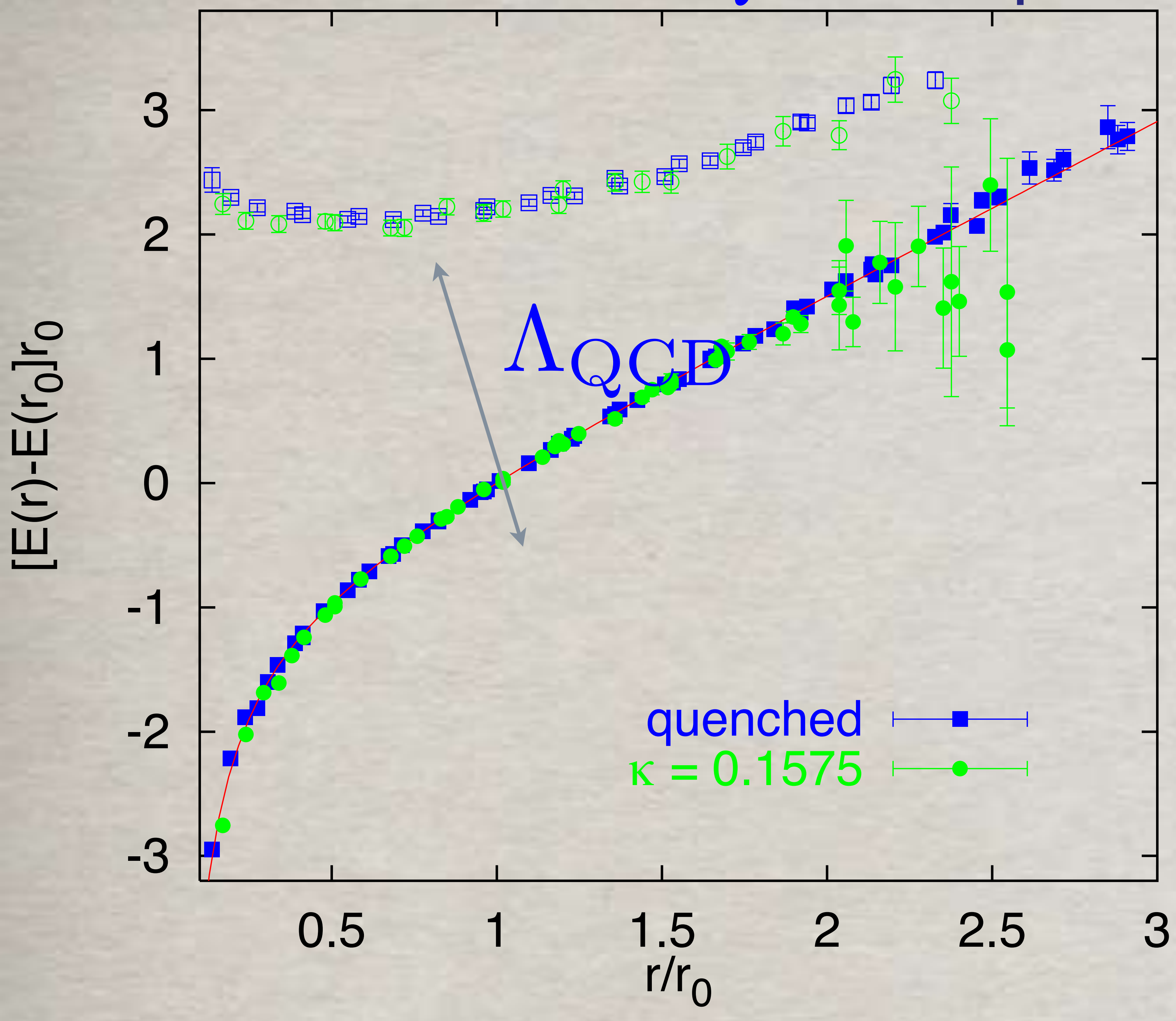
Brambilla Pineda Soto Vairo 00

$\Rightarrow$ The singlet quarkonium field S of energy mv^2 is the only the degree of freedom of pNRQCD (up to ultrasoft light quarks, e.g. pions).

Bali et al. 98 $mv \sim \Lambda_{QCD}$



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$$\mathcal{L} = \mathrm{Tr}\left\{ \mathrm{S}^\dagger \left(i\partial_0 - \frac{\mathbf{p}^2}{m} - V_s \right) \mathrm{S} \right\} \quad + \Delta\mathcal{L}(\text{US light quarks})$$

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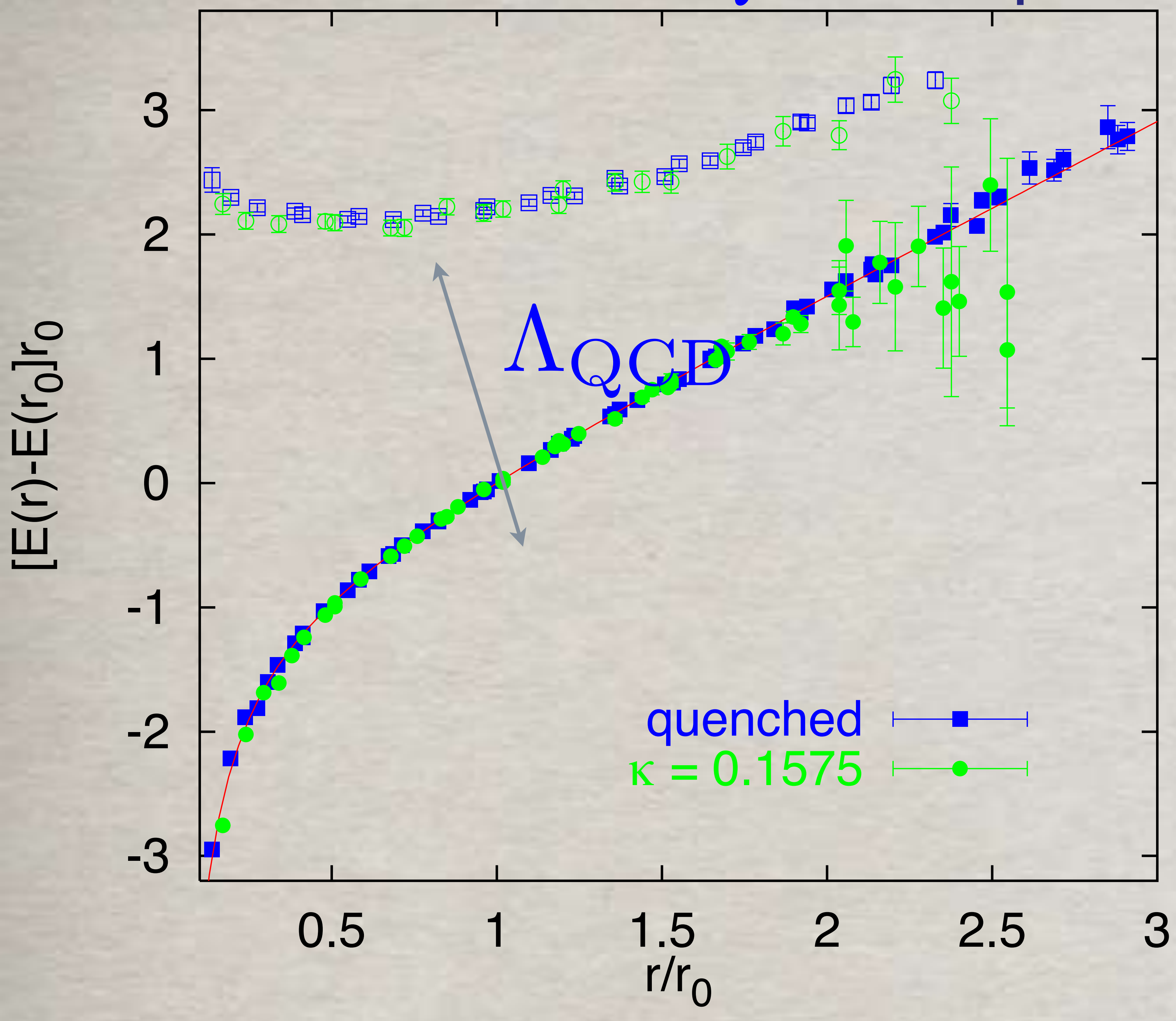

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- A pure potential description emerges from the EFT **however this is not the constituent quark model, alphas and masses are the QCD fundamental parameters**

- The potentials $V = \mathrm{Re}V + ImV$ from QCD in the matching: get spectra and decays

- We obtain the form of the nonperturbative potentials V in terms of generalized Wilson loops (stat that are low energy pure gluonic correlators: all the flavour dependence is pulled out

The singlet potential has the general structure

$$V = V_0 + \frac{1}{m} V_1 + \frac{1}{m^2}(V_{SD} + V_{VD})$$

static (V_0), spin dependent (V_{SD}), velocity dependent (V_{VD})

the fact that spin dependent corrections appear at order 1/m^2 is called Heavy Quark Spin Symmetry

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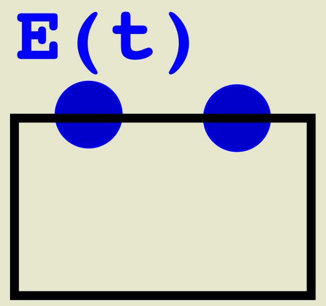
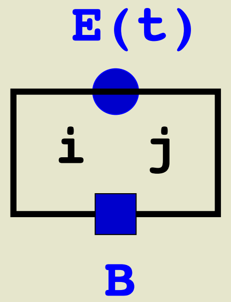
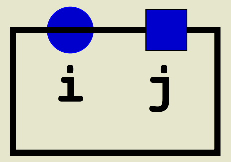
$V^{(1)} = -\frac{1}{2}\int_0^\infty dt\, t\, \langle$  $\rangle$

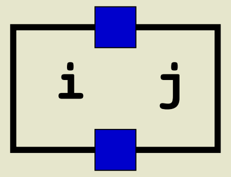
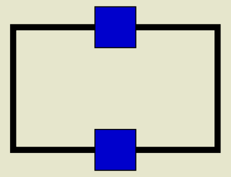
gauge invariant wilson loops can be calculated also in QCD vacuum model and large N (-> see Giancarlo talks)

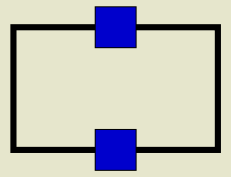
Pineda Vairo PRD 63 (2001) 054007

Brambilla Pineda Soto Vairo PRD 63 (2001) 014023

$V^{(2)}_{\rm SD} = -\frac{r^k}{r^2} c_F\, \epsilon^{kij}\, i \int_0^\infty dt\, t \langle$  $\rangle\ \mathbf{L}_1 \cdot \mathbf{S}_2 + (1 \leftrightarrow 2)\ |V^{(2)}_{LS}$

$-\frac{r^k}{r^2}\Big(c_F\, \epsilon^{kij}\, i \int_0^\infty dt\, t\, \langle$  $\rangle - \frac{2c_F - 1}{2}\nabla^k V^{(0)}\Big)\mathbf{L}_1 \cdot \mathbf{S}_1 + (1 \leftrightarrow 2)\ |V^{(1)}_{LS}$

$-c_F^2 \hat{r}_i \hat{r}_j i \int_0^\infty dt \Big(\langle$  $\rangle - \frac{\delta_{ij}}{3} \langle$  $\rangle \Big) \Big(\mathbf{S}_1 \cdot \mathbf{S_2} - 3(\mathbf{S}_1 \cdot \hat{\mathbf{r}})(\mathbf{S_2} \cdot \hat{\mathbf{r}})\Big)\ |V_T$

$+\Big(\frac{2}{3} c_F^2 i \int_0^\infty dt \langle$  $\rangle - 4\Big(d_{sv} + \frac{4}{3} d_{vv}\Big)\ \delta^{(3)}(\mathbf{r})\Big)\mathbf{S}_1 \cdot \mathbf{S_2}\ |V_S$

$c_F = 1 + \alpha_s/\pi(13/6 + 3/2 \ln m/\mu) + \ldots),\ d_{sv,vv} = O(\alpha_s^2)$ from NRQCD.

The singlet potential has the general structure

$$V = V_0 + \frac{1}{m} V_1 + \frac{1}{m^2}(V_{SD} + V_{VD})$$

static — spin dependent — velocity dependent

the fact that spin dependent corrections appear at order 1/m^2 is called Heavy Quark Spin Symmetry

E(t)

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Pineda Vairo PRD 63 (2001) 054007

Brambilla Pineda Soto Vairo PRD 63 (2001) 014023

E(t), B

$$V_{\rm SD}^{(2)} = -\frac{r^k}{r^2} c_F\, \epsilon^{kij}\, i \int_0^\infty dt\, t \langle \square_{ij} \rangle\, \mathbf{L}_1 \cdot \mathbf{S}_2 + (1 \leftrightarrow 2)\ |V_{LS}^{(2)}$$

$$-\frac{r^k}{r^2}\left(c_F\, \epsilon^{kij}\, i \int_0^\infty dt\, t\, \langle \square_{ij} \rangle - \frac{2c_F - 1}{2}\nabla^k V^{(0)}\right)\mathbf{L}_1 \cdot \mathbf{S}_1 + (1 \leftrightarrow 2)\ |V_{LS}^{(1)}$$

$$-c_F^2 \hat{r}_i \hat{r}_j i \int_0^\infty dt \left(\langle \square_{ij} \rangle - \frac{\delta_{ij}}{3}\langle \square \rangle\right)\left(\mathbf{S}_1 \cdot \mathbf{S}_2 - 3(\mathbf{S}_1 \cdot \hat{\mathbf{r}})(\mathbf{S}_2 \cdot \hat{\mathbf{r}})\right)\ |V_T$$

$$+\left(\frac{2}{3} c_F^2 i \int_0^\infty dt \langle \square \rangle - 4\left(d_{sv} + \frac{4}{3} d_{vv}\right)\delta^{(3)}(\mathbf{r})\right)\mathbf{S}_1 \cdot \mathbf{S}_2\ |V_S$$

$c_F = 1 + \alpha_s/\pi(13/6 + 3/2 \ln m/\mu) + \ldots)$, $d_{sv,vv} = O(\alpha_s^2)$ from NRQCD.

- the potentials contain the contribution of the scale m inherited from NRQCD matching coefficients—> they cancel any QM divergences, good UV behaviour
- the nonperturbative part is factorized and depends only on the glue —> only one lattice calculation to get the dynamics and the observables instead of an ab initio calculation of multiple Green functions

pNRQCD can describe also quarkonium production and, together with open quantum systems, the nonequilibrium evolution of quarkonium in medium (in heavy ions)

—> **which has implications on the fact that BOEFT could do the same**

X Y Z : close or above the quarkonium strong decay threshold

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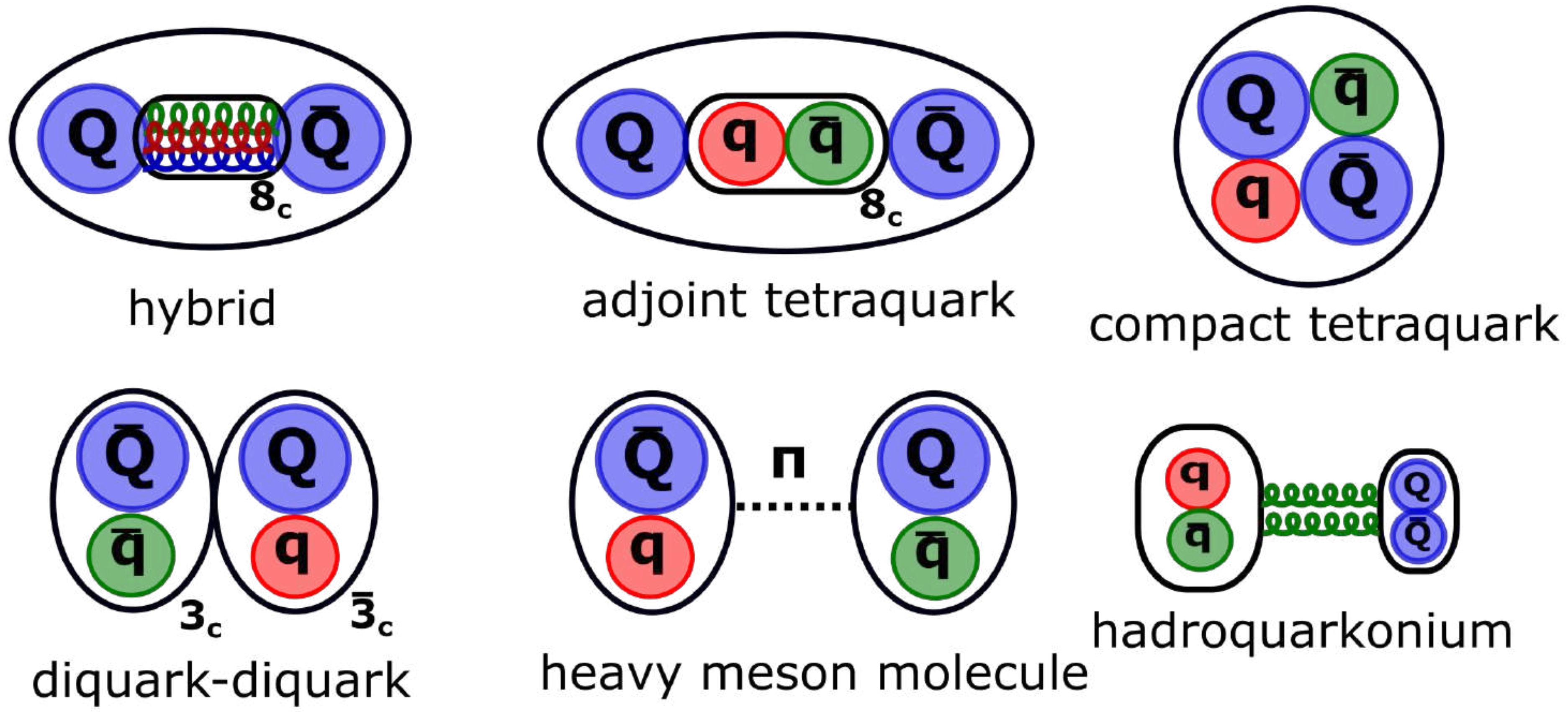

depending on the underlying QCD dynamics

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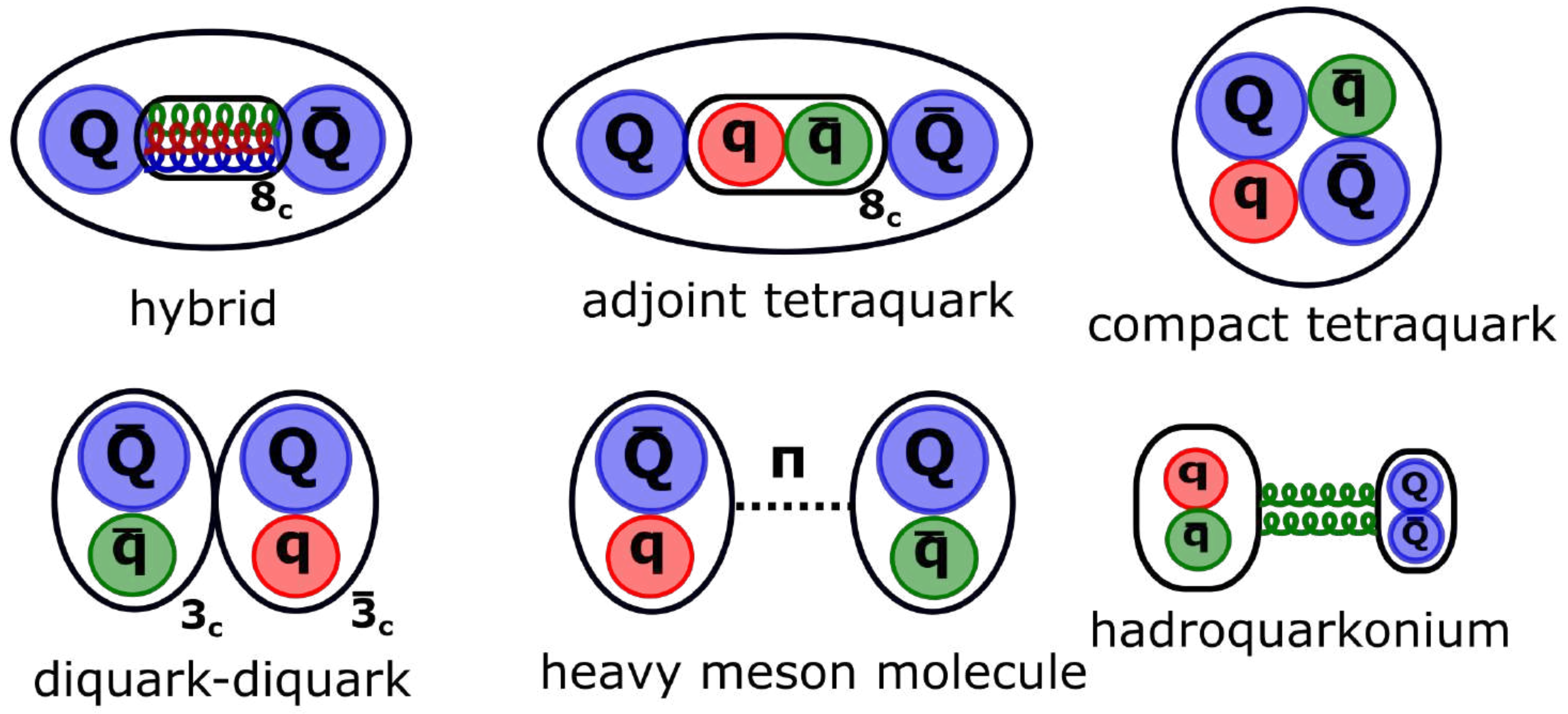


depending on the underlying QCD dynamics

Still: m is the bigger scale —> NRQCD is still valid

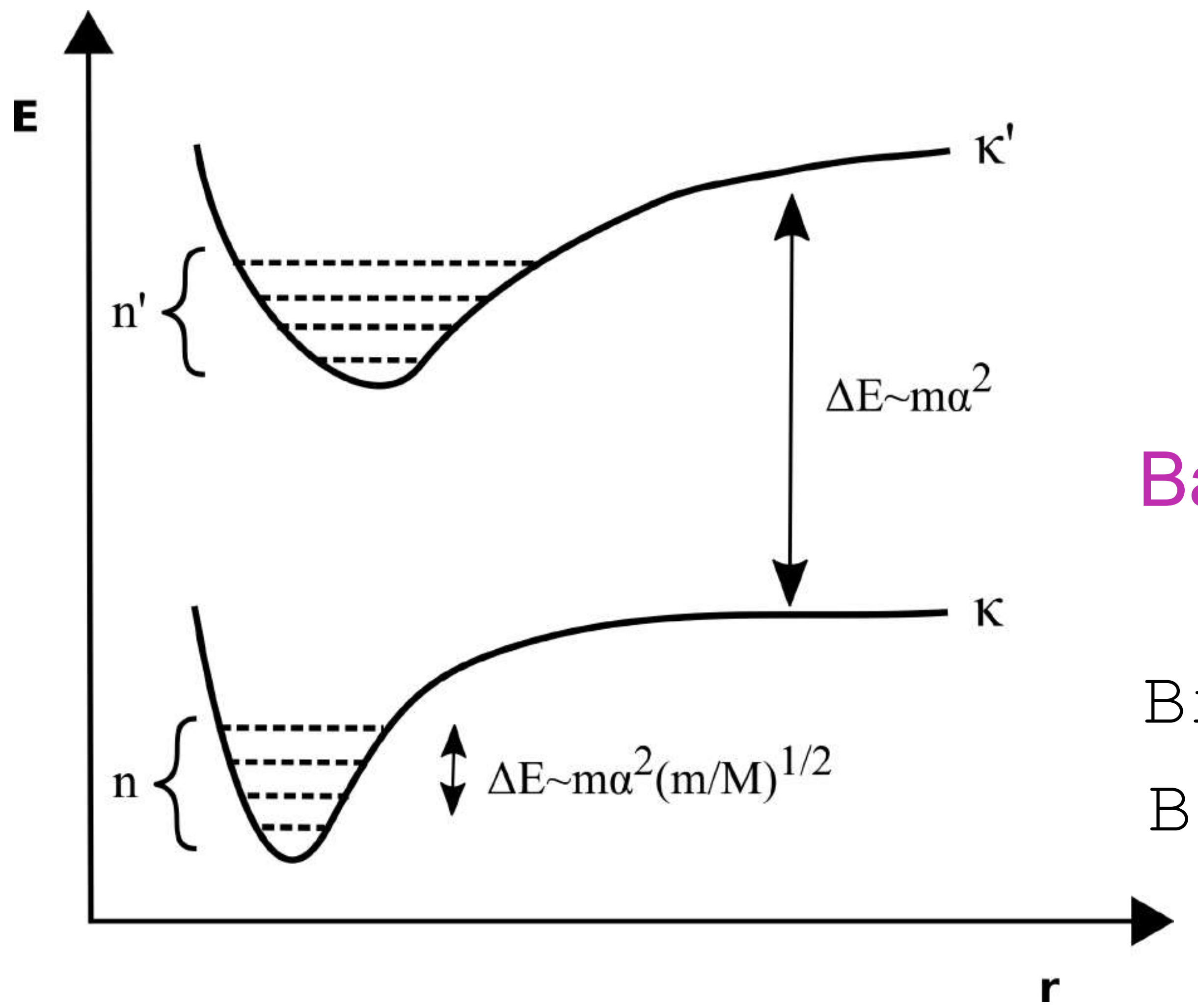
another separation of scales allows to construct an EFT —> BOEFT

BOEFT: EFT for nonrelativistic pairs and light d.o.f.

Consider bound states of two nonrelativistic particles and some light d.o.f., e.g., molecules/quarkonium hybrids ($Q\bar{Q}g$ states)or tetraquarks ($Q\bar{Q}q\bar{q}$ states):

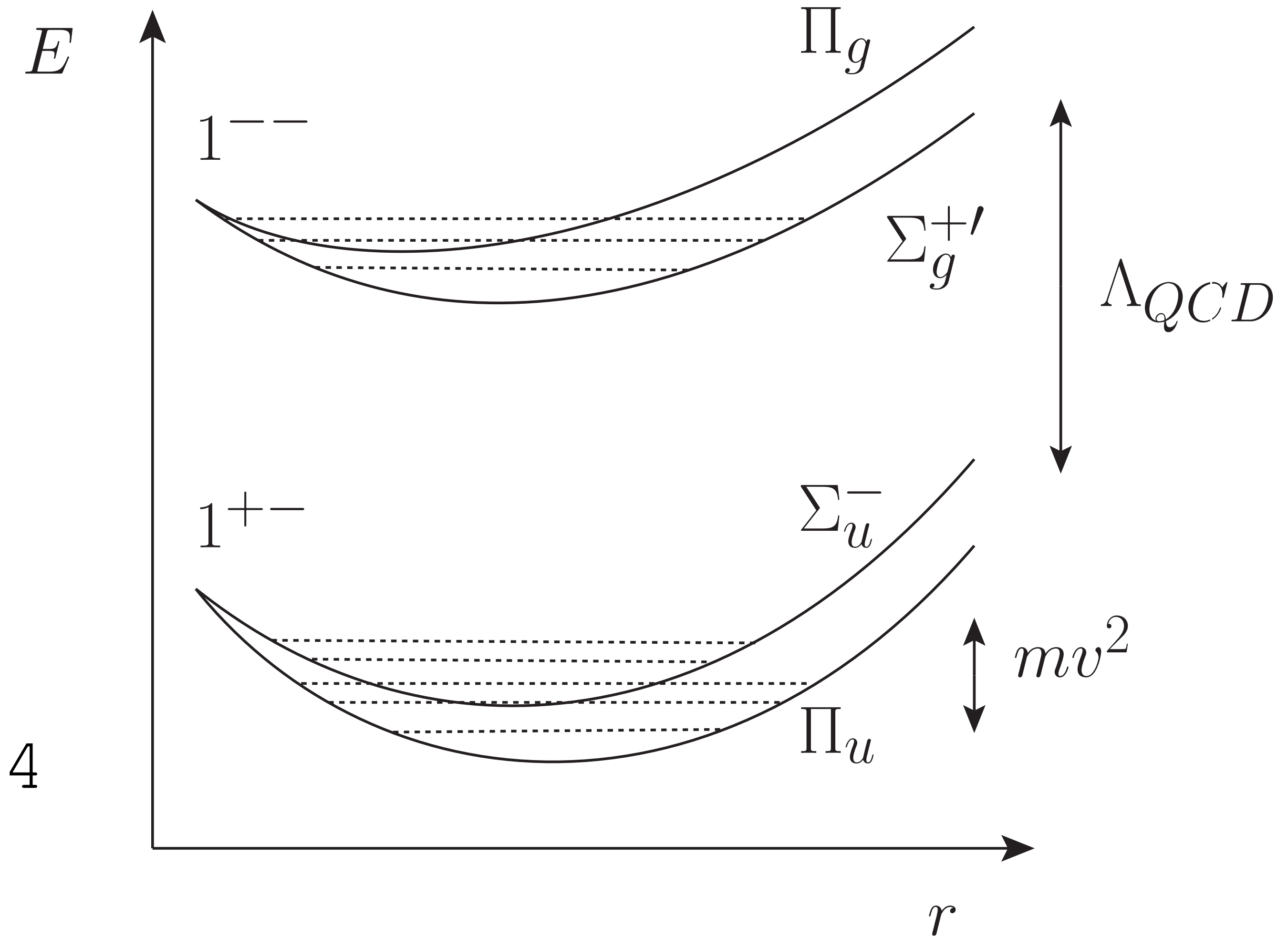
- electron/gluon fields change adiabatically in the presence of heavy quarks/nuclei. The heavy quarks/nuclei interaction may be described at leading order in the non-relativistic expansion by an effective potential V_κ between static sources, where κ labels different excitations of the light d.o.f.
- a plethora of states can be built on each of the potentials V_κ by solving the corresponding Schrödinger equation.

This picture goes also under the name of Born-Oppenheimer approximation. Starting from pNRQED/pNRQCD the Born-Oppenheimer approximation can be made rigorous and cast into a suitable nonrelativistic EFT called Born–Oppenheimer EFT (BOEFT).



Lattice evaluation of the QCD static energies:
Michael et al. 1983,
Juge, Kuti, Morningstar 1997, 1998,
Bali Pineda 2004, Capitani, Philipsen, Reisinger,
Riehl, Wagner 2018

Braaten PRL 111 (2013) 162003
Braaten Langmack Smith PRD 90 (2014) 014044



Focus on hybrids

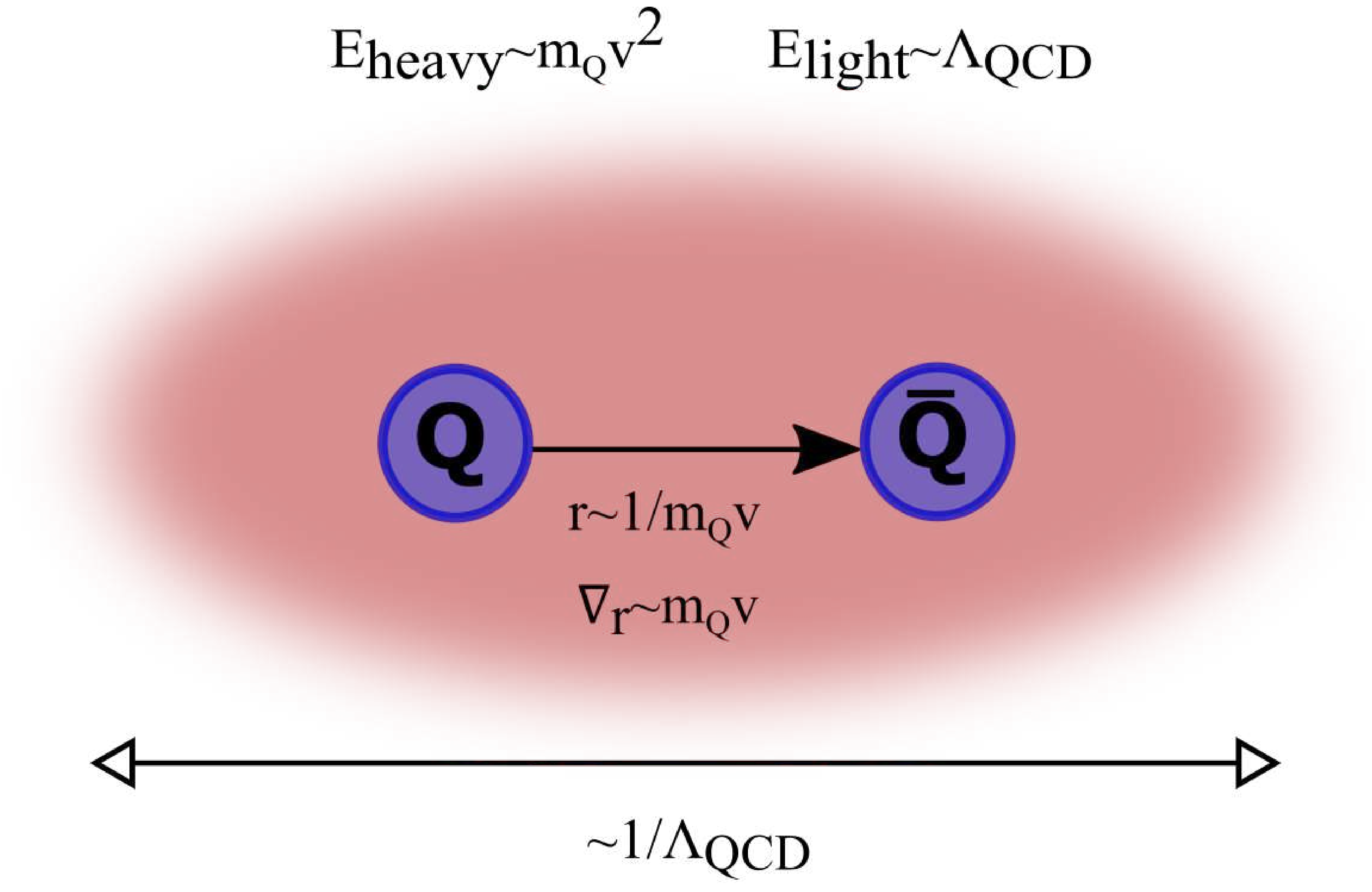
two different scales

$\Lambda_{\rm QCD} \gg mv^2$

we proceed to integrate out 1/r and then $\Lambda_{\rm QCD}$
(or simultaneously see Soto, Tarrus)

- 2005.00552

$\Lambda_{\rm QCD}$



analogous to

$E_{electrons} \gg E_{nuclei}$

in QED

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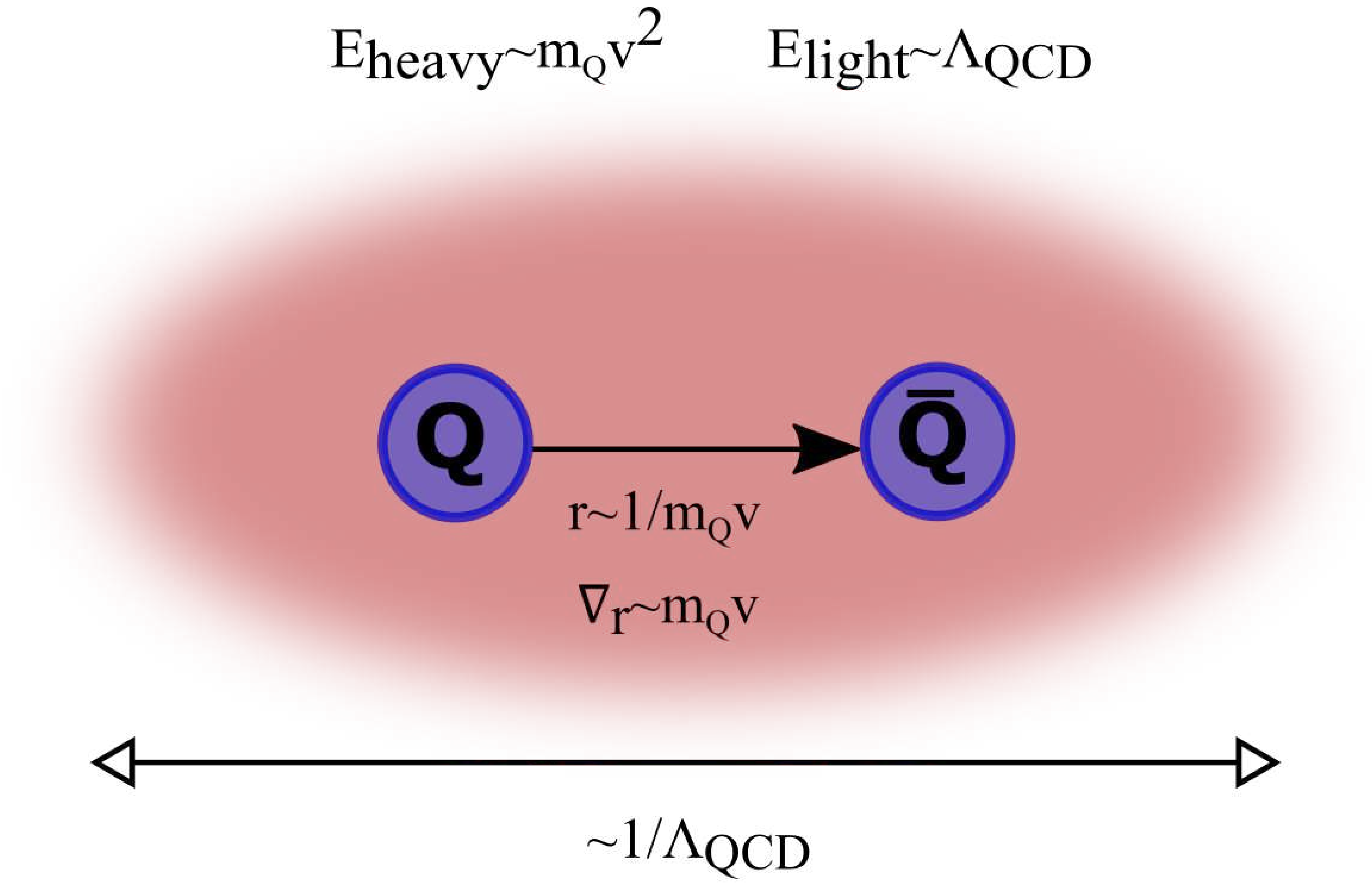


analogous to

$E_{electrons} \gg E_{nuclei}$

in QED

$\Lambda_{\rm QCD}$ is nonperturbative but we can use the lattice to calculate the appropriate gluonic static energies (corresponding in molecular physics to the electronic static energies)

Focus on hybrids

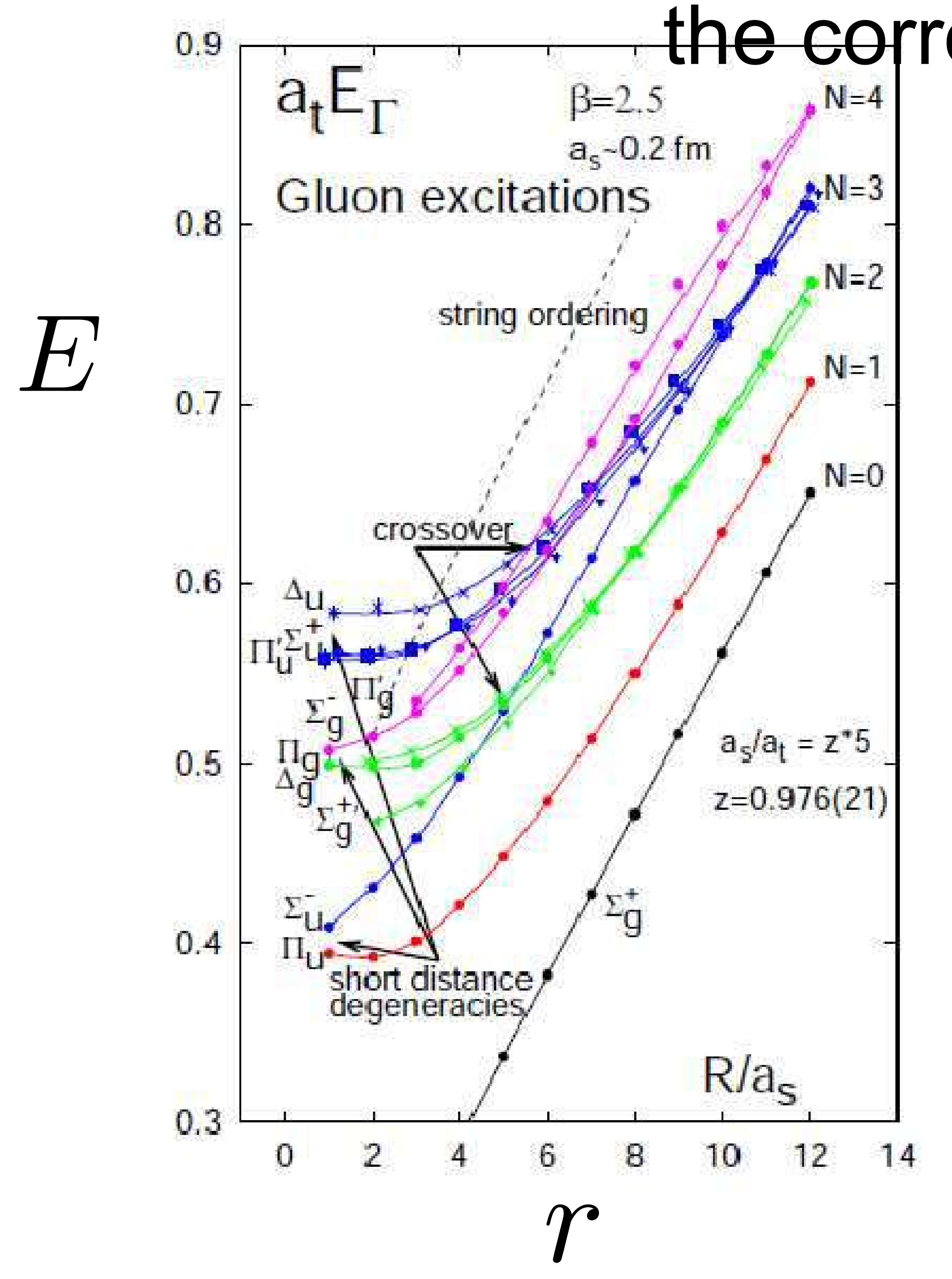
We need the static energies for the lattice

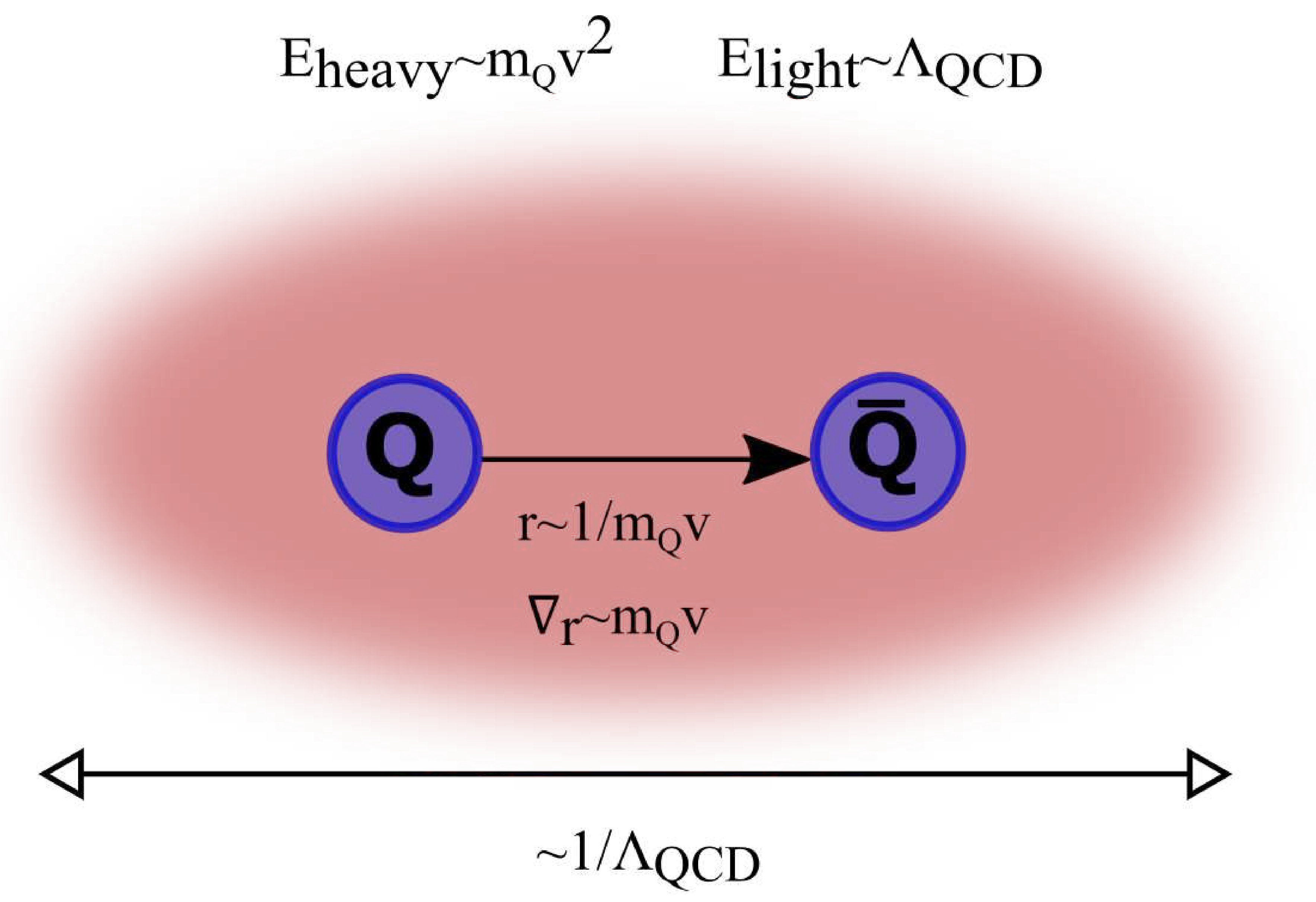
wilson loop

$$E_n^{(0)}(r) = \lim_{T\to\infty} \frac{i}{T} \log\langle X_n, T/2 | X_n, -T/2\rangle$$

$$|X_n\rangle = \chi(\mathbf{x_2})\phi(\mathbf{x_2},\mathbf{R})T^a H^a(\mathbf{R})\phi(\mathbf{R},\mathbf{x}_1)\psi^\dagger(\mathbf{x_1})|\text{vac}\rangle$$

Phi wilson lines and H gluonic operator with the correct quantum numbers





- Σ_g^+ is the ground state potential that generates the standard quarkonium states.
- The rest of the static energies correspond to excited gluonic states that generate hybrids.
- The two lowest hybrid static energies are Π_u and Σ_u^-, they are nearly degenerate at short distances.

Bali Pineda PRD69 (2004) 094001

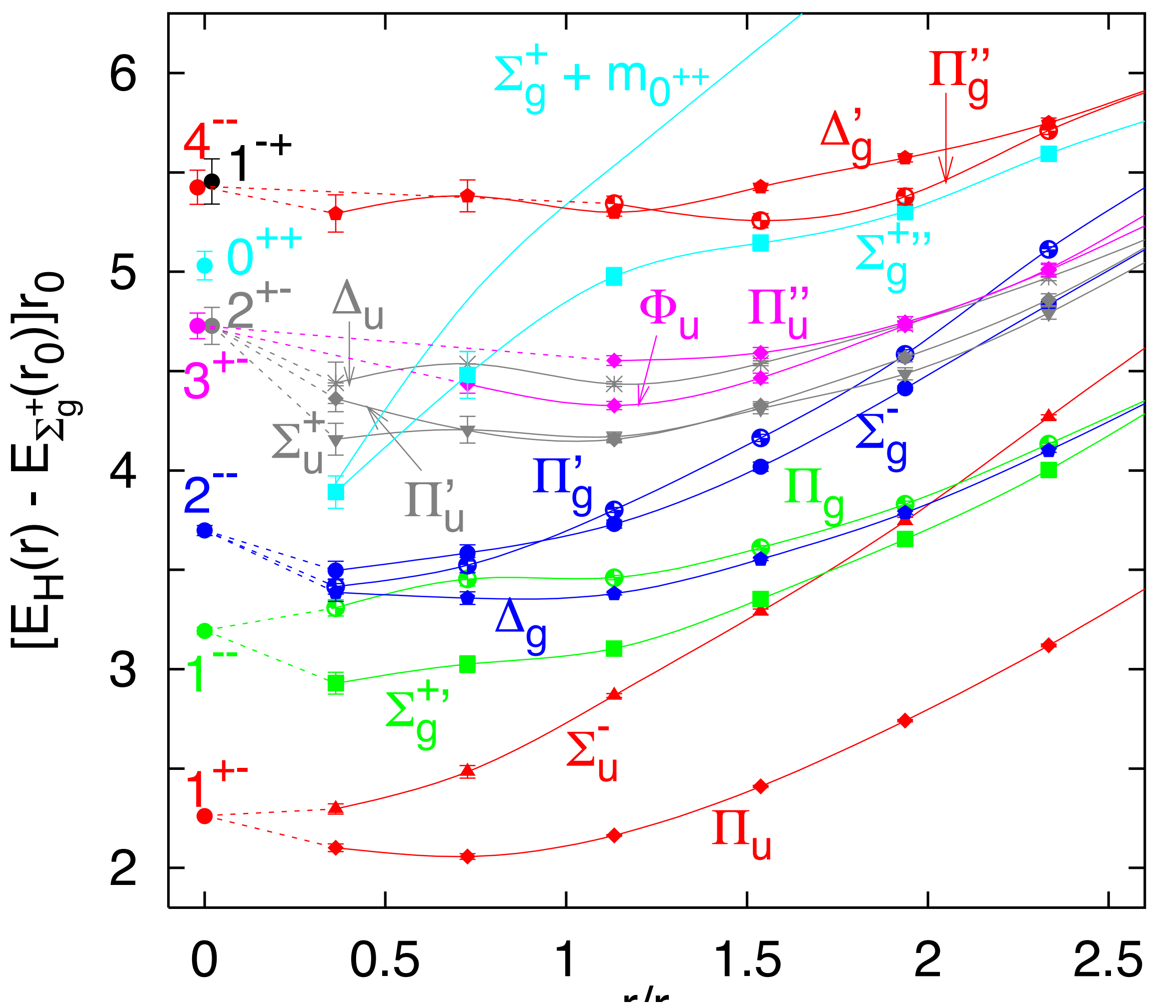
- Juge Kuti Morningstar PRL 90 (2003) 161601
  Capitani Philipsen Reisinger Riehl Wagner PRD 99 (2019) 03450

Schlosser, Wagner 2111.00741

We understand the static energies —>

The BOEFT characterises the hybrids static energy for short distance

In the short-range hybrids become gluelumps, i.e., quark-antiquark octets, O^a, in the presence of a gluonic field, H^a: $H(R,r,t) = H^a(R,t)O^a(R,r,t)$.



the hybrid static energy can be written as a (multipole) expansion in r:

$$E_g = \frac{\alpha_s}{6r} + \Lambda_g + a_g r^2 + \ldots$$

octet potential

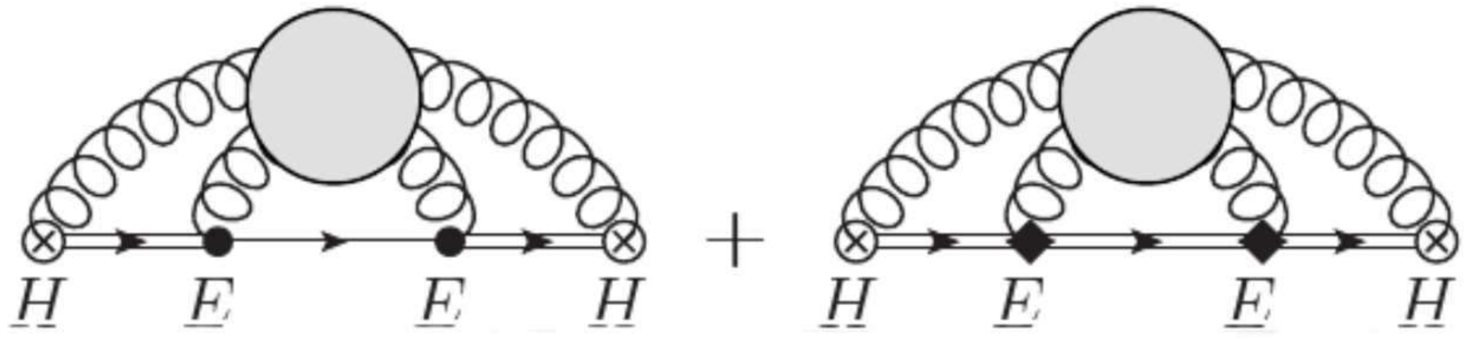
non perturbative coefficient

Λ_g is the gluelump mass: $\Lambda_g = \lim_{T\to\infty} \frac{i}{T} \ln\langle H^a(T/2)\phi^{\mathrm{adj}}_{ab}(T/2,-T/2)H^b(-T/2)\rangle$

calculated on the lattice

Foster Michael PRD 59 (1999) 094509
Bali Pineda PRD 69 (2004) 094001
Lewis Marsh PRD 89 (2014) 014502

a_g can be expressed as field correlators (single line = singlet, double line = octet), e.g.,



In the limit $r \to 0$ more symmetry: $D_{\infty h} \to O(3)\times C$

- Several Λ_η^σ representations contained in one J^{PC} representation:
- Static energies in these multiplets have same $r \to 0$ limit.

The gluelump multiplets Σ_u^-, Π_u; $\Sigma_g^{+\prime}$, Π_g; Σ_g^-, Π_g', Δ_g; Σ_u^+, Π_u', Δ_u are degenerate.

Gluonic excitation operators up to dim 3

Λ_η^σ	K^{PC}	H^a
Σ_u^-	1^{+-}	$\mathbf{r}\cdot\mathbf{B}$, $\mathbf{r}\cdot(\mathbf{D}\times\mathbf{E})$
Π_u	1^{+-}	$\mathbf{r}\times\mathbf{B}$, $\mathbf{r}\times(\mathbf{D}\times\mathbf{E})$
$\Sigma_g^{+\prime}$	1^{--}	$\mathbf{r}\cdot\mathbf{E}$, $\mathbf{r}\cdot(\mathbf{D}\times\mathbf{B})$
Π_g	1^{--}	$\mathbf{r}\times\mathbf{E}$, $\mathbf{r}\times(\mathbf{D}\times\mathbf{B})$
Σ_g^-	2^{--}	$(\mathbf{r}\cdot\mathbf{D})(\mathbf{r}\cdot\mathbf{B})$
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Δ_g	2^{--}	$(\mathbf{r}\times\mathbf{D})^i(\mathbf{r}\times\mathbf{B})^j+(\mathbf{r}\times\mathbf{D})^j(\mathbf{r}\times\mathbf{B})^i$
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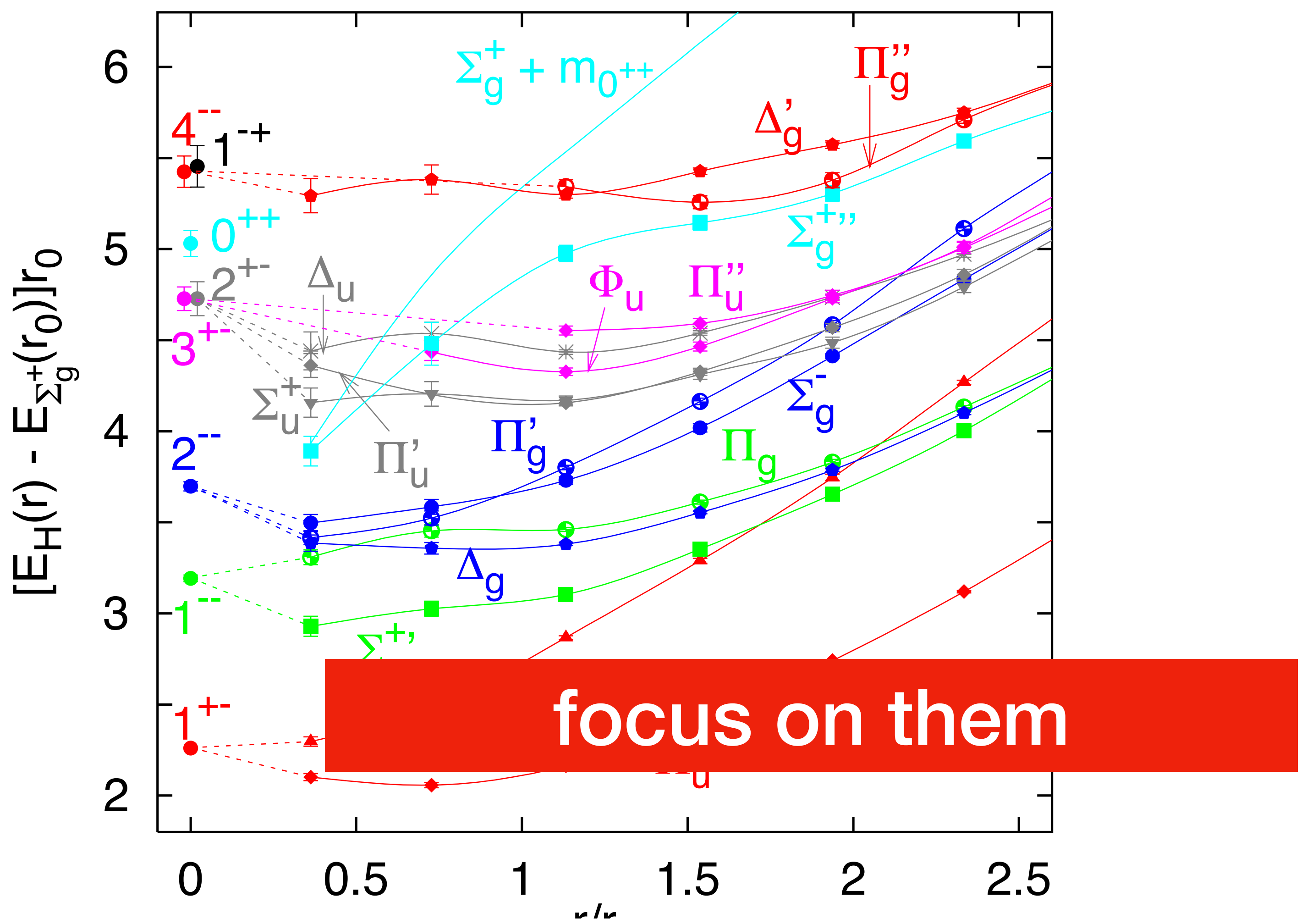

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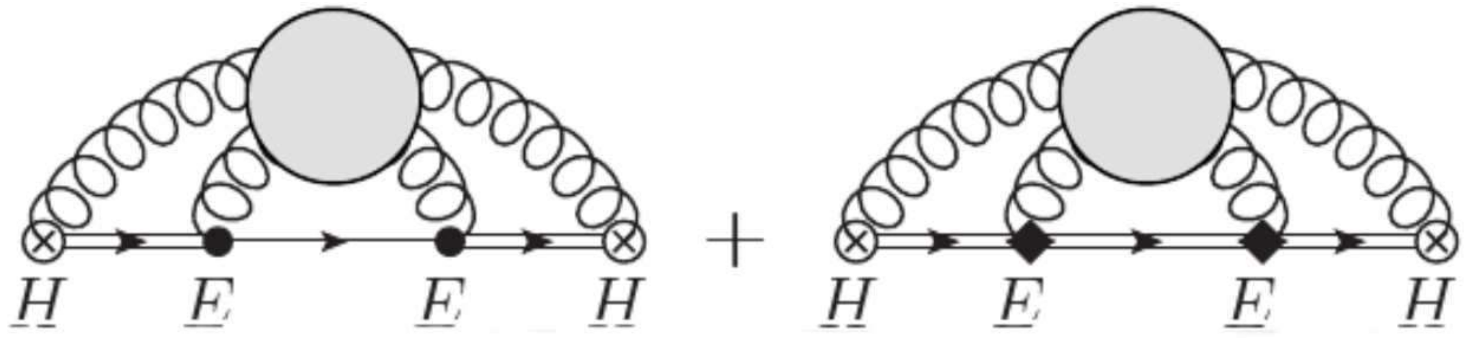

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$(\Lambda_\eta^\sigma = \Sigma_u^-, \Pi_u)$

Hybrids Multiplets

We consider hybrids that are excitations of the lowest lying static energies Π_u and Σ_u^-. In the $r \to 0$ limit Π_u and Σ_u^- are degenerate and correspond to a gluonic operator with quantum numbers 1^{+-}.

Multiplet	T	$J^{PC}(S=0)$	$J^{PC}(S=1)$	E_Γ
H_1	1	1^{--}	$(0,1,2)^{-+}$	$E_{\Sigma_u^-}, E_{\Pi_u}$
H_2	1	1^{++}	$(0,1,2)^{+-}$	E_{Π_u}
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the J^PCquantum numbers come from the properties of the solution of the coupled Schroedinger eqs. in BOEFT

T is the sum of the orbital angular momentum of the quark-antiquark pair and the gluonic angular momentum; $T=0$ state turns out not to be the lowest mass state.

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$$P^{i\dagger}_{\kappa\lambda} O^a(\mathbf{r},\mathbf{R},t) H^{ia}_\kappa(\mathbf{R},t) = Z_\kappa \Psi_{\kappa\lambda}(\mathbf{r},\mathbf{R},t)$$

we use $\Psi_{\kappa\lambda}(\mathbf{r},\mathbf{R},t)$ as degree of freedom in BOEFT

BOEFT for E_{Π_u} and $E_{\Sigma_u^-}$ hybrids

- Berwein Brambilla Tarrus Vairo PRD 92 (2015) 114019
  Oncala Soto PRD 96 (2017) 014004
  Brambilla Krein Tarrus Vairo PRD 97 (2018) 016016

$$\mathcal{L}_{\text{BOEFT for } 1+-} = \int d^3r \sum_{\lambda\lambda'} \text{Tr}\left\{\Psi^\dagger_{1+-\lambda}\left(i\partial_0 - V_{1+-\lambda\lambda'}(r) + \hat{r}^{i\dagger}_\lambda \frac{\boldsymbol{\nabla}^2_r}{m}\hat{r}^i_{\lambda'}\right)\Psi_{1+-\lambda'}\right\}$$

- $\lambda = \pm 1, 0;\quad \hat{r}^i_0 = \hat{r}^i$ and $\hat{r}^i_{\pm 1} = \mp\left(\hat{\theta}^i \pm i\hat{\phi}^i\right)/\sqrt{2}$.
- $V_{1+-\lambda\lambda'} = V^{(0)}_{1+-\lambda\lambda'} + \dfrac{V^{(1)}_{1+-\lambda\lambda'}}{m} + \dfrac{V^{(2)}_{1+-\lambda\lambda'}}{m^2} + \cdots$
- For the static potential: $V^{(0)}_{1+-\lambda\lambda'} = \delta_{\lambda\lambda'} V^{(0)}_{1+-\lambda}$, with $V^{(0)}_{1+-0} = E_{\Sigma_u^-}$, $V^{(0)}_{1+-\pm 1} = E_{\Pi_u}$ → fitted from the lattice hybrids static energies

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fitted from the lattice hybrids static energies

The LO e.o.m. for the fields $\Psi^\dagger_{1+-\lambda}$ are a set of coupled Schrödinger equations:

$$i\partial_0\Psi_{1+-\lambda} = \left[\left(-\frac{\boldsymbol{\nabla}_r^2}{m} + V^{(0)}_{1+-\lambda}\right)\delta_{\lambda\lambda'} - \sum_{\lambda'} C^{\mathrm{nad}}_{1+-\lambda\lambda'}\right]\Psi_{\kappa\lambda'}$$

The eigenvalues $\mathcal{E}_N$ give the masses M_N of the states as $M_N = 2m + \mathcal{E}_N$.

$$\hat{r}^{i\dagger}_\lambda\left(\frac{\boldsymbol{\nabla}_r^2}{m}\right)\hat{r}^i_{\lambda'} = \delta_{\lambda\lambda'}\frac{\boldsymbol{\nabla}_r^2}{m} + C^{\mathrm{nad}}_{1+-\lambda\lambda'}$$

with $C^{\mathrm{nad}}_{1+-\lambda\lambda'} = \hat{r}^{i\dagger}_\lambda\left[\dfrac{\boldsymbol{\nabla}_r^2}{m}, \hat{r}^i_{\lambda'}\right]$ called the nonadiabatic coupling.

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fitted from the lattice hybrids static energies

$$\left[-\frac{1}{mr^2}\,\partial_r r^2\partial_r + \frac{1}{mr^2}\begin{pmatrix} l(l+1)+2 & 2\sqrt{l(l+1)} \\ 2\sqrt{l(l+1)} & l(l+1)\end{pmatrix} + \begin{pmatrix} E^{(0)}_{\Sigma} & 0 \\ 0 & E^{(0)}_{\Pi}\end{pmatrix}\right]\begin{pmatrix}\psi^{(N)}_{\Sigma} \\ \psi^{(N)}_{-\Pi}\end{pmatrix} = \mathcal{E}_N\begin{pmatrix}\psi^{(N)}_{\Sigma} \\ \psi^{(N)}_{-\Pi}\end{pmatrix}$$

$$\left[-\frac{1}{mr^2}\,\partial_r\, r^2\,\partial_r + \frac{l(l+1)}{mr^2} + E^{(0)}_{\Pi}\right]\psi^{(N)}_{+\Pi} = \mathcal{E}_N\,\psi^{(N)}_{+\Pi}$$

Mixing remove the degeneration among opposite parity states:
->Lambda doubling
existing also in molecular physics

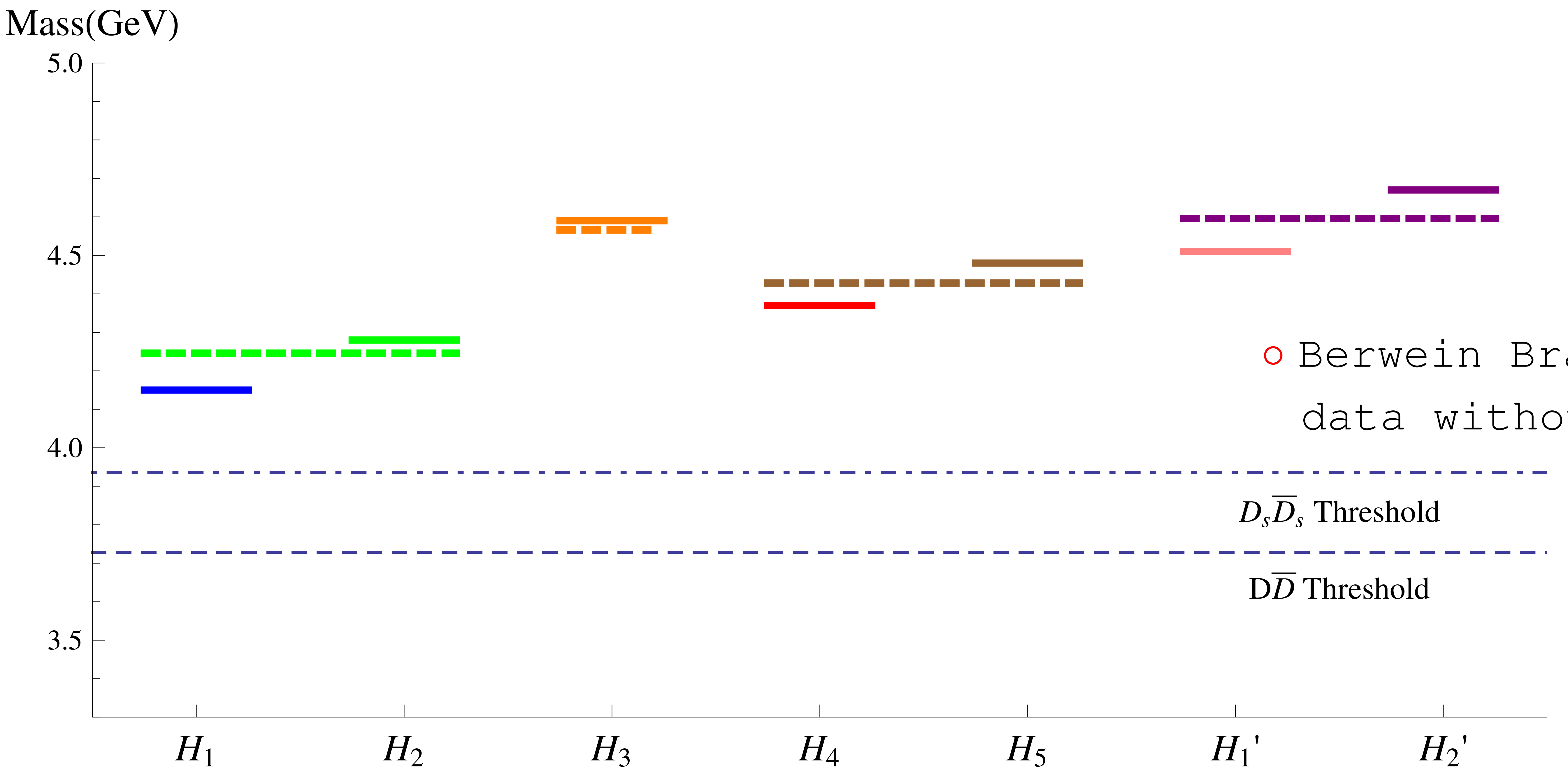
- $l(l+1)$ is the eigenvalue of angular momentum $\boldsymbol{L}^2 = \left(\boldsymbol{L}_{Q\bar{Q}} + \boldsymbol{L}_g\right)^2$
- the two solutions correspond to **opposite parity** states: $(-1)^l$ and $(-1)^{l+1}$
- corresponding eigenvalues under charge conjugation: $(-1)^{l+s}$ and $(-1)^{l+s+1}$

Spectrum: general consideration

- The Schrödinger equation mixes states with the same parity. A consequence is Λ-doubling, i.e., the lifting of degeneracy between states with opposite parity. This happens also in molecular physics, however, there Λ-doubling is a subleading effect, while it is a LO effect in the quarkonium hybrid spectrum.
- The eigenstates are organized in the multiplets H_1, H_2, ... Neglecting off-diagonal terms, the multiplets H_1 and H_2 would be degenerate.
- We compute the spectrum using quark masses in the renormalon subtraction (RS) scheme: $m_{c\,\mathrm{RS}} = 1.477(40)$ GeV and $m_{b\,\mathrm{RS}} = 4.863(55)$ GeV.

The gluelump masses, which enter in the normalization of the hybrid potentials, have been computed in the same scheme and assigned an uncertainty of ±0.15 GeV, which is the largest source of uncertainty in the hybrid masses.

Spectrum: with mixing and Λ-doubling

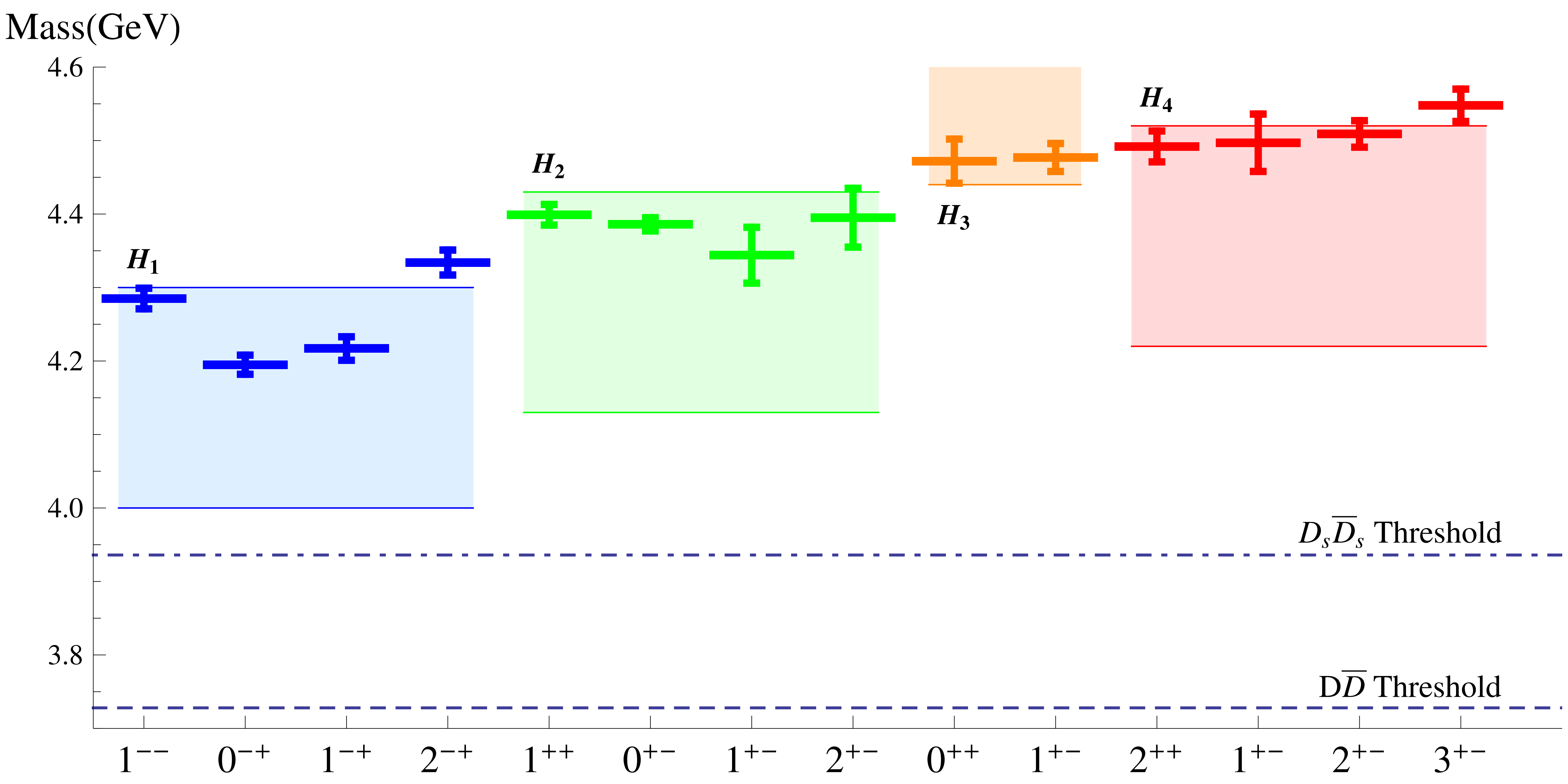


charmonium hybrids

○ Berwein Brambilla Tarrus Vairo PRD 92 (2015) 114019
data without mixing (dashed) from Braaten et al PRD 90 (2014)

in BO papers
without the BOEFT
masses of opposite parity
states are degenerate

Charmonium hybrid states vs direct lattice data



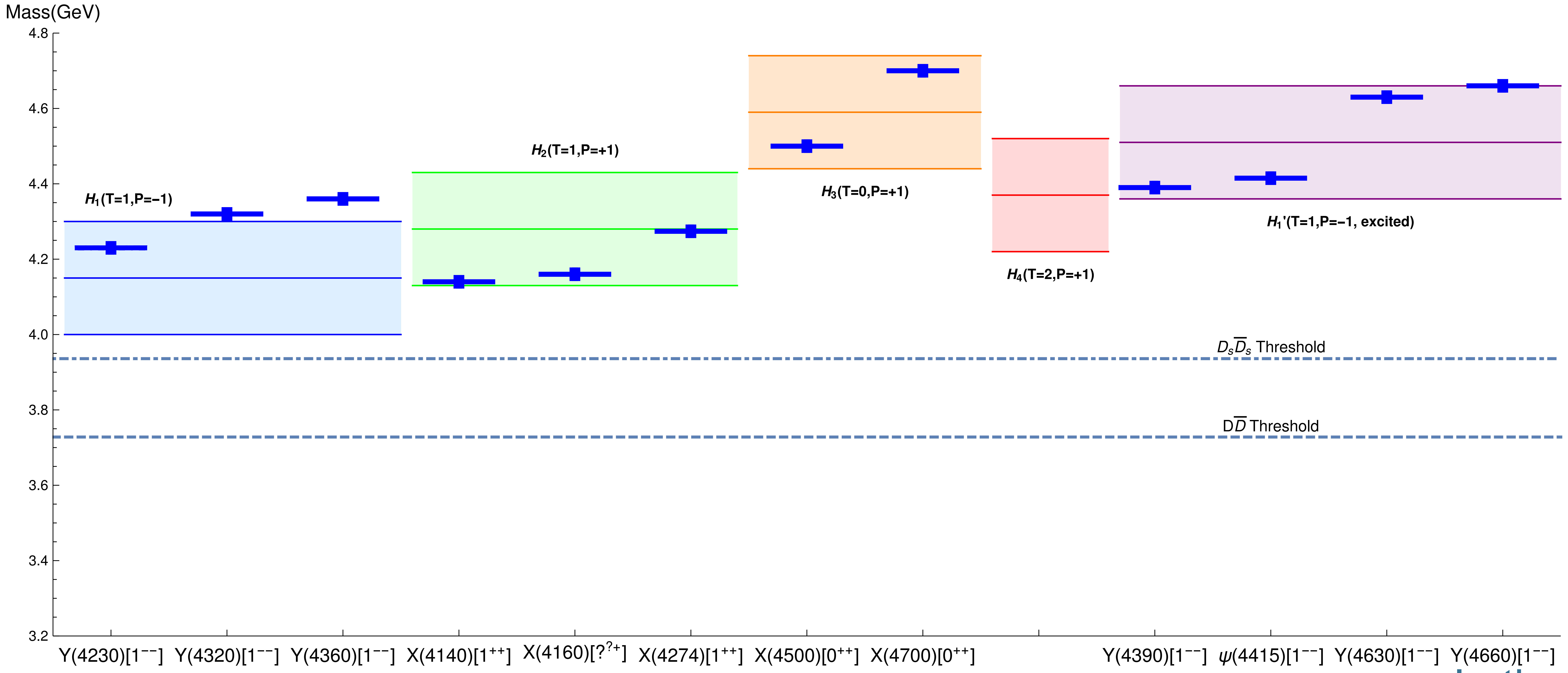
Lattice (crosses) confirms Lambda doubling (H_1 not degenerate with H_2)

Bands BOEFT predict -uncertainty comes from the uncertainty on the mass of the gluelump

- Berwein Brambilla Tarrus Vairo PRD 92 (2015) 114019
  lattice data from the Hadron Spectrum coll JHEP 1207 (2012) 126
  [2+1 flavors, $m_\pi = 400$ MeV]

Quarkonium hybrid states vs experiments I

neutral isoscalar states in charmonium with matching quantum numbers



- Berwein Brambilla Tarrus Vairo PRD 92 (2015) 114019 updated in Brambilla Eidelman Hanhart Nefediev Shen Thomas Vairo Yuan arXiv:1907.11747

In the same way bottomonium and bc hybrids multiplets are obtained, depend on the same gluelump masses

Quarkonium hybrid states vs experiments I

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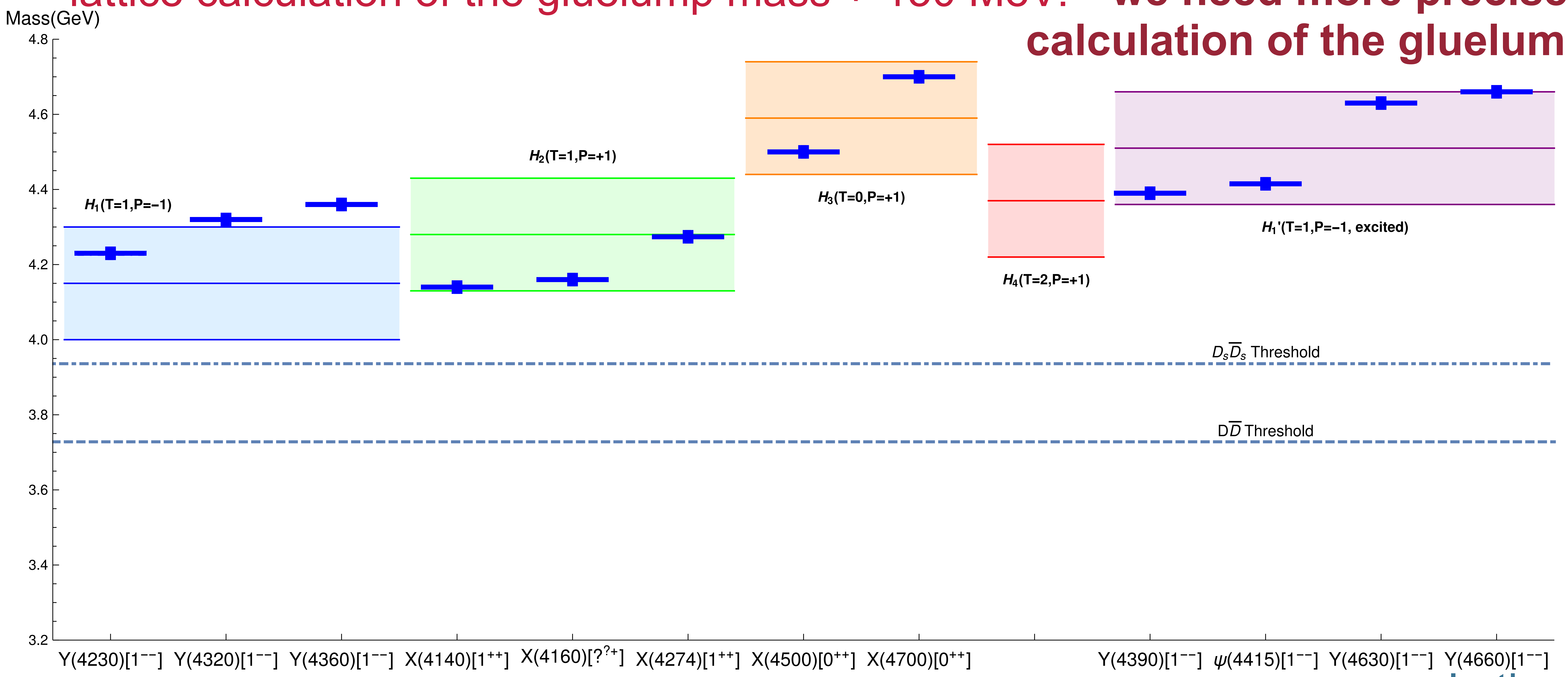
band in our H multiplet masses comes form the error on the lattice calculation of the gluelump mass +-150 MeV: **we need more precise lattice calculation of the gluelump masses !**



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Quarkonium hybrid states vs experiments II

- Promising candidates for charmonium hybrids or for states with a large hybrid component are the $Y(4230)$ and $Y(4390)$ because of their significant width into $\pi^+\pi^- h_c$. This decay does not need spin flipping of the heavy quark-antiquark pair, which is in a spin zero state. Spin-flipping terms are suppressed in the heavy quark limit. Nevertheless, mixing with spin one quarkonium states happens already at order $\Lambda^2_{\text{QCD}}/m_h$. This possibly large mixing may allow for significant widths also into final states with spin one quarkonia, in particular $\pi^+\pi^- J/\psi$.
  - Oncala Soto PRD 96 (2017) 014004
- From the experimental side, candidate states of bottomonium hybrids in the H_1 or H_1' multiplets are the $\Upsilon(10860)$ $[1^{--}]$, with a mass of $M_{\Upsilon(10860)} = (10891.1 \pm 3.2^{+0.6}_{-1.7})$ MeV and the $\Upsilon(11020)$ $[1^{--}]$, with a mass of $M_{\Upsilon(11020)} = (10987.5^{+6.4}_{-2.5}{}^{+9.0}_{-2.1})$ MeV
  - Belle coll PRD 93 (2016) 011101

  To these we can add the recently observed signal by Belle with a mass of $M_{\Upsilon(10750)} = (10752.7 \pm 5.9\,^{+0.7}_{-1.1})$ MeV, which may also qualify as an H_1 multiplet bottomonium hybrid candidate.
  - Belle coll arXiv:1905.05521

Tarrus Passemar 2104.03975
studies transition of these states identified as hybrids to quarkoniumin BOEFT—>
results suggest 11020 being a hybrid

besides the spectrum we need:

- relativistic corrections, especially spin dependent potentials
- mixing with quarkonium, decays and transitions: what is the width of these states?
- production
- nonequilibrium evolution of X Y Z in medium

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BOEFT gives or has the potential to give all of that to us!

The **BOEFT** gives a prescription to calculate the **hybrids spin dependent potentials at order 1/m and 1/m^2**

1/m

$$V^{(1)}_{1+-\lambda\lambda'\,\mathrm{SD}}(\boldsymbol{r}) = V_{SK}(r)\left(\hat{r}^{i\dagger}_{\lambda}\boldsymbol{K}^{ij}\hat{r}^{j}_{\lambda'}\right)\cdot\boldsymbol{S} + V_{SK\,b}(r)\left[\left(\boldsymbol{r}\cdot\hat{\boldsymbol{r}}^{\dagger}_{\lambda}\right)\left(r^{i}\boldsymbol{K}^{ij}\hat{r}^{j}_{\lambda'}\right)\cdot\boldsymbol{S} + \left(r^{i}\boldsymbol{K}^{ij}\hat{r}^{j\dagger}_{\lambda}\right)\cdot\boldsymbol{S}\left(\boldsymbol{r}\cdot\hat{\boldsymbol{r}}_{\lambda'}\right)\right]$$

$$\mathbf{S} = \mathbf{S}_1 + \mathbf{S}_2$$

$$S_{12} = 12(\mathbf{S}_1\cdot\hat{\mathbf{r}})(\mathbf{S}_2\cdot\hat{\mathbf{r}}) - 4(\mathbf{S}_1\cdot\mathbf{S}_2)$$

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$$V^{(2)}_{1+-\lambda\lambda'\,\mathrm{SD}}(\boldsymbol{r}) = V^{(2)}_{LS\,a}(r)\left(\hat{r}^{i\dagger}_{\lambda}\boldsymbol{L}\,\hat{r}^{i}_{\lambda'}\right)\cdot\boldsymbol{S} + V^{(2)}_{LS\,b}(r)\hat{r}^{i\dagger}_{\lambda}\left(L^{i}S^{j} + S^{i}L^{j}\right)\hat{r}^{j}_{\lambda'} + V^{(2)}_{S^2}(r)\boldsymbol{S}^2\delta_{\lambda\lambda'} + V^{(2)}_{S_{12}\,a}(r)S_{12}\delta_{\lambda\lambda'} + V^{(2)}_{S_{12}\,b}(r)\hat{r}^{i\dagger}_{\lambda}\hat{r}^{j}_{\lambda'}\left(S^{i}_{1}S^{j}_{2} + S^{i}_{2}S^{j}_{1}\right)$$

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Features:

- New spin structures with respect to the quarkonium case: all terms at order 1/m and two terms at order 1/m^2

Differently from the quarkonium case, the hybrid potential gets a first contribution already at order $\Lambda^2_{\mathrm{QCD}}/m_h$. The corresponding operator does not contribute at LO to matrix elements of quarkonium states as its projection on quark-antiquark color singlet states vanishes. Hence, spin splittings are remarkably less suppressed in heavy quarkonium hybrids than in heavy quarkonia.

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Mixing with quarkonium via spin may also enhanced and decay to different spin states may be enhanced

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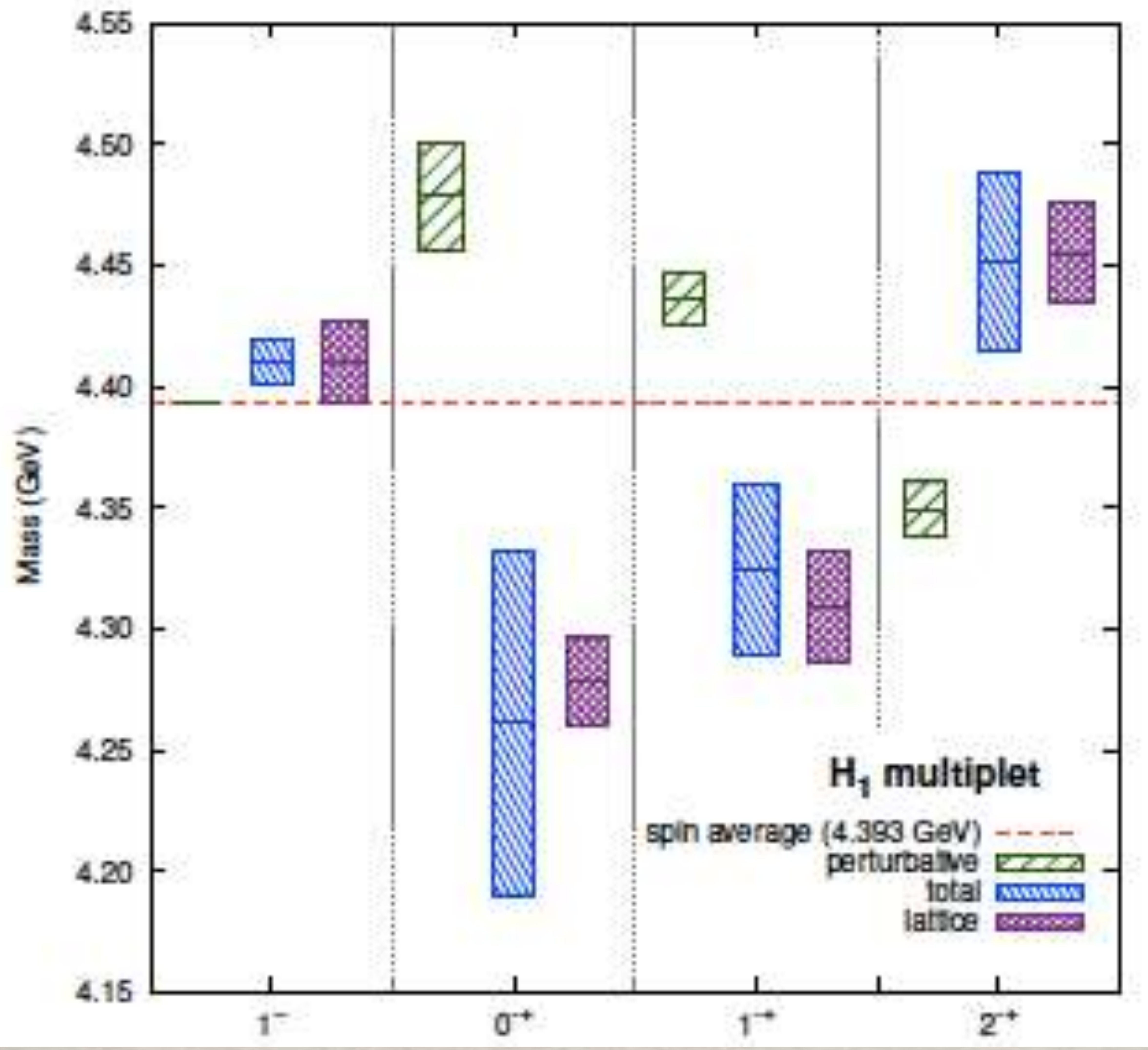
USE LATTICE CALCULATION OF THE CHARMONIUM SPIN MULTIPLETS TO EXTRACT the 6 UNKNOWNs and PREDICT THE BOTTOMONIUM SPIN MULTIPLETS, learn also about the DYNAMICS

Charmonium Hybrids Multiplets H_1



lattice data from (violet) from

G. K. C. Cheung, C. O'Hara, G. Moir, M. Peardon, S. M. Ryan, C. E. Thomas, and D. Tims (Hadron Spectrum), JHEP **12**, 089 (2016), arXiv:1610.01073 [hep-lat].

with a pion of about 240 MeV

- Brambilla Lai Segovia Tarrus Vairo PRD 99 (2019) 014017

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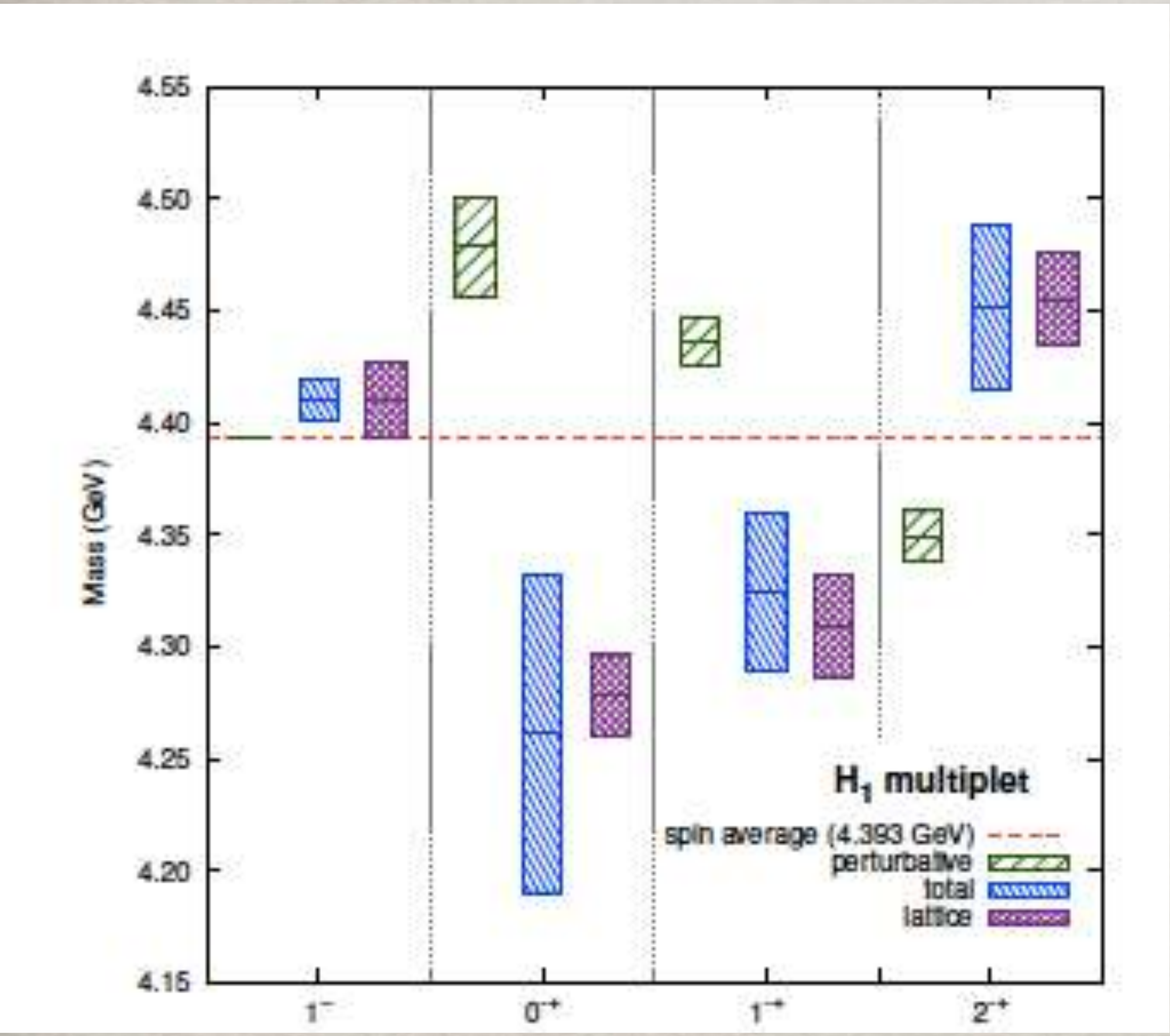

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estimated by the parametric size of higher order corrections, m alpha_s^5 for the perturbative part, powers of Lambda_qcd/m for the nonperturbative part, plus the statistical error on the fit

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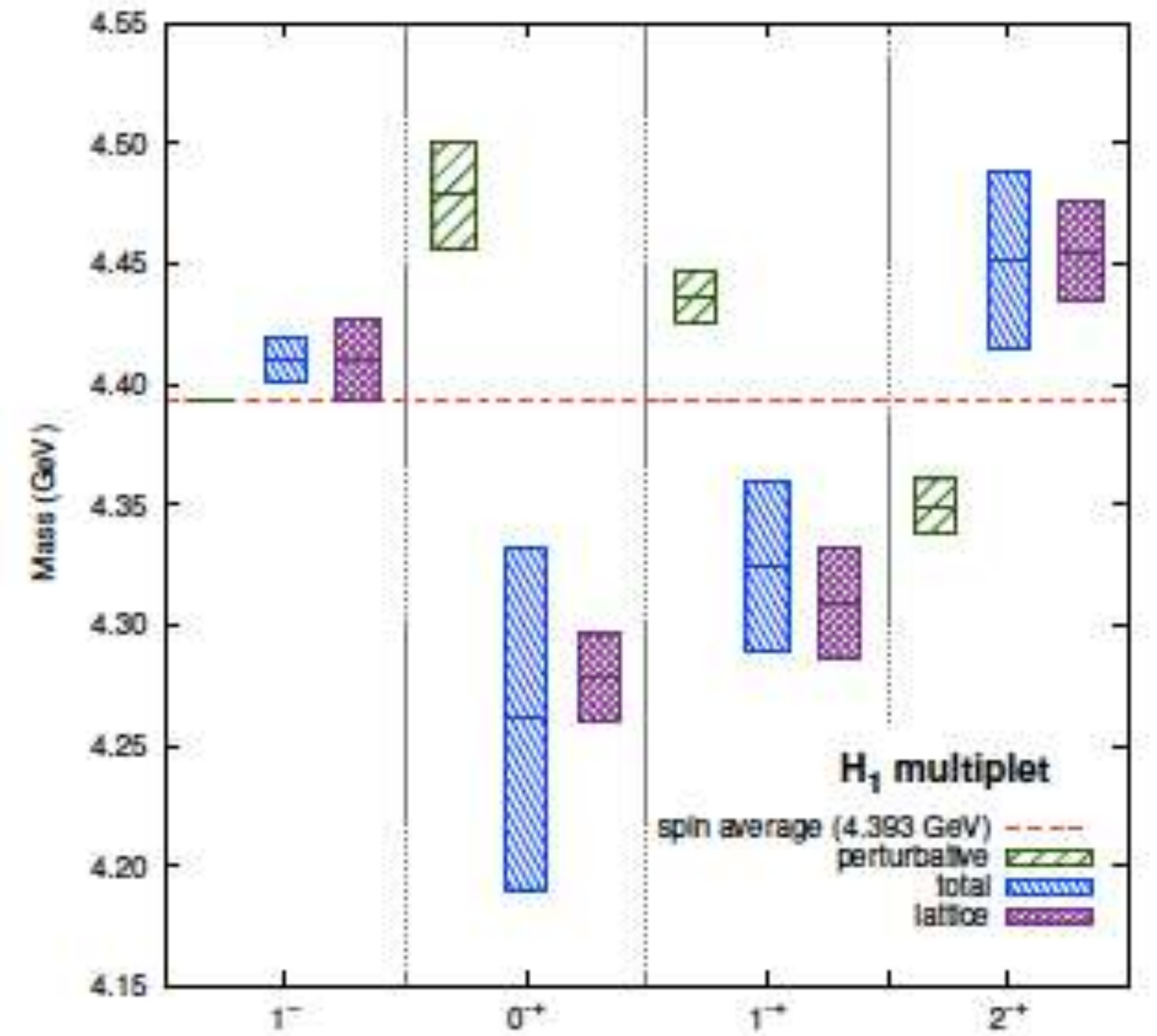

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the perturbative part produces a pattern opposite to the lattice and to ordinary quarkonia —> discrepancy can be reconciled thanks to the nonperturbative parts, especially the one at order 1/m which goes like Lambda^2/m and is parametrically larger than the perturbative contribution at order m v^4

which is interesting as some models are taking the spin interaction from perturbation theory with a constituent gluon

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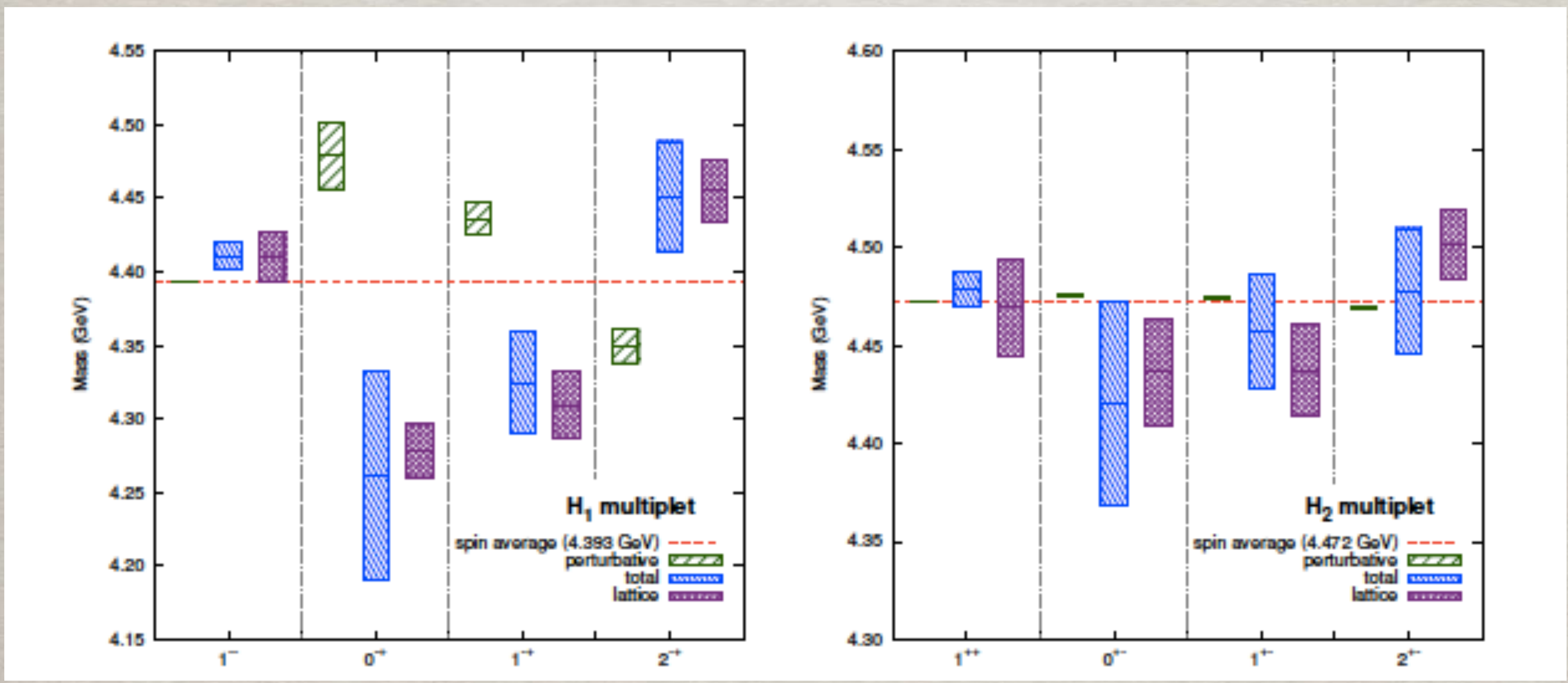
Charmonium Hybrids Multiplets H_1 and H_2



H_1 and H_2 corresponds to l=1 and are negative and positive parity resp. The mass splitting between H_1 and H_2 is a result of lambda-doubling

H_3 and H_4 are also calculated

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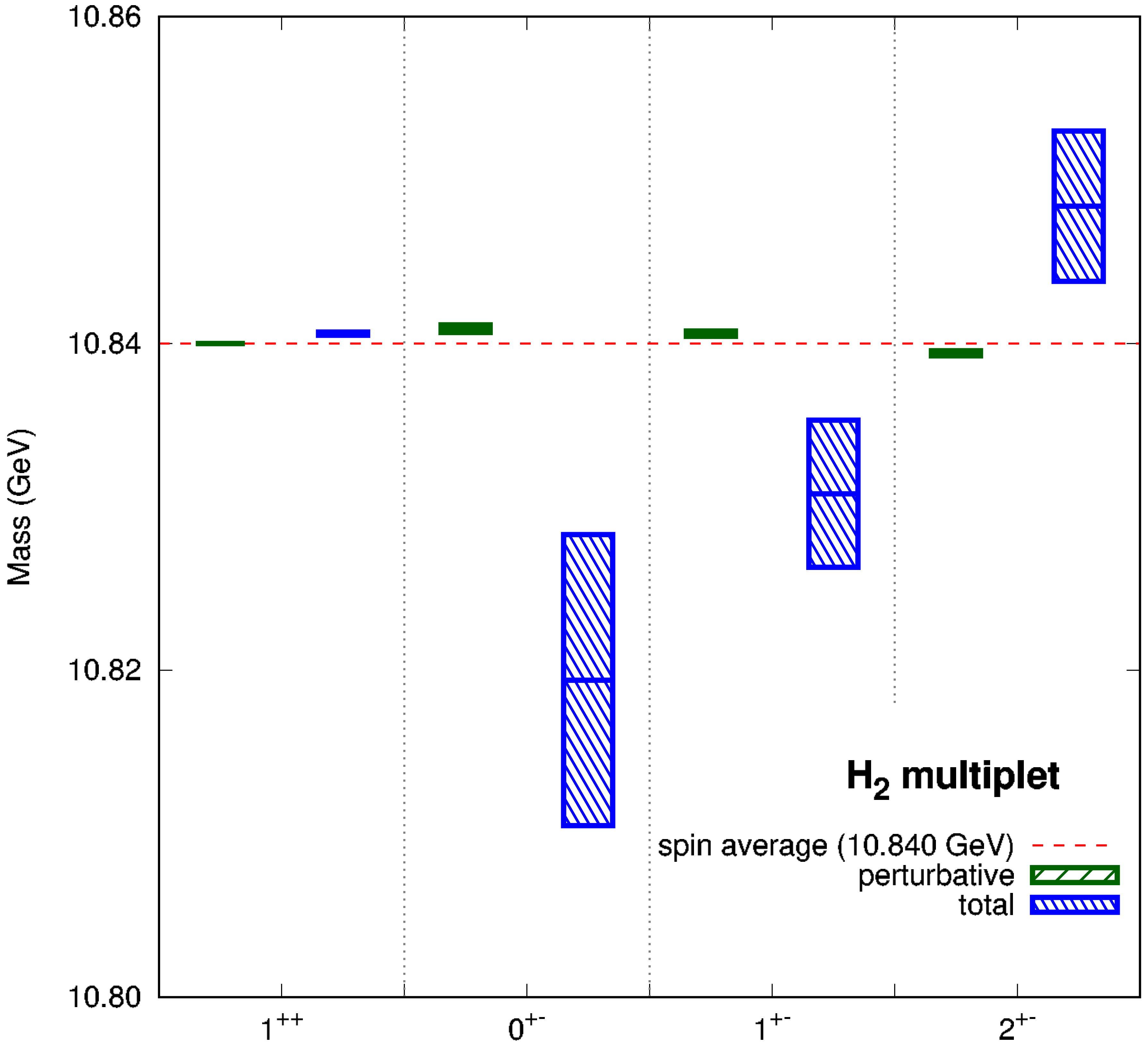
here you find predictions for all H multiplets

Bottomonium hybrid spin splittings

thanks to the BOEFT factorizatio we can fix the nonperturbative unknowns from a charmonium hybrid calculationthe nonperturbative low energy unknownsdo not depend on the flavor: we can predict the bottomonium hybrids splin splittings



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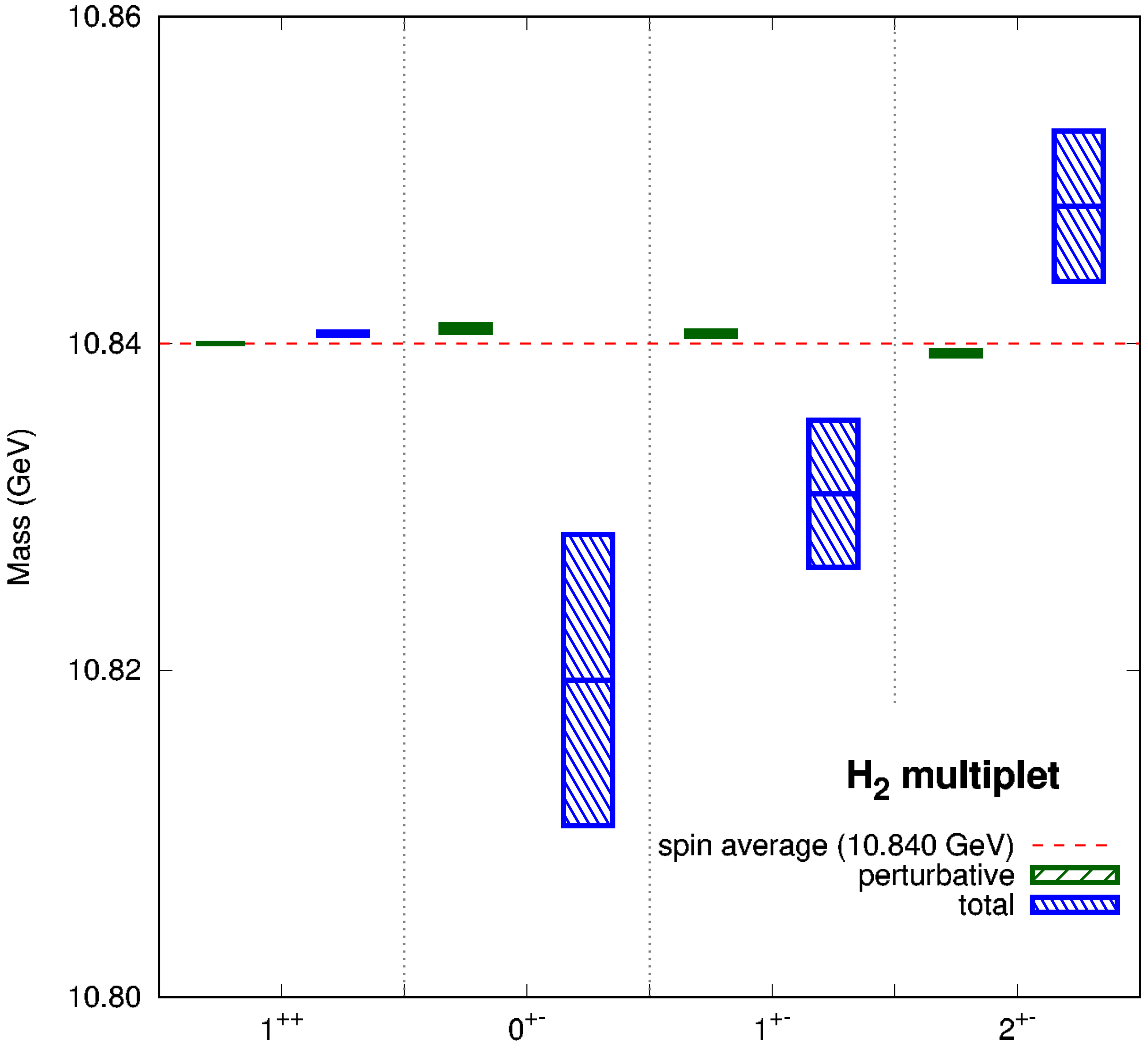

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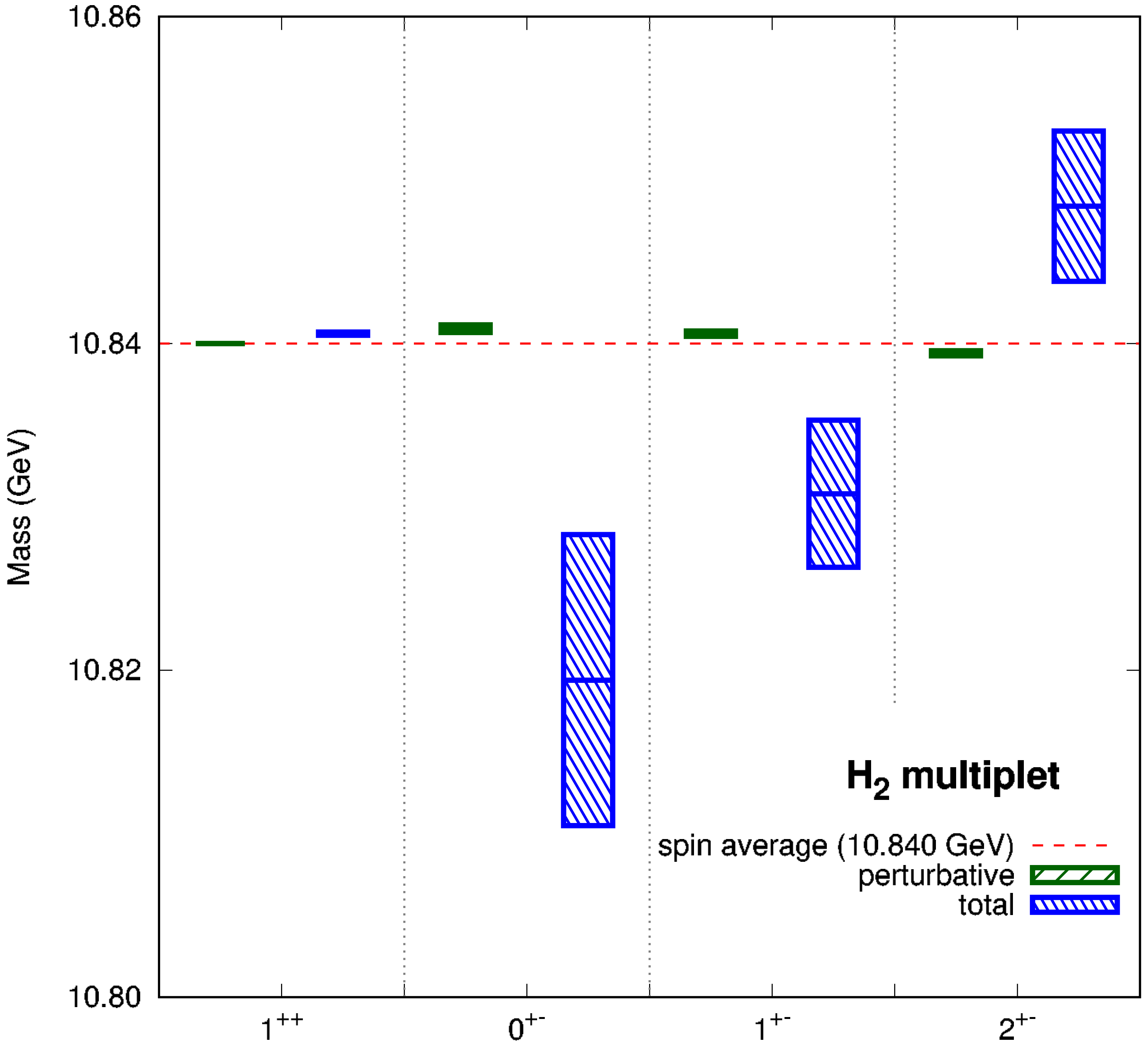

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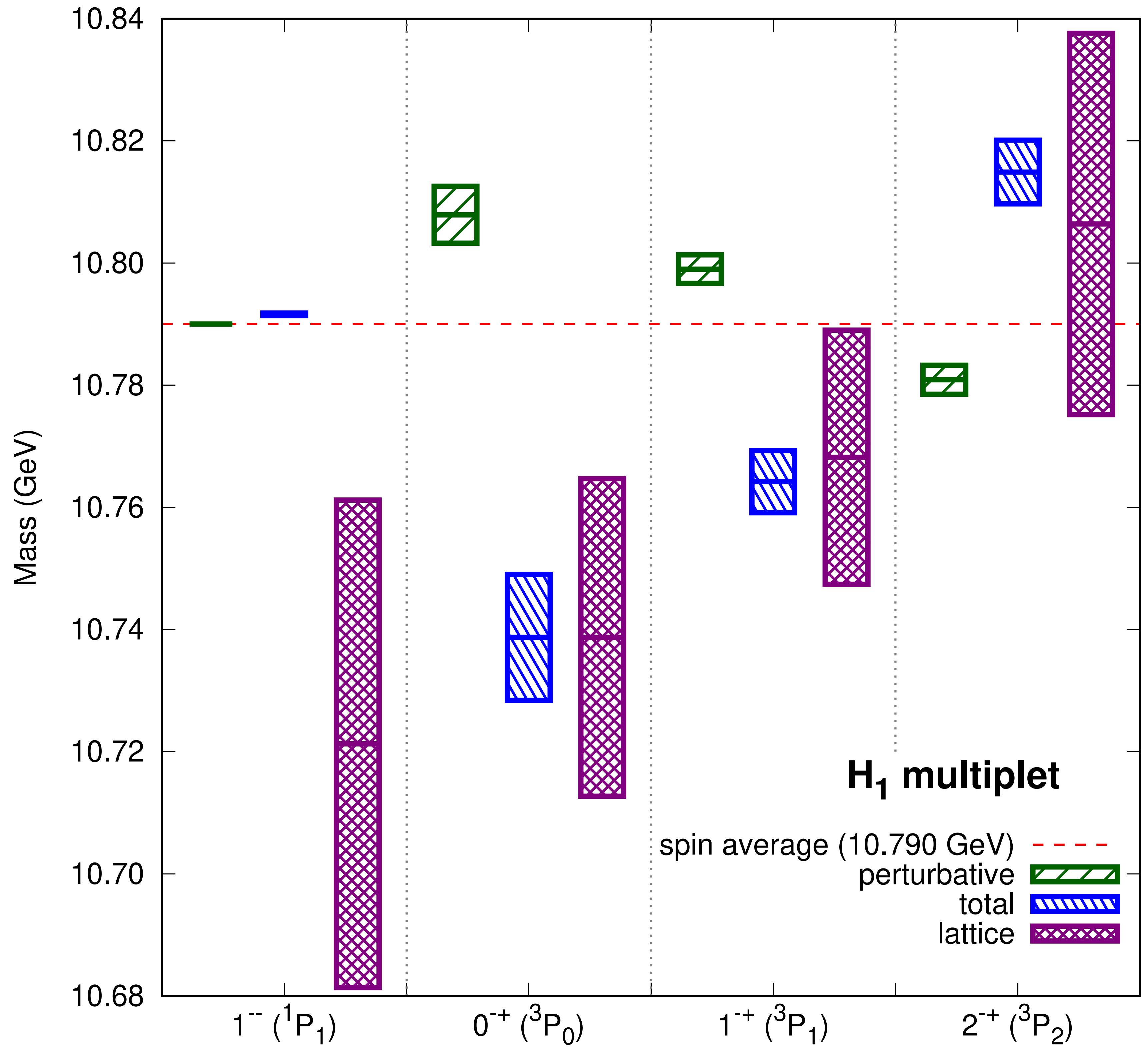

blue BOEFT predictions (more precise), violet actual lattice calculation

- Ryan et al arXiv:2008.02656 [2+1 flavors, $m_\pi = 400$ MeV] unpublished plot by J. Segovia

Mixing

The 1/m operator giving origin to the 1/m spin potential in heavy hybrids
is also responsible for a mixing between spin 0 (1) hybrids and spin 1 (0) quarkonia

Oncala Soto 1702.03900

in dependence of the strength of the mixing, which is of order Λ^2_{QCD}/m_h and non-perturbative, why some hybrid candidates appear to decay both into $\pi^+\pi^- J/\psi$ and $\pi^+\pi^- h_c$.

—> **spin violating decays**

This effect is encoded in a single generalised
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Decays to lowest lying quarkonia —> get information on the hybrids widths!

BOEFT allows to study hybrids semi inclusive decays to quarkonium + X

$$\Gamma_{H\to S} = -2\,\langle H|{\rm Im}\Delta V|H\rangle.$$

we are currently calculating all spin conserving and spin flipping decays for charmonium and bottomonium hybrids

N. B., A. Mohapatra, W.K. Lai, A. Vairo

Decays from hybrids to quarkonium

	$nL_T \to n'L'$	ΔE (MeV)	Γ (MeV)
H_3	$c\bar{c}$ sector		
	$1P_0 \to 2S$	808	7.5(7.4)
H_1	$2(S/D)_1 \to 1P$	861	22(19)
	$4(S/D)_1 \to 1P$	1224	23(15)
	$b\bar{b}$ sector		
	$1P_0 \to 1S$	1569	44(23)
	$1P_0 \to 2S$	1002	15(9)
	$2P_0 \to 2S$	1290	2.9(1.3)
H_3	$2P_0 \to 3S$	943	15(12)
	$4P_0 \to 1S$	2337	53(25)
	$4P_0 \to 2S$	1770	18(7)
	$4P_0 \to 3S$	1423	7.4(4.1)
	$2(S/D)_1 \to 1P$	977	17(8)
H_1	$3(S/D)_1 \to 1P$	1176	29(14)
	$3(S/D)_1 \to 2P$	818	5(3)
	$4(S/D)_1 \to 2P$	891	33(25)
	$5(S/D)_1 \to 1P$	1376	18(7)
	$5(S/D)_1 \to 2P$	1018	14(8)

Tetraquarks and pentaquarks

BOEFT can be used to describe any system made by two heavy quarks bound adiabatically with some light degrees of freedom: glue (hybrids) or light quarks (tetraquarks, pentaquark)

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steps go as before:

identify the symmetries, identify the interpolating operators O_n and define the static energies

$$\mathcal{O}_n(t,\, \boldsymbol{r},\, \boldsymbol{R}) = \chi(t,\, \boldsymbol{R}-\boldsymbol{r}/2)\phi(t,\, \boldsymbol{R}-\boldsymbol{r}/2, \boldsymbol{R})H_n(t,\, \boldsymbol{R})\phi(t,\, \boldsymbol{R}, \boldsymbol{R}+\boldsymbol{r}/2)\psi^\dagger(t,\, \boldsymbol{R}+\boldsymbol{r}/2)$$

$$E_n^{(0)}(r) = \lim_{T\to\infty} \frac{i}{T} \log\langle \mathcal{O}_n(T,\, \boldsymbol{r},\, \boldsymbol{R})|\mathcal{O}_n(0,\, \boldsymbol{r},\, \boldsymbol{R})\rangle$$

.. Examples of gluonic operators and light-quark operators for quarkonium hybrids and tetraquarks respectively, $\boldsymbol{q} = (u,\, d)$ and τ^a are isospin Pauli matrices.

Λ_η^σ	κ	H	$H = H^a T^a (I=0,\, I=1)$
Σ_g^+	0^{++}	$\mathbb{1}$	$\bar{\boldsymbol{q}} T^a(\mathbb{1},\, \boldsymbol{\tau})\boldsymbol{q}$
Σ_u^-	1^{+-}	$\hat{\mathbf{r}}\cdot\mathbf{B}$	$\bar{\boldsymbol{q}}\, [(\hat{\boldsymbol{r}}\times\gamma)\cdot,\, \gamma]\, T^a(\mathbb{1},\, \boldsymbol{\tau})\boldsymbol{q}$
Π_u	1^{+-}	$\hat{\mathbf{r}}\times\mathbf{B}$	$\bar{\boldsymbol{q}}\, [\hat{\boldsymbol{r}}\cdot\gamma,\, \gamma]\, T^a(\mathbb{1},\, \boldsymbol{\tau})\boldsymbol{q}$
$\Sigma_g^{+\prime}$	1^{--}	$\hat{\mathbf{r}}\cdot\mathbf{E}$	$\bar{\boldsymbol{q}}\, (\hat{\boldsymbol{r}}\cdot\gamma) T^a(\mathbb{1},\, \boldsymbol{\tau})\boldsymbol{q}$
Π_g	1^{--}	$\hat{\mathbf{r}}\times\mathbf{E}$	$\bar{\boldsymbol{q}}\, (\hat{\boldsymbol{r}}\times\gamma) T^a(\mathbb{1},\, \boldsymbol{\tau})\boldsymbol{q}$

N.B. G. Krein, J. Tarrus, A. Vairo 1707.09647. J. Tarrus 1901. 09761, J. Soto, J. Tarrus 20005.00552

BOEFT for $I = 1$ tetraquarks

○ Tarrus arXiv:1901.09761

$$\Gamma_\mu = \left(u^\dagger \partial_\mu u + u \partial_\mu u^\dagger\right)/2 \text{ and } u = \exp(i\boldsymbol{\pi}\cdot\boldsymbol{\tau}/(2$$

$$D_\mu \boldsymbol{Z} = \partial_\mu + [\Gamma_\mu, \boldsymbol{Z}]$$

$$\begin{aligned}\mathcal{L}_{\text{BOEFT for } I=1} = & \int d^3r \, \mathrm{Tr}\left\{Z^\dagger_{0+-}\left(iD_0 - V^{\text{tetra}}_{\Sigma_g^+}(r) + \frac{\boldsymbol{\nabla}_r^2}{m_h}\right) Z_{0+-}\right\} \\ & + \int d^3r \sum_{\lambda\lambda'} \mathrm{Tr}\left\{Z^\dagger_{1+-\lambda}\left(iD_0 - V^{\text{tetra}}_{1+-\lambda\lambda'}(r) + \hat{r}^{i\dagger}_\lambda \frac{\boldsymbol{\nabla}_r^2}{m_h} \hat{r}^i_{\lambda'}\right) Z_{1+-\lambda'}\right\} \\ & + \int d^3r \sum_{\lambda\lambda'} \mathrm{Tr}\left\{Z^\dagger_{1--\lambda}\left(iD_0 - V^{\text{tetra}}_{1--\lambda\lambda'}(r) + \hat{r}^{i\dagger}_\lambda \frac{\boldsymbol{\nabla}_r^2}{m_h} \hat{r}^i_{\lambda'}\right) Z_{1--\lambda'}\right\} \\ & + \text{terms with higher orbital momentum and mixing of states}\end{aligned}$$

with the isovector field

$$Z_\kappa = Z^i_\kappa \sigma^i = \begin{pmatrix} Z^0_\kappa & \sqrt{2} Z^+_\kappa \\ \sqrt{2} Z^-_\kappa & -Z^0_\kappa \end{pmatrix}$$

needs lattice calculations of tetraquarks static energies

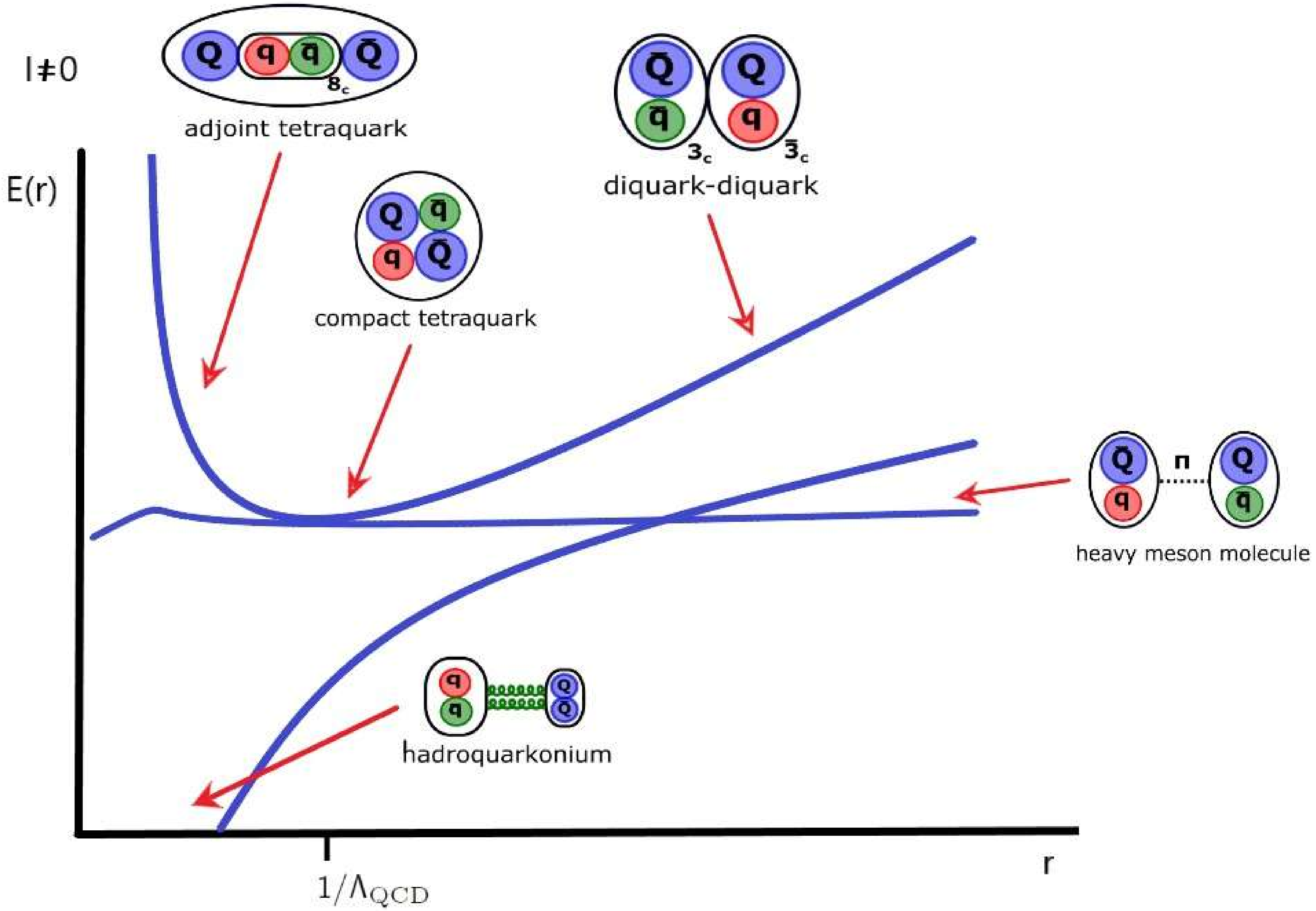
The direct use of the $I = 1$ BO effective Lagrangian is limited by the fact that the potentials have not, even in their static limit, been measured on the lattice. Hence, the situation is different from the hybrid case, where static hybrid energies are known since long time.

I=1 S. Prevlosek, H. Bahtiyar, J. Petrovich eprint: 1912.02656

I=0 Bicudo Cichy Peters Wagner PRD 93 (2016) 034501

The BOEFT contains all configurations: what dominates and where depends on the QCD dynamics

Static energies for $I \neq 0$ (schematic):



The static energies are defined in BOEFT that
gives the appropriate set of operators to be used
and could describe the short distance limit.
Being nonperturbative objects E(r) should be calculated
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Figure from J. Tarrus

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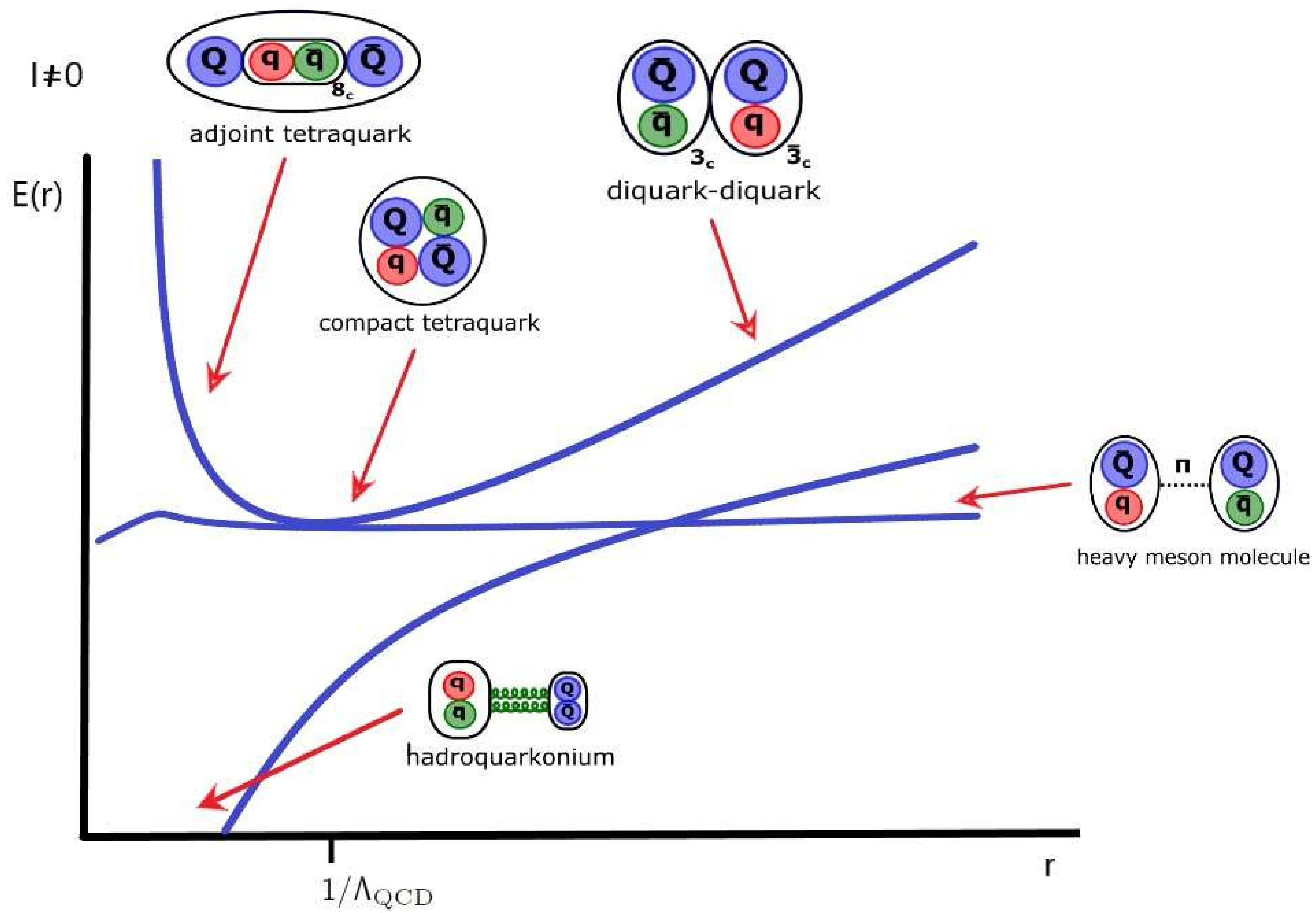
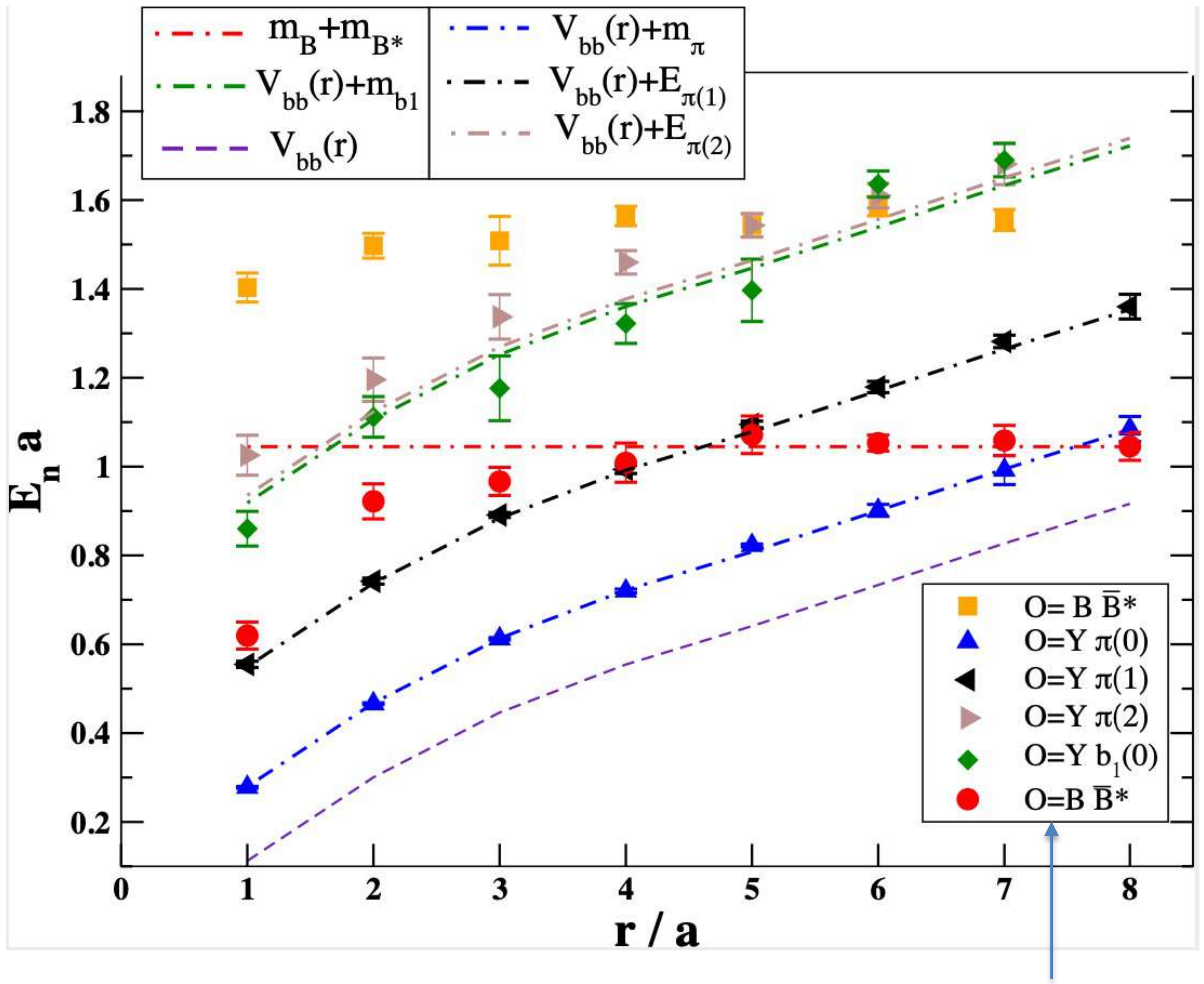


$\bar{b}\,b\,\bar{d}\,u$ Z_b channel $\vec{S}_h = \vec{S}_b + \vec{S}_{\bar{b}}$

Eigen-energies $E_n(r)$: channel S_h=1, CP=-1, ε=-1



m_B+m_B

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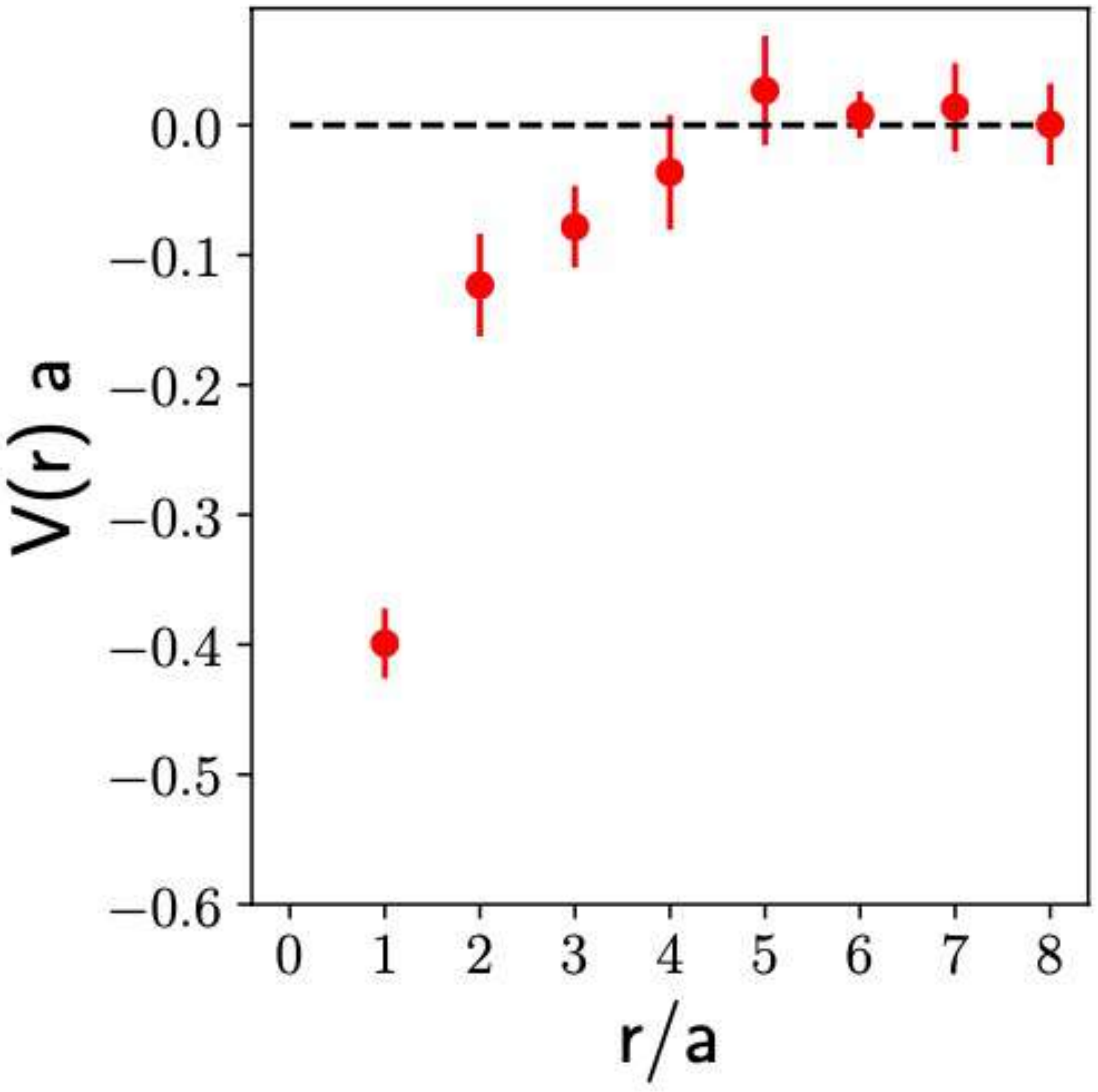
Figure from J. Tarrus

Figure from S. Prevlosek

The BOEFT contains all configurations: what dominates and where depends on the QCD dynamics

Binding configuration found on the lattice



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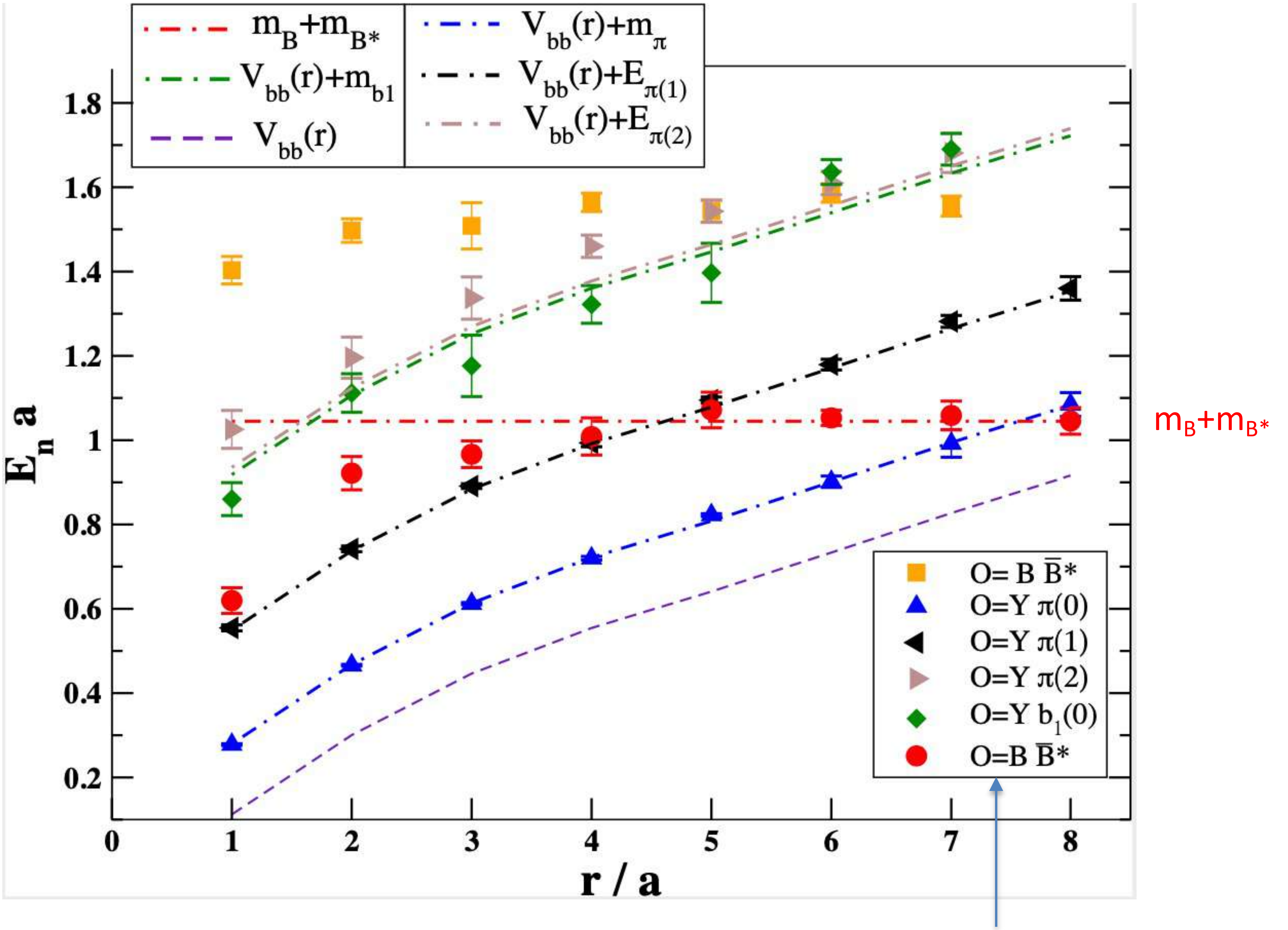

Figure from S. Prevlosek

Outlook

NREFTs and lattice   allows us to describe the physics of quarkonium away from the strong decay threshold in quantum field theory: higher order perturbative calculation can be performed and quarkonium can be used for precision physics/ factorisation allows to systematically studyconfinement

NREFTs and lattice and open quantum system allows us to describe the nonequilibrium evolution quarkonium in the quark gluon plasma

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Combining BOEFT + open quantum systems one can attempt to study the X Y Z in heavy ion collisions

spare slides

Applications of strongly coupled pNRQCD include: **Quarkonium Production at LHC**

NRQCD factorization formula for quarkonium production
valid for large p_T

Bodwin Braaten Lepage 1995

cross section

$$\sigma(H) = \sum_n F_n \langle 0 | \mathcal{O}_n^H | 0 \rangle.$$

short distance coefficients
partonic hard scattering cross section
convoluted with parton distribution

long distance matrix elements
(LDME)
give the probability of a qqbar pair with certain quantum number to evolve into a final quarkonium H

they are vacuum expectation values of four fermion operators with color singlet and color octet contributions and a projection over quarkonium plus X in the middle

Intense work in the theory community, within QCD, NRQCD and SCET,

Qiu, Nayak, Sterman, Butenschon Kniehl , Bodwin , Hee Soh, Chung, J. Lee, Kuang Ta Chao, Y. Q. Ma, Gong Wang, Fleming, Mehen, Yu Jia, Braaten, Lansberg, Leibovich, Rothstein…

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One problem is the proliferation of LDMEs: nonperturbative objects
that cannot be evaluated on the lattice
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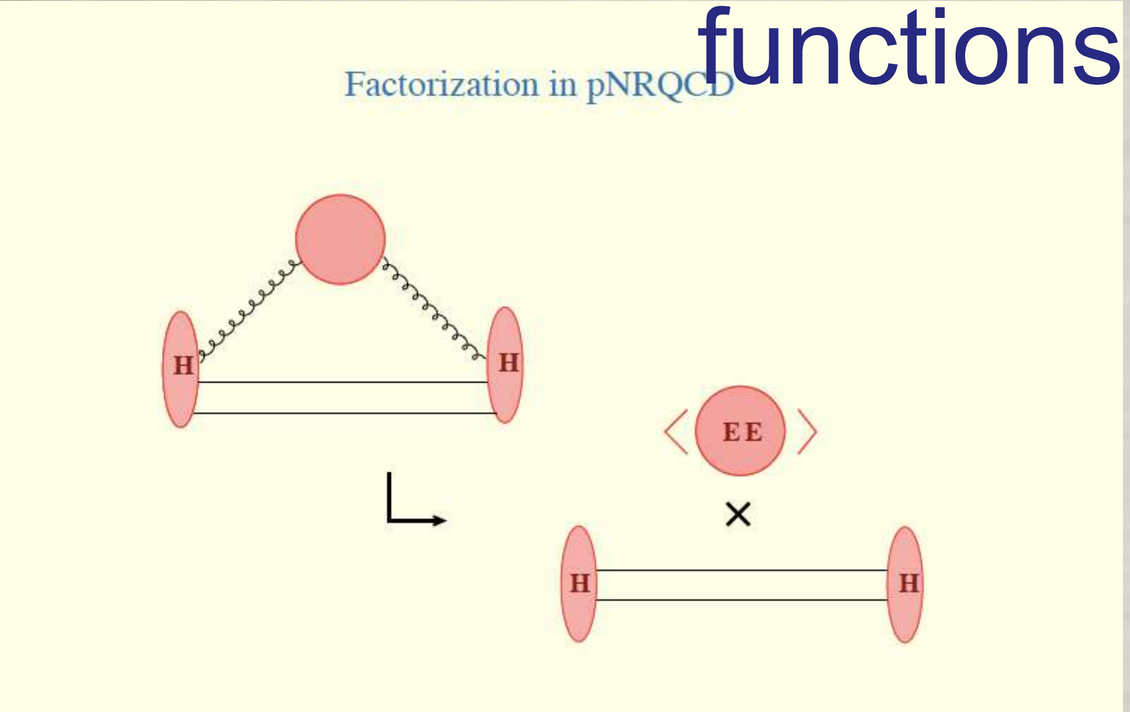
Factorization of LDMEs in pNRQCD : the NRQCD LDMEs are factorized in terms of wave functions and universal nonperturbative correlators depending only on the glue



- The number of nonperturbative unknowns is reduced by half
- The nonperturbative unknowns are correlators of gluonic fields that can be calculated on the lattice

N.B. Chung Vairo 2007.07613, 2106.09417

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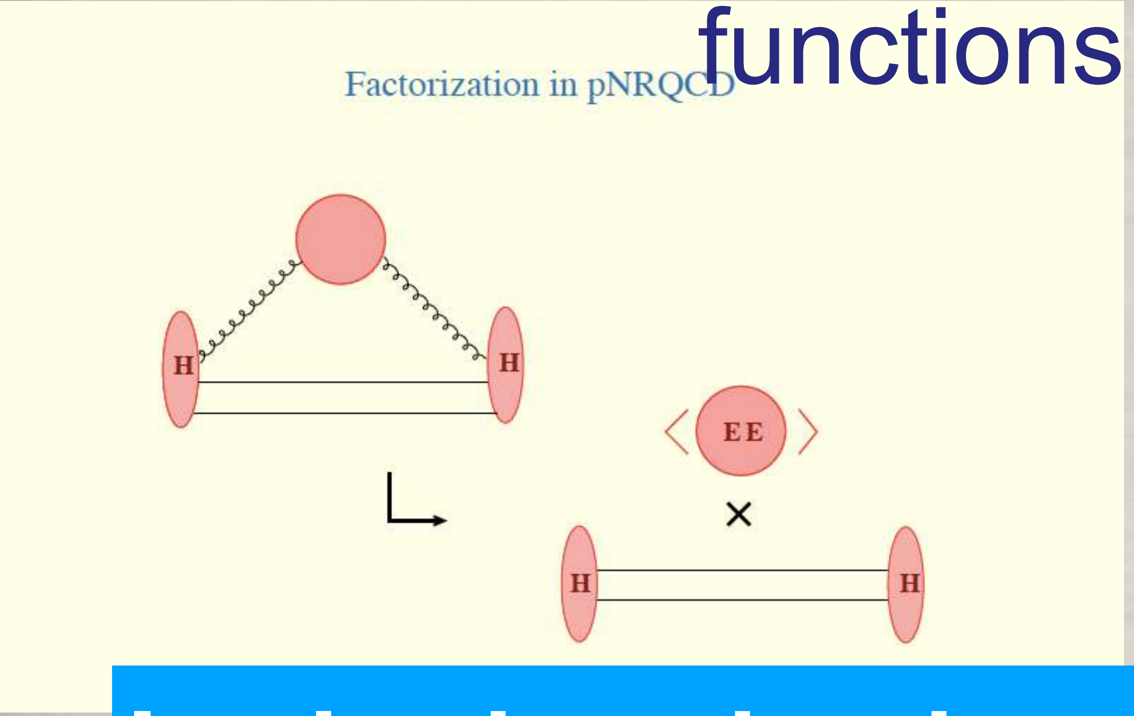

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Inclusive hadroproduction of p wave quarkonia

$$\sigma_{\chi_{QJ}+X} = (2J+1)\sigma_{Q\bar{Q}(^3P_J^{[1]})}\langle \mathcal{O}^{\chi_{Q0}}(^3P_0^{[1]})\rangle$$

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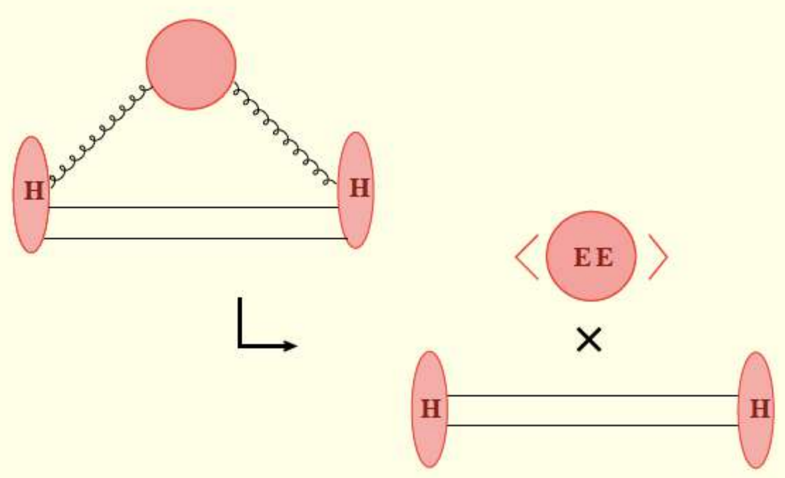

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could be evaluated on the lattice, similar to TMDs

- The dimensionless correlator $\mathcal{E}$ is defined in terms of chromoelectric fields gE with Wilson lines Φ extending to infinity in the ℓ direction.

$$\mathcal{E} = \frac{3}{N_c}\int_0^\infty t\,dt \int_0^\infty t'\,dt' \langle\Omega|\Phi_\ell^{\dagger ab}\Phi_0^{\dagger da}(0,t) gE^{d,i}(t) gE^{e,i}(t')\Phi_0^{ec}(t',0)\Phi_\ell^{bc}|\Omega\rangle$$

- $\mathcal{E}$ has a ***one-loop scale dependence*** that is ***consistent with the evolution equation for NRQCD matrix elements***

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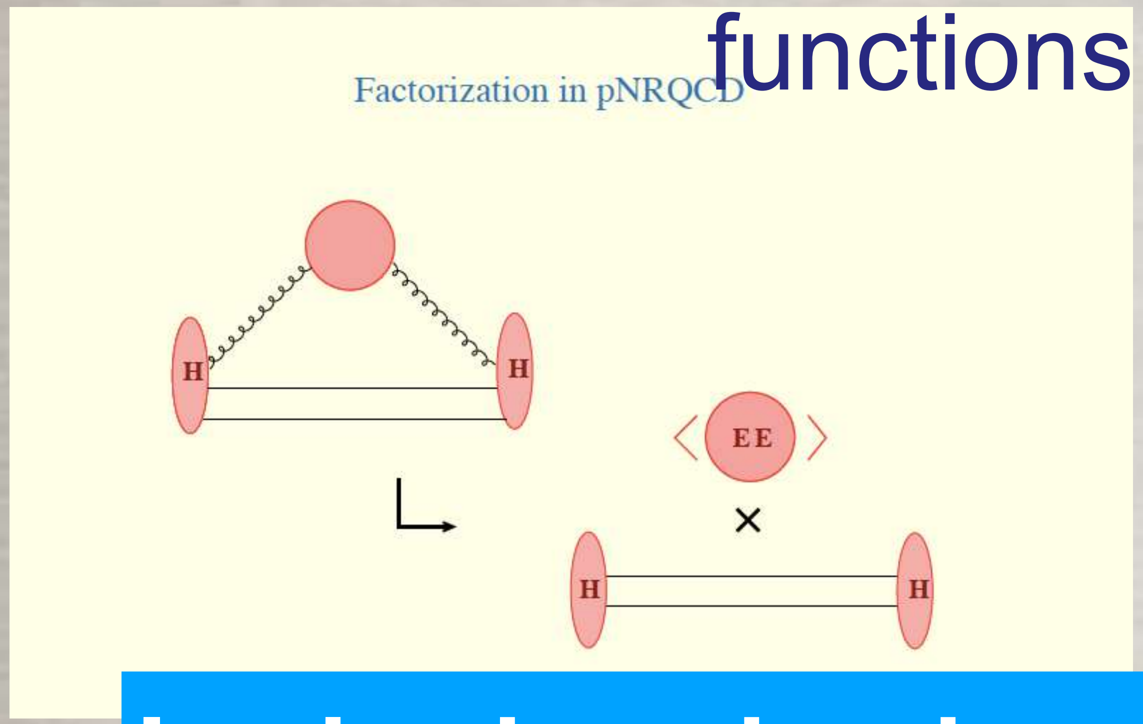

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$\mathcal{E}$ is a universal quantity that ***does not depend on quark flavor or radial excitation***. Determination of $\mathcal{E}$ directly leads to ***determination of all χ_{cJ} and $\chi_{bJ}(nP)$ cross sections, as well as h_c and h_b production rates***.

->good description of data at ATLAS and CMS

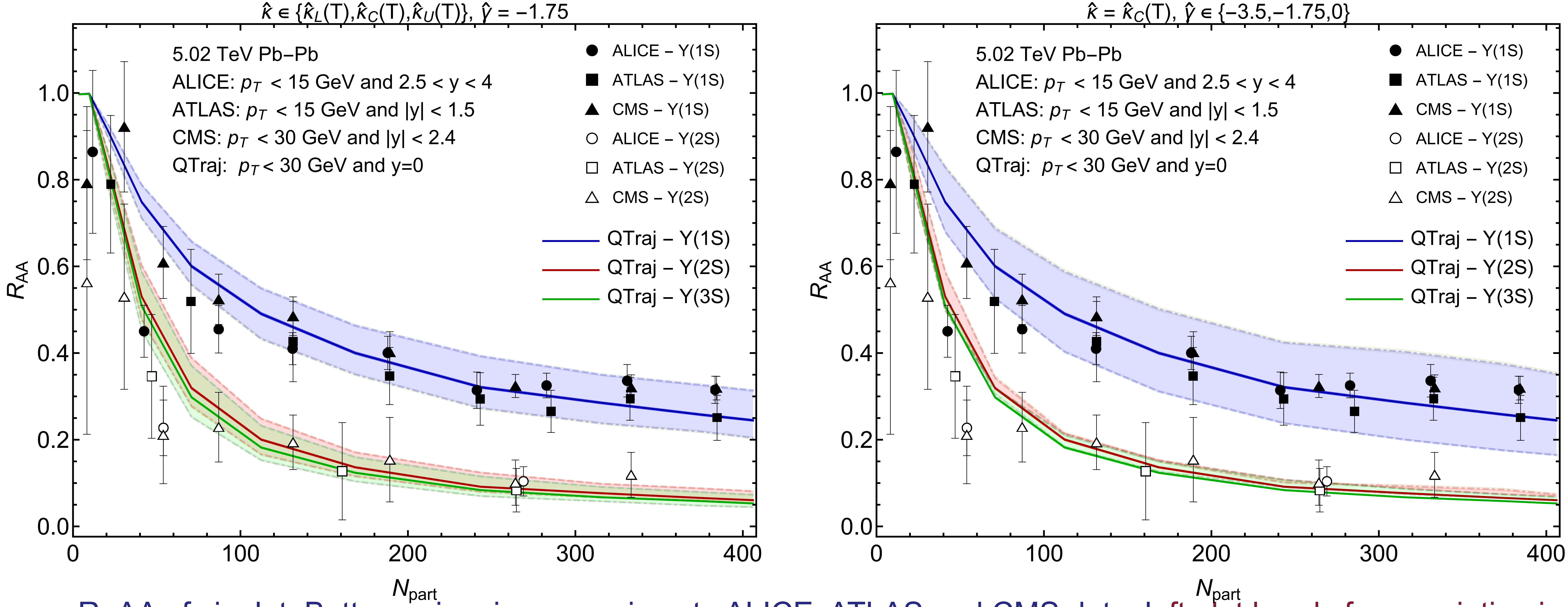
N.B. Chung Vairo 2007.07613, 2106.09417

nonequilibrium evolution of quarkonium in medium: nuclear modification factor R_AA

We compute the nuclear modification factor R_{AA}:

$$R_{AA}(nS) = \frac{\langle n, \mathbf{q} | \rho_s(t_F; t_F) | n, \mathbf{q} \rangle}{\langle n, \mathbf{q} | \rho_s(0; 0) | n, \mathbf{q} \rangle}$$

calculation with no free parameters, results depends on kappa function of T (calculated on the lattice) and gamma (extracted from the lattice)



R_AA of singlet Bottomonium in comparison to ALICE, ATLAS and CMS data, left plot bands from variation in kappa, right plot variation in gamma —> we can use R_AA to learn about the QGP!

N.B. Escobedo , Strickland, Vairo, Vander Griend, Weber, 2012.01240