Lattice QCD and tetraquarks

talk at *Double charm tetraquarks and other exotics*, IP2I, Lyon

Pedro Bicudo

CeFEMA, Departamento de Física,
Instituto Superior Técnico
Universidade de Lisboa, Portugal

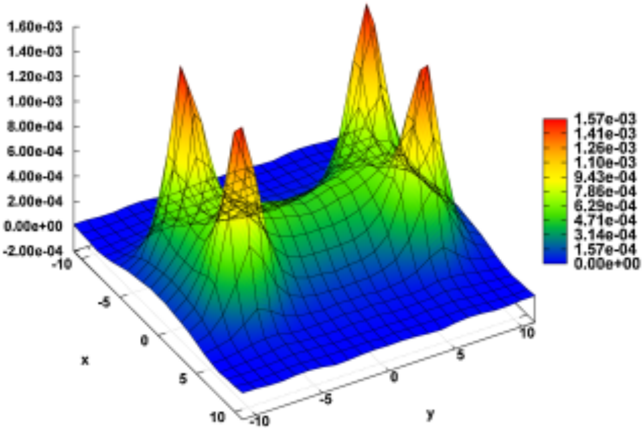
bicudo@tecnico.ulisboa.pt

November 23, 2021

Abstract

We discuss how lattice QCD can contribute to the understanding and prediction of tetraquarks and review some of our work.

This work was partly done with J. Emílio Ribeiro, Felipe Llanes-Estrada, Marco Cardoso, Nuno Cardoso, Marc Wagner, Antje Peters, Lasse Müller and in other shorter collaborations.



Introduction: homage, models, experiments and lattice QCD

Different lattice QCD approaches to study tetraquarks

Results with static heavy quarks for the T_{QQ} family of tetraquarks

Outlook for tetraquarks with lattice QCD

Introduction: homage to Jean-Marc Richard
and why I moved from quark models with χSB to lattice QCD

Figure: View from Les Houches Physics School

I first met Jean-Marc Richard in March 1987, when he was organizing the Workshop at Les Houches "The Elementary structure of matter". It was my first international meeting and the best I ever attended.

Jean-Marc was proposing his mechanism for tetraquark stability, he realized Tcc, and other tetraquarks with two heavy quarks, were the most promising ones for future discoveries. Then, most of the community was starting to disbelieve in exotics, because no experimental evidence for exotics was yet confirmed. It still took many more years of waiting, but finally there are more than 40 different experimental evidences of different tetraquarks, most of them with two heavy quarks, and finally the T_{cc} was observed.

I would like to explain why, from quark models I am doing lattice QCD.

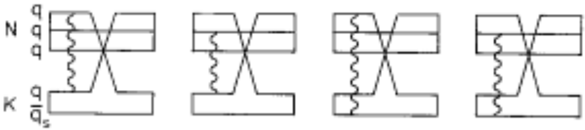


Figure: Example of a diagram computed for K-N exotic scattering.

My PhD supervisor was J. Emílio Ribeiro. His thesis, supervised by Richard Dalitz in 1978, was on the microscopic derivation of the N-N potential using the Resonating Group Method. My first work was as well on microscopic study of two-hadron systems, all the way computed from quark models, using the Resonating Group Method [14]. This is already a difficult work, since we would use systems of 4, 5 or 6 particles, with complicated potentials and colour, flavour, spin and Jacobi coordinates.

Then Emilio proposed to include spontaneous chiral symmetry breaking (SχSB) in quark models, since it would impact on the microscopic derivation of hadron-hadron interactions.

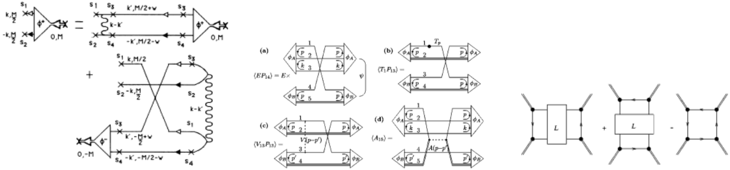


Figure: Including spontaneous ξSB in the microscopic studies of hadrons.

In the 1987 meeting of Les Houches I also met Olivier Pène who was already pursuing the same goal. We succeeded in implementing chiral symmetry breaking, in 1989, with one of the pioneer works with Dyson-Schwinger diagrammatic techniques and the Bogolubov--Valatin rotation of second quantized fields. This framework is ideal to study multiquarks because we can address any hadrons with any number of quarks, including the quasi-Goldstone boson, the pion. It is a very beautiful approach that complies with the chiral theorems.

However SχSB makes our work even harder. The model is also hard to calibrate due to the small number of parameters. In 2003 we worked with heptaquarks and this was already a herculean task. Thus in 2008 I started a lattice QCD group in Lisboa, to have an absolutely first principle computation and avoid model calibration.

Different lattice QCD approaches to study tetraquarks

In lattice QCD, when the computations are precise enough, with a small lattice spacing a an large volume $a^4 N_x N_y N_z N_t$ we get results arbitrarily close to the experiment. This is the main proof that QCD is indeed the fundamental theory of strong interactions.
In lattice QCD there are few parameters, just the current/bare quark masses. The confinement scale emerges, with dimensional transmutation, from quantization.
Moreover in models we have few clues on how to address gluons, whereas gluons are the easiest part of lattice QCD.

However lattice QCD also has its technical problems: an obvious problem is that lattice QCD is a brute force method, requiring HPC both in computational power and in coding efficiency. Results are limited by the size of the lattice $< 100^4$. Thus there are lattice QCD groups only in rich enough countries, as in experimental physics.
Lattice QCD computes the energy of observables, extracted from the evolution operator of correlation matrices in Euclidian space. From them we can also, only indirectly, study some important theoretical structures: potentials, wavefunctions or scattering matrices.

To bypass its limitations, there are different approaches to use lattice QCD computations to address tetraquarks and other exotics

1. first, during the θ^+ time, lattice QCD computed potentials for tetraquarks and pentaquarks using static quarks, by Okiharu et al. [29] and Alexandrou et al. [4]; static quarks are inexpensive in lattice QCD;

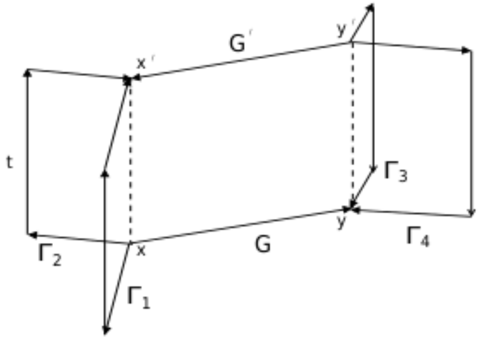


Figure: The Wilson loop to compute the potential for the $QQ\bar{Q}\bar{Q}$ system

2. then, after the Z_b was discovered there was a new and more lasting interest in tetraquarks from lattice QCD, the colour electric and colour magnetic square field densities can also be studied with four static quarks, by Cardoso et al. [21, 22];

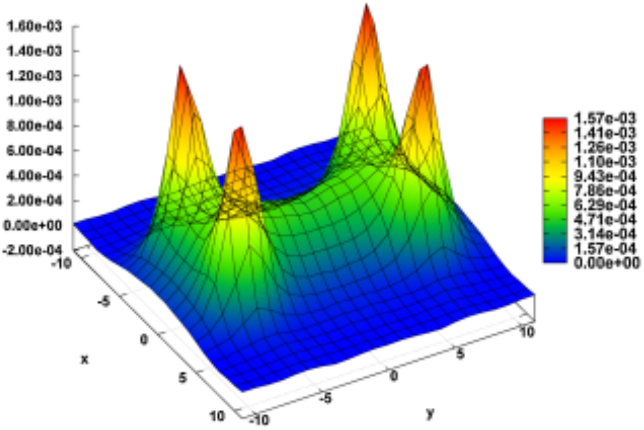


Figure: Tetraquark field density computed with four static quarks

3. in the 1987 meeting at Les Houches I also met Ernest Moniz who was studying resonances with flip-flop potentials which energy may either correspond to a tetraquark or to a meson pair; in lattice QCD, there is evidence of the flip-flop in the field densities as studied by Cardoso et al. [19]; and also in flip-flop potentials [7], including its colour dependence;

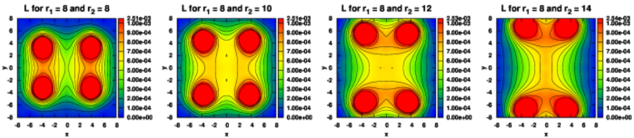
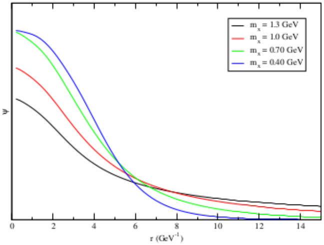


Figure: Evidence of flip-flop of the flux tube in the colour field densities for the $QQ\bar{Q}\bar{Q}$ system



We also utilized the flip-flop potentials derived from lattice QCD, to study the binding of T_{bb} systems. However the potential still needs to be improved since it does not include spin [6];

4. at the same time, Wagner [36, 37] was studying the interaction between two static-light mesons, including two static antiquarks and two light quarks, combining the expensive dynamical quark techniques with light quarks;

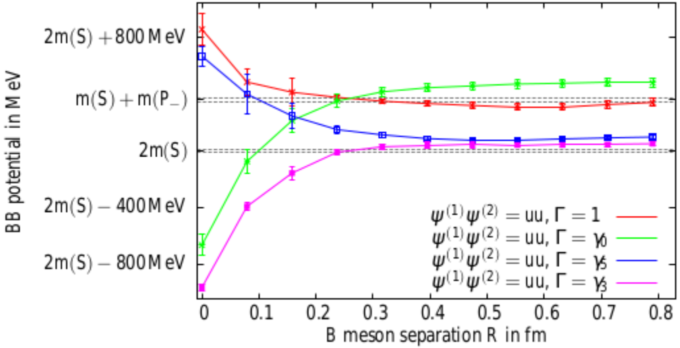


Figure: Potential between two static-light mesons, two static antiquark case

The results obtained with this potential boosted the interest in T_{bb}, T_{cb}, T_{cc} . . . We will report on this in the next section.

5. this is technically related to the study of string breaking, which first motivation was independent of tetraquarks, with coupled potentials between in a static quark-antiquark channel and two static-light mesons by Bali et al. [5] and by Bulava et al. [18], these studies are now continued by Prelovsek et al. [32], motivated by the Z_b tetraquarks;

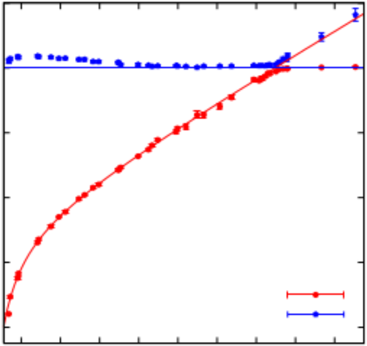


Figure: Potential eigenvalues for a quarkonium and two static-light mesons

6. after Z_c was discovered, full dynamical lattice QCD calculations were performed, searching for new energy levels in the excited spectrum by Prelovsek et al. [34] and Esposito et al. [23];

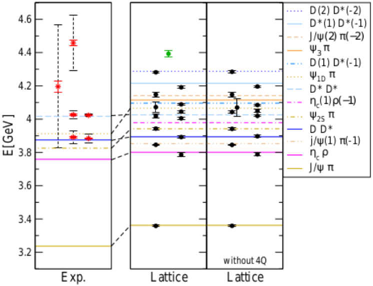


Figure: Potential eigenvalues for a quarkonium and two static-light mesons

this approach does not find any evidence for Z_c, notice these states are resonances, not boundstates; besides they are resonances high in the spectrum, decaying to a large number of channels; and the Lüscher technique cannot as well be applied to these resonances;

thus the numerous tetraquarks and pentaquarks of the Z family so far escape evidence from lattice QCD computations;

7. after the T_{bb} lattice QCD predictions by Bicudo et al. a new approach was applied to search for bound states in systems with light quarks and heavy bottom quarks: the Heavy Quark Effective Theory (HQET) or NRQCD, since bottom quarks are too heavy to be used as fully relativistic dynamical quarks in lattice QCD, by Francis et al. [25], Leskovec et al. [28] and Junnarkar et al. [27];

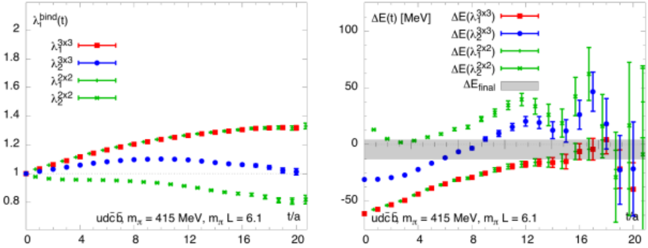


Figure: Evidence for binding of T_{cb} with dynamical light and charm quarks and HQET bottom quarks, this approach finds more binding than the one of using static heavy quarks + Born-Oppenheimer approximation

8. finally, when there are few open channels, close to the lowest threshold, lattice QCD is able to compute phase shifts from the energy spectrum of hadrons with momentum, with the Lüscher method, and this has been applied to the scattering of $D - \bar{D}^{(*)}$ mesons by Prelovsek et al . [35, 33, 31] for instance to study $X(3872)$ and charmonium;

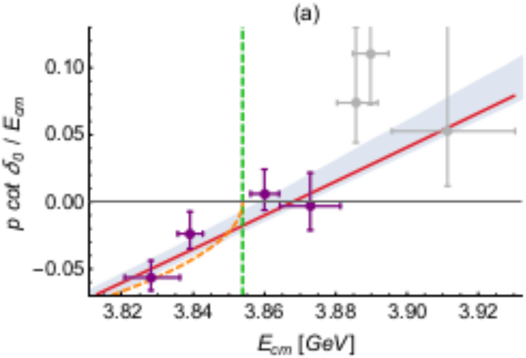


Figure: Phase shifts computed in lattice QCD with the Lüscher method, in this case for charmonium, with evidence for a boundstate

Results with lattice QCD static potentials and B-O aprox.

Before the recent result of LHCb, lattice QCD new results boosted the interest in T_{QQ}.

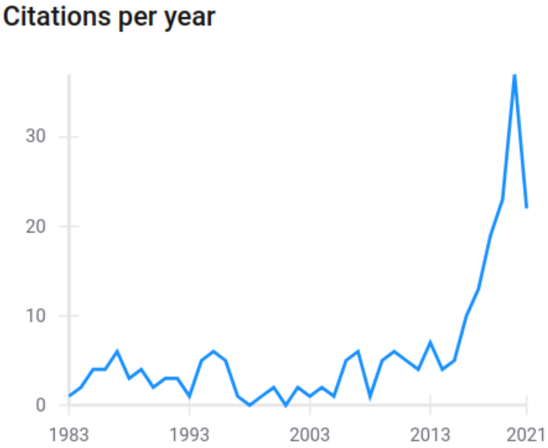


Figure: Citations of Ader, Richard and Taxil (1982) [3]: the interest was apparently boosted by lattice QCD new results, on a T_{bb} and other T_{QQ}.

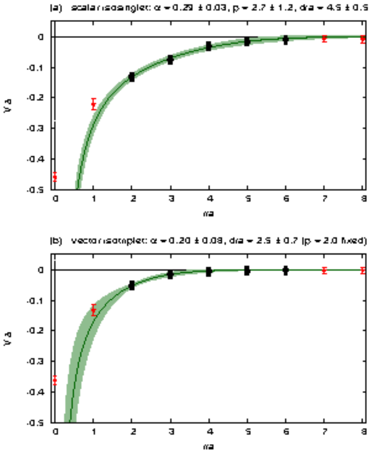
There are tensions between lattice QCD approaches to tetraquarks.

state	qnumber	δmass	lattice approach	year
$Tcc(3874)$	$ud\bar{c}\bar{c}$?	-0.360 ± 0.044 MeV	LHCb [1, 2], width 48 ± 16 KeV to $D^0 D^{*+}$	2021
	$ud\bar{c}\bar{c}\ 1^+$	-23 ± 11 MeV	dynamical lattice QCD [27]	2018
$Tccs$	$us\bar{c}\bar{c}\ 1^+$	-8 ± 8 MeV	dynamical lattice QCD [27]	2018
Tbb	$ud\bar{b}\bar{b}\ 1^+$	-90 ± 43 MeV	static lattice QCD [16, 11, 12, 15, 9, 13]	2012, 2015 ...
		~ -113 MeV	heavy quark lattice QCD [24, 25, 26]	2016, 2018 ...
		-143 ± 34 MeV	heavy quark lattice QCD [27]	2018
		-128 ± 34 MeV	heavy quark lattice QCD [28, 30]	2019
	$ud\bar{b}\bar{b}\ 0^+$	-50.0 ± 5.1 MeV	static lattice QCD [17]	2012
		-5 ± 18 MeV	heavy quark lattice QCD [27]	2018
$Tbbs$	$us\bar{b}\bar{b}$, $bs\bar{b}\bar{b}\ 1^+$	~ -36 MeV	heavy quark lattice QCD [24, 25, 26]	2016, 2018 ...
		-87 ± 32 MeV	heavy quark lattice QCD [27]	2018
		~ -80 MeV	heavy quark lattice QCD [30]	2021
$Tbbc$	$uc\bar{b}\bar{b}\ 1^+$	-6 ± 11 MeV	heavy quark lattice QCD [27]	2018
$Tbbcs$	$sc\bar{b}\bar{b}\ 1^+$	-8 ± 3 MeV	heavy quark lattice QCD [27]	2018

Table: The resonance binding energies for Tetraquark boundstates, or very narrow resonances, in lattice QCD (together with the experimental result), where we detail the approach to address the heavy quarks (static, heavy quark effective theory, or dynamical). These results are from 2012 to 2021.

Tensions in negative/ positive evidences, are not shown in the table.

To let you judge where the tension may come from, we now detail our work, for T_{bb} and related states, and for bottonomium, using static potentials computed with static heavy quarks and dynamical light quarks + dynamics provided to the heavy quarks in the Schrödinger equation.



These are the lattice QCD potentials obtained in 2010, top scalar-isoscalar and bottom vector-isovector. Notice, with static quarks, the potentials don't depend on the heavy quark spin, since they are in the infinite mass limit.
We include them in the Schrödinger equation, we need to:

- relax the infinite mass limit, with a finite heavy mass, say m_b or (less satisfactory) m_c, either the quark mass (bare current mass or the constituent mass used in quark models) or the B meson mass which are relatively similar,
- fit the potential with a continuous curve.

To fit the potential points with a curve we need an ansatz, we are educated by what we know about QCD, assymptotic freedom at small distances and infrared slavery at large distances.

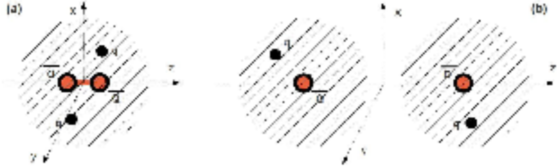


Figure: The picture of perturbative one-gluon exchange at short distances and of meson wavefunction screening at large distances

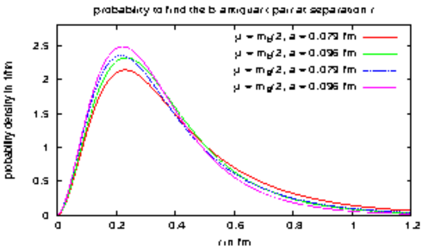
We expect a Coulomb potential at short distances, and the two heavy antiquarks are in the groundstate if they are in a triplet colour state 3, s-wave and thus spin 1. At large distances we expect a screening typical of the static-light wavefunction, say Gaussian or exponential. Our ansatz with best fits is,

$$V(r) = -\frac{\alpha}{r} \exp\left(- \left(\frac{r}{d} \right)^{p} \right) , \tag{1}$$

The error bar of the fit is reflected in the band in the plot. Results are summarized in Table 2.

channel	α	d/a	p	χ^2/dof
scalar isosinglet	0.293(33)	4.51(54)	2.74(1.20)	0.35
vector isotriplet	0.201(77)	2.48(69)	2.0 (fixed)	0.06

Table: χ^2 minimizing fit results of the ansatz (1) to the lattice static antiquark-antiquark potential; fitting range $2 \leq r/a \leq 6$; lattice spacing $a \approx 0.079\,\mathrm{fm}$



This is the wavefunction of our only solution, scalar-isoscalar for the light quarks. This was obtained with a $m_\pi \approx 340\,\mathrm{MeV}$), decreasing the light quark masses to their physical values should increase the light cloud radius and lead to a stronger binding.

μ	a	$E - 2m_B$	confidence for binding
$m_b/2$	$0.079\,\mathrm{fm}$	$-30(17)\,\mathrm{MeV}$	$1.76\,\sigma$, 96% binding
$m_b/2$	$0.096\,\mathrm{fm}$	$-49(17)\,\mathrm{MeV}$	$2.88\,\sigma$, 100% binding
$m_B/2$	$0.079\,\mathrm{fm}$	$-38(18)\,\mathrm{MeV}$	$2.11\,\sigma$, 98% binding
$m_B/2$	$0.096\,\mathrm{fm}$	$-57(19)\,\mathrm{MeV}$	$3.00\,\sigma$, 100% binding

Table: Binding energy and confidence for the existence of a heavy-heavy-light-light tetraquark for the scalar isosinglet channel.

Moreover,

- the quantum numbers are then $I(J^P) = 0(1^+)$;
- further calculations extrapolating to the physical quark mass increases binding energy to -90 ± 45 MeV;
- approximate spin effects reduces binding slightly to -60 ± 45 MeV, and the channel is the one of a BB^* binding;
- other tetraquarks do not bind: $ls\bar{b}\bar{b}$, $lc\bar{b}\bar{b}$, $sc\bar{b}\bar{b}$, $ll\bar{c}\bar{b}$, $ll\bar{c}\bar{c}$;
- but there is a p-wave resonance with the same flavour $ud\bar{b}\bar{b}$ with $I(J^P) = 0(1^-)$, mass 10576 ± 4 MeV and decay width $\Gamma = 112^{+90}_{-103}$ MeV.

And we also tested, solving the generalized eigenvalue problem, with operators which are not orthogonal, what operator contributes more at each distance between the static quarks. Notice at $R = 0$ the diquark-antidiquark operator is exactly identical to the meson-meson operator. Then, the larger the distance the smaller the overlap. We get that at large distances meson-meson dominates while at small distances diquark-antidiquark dominates. Moreover for the light quarks the spinor matrix $(1+\gamma_0)\gamma_5$ dominates [13].

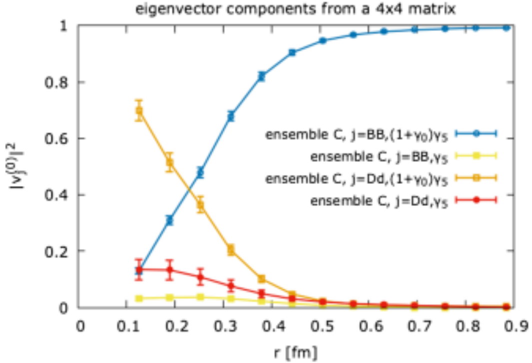
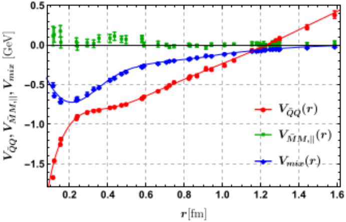
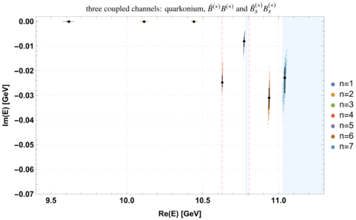
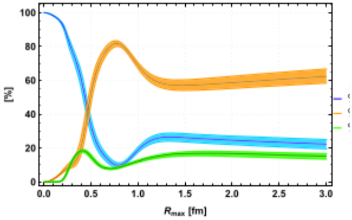


Figure: Contribution of the operators to the lightest correlation eigenvalue.







We also briefly review our results [8, 10] for bottomonium from string breaking static potentials, as a warm up for the study [20] of the $Z_b, Z_c \ldots$ family

We find an extra state compatible with the $\Upsilon(10753)$. We respectively show,

- the diagonal and off diagonal potentials reconstructed from string breaking potentials, cumpling the quarkonium $b\bar{b}$ to meson-meson $B^{(*)}\bar{B}^{(*)}$;
- the poles of the s matrix in the complex energy plane for bottomonium coupled to meson-meson channels;
- the meson-meson versus quarkonium composition of $\Upsilon(10753)$, our only state which is mostly meson-meson, in particular dominated by $B^{(*)}\bar{B}^{(*)}$.

Outlook for tetraquarks with lattice QCD

Future directions:

- with the growing interest in tetraquarks, and the tension between different approaches, the precision lattice QCD computations should definitely enter the game of bound tetraquarks such as T_{cc}, T_{bc} and T_{bb} ... with a large *engagement* of the community.
- All the different approaches can find room for improvement, for instance spin dependence can be included in the static approach.
- The lattice QCD community will also probably address theoretical questions on the properties of tetraquarks.
- However, for the Z_b, Z_c ... family, high in the spectrum, with many decay coupled channels, technical advances are still necessary to study them with lattice QCD.

MERCI !!!

[1] Roel Aaij et al. Observation of an exotic narrow doubly charmed tetraquark. 9 2021.

[2] Roel Aaij et al. Study of the doubly charmed tetraquark T_{cc}^{+}. 9 2021.

[3] J. P. Ader, J. M. Richard, and P. Taxil. DO NARROW HEAVY MULTI - QUARK STATES EXIST? *Phys. Rev.*, D25:2370, 1982.

[4] C. Alexandrou and G. Koutsou. The Static tetraquark and pentaquark potentials. *Phys.Rev.*, D71:014504, 2005.

[5] Gunnar S. Bali, Hartmut Neff, Thomas Duessel, Thomas Lippert, and Klaus Schilling. Observation of string breaking in QCD. *Phys. Rev.*, D71:114513, 2005.

[6] P. Bicudo and M. Cardoso. Tetraquark bound states and resonances in the unitary and microscopic triple string flip-flop quark model, the light-light-antiheavy-antiheavy $qq\bar{Q}\bar{Q}$ case study. *Phys. Rev. D*, 94(9):094032, 2016.

[7] P. Bicudo, M. Cardoso, O. Oliveira, and P. J. Silva. Lattice QCD static potentials of the meson-meson and tetraquark systems computed with both quenched and full QCD. *Phys. Rev. D*, 96(7):074508, 2017.

[8] Pedro Bicudo, Marco Cardoso, Nuno Cardoso, and Marc Wagner. Bottomonium resonances with $I = 0$ from lattice QCD

correlation functions with static and light quarks. *Phys. Rev. D*, 101(3):034503, 2020.

[9] Pedro Bicudo, Marco Cardoso, Antje Peters, Martin Pflaumer, and Marc Wagner. $ud\bar{b}\bar{b}$ tetraquark resonances with lattice QCD potentials and the Born-Oppenheimer approximation. *Phys. Rev. D*, 96(5):054510, 2017.

[10] Pedro Bicudo, Nuno Cardoso, Lasse Müller, and Marc Wagner. Computation of the quarkonium and meson-meson composition of the $\Upsilon(nS)$ states and of the new $\Upsilon(10753)$ Belle resonance from lattice QCD static potentials. *Phys. Rev. D*, 103(7):074507, 2021.

[11] Pedro Bicudo, Krzysztof Cichy, Antje Peters, Björn Wagenbach, and Marc Wagner. Evidence for the existence of $ud\bar{b}\bar{b}$ and the non-existence of $ss\bar{b}\bar{b}$ and $cc\bar{b}\bar{b}$ tetraquarks from lattice QCD. 2015.

[12] Pedro Bicudo, Krzysztof Cichy, Antje Peters, and Marc Wagner. BB interactions with static bottom quarks from Lattice QCD. *Phys. Rev. D*, 93(3):034501, 2016.

[13] Pedro Bicudo, Antje Peters, Sebastian Velten, and Marc Wagner. Importance of meson-meson and of diquark-antidiquark creation operators for a $\bar{b}\bar{b}ud$ tetraquark. *Phys. Rev. D*, 103(11):114506, 2021.

[14] Pedro Bicudo and Jose Emilio Ribeiro. $K - N$ Exotic S Channel I = 1, 0 Phase Shifts: A Test of the Nonannihilating Quark Quark Potential. *Z. Phys. C*, 38:453, 1988.

[15] Pedro Bicudo, Jonas Scheunert, and Marc Wagner. Including heavy spin effects in the prediction of a $\bar{b}\bar{b}ud$ tetraquark with lattice QCD potentials. *Phys. Rev. D*, 95(3):034502, 2017.

[16] Pedro Bicudo and Marc Wagner. Lattice QCD signal for a bottom-bottom tetraquark. *Phys.Rev.*, D87(11):114511, 2013.

[17] Zachary S. Brown and Kostas Orginos. Tetraquark bound states in the heavy-light heavy-light system. *Phys.Rev.*, D86:114506, 2012.

[18] John Bulava, Ben Hörz, Francesco Knechtli, Vanessa Koch, Graham Moir, Colin Morningstar, and Mike Peardon. String breaking by light and strange quarks in QCD. *Phys. Lett. B*, 793:493–498, 2019.

[19] M. Cardoso, N. Cardoso, and P. Bicudo. Variational study of the flux tube recombination in the two quarks and two quarks system in Lattice QCD. *Phys.Rev.*, D86:014503, 2012.

[20] Marco Cardoso and Pedro Bicudo. Microscopic calculation of the decay of Jaffe-Wilczek tetraquarks, and the Z(4433). *AIP Conf.Proc.*, 1030:352, 2008.

[21] N. Cardoso, M. Cardoso, and P. Bicudo. Colour Fields Computed in SU(3) Lattice QCD for the Static Tetraquark System. *Phys.Rev.*, D84:054508, 2011.

[22] Nuno Cardoso and Pedro Bicudo. Color fields of the static pentaquark system computed in SU(3) lattice QCD. *Phys.Rev.*, D87(3):034504, 2013.

[23] A. Esposito, M. Papinutto, A. Pilloni, A. D. Polosa, and N. Tantalo. Doubly charmed tetraquarks in B_c and Ξ_{bc} decays. *Phys. Rev. D*, 88(5):054029, 2013.

[24] Anthony Francis, Renwick J. Hudspith, Randy Lewis, and Kim Maltman. Lattice Prediction for Deeply Bound Doubly Heavy Tetraquarks. *Phys. Rev. Lett.*, 118(14):142001, 2017.

[25] Anthony Francis, Renwick J. Hudspith, Randy Lewis, and Kim Maltman. Evidence for charm-bottom tetraquarks and the mass dependence of heavy-light tetraquark states from lattice QCD. *Phys. Rev. D*, 99(5):054505, 2019.

[26] R. J. Hudspith, B. Colquhoun, A. Francis, R. Lewis, and K. Maltman. A lattice investigation of exotic tetraquark channels. *Phys. Rev. D*, 102:114506, 2020.

[27] Parikshit Junnarkar, Nilmani Mathur, and M. Padmanath. Study of doubly heavy tetraquarks in Lattice QCD. *Phys. Rev. D*, 99(3):034507, 2019.

[28] Luka Leskovec, Stefan Meinel, Martin Pflaumer, and Marc Wagner. Lattice QCD investigation of a doubly-bottom $\bar{b}\bar{b}ud$ tetraquark with quantum numbers $I(J^P) = 0(1^+)$. *Phys. Rev. D*, 100(1):014503, 2019.

[29] Fumiko Okiharu, Hideo Suganuma, and Toru T. Takahashi. First study for the pentaquark potential in SU(3) lattice QCD. *Phys. Rev. Lett.*, 94:192001, 2005.

[30] Martin Pflaumer, Luka Leskovec, Stefan Meinel, and Marc Wagner. Existence and Non-Existence of Doubly Heavy Tetraquark Bound States. In *38th International Symposium on Lattice Field Theory*, 8 2021.

[31] Stefano Piemonte, Sara Collins, Daniel Mohler, M. Padmanath, and Sasa Prelovsek. Charmonium resonances with $J^{PC} = 1^{--}$ and 3^{--} from $\bar{D}D$ scattering on the lattice. *Phys. Rev. D*, 100(7):074505, 2019.

[32] S. Prelovsek, H. Bahtiyar, and J. Petkovic. Zb tetraquark channel from lattice QCD and Born-Oppenheimer approximation. *Phys. Lett. B*, 805:135467, 2020.

[33] Sasa Prelovsek, Sara Collins, Daniel Mohler, M. Padmanath, and Stefano Piemonte. Charmonium-like resonances with $J^{PC} = 0^{++}, 2^{++}$ in coupled $\mathrm{D}\overline{\mathrm{D}}$, $\mathrm{D_s}\overline{\mathrm{D}}_\mathrm{s}$ scattering on the lattice. *JHEP*, 06:035, 2021.

[34] Sasa Prelovsek, C.B. Lang, Luka Leskovec, and Daniel Mohler. Study of the Z_c^+ channel using lattice QCD. 2014.

[35] Sasa Prelovsek and Luka Leskovec. Evidence for X(3872) from DD* scattering on the lattice. *Phys. Rev. Lett.*, 111:192001, 2013.

[36] Marc Wagner. Forces between static-light mesons. *PoS*, LATTICE2010:162, 2010.

[37] Marc Wagner. Static-static-light-light tetraquarks in lattice QCD. *Acta Phys. Polon. Supp.*, 4:747–752, 2011.