

The $I(J^\pi) = 0(1^+)$ T_{bb}^- tetraquark structure and weak decay

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Based on Physics Letters B 800 (2020) 135073.

Outline

- Theoretical evidence for the existence of the $I(J^\pi) = 0(1^+)$ T_{bb}^- tetraquark
- T_{bb}^- in the constituent quark model
- T_{bb}^- weak decay
- Summary

Theoretical evidence

- In J.P. Ader, J.-M. Richard and P. Taxil, Phys. Rev. D25 (1982) 2370, it is shown how $QQ\bar{Q}'\bar{Q}'$ can be stable if the ratio of quark masses is large enough.
- S. Zouzou et al., Z. Phys. C 30 (1986) 457, they already found that some states with $bb\bar{u}\bar{d}$ quark content were bound. See also B- Silvestre-Brac and C. Semay, Z.Phys. C 57 (1993) 273
- J. Carlson, L. Heller and J.A. Tjon, Phys. Rev. D 37 (1988) 744, derived a static potential from the Born-Oppenheimer approximation to the MIT bag model and found that the $I(J^\pi) = 0(1^+)$ $bb\bar{u}\bar{d}$ tetraquark was bound by 70 MeV.
- A.V. Manohar and M.B. Wise, Nucl. Phys. B399 (1993) 17, show that the long-range one-pion exchange potential between ground state $Q\bar{q}$ meson may be sufficiently attractive in the $Q = b$ case to produce weakly bound two-meson states.
- S. Pepin et al., Phys. Lett. B 393 (1997) 119, use a model where the chromomagnetic hyperfine interaction is replaced by pseudoscalar-meson exchange contributions. They find that such an interaction binds a heavy tetraquark system $QQqq$ ($Q = c, b$; $q = u, d$) by 0.2-0.4 GeV.
- D.M. Brink and Fl. Stancu, Phys. Rev. D 57 (1998) 6778, within a simple nonrelativistic potential model found the $I(J^\pi) = 0(1^+)$ $bb\bar{u}\bar{d}$ tetraquark to be bound by 100 MeV.

Theoretical evidence II

- J. Vijande et al., Eur. Phys. J. A 19 (2004) 383, using a chiral constituent quark model found the $\bar{b}\bar{b}qq$ ($q = u, d$) to be bound by 340 MeV.
- F.S. Navarra, M. Nielsen and S.-H. Lee, Phys. Lett. B 649 (2007) 166, used QCD sum rules and found the $I(J^\pi) = 0(1^+)$ $bb\bar{u}\bar{d}$ below threshold.
- D. Ebert et al., Phys. Rev. D 76, 114015 (2007), using the relativistic quark model in the framework of the diquark-antidiquark picture also found the $I(J^\pi) = 0(1^+)$ $bb\bar{u}\bar{d}$ below threshold.
- M. Zhang, H.-X. Zhang and Z.-Y. Zhang, Commun. Theor. Phys. 50, (2008) 437, use a chiral SU(3) quark model find a smaller binding of around 30 MeV.
- J. Vijande, A. Valcarce and N. Barnea, Phys. Rev. D 79 (2009) 074010, use the hyperspherical formalism and two different potentials. In each case the $I(J^\pi) = 0(1^+)$ $ud\bar{b}\bar{b}$ tetraquark is below threshold.
- M.-L. Du et al, Phys. Rev. D87 (2013) 014003, use a QCD sum rule approach concluding that $bb\bar{q}\bar{q}$ tetraquarks are stable.

Theoretical evidence III

- P. Bicudo et al., Phys. Rev. D 93 (2016) 034501, use lattice QCD to derive an isospin, spin and parity dependent potential between a pair of two B mesons. Using the attractive channels they find an indication for a $\bar{b}\bar{b}ud$ tetraquark.
- M. Karliner and J.L. Rosner, Phys. Rev. Lett. 119 (2017) 202001, use the assumption, validated by their successful prediction of the Ξ_{cc} mass, that the binding energy of two heavy quarks in a color-antitriplet QQ state is half that of $Q\bar{Q}$ in a color-singlet, to predict the $I(J^\pi) = 0(1^+)$ $ud\bar{b}\bar{b}$ tetraquark mass at 10389 ± 12 MeV. It is 215 MeV below threshold being stable not only against strong but also e.m. interaction.
- E. J. Eichten and C. Quigg, Phys. Rev. Lett 119 (2017) 202002, using heavy quark symmetry relations and finite-mass corrections show that the $I(J^\pi) = 0(1^+)$ $bb\bar{u}\bar{d}$ tetraquark is stable against strong decays.
- S.-Q. Luo et al, Eur. Phys. J C (2017) 77:709, study the mass of the $qq\bar{Q}\bar{Q}$ tetraquarks states with the color-magnetic interaction. They find the $I(J^\pi) = 0(1^+)$ $bb\bar{u}\bar{d}$ tetraquark is stable against strong and e.m. interactions.

Theoretical evidence IV

- In A. Francis et al., Phys. Rev. Lett. 118 (2017) 142001, the authors investigate the possibility of $qq\bar{b}\bar{b}$ tetraquark bound states using lattice QCD. They find that the $I(J^\pi) = 0(1^+)$ $ud\bar{b}\bar{b}$ tetraquark lies 189(10)(3) MeV below the corresponding two-meson threshold, again stable against strong and e.m. interaction.
- J.-M. Richard, A. Valcarce and J. Vijande, Phys. Rev. C 97 (2018) 035211, made a critical analysis of different strategies in the literature. They emphasize the role of spin effects and color mixing in order to get below threshold $bb\bar{u}\bar{d}$ states.
- P. Junnarkar et al., Phys. Rev. D 99 (2019) 034507, also using lattice QCD find a $I(J^\pi) = 0(1^+)$ $ud\bar{b}\bar{b}$ tetraquark with a binding energy of 143(34) MeV.
- S.S. Agaev et al., Phys. Rev. D 99 (2019) 033002, also find a stable $bb\bar{u}\bar{d}$ tetraquark using QCD two-point sum rules.
- Q. Meng et al., Phys. Lett. B 814 (2021) 136095, within a constituent quark model find the $I(J^\pi) = 0(1^+)$ $ud\bar{b}\bar{b}$ tetraquark is bound by 173 MeV.

There is evidence from relativistic and nonrelativistic quark models, QCD sum rules and lattice QCD for the existence of a $I(J^\pi) = 0(1^+)$ $ud\bar{b}\bar{b}$ tetraquark stable against strong and e.m. decays.

T_{QQ} state

$$\begin{aligned}
& \left| T, \lambda \vec{P} \right\rangle_{NR} = \sqrt{2E(\vec{P})} \int d^3 p_x \int d^3 p_y \int d^3 p_z \frac{1}{\sqrt{N_Q}} \frac{1}{\sqrt{N_q}} \sum_{\alpha_1, \alpha_2, \alpha_3, \alpha_4} \hat{\phi}_{\alpha_1, \alpha_2, \alpha_3, \alpha_4}^{(T, \lambda)} (\vec{p}_x, \vec{p}_y, \vec{p}_z) \\
& \times \frac{(-1)^{1/2-s_3} (-1)^{1/2-s_4}}{(2\pi)^{9/2} \sqrt{2E_{f_1}(\vec{p}_1) 2E_{f_2}(\vec{p}_2) 2E_{f_3}(\vec{p}_3) 2E_{f_4}(\vec{p}_4)}} \\
& \times \left| Q, \alpha_1 \vec{p}_1 = \frac{m_{f_1}}{M} \vec{P} + \vec{p}_x + \frac{m_{f_1}}{m_{f_1} + m_{f_2}} \vec{p}_z \right\rangle \left| Q, \alpha_2 \vec{p}_2 = \frac{m_{f_2}}{M} \vec{P} - \vec{p}_x + \frac{m_{f_2}}{m_{f_1} + m_{f_2}} \vec{p}_z \right\rangle \\
& \times \left| \bar{q}, \alpha_3 \vec{p}_3 = \frac{m_{f_3}}{M} \vec{P} + \vec{p}_y - \frac{m_{f_3}}{m_{f_3} + m_{f_4}} \vec{p}_z \right\rangle \left| \bar{q}, \alpha_4 \vec{p}_4 = \frac{m_{f_4}}{M} \vec{P} - \vec{p}_y - \frac{m_{f_4}}{m_{f_3} + m_{f_4}} \vec{p}_z \right\rangle
\end{aligned}$$

with $\alpha \equiv (c, f, s)$ and $\vec{p}_{x,y,z}$ conjugate momenta to the relative coordinates

$$\begin{aligned}
\vec{x} &= \vec{r}_1 - \vec{r}_2 \\
\vec{y} &= \vec{r}_3 - \vec{r}_4, \\
\vec{z} &= \frac{m_{f_1} \vec{r}_1 + m_{f_2} \vec{r}_2}{m_{f_1} + m_{f_2}} - \frac{m_{f_3} \vec{r}_3 + m_{f_4} \vec{r}_4}{m_{f_3} + m_{f_4}}.
\end{aligned}$$

T_{QQ} state II

The states are normalized such that

$$\left\langle T, \lambda' \vec{P}' \mid T, \lambda \vec{P} \right\rangle_{NR} = \delta_{\lambda \lambda'} (2\pi)^3 2E(\vec{P}) \delta(\vec{P} - \vec{P}'),$$

which requires

$$\begin{aligned} & \int d^3 p_x \int d^3 p_y \int d^3 p_z \sum_{\alpha_1 \alpha_2, \alpha_3 \alpha_4} \left(\hat{\phi}_{\alpha_1, \alpha_2, \alpha_3, \alpha_4}^{(T, \lambda')} (\vec{p}_x, \vec{p}_y, \vec{p}_z) \right)^* \hat{\phi}_{\alpha_1, \alpha_2, \alpha_3, \alpha_4}^{(T, \lambda)} (\vec{p}_x, \vec{p}_y, \vec{p}_z) \\ &= \int d^3 x \int d^3 y \int d^3 z \sum_{\alpha_1, \alpha_2, \alpha_3, \alpha_4} \left(\phi_{\alpha_1, \alpha_2, \alpha_3, \alpha_4}^{(T, \lambda')} (\vec{x}, \vec{y}, \vec{z}) \right)^* \phi_{\alpha_1, \alpha_2, \alpha_3, \alpha_4}^{(T, \lambda)} (\vec{x}, \vec{y}, \vec{z}) \\ &= \delta_{\lambda \lambda'} \end{aligned}$$

I(J^π) = 0(1^+) T_{bb}^- wave function

$$\begin{aligned}
& \hat{\phi}_{\alpha_1, \alpha_2, \alpha_3, \alpha_4}^{(T, \lambda)}(\vec{p}_x, \vec{p}_y, \vec{p}_z) = \delta_{f_1 b} \delta_{f_2 b} \frac{-1}{\sqrt{2}} (\delta_{f_3 d} \delta_{f_4 u} - \delta_{f_3 u} \delta_{f_4 d}) \\
& \times \left\{ \frac{1}{2\sqrt{3}} \left[(\delta_{c_1 1} \delta_{c_2 2} - \delta_{c_1 2} \delta_{c_2 1})(\delta_{c_3 1} \delta_{c_4 2} - \delta_{c_3 2} \delta_{c_4 1}) \right. \right. \\
& \quad + (\delta_{c_1 1} \delta_{c_2 3} - \delta_{c_1 3} \delta_{c_2 1})(\delta_{c_3 1} \delta_{c_4 3} - \delta_{c_3 3} \delta_{c_4 1}) + (\delta_{c_1 2} \delta_{c_2 3} - \delta_{c_1 3} \delta_{c_2 2})(\delta_{c_3 2} \delta_{c_4 3} - \delta_{c_3 3} \delta_{c_4 2}) \left. \right] \\
& \times \left[(1/2, 1/2, 1; s_1, s_2, S_z)(1/2, 1/2, 0; s_3, s_4, 0) \Phi_{SS}^{(1)}(\vec{p}_x, \vec{p}_y, \vec{p}_z) \right. \\
& \quad + (1/2, 1/2, 0; s_1, s_2, 0)(1/2, 1/2, 1; s_3, s_4, S_z) \Phi_{AA}^{(2)}(\vec{p}_x, \vec{p}_y, \vec{p}_z) \\
& \quad \left. + \Phi_{SA}^{(3)}(\vec{p}_x, \vec{p}_y, \vec{p}_z) \sum_{m, m'} (1, 1, 1; m, m', S_z)(1/2, 1/2, 1; s_1, s_2, m)(1/2, 1/2, 1; s_3, s_4, m') \right] \\
& + \dots
\end{aligned}$$

I(J^π) = 0(1^+) T_{bb}^- wave function II

$$\begin{aligned}
& \cdots + \frac{1}{\sqrt{6}} \left[\delta_{c_1 1} \delta_{c_2 1} \delta_{c_3 1} \delta_{c_4 1} + \delta_{c_1 2} \delta_{c_2 2} \delta_{c_3 2} \delta_{c_4 2} + \delta_{c_1 3} \delta_{c_2 3} \delta_{c_3 3} \delta_{c_4 3} \right. \\
& + \frac{1}{2} (\delta_{c_1 1} \delta_{c_2 2} + \delta_{c_1 2} \delta_{c_2 1}) (\delta_{c_3 1} \delta_{c_4 2} + \delta_{c_3 2} \delta_{c_4 1}) + \frac{1}{2} (\delta_{c_1 1} \delta_{c_2 3} + \delta_{c_1 3} \delta_{c_2 1}) (\delta_{c_3 1} \delta_{c_4 3} + \delta_{c_3 3} \delta_{c_4 1}) \\
& \left. + \frac{1}{2} (\delta_{c_1 2} \delta_{c_2 3} + \delta_{c_1 3} \delta_{c_2 2}) (\delta_{c_3 2} \delta_{c_4 3} + \delta_{c_3 3} \delta_{c_4 2}) \right] \\
& \times \left[(1/2, 1/2, 1, s_1, s_2, S_z) (1/2, 1/2, 0; s_3, s_4, 0) \Phi_{AA}^{(4)}(\vec{p}_x, \vec{p}_y, \vec{p}_z) \right. \\
& + (1/2, 1/2, 0; s_1, s_2, 0) (1/2, 1/2, 1, s_3, s_4, S_z) \Phi_{SS}^{(5)}(\vec{p}_x, \vec{p}_y, \vec{p}_z) \\
& \left. + \Phi_{AS}^{(6)}(\vec{p}_x, \vec{p}_y, \vec{p}_z) \sum_{m, m'} (1, 1, 1; m, m', S_z) (1/2, 1/2, 1; s_1, s_2, m) (1/2, 1/2, 1; s_3, s_4, m') \right] \}
\end{aligned}$$

For the wave function to be normalized one needs that

$$\begin{aligned}
& \int d^3 p_x \int d^3 p_y \int d^3 p_z \left(|\Phi_{SS}^{(1)}(\vec{p}_x, \vec{p}_y, \vec{p}_z)|^2 + |\Phi_{AA}^{(2)}(\vec{p}_x, \vec{p}_y, \vec{p}_z)|^2 + |\Phi_{SA}^{(3)}(\vec{p}_x, \vec{p}_y, \vec{p}_z)|^2 \right. \\
& \left. + |\Phi_{AA}^{(4)}(\vec{p}_x, \vec{p}_y, \vec{p}_z)|^2 + |\Phi_{SS}^{(5)}(\vec{p}_x, \vec{p}_y, \vec{p}_z)|^2 + |\Phi_{AS}^{(6)}(\vec{p}_x, \vec{p}_y, \vec{p}_z)|^2 \right) = 1.
\end{aligned}$$

$I(J^\pi) = 0(1^+)$ T_{bb}^- wave function III

$$\Phi^{(j)}(\vec{p}_x, \vec{p}_y, \vec{p}_z) = \frac{1}{(2\pi)^{9/2}} \int d^3x \int d^3y \int d^3z e^{-i(\vec{p}_x \cdot \vec{x} + \vec{p}_y \cdot \vec{y} + \vec{p}_z \cdot \vec{z})} \phi^{(j)}(\vec{x}, \vec{y}, \vec{z})$$

and

$$\phi^{(j)}(\vec{x}, \vec{y}, \vec{z}) = \sum_{k=1}^5 \lambda_k^{(j)} e^{-a_k^{(j)} \vec{x}^2 - b_k^{(j)} \vec{y}^2 - c_k^{(j)} \vec{z}^2 - d_k^{(j)} \vec{x} \cdot \vec{y} - e_k^{(j)} \vec{x} \cdot \vec{z} - f_k^{(j)} \vec{y} \cdot \vec{z}}$$

to which one has to apply one of the four symmetry operators

$$P_{SS} = (1 + P_{12})(1 + P_{34})$$

$$P_{AA} = (1 - P_{12})(1 - P_{34})$$

$$P_{SA} = (1 + P_{12})(1 - P_{34})$$

$$P_{AS} = (1 - P_{12})(1 + P_{34})$$

$I(J^\pi) = 0(1^+)$ T_{bb}^- wave function IV

The full set of parameters that describe the wave function are determined by minimizing the energy.

Apart from kinetic terms the Hamiltonian consist of two-body potential terms for which the AL1 potential from C. Semay and B. Silvestre-Brac, Z. Phys. C 61 (1994) 271, is used

$$V_{ij}^{qq}(r) = -\frac{3}{16} \vec{\lambda}_i^c \cdot \vec{\lambda}_j^c \left(-\frac{\kappa}{r} + \lambda r - \Lambda + \frac{2\pi}{3m_i m_j} \kappa' \frac{e^{-r^2/x_0^2}}{\pi^{3/2} x_0^3} \vec{\sigma}_i \vec{\sigma}_j \right), \quad x_0 = A \left(\frac{2m_i m_j}{m_i + m_j} \right)^{-B}$$

For $q\bar{q}$

$$-\frac{3}{16} \vec{\lambda}_i^c \cdot \vec{\lambda}_j^c \rightarrow \frac{3}{16} \vec{\lambda}_i^c \cdot \vec{\lambda}_j^{cT}$$

Establishing the tetraquark structure

M_Q [MeV]	$M_1 + M_2$ [MeV]	$M_{T_{QQ}}$ [MeV]	ΔE [MeV]	$P[\bar{3}3\rangle]$	$P[6\bar{6}\rangle]$	$P[11\rangle]$	$P[88\rangle]$	P_{MM^*}	$P_{M^*M^*}$
5227	10644	10493	-151	0.967	0.033	0.344	0.656	0.561	0.439
4549	9290	9163	-126	0.955	0.045	0.348	0.652	0.597	0.403
3871	7936	7835	-100	0.930	0.070	0.357	0.643	0.646	0.354
3193	6582	6511	-71	0.885	0.115	0.372	0.628	0.730	0.270
2515	5230	5189	-41	0.778	0.222	0.407	0.593	0.795	0.205
1836	3878	3865	-13	0.579	0.421	0.474	0.526	0.880	0.120
1158	2534	2552	> 0	0.333	0.667	0.556	0.444	1.000	0.000

Within the AL1 potential

$$m_u = m_d = 315 \text{ MeV} , \quad m_c = 1836 \text{ MeV} , \quad m_b = 5277 \text{ MeV}$$

See J. Vijande and A. Valcarce, Phys. Rev. C80 (2009) 035204 to see how the $P_{M_1 M_2}$ are defined.

Establishing the tetraquark structure

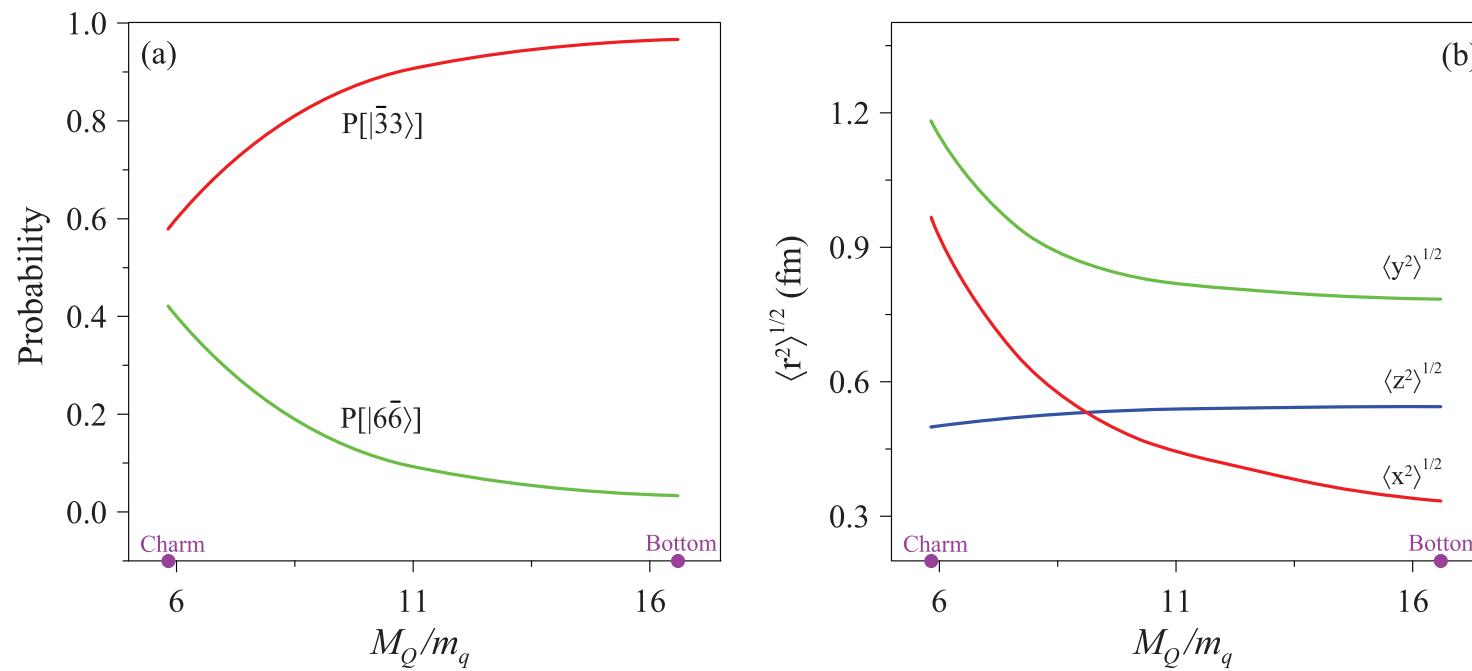
M_Q [MeV]	ΔE [MeV]	$\langle x^2 \rangle^{1/2}$ [fm]	$\langle y^2 \rangle^{1/2}$ [fm]	$\langle z^2 \rangle^{1/2}$ [fm]	$X_{T_{QQ}}$ [fm]
5227	-151	0.334	0.784	0.544	0.226
4549	-126	0.362	0.791	0.544	0.242
3871	-100	0.411	0.806	0.541	0.268
3193	-71	0.475	0.833	0.536	0.301
2515	-41	0.621	0.919	0.523	0.369
1836	-13	0.966	1.181	0.499	0.530
1158	> 0	$\gg 1$	$\gg 1$	0.470	$\gg 1$

with X_T the root-mean-square radius of the tetraquark defined as

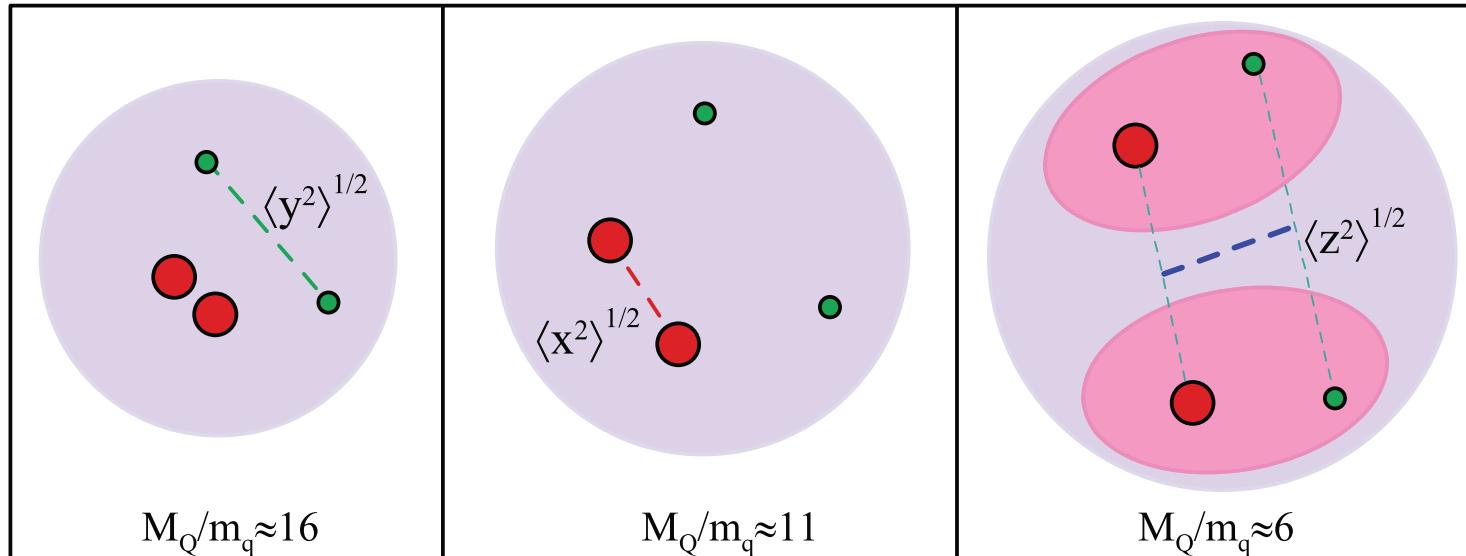
$$X_T = \left[\frac{\sum_{i=1}^4 m_i \langle (\vec{r}_i - \vec{R})^2 \rangle}{\sum_{i=1}^4 m_i} \right]^{1/2},$$

with \vec{R} the center of mass vector.

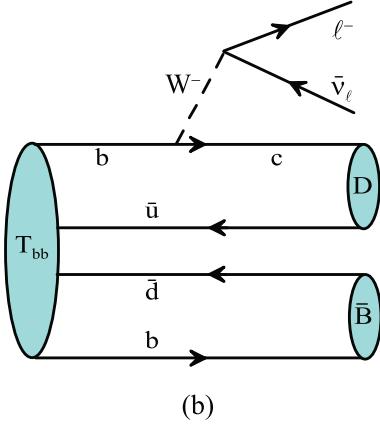
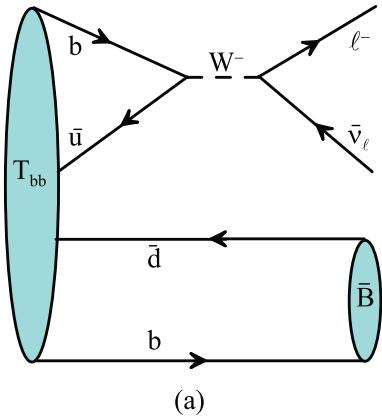
Establishing the tetraquark structure



Establishing the tetraquark structure



Semileptonic decays



$$\begin{aligned}
 h_{\rho}^{T \rightarrow B} &= 4\sqrt{m_T E_B} \iint d\mathbf{p}_x d\mathbf{p}_z \\
 &\times \sum_{\alpha_1, \alpha_2, \alpha_3, \alpha_4} \left[\hat{\phi}_{\alpha_2, \alpha_4}^{(B, \lambda')} \left(\frac{m_b}{m_b + m_u} \mathbf{P}_B + \mathbf{p}_x - \frac{1}{2} \mathbf{p}_z \right) \right]^* \\
 &\times \hat{\phi}_{\alpha_1, \alpha_2, \alpha_3, \alpha_4}^{(T, \lambda)} (\mathbf{p}_x, -\mathbf{p}_x - \mathbf{P}_B, \mathbf{p}_z) \\
 &\times \frac{(-1)^{1/2-s_3}}{2\sqrt{E_u E_b}} \bar{v}_u^{s_3} \left(-\mathbf{P}_B - \mathbf{p}_x - \frac{1}{2} \mathbf{p}_z \right) \gamma_\rho (1 - \gamma_5) \\
 &\times u_b^{s_1} \left(\mathbf{p}_x + \frac{1}{2} \mathbf{p}_z \right) \delta_{c_1 c_3} \delta_{f_1 b} \delta_{f_3 u},
 \end{aligned}$$

$$\begin{aligned}
 h_{\rho}^{T \rightarrow BD} &= 4(2\pi)^{3/2} \sqrt{2m_T E_B E_D} \sum_{\alpha_2, \alpha_3, \alpha_4, \alpha_5} \iint d\mathbf{p}_x d\mathbf{p}_z \\
 &\times \left[\hat{\phi}_{\alpha_5, \alpha_3}^{(D, \lambda'')} \left(-\frac{m_u}{m_c + m_u} \mathbf{P}_D - \mathbf{P}_B - \mathbf{p}_x - \frac{1}{2} \mathbf{p}_z \right) \right]^* \\
 &\times \left[\hat{\phi}_{\alpha_2, \alpha_4}^{(B, \lambda')} \left(\frac{m_b}{m_b + m_u} \mathbf{P}_B + \mathbf{p}_x - \frac{1}{2} \mathbf{p}_z \right) \right]^* \\
 &\times \sum_{\alpha_1} \hat{\phi}_{\alpha_1, \alpha_2, \alpha_3, \alpha_4}^{(T, \lambda)} (\mathbf{p}_x, -\mathbf{p}_x - \mathbf{P}_B, \mathbf{p}_z) \\
 &\times \frac{1}{2\sqrt{E_c E_b}} \bar{u}_c^{s_5} \left(\mathbf{P}_D + \mathbf{P}_B + \mathbf{p}_x + \frac{1}{2} \mathbf{p}_z \right) \gamma_\rho (1 - \gamma_5) \\
 &\times u_b^{s_1} \left(\mathbf{p}_x + \frac{1}{2} \mathbf{p}_z \right) \delta_{c_1 c_5} \delta_{f_1 b} \delta_{f_5 c}.
 \end{aligned}$$

For meson states we similarly have

$$\begin{aligned} \left| M, \lambda \vec{P} \right\rangle_{NR} &= \sqrt{2E(\vec{P})} \int d^3 p \sum_{\alpha_1, \alpha_2} \hat{\phi}_{\alpha_1, \alpha_2}^{(M, \lambda)}(\vec{p}) \\ &\times \frac{(-1)^{1/2-s_2}}{(2\pi)^{3/2} \sqrt{2E_{f_1} 2E_{f_2}}} |q, \alpha_1 \vec{p}_1 = \frac{m_{f_1} \vec{P}}{m_{f_1} + m_{f_2}} - \vec{p}\rangle |\bar{q}, \alpha_2 \vec{p}_2 = \frac{m_{f_2} \vec{P}}{m_{f_1} + m_{f_2}} + \vec{p}\rangle \end{aligned}$$

normalized to

$$\left\langle M, \lambda' \vec{P}' \left| M, \lambda \vec{P} \right\rangle_{NR} \right\rangle = \delta_{\lambda \lambda'} (2\pi)^3 2E(\vec{P}) \delta(\vec{P} - \vec{P}'),$$

which requires

$$\int d^3 p \sum_{\alpha_1 \alpha_2} \left(\hat{\phi}_{\alpha_1, \alpha_2}^{(M, \lambda')}(\vec{p}) \right)^* \hat{\phi}_{\alpha_1, \alpha_2}^{(M, \lambda)}(\vec{p}) = \int d^3 x \sum_{\alpha_1, \alpha_2} \left(\phi_{\alpha_1, \alpha_2}^{(M, \lambda')}(\vec{x}) \right)^* \phi_{\alpha_1, \alpha_2}^{(M, \lambda)}(\vec{x}) = \delta_{\lambda \lambda'}$$

The orbital part in coordinate space given as a sum of three gaussians on the relative coordinate

Semileptonic decays II

Final state	$\Gamma [10^{-15} \text{ GeV}]$
$B^{\star-} e^- \bar{\nu}_e$	0.0365 ± 0.0004
$\bar{B}^0 e^- \bar{\nu}_e$	0.0394 ± 0.0006
$B^{\star-} \mu^- \bar{\nu}_\mu$	0.0355 ± 0.0004
$\bar{B}^0 \mu^- \bar{\nu}_\mu$	0.0396 ± 0.0006
$B^{\star-} \tau^- \bar{\nu}_\tau$	0.0355 ± 0.0004
$\bar{B}^0 \tau^- \bar{\nu}_\tau$	0.0396 ± 0.0006

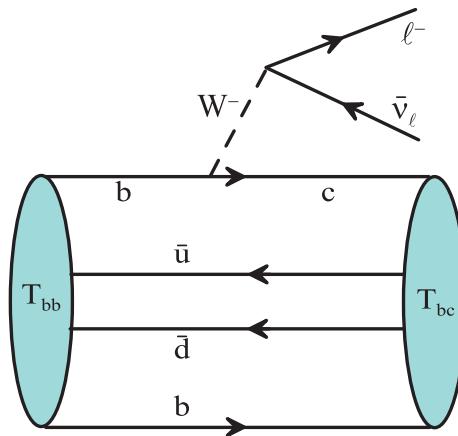
Final state	$\Gamma [10^{-15} \text{ GeV}]$	Final state	$\Gamma [10^{-15} \text{ GeV}]$
$B^{\star-} D^{*+} \ell^- \bar{\nu}_\ell$	9.02 ± 0.07	$B^{\star-} D^{*+} \tau^- \bar{\nu}_\tau$	1.55 ± 0.01
$\bar{B}^{\star 0} D^{*0} \ell^- \bar{\nu}_\ell$		$\bar{B}^{\star 0} D^{*0} \tau^- \bar{\nu}_\tau$	
$B^{\star-} D^+ \ell^- \bar{\nu}_\ell$	3.59 ± 0.03	$B^{\star-} D^+ \tau^- \bar{\nu}_\tau$	0.727 ± 0.005
$\bar{B}^{\star 0} D^0 \ell^- \bar{\nu}_\ell$		$\bar{B}^{\star 0} D^0 \tau^- \bar{\nu}_\tau$	
$B^- D^{*+} \ell^- \bar{\nu}_\ell$	4.63 ± 0.05	$B^- D^{*+} \tau^- \bar{\nu}_\tau$	0.86 ± 0.007
$\bar{B}^0 D^{*0} \ell^- \bar{\nu}_\ell$		$\bar{B}^0 D^{*0} \tau^- \bar{\nu}_\tau$	
$B^- D^+ \ell^- \bar{\nu}_l$	1.92 ± 0.02	$B^- D^+ \tau^- \bar{\nu}_\tau$	0.409 ± 0.003
$\bar{B}^0 D^0 \ell^- \bar{\nu}_\ell$		$\bar{B}^0 D^0 \tau^- \bar{\nu}_\tau$	

with $\ell = e, \mu$.

In the latter case, the decay into vector mesons is favored with respect to the corresponding decays into pseudoscalars ones. Other similar channels, not shown, have widths that are smaller by one order of magnitude.

Semileptonic decays III

We also evaluated the decay



$$\begin{aligned}
 h_\rho^{T \rightarrow T_{bc}} = & 2\sqrt{2m_T E_{T_{bc}}} \sum_{\alpha_2, \alpha_3, \alpha_4, \alpha_5} \iiint d\mathbf{p}_x d\mathbf{p}_y d\mathbf{p}_z \\
 & \times \left[\hat{\phi}_{\alpha_2, \alpha_5}^{(T_{bc}, \lambda')} \left(-\mathbf{p}_x - \frac{m_b - m_c}{2(m_b + m_c)} \mathbf{p}_z - \frac{m_b}{m_b + m_c} \mathbf{P}_{T_{bc}}, \mathbf{p}_y, \right. \right. \\
 & \left. \left. \mathbf{p}_z + \frac{2m_u}{m_b + m_c + 2m_u} \mathbf{P}_{T_{bc}} \right) \right]^* \\
 & \times \sum_{\alpha_1} \hat{\phi}_{\alpha_1, \alpha_2}^{(T, \lambda)} (\mathbf{p}_x, \mathbf{p}_y, \mathbf{p}_z) \frac{1}{2\sqrt{E_c E_b}} \bar{u}_c^{s_5} \left(\mathbf{P}_{T_{bc}} + \mathbf{p}_x + \frac{1}{2} \mathbf{p}_z \right) \\
 & \times \gamma_\rho (1 - \gamma_5) u_b^{s_1} \left(\mathbf{p}_x + \frac{1}{2} \mathbf{p}_z \right) \delta_{c_1 c_5} \delta_{f_1 b} \delta_{f_5 c}. \quad (1)
 \end{aligned}$$

The $I(J^\pi) = 0(0^+)$ T_{bc} tetraquark was found to be stable against strong and e.m. decay with a binding energy of ~ 23 MeV in T.F. Caramés, J. Vijande and V. Valcarce, Phys. Rev. D 99 (2019) 014006. This finding is supported by the Lattice QCD result of A. Francis et al., Phys. Rev. D 99 (2019) 054505.

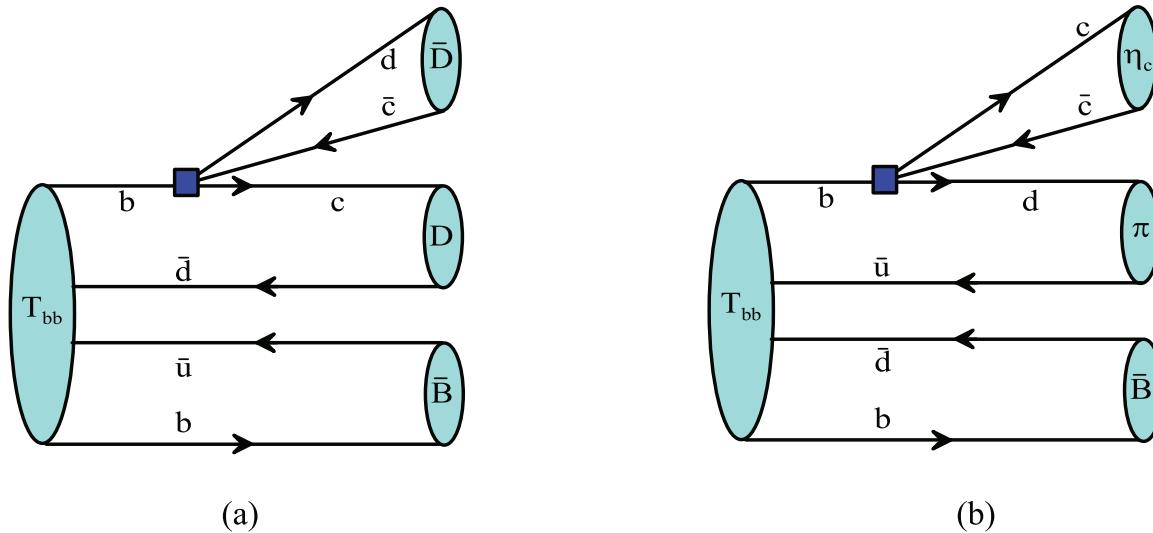
This decay channel was claimed to be the dominant one in S.S. Agaev et al., Phys. Rev. D 99 (2019) 033002 where using a QCD three-point sum rule approach they got $\Gamma = (7.17 \pm 1.23) \times 10^{-11}$ GeV.

We get a very different result.

Final state	$\Gamma [10^{-15} \text{ GeV}]$
$T_{bc} e^- \nu_e$	3.06 ± 0.03
$T_{bc} \mu^- \nu_\mu$	3.02 ± 0.02
$T_{bc} \tau^- \nu_\tau$	1.40 ± 0.01

Four orders of magnitude smaller.

Nonleptonic decays



To evaluate these type of decays we use the effective four-quark Hamiltonian

$$H_{\text{eff}} = \frac{G_F}{\sqrt{2}} \left\{ V_{cb} \left[c_1(\mu) Q_1^{cb} + c_2(\mu) Q_2^{cb} \right] + V_{ub} \left[c_1(\mu) Q_1^{ub} + c_2(\mu) Q_2^{ub} \right] + \text{h.c.} \right\},$$

in P. Colangelo and F. de Fazio Phys. Rev. D 61 (2000) 034012 where c_1 , c_2 are scale-dependent Wilson coefficients, and Q_1^{ib} , Q_2^{ib} , $i = u, c$, are local four-quark operators of the current-current type given by

Nonleptonic decays II

$$\begin{aligned}
Q_1^{ib} &= \bar{\Psi}_i(0)\gamma_\mu(1 - \gamma_5)\Psi_b(0) \left[V_{ud}^* \bar{\Psi}_d(0)\gamma^\mu(1 - \gamma_5)\Psi_u(0) + V_{us}^* \bar{\Psi}_s(0)\gamma^\mu(1 - \gamma_5)\Psi_u(0) \right. \\
&\quad \left. + V_{cd}^* \bar{\Psi}_d(0)\gamma^\mu(1 - \gamma_5)\Psi_c(0) + V_{cs}^* \bar{\Psi}_s(0)\gamma^\mu(1 - \gamma_5)\Psi_c(0) \right], \\
Q_2^{ib} &= \bar{\Psi}_d(0)\gamma_\mu(1 - \gamma_5)\Psi_b(0) \left[V_{ud}^* \bar{\Psi}_i(0)\gamma^\mu(1 - \gamma_5)\Psi_u(0) + V_{cd}^* \bar{\Psi}_i(0)\gamma^\mu(1 - \gamma_5)\Psi_c(0) \right] \\
&\quad + \bar{\Psi}_s(0)\gamma_\mu(1 - \gamma_5)\Psi_b(0) \left[V_{us}^* \bar{\Psi}_i(0)\gamma^\mu(1 - \gamma_5)\Psi_u(0) + V_{cs}^* \bar{\Psi}_i(0)\gamma^\mu(1 - \gamma_5)\Psi_c(0) \right]
\end{aligned}$$

We work in factorization approximation and include the Fierz rearranged contribution which amounts to substitute

$$c_1 \rightarrow a_1 = c_1 + \frac{1}{N_c} c_2 = 1.14 , \quad c_2 \rightarrow a_2 = c_2 + \frac{1}{N_c} c_1 = -0.20$$

evaluated at the m_b scale.

Nonleptonic decays results

Final state	$\Gamma [10^{-15} \text{ GeV}]$	Final state	$\Gamma [10^{-15} \text{ GeV}]$
$B^* - D^* + D_s^-$	4.00 ± 0.06	$B^- D^* + D_s^{*-}$	3.15 ± 0.05
$\bar{B}^* 0 D^* 0 D_s^-$		$\bar{B}^0 D^* 0 D_s^{*-}$	
$B^* - D^* + D_s^{*-}$	6.50 ± 0.09	$B^- D^+ D_s^{*-}$	1.20 ± 0.02
$\bar{B}^* 0 D^* 0 D_s^{*-}$		$\bar{B}^0 D^0 D_s^{*-}$	
$B^* - D^+ D_s^-$	2.57 ± 0.04	$B^* - D^* + \rho^-$	3.57 ± 0.09
$\bar{B}^* 0 D^0 D_s^-$		$B^* - D^* + \pi^-$	1.28 ± 0.03
$B^* - D^+ D_s^{*-}$	2.32 ± 0.03	$B^* - D^+ \rho^-$	1.70 ± 0.04
$\bar{B}^* 0 D^0 D_s^{*-}$		$B^* - D^+ \pi^-$	0.70 ± 0.02
$B^- D^* + D_s^-$	2.78 ± 0.05	$B^- D^* + \rho^-$	2.01 ± 0.05
$\bar{B}^0 D^* 0 D_s^-$		$B^- D^* + \pi^-$	0.77 ± 0.03

Again, decay into vector mesons is favored.

All the processes shown contain a $b \rightarrow c$ vertex (which means $|V_{bc}|$ and a_1 factors) plus a $|V_{cs}|$ or $|V_{ud}|$ CKM matrix element.

Other similar decays, not shown, are suppressed by at least a factor of ten.

Total decay width

- Our estimate gives

$$\Gamma \gtrsim 9 \times 10^{-14} \text{ GeV}$$

- At variance with the crude estimate, based on an effective semileptonic decay assumption, from M. Karliner and J.L. Rosner, Phys. Rev. Lett. 119 (2017) 202001 where they get

$$\Gamma \approx 1.8 \times 10^{-12} \text{ GeV}$$

- We are also at variance with the more recent value $\Gamma = (7.72 \pm 1.23) \times 10^{-11} \text{ GeV}$ obtained by S.S. Agaev et al., Eur. Phys. J. A (2021) 57:106 , which is still dominated by the $T_{bb} \rightarrow T_{bc}$ semileptonic decay.

Summary

- There is strong theoretical evidence and consensus on the existence of a $I(J^\pi) = 0(1^+)$ T_{bb}^- tetraquark which is stable against strong and e.m. interactions.
- As to its width
 - There are just a few estimates of the width that range from $\approx 8 \times 10^{-11}$ GeV down to $\approx 9 \times 10^{-14}$ GeV. A huge theoretical uncertainty.
 - There is no agreement on which decay channel is the dominant one.

There seems to be plenty of room for new determinations of Γ .