Double charm tetraquark and other exotics


## Exotics in QCD

## Sum Rules

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# Candidates to Exotic Charmonium States 

| State | $m(\mathrm{MeV})$ | $\Gamma(\mathrm{MeV})$ | $J^{p c}$ | Process (mode) | experiment | Year |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $x(3872)$ | $71.69 \pm 0.17$ <br> st one | < 1.2 | $1^{++}$ | $\begin{gathered} B \rightarrow K\left(\pi^{+} \pi^{-} J / \psi\right) \\ p \bar{p} \rightarrow\left(\pi^{+} \pi^{-} J / \psi\right)(\ldots) \\ B \rightarrow K(\omega J / \psi) \\ B \rightarrow K\left(D^{0} \bar{D}^{\varphi}\right) \\ B \rightarrow K(\gamma J / \psi) \\ B \rightarrow K(\gamma \psi(2 S) \\ e^{+} e^{-} \rightarrow \pi^{+} \pi^{-} J / \psi \\ p p \rightarrow\left(\pi^{+} \pi^{-} J / \psi\right)(\ldots) \end{gathered}$ | Belle [22-24], BaBar [25] <br> CDF [26-28], DØ [29] <br> Belle [30], BaBar [31] <br> Belle [32, 33], BaBar [34] <br> Belle [30], BaBar [35, 36] <br> BaBar [36], LHCb [37] <br> BESIII [38] <br> LHCb [39, 40], CMS [41] | 2003 |
| $Z_{e}^{+}(3900)$ | $3886.6 \pm 2.4$ | $28.2 \pm 2.6$ | $1^{+-}$ | $\begin{aligned} Y(4260) & \rightarrow\left(J / \psi \pi^{+}\right) \pi^{-} \\ Y(4260) & \rightarrow\left(D D^{+}\right)^{+} \pi^{-} \end{aligned}$ | BESIII [42], Belle [43], CLEO-c [44]] BESIII [45] | 2013 |
| $Y$ (3940) | $3918.4 \pm 1.9$ | $20 \pm 5$ | 0/2++ | $\begin{aligned} B & \rightarrow K(J / \psi \omega) \\ e^{+} e^{-} & \rightarrow e^{+} e^{-}(\omega J / \psi) \end{aligned}$ | Belle [46], BaBar [31, 47] <br> Belle [48], BaBar [49] | 2004 |
| $X$ (3940) | $3942+8$ | $37+17$ | $p^{++}$ | $\begin{gathered} e^{+} e^{-} \rightarrow J / \psi(\ldots) \\ e^{+} e^{-} \rightarrow J / \psi\left(D D^{*}\right) \end{gathered}$ | Belle [50] <br> Belle [51] | 2005 |
| $Y(4008)$ | $3891 \pm 42$ | $255 \pm 42$ | $1^{--}$ | $e^{+} e^{-} \rightarrow \pi^{+} \pi^{-} J / \psi$ | Belle [43, 52], BESIII [53] | 2007 |
| $\mathrm{Z}_{e}^{+}(4020)$ | $4024.1 \pm 1.9$ | $13 \pm 5$ | ? | $\begin{gathered} e^{+} e^{-} \rightarrow \pi^{-}\left(\pi^{+} h_{c}\right) \\ Y(4260) \rightarrow \pi^{-}\left(D^{+} \bar{D}^{+}\right)^{+} \end{gathered}$ | BESIII [54] <br> BESIII [55] | 2013 |
| $Z_{1}^{+}(4050)$ | $4051{ }_{-43}^{124}$ | $82_{-55}^{+51}$ | $?^{7-}$ | $B \rightarrow K\left(\pi^{+} \chi_{c l}(1 P)\right)$ | Belle [56], BaBar [57] | 2008 |
| $\mathrm{Z}_{e}^{+}(4055)$ | $4054 \pm 3$ | 45 | (? ${ }^{\text {?-- }}$ | $e^{+} e^{-} \rightarrow \pi^{-}\left(\pi^{*} \phi(2 S)\right)$ | Belle [58] | 2014 |
| $\mathrm{Z}_{e}(4100)$ | $\left(4096 \pm{ }_{-32}{ }^{28}\right)$ | $152_{-45}^{+75}$ ) | $0^{++} / 1^{+}$ | $B^{0} \rightarrow K^{+}\left(\pi^{-} \eta_{c}(1 S)\right)$ | LHCb [59] | 2018 |
| $Y(4140)$ | $4146.8 \pm 2.4$ | $22_{-7}^{+8}$ | $1^{++}$ | $B \rightarrow K(\phi J / \psi)$ | CDF [60, 61], D0 [62], LHCb [63], BESIII [64, 65] | 2009 |
| $X$ (4160) | 4156 -29 | ${ }^{139} 9_{-65}^{+113}$ | $?^{7+}$ | $e^{+} e^{-} \rightarrow J / \psi\left(D^{*} D^{*}\right)$ | Belle [51] | 2007 |
| $\mathrm{Z}_{\varepsilon}^{*}(4200)$ | $4196{ }_{-36} 35$ | $370 \times 10$ | $1^{+-}$ | $B \rightarrow K\left(\pi^{+} J / \psi\right)$ | Belle [66] | 2014 |
| $Y(4220)$ | $4218{ }_{-4}^{5}$ | $59{ }_{-12}$ | $1^{--}$ | $e^{+} e^{-} \rightarrow x_{00} \omega$ | BESIII [67] peemikel | 2014 |
| $\mathrm{Z}_{2}^{+}(4250)$ | $4248{ }_{-45}^{+185}$ | $177_{-721}^{+321}$ | ? ${ }^{\text {+ }}$ | $B \rightarrow K\left(\pi^{+} \chi_{c 1}(1 P)\right)$ | Belle [56], BaBar [57] | 2008 |
| $Y(4260)$ | $4230 \pm 8$ | $55 \pm 19$ | 1-- | $\begin{gathered} e^{+} e^{-} \rightarrow \pi^{+} \pi^{-} J / \psi \\ e^{+} e^{-} \rightarrow K^{+} K^{-} J / \psi \\ e^{+} e^{-} \rightarrow \pi^{0} \pi^{0} J / \psi \\ e^{+} e^{-} \rightarrow Z_{c}(3900)^{ \pm} \pi^{*} \end{gathered}$ | ```BaBar [71, 72], CLEO-c [73], Belle [43, 52], BESIII [53] CLEO-c [74], BESIII [45] CLEO-c [74] Belle [43], BESIII [42]``` | 2005 |
| $X(4350)$ | $4350.6{ }_{-5.1}^{+4.6}$ | $13.3+18.4$ | $?^{7+}$ | $e^{+} e^{-} \rightarrow \phi J / \psi$ | Belle [75] | 2009 |
| $Y(4360)$ | $4368 \pm 13$ | $96 \pm 7$ | $1^{--}$ | $\begin{aligned} & e^{+} e^{-} \rightarrow \pi^{+} \pi^{-} \phi(2 S) \\ & e^{+} e^{-} \rightarrow \pi^{+} \pi^{-} J / \psi \end{aligned}$ | BaBar [76, 77]. Belle [58, 78], BESIII [69] BESIII [53] | 2007 |
| $Y(4390)$ | $4391.5_{-78}^{473}$ | $139.5{ }_{-207}^{+163}$ | $1^{--}$ | $e^{+} e^{-} \rightarrow h_{e} \pi^{+} \pi^{-}$ | BESIII [68] | 2016 |
| $\mathrm{Z}^{+}(4430)$ | $4478{ }_{-18}^{+15}$ | $181 \pm 31$ | $1^{+-}$ | $\begin{gathered} B \rightarrow K^{-}\left(\pi^{+} \Psi(2 S)\right) \\ B \rightarrow K^{-}\left(\pi^{+} J / \phi\right) \end{gathered}$ | Belle [79-81], BaBar [82], LHCb [83] <br> Belle [66], BaBar [82] | 2007 |
| $X$ (4630) | $4634_{-11}^{+9}$ | $92_{-32}^{+41}$ | $1^{--}$ | $e^{+} e^{-} \rightarrow \Lambda_{e}^{+} \Lambda_{e}^{-}$ | Belle [84] | 2008 |
| $Y(4660)$ | $4643 \pm 9$ | $72 \pm 11$ | $1^{--}$ | $e^{+} e^{-} \rightarrow \pi^{+} \pi^{-} \psi(2 S)$ | Belle [ 58,78$]$, BaBar [77] | 2007 |

## Hadrons discovered at LHC:

62 new hadrons were discovered at LHC, including many exotic states


## Exotic Spectroscopy: "my" overview

- 2003: Discovery of $X(3872)$ by Belle started a new era in exotic spectroscopy.


$M_{x}=(3871.65 \pm 0.06) \mathrm{MeV}$ $\Gamma=(1.19 \pm 0.21) \mathrm{MeV}$

$$
J^{P C}=1^{++}
$$

$\chi_{c 1}(3872)$
also known as $X(3872)$

- 2007: Observation of $Z^{+}(4430)$ by Belle: the firs $\dagger$ charged tetraquark state. Not confirmed by BaBar in 2009. Confirmed by LHCb in 2014

$$
\begin{gathered}
M=(4478 \pm 16) \mathrm{MeV} \\
\Gamma=(181 \pm 31) \mathrm{MeV} \\
\mathrm{JP}^{\mathrm{P}}=1^{+}
\end{gathered}
$$



Maiani et al. (arXiv:0708.3997): four-quark radial excitation of the $1^{+}$charged state ( $X(3872)$ partner)


- 2013: Observation of $Z_{c}{ }^{+}(3900)$ by Belle and BESIII



# $Z_{c}{ }^{+}(3900) \quad$ B E S III <br> arXiv:1303.5949 arXiv:1304.0121 

$$
\begin{gathered}
Y(4260) \rightarrow\left(J / \psi \pi^{+}\right) \pi^{-} \\
M=(3887.1 \pm 2.6) \mathrm{MeV} \\
\Gamma=\left(\begin{array}{c}
28.4 \pm 2.6) \mathrm{MeV} \\
\mathrm{~J}^{\mathrm{P}}=1^{+}
\end{array}\right.
\end{gathered}
$$

## SuSyLFHQCD

MN, Brodsky, Téramond, Dosch, Navarra, Zou (arXiv:1805.11567)


## SuSyLFHQCD

MN, Brodsky, Téramond, Dosch, Navarra, Zou (arXiv:1805.11567)


- 2011: Observation of $\mathrm{Z}_{\mathrm{b}}{ }^{+}(10610)$ by Belle: the first beauty charged tetraquark state: $\mathrm{J}^{P}=1^{+}, \mathrm{M}=(10,607 \pm 2) \mathrm{MeV}, \Gamma=(18.4 \pm 2.4) \mathrm{MeV}$

$$
M_{B^{*}}+M_{B}=10,605 \mathrm{MeV} \Rightarrow Z_{b}^{+}(10610) \leftrightarrow Z_{c}^{+}(3900)
$$

$2 b$ or not to be, Navarra, MN, Richard (arXiv:1108.1230)


- 2015: Observation of the first charmed pentaquark states, cc̄uud, by LHCb

- 2020: Observation of the $X(c c \bar{c} \bar{c})$ by LHCb.

- 2020: Observation of $\mathrm{Z}_{c s}-(3985) \rightarrow \mathrm{D}^{-{ }^{-(*)}} \mathrm{D}^{0(*)}$ by BESIII. The first strange charged tetraquark state.

- 07/2021: Observation of $\mathrm{T}_{c c}$ : a double charmed tetraquark arxiv:2109.01038v2 [hepeex] 3 Sep 2021




## QCD Sum Rule

Fundamental Assumption: Principle of Duality

$$
\Pi(q)=i \int d^{4} x e^{i q \cdot x}\langle 0| T\left[j(x) j^{\dagger}(0)\right]|0\rangle
$$

Theoretical side
quark level
quark and gluon
degrees of freedom

Wilson OPE

Phenomenological side
hadron level
hadron parameters (masses, couplings, form-factors,...)
dispersion relation

$$
\Pi_{i}^{p h e n} \leftrightarrow \Pi_{i}^{O P E}
$$

$$
\Pi^{p h e n}=-\lambda^{2} \frac{1}{m^{2}-q^{2}}+\text { continuum }
$$

$\lambda \Rightarrow$ coupling current-state

$\mathrm{s}_{0}$ : continuum parameter
$\Pi^{p h e n}\left(Q^{2}\right) \leftrightarrow \Pi^{O P E}\left(Q^{2}\right)$

valid at small $Q^{2}$
valid at large $Q^{2}$

## To improve the matching $\Rightarrow$ Borel transform

Borel Transform
eliminates subtraction terms suppresses higher order condensates increases importance pole contribution

$$
\begin{aligned}
& \lambda^{2} e^{-m^{2} / M^{2}}=\int_{s_{m i n}}^{s_{0}} d s e^{-s / M^{2}} \rho^{O P E}(s) \\
& m^{2}=\frac{\int_{s_{m i n}}^{s_{0}} d s s \rho_{i}^{O P E}(s) e^{-s / M^{2}}}{\int_{s_{m i n}}^{s_{0}} d s \rho_{i}^{O P E}(s) e^{-s / M^{2}}}
\end{aligned}
$$

## Good Sum Rule $\Rightarrow$ Borel window such that:

- pole contribution > continuum contribution
- good OPE convergence
- good Borel stability

OPE side: condensates $\left\{\begin{array}{l}\text { quark condensate } \\ \text { gluon condensate } \\ \text { mixed condensates } \\ \text { four-quark condensate } .\end{array}\right.$

## With QCDSR one can get anything




## QCDSR calculation for $X^{+}(5568)$ mass

Khemchandani, MN, Zanetti: arXiv:1602.09041
tetraquark current with $\mathrm{J}^{\mathrm{P}}=0^{+}$

$$
j_{X}=\varepsilon_{a b c} \varepsilon_{d e c}\left(u_{a}^{T} C \gamma_{5} b_{b}\right)\left(\bar{d}_{d} \gamma_{5} C \bar{s}_{e}^{T}\right)
$$

$$
\begin{gathered}
\Pi(q)=i \int d^{4} x e^{i q \cdot x \cdot\langle 0| j_{X}(x) j_{X}^{\dagger}(0)|0\rangle} \\
\Pi(x)=f(S^{u}(x) \underbrace{\left.S^{d}(-x) S^{s}(x) S^{b}(-x)\right)}_{\left(\begin{array}{c}
\text { OPE for the quark } \\
\text { propagator }
\end{array}\right.}
\end{gathered}
$$





Khemchandani, MN, Zanetti
Phys. Rev. D 93, 096011 (2016)

## $m_{x}=(6.39 \pm 0.10) \mathrm{GeV}$

not compatible with $X(5568)$ mass!

LHCb (Phys. Rev. Lett. 118, 109904 (2017)), CDF (Phys. Rev. Lett. 120, 202006 (2018)) and ATLAS (Phys. Rev. Lett. 120, 202007(2018)): no structure is found from $B_{s}{ }^{0} \pi^{+}$from threshold up to 6000 MeV

## First QCD Sum Rule Calculation for $X(3872)$

Matheus, Narison, MN, Richard: tetraquark current (PRD75(07)014005)

$$
j_{\mu}=\frac{i \epsilon_{a b c} \epsilon_{d e c}}{\sqrt{2}}\left[\left(q_{a}^{T} C \gamma_{5} c_{b}\right)\left(\bar{q}_{d} \gamma_{\mu} C \bar{c}_{e}^{T}\right)+\left(q_{a}^{T} C \gamma_{\mu} c_{b}\right)\left(\bar{q}_{d} \gamma_{5} C \bar{c}_{e}^{T}\right)\right]
$$





Lee, MN, Wiedner: $D^{0} \bar{D}^{* 0}$ molecular current (arXiv:0803.1168)

$$
\begin{gathered}
j_{\mu}^{(q, m o l)}(x)=\frac{1}{\sqrt{2}}\left[\left(\bar{q}_{a}(x) \gamma_{5} c_{a}(x) \bar{c}_{b}(x) \gamma_{\mu} q_{b}(x)\right)-\left(\bar{q}_{a}(x) \gamma_{\mu} c_{a}(x) \bar{c}_{b}(x) \gamma_{5} q_{b}(x)\right)\right] \\
m_{X}=(3.87 \pm 0.07) \mathrm{GeV}
\end{gathered}
$$

## double-ratio sum rules:

$$
r_{m o l / 3}=\frac{M_{m o l}}{M_{3}}
$$

Narison, Navarra, MN, PRD83(11)016004
differences smaller than $0.01 \%$


Since $T_{c c}(c c \bar{q} \bar{q})$ can be described by a DD* current and $X(3872)$ ( $c \bar{c} q \bar{q})$ by a $\bar{D} D^{*}$ current, similar comparison can also be made for $X$ and $T_{c c}$ states.


Dias, Narison, Navarra, MN, Richard (arXiv:1105.5630)
Using the $X(3872)$ experimental mass our prediction for the $T_{c c}$ mass is:

$$
M_{T_{c c}}=(3872.2 \pm 39.5) \mathrm{MeV}
$$

## First QCD Sum Rule Prediction for $T_{c c}$

Navarra, MN, Lee, hep-ph/0703071

$$
\boldsymbol{T}_{c c}^{+}([c c][\bar{u} \bar{d}]) J^{P}=\mathbf{1}^{+}
$$

Tetraquark current with $J^{P}=1^{+} \nearrow^{\text {light antidiquark: } \epsilon_{a b c}\left[\bar{u}_{b} \gamma_{5} C d_{c}^{T}\right]}$
$\searrow_{\text {heavy diquark: } \epsilon_{a e f}\left[c_{e}^{T} C \gamma_{\mu} c_{f}\right]}$


$$
m_{T_{c c}}=(4.0 \pm 0.2) \mathrm{GeV}
$$

Prediction for $\mathrm{T}_{\mathrm{b}}$ :

$$
m_{T_{b b}}=(10.2 \pm 0.3) \mathrm{GeV}
$$

## Prediction for a $\mathbf{Z c s}^{+}{ }^{+}$state from QCD Sum Rule

Lee, MN, Wiedner, arXiv:0803.1168, $D_{s}{ }^{+} \bar{D}^{*}$ molecular current

$$
j_{\mu}=\frac{i}{\sqrt{2}}\left[\left(\bar{s}_{a} \gamma_{5} c_{a}\right)\left(\bar{c}_{b} \gamma_{\mu} d_{b}\right)-\left(\bar{s}_{a} \gamma_{\mu} c_{a}\right)\left(\bar{c}_{b} \gamma_{5} d_{b}\right)\right] \quad\left(J^{P}=1^{+}\right)
$$





Decay width from QCD Sum Rule


## Decay width $\quad \mathrm{X}(3872) \rightarrow \mathrm{J} / \psi \pi \pi$

(Navarra, MN, PLB639 (06)272)

## coupling constant



OPE side:

(c)

(d)
(b)

(e)
$\Gamma(X \rightarrow J / \psi(n \pi)) \longrightarrow g_{X \psi V}$
Problem: decay width $X \rightarrow J / \psi \pi \pi$
$\sim 50 \mathrm{MeV}$

## How to solve this problem?



> If $X(3872)$ is a genuine tetraquark state, only colorconected diagrams will contribute

$$
\Gamma_{C C}(X \rightarrow J / \psi(n \pi))=(0.7 \pm 0.2) \mathrm{MeV}
$$

Navarra, MN,(PLB639(06)272)

Compatible with the experimental X (3872) width: $\Gamma<1.2 \mathrm{MeV}$

QCDSR Results for decay widths of the $\mathbf{Z}_{\mathrm{c}}{ }^{+}(3900)$

Dias, Navarra, MN, Zanetti arXiv:1304.6433

OPE side: only color-connected diagrams

| Vertex | coupling constant $(\mathrm{GeV})$ | decay width $(\mathrm{MeV})$ |
| :---: | :---: | :---: |
| $Z_{c}^{+}(3900) J / \psi \pi^{+}$ | $3.89 \pm 0.56$ | $29.1 \pm 8.2$ |
| $Z_{c}^{+}(3900) \eta_{c} \rho^{+}$ | $4.85 \pm 0.81$ | $27.5 \pm 8.5$ |
| $Z_{c}^{+}(3900) D^{+} \bar{D}^{* 0}$ | $2.5 \pm 0.3$ | $3.2 \pm 0.7$ |
| $Z_{c}^{+}(3900) \bar{D}^{0} D^{*+}$ | $2.5 \pm 0.3$ | $3.2 \pm 0.7$ |

## $\Gamma_{Z_{c}^{+}}=(63 \pm 18) \mathrm{MeV}$

$$
\begin{aligned}
& \Gamma_{Z_{c}^{+}}^{B E S}=(46 \pm 22) \mathrm{MeV} \\
& \Gamma_{Z_{c}^{+}}^{B E L L E}=(63 \pm 35) \mathrm{MeV}
\end{aligned}
$$

Very good agreement

QCDSR Prediction for the decay width of $\mathrm{Z}_{\text {cs }}{ }^{+}$

Again only color-connected diagrams were considered

Dias, Liu, MN, arXiv:1307.7100


| Vertex | coupling constant $(\mathrm{GeV})$ | decay width $(\mathrm{MeV})$ |
| :---: | :---: | :---: |
| $Z_{c s}^{+} J / \psi K^{+}$ | $2.58 \pm 0.30$ | $11.2 \pm 3.5$ |
| $Z_{c s}^{+} \eta_{c} K^{*+}$ | $3.4 \pm 0.3$ | $10.8 \pm 6.2$ |
| $Z_{c s}^{+} D_{s}^{+} \bar{D}^{* 0}$ | $1.4 \pm 0.3$ | $1.5 \pm 1.5$ |
| $Z_{c s}^{+} \bar{D}^{0} D_{s}^{*+}$ | $1.4 \pm 0.4$ | $1.4 \pm 1.4$ |

$$
\Gamma_{Z_{c s}}=(24.9 \pm 12.6) \mathrm{MeV} \quad \Gamma \mathrm{BES}=\underset{\text { arxiv:2011.07855 }}{(12.8 \pm 4.7) \mathrm{MeV}}
$$

Very good agreement

## Conclusions

Exotic states are real, and QCDSR calculations can be trusted!
$X(3872) \rightarrow$ The first and best studied charmonium tetraquark candidate. Its mass and decay width can be well described in QCDSR.
$\mathrm{Z}_{\mathrm{c}}{ }^{+}(3900) \rightarrow$ For sure a tetraquark state. Its mass and decay width can be well described in QCDSR.
$Z_{c}{ }^{+}(4430) \rightarrow$ First observed charged tetraquark state. Possible first radial excitation of $Z_{c}{ }^{+}(3900)$
$\mathrm{Z}_{\mathrm{cs}}{ }^{+}(3985) \rightarrow$ first $[c \bar{c} u \bar{s}]$ observed $\mathrm{J}^{P}=1^{+}$ tetraquark state. Predictions by QCDSR are consistent with its mass and decay width.
$\mathrm{T}_{c \mathrm{c}}{ }^{+}(3875) \rightarrow$ The star of this conference, and the most expected tetraquark state. First prediction in 1987. First QCDSR prediction in 2007. Finally observed by LHCb in 2021.


Thank you!

