Tetra-quarks, penta-quarks and exotics
Old and new views

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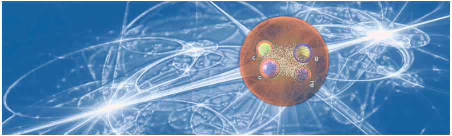
Theory Division, CERN, CH-1211 Geneva 23 - Switzerland



November 22, 2021

Bibliography I

- This talk is based on a few old and more recent papers

Existence of a “family of multi-quark states” conjectured in
G. C. Rossi and G. Veneziano
Nucl. Phys. B **123** (1977) 507

See also the old experimental & theoretical review paper
L. Montanet, G. C. Rossi and G. Veneziano
Phys. Rept. **63** (1980) 149.

Baryonium states isospin mixing
G. C. Rossi and G. Veneziano
Phys. Lett. **70B** (1977) 255
Phys. Lett. **597B** (2004) 338

Baryonium states emerge in the large ’t Hooft coupling limit
G. C. Rossi and G. Veneziano
JHEP **1606** (2016) 041

Bibliography II

- Since 1977 ∞-ly many other theoretical papers were published

- R. L. Jaffe, Phys. Rev. Lett. **38** (1977), 195 and Phys. Rev. D **15** (1977), 267
- R. L. Jaffe, Phys. Rev. D **17** (1978), 1444
- H. J. Lipkin, Phys. Lett. B **172** (1986), 242-247
- H. J. Lipkin, Phys. Lett. B **195** (1987), 484-488
- S. Pepin, F. Stancu, M. Genovese, J. M. Richard, Phys. Lett. B **393** (1997), 119
- S. H. Lee and S. Yasui, Eur. Phys. J. C **64** (2009), 283-295
- J. M. Richard, Few Body Syst. **57** (2016) no.12, 1185-1212
- M. Karliner and J. L. Rosner, Phys. Rev. Lett. **119** (2017) no.20, 202001
- W. Park, S. Noh and S. H. Lee, Nucl. Phys. A **983** (2019), 1-19
- L. Maiani, A. D. Polosa and V. Riquer, Phys. Rev. D **100** (2019) no.7, 074002
- N. Brambilla *et al.*, Phys. Rept. **873** (2020), 1-154
- A. Ali, L. Maiani, A. D. Polosa, “Multiquark Hadrons” doi:10.1017/9781316761465
- ... plus many more papers that could not fit in this page!

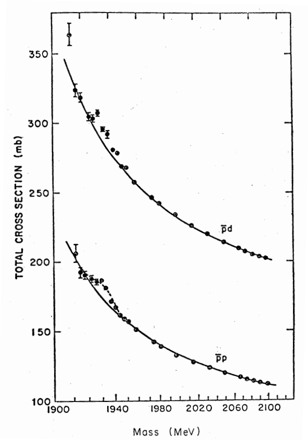
Outline of the talk

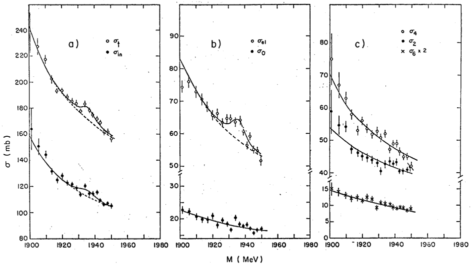
- A bit of (pre-)history
- A unified QCD description of color singlet quark states capable of encompassing
  - ordinary mesons & baryons
  - multi-quark states $\to$ “baryonium states” $\sim$ hidden baryon number
- Some phenomenological consideration
  - Exotics
  - P_c & photoproduction
  - A crude mass formula
- Conclusions

A bit of (pre-)history

S(1930) first narrow ($\Gamma \sim 4$) bump, a $[2q\,2\bar{q}]$ state?

1974 $\rightarrow$ S(1930) $qq\bar{q}\bar{q}$ has today disappeared, but
2020 $\rightarrow$ X($p\bar{p}$) narrow structures just below $p\bar{p}$ thr.
newly born $qq\bar{q}\bar{q}$'s (H. Jiang, BES III)?





- A. S. Carroll *et al.*, PRL **32** (1974) 247
"Observation of structure in $\bar{p}p$ and $\bar{p}d$ total cross-sections below 1.1 GeV/c"
@ BNL

- V. Chaloupka *et al.*, Phys. Lett. **61B** (1976) 487
"Measurement of the total and partial $\bar{p}p$ cross-section between 1901 and 1950 MeV"
@ CERN

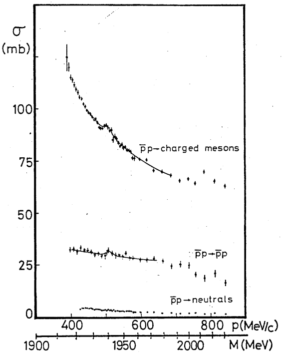


FIG. 4

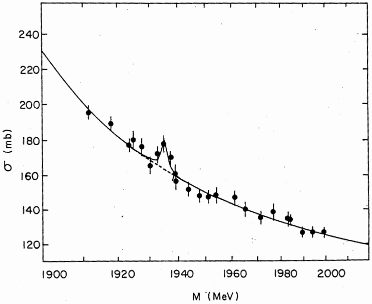
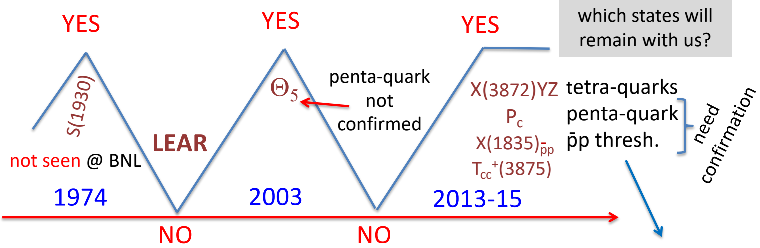


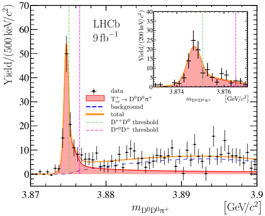
FIG. 5

- W. Bruckner *et al.*, Phys. Lett. **67B** (1977) 222 "Observation of a narrow resonance near the $\bar{p}p$ threshold" @ CERN
- S. Sakamoto *et al.*, NP B **158** (1979) 410 "Observation of a narrow resonance with a mass of 1930 MeV in $\bar{p}p$ total cross-section" @ BNL

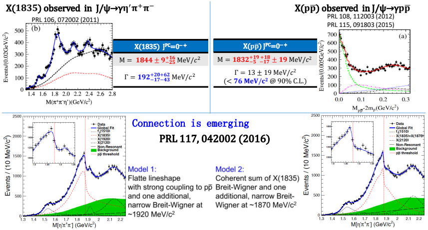
The roller-coaster of multi-quark states

Rise and fall of narrow multi-quark states



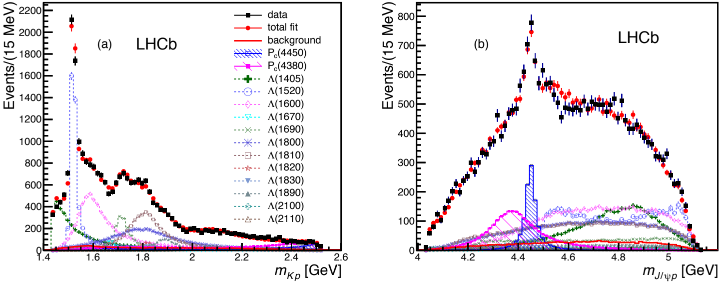


$T_{cc}^+(3875)$

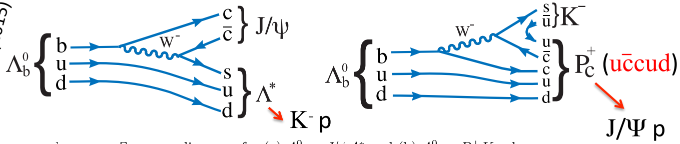


Courtesy of Beijiang Liu (BESIII)

R. Aaij *et al.* [LHCb Collaboration], 2015



PRL **115**, 072001 (2015)



Feynman diagrams for (a) $\Lambda_b^0 \to J/\psi \Lambda^*$ and (b) $\Lambda_b^0 \to P_c^+ K^-$ decay.

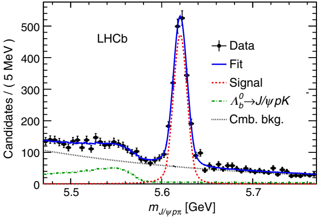
$M(P_{Ic}^{+}) = 4450$ $J^P = 5/2^+$ $\Gamma = 39$ MeV

$M(P_{IIc}^{+}) = 4380$ $J^P = 3/2^-$ $\Gamma = 205$ MeV

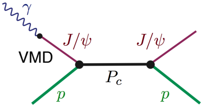
Need for confirmation

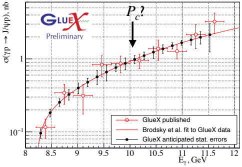
- By the same collaboration, PRL **117**, 082003 (2016)

R. Aaij *et al.**
(LHCb Collaboration)



- Not seen in photoproduction, 2020 (courtesy of M.R. Shepherd)





A unified picture of hadrons

emerges from

QCD color string + “planarity”

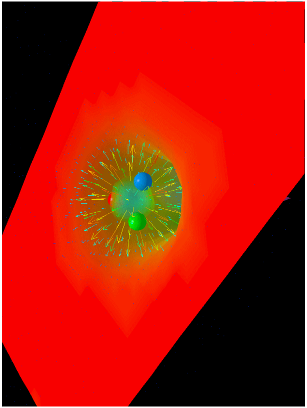
encompassing

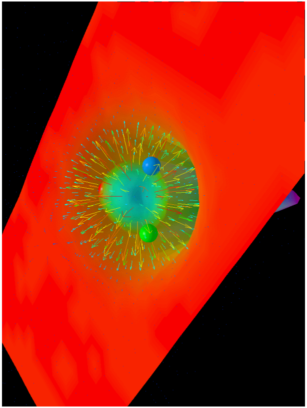
- ordinary mesons & baryons
- multi-quark states $\rightarrow$ baryonium states

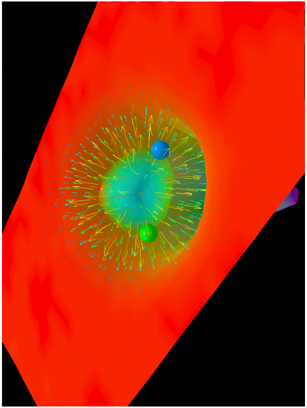
"Topological" expansion of hadronic amplitudes

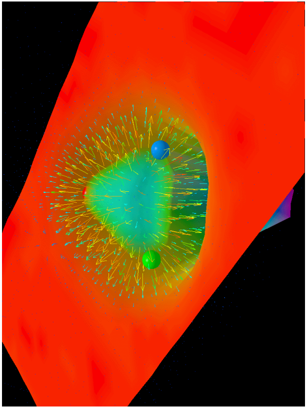
Multi-quark states emerge in a "topological" expansion
at the level of the leading planar and dual amplitudes

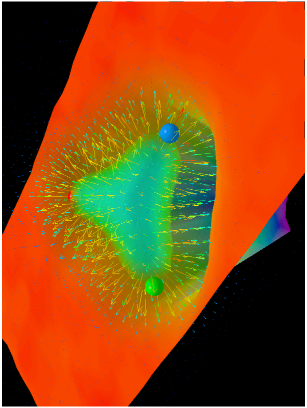
- "Planarity" (dominance of "planar" diagrams) & "planar duality"
  - Duality (Rosner, 1968) can be extended to include
    - $MM \to MM$, $MB \to MB$, $B\bar{B} \to B\bar{B}$ amplitudes
    - Duality constraints require introducing notion of junction for B's
- "Planarity" emerges from theoretical as well as lattice results
  - Large N-expansions
    - from either large N_c @ $\lambda = g^2 N_c$ fix 't Hooft 1974
      or topological expansion Veneziano 1976
    - Adding duality Rossi & Veneziano 1977, Witten 1979
    - one gets $\to$ Y-shaped baryons & multi-quark (baryonium) states
  - Large $\lambda = g^2 N_c$ expansion Rossi & Veneziano 2016
    - equivalent in QCD lattice regularization to large g^2 expansion
    - $\to$ minimal "area" diagrams dominate scattering amplitudes
  - Lattice data confirm (see movie)
    - Y-shaped colour flux tube Bissey et al. 2006
    - with junction located at the solution of the Fermat–Torricelli problem, which is the "*locus*" at minimal distance from N ($= 3$) given points

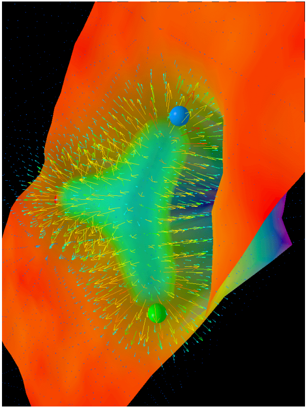


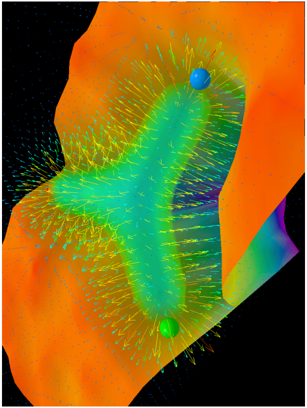


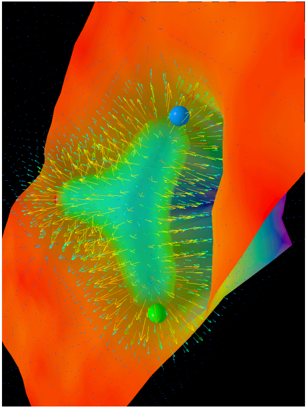


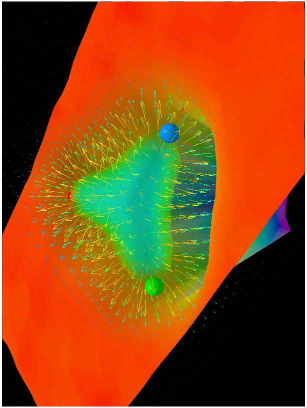


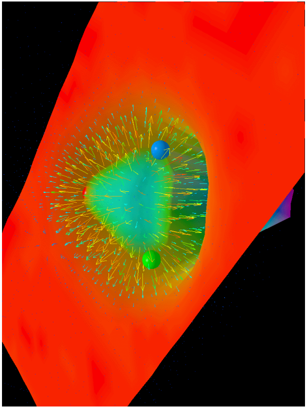


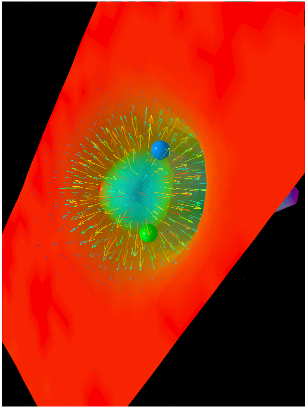


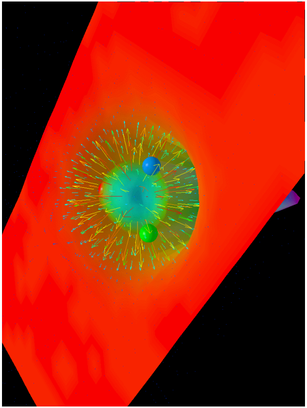




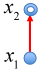








Lowest order building blocks we start with are

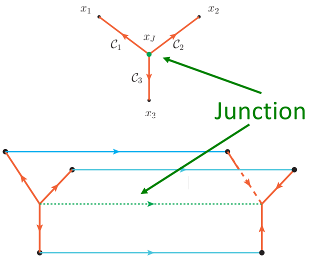


Meson → $M(x_1,x_2) = \frac{1}{\sqrt{3}} \bar{q}_k(x_1) C(x_1,x_2)^k_j q^j(x_2)$

$$C(x_1,x_2)^k_j = P\exp\left(ig\int_{x_1}^{x_2} A_\mu(z)\,dz_\mu\right)\Big|^k_j$$

Propagator → $\langle M(x_1,x_2) M^+(y_1,y_2) \rangle$

gauge invariance + planarity



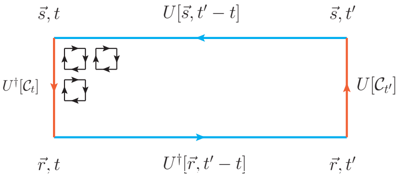
Baryon → $B(x_1,x_2,x_3) =$

$$\frac{\varepsilon^{i_1 i_2 i_3}}{\sqrt{3!}} q_{k_1}(x_1) C(x_1,\underline{x_J})^{k_1}_{i_1} q_{k_2}(x_2) C(x_2,\underline{x_J})^{k_2}_{i_2} q_{k_3}(x_3) C(x_3,\underline{x_J})^{k_3}_{i_3}$$

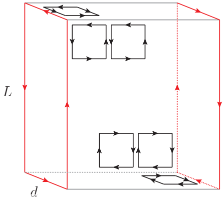
Propagator → $\langle B(x_1,x_2,x_3) B^+(y_1,y_2,y_3) \rangle$

Unified picture of hadrons → Tiling

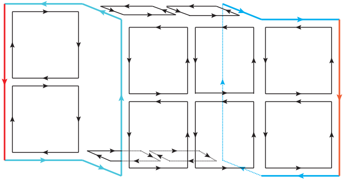
- In QCD lattice regularization, at large g^2, planarity is leading topology
- amplitudes span surfaces of minimal area
- corrections are O($(g^2 N_c)^{-1}, N_c^{-1}$)



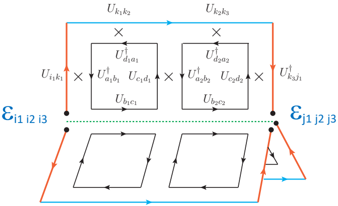
meson



glueball



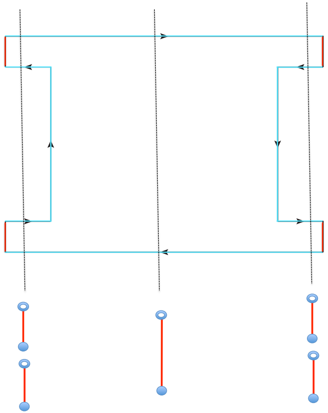
glueball-meson mixing



baryon

Scattering amplitudes

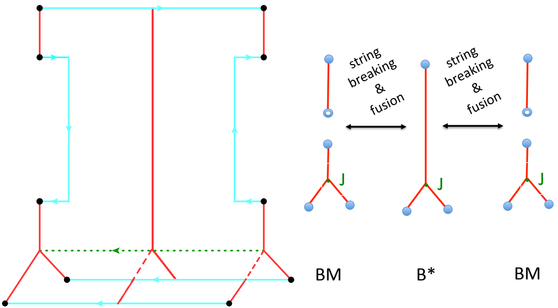
- Gluing & stretching Meson sheets $\Longrightarrow$ *MM* $\to$ *MM* amplitude



- *s*-channel mesons dual to *t*-channel mesons
- leading dynamics proceeds via string breaking $\leftrightarrow$ string fusion

Scattering amplitudes

- Gluing Meson to Baryon sheets $\Longrightarrow$ *MB* $\rightarrow$ *MB* amplitude

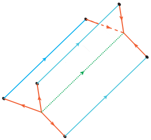


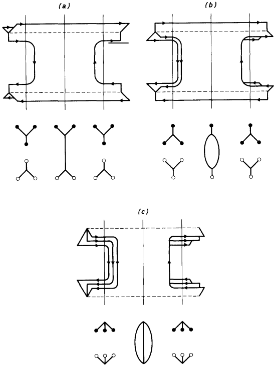
- *s*-channel baryons dual to *t*-channel mesons
- leading dynamics proceeds via string breaking $\leftrightarrow$ string fusion

$B\bar{B} \to B\bar{B}$ amplitudes

- Gluing together baryon sheets $\Longrightarrow$ $B\bar{B} \to B\bar{B}$ scattering
  - Junction flow $\leftrightarrow$ baryon number flow

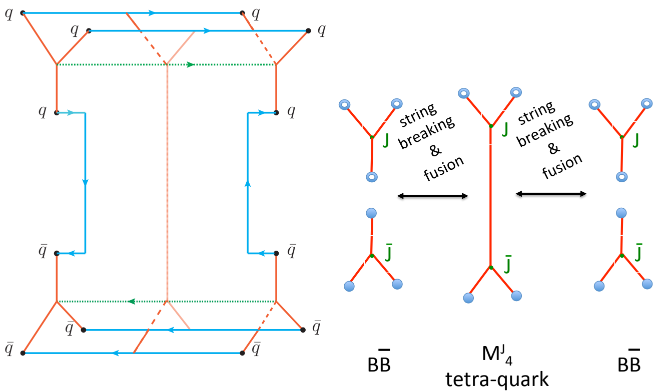
$N_c=3$





  - New intermediate *s*-channel states emerge $\to$ Baryonium states (states with hidden baryon number) M_4^J, M_2^J, M_0^J
  - *t*-channel intermediate states are 1, 2 and 3 $q\bar{q}$-jets
- $B\bar{B} \to B\bar{B}$ annihilation: just turn by 90° the above diagrams
  - Junction annihilation $\leftrightarrow$ baryon number annihilation

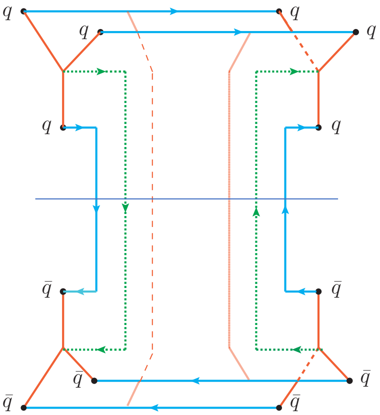
Leading (planar) $B\bar{B} \to B\bar{B}$ scattering

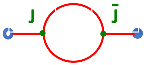


- s-channel M_4^J tetra-quark dual to a t-channel $q\,\bar{q}$ meson
- All sort of exotic s-channel M_4^J states are predicted
  Rossi Veneziano, 1977
- By string breaking M_4^J (baryonium) is coupled to $\bar{B}B$

Leading (planar) $B\bar{B} \to B\bar{B}$ annihilation

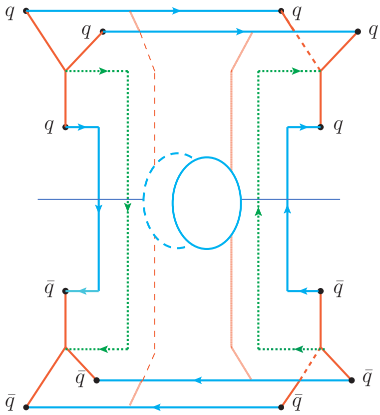
- Same flavour flow as before

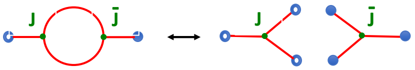




- s-channel 2 $q\,\bar{q}$ jets dual to a t-channel M_2^J baryonium meson

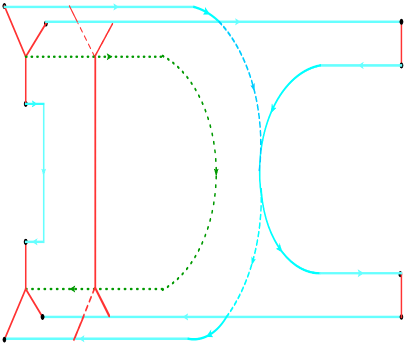
Loop correction to $B\bar{B} \to B\bar{B}$ annihilation

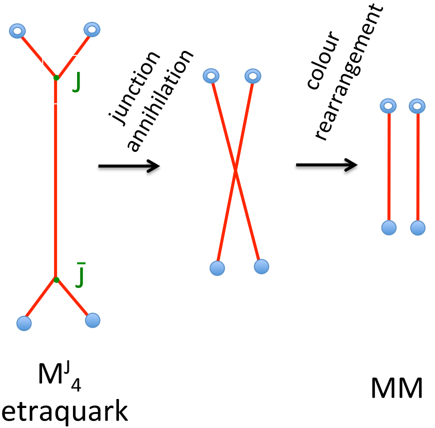




- s-channel 2 $q\,\bar{q}$ jets dual to a t-channel M_2^J baryonium meson
- insertion of two quark loops allows $M_2^J \to B\bar{B}$ decay (one quark loop $\to$ mixing between M_2^J and M_4^J)

Subleading (non-planar) $B\bar{B} \rightarrow MM$ annihilation

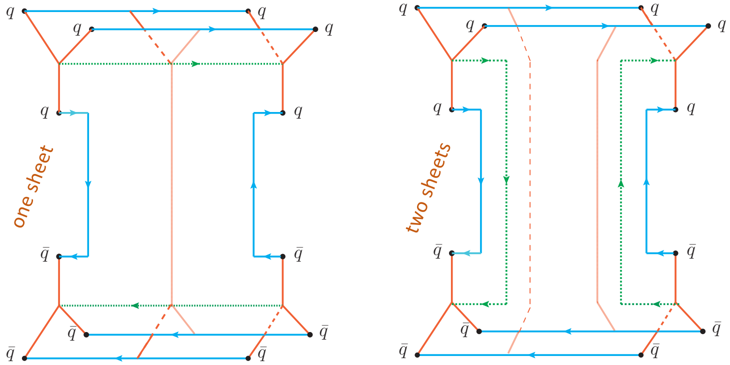




- The (dynamically disfavoured?) $M_4^J \rightarrow MM$ decay
- with junction annihilation and color rearrangement
  $\rightarrow$ violation of what we may call the J-Zweig rule

Scattering vs. annihilation

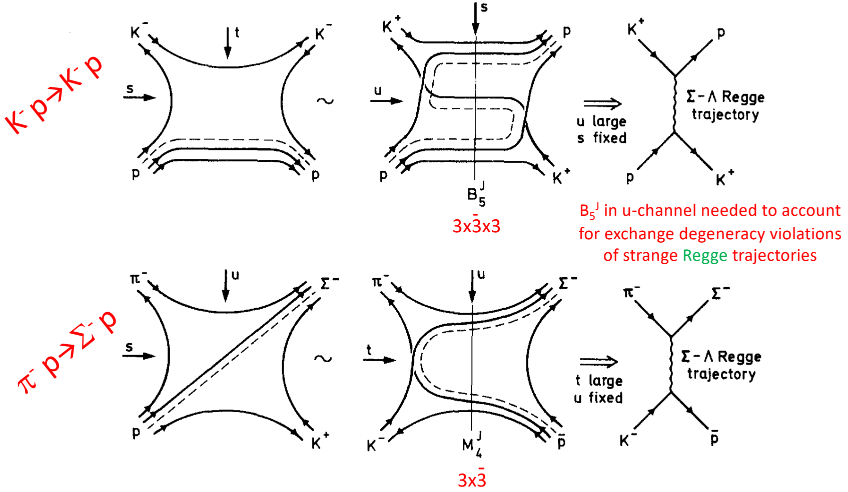
Following flavour flow is not enough ...



... need to specify topology, i.e. Junction flow (green sewing lines)

Duality and multi-quark states

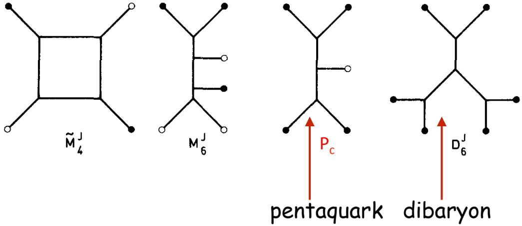
Multi-quark states are needed already at lowest order in $MB \to MB$



(figure taken from the 1980 Rossi–Veneziano Physics Report)

A whole family of states is predicted

- Other multi-quark, possibly exotic, states Rossi Veneziano, 1980
  - First time penta-quark states are mentioned

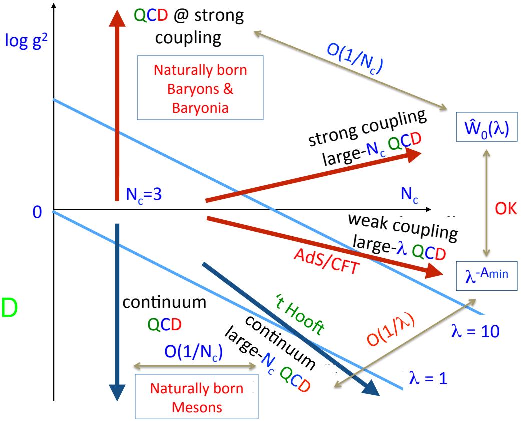


- Our conjecture
  - Fully baryonic decay (via string breaking) is dynamically favoured
  - If kinematics forbids it, baryonium states can be unusually narrow
- Many other challenging interpretations are on the market
  - Jaffe 1977 - Jaffe & Wilczek 2003
  - Miyazawa, 1979
  - Maiani, Piccinini, Polosa & Riquer 2005
  - Matheus, Narison, Nielsen & Richard 2006
  - Karliner & Lipkin 2008 - Karliner & Rosner 2015
  - . . .

The interesting limits of QCD

- The limits we have good theoretical control of are
  - PT, small g^2 fixed N_c
  - 't Hooft large-N_c limit (fixed N_f and fixed $\lambda = g^2 N_c$)
  - (lattice) strong coupling limit $g^2 \to \infty$, fixed N_c
  - large λ limit with N_c large and g^2 possibly small (AdS/CFT)

- Question to what extent we can extrapolate
  - from large $g^2 N_c$
    - strong coupling QCD
  - to small g^2 & N_c
    - continuum QCD

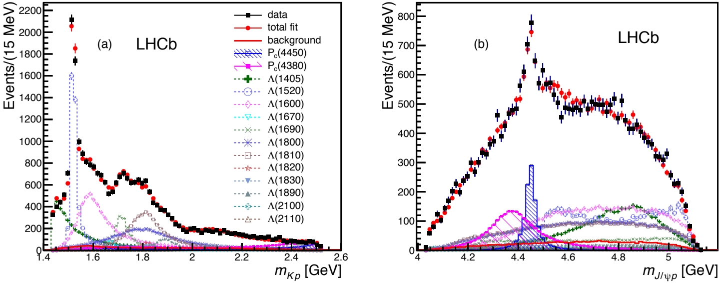


Some phenomenological considerations

- P_c photoproduction in the baryonium picture
- A mass formula for narrow baryonium states

Experimental evidence for the P_c $[qq\bar{q}qq]$ state

R. Aaij *et al.* [LHCb Collaboration]



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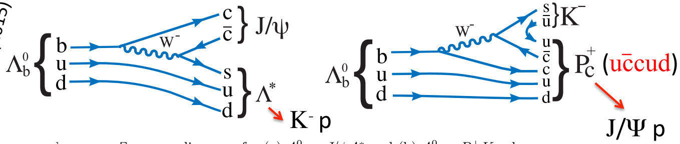


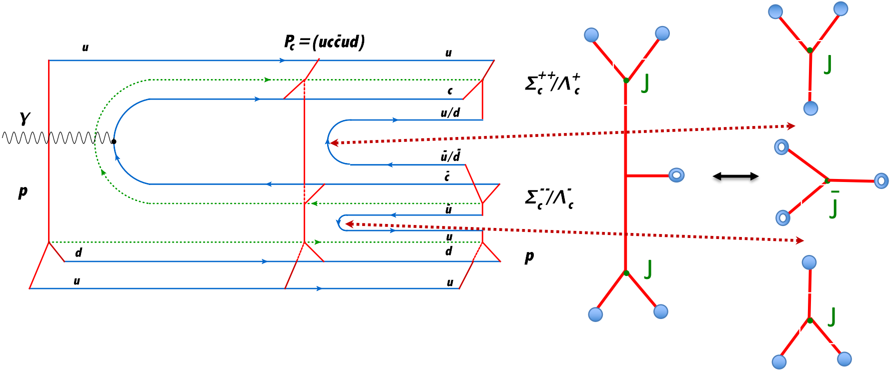
Feynman diagrams for (a) $\Lambda_b^0 \to J/\psi \Lambda^*$ and (b) $\Lambda_b^0 \to P_c^+ K^-$ decay.

$M(P_{Ic}^+) = 4450$ $J^P=5/2^+$ $\Gamma = 39$ MeV

$M(P_{IIc}^+) = 4380$ $J^P=3/2^-$ $\Gamma = 205$ MeV

P_c photoproduction & string breaking decays

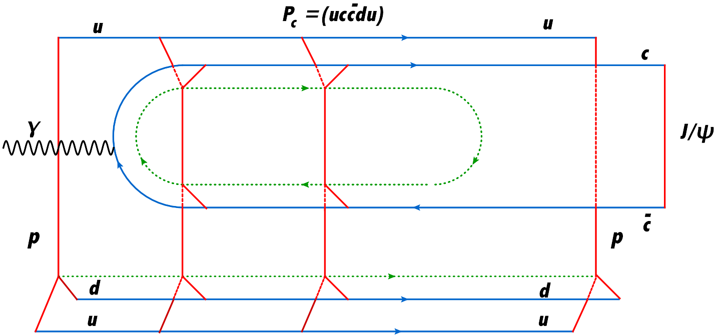
- Kinematically forbidden
  - $P_c \not\to \Lambda_c^+ + \Lambda_c^- + p$
  - $P_c \not\to \Sigma_c^{++} + \Sigma_c^{--} + p$



- dynamically favoured, but kinematically forbidden
  - $M_{P_c} \sim 4450 < 2M_{\Lambda_c^+} + M_p \sim 2 \times 2286 + 938 = 5510$ MeV
  - $M_{P_c} \sim 4450 < 2M_{\Sigma_c^{++}} + M_p \sim 2 \times 2454 + 938 = 5846$ MeV

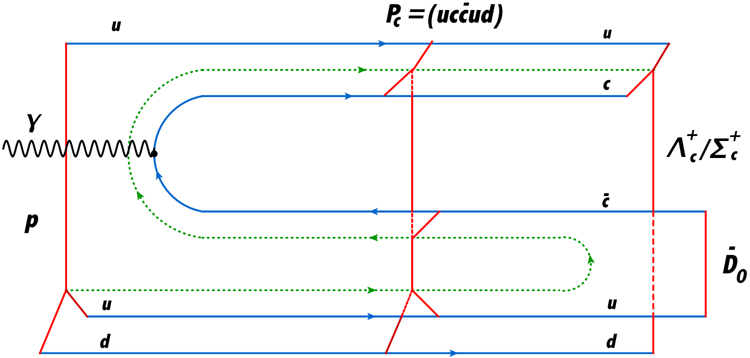
P_c photoproduction & Junction annihilation decay - I

- Kinematically allowed
  - $P_c \to J/\psi + p$
  - closed Junction loop ("bathtub" diagrams)



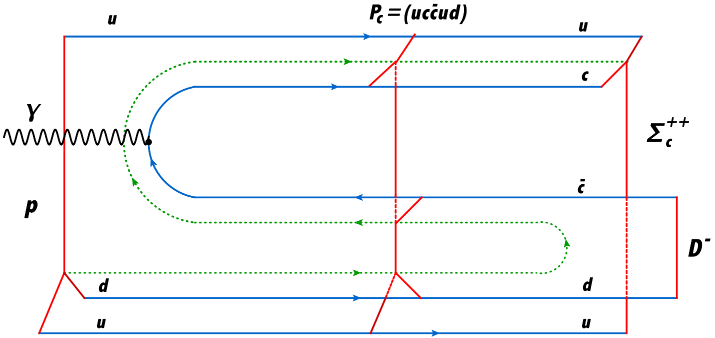
- $M_{P_c} \sim 4450 > M_{J/\psi} + M_p \sim 3097 + 938 = 4035$ MeV
- This is the decay with the largest available phase-space

- Kinematically allowed
  - $P_c \to \Lambda_c^+/\Sigma_c^+ + \bar{D}_0$
  - no Junction loops ("snake" diagrams)



- $M_{P_c} \sim 4450 > M_{\Lambda_c^+} + M_{\bar{D}_0} \sim 2286 + 1865 = 4151$ MeV
- $M_{P_c} \sim 4450 > M_{\Sigma_c^+} + M_{\bar{D}_0} \sim 2454 + 1865 = 4319$ MeV

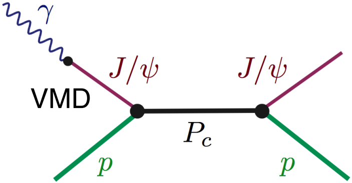
- Another kinematically allowed decay
  - $P_c \to \Sigma_c^{++} + D^-$
  - no Junction loops ("snake" diagrams)

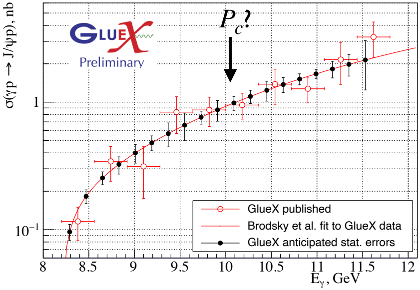


- $M_{P_c} \sim 4450 > M_{\Sigma_c^{++}} + M_{D^-} \sim 2454 + 1865 = 4319$ MeV

The Gluex photoproduction experiment

We can only recall the recent negative result





- $P_c \to J/\psi + p$
- $P_c \to \Lambda_c^+/\Sigma_c^+ + \bar{D}_0$
- $P_c \to \Sigma_c^{++} + D^-$

A mass formula for narrow baryonium states?

- With an eye to the supposedly dynamically dominant hadronic string breaking (HSB) decay mode of baryonium states, we are led to conjecture the oversimplified mass formulae

$$M^{J\bar{J}}_{qq\bar{q}\bar{q}} + \Delta_{HSB} = M_{B_1} + M_{\overline{B_2}}\,,$$
$$M^{J\bar{J}J}_{qq\bar{q}qq} + 2\Delta_{HSB} = M_{B_1} + M_{\overline{B_2}} + M_{B_3}\,,$$

  for narrow tetra- and penta-quark states with mass below their $B\overline{B}$ or $B\overline{B}B$ threshold
- Δ_{HSB} is the bit of energy one needs to provide to let the multi-quark state decaying into baryons
- We expect Δ_{HSB} of the order of some QCD scale and somehow "universal", if the above mass formulae has to be of any use
- Much more refined "mass formulae" have been elaborated by e.g. Karliner & Rosner - see also references in slide 3

A few applications - Narrow states

- $T_{cc}^{+}(3875)[cc\bar{u}\bar{d}]$ - $\Gamma = 0.409 \pm 0.163$ MeV
$$\Delta_{HSB} = \left(M_{\Xi_{cc}[ccu]} + M_{\bar{p}[\bar{u}\bar{u}\bar{d}]}\right) - M^{J\bar{J}}_{cc\bar{u}\bar{d}} =$$
$$= (3621 + 938) - 3875 = 4559 - 3875 = \underline{684 \text{ MeV}}$$
- $\chi_c \equiv X(3872)[cu\bar{c}\bar{u}]$ - $\Gamma = 1.2 \pm 0.2$ MeV.
$$\Delta_{HSB} = \left(M_{\Lambda_c^+[cud]} + M_{\Lambda_c^-[\bar{c}\bar{u}\bar{d}]}\right) - M^{J\bar{J}}_{c\bar{c}u\bar{u}} =$$
$$= (2286 + 2286) - 3872 = 4572 - 3872 = \underline{700 \text{ MeV}}$$
- $Y_b(10888)[bq\bar{b}\bar{q}]$ - $\Gamma = 50 \pm 10$ MeV
$$\Delta_{HSB} = \left(M_{\Sigma_b^+[buu]} + M_{\overline{\Sigma}_b^-[\bar{b}\bar{u}\bar{u}]}\right) - M^{J\bar{J}}_{bu\bar{b}\bar{u}} =$$
$$= (5810 + 5810) - 10888 = 11620 - 10888 = \underline{732 \text{ MeV}}$$
- $P_c^+(4450)[uc\bar{c}ud]$ - $\Gamma = 6.4 \pm 4$ MeV
$$2\Delta_{HSB} = \left(M_{\Sigma_c^+[cud]} + M_{\Sigma_c^-[\bar{c}\bar{u}\bar{d}]} + M_{p[uud]}\right) - M^{J\bar{J}J}_{uc\bar{c}ud} =$$
$$= (2453 + 2453 + 938) - 4457 = 5844 - 4457 = \underline{1387 \text{ MeV}}$$

Note - $\underline{\Delta_{HSB} \sim 700 \text{ MeV}}$ is what you need to create a $\bar{q}q$ pair and a QCD string bit, something that looks very much like a ρ ($m_\rho \sim 770$ MeV)!

A few more applications - Not so narrow states

- $Z_{sc}(3982)[cs\bar{c}\bar{u}]$ - $\Gamma \sim 15 \pm 5$ MeV

$$\Delta_{HSB} = \left(M_{\Xi_c^+[csu]} + M_{\Sigma_c^{--}[\bar{c}\bar{u}\bar{u}]} \right) - M^{J\bar{J}}_{[cs\bar{c}\bar{u}]} =$$
$$= (2468 + 2454) - 3982 = 4922 - 3982 = 940 \text{ MeV}$$

- $X_0(2900)[du\bar{c}\bar{s}]$ - $\Gamma \sim 57 \pm 12$ MeV

$$\Delta_{HSB} = \left(M_{p[duu]} + M_{\Xi_c^-[\bar{u}\bar{c}\bar{s}]} \right) - M^{J\bar{J}}_{du\bar{c}\bar{s}} =$$
$$= (938 + 2468) - 2866 = 3460 - 2866 = 594 \text{ MeV}$$

- $Z^-(4430)[cd\bar{c}\bar{u}]$ - $\Gamma \sim 180$ MeV

$$\Delta_{HSB} = \left(M_{\Sigma_c^0[cdd]} + M_{\Sigma_c^-[\bar{d}\bar{c}\bar{u}]} \right) - M^{J\bar{J}}_{cd\bar{c}\bar{u}} =$$
$$= (2455 + 2455) - 4430 = 4910 - 4430 = 480 \text{ MeV}$$

- $X(6900)[cc\bar{c}\bar{c}]$ - $\Gamma \sim 80 \div 168$ MeV

$$\Delta_{HSB} = \left(M_{\Xi_{cc}^{++}[ccu]} + M_{\Xi_{cc}^{--}[\bar{u}\bar{c}\bar{c}]} \right) - M^{J\bar{J}}_{cc\bar{c}\bar{c}} =$$
$$= (3621 + 3621) - 6900 = 7242 - 6900 = 342 \text{ MeV}$$

Note - $Z_{sc}(3982)[cs\bar{c}\bar{u}]$ is lighter than $Z^-(4430)[cd\bar{c}\bar{u}]$ despite $s \leftrightarrow d$

A prediction

- M. Karliner & J.L. Rosner in PRL **119** (2017) 20, 202001 predict
  - T$[bb\bar{u}\bar{d}]$, $J^P = 1^+$, $M_{T[bb\bar{u}\bar{d}]} = 10389 \pm 12$ MeV

baryonic quarks	MeV
$2m_b^b$	10087.0
$2m_q^b$	726.0
$a_{bb}/(m_b^b)^2$	7.8
$-3a/(m_q^b)^2$	-150.0
bb binding	-281.4
Total	10389 ± 12

  - $\rightarrow$ stable under strong interactions because
  - $M_{T[bb\bar{u}\bar{d}]} = 10389$ lies
    - 215 MeV below the $B^- \bar{B}^{\star 0}$ threshold ~ 10604 MeV
    - 170 MeV below the $B^- \bar{B}^0 \gamma$ threshold ~ 10559 MeV
- We suggest the (weaker) relation between $M_{T[bb\bar{u}\bar{d}]}$ and $M_{\Xi^0_{bb}[bbu]}$

$$M_{T[bb\bar{u}\bar{d}]} = M_{\Xi^0_{bb}[bbu]} + M_{\bar{p}} - 700 = M_{\Xi^0_{bb}[bbu]} + 238$$

Conclusions

- Starting from a planar approximation, a systematic expansion of QCD is constructed, providing a unified picture of hadrons where
  - a new family of (possibly exotic) multi-quark hadrons emerges based on the notion of Junction
    - tetra-quarks $B = 0$ $J\bar{J}$
    - penta-quarks $B = 1$ $J\bar{J}J$
    - di-baryons $B = 2$ $JJ\bar{J}J$
    - ...
  - Conjectured dynamically favoured decay is by string breaking (as opposed to J-$\bar{J}$ annihilation & colour rearrangement, $\not{J}$-Zweig)
  - if kinematically forbidden, baryonium can be unusually narrow
    - opposite to $X(4630)$, it's above $\Lambda_c^+\Lambda_c^-$ threshold & large $\Gamma \sim 180$ MeV
- Photoproduction is a suitable tool to produce penta-quarks
  - the paradigmatic case of $P_c(4450)$
- A crude mass formula for narrow baryonium states is proposed
- Most probably multi-quark states are a superposition of
  - baryonium-like resonances Rossi Veneziano 1977
  - diquark–anti-diquark bound states Jaffe 1977, Maiani *et al.* 2005
  - molecular di-meson configurations Karliner Rosner 2005

Thanks for your attention