

T_{QQ} in Quark and Diquark Models

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Workshop on Double charm tetraquark and other exotics
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1. Introduction: Tetraquark and Color Confinement
 2. Production of T_{cc} in NRQCD: Heavy-Diquark picture
 3. Chiral Diquark picture of Tetraquarks
 4. Doubly-Heavy Tetraquarks in the Quark Model

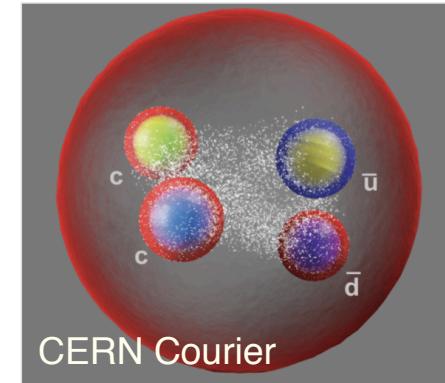
Introduction Tetraquark and Color Confinement

Doubly-Heavy Tetraquark (DHTQ)

Doubly-Heavy Tetraquark (DHTQ)

$$T_{QQ} = Q\bar{Q} q\bar{q}$$

($Q = c$, or b , $q = u, d$, or s)



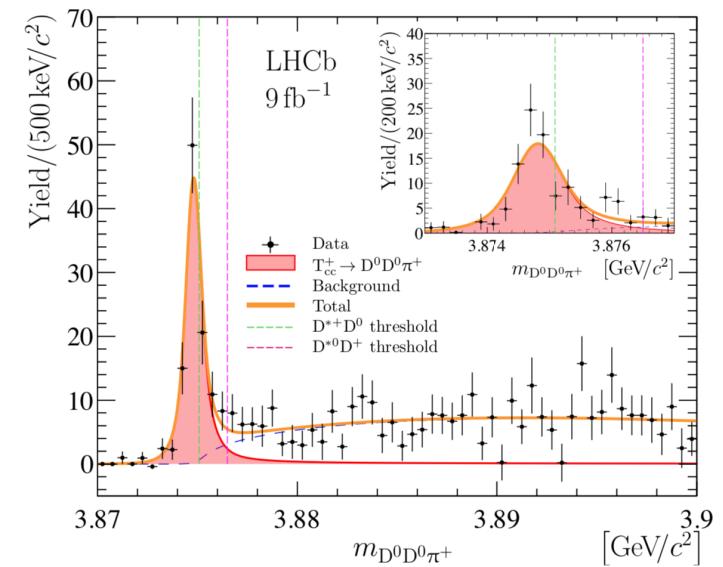
CERN Courier

First observation of T_{cc} at LHCb, arXiv:2109.01038, 2109.01056

BW peak fit

$\delta m = -273 \pm 61 \text{ keV}/c^2$, $\Gamma = 410 \pm 165 \text{ keV}$

from $D^{*+}D^0$ threshold



$T_{cc}^+ \rightarrow D^0 D^0 \pi^+$

Doubly-Heavy Tetraquark (DHTQ)

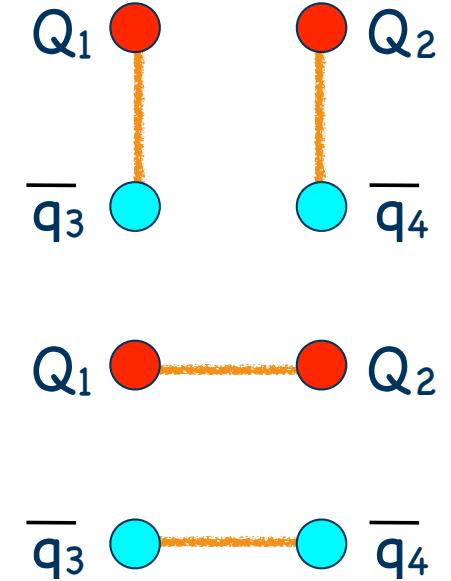
- # Why is this observation so exciting? *New, but more . .*
- # What does this mean to QCD?
- # My personal answer :

Color configurations in multi-quark ($N \geq 4$) systems and associated color confinement dynamics are not determined uniquely.
The tetra-quark, the simplest multi-quark, will clarify the quark color dynamics and will tell us which of the various quark confinement potentials (models) is correct.
- # Tetraquark is the simplest system for this study.

All heavy tetraquarks may also be good for this purpose, but they may not have bound states, but resonances only.
=> *Doubly heavy tetraquark*

Tetraquark - color configurations

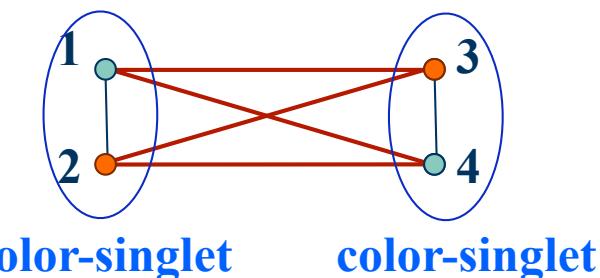
- # Color singlet $Q_1 Q_2 q_3^{\bar{b}a} q_4^{\bar{b}a}$ system has two independent color configurations.
- # Choices of the basis states
 - Two-meson channels:
 $(Q_1 q_3^{\bar{b}a})_1 (Q_2 q_4^{\bar{b}a})_1, (Q_1 q_4^{\bar{b}a})_1 (Q_2 q_3^{\bar{b}a})_1$
 - Singlet + hidden color states:
 $(Q_1 q_3^{\bar{b}a})_1 (Q_2 q_4^{\bar{b}a})_1, (Q_1 q_3^{\bar{b}a})_8 (Q_2 q_4^{\bar{b}a})_8$
 - Diquarks with color 3^{bar} and 6:
 $(Q_1 Q_2)_{3\bar{b}a} (q_3^{\bar{b}a} q_4^{\bar{b}a})_3, (Q_1 Q_2)_6 (q_3^{\bar{b}a} q_4^{\bar{b}a})_{6\bar{b}a}$
- # While these bases are equivalent, *i.e.*, related by linear transforms, physical pictures are often different from each other.
- # Color configurations will determine “*how quarks are confined in multi-quark systems*”.



Two-body confinement

■ Two-body confinement with color saturation

$$V = \sum_{i < j} V(r_{ij}) = \sum_{i < j} (\lambda_i \cdot \lambda_j) (-a) r_{ij} \quad (\lambda_i \cdot \lambda_j) = \sum_{\alpha} \lambda_i^{\alpha} \lambda_j^{\alpha}$$



$$\sigma = \frac{16}{3}a \sim 1 \text{ GeV/fm}$$

■ No confinement between two color-singlet systems.

$$V(R) = \left\langle \sum_{i \in (1,2), j \in (3,4)} (\lambda_i \cdot \lambda_j) (-a) r_{ij} \right\rangle \sim - \langle (\lambda_1 + \lambda_2) \cdot (\lambda_3 + \lambda_4) \rangle a R = 0$$

■ It, however, induces long-range color van der Waals force, $\sim -1/R^3$.

T. Appelquist, W. Fischler, Phys. Lett. B77, 405 (1978)

R.S. Willey, Phys. Rev. D18, 270 (1978)

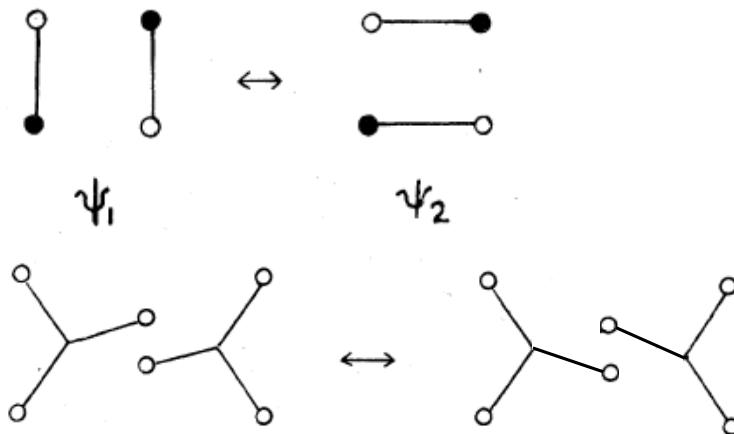
S. Matsuyama, H. Miyazawa, Prog. Theor. Phys. 61, 942 (1979)

String Flip-Flop model

- # “Reconnection of strings and quark matter”, H. Miyazawa, PR D20, 2953 (1979).

$$V_{\text{string}} = \sigma \times \underset{\text{links}}{\text{Min}} \sum r_{\text{link}}$$

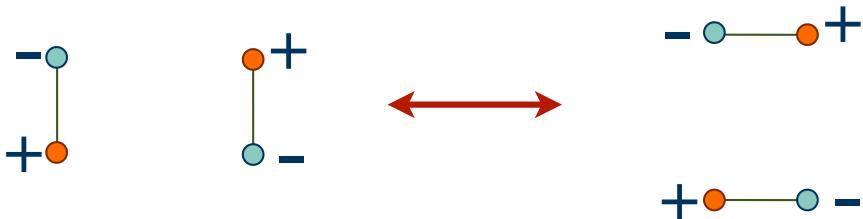
“String Flip-Flop” model



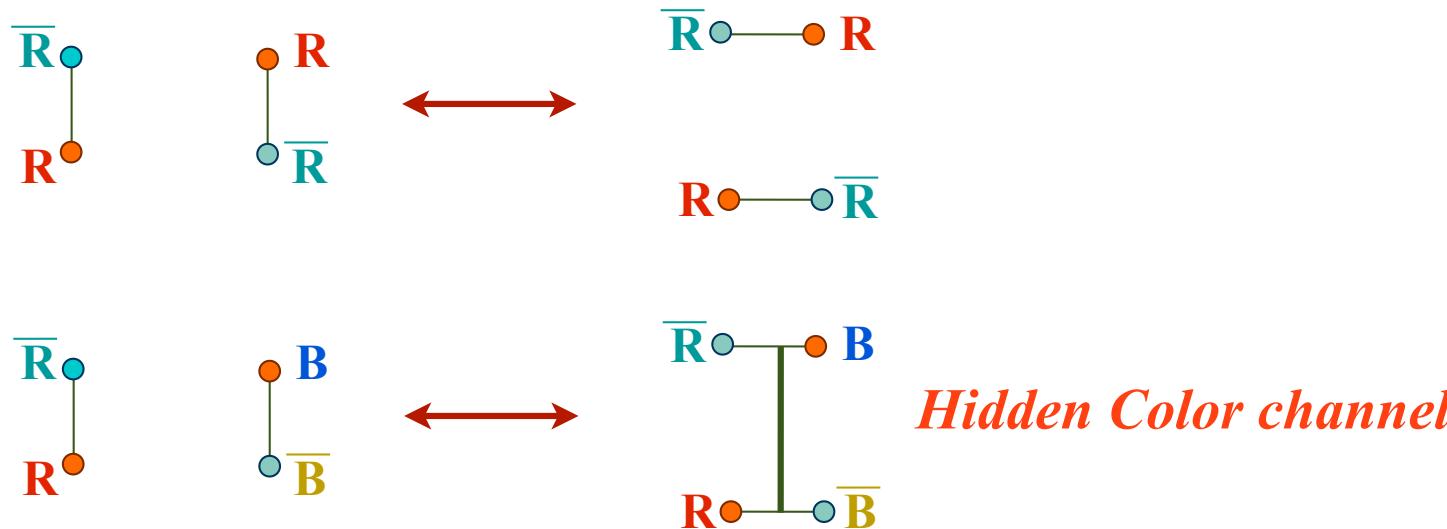
- # The string FF model is free from the difficulty of the color van-der-Waals force.
- # Similar “string-type” confinement potential models for multi-quark systems were proposed.
O.W. Greenberg, J. Hietarinta, Phys. Lett. B 86, 309 (1979)
N. Isgur, J. E. Paton, Phys. Lett. B 124, 247 (1983)
F. Lenz, et al., Annals Phys. 170, 65 (1986).
J. Vijande, A. Valcarce, J.M. Richard, Phys. Rev. D 85, 014019 (2012).

String Flip-Flop model

- # The string FF model works very well for the U(1) color.



- # In the color SU(3) theory, ...



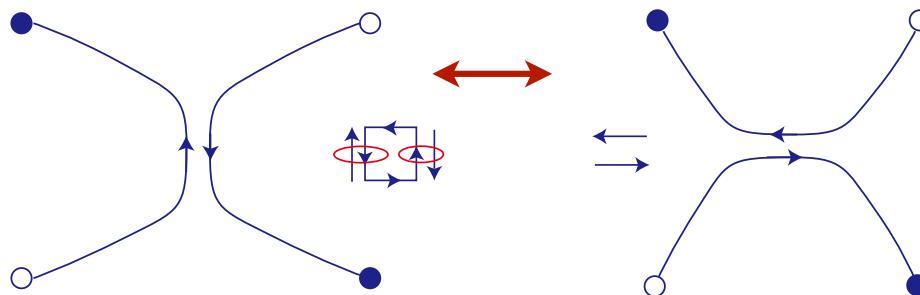
- # As the HC channels are always mixed with color-singlet channels, they are not simply omitted.

What does QCD tell us?

Flip-flop of QCD strings

We cannot assign a specific color to each quark, because gauge rotations can change the color of quark.

Consider the strong coupling expansion for simplicity.



- # Appropriate color recombination is automatically chosen by the gauge link action. *Hidden color channels seem not appear.*
- # How do color-dependent quark potentials describe the quark confinement in QCD? - *not yet answered* -

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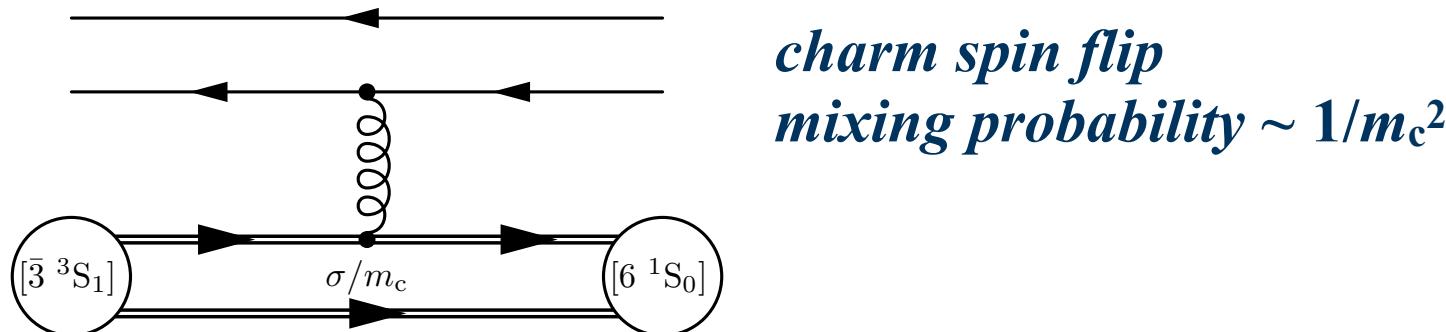
Production of T_{cc} in NRQCD Heavy-Diquark picture

Heavy Diquark picture of DHTQ

- # Heavy-Diquark picture of T_{QQ}
Two orthogonal color configurations
 $(QQ)_{3\bar{3}} + (q^{\bar{3}}q^{\bar{3}})_3$
or
 $(QQ)_6 + (q^{\bar{6}}q^{\bar{6}})_{6\bar{3}}$



- # For the same flavor heavy quarks, $Q=c$, T_{cc} , the Pauli principle requires the spin and color of QQ are entangled for the lowest states as $(3^{\bar{3}}, S=1)$ and $(6, S=0)$.
Mixing of these two configurations is suppressed by the heavy-quark spin symmetry.



Heavy Diquark picture of DHTQ

T_{cc} ($I=0$, $J^\pi=1^+$) of color $\bar{3}$ and 6 configurations

S.H. Lee, S. Yasui, W. Liu, C.M. Ko, Eur. Phys. J. C54, 259 (2008)

J. Vijande, A. Valcarce, Phys. Rev. C80, 035204 (2009)

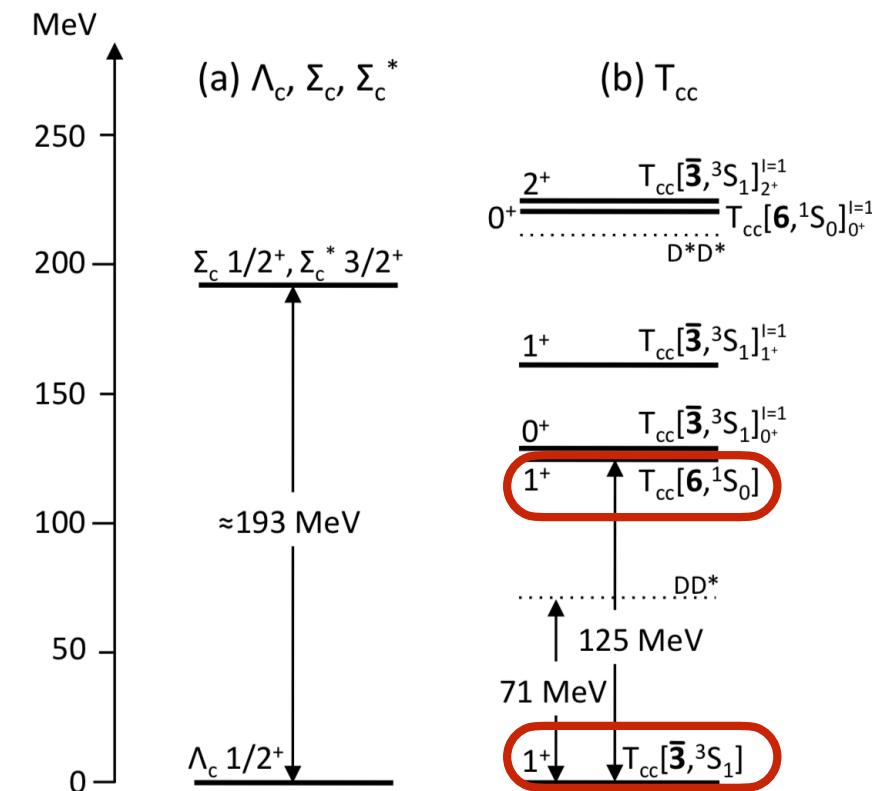
T. Hyodo, Y.R. Liu, M. Oka, K. Sudoh, S. Yasui, Phys. Lett. B721, 56 (2013)

T. Hyodo, Y.R. Liu, MO, S. Yasui, ArXiv: 1708.05169 (2017)

Spectrum of T_{cc} in the diquark picture
is studied in a simple color-spin
interaction model.

$$H_{\text{int}} = \sum_{i < j} \frac{C_H}{m_i m_j} \left(-\frac{3}{8} \right) \vec{\lambda}_i \cdot \vec{\lambda}_j \vec{s}_i \cdot \vec{s}_j$$

$T_{cc}[6]$ is ~ 125 MeV above $T_{cc}[\bar{3}]$
 $\Leftrightarrow 193$ MeV for Λ_c - $\Sigma_c^{(*)}$.



Production of DHTQ

- # Belle (KEKB) observed double-charm-pair productions in e+e- collision. *K. Abe, et al, Belle Collaboration, Phys. Rev. Lett. 89, 142001 (2002)*
- # We applied NRQCD to the production of DHTQ
NRQCD (factorization and expansion in v_Q)
hard process $e^+e^- \rightarrow cc [J^P]$ (perturbative)
soft process $cc \rightarrow T_{cc}$ (HQEFT, velocity $v = p/m_c$)

$$\sigma \sim \sum_k \frac{f_k(\alpha_s)}{\text{hard}} \left| \langle H | \mathcal{O}_k(v) | 0 \rangle \right|^2$$

soft

G.T. Bodwin, E. Braaten, G.P. Lepage, Phys. Rev. D51, 1125 (1995)
A. Petrelli, et al, Nucl. Phys. B514, 245 (1998)

E. Braaten, J. Lee, Phys. Rev. D67, 054007 (2003).

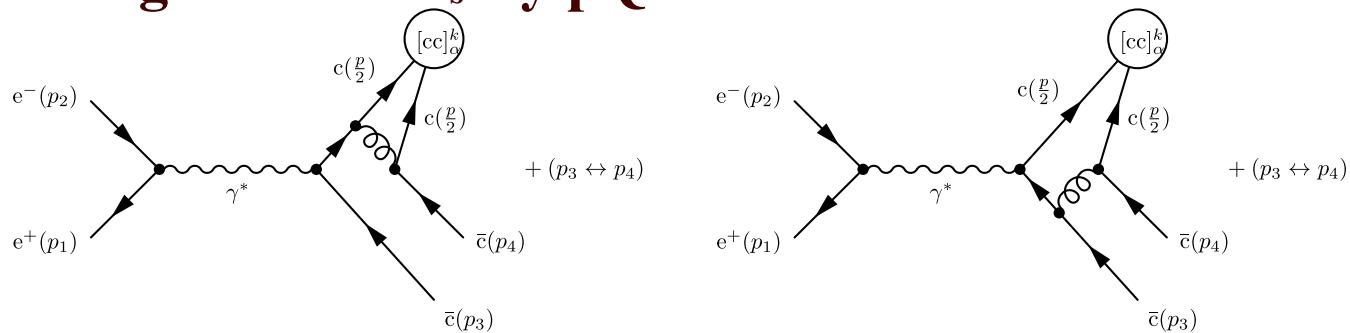
K.Y. Liu, Z.G. He, K.T. Chao, Phys. Lett. B557, 45 (2003)

Production of DHTQ

■ Cross section

$$d\sigma_\alpha(e^+e^- \rightarrow T_{cc}[\alpha] + X) = \sum_k \frac{d\hat{\sigma}(e^+e^- \rightarrow [cc]_\alpha^k + \bar{c} + \bar{c})}{\text{blue line}} \frac{|\langle T_{cc} + X | [cc]_\alpha^k | 0 \rangle|^2}{\text{red line}}$$

Hard part: leading order in α_s by pQCD



Soft part can be factorized: leading order in $v \rightarrow 0$.

$$\left| \langle T_{cc} + X | [cc]_\alpha^k | 0 \rangle \right|^2 \Big|_{k=\text{LO}} = \begin{cases} h_3 & \text{for } \alpha = [\bar{\mathbf{3}}, {}^3S_1] \\ h_6 & \text{for } \alpha = [\mathbf{6}, {}^1S_0] \end{cases}$$

$$h_3(\Xi_{cc}[\bar{\mathbf{3}}]) = \text{Prob } (cc \rightarrow \Xi_{cc}) \times |R_{cc}^{\Xi_{cc}}(0)|^2 / 4\pi,$$

$$h_3(T_{cc}[\bar{\mathbf{3}}]) = \text{Prob } (cc \rightarrow T_{cc}[\bar{\mathbf{3}}, {}^3S_1]) \times |R_{cc}^{T_{cc}[\bar{\mathbf{3}}, {}^3S_1]}(0)|^2 / 4\pi,$$

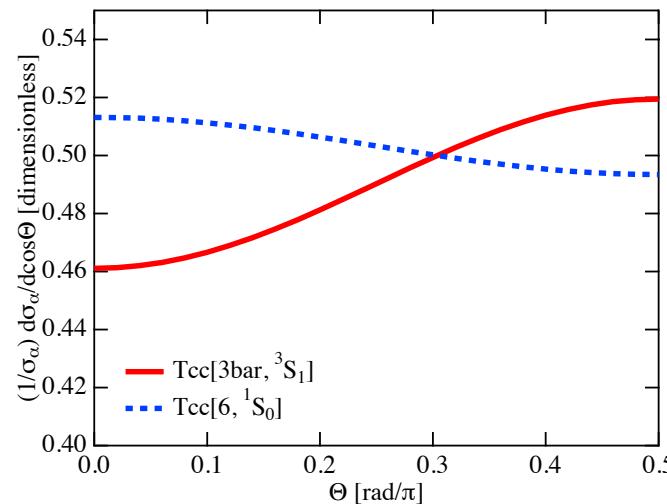
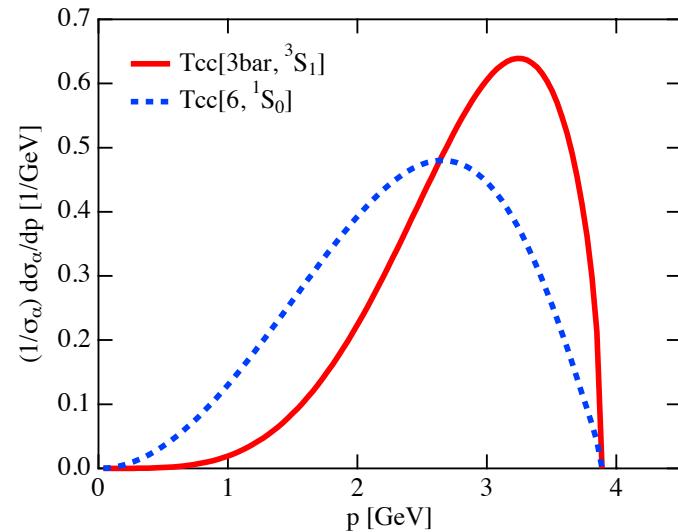
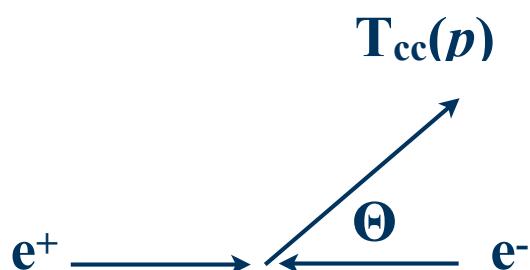
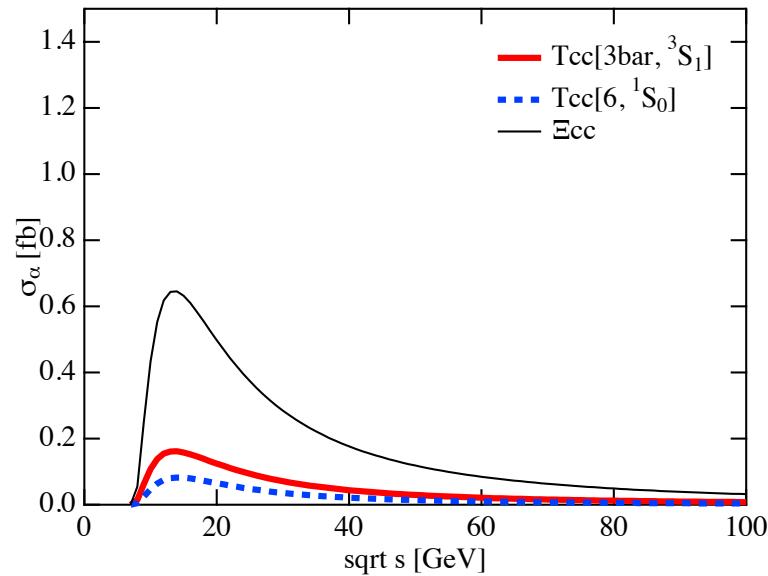
$$h_6(T_{cc}[\mathbf{6}]) = \text{Prob } (cc \rightarrow T_{cc}[\mathbf{6}, {}^1S_0]) \times |R_{cc}^{T_{cc}[\mathbf{6}, {}^1S_0]}(0)|^2 / 4\pi.$$

T. Hyodo, et al., Phys. Lett. B721, 56 (2013), ArXiv: 1708.05169 (2017)

Production of DHTQ

■ Double charm production in $e^+ + e^-$ at $\sqrt{s} = 10.6 \text{ GeV}$

Production cross sections for $T_{cc}[3\bar{b}]$, $T_{cc}[6]$ vs Ξ_{cc}



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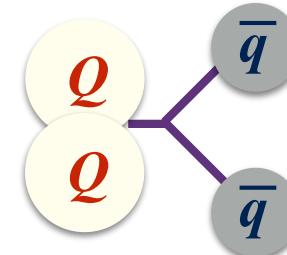
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Chiral Diquark picture of Tetraquarks

Diquark picture of tetraquarks

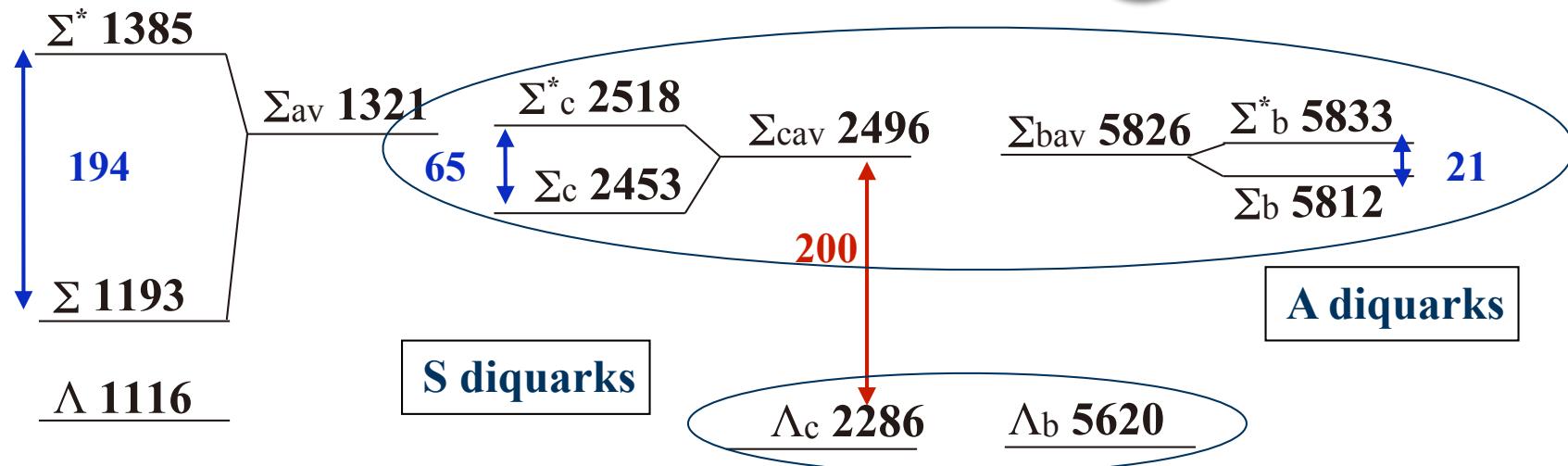
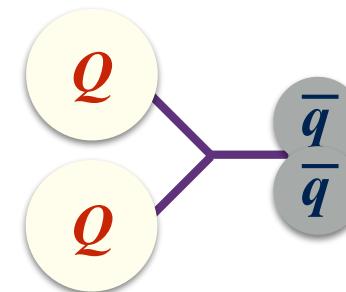
Diquark in baryons vs tetraquarks

$$\Xi_{QQ} = (QQ) q \Leftrightarrow T_{QQ} = (QQ) (q^{\bar{}} q^{\bar{}})$$



Or alternatively

$$\Lambda_Q = Q (ud)_0 \Leftrightarrow T^{\bar{}}_{Q\bar{Q}} = Q^{\bar{}} Q^{\bar{}} (ud)_0$$



Chiral Effective Theory of Diquarks

- # Chiral $SU(3)_R \times SU(3)_L$ effective theory for diquarks
M. Harada, Y.R. Liu, M.O., K. Suzuki, Phys. Rev. D101, 054038 (2020)
Y. Kim, E. Hiyama, M.O., K. Suzuki, Phys. Rev. D102, 014004 (2020)
Y. Kawakami, M. Harada, M.O., K. Suzuki, Phys. Rev. D102, 114004 (2020)
Y. Kim, Y.R. Liu, M.O., K. Suzuki, Phys. Rev. D 104, 054012 (2021)
An effective chiral diquark theory a la linear sigma model is constructed and derive mass relations of diquarks
- # Color $\bar{3}$, Spin 0 diquarks in $(\bar{3}, 1) + (1, \bar{3}) \Rightarrow S(\bar{3}) + P(\bar{3})$
 $d_{iR}^a \equiv \epsilon_{ijk} (q_{jR}^T C q_{kR})^{\bar{3}}$ $d_{iL}^a \equiv \epsilon_{ijk} (q_{jL}^T C q_{kL})^{\bar{3}}$ $(\bar{3}, 1) + (1, \bar{3})$
- # Color $\bar{3}$, Spin 1+/1- diquarks in $(3, 3) \Rightarrow A(6) + V(\bar{3})$
 $d_{ij}^{\mu a} \equiv \epsilon_{abc} (q_{iL}^{bT} C \gamma^\mu q_{jR}^c) = \epsilon_{abc} (q_{jR}^{bT} C \gamma^\mu q_{iL}^c)$ $(3, 3)$
- # Parity eigenstates
 $S_i^a = d_{iR}^a - d_{iL}^a = \epsilon_{ijk} (q_j^T C \gamma_5 q_k)^{\bar{3}}$ $P_i^a = d_{iR}^a + d_{iL}^a = \epsilon_{ijk} (q_j^T C q_k)^{\bar{3}}$
 $d_{V[ij]}^{\mu a} = d_{ij}^{\mu a} - d_{ji}^{\mu a} = \epsilon_{abc} (q_i^{bT} C \gamma^\mu \gamma^5 q_j^c)$
 $d_{A\{ij\}}^{\mu a} = d_{ij}^{\mu a} + d_{ji}^{\mu a} = \epsilon_{abc} (q_i^{bT} C \gamma^\mu q_j^c)$

Chiral Effective Theory of Diquarks

M. Harada, Y.R. Liu, M.O., K. Suzuki, PR D101, 054038 (2020)

$$\begin{aligned}\mathcal{L} = & \mathcal{D}_\mu d_{R,i} (\mathcal{D}^\mu d_{R,i})^\dagger + \mathcal{D}_\mu d_{L,i} (\mathcal{D}^\mu d_{L,i})^\dagger \\ & - m_0^2 (d_{R,i} d_{R,i}^\dagger + d_{L,i} d_{L,i}^\dagger) \quad \text{chiral invariant mass term} \\ & - \frac{m_1^2}{f} (d_{R,i} \Sigma_{ij}^\dagger d_{L,j}^\dagger + d_{L,i} \Sigma_{ij} d_{R,j}^\dagger) \quad \text{U_A(1) anomaly} \\ & - \frac{m_2^2}{2f^2} \epsilon_{ijk} \epsilon_{lmn} (d_{R,k} \Sigma_{\ell i} \Sigma_{mj} d_{L,n}^\dagger + d_{L,k} \Sigma_{\ell i}^\dagger \Sigma_{mj}^\dagger d_{R,n}^\dagger) \\ & + \frac{1}{4} \text{Tr} [\partial^\mu \Sigma^\dagger \partial_\mu \Sigma] + V(\Sigma). \quad \Sigma_{ij} \equiv \sigma_{ij} + i\pi_{ij}\end{aligned}$$

- # For the SSB vacuum $\langle \Sigma \rangle = f$, the mass term of the right and left diquarks are given by

$$M(0^+) = \sqrt{m_0^2 - m_1^2 - m_2^2}, \quad M(0^-) = \sqrt{m_0^2 + m_1^2 + m_2^2},$$

Chiral Effective Theory of Diquarks

SU(3) breaking and inverse mass hierarchy $A \equiv \frac{f_s}{f_\pi} \left(1 + \frac{m_s}{g_s f_s}\right) \sim \frac{5}{3}$
i=3 (ud)

$$M_3(0^+) = \sqrt{m_0^2 - Am_1^2 - m_2^2}, \quad M_3(0^-) = \sqrt{m_0^2 + Am_1^2 + m_2^2}.$$

i=1,2 (ds), (us)

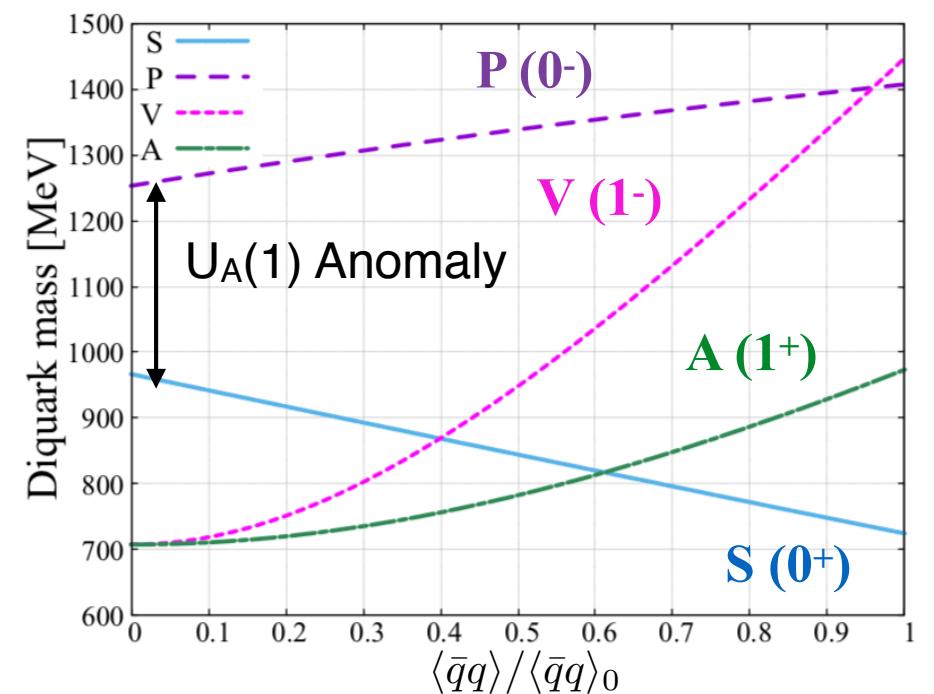
$$M_1(0^+) = M_2(0^+) = \sqrt{m_0^2 - m_1^2 - Am_2^2}, \quad M_1(0^-) = M_2(0^-) = \sqrt{m_0^2 + m_1^2 + Am_2^2},$$

$$M_1(0^-) < M_3(0^-)$$

(ds), (us) (ud)

Inverse Mass Hierarchy

*M. Harada, Y.R. Liu, M.O., K. Suzuki,
 Phys. Rev. D101, 054038 (2020)*



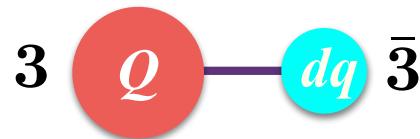
Y. Kim, Y.R. Liu, M.O., K. Suzuki, Phys. Rev. D 104, 054012 (2021)

Diquark-Heavy-Quark model

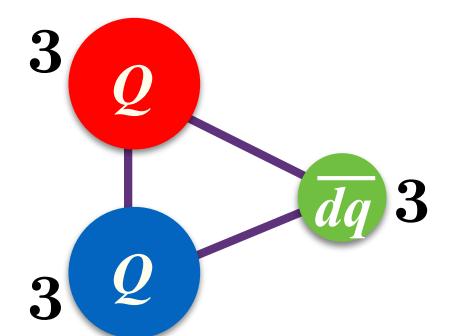
Diquark-Heavy-quark potential

A linear + Coulomb potential between Q - dq for baryon

$$V(r) = -\frac{\alpha}{r} + \lambda r + C,$$



2-body potentials between Q - Q and Q - dq^{bar}
for tetraquark



α	$\lambda(\text{GeV}^2)$	$C_c(\text{GeV})$	$C_b(\text{GeV})$	$M_c(\text{GeV})$	$M_b(\text{GeV})$
$(2/3) \times 90/\mu$	0.165	-0.58418362	-0.58829590	1.750	5.112

B. Silvestre-Brac, C. Semay, Z. Phys. C 59, 457 (1993)

T. Yoshida, E. Hiyama, A. Hosaka, M. Oka, K. Sadato, PR D 92, 114029 (2015)

$$M_{(ud)}(0^+) = 725 \text{ MeV} \quad M_{(ud)}(0^-) = 1265 \text{ MeV}$$

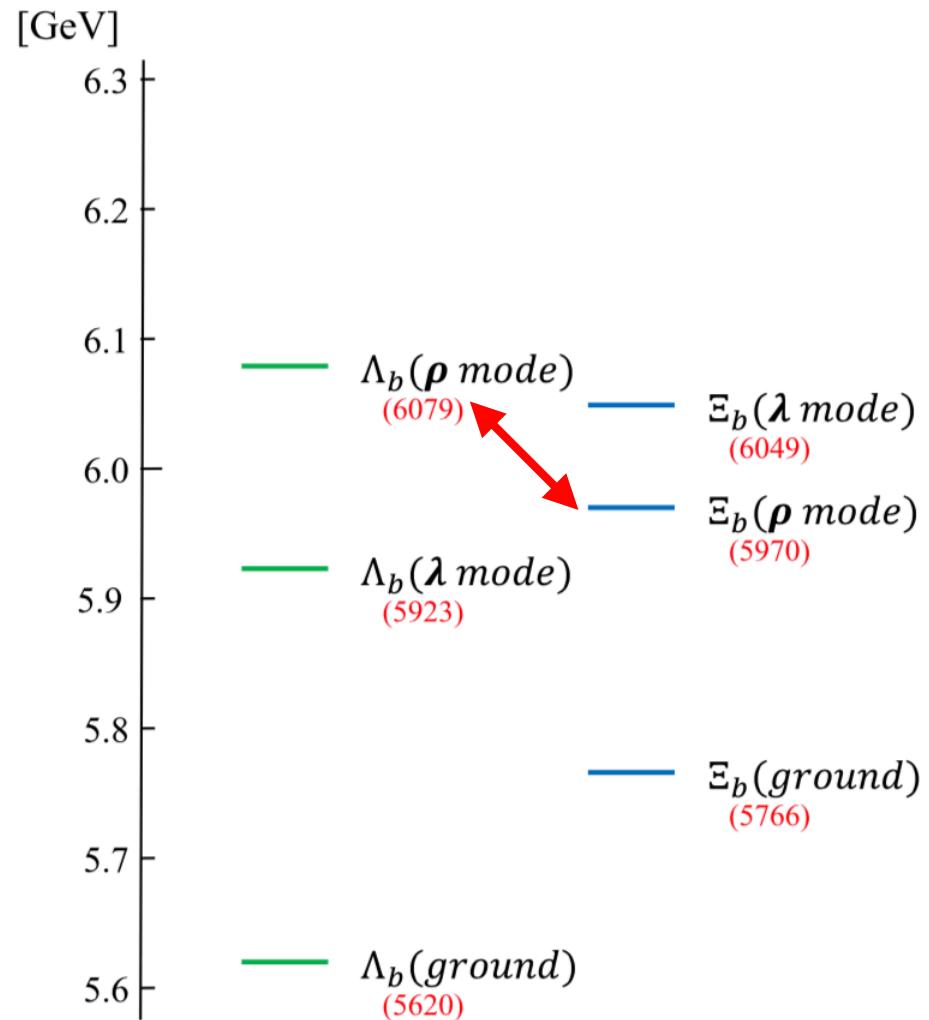
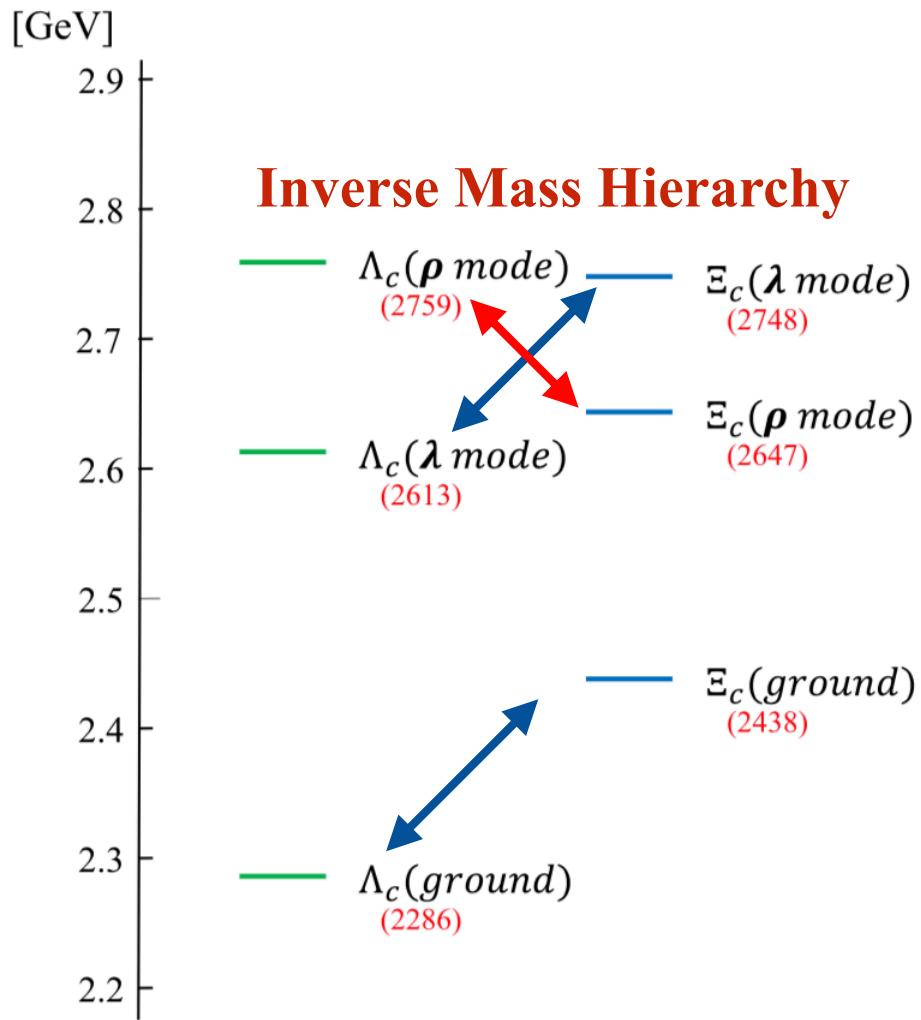
$$M_{(us)}(0^+) = 906 \text{ MeV} \quad M_{(us)}(0^-) = 1142 \text{ MeV}$$

$$M_{(qq)}(1^+) = 974 \text{ MeV} \quad M_{(qq)}(1^-) = 1447 \text{ MeV}$$

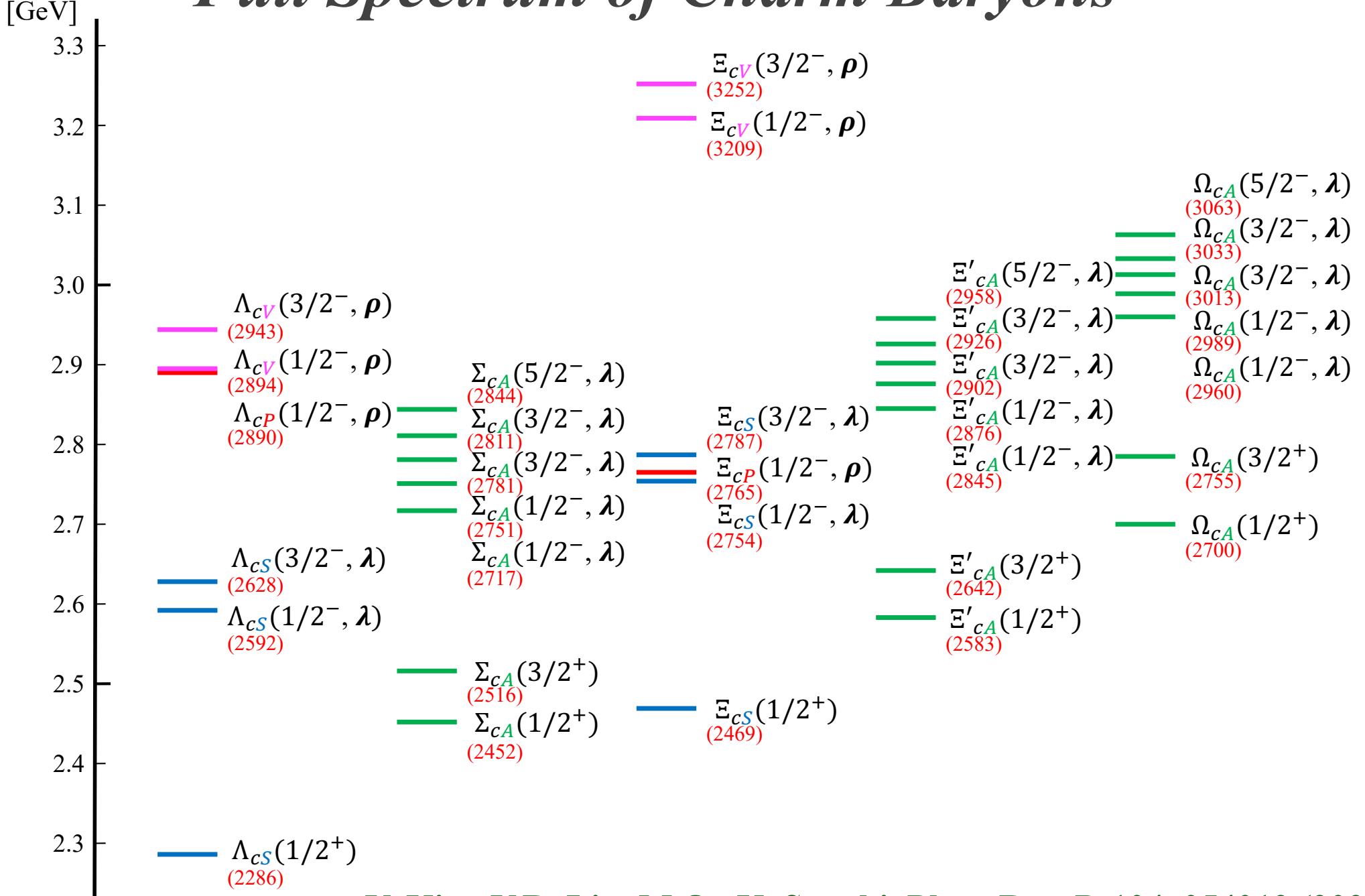
$$M_{(qs)}(1^+) = 1116 \text{ MeV} \quad M_{(ss)}(1^+) = 1242 \text{ MeV}$$

Inverse mass hierarchy

Y. Kim, E. Hiyama, M. Oka, K. Suzuki, *PRD 102, 014004 (2020)*

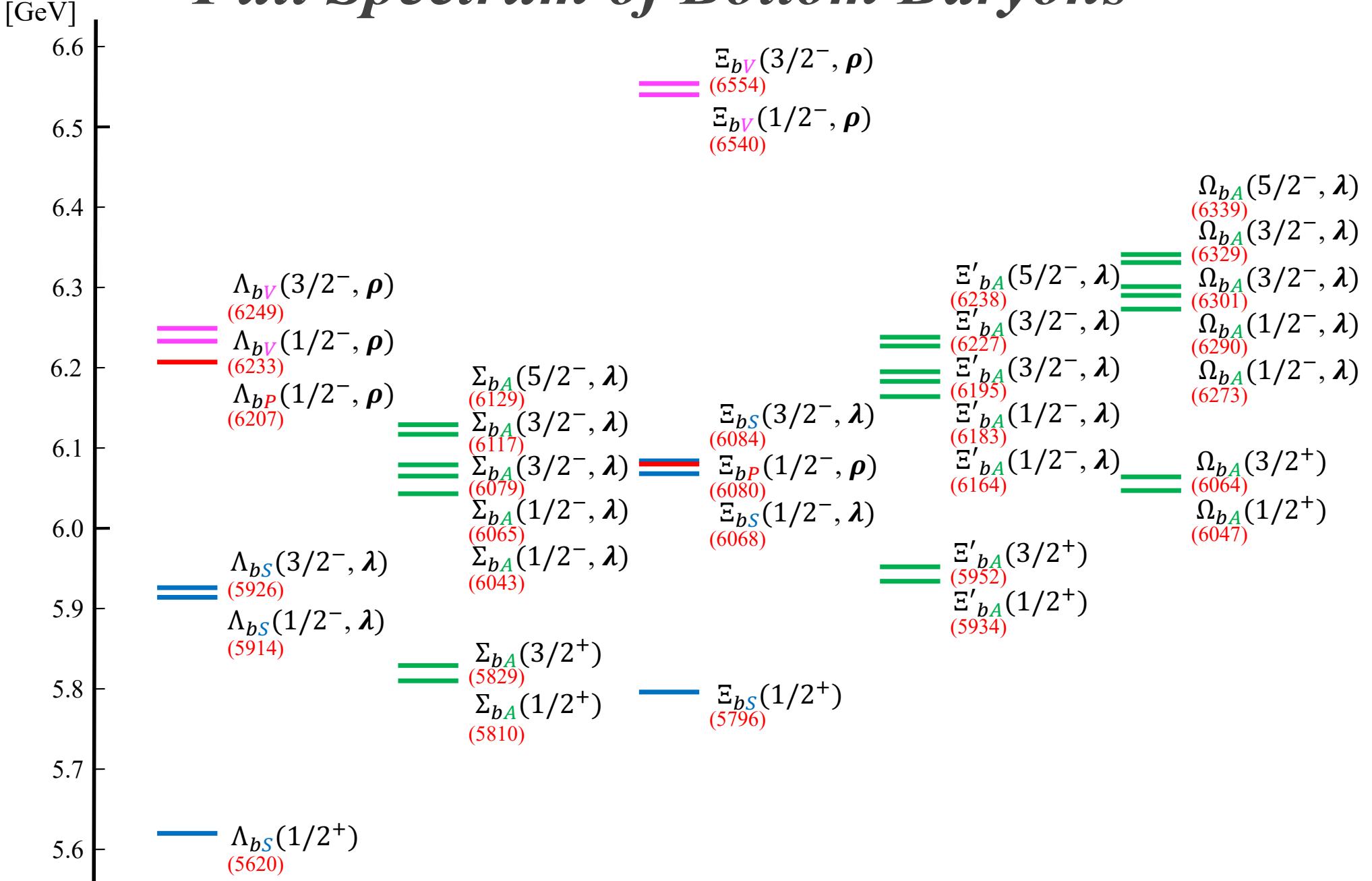


Full Spectrum of Charm Baryons



Y. Kim, Y.R. Liu, M.O., K. Suzuki, Phys. Rev. D 104, 054012 (2021)

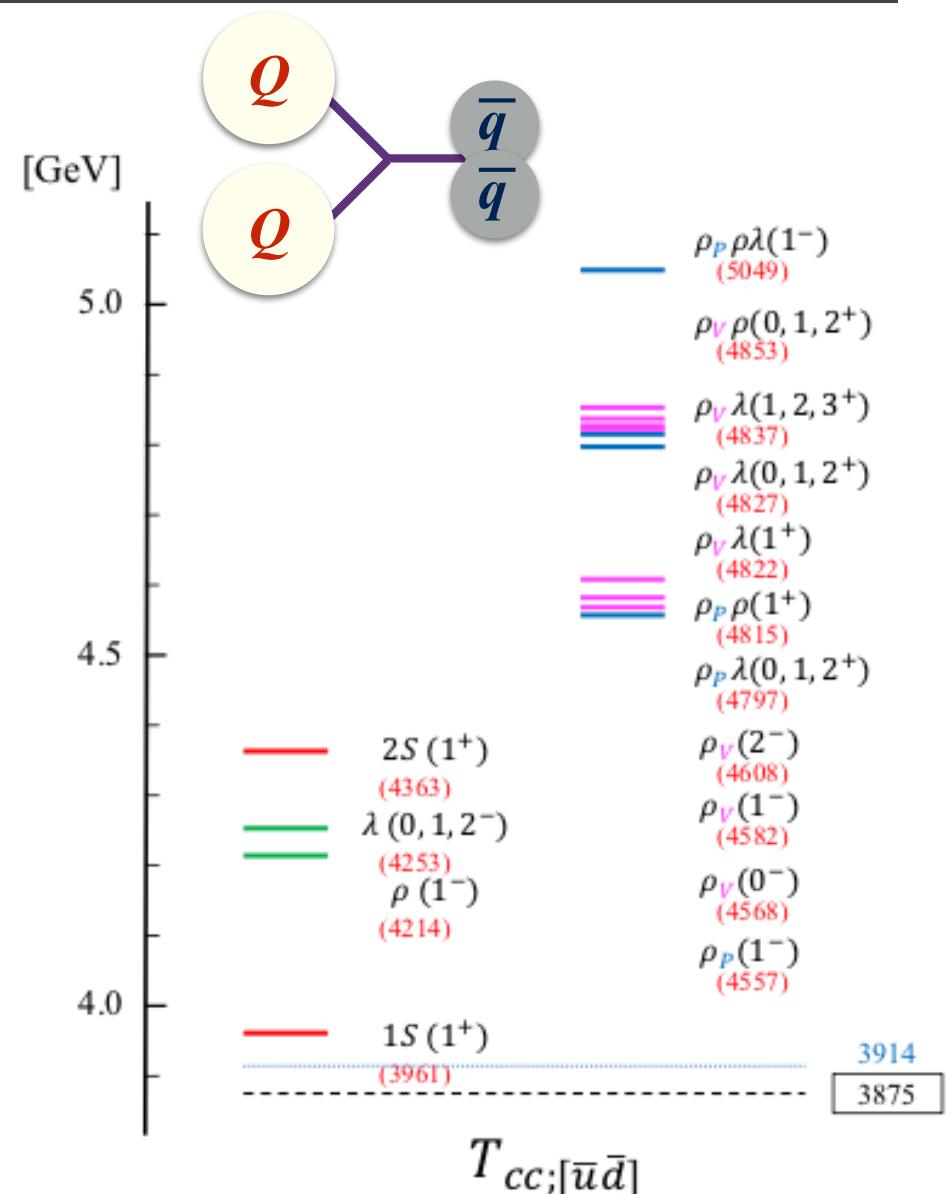
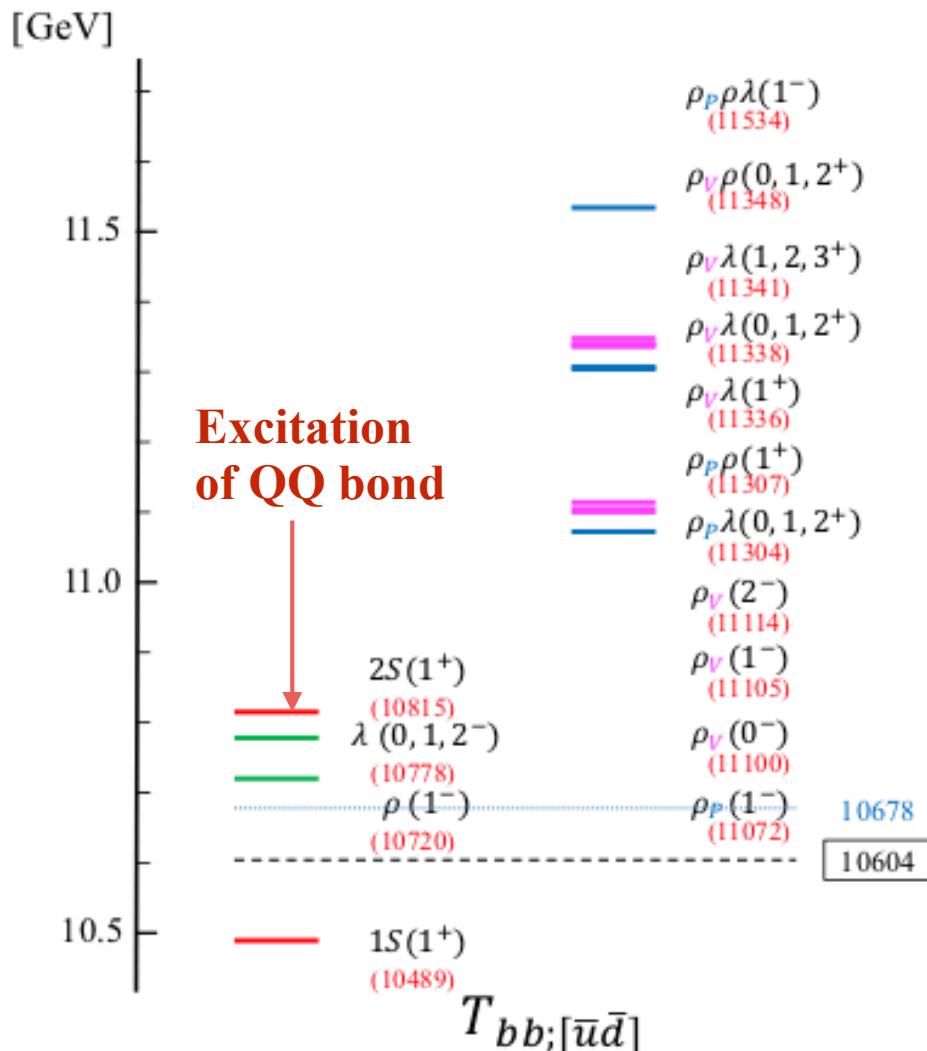
Full Spectrum of Bottom Baryons



Y. Kim, Y.R. Liu, M.O., K. Suzuki, Phys. Rev. D 104, 054012 (2021)

Doubly Heavy Tetraquarks

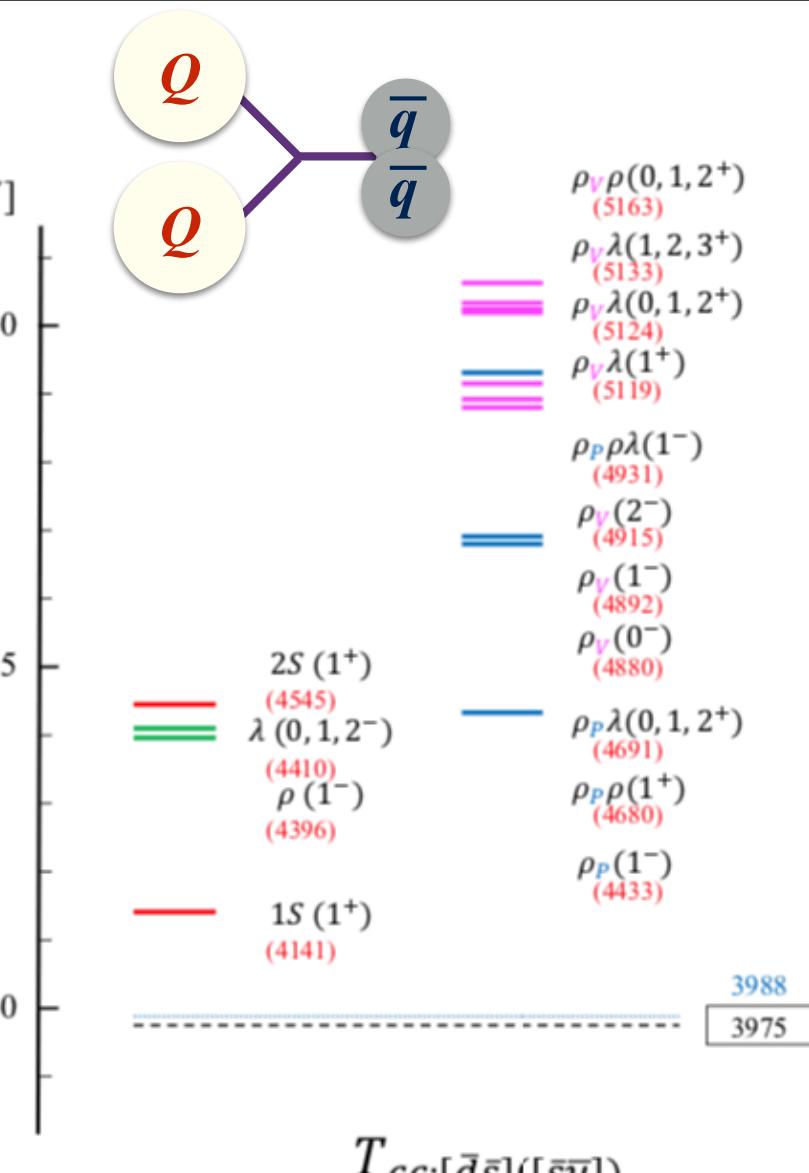
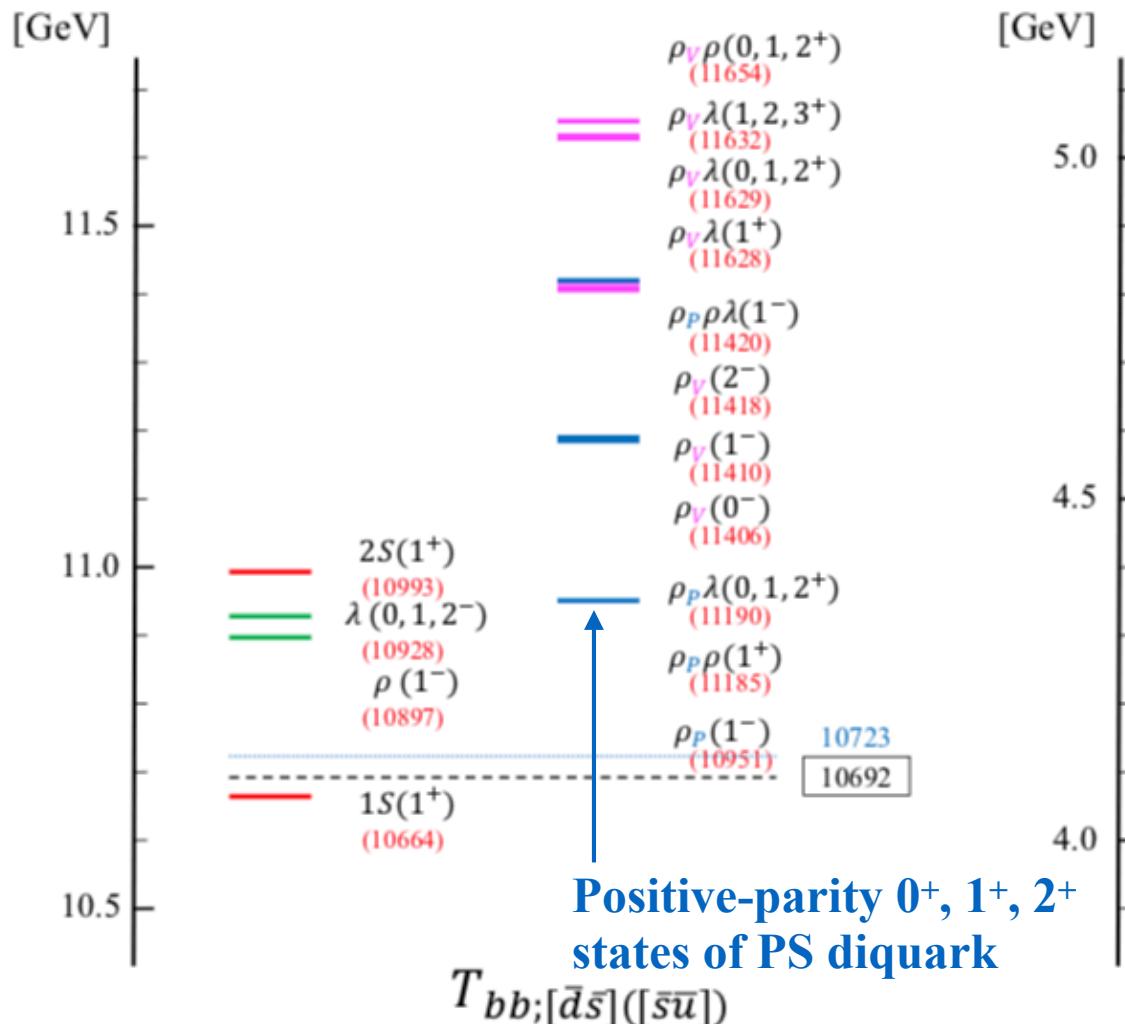
Flavor SU(3) 3 (I=0) states



Y. Kim, M.O., K. Suzuki, *in preparation*

Doubly Heavy Tetraquarks

■ I=1/2 Strange states



Y. Kim, M.O., K. Suzuki, *in preparation*

Comparison: masses of 1S ground states with prior researches

- [1] Q. Meng, E. Hiyama, A. Hosaka, M. Oka, P. Gubler, and K.U. Can, T.T. Takahashi, and H.S. Zong,
“Stable double-heavy tetraquarks: spectrum and structure”, Phys. Lett. B, 814.136095. (2021).
- [2] Eric Braaten, Li-Ping He, and Abhishek Mohapatra,
“Masses of doubly heavy tetraquarks with error bars,” Phys. Rev. D 103, 016001 (2021).
- [3] E.J. Eichten and C. Quigg,
“Heavy-quark symmetry implies stable heavy tetraquark mesons $QQ\bar{q}\bar{q}$ ”, Phys. Rev. Lett. 119, 202002 (2017).
- [4] A. Francis, R. J. Hudspith, R. Lewis and K. Maltman,
“Lattice prediction for Deeply Bound Doubly Heavy Tetraquarks”, Phys. Rev. Lett. 118, 142001 (2017).
- [5] P. Junnarkar, N. Mathur and M. Padmanath
“A study of doubly heavy tetraquarks in Lattice QCD”, Phys. Rev. D 99, 034507 (2019).

⌘ Unit: MeV

Particle ($\mathbf{1}^+$)	$T_{bb;[\bar{u}\bar{d}]}$	$T_{bb;[\bar{d}\bar{s}]([\bar{s}\bar{u}])}$	$T_{cc;[\bar{u}\bar{d}]}$	$T_{cc;[\bar{d}\bar{s}]([\bar{s}\bar{u}])}$
(Threshold)	10604	10692	3876	3977
Ref. [1]	10444 (-160)	10625 (-67)	3865 (-11)	• • •
Ref. [2]	10471 (-133)	10644 (-48)	3947 (+71)	4124 (+147)
Ref. [3]	10482 (-122)	10643 (-49)	3978 (+102)	4156 (+179)
Ref. [4]	10415 (-189)	10594 (-98)	• • •	• • •
Ref. [5]	10461 (-143)	10605 (-87)	3853 (-23)	3969 (-8)
This work	10489 (-115)	10664 (-28)	3961 (+85)	4141 (+164)

Mostly Bound

Bound? Unbound?

Contents

1. Introduction: Tetraquark and Color Confinement
2. Production of T_{cc} in NRQCD: Heavy-Diquark picture
3. Chiral Diquark picture of Tetraquarks
4. Doubly-Heavy Tetraquarks in the Quark Model

Doubly-Heavy Tetraquarks in the Quark Model

Doubly heavy tetraquarks

- # Long-time predictions of bound doubly-heavy tetraquarks (QQ q^{bar}q^{bar}).
Zouzou, Silvestre-Brac, Gignoux, Richard (1986)
Brink, Stancu, (1998)
Karliner, Rosner (2017), Eichten, Quigg (2017)
- # A quark model study of bound and resonance spectrum of DHTQ states is carried out for *two-body confinement plus Coulomb and spin-dependent forces* a la *Silvestre-Brac, Semay, ZP C59, 457 (1993)*.
- # We employ the Gaussian Expansion Method with high precision, and resonances are analyzed by the real scaling method.
Q. Meng, E. Hiyama, A. Hosaka. M.O. et al., PLB 814 (2021) 136095
Q. Meng, M. Harada, E. Hiyama, A. Hosaka. M.O., ArXiv 2106.11868
- # Such a calculation becomes a quark model standard in comparing with experiment as well as lattice QCD calculations. It is also desirable to check other types of multi-quark confinements.

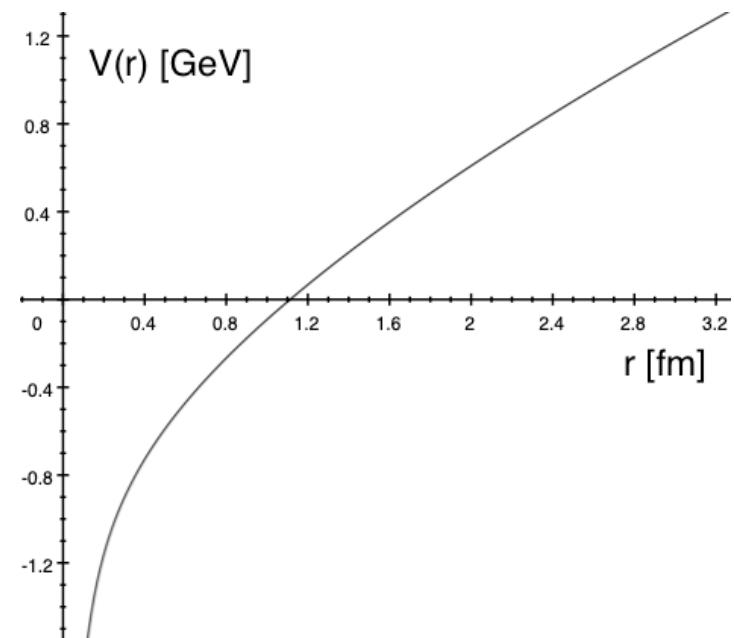
- Hamiltonian
$$H = \sum_i^4 \left(m_i + \frac{\mathbf{p}_i^2}{2m_i} \right) - T_G - \frac{3}{16} \sum_{i < j=1}^4 \sum_a^8 \left((\lambda_i^a \cdot \lambda_j^a) V_{ij}(\mathbf{r}_{ij}) \right)$$

- Semay and Silvestre-Brac potential

$$V_{ij}(\mathbf{r}) = -\frac{\kappa}{r} + \lambda r^p - \Lambda + \frac{2\pi\kappa'}{3m_i m_j} \frac{\exp(-r^2/r_0^2)}{\pi^{3/2} r_0^3} \boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j$$

$$r_0(m_i, m_j) = A \left(\frac{2m_i m_j}{m_i + m_j} \right)^{-B}$$

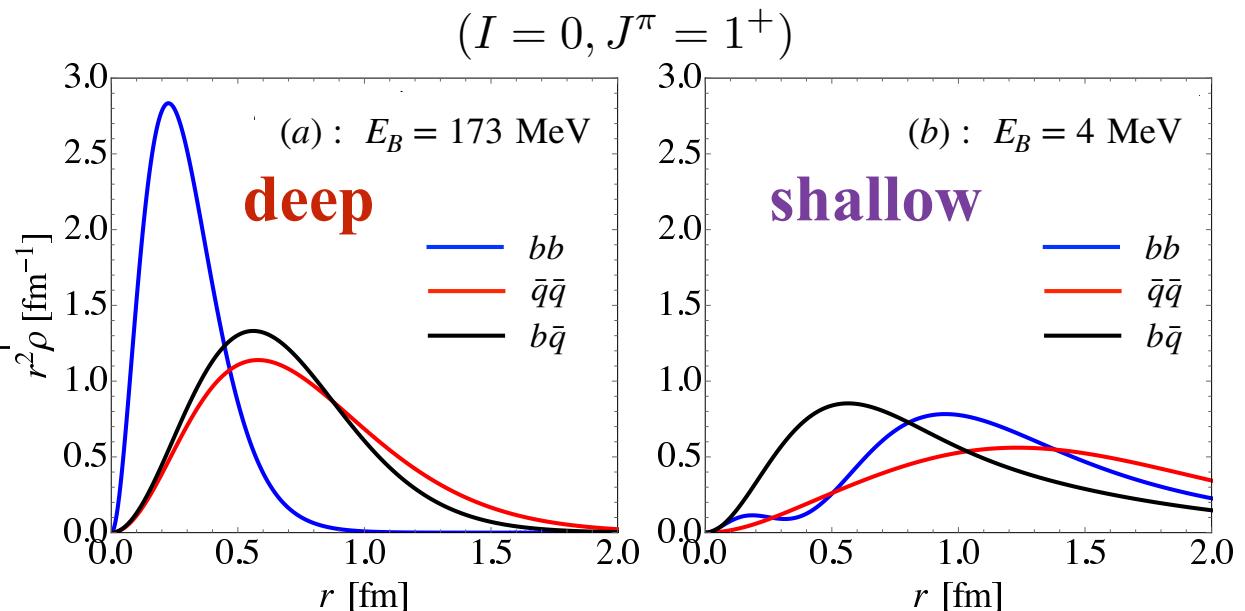
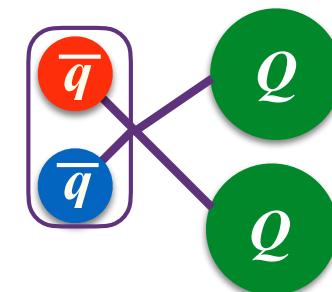
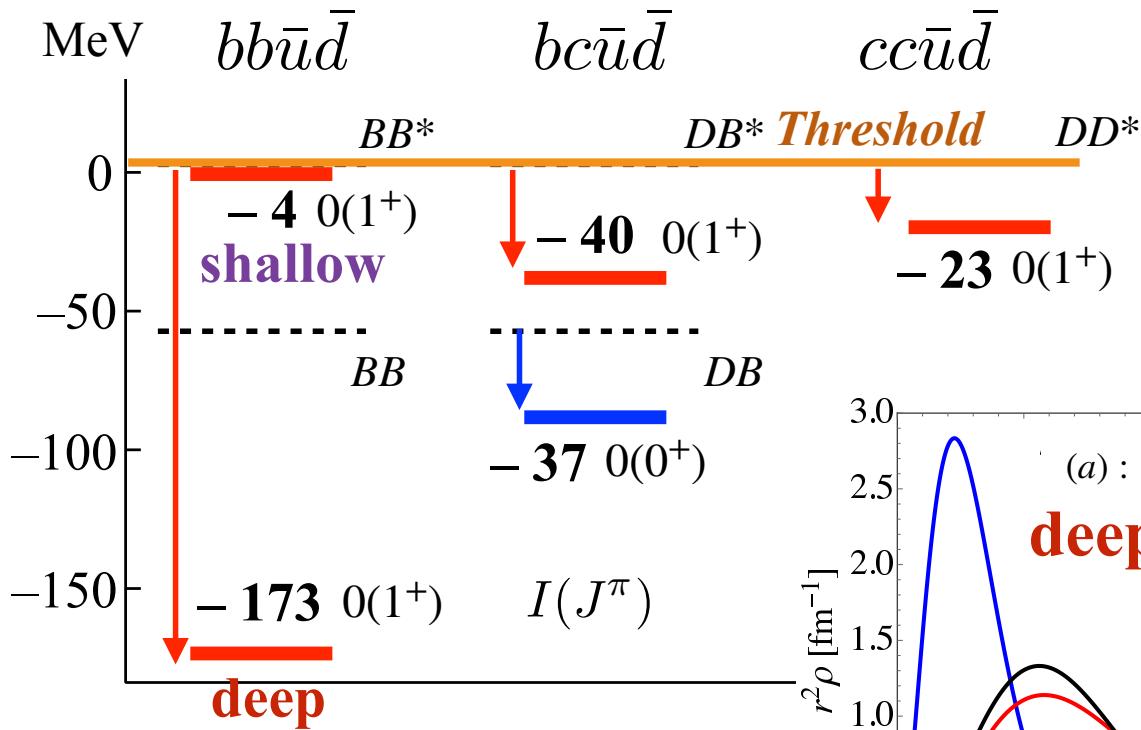
Parameters	Masses (MeV)		
		Cal	Exp
$m_{u,d}$ (GeV)	0.277	$\eta_b(0^-)$	9375
m_s (GeV)	0.593	$\Upsilon(1^-)$	9433
m_c (GeV)	1.826	$\eta_c(0^-)$	2984
m_b (GeV)	5.195	$J/\psi(1^-)$	3102
p	2/3	$B^-(0^-)$	5281
κ	0.4222	$B^{*-}(1^-)$	5336
κ'	1.7925	$B_s(0^-)$	5348
λ (GeV $^{5/3}$)	0.3798	$B_s^*(1^-)$	5410
Λ (GeV)	1.1313	$D^-(0^-)$	1870
A (GeV B^{-1})	1.5296	$D^{*-}(1^-)$	2018
B	0.3263		2010



Doubly heavy tetraquarks

Double Heavy Tetraquarks $Q\bar{Q}u\bar{d}$ ($I=0, J^\pi=1^+$)

Qi Meng et al., Physics Letters B 814 (2021) 136095



Doubly heavy tetraquarks

We have found 7 bound DHTQ states and compared with recent Lattice QCD.

Qi Meng et al., Physics Letters B 814 (2021) 136095

$I(J^P)$	Our work	[1]	[2]	[3]	[4]	[5]
$bb\bar{q}\bar{q}$	0(1^+)	-173	-189 ± 13	-143 ± 34	-	-186 ± 15
$bc\bar{q}\bar{q}$	0(1^+)	-40	-	-	13 ± 3	-
$cc\bar{q}\bar{q}$	0(1^+)	-23	-	-23 ± 11	-	-
$bs\bar{q}\bar{q}$	0(1^+)	-5	-	-	16 ± 2	-
$bb\bar{s}\bar{q}$	$\frac{1}{2}(1^+)$	-59	-98 ± 10	-87 ± 32	-	-
$bb\bar{q}\bar{q}$	1(0^+)	N	-	-5 ± 18	-	-
$bc\bar{q}\bar{q}$	0(0^+)	-37	-	-	17 ± 3	-
$cc\bar{q}\bar{q}$	1(0^+)	N	-	26 ± 11	-	-
$bs\bar{q}\bar{q}$	0(0^+)	-7	-	-	18 ± 2	-

[1] Francis et al. (2017)

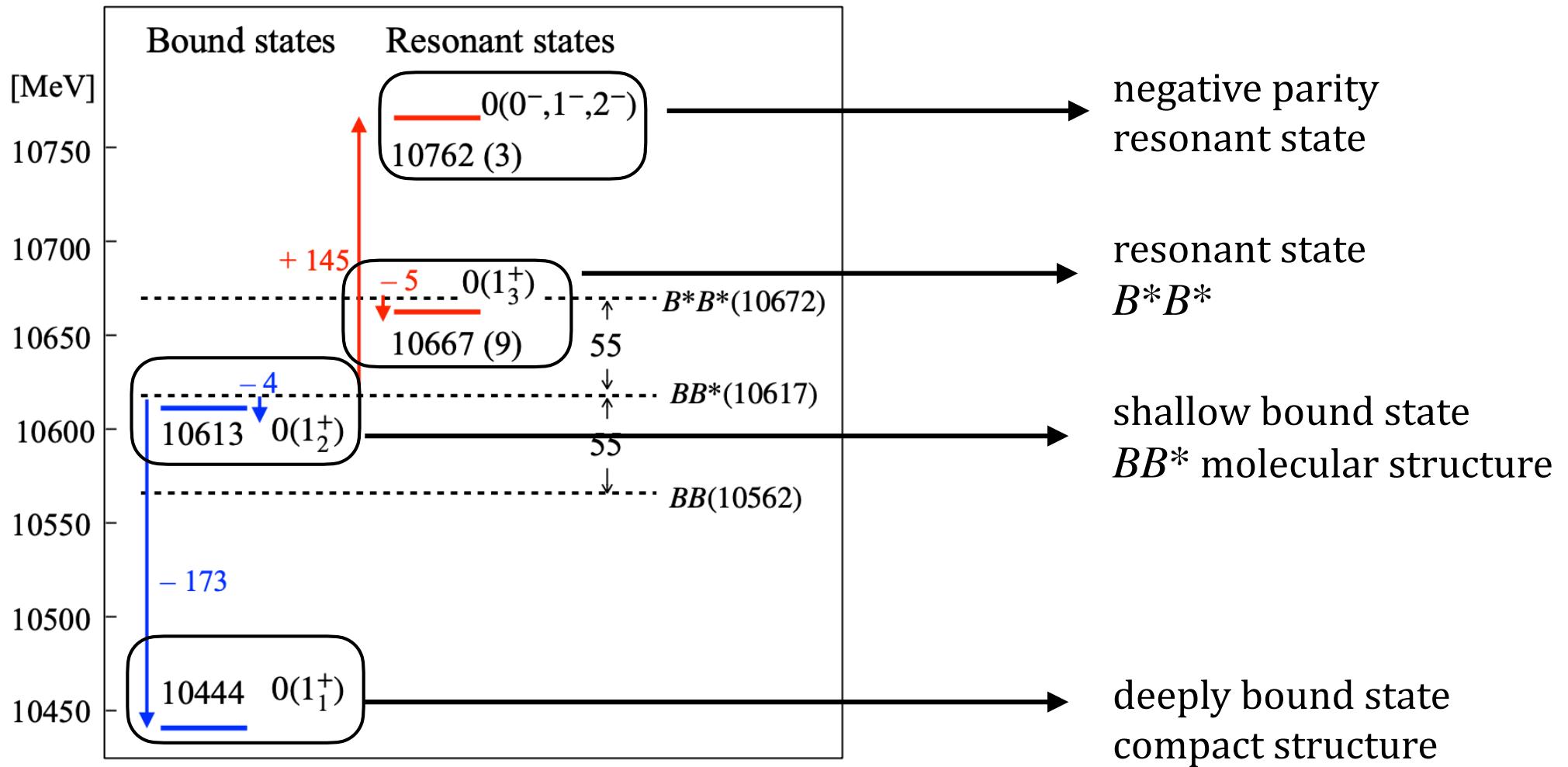
[4] Mohanta et al. (2020)

[2] Junnarkar et al. (2019)

[5] Leskovec et al. (2019)

[3] Hudspith et al. (2020)

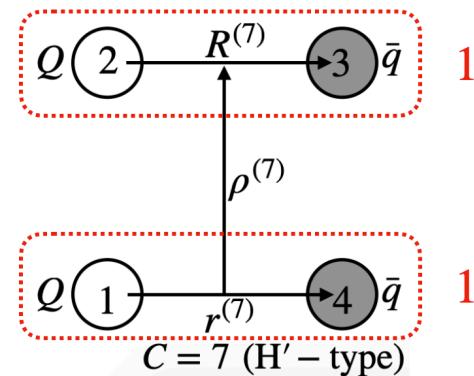
DHTQ resonances



Qi Meng et al., arXiv:2106.11868

Real Scaling (stabilization) method

- scattering channel

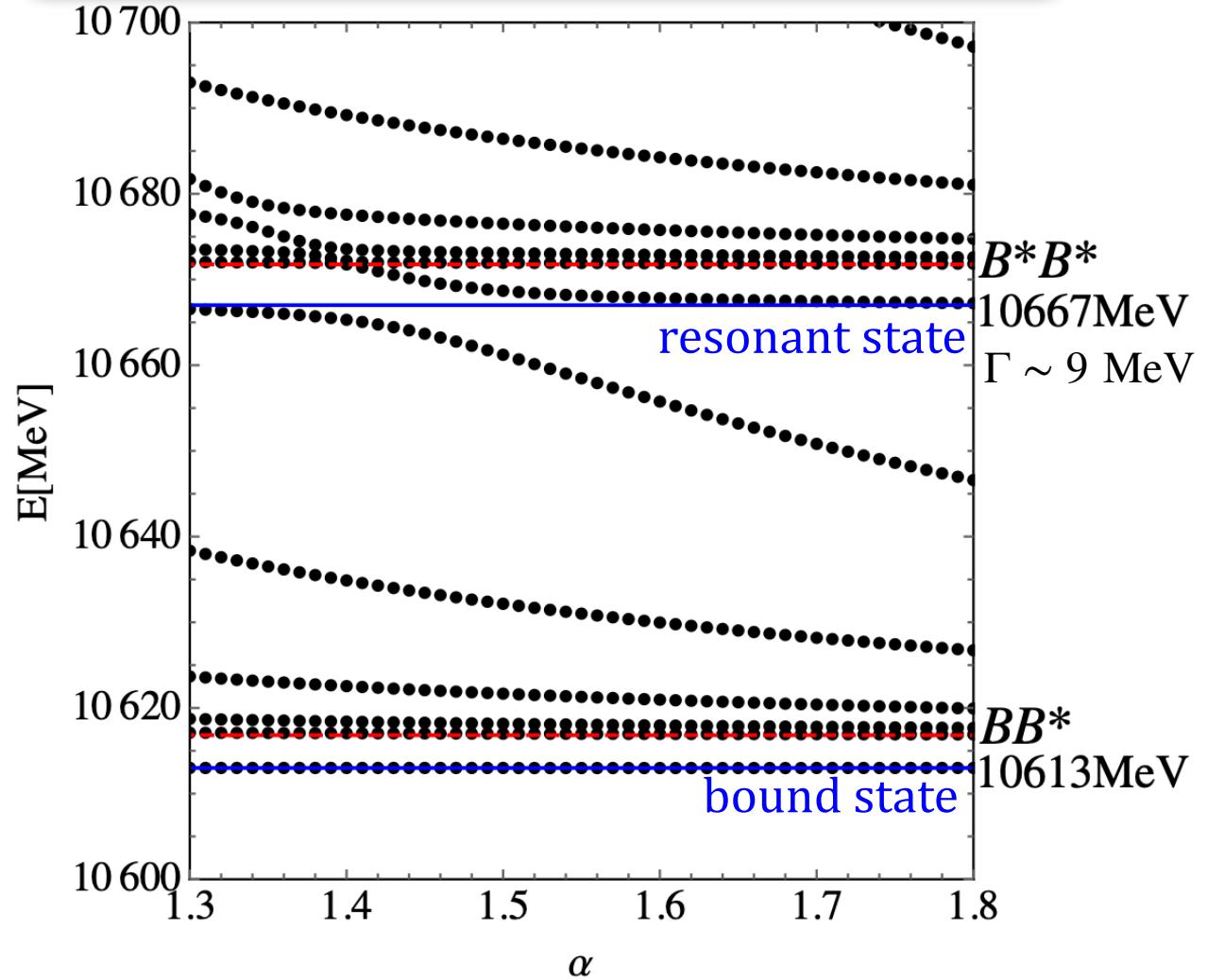


scale $\rho_n^{(c)}$ by multiplying α

$$\rho_n^{(7)} \rightarrow \alpha \rho_n^{(7)}$$

stabilization plot

stabilization plot for $bb\bar{q}\bar{q}$ $I(J^P) = 0(1^+)$



Qi Meng et al., arXiv:2106.11868

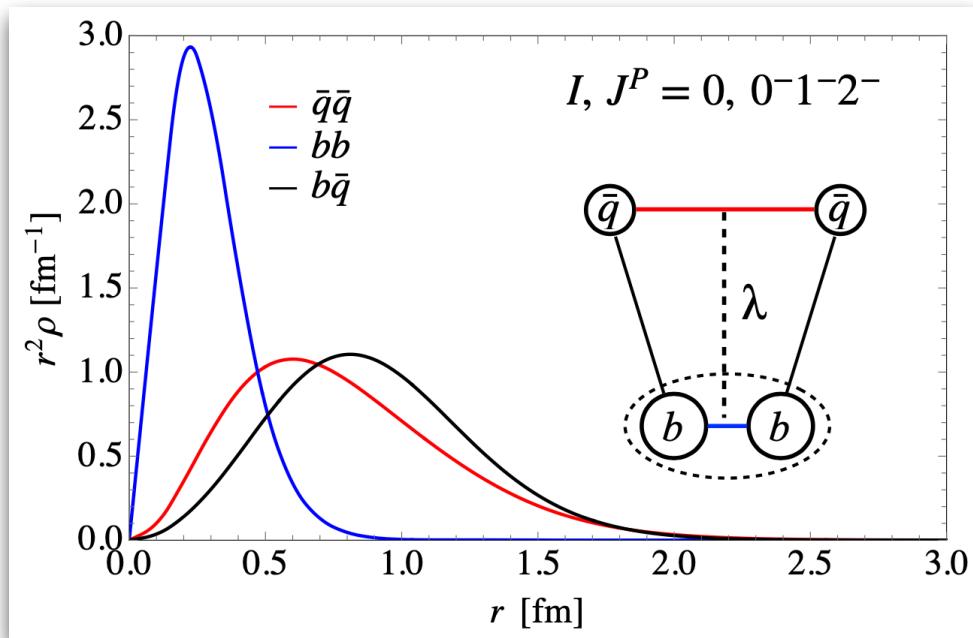
DHTQ resonances

$bb\bar{q}\bar{q}$ tetraquark negative-parity resonances

$E = 10762\text{MeV}$ ($\Gamma \sim 3\text{MeV}$) 200MeV above BB threshold

$I(J^P) = 0(0^-, 1^-, 2^-)$

λ -mode L=1 excitation



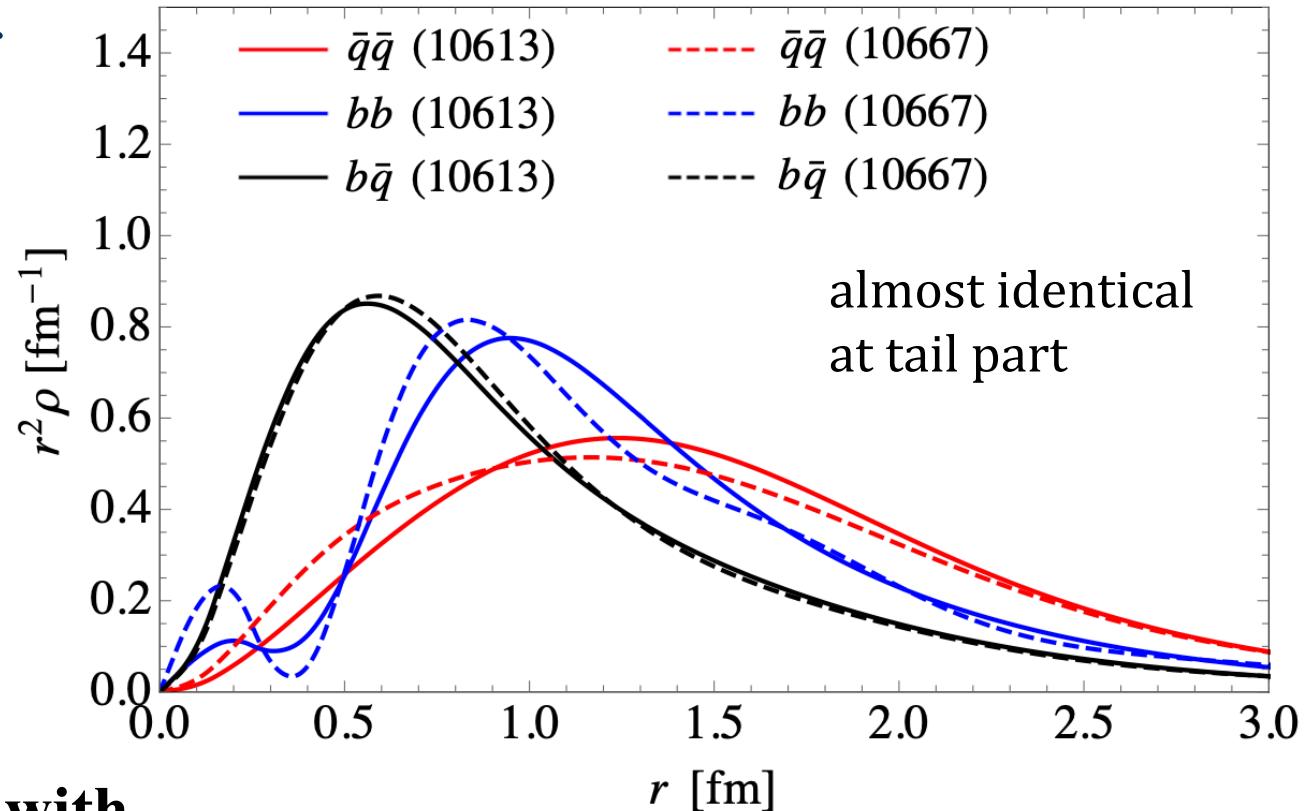
Qi Meng et al., arXiv:2106.11868

DHTQ resonances

Density distribution of

1^+_2 (solid line)

1^+_3 (dash line)



overlap probabilities with
 BB^* and B^*B^*

	$P(B^*B^*)/P(BB^*)$
1^+_2 (10613)	0.02
1^+_3 (10667)	2.40

dominated by BB^*
dominated by B^*B^*

Qi Meng et al., arXiv:2106.11868

DHTQ resonances

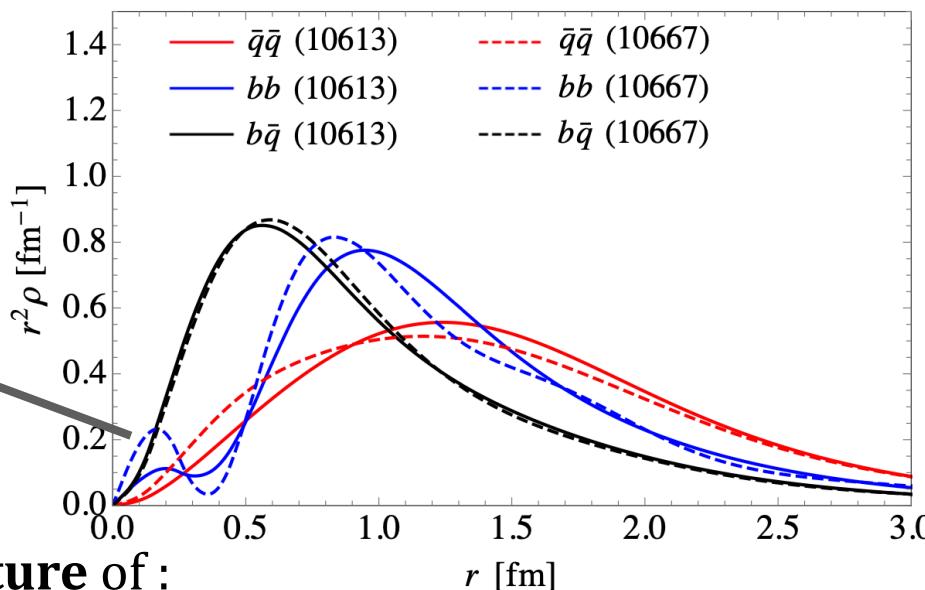
expectation values of color operator $\langle \lambda_{b1} \cdot \lambda_{b2} \rangle$

	$\langle \lambda_{b1} \cdot \lambda_{b2} \rangle$
1^+_2 (10613)	~ 0
1^+_3 (10667)	-0.63

nodal excitation of bb
in 1^+_3

1^+_2 : almost purely BB^*

1^+_3 : contains a significant color $\bar{3} - 3$ at short distance
 $bb(2S)$ combined with ground $\bar{q}\bar{q}$



resonant state 1^+_3 is an **admixture** of :
compact $bb(2S)\bar{q}\bar{q}$ tetraquark and **extended B^*B^* molecular component**

Qi Meng et al., arXiv:2106.11868

Conclusion

- # Doubly Heavy Tetraquarks will bring us with some important information about the confinement of colored quarks, which is not well understood for multi-quark configurations.
- # We have found several bound DHTQ states. T_{bb} ($J^\pi=1^+$, $I=0$) is deepest, while T_{bc} , T_{cc} may also be bound but are shallower.
- # In T_{bb} ($J^\pi=1^+$, $I=0$) channel, we found two bound and one resonance states. The deep ($>100\text{MeV}$) bound state may be a compact tetra-quark state, while the shallow bound and resonance states contain a large fraction of molecular component of $\text{BB}^*/\text{B}^*\text{B}^*$.
- # Chiral diquark picture also predicts a bound T_{bb} , and series of excited states. Excitations of the bb relative motion lie below the excitation of light diquarks, which is consistent with the four-quark calculation.