## Constituent quark model. What have we learned?


J. Vijande PhD

University of Valencia, Spain

## Double charm tetraquarks and other exotics

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## Double charm tetraquarks and other exotics

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First, let me apologize if I do not quote your favorite paper.

## Double charm tetraquarks and other exotics

- And second, I will mostly focus this talk on what can we learn from the constituent quark model approach in the double heavy four-quark sector.
- Topics I will not cover
- QCD sum rules.
- Lattice QCD.
- Dinamically generated resonances.
- Phenomenological mass-based relations.
- etc...


## Double heavy tetraquarks and very little, if any, about other exotics

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- etc...


## The basics



- The constituent quark model have (probably surprisingly) described rather well mesons and baryons as composite objects made of constituent valence quarks

$$
m_{c} \approx 1.3 \mathrm{GeV} \quad m_{b} \approx 5 \mathrm{GeV} \quad m_{u} \approx 0.3 \mathrm{GeV}
$$

interacting by means of a potential, normally pairwise, but not always.

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interacting by means of a potential, normally pairwise, but not always.

- A four-quark state is the simplest object with a non-trivial color structure.



## The basics



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## In the begining. The year 1982

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PHYSICAL REVIEW D VOLUME 25, NUMBER 9 9 Do narrow heavy multiquark states exist?
J.-P. Ader J.-M. Richard $\quad$ P. Taxil
(Received 11 August 1981)

| We discuss the existence of states made of four heavy quarks in the context of poten- |
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PHYSICAL REVIEW D VOLUME 25, NUMBER 9 1 MAY 1982

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We discuss the existence of states made of four heavy quarks in the context of potential models already used in the study of heavy mesons and baryons. [...]
the $\alpha$ particle lies below the threshold for the decay into two deuterons. In quark physics, one of the most important problems today, experimentally and theoretically, is whether or not narrow multiquark states do exist. In this paper we do not in-
$C \overline{C C}$
$C U \overline{C d}$

## In the begining. The year 1982

PHYSICAL REVIEW D VOLUME 25, NUMBER 9 1 MAY 1982

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We discuss the existence of states made of four heavy quarks in the context of potential models already used in the study of heavy mesons and baryons. [...]

The authors consider linear+coulomb and power-law potentials and a variational approach using a harmonic oscillator wave function .

More complex options are included for the all-heavy four-quark states (chromomagnetic interaction, bag model, negative parity states, etc...)

## $C C \overline{C C}$

$c u \bar{c} \bar{d}$

## $c c \bar{u} \bar{d}$

$$
\begin{aligned}
V_{Q \bar{Q}}^{\mathrm{I}}(r) & =-\frac{16}{3} V_{8}^{\mathrm{I}}(r) \\
& =-\frac{4}{3} \frac{\alpha_{s}}{r}+\lambda r,
\end{aligned}
$$

$$
V_{Q \bar{Q}}^{\mathrm{II}}(r)=-\frac{16}{3} V_{8}^{\mathrm{II}}(r)=A+B r^{\beta} .
$$

# In the begining. The year 1982 

PHYSICAL REVIEW D VOLUME 25, NUMBER 9 1 MAY 1982

## Do narrow heavy multiquark states exist?

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of the shape of the confining potential $V_{8}$. Using phenomenological interactions, we found for instance the first $c c \overline{c \bar{c}}$ state around 300 MeV above the threshold made of two charmonia, and the spin-independent corrections do not appreciably reduce this gap.
first threshold is $\psi \chi$ instead of $\eta_{c} \eta_{c}$. Even so, we did not find any narrow $c c \overline{c c} P$ state emerging from our calculation.
our qualitative conclusions are certainly rather general. The cryptoexotic configuration $Q Q^{\prime} \bar{Q} \bar{Q}^{\prime}$, lies above its lowest dissociation threshold $Q \bar{Q}+Q^{\prime} \bar{Q}^{\prime}$. On the other hand, the genuine exotic $Q Q \bar{Q}$ ' $\bar{Q}$ ' can be stable against dissociation if the ratio of the quark masses is large enough. Our predictions

## Exploring numerical methods

Z. Phys. C - Particles and Fields 30, 457-468 (1986)
Four-Quark Bound States
S. Zouzou ${ }^{1}$, B. Silvestre-Brac ${ }^{2}$, C. Gignoux ${ }^{2}$, J.M. Richard ${ }^{3}{ }^{\star}$
${ }^{1}$ Laboratoire de Physique Théorique des Particules Elémentaires, Université Pierre et Marie Curie, F-75230 Paris, France and
Division de Physique Théorique, IPN, F-91406 Orsay, France $_{2}$ Institut des Sciences Nucléairs, F-38026 Grenoble, France
${ }^{3}$ Institut Laue-Langevin, F-38042 Grenoble, France
Received 29 October 1985

The authors search bound states with central forces only, by comparing three methods: a gaussian parametrization of the wave-function, the harmonic oscillator expansion and the hyperspherical expansion. They include spin-spin terms and virtual meson-meson configurations.

## Exploring numerical methods

| Z. Phys. C - Particles and Fields 30, 457-468 (1986) | 2 and Fields <br> (C) Springer-Verlag 1986 |
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| Received 29 October 1985 |  |

seems hardly" "defeatable" in our model. On the other hand, the genuine exotic ( $Q Q \bar{q} \bar{q}$ ) can take advantage of the asymmetry in the quark masses (with e.g. $r(Q Q) \ll r(Q \bar{q}) \simeq r(\bar{q} \bar{q})$ ) and benefit from the strong attraction between the two heavy quarks, whereas in its threshold, $(Q \bar{q})+(Q \bar{q})$, the heavy quarks do not interact together. This is why we consider systems combining various flavours in our search for stable multiquarks.

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The authors search bound states with central forces only, by comparing three methods: a gaussian parametrization of the wave-function, the harmonic oscillator expansion and the hyperspherical expansion. They include spin-spin terms and virtual meson-meson configurations.


Using the Bhaduri potential they identified the S=1 I=0 case as the most promising candidate for a bound state.

## Potentials derived from the MIT Bag model

| PHYSICAL REVIEW D | VOLUME 35, NUMBER 3 | 1 FEBRUARY 1987 |
| :---: | :---: | :---: |
|  | existence of stable di |  |
| L. Heller <br> Theoretical Division, Los Alamos National Laboratory, University of California, Los Alamos, New Mexico 87545 |  |  |
| J. A. Tjon <br> Theoretical Division, Los Alamos National Laboratory, University of California, Los Alamos, New Mexico 87545 and Institute for Theoretical Physics, P.O. Box 80.006, 3508 TA Utrecht, The Netherlands* (Received 11 August 1986) |  |  |
| PHYSICAL REVIEW D | VOLUME 37, NUMBER 3 | 1 FEBRUARY 1988 |
| Stability of dimesons |  |  |
| J. Carlson and L. Heller <br> Theoretical Division, Los Alamos National Laboratory, Los Alamos, New Mexico 87545 |  |  |
|  | J. A. Tjon <br> ysics, P.O. Box $80.006,3508$ TA |  |



The bound-state problem of two- and fourquarks with coupled channels in color space is studied, using a potential derived from the MIT bag model.



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## A systematic analysis

| Z. Phys. C 57, 273-282 (1993) |  |
| :---: | :---: |
| Systematics of $L=0 \boldsymbol{q}^{\mathbf{2}} \overline{\boldsymbol{q}}^{\mathbf{2}}$ systems | Tzing wa Particles and Fields <br> © Springer-Verlag 1993 |
| B. Silvestre-Brac ${ }^{1}$, C. Semay ${ }^{2, \star}$ |  |
| Z. Phys. C 59, 457-470 (1993) |  |
| Spectrum and decay properties of diquonia B. Silvestre-Brac ${ }^{1}$, C. Semay ${ }^{2 . \star}$ | ZEITSCHRIFT FÜR PHYSIK C |
| Z. Phys. C 61, 271-275 (1994) | ZEITSCHRIFT |
| Diquonia and potential models | FÜR PHYSIK C <br> © Springer-Verlag 1994 |
| C. Semay ${ }^{1, \star}$, B. Silvestre-Brac ${ }^{2}$ |  |

Using the interquark potential due to Bhaduri et al., the energies of all $L=0,1,2,3$ four-quark states are calculated for any value of the total $S$ and $I$ and for $\mathrm{q}=\mathrm{u}, \mathrm{d}, \mathrm{s}, \mathrm{c}, \mathrm{b}$ using a harmonic oscillator basis up to $7 / 8$ quanta. Natural parity is considered.

This implies 924 combinations.

## A systematic analysis

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| Nature | $I$ | $J=S$ | $E_{d}(\mathrm{MeV})$ | $\Delta(\mathrm{MeV})$ |
| :--- | :--- | :--- | :---: | :---: |
| $n n \bar{b} \bar{b}$ | 0 | 1 | 10525 | -131 |
| $n s \bar{b} \bar{b}$ | $1 / 2$ | 1 | 10680 | -40 |
| $n n \bar{c} \bar{b}$ | 0 | 1 | 7244 | 1 |
| $n n \bar{c} \bar{b}$ | 0 | 0 | 7206 | 11 |
| $n n \bar{c} \bar{c} \bar{b}$ | 0 | 1 | 3931 | 19 |
| $n n \bar{b} \bar{b}$ | 1 | 2 | 10735 | 30 |
| $n s \bar{b} \bar{b}$ | $1 / 2$ | 2 | 10816 | 48 |
| $n n \bar{s} \bar{c}$ | 0 | 2 | 2975 | 49 |
| $n n \bar{s} \bar{b}$ | 0 | 2 | 6306 | 49 |
| $n n \bar{c} \bar{b}$ | 0 | 2 | 7422 | 49 |
| $n n \bar{n} \bar{n}$ | 0 | 2 | 1605 | 51 |
| $n n \bar{n} \bar{b}$ | $1 / 2$ | 2 | 6181 | 52 |
| $n n \bar{n} \bar{s}$ | $1 / 2$ | 2 | 1734 | 52 |
| $n n \bar{b} \bar{b}$ | 1 | 1 | 10712 | 56 |
| $n s \bar{n} \bar{s} \bar{s}$ | 0,1 | 2 | 1854 | 59 |
| $n s \overline{\bar{c}} \bar{b}$ | $1 / 2$ | 2 | 7496 | 59 | using a harmonic oscillator basis up to $7 / 8$ quanta. Natural parity is considered.

This implies 924 combinations.

## Improving the numerical methods

A different approach based on Gaussian variational wave functions including combinations of three different radial coordinates is considered.

```
PHYSICAL REVIEW D
```

VOLUME 57, NUMBER 11
Tetraquarks with heavy flavors

## D. M. Brink

Dipartimento di Fisica, Università degli Studi di Trento, I-38050 Povo (Trentol, Italy

## Fl. Stancu

Université de Liège, Institut de Physique B5, Sart Tilman, B-4000 Liège 1, Belgium

1 JUNE 1998
$E(q q \bar{b} \bar{b})(\mathrm{MeV})$

| $S I$ | 1 Gaussian | 5 Gaussians | Brac-Semay | Threshold | $E-E_{T}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 10 | 10577.7 | 10558.1 | 10525 | $B+B^{*}$ | -98.9 |
| 01 | 10802.4 | 10766.2 |  | $B+B$ | 156.2 |
| 11 | 10812.1 | 10774.1 | 10712 | $B+B^{*}$ | 117.1 |
| 21 | 10831.5 | 10789.8 | 10735 | $B^{*}+B^{*}$ | 85.8 |

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First detailed study of typical radii and radial properties.


|  | Threshold | $N_{\max }=140$ | Ref. [3] |
| :---: | :---: | :---: | :---: |
| Bhaduri | 3905.3 | 3904.7 | 3931 |
| AL1 | 3878.6 | 3875.9 | 3892 |

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## Exploring constituent quark models



| Eur. Phys. J. C 49, 743-754 (2007) <br> DOI $10.1140 /$ epjc/sl0052-006-0142-1 | THE EUROPEAN |
| :--- | :--- |
| Regular Article - Theoretical Physics | PHYSICAL JOURNAL C |
|  |  |
| Chromomagnetism, flavour symmetry breaking |  |
| and S-wave tetraquarks |  |
| F. Buccella ${ }^{1, \mathrm{a}}$, H. Høgaasen ${ }^{2, \mathrm{~b}}$, , J.-M. Richard ${ }^{3, c}$, P. Sorba ${ }^{4, \mathrm{~d}}$ |  |

These systems were studied with a potential model fitted in the baryon spectrum that includes meson-exchange forces between quarks and entirely neglects the chromomagnetic interaction.

| System | $\left(C_{1}\right)+$ OME | $\left(C_{2}\right)+$ OME | Ref. $\|4\|$ |
| :--- | :--- | :--- | ---: |
| $c c \bar{q} \bar{q}$ | -0.185 | -0.332 | 0.019 |
| $b b \bar{q} \bar{q}$ | -0.226 | -0.497 | -0.135 |

A detailed formalism is presented to fully account for flavoursymmetry breaking in the chromomagnetic interaction together with its application to four-quark systems.

[^0]
## Exploring constituent quark models

| Eur. Phys. J. A 19, 383-389 (2004) DOI 10.1140/epja/i2003-10128-9 | THE EUROPEAN PhYSICAL JOURNAL A |
| :---: | :---: |
| Tetraquarks in a chiral constituent-quark model |  |
|  | vestre-Brac ${ }^{2}$ |

## Exploring constituent quark models

| Eur. Phys. J. A 19/ 383-389 (2004) THE EUROPEAN <br> DOI 10.1140/epia/i2003-10128-9 PHYSICAL JOURNAL A | A different constituent quark model incorporating meson exchanges between light quarks on top of gluon exchange was explored |
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| Tetraquarks in a chiral constituent-quark model |  |

Institute of Physics Publishing Journal of Physics G: Nuclear and Particle Physics
J. Phys. G: Nucl. Part. Phys. 31 (2005) 481-506 doi:10.1088/0954-3899/31/5/017

## Constituent quark model study of the meson spectra

J Vijande, F Fernández and A Valcarce

## Exploring constituent quark models

| Eur. Phys. J. A 19, 383-389 (2004) | THE EUROPEAN |
| :--- | :--- |
| DOI $10.1140 /$ epja/i2003-10128-9 | PHYSICAL JOURNAL A |
|  |  |

Tetraquarks in a chiral constituent-quark model
J. Vijande ${ }^{1, \mathrm{a}}$, F. Fernández ${ }^{1}$, A. Valcarce ${ }^{1}$, and B. Silvestre-Brac ${ }^{2}$

Institute of Physics Publishing
Journal of Physics G: Nuclear and Particle Physics
J. Phys. G: Nucl. Part. Phys. 31 (2005) 481-506
doi:10.1088/0954-3899/31/5/017
A different constituent quark model incorporating meson exchanges between light quarks on top of gluon exchange was explored
A variational method based on a Gaussian expansion was considered.

$$
\begin{aligned}
& R_{s}(1234)=\sum_{i=1}^{n} \beta_{s}^{(i)} R_{s}^{(i)}= \\
& \sum_{i=1}^{n} \beta_{s}^{(i)} e^{-a_{s}^{(i)} \vec{x}^{2}-b_{s}^{(i)} \vec{y}^{2}-c_{s}^{(i)} \vec{z}^{2}-d_{s}^{(i)} \vec{x} \vec{y}-e_{s}^{(i)} \vec{x} \vec{z}-f_{s}^{(i)} \vec{y} \vec{z}}
\end{aligned}
$$

## Constituent quark model study of the meson spectra

## Exploring constituent quark models

| Eur. Phys. J. A 19, 383-389 (2004) | THE EUROPEAN |
| :--- | :--- |
| DOI $10.1140 /$ epja/i2003-10128-9 | PHYSICAL JOURNAL A |
|  |  |

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Journal of Physics G: Nuclear and Particle Physics
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$$
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& \left.\sum_{i=1}^{n} \beta_{s}^{(i)} e^{-a_{s}^{(i)} \vec{x}^{2}-b_{s}^{(i)} \vec{y}^{2}-c_{s}^{(i)} \vec{z}^{2}-d^{(2)} \vec{x} y}-e^{(i)} \vec{x}_{z} z-f^{(i)} \vec{z} z\right)
\end{aligned}
$$

## Constituent quark model study of the meson spectra

## Exploring constituent quark models



## Exploring constituent quark models

[^1]We revisit the same sector using more powerful numerical techniques:

- A variational method using generalized gaussians with all non-diagonal terms (relative $l \neq 0$ )
- A hyperspherical harmonic formalism (up to $K=30$ )


## Exploring constituent quark models

PHYSICAL REVIEW D 76, 094027 (2007)
Are there compact heavy four-quark bound states?
J. Vijande, ${ }^{1}$ E. Weissman, ${ }^{2}$ A. Valcarce, ${ }^{3}$ and N. Barnea ${ }^{2,4}$

PHYSICAL REVIEW D 79, 074010 (2009)
Exotic meson-meson molecules and compact four-quark states

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J. Vijande, ,}\mp@subsup{}{}{1,2}\mathrm{ A. Valcarce, ,}\mp@subsup{}{}{2}\mathrm{ and N. Barnea 3,4
```

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- A variational method using generalized gaussians with all non-diagonal terms (relative $l \neq 0$ )
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## Exploring constituent quark models

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```

| $(L, S, I)$ | Reference [14] | HH $\left(\sum_{i} \ell_{i}=0\right)$ | HH |
| :--- | :---: | :---: | :---: |
| $(0,0,1)$ | 4155 | 4154 | 3911 |
| $(0,1,0)$ | 3927 | 3926 | 3860 |
| $(0,1,1)$ | 4176 | 4175 | 3975 |
| $(0,2,1)$ | 4195 | 4193 | 4031 |

We revisit the same sector using more powerful numerical techniques:

- A variational method using generalized gaussians with all non-diagonal terms (relative $l \neq 0$ )
- A hyperspherical harmonic formalism (up to $K=30$ )



## Exploring constituent quark models

PHYSICAL REVIEW D 76, 094027 (2007)

## Are there compact heavy four-quark bound states?

J. Vijande, ${ }^{1}$ E. Weissman, ${ }^{2}$ A. Valcarce, ${ }^{3}$ and N. Barnea ${ }^{2,4}$

PHYSICAL REVIEW D 79, 074010 (2009)
Exotic meson-meson molecules and compact four-quark states

```
J. Vijande, ,}\mp@subsup{}{}{1,2}\mathrm{ A. Valcarce, ,}\mathrm{ and N. Barnea }\mp@subsup{}{}{3,4
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## Exploring constituent quark models

| Are there <br> J. Vijande, <br> Exotic meson-m | ICAL REVIE pact heav Weissman, ${ }^{2}$ <br> ICAL REVIE n molecul <br> de, ${ }^{1,2}$ A. Val | 76, 09 <br> our-qu <br> Valcarce <br> 79, 074 <br> nd con <br> ${ }^{2}{ }^{2}$ and | (2007) <br> bound states? <br> nd N. Barnea ${ }^{2,4}$ <br> (2009) <br> act four-quark <br> arnea ${ }^{3,4}$ |
| :---: | :---: | :---: | :---: |
| Quark content | $J^{P}(L, S, I)$ | Model | Decay mode |
| $c c \bar{n} \bar{n}$ | $1^{+}(0,1,0)$ | $\begin{aligned} & \mathrm{CQC} \\ & \mathrm{BCN} \end{aligned}$ | Weak <br> Electromagnetic |
| $b b \bar{n} \bar{n}$ | $1^{+}(0,1,0)$ | $\begin{aligned} & \mathrm{CQC} \\ & \mathrm{BCN} \end{aligned}$ | Weak <br> Weak |
|  | $3^{-}(1,2,1)$ | $\begin{aligned} & \mathrm{CQC} \\ & \mathrm{BCN} \end{aligned}$ | Electromagnetic <br> Electromagnetic |
|  | $0^{+}(0,0,0)$ | CQC | Electromagnetic |
|  | $1^{-}(1,0,0)$ | $\begin{aligned} & \mathrm{BCN} \\ & \mathrm{CQC} \end{aligned}$ | Electromagnetic Weak |



## Beyond pairwise interactions

## PHYSICAL REVIEW D 76, 114013 (2007)

## Stability of multiquarks in a simple string model

J. Vijande, ${ }^{1,2, *}$ A. Valcarce, ${ }^{2, \uparrow}$ and J.-M. Richard ${ }^{3, *}$


Flip-Flop model

$$
\begin{aligned}
& V_{f}=\lambda \min \left(r_{13}+r_{24}, r_{23}+r_{14}\right) \\
& V_{s}=\min \left(V_{f}, V_{b}\right) . \\
& V_{b}=\lambda \min _{k, \ell}\left(r_{1 k}+r_{2 k}+r_{k \ell}+r_{\ell 3}+r_{\ell 4}\right)
\end{aligned}
$$

Butterfly model


## Beyond pairwise interactions

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## Beyond pairwise interactions

## PHYSICAL REVIEW D 94, 094032 (2016)

Tetraquark bound states and resonances in a unitary microscopic quark model: A case study of bound states of two light quarks and two heavy antiquarks

```
P. Bicudo \({ }^{*}\) and M. Cardoso \({ }^{\dagger}\)
```

> To summarize, we obtain tetraquark bound states on the $q q \bar{b} \bar{b}$ system, with quantum numbers $0^{+}$for $s$ and $c$ quarks, or light quarks with $I_{12}=1$. For light quarks with $I_{12}=0$, we obtain bound states with quantum numbers $1^{+}$.
> We also tried to find bound states for the $q q \bar{c} \bar{c}$ system, but we were unable to find them when the lightest quarks have constituent masses equal to or larger than the ones of light quarks $m_{q} \geq 400 \mathrm{MeV}$.


The ground state potential for a system composed of two quarks and two antiquarks is well fitted by a string flipflop potential.


## What about the probabilities?

| PHYSICAL REVIEW C 80, 035204 (2009) |
| :---: |
| Probabilities in nonorthogonal bases: Four-quark systems |
| J. Vijande $^{1}$ and A. Valcarce ${ }^{2}$ |

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\begin{aligned}
& {\left[\left(q_{1} q_{2}\right)\left(\bar{q}_{3} \bar{q}_{4}\right)\right] \equiv\left\{\left|\overline{3}_{12} 3_{34}\right\rangle,\left|6_{12} \overline{6}_{34}\right\rangle\right\} \equiv\left\{|\overline{3} 3\rangle_{c}^{12},|6 \overline{6}\rangle_{c}^{12}\right\}} \\
& {\left[\left(q_{1} \bar{q}_{3}\right)\left(q_{2} \bar{q}_{4}\right)\right] \equiv\left\{\left|1_{13} 1_{24}\right\rangle,\left|8_{13} 8_{24}\right\rangle\right\} \equiv\left\{|11\rangle_{c},|88\rangle_{c}\right\}} \\
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\end{aligned}
$$

Physical interpretation

Journal of Mathematical Chemistry 5(1990)323-357

THEORY OF PROJECTED PROBABILITIES ON NON-ORTHOGONAL STATES: APPLICATION TO ELECTRONIC POPULATIONS IN MOLECULES
R.S. MANNING* and N. De LEON ${ }^{\star}{ }^{\dagger}$

$$
\begin{aligned}
|\Psi\rangle & =\alpha|11\rangle_{c}+\beta|88\rangle_{c} \\
& =\alpha|11\rangle_{c}+\beta\left(\chi\left|1^{\prime} 1^{\prime}\right\rangle_{c}+\delta\left|8^{\prime} 8^{\prime}\right\rangle_{c}\right) \\
& =\alpha|11\rangle_{c}+\gamma\left|1^{\prime} 1^{\prime}\right\rangle_{c}+\mu\left|8^{\prime} 8^{\prime}\right\rangle_{c} \\
& =\alpha|11\rangle_{c}+\gamma\left|1^{\prime} 1^{\prime}\right\rangle_{c}+\mu\left(\varpi|11\rangle_{c}+\varsigma|88\rangle_{c}\right) \\
& =\ldots . . .=\wp_{\{11\rangle_{c}}|\Psi\rangle+\wp_{\left[11^{\prime}\right\rangle_{c}}|\Psi\rangle
\end{aligned}
$$

## What about the probabilities?

## PHYSICAL REVIEW C 80, 035204 (2009) <br> Probabilities in nonorthogonal bases: Four-quark systems

J. Vijande ${ }^{1}$ and A. Valcarce ${ }^{2}$


| $\left(S_{T}, I\right)$ | $(0,1)$ | $(1,1)$ | $(1,0)$ | $(1,0)$ | $(0,0)$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Flavor | $c c \bar{n} \bar{n}$ | $c c \bar{n} \bar{n}$ | $c c \bar{n} \bar{n}$ | $b b \bar{n} \bar{n}$ | $b b \bar{n} \bar{n}$ |
| Energy | 3877 | 3952 | 3861 | 10395 | 10948 |
| Threshold | $\left.D D\right\|_{S}$ | $\left.D D^{*}\right\|_{S}$ | $\left.D D^{*}\right\|_{S}$ | $\left.B B^{*}\right\|_{S}$ | $\left.B_{1} B\right\|_{P}$ |
| $\Delta_{E}$ | +5 | +15 | -76 | -217 | -153 |
| $P\left[\|\overline{3}\rangle_{c}^{12}\right]$ | 0.333 | 0.333 | 0.881 | 0.974 | 0.981 |
| $P\left[\|6 \overline{6}\rangle_{c}^{12}\right]$ | 0.667 | 0.667 | 0.119 | 0.026 | 0.019 |
| $P\left[\|11\rangle_{c}\right]$ | 0.556 | 0.556 | 0.374 | 0.342 | 0.340 |
| $P\left[\|88\rangle_{c}\right]$ | 0.444 | 0.444 | 0.626 | 0.658 | 0.660 |
| $P_{M M}$ | 1.000 | - | - | - | 0.254 |
| $P_{M M^{*}}$ | - | 1.000 | 0.505 | 0.531 | - |
| $P_{M^{*} M^{*}}$ | 0.000 | 0.000 | 0.495 | 0.469 | 0.746 |
| $\left\langle x^{2}\right\rangle^{1 / 2}$ | 60.988 | 13.804 | 0.787 | 0.684 | 0.740 |
| $\left\langle y^{2}\right\rangle^{1 / 2}$ | 60.988 | 13.687 | 0.590 | 0.336 | 0.542 |
| $\left\langle z^{2}\right\rangle^{1 / 2}$ | 0.433 | 0.617 | 0.515 | 0.503 | 0.763 |
| $R M_{4 q}$ | 30.492 | 6.856 | 0.363 | 0.217 | 0.330 |
| $\Delta_{R}$ | 69.300 | 11.640 | 0.799 | 0.700 | 0.885 |

## What about the probabilities?

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## PHYSICAL REVIEW C 80, 035204 (2009) <br> Probabilities in nonorthogonal bases: Four-quark systems

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## Compact or meson-meson configuration?

Physics Letters B 699 (2011) 291-295
Doubly charmed exotic mesons: A gift of nature?
T.F. Caramés ${ }^{\text {a }}$, A. Valcarce ${ }^{\text {a,* }}$, J. Vijande ${ }^{\text {b }}$

$$
|c c \bar{n}\rangle=\alpha_{1}|\overline{3} 3\rangle+\ldots+\alpha_{2}|6 \overline{6}\rangle+\ldots ? ?
$$

In this work the meson-meson configuration is solved by means of the Lippmann-Schwinger equation using the same interaction as the four-quark problem.


## Few-body dynamics

## PHYSICAL REVIEW C 97, 035211 (2018)

Few-body quark dynamics for doubly heavy baryons and tetraquarks
Jean-Marc Richard, ${ }^{1, *}$ Alfredo Valcarce, ${ }^{2, \dagger}$ and Javier Vijande ${ }^{3, \ddagger}$

A very delicate interplay between color and spin configurations.


## Few-body dynamics

## PHYSICAL REVIEW C 97, 035211 (2018)

## Few-body quark dynamics for doubly heavy baryons and tetraquarks

Jean-Marc Richard, ${ }^{1,{ }^{*}}$ Alfredo Valcarce, ${ }^{2, \dagger}$ and Javier Vijande ${ }^{3, \ddagger}$

The treatment of the four-body dynamics for double-charm tetraquarks is discussed. The variational and Born-Oppenheimer approximations together with the Hall-Post inequalities give energies very close to the exact ones, while the diquark approximation might be more problematic.


## Will the relativistic kinematics increase the number of stable multiquarks?

PHYSICAL REVIEW D 102, 034012 (2020)

Masses of doubly heavy tetraquarks $T_{Q Q^{\prime}}$ in a relativized quark model Qi-Fang Lü®, ${ }^{1,2,3, *}$ Dian-Yong Chen, ${ }^{4, \dagger}$ and Yu-Bing Dong ${ }^{5,6,7,{ }^{*}}$


The authors investigate the mass spectra using the relativized quark model proposed by Godfrey, Capstick, and Isgur.

The spatial wave function is expanded in terms of a set of Gaussian basis functions where the Gaussian size parameters are taken in geometric progression



## Will the relativistic kinematics increase the number of stable multiquarks?

## PHYSICAL REVIEW D 103, 054020 (2021)

Effect of relativistic kinematics on the stability of multiquarks Jean-Marc Richard $\odot^{*}$ Alfredo Valcarce $\odot^{\dagger}$ Javier Vijande $\odot^{\ddagger}$

In this case the threshold is made of two ( $q Q$ ) mesons while in the four-quark state there are $(q q),(Q Q)$ and four $(q Q)$ interactions. Who will benefit more from the relativistic dynamics?

We consider the AL1 potential properly re-parametrized in the SR case for keeping the description of the meson spectra.


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## Conclusions

- The constituent quark model predicts a clear bound state, $b b \bar{u} \bar{d}$, and another one, $c c \bar{u} \bar{d}$, just below threshold with $(I) J^{P}=(0) 1^{+}$. Some particular models may point to the existence of about five more bound states.
- There is not an overwhelming abundance of bound states within the constituent quark model.
- The numerical methods required should be able to handle short- and long-range correlations, i.e. a meson-meson structures together with a more clusterized behaviour.
- Approximations and simplifications in the colour-spin structure should be done carefully.
- We should double check whether our findings are entirely due to our hypothesis and aproximations before extracting any general conclusion.


[^0]:    For ( $Q Q \bar{q} \bar{q}$ ) with identical heavy quarks, the chromomagnetic interaction is optimal for $J^{P}=1^{+}$, since the Pauli principle forbids the $0^{+}$eigenstates with the lowest eigenvalue of $H_{\mathrm{CM}} \cdot[\ldots]$
    [...] The very large value of the mass ratio $M_{Q} / m_{n}$, where $M_{Q}^{-1}$ is the average of the inverse masses $m_{c}$ and $m_{b},{ }^{3}$ presumably gives binding or almost binding from the sole chromoelectric effects. The chromomagnetic interaction is also favourable, and, if alone, would give a binding of more than 100 MeV .

[^1]:    PHYSICAL REVIEW D 76, 094027 (2007)
    Are there compact heavy four-quark bound states?
    J. Vijande, ${ }^{1}$ E. Weissman, ${ }^{2}$ A. Valcarce, ${ }^{3}$ and N. Barnea ${ }^{2,4}$

    PHYSICAL REVIEW D 79, 074010 (2009)
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    \text { J. Vijande, }{ }^{1,2} \text { A. Valcarce, }{ }^{2} \text { and N. Barnea }{ }^{3,4}
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