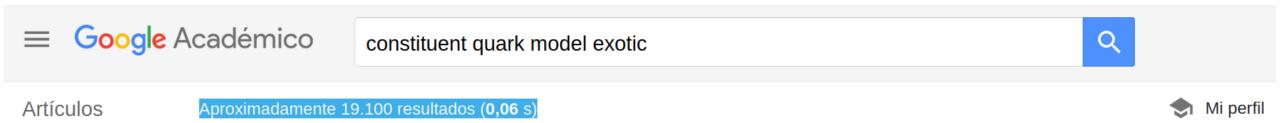
Constituent quark model. What have we learned?

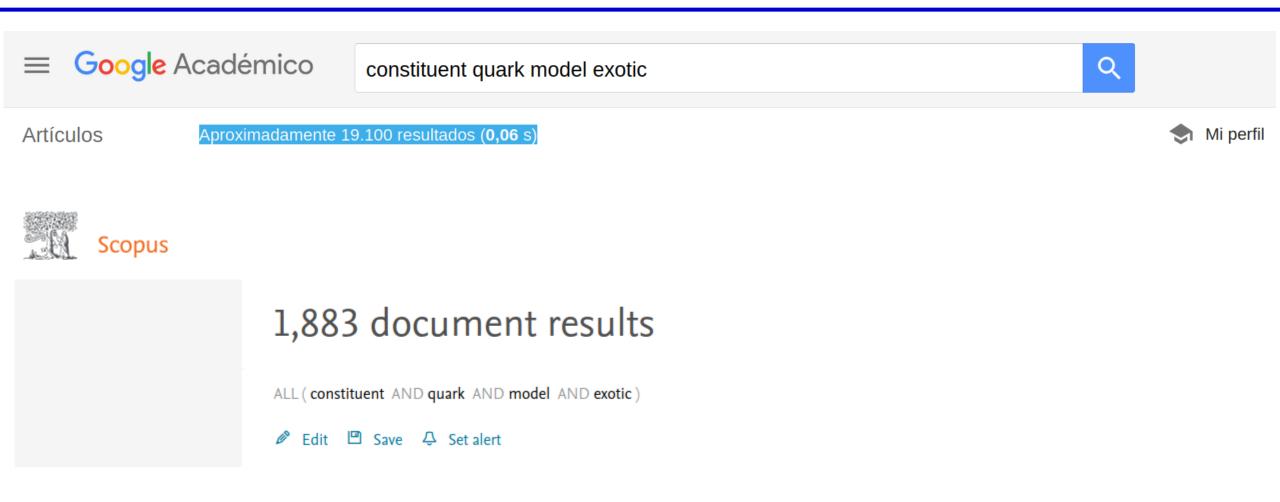


J. Vijande PhD University of Valencia, Spain

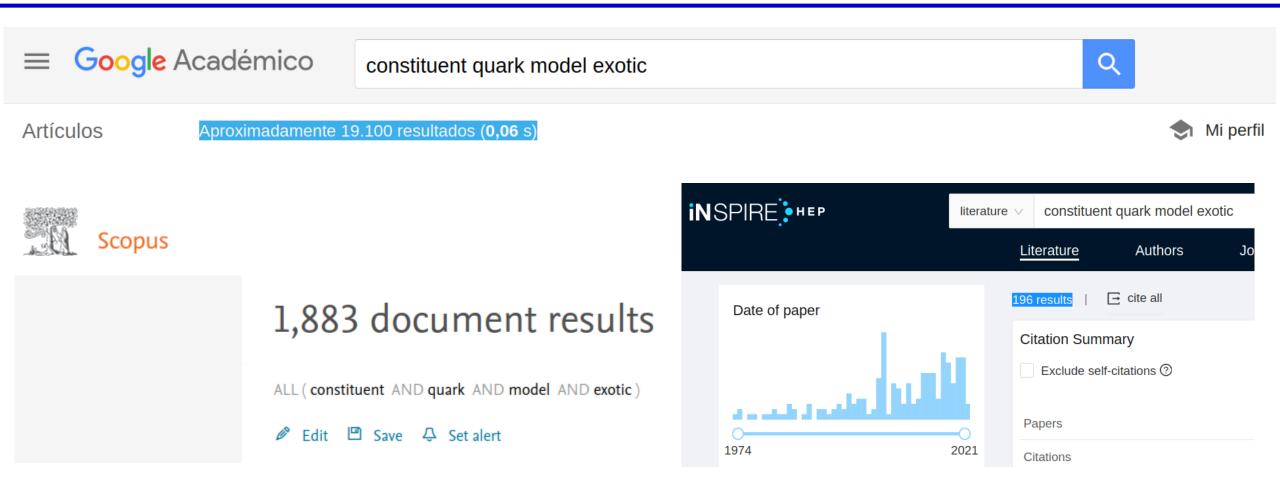




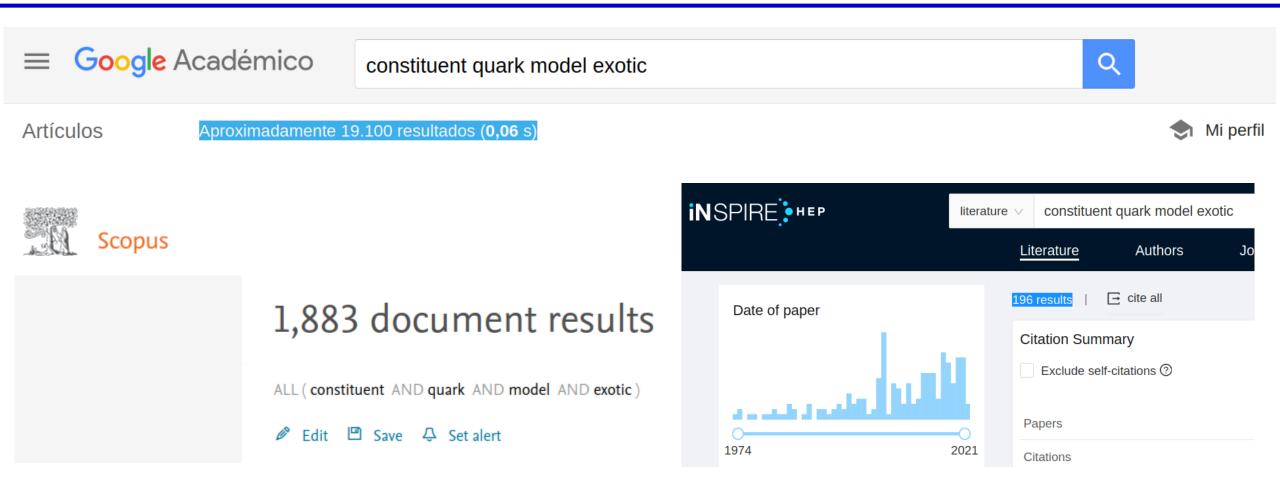












First, let me apologize if I do not quote your favorite paper.



- And second, I will mostly focus this talk on what can we learn from the constituent quark model approach in the double heavy four-quark sector.
- Topics I will not cover
 - QCD sum rules.
 - Lattice QCD.
 - Dinamically generated resonances.
 - Phenomenological mass-based relations.
 - etc...

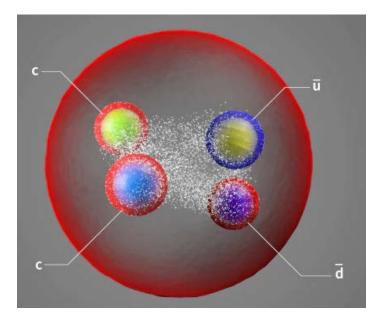


Double heavy tetraquarks and very little, if any, about other exotics

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The basics

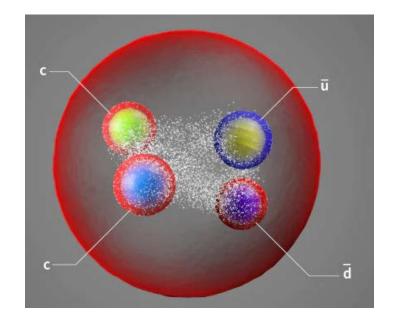


• The constituent quark model have (probably surprisingly) described rather well mesons and baryons as composite objects made of constituent valence quarks

$$m_c \approx 1.3 \, GeV$$
 $m_b \approx 5 \, GeV$ $m_u \approx 0.3 \, GeV$

interacting by means of a potential, normally pairwise, but not always.

The basics

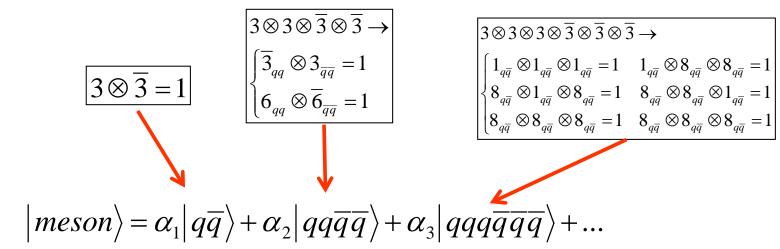


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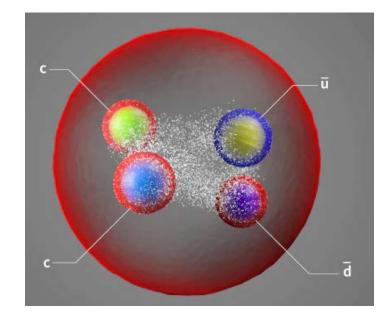
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• A four-quark state is the simplest object with a non-trivial color structure.



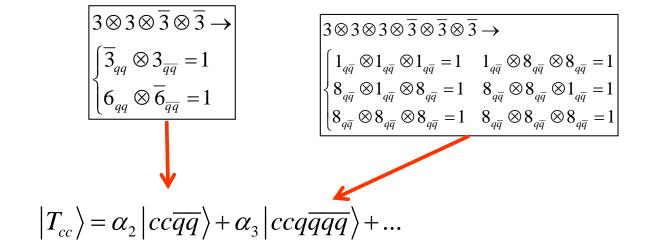
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But for charm equal $\pm 2 \rightarrow$

























PHYSICAL REVIEW D

VOLUME 25, NUMBER 9

1 MAY 1982

Do narrow heavy multiquark states exist?

J.-P. Ader J.-M. Richard P. Taxil

(Received 11 August 1981)

We discuss the existence of states made of four heavy quarks in the context of potential models already used in the study of heavy mesons and baryons.[...]

 $cc\overline{cc}$

cucd

 $cc\overline{u}\overline{d}$



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the α particle lies below the threshold for the decay into two deuterons. In quark physics, one of the most important problems today, experimentally and theoretically, is whether or not narrow multiquark states do exist. In this paper we do not in-

 $cc\overline{cc}$

 $cu\overline{c}\overline{d}$

 $cc\overline{u}\overline{d}$



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We discuss the existence of states made of four heavy quarks in the context of potential models already used in the study of heavy mesons and baryons.[...]

The authors consider **linear+coulomb** and **power-law** potentials and a **variational approach** using a harmonic oscillator wave function .

More complex options are included for the all-heavy four-quark states (chromomagnetic interaction, bag model, negative parity states, etc...)

$$cc\overline{cc}$$
 $cu\overline{c}\overline{d}$

$$cc\overline{u}\overline{d}$$

$$V_{Q\bar{Q}}^{I}(r) = -\frac{16}{3} V_{8}^{I}(r)$$

= $-\frac{4}{3} \frac{\alpha_{s}}{r} + \lambda r$,
 $V_{Q\bar{Q}}^{II}(r) = -\frac{16}{3} V_{8}^{II}(r) = A + Br^{\beta}$.

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 $cc\overline{u}\overline{d}$

of the shape of the confining potential V_8 . Using phenomenological interactions, we found for instance the first $cc\bar{c}c$ state around 300 MeV above the threshold made of two charmonia, and the spin-independent corrections do not appreciably reduce this gap.

first threshold is $\psi \chi$ instead of $\eta_c \eta_c$. Even so, we did not find any narrow $cc\bar{c}c$ P state emerging from our calculation.

our qualitative conclusions are certainly rather general. The cryptoexotic configuration $QQ'\bar{Q}\bar{Q}'$, lies above its lowest dissociation threshold $Q\bar{Q} + Q'\bar{Q}'$. On the other hand, the genuine exotic $QQ\bar{Q}'\bar{Q}'$ can be stable against dissociation if the ratio of the quark masses is large enough. Our predictions



Z. Phys. C - Particles and Fields 30, 457-468 (1986)



Four-Quark Bound States

S. Zouzou¹, B. Silvestre-Brac², C. Gignoux², J.M. Richard³*

- Laboratoire de Physique Théorique des Particules Elémentaires, Université Pierre et Marie Curie, F-75230 Paris, France and Division de Physique Théorique, IPN, F-91406 Orsay, France
- ² Institut des Sciences Nucléaires, F-38026 Grenoble, France
- 3 Institut Laue-Langevin, F-38042 Grenoble, France

Received 29 October 1985

The authors search bound states with central forces only, by comparing **three methods**: a gaussian parametrization of the wave-function, the harmonic oscillator expansion and the hyperspherical expansion. They include **spin-spin** terms and virtual **meson-meson configurations**.



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seems hardly "defeatable" in our model. On the other hand, the genuine exotic $(QQ\bar{q}\bar{q})$ can take advantage of the asymmetry in the quark masses (with e.g. $r(QQ) \ll r(Q\bar{q}) \simeq r(\bar{q}\bar{q})$) and benefit from the strong attraction between the two heavy quarks, whereas in its threshold, $(Q\bar{q})+(Q\bar{q})$, the heavy quarks do not interact together. This is why we consider systems combining various flavours in our search for stable multiquarks.



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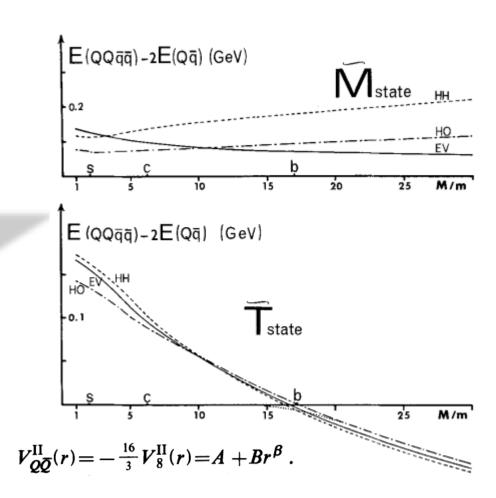


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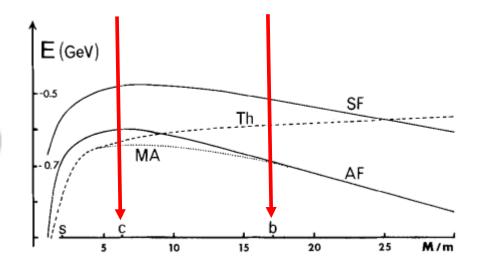


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Using the Bhaduri potential they identified the S=1 I=0 case as the most promising candidate for a bound state.

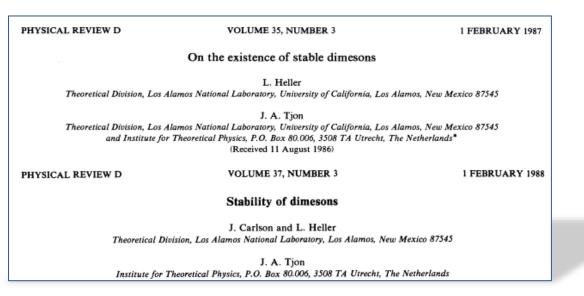


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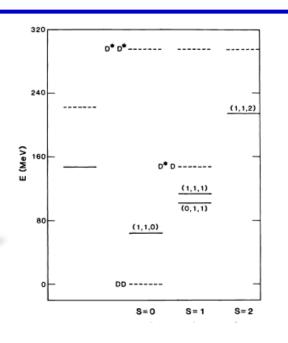
Potentials derived from the MIT Bag model

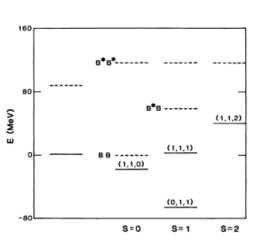


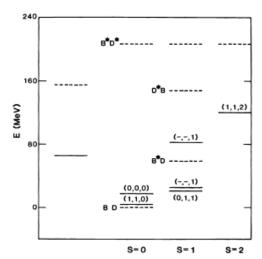
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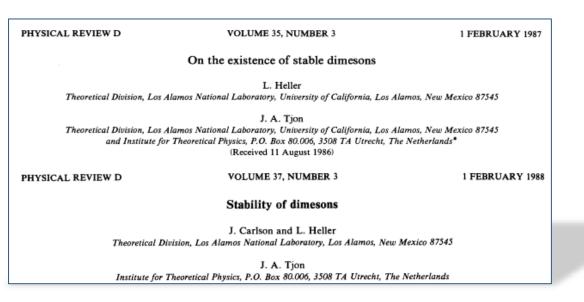








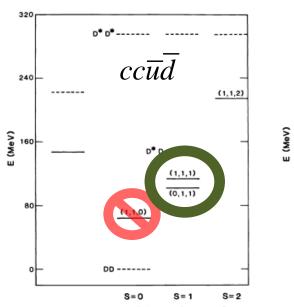
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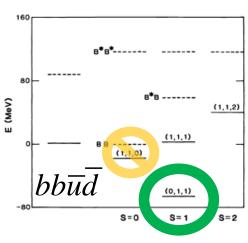


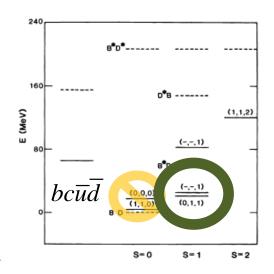
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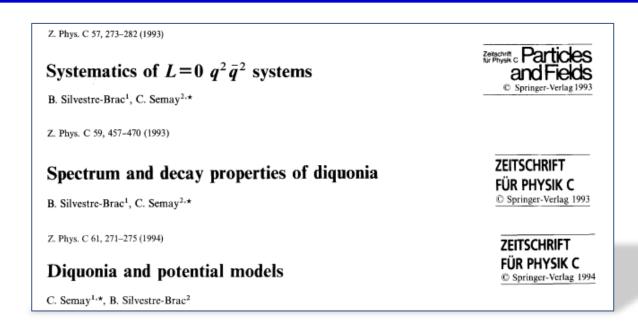








A systematic analysis

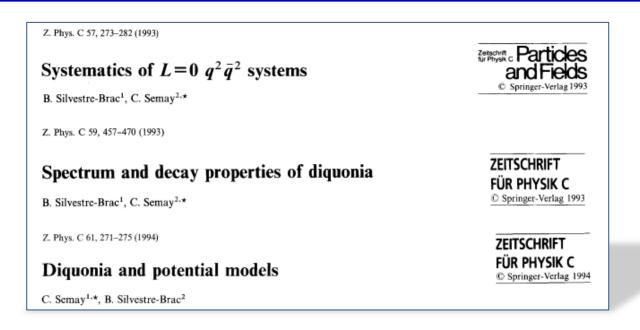


Using the interquark potential due to Bhaduri *et al.*, the energies of all L = 0,1,2,3 four-quark states are calculated for any value of the total S and I and for q = u, d, s, c, b using a harmonic oscillator basis up to 7/8 quanta. Natural parity is considered.

This implies 924 combinations.



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Nature	I	J = S	$E_d(MeV)$	∆(MeV)
$nnb\overline{b}$	0	1	10525	-131
$nsb\overline{b}$	1/2	1	10680	-40
$nn\bar{c}\bar{b}$	0	1	7244	1
$nn\bar{c}\bar{b}$	0	0	7206	11
$nn\bar{c}\bar{c}$	0	1	3931	19
$nn\overline{b}\overline{b}$	1	2	10735	30
nsbb	1/2	2	10816	48
$nn\bar{s}\bar{c}$	0	2	2975	49
nnsb	0	2	6306	49
$nn\bar{c}\bar{b}$	0	2	7422	49
nnññ	0	2	1605	51
$nn\bar{n}\bar{b}$	1/2	2	6181	52
$nn\bar{n}\bar{s}$	1/2	2	1734	52
$nn\overline{b}\overline{b}$	1	1	10712	56
nsīns	0, 1	2	1854	59
$ns\bar{c}\bar{b}$	1/2	2	7496	59

This implies 924 combinations.



Improving the numerical methods

A different approach based on Gaussian variational wave functions including combinations of three different radial coordinates is considered.

PHYSICAL REVIEW D	VOLUME 57, NUMBER 11	1 JUNE 1998
	Tetraquarks with heavy flavors	
Dip	D. M. Brink partimento di Fisica, Università degli Studi di Trento, I-38050 Povo (Trento), Italy	
Uı	Fl. Stancu niversité de Liège, Institut de Physique B5, Sart Tilman, B-4000 Liège 1, Belgium	

	$E(qq\bar{b}\bar{b})$ (MeV)						
SI	1 Gaussian	5 Gaussians	Brac-Semay	Threshold	$E-E_T$		
10	10 577.7	10 558.1	10 525	B+B*	-98.9		
01	10 802.4	10 766.2		B+B	156.2		
11	10 812.1	10 774.1	10 712	B+B*	117.1		
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Few-Body Systems 35, 175-196 (2004) DOI 10.1007/s00601-004-0068-9



The $T_{cc} = DD^*$ Molecular State

D. Janc 1,* and M. Rosina 1,2,**

First detailed study of typical radii and radial properties.

•	

$bb\overline{u}d$								
	IS	Threshold [Bh]	$N_{\text{max}} = 90$ [Bh]	Ref. [3] [Bh]	Threshold [AL1]	$N_{\text{max}} = 90$ [AL1]	Ref. [4] [AL1]	
	01	10650.9	10518.9	10525	10644.1	10503.9	10509	
	10	10601.4	10601.4	>10642	10587.0	10587.0	_	
	11	10650.9	10650.9	10712	10644.1	10644.1	-	

 $cc\overline{ud}$ S = 1 I = 0

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 Bhaduri
 3905.3
 3904.7
 3931

 AL1
 3878.6
 3875.9
 3892



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6 February 1997

PHYSICS LETTERS B

Physics Letters B 393 (1997) 119-123

Tetraquarks with colour-blind forces in chiral quark models

S. Pepin a,1, Fl. Stancu a,2, M. Genovese b,3, J.-M. Richard b,4

 Université de Liège, Institut de Physique, B.5. Sart Tilman, B-4000 Liège 1, Belgium
 Institut des Sciences Nucléaires, Université Joseph Fourier-IN2P3-CNRS, 53, avenue des Martyrs, F-38026 Grenoble Cedex, France These systems were studied with a potential model fitted in the baryon spectrum that includes meson-exchange forces between quarks and entirely neglects the chromomagnetic interaction.

System	(C_1) + OME	(C_2) + OME	Ref. [4
$ccar{q}ar{q}$	-0.185	-0.332	0.019
$bbar{q}ar{q}$	-0.226	-0.497	-0.135

Eur. Phys. J. C 49, 743-754 (2007)DOI 10.1140/epjc/s10052-006-0142-1

THE EUROPEAN
PHYSICAL JOURNAL C

Regular Article - Theoretical Physics

Chromomagnetism, flavour symmetry breaking and S-wave tetraquarks

F. Buccella^{1,a}, H. Høgaasen^{2,b}, J.-M. Richard^{3,c}, P. Sorba^{4,d}

A detailed formalism is presented to fully account for flavoursymmetry breaking in the chromomagnetic interaction together with its application to four-quark systems.

For $(QQ\bar{q}\bar{q})$ with identical heavy quarks, the chromomagnetic interaction is optimal for $J^P=1^+$, since the Pauli principle forbids the 0^+ eigenstates with the lowest eigenvalue of $H_{\rm CM}$. [...]

[...] The very large value of the mass ratio M_Q/m_n , where M_Q^{-1} is the average of the inverse masses m_c and m_b , presumably gives binding or almost binding from the sole chromoelectric effects. The chromomagnetic interaction is also favourable, and, if alone, would give a binding of more than 100 MeV.



Eur. Phys. J. A **19**, 383–389 (2004) DOI 10.1140/epja/i2003-10128-9

THE EUROPEAN
PHYSICAL JOURNAL A

Tetraquarks in a chiral constituent-quark model

J. Vijande^{1,a}, F. Fernández¹, A. Valcarce¹, and B. Silvestre-Brac²



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$$\sum_{i=1}^{n} \beta_s^{(i)} e^{-a_s^{(i)} \vec{x}^2 - b_s^{(i)} \vec{y}^2 - c_s^{(i)} \vec{z}^2 - d_s^{(i)} \vec{x} \vec{y} - e_s^{(i)} \vec{x} \vec{z} - f_s^{(i)} \vec{y} \vec{z}}$$



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(S,I)		(0,0)	(0,1)	(1,0)	(1,1)	(2,0)	(2,1)
$nnar{s}ar{s}$	E_T	2396	1858	1696	1934	2672	1993
	ΔE	+1404	+866	+291	+530	+852	+173
$nnar{c}ar{c}$	E_T	4508	4155	3927	4176	4852	4195
	ΔE	+742	+389	+34	+283	+833	+175
$nnar{b}ar{b}$	E_T	10975	10682	10424	10685	11321	10693
	ΔE	+413	+120	-178	+83	+679	+51

A different constituent quark model incorporating meson exchanges between light quarks on top of gluon exchange was explored

A variational method based on a Gaussian expansion was considered.

$$R_{s}(1234) = \sum_{i=1}^{n} \beta_{s}^{(i)} R_{s}^{(i)} = \sum_{i=1}^{n} \beta_{s}^{(i)} e^{-a_{s}^{(i)} \vec{x}^{2} - b_{s}^{(i)} \vec{y}^{2} - c_{s}^{(i)} \vec{z}^{2} - d_{s}^{(i)} \vec{x} \vec{y} - e_{s}^{(i)} \vec{x} \vec{z} - f_{s}^{(i)} \vec{y} \vec{z}$$



PHYSICAL REVIEW D 76, 094027 (2007)

Are there compact heavy four-quark bound states?

J. Vijande, 1 E. Weissman, 2 A. Valcarce, 3 and N. Barnea^{2,4}

PHYSICAL REVIEW D 79, 074010 (2009)

Exotic meson-meson molecules and compact four-quark states

J. Vijande, 1,2 A. Valcarce, 2 and N. Barnea 3,4

- A variational method using generalized gaussians with all non-diagonal terms (relative $l \neq 0$)
- A hyperspherical harmonic formalism (up to K = 30)



PHYSICAL REVIEW D 76, 094027 (2007)

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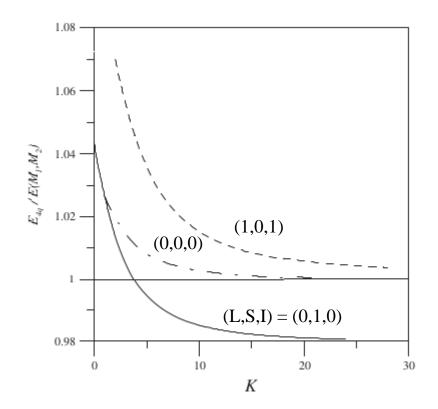
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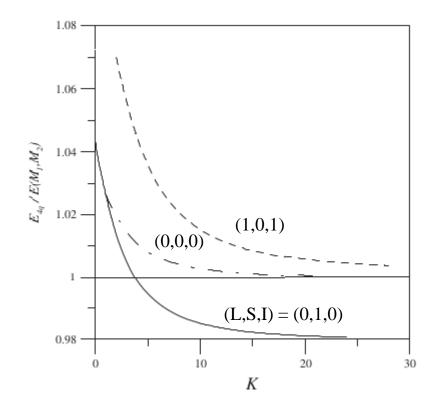
PHYSICAL REVIEW D 79, 074010 (2009)

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(L, S, I)	Reference [14]	$\mathrm{HH}\;(\sum_{i}\ell_{i}=0)$	НН
(0,0,1)	4155	4154	3911
(0,1,0)	3927	3926	3860
(0,1,1)	4176	4175	3975
(0,2,1)	4195	4193	4031

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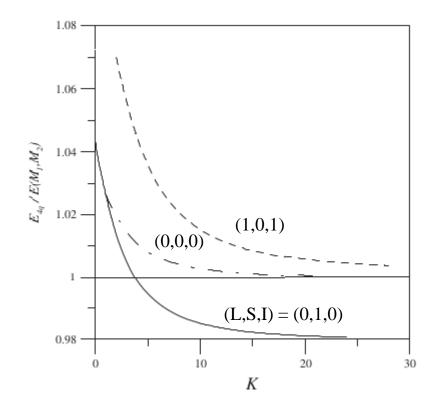
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PHYSICAL REVIEW D 76, 094027 (2007)

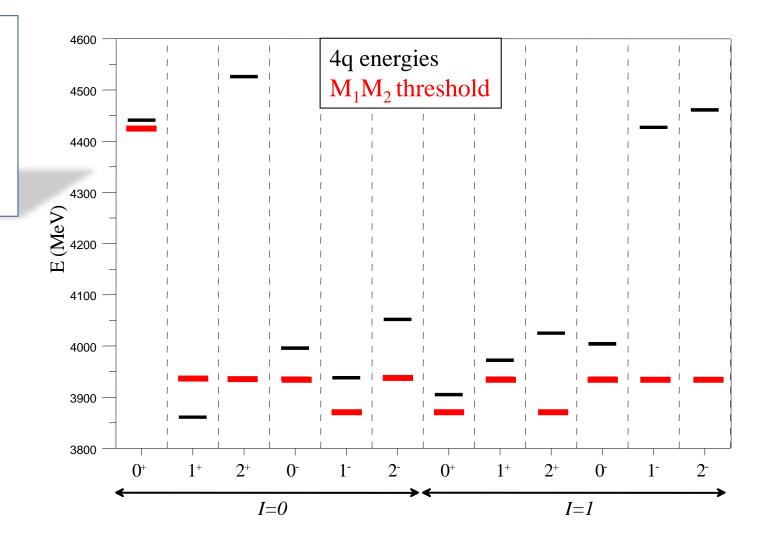
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PHYSICAL REVIEW D 76, 094027 (2007)

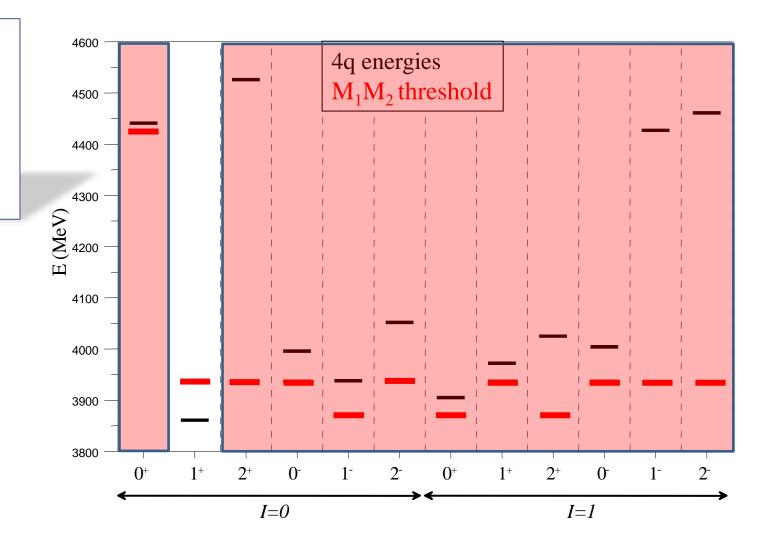
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PHYSICAL REVIEW D 76, 094027 (2007)

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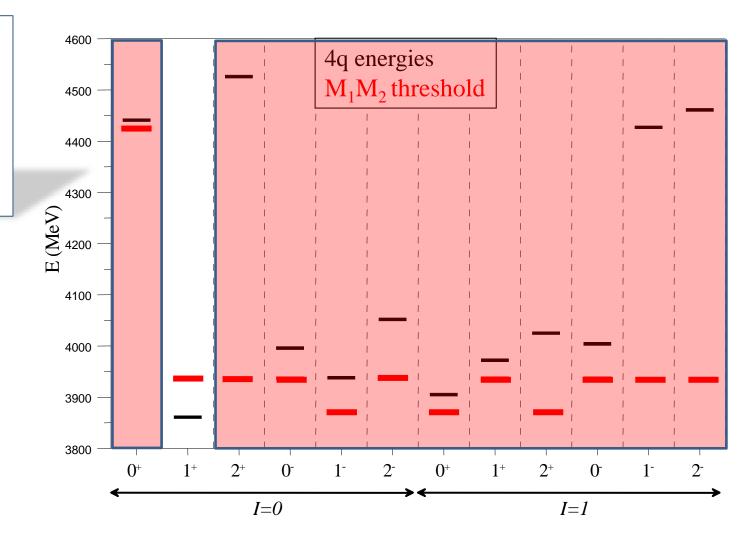
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PHYSICAL REVIEW D 79, 074010 (2009)

Exotic meson-meson molecules and compact four-quark states

J. Vijande, 1,2 A. Valcarce, 2 and N. Barnea 3,4

Quark content	$J^{P}(L, S, I)$	Model	Decay mode
$cc\bar{n}\bar{n}$	1+(0, 1, 0)	CQC	Weak
		BCN	Electromagnetic
$bb\bar{n}\bar{n}$	$1^+(0, 1, 0)$	CQC	Weak
		BCN	Weak
	$3^{-}(1, 2, 1)$	CQC	Electromagnetic
		BCN	Electromagnetic
	$0^+(0,0,0)$	CQC	Electromagnetic
		BCN	Electromagnetic
	1-(1, 0, 0)	CQC	Weak



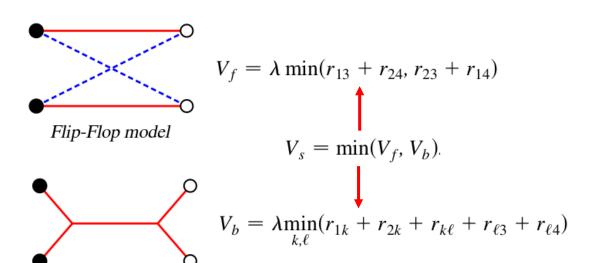


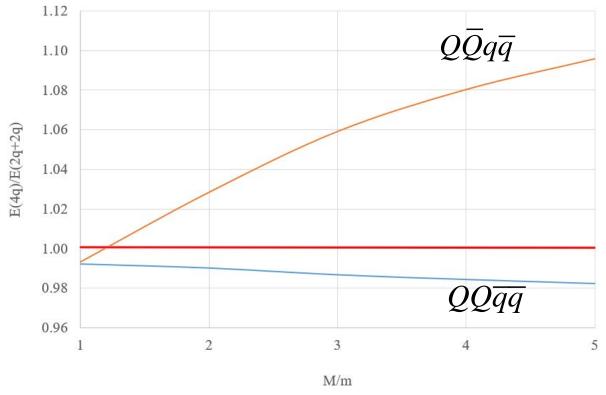
Beyond pairwise interactions

PHYSICAL REVIEW D 76, 114013 (2007)

Stability of multiquarks in a simple string model

J. Vijande, 1,2,* A. Valcarce, 2,† and J.-M. Richard 3,\$







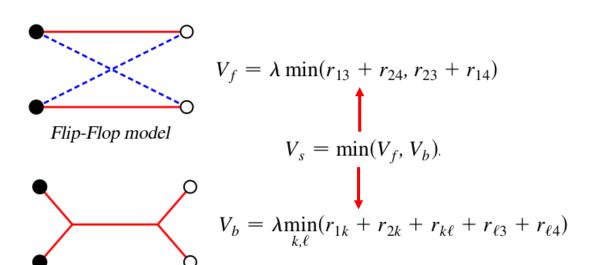
Butterfly model

Beyond pairwise interactions

PHYSICAL REVIEW D 76, 114013 (2007)

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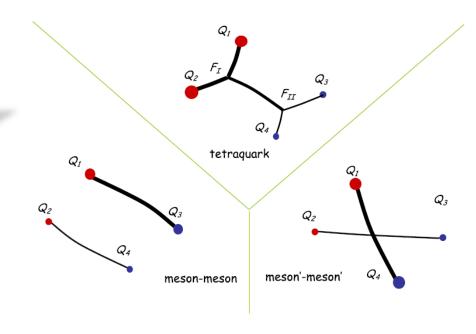
PHYSICAL REVIEW D **94,** 094032 (2016)

Tetraquark bound states and resonances in a unitary microscopic quark model: A case study of bound states of two light quarks and two heavy antiquarks

P. Bicudo* and M. Cardoso*

To summarize, we obtain tetraquark bound states on the $qq\bar{b}\bar{b}$ system, with quantum numbers 0^+ for s and c quarks, or light quarks with $I_{12} = 1$. For light quarks with $I_{12} = 0$, we obtain bound states with quantum numbers 1^+ .

We also tried to find bound states for the $qq\bar{c}\bar{c}$ system, but we were unable to find them when the lightest quarks have constituent masses equal to or larger than the ones of light quarks $m_q \ge 400$ MeV.



The ground state potential for a system composed of two quarks and two antiquarks is well fitted by a string flipflop potential.



 $bb\overline{u}d$

 $bb\overline{u}d$

 $bb\overline{ss}$

 $bb\overline{c}\overline{c}$

$$(S=1, I=0)$$
 $(S=0, I=1)$ $(S=0, I=0)$ $(S=0, I=0)$

$$(S = 0, I = 0)$$

$$(S = 0, I = 0)$$

PHYSICAL REVIEW C 80, 035204 (2009)

Probabilities in nonorthogonal bases: Four-quark systems

$$\begin{bmatrix} (q_1q_2)(\overline{q}_3\overline{q}_4) \end{bmatrix} \equiv \left\{ |\overline{3}_{12}3_{34}\rangle, |6_{12}\overline{6}_{34}\rangle \right\} \equiv \left\{ |\overline{3}3\rangle_c^{12}, |6\overline{6}\rangle_c^{12} \right\}$$

$$\begin{bmatrix} (q_1\overline{q}_3)(q_2\overline{q}_4) \end{bmatrix} \equiv \left\{ |1_{13}1_{24}\rangle, |8_{13}8_{24}\rangle \right\} \equiv \left\{ |11\rangle_c, |88\rangle_c \right\}$$

$$\begin{bmatrix} (q_1\overline{q}_4)(q_2\overline{q}_3) \end{bmatrix} \equiv \left\{ |1_{14}1_{23}\rangle, |8_{14}8_{23}\rangle \right\} \equiv \left\{ |1'1'\rangle_c, |8'8'\rangle_c \right\}$$



PHYSICAL REVIEW C 80, 035204 (2009)

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\begin{bmatrix} (q_1\overline{q}_3)(q_2\overline{q}_4) \end{bmatrix} \equiv \left\{ |1_{13}1_{24}\rangle, |8_{13}8_{24}\rangle \right\} \equiv \left\{ |11\rangle_c, |88\rangle_c \right\} \\
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PHYSICAL REVIEW C 80, 035204 (2009)

Probabilities in nonorthogonal bases: Four-quark systems

J. Vijande¹ and A. Valcarce²

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\end{array}$$

Physical interpretation



PHYSICAL REVIEW C 80, 035204 (2009)

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Physical interpretation

Journal of Mathematical Chemistry 5(1990)323-357

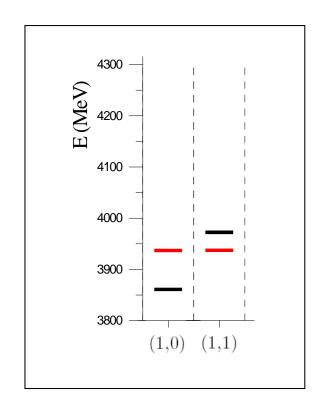
THEORY OF PROJECTED PROBABILITIES ON NON-ORTHOGONAL STATES: APPLICATION TO ELECTRONIC POPULATIONS IN MOLECULES

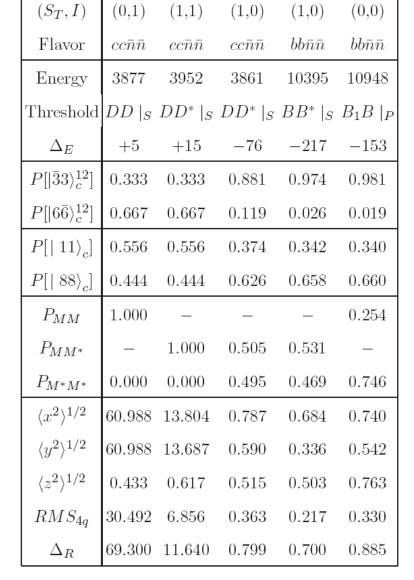
R.S. MANNING* and N. De LEON*†

$$\begin{aligned} |\Psi\rangle &= \alpha |11\rangle_c + \beta |88\rangle_c \\ &= \alpha |11\rangle_c + \beta (\chi |1'1'\rangle_c + \delta |8'8'\rangle_c) \\ &= \alpha |11\rangle_c + \gamma |1'1'\rangle_c + \mu |8'8'\rangle_c \\ &= \alpha |11\rangle_c + \gamma |1'1'\rangle_c + \mu (\varpi |11\rangle_c + \varsigma |88\rangle_c) \\ &= \dots = \wp_{|11\rangle_c} |\Psi\rangle + \wp_{|1'1'\rangle_c} |\Psi\rangle \end{aligned}$$

PHYSICAL REVIEW C 80, 035204 (2009)

Probabilities in nonorthogonal bases: Four-quark systems

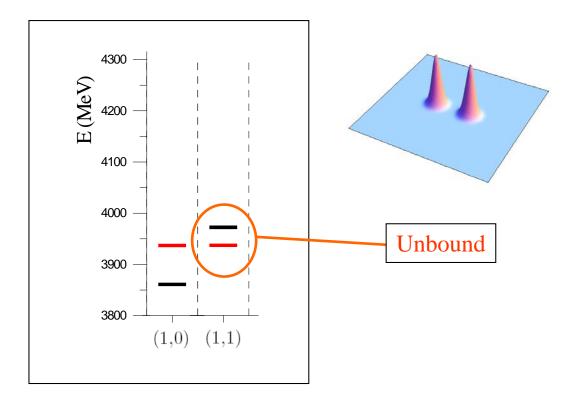






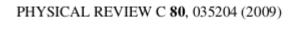
PHYSICAL REVIEW C 80, 035204 (2009)

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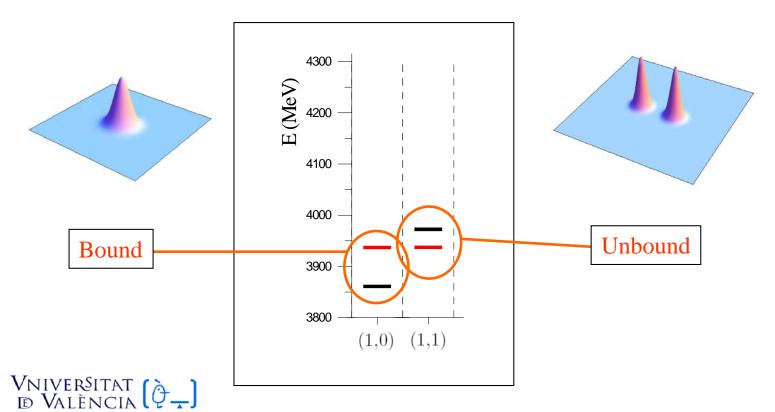


(S_T, I)	(0,1)	(1,1)	(1,0)	(1,0)	(0,0)
Flavor	$cc\bar{n}\bar{n}$	$cc\bar{n}\bar{n}$	$cc\bar{n}\bar{n}$	$bb\bar{n}\bar{n}$	$bb\bar{n}\bar{n}$
Energy	3877	3952	3861	10395	10948
Threshold	$DD \mid_{S}$	$DD^* _S$	$DD^* \mid_S$	$BB^* \mid_S$	$B_1B\mid_P$
Δ_E	+5	$\left(+15\right)$	-76	-217	-153
$P[\bar{3}3\rangle_c^{12}]$	0.333	0.333	0.881	0.974	0.981
$P[6\bar{6}\rangle_c^{12}]$	0.667	0.667	0.119	0.026	0.019
$P[\mid 11\rangle_c]$	0.556	0.556	0.374	0.342	0.340
$P[\mid 88\rangle_c]$	0.444	0.444	0.626	0.658	0.660
P_{MM}	1.000	-	_	_	0.254
P_{MM^*}	_	1.000	0.505	0.531	_
$P_{M^*M^*}$	0.000	0.000	0.495	0.469	0.746
$\langle x^2 \rangle^{1/2}$	60.988	13.804	0.787	0.684	0.740
$\langle y^2 \rangle^{1/2}$	60.988	13.687	0.590	0.336	0.542
$\langle z^2 \rangle^{1/2}$	0.433	0.617	0.515	0.503	0.763
RMS_{4q}	30.492	6.856	0.363	0.217	0.330
Δ_R	69.300	11.640	0.799	0.700	0.885





Probabilities in nonorthogonal bases: Four-quark systems



(S_T, I)	(0,1)	(1,1)	(1,0)	(1,0)	(0,0)
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Compact or meson-meson configuration?

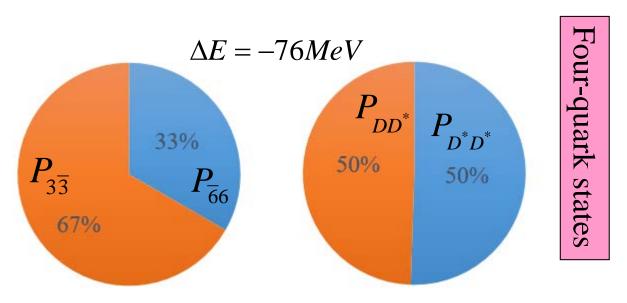
Physics Letters B 699 (2011) 291–295

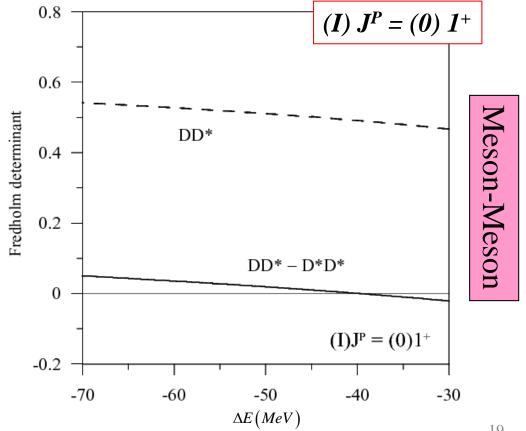
Doubly charmed exotic mesons: A gift of nature?

T.F. Caramés ^a, A. Valcarce ^{a,*}, J. Vijande ^b

$$\left|cc\overline{nn}\right\rangle = \alpha_1 \left|\overline{3}3\right\rangle + \dots + \alpha_2 \left|6\overline{6}\right\rangle + \dots = \alpha_1 \left|DD^*\right\rangle + \alpha_2 \left|D^*D^*\right\rangle + \dots$$

In this work the meson-meson configuration is solved by means of the Lippmann-Schwinger equation using the same interaction as the four-quark problem.





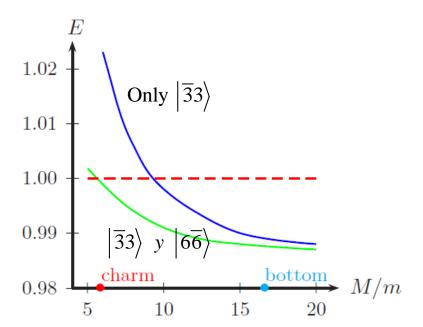


Few-body dynamics

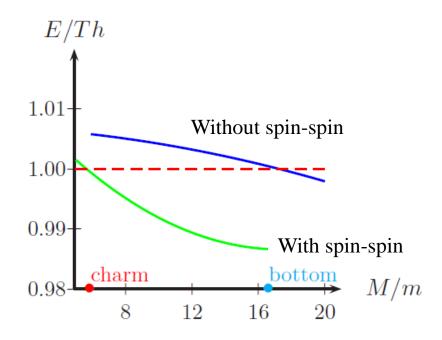
PHYSICAL REVIEW C 97, 035211 (2018)

Few-body quark dynamics for doubly heavy baryons and tetraquarks

Jean-Marc Richard, 1,* Alfredo Valcarce, 2,† and Javier Vijande 3,‡



A very delicate interplay between color and spin configurations.





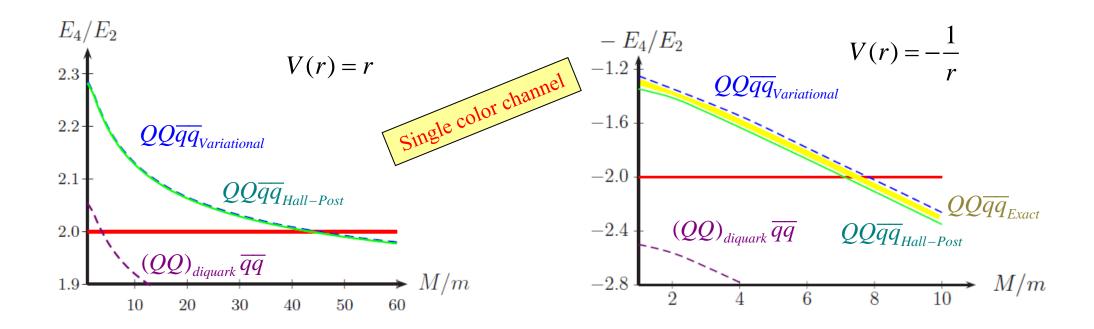
Few-body dynamics

PHYSICAL REVIEW C 97, 035211 (2018)

Few-body quark dynamics for doubly heavy baryons and tetraquarks

Jean-Marc Richard, 1,* Alfredo Valcarce, 2,† and Javier Vijande 3,‡

The treatment of the four-body dynamics for double-charm tetraquarks is discussed. The **variational** and **Born-Oppenheimer** approximations together with the **Hall-Post** inequalities give energies very **close to the exact ones**, while the **diquark** approximation might be **more problematic**.





Will the relativistic kinematics increase the number of stable multiquarks?

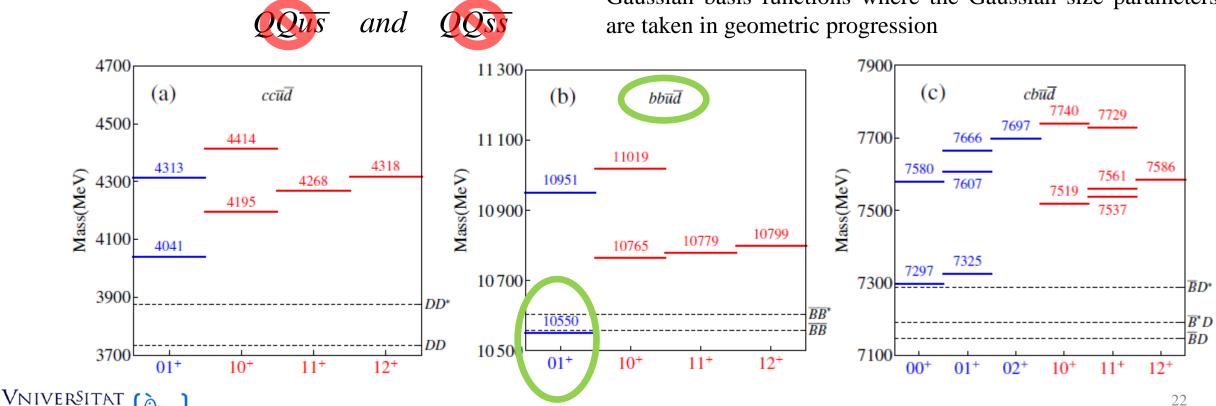
PHYSICAL REVIEW D 102, 034012 (2020)

Masses of doubly heavy tetraquarks $T_{QQ'}$ in a relativized quark model

Qi-Fang Lü⁰, 1,2,3,* Dian-Yong Chen, 4,† and Yu-Bing Dong 5,6,7,‡

The authors investigate the mass spectra using the relativized quark model proposed by Godfrey, Capstick, and Isgur.

The spatial wave function is expanded in terms of a set of Gaussian basis functions where the Gaussian size parameters



Will the relativistic kinematics increase the number of stable multiquarks?

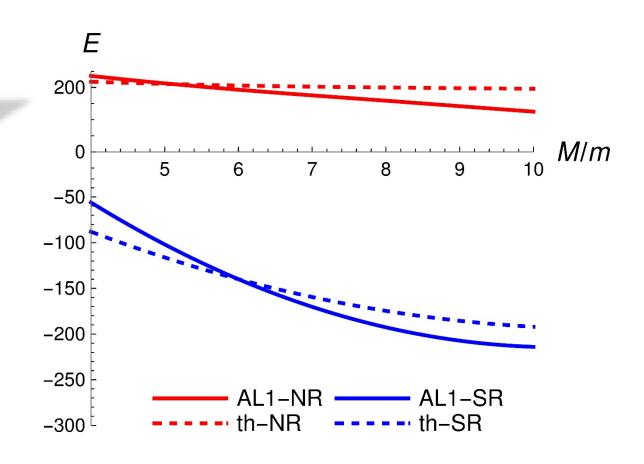
PHYSICAL REVIEW D 103, 054020 (2021)

Effect of relativistic kinematics on the stability of multiquarks

Jean-Marc Richard Alfredo Valcarce Tavier Vijande Tavier Vijande

In this case the threshold is made of two (qQ) mesons while in the four-quark state there are (qq), (QQ) and four (qQ) interactions. Who will benefit more from the relativistic dynamics?

We consider the AL1 potential properly re-parametrized in the SR case for keeping the description of the meson spectra.





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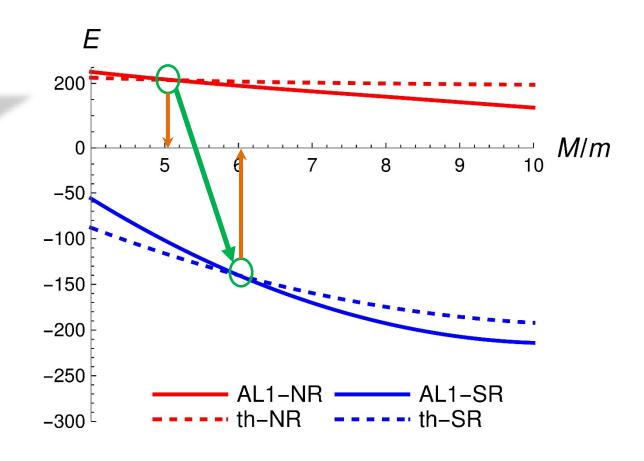
PHYSICAL REVIEW D 103, 054020 (2021)

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Conclusions

- The constituent quark model predicts a clear bound state, $bb\overline{u}\overline{d}$, and another one, $cc\overline{u}\overline{d}$, just below threshold with $(I)J^P=(0)I^+$. Some particular models may point to the existence of about five more bound states.
 - There is not an overwhelming abundance of bound states within the constituent quark model.
- The numerical methods required should be able to handle short- and long-range correlations, i.e. a meson-meson structures together with a more *clusterized* behaviour.
- Approximations and simplifications in the colour-spin structure should be done carefully.
 - We should double check whether our findings are entirely due to our hypothesis and aproximations before extracting any general conclusion.

