CERN COURIER

CULTURE AND HISTORY | MEETING REPORT

50 years of the GIM mechanism

24 January 2020

Hong-Jian He, John Ellis, John Iliopoulos, Sheldon Lee Glashow, Verónica Ri
and Luciano Maiani at a celebration of 50 years of the GIM mechanism in Shanghai. Credit: J Liu

- November 1st 1969: newly married, I arrive in Harvard, to be a Sheldon Glashow's postdoc
- I found Shelly and the other postdoc, John Iliopoulos, discussing about higher order weak interactions, Cabibbo theory and the like
- …as I had done till few days before in Roma with Nicola Cabibbo.
- a lucky circumstance… the beginning of our story

You may find my story here

Eur. Phys. J. H 42, 611–661 (2017)
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THE EUROPEAN PHYSICAL JOURNAL H

Oral history interview

The Charm of Theoretical Physics (1958–1993)*

Luciano Maiani[1] and Luisa Bonolis[2,a]

1. Late nineteen-sixties...

...hopes of a basic theory for strong, e.m. and weak interactions

•well established results:

-Gell-Mann-Zweig quarks in 3 flavours (baryons=qqq, etc.)

-u and d quark masses very small (~MeV) from chiral symmetry breaking

-Cabibbo theory of semileptonic decays, ΔS=0,1:

$$q = \begin{bmatrix} u \\ d \\ s \end{bmatrix}$$

$$\mathcal{L}_F = \frac{G_F}{\sqrt{2}} J^\lambda J^+_\lambda$$

$$J^\lambda = \bar{\nu}_e \gamma^\lambda (1-\gamma_5) e + \bar{\nu}_\mu \gamma^\lambda (1-\gamma_5)\mu + \bar{u}\gamma^\lambda (1-\gamma_5) d_C$$

$$d_C = \cos\theta d + \sin\theta s$$

quarks in one weak doublet:

$$\begin{pmatrix} u \\ d_C \end{pmatrix}_L ; (s_C)_L; d_R; u_R; s_R$$

•clouds:

-do quark clash with Fermi-Dirac statistics? first ideas about color (Han-Nambu)

-basic strong interactions: *gluon (abelian) mediated* ? *dual-like* (Veneziano model)?

-Fermi theory not renormalizable. W boson? strong interaction form factors?

•Schwinger's ideas about EW unification and Yang-Mills theory

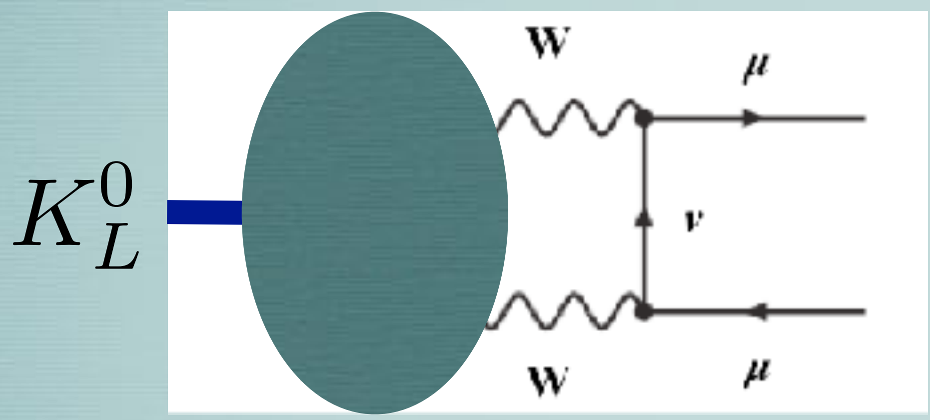
- Glashow: SU(2)⊗U(1) (1961)

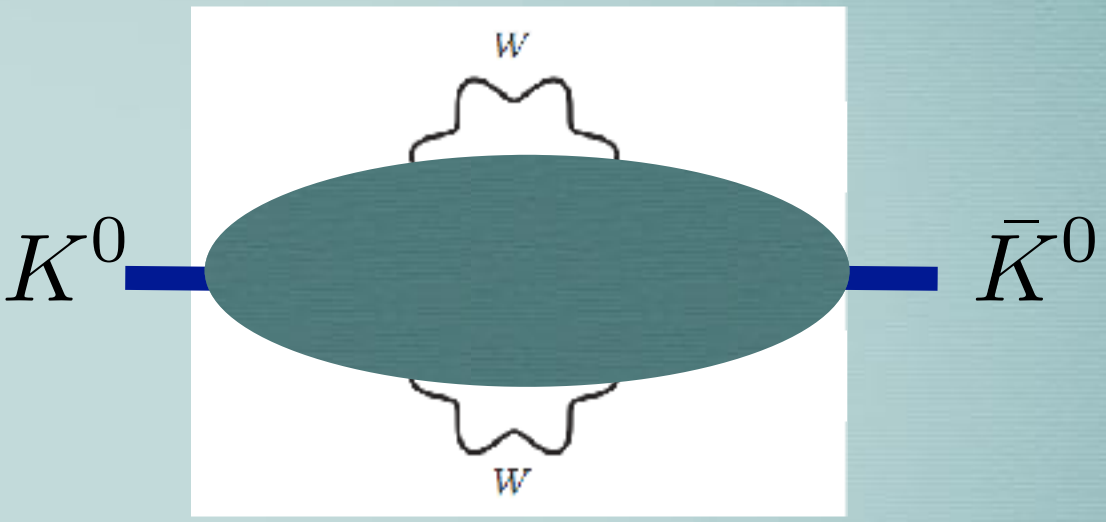
- Brout-Englert-Higgs Mechanism (1965) → Weinberg-Salam (1967)

•embedding Cabibbo theory in SU(2) ⊗ U(1) leads to unobserved Flavor Changing Neutral Currents: does Unification work for leptons only ?

1. The G(GΛ²) puzzle, 1968

•The discussion on higher order weak interactions was opened in 1968 by a calculation by Boris Ioffe and Evgeny Shabalin, indicating that ΔS= ± 1 neutral currents and ΔS=2 amplitudes would result from higher order weak interactions, ***even in a theory with a charged W only***





• the amplitudes were found to be divergent, to order $G(G\Lambda^2)$, and in disagreement with experiments, unless limited by an ultraviolet cut-off, determined to be $\Lambda \approx$ 3-4 GeV from Δm_K;

•result is based on current algebra commutators and shows that hadron form factors cannot help: ***current commutators imply hard constituents***;

•Similar results were found by F. Low and by R. Marshak *et al*.

first attempts

- Attempts were made during 1968-69 to make the amplitude more convergent:

  - introducing more than one Intermediate Vector Boson (Gell-Mann, Low, Kroll, Ruderman) (too many were needed);

  - introducing negative metrics (ghost) states (T.D.Lee and G.C. Wick), of mass $\simeq \Lambda$!

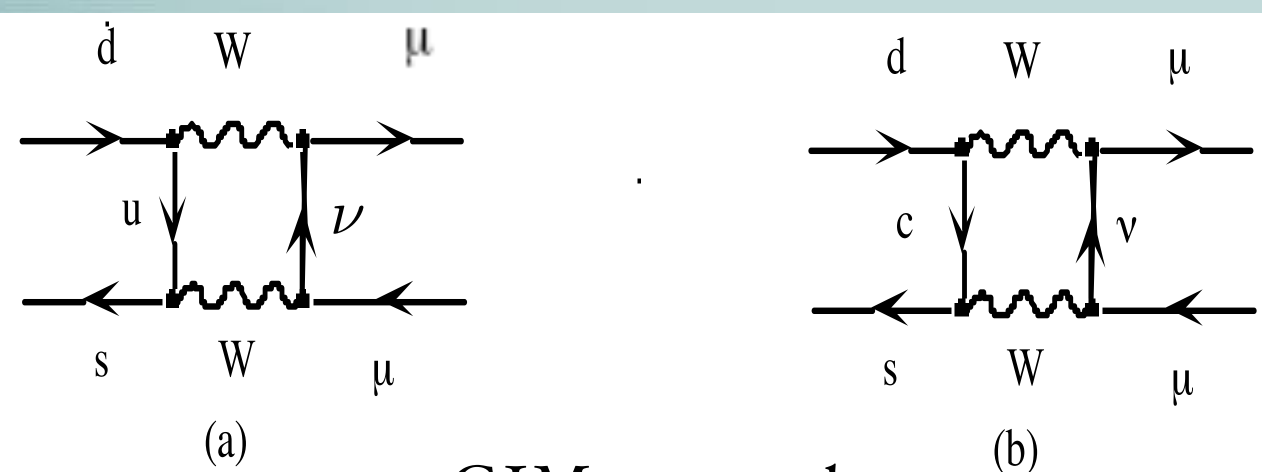
- it was realised that quadratically divergent amplitudes of order $G\Lambda^2$ would also arise, in the IVB theory, with potential violations of strong interaction symmetries (parity, isospin, SU(3) and strangeness).
- C. Bouchiat, J. Iliopoulos and J. Prentki (1968): with chiral SU(3) $\otimes$ SU(3) breaking described by a (3,3bar) representation, leading divergent terms that violate strangeness and parity are of the form $\partial^\mu J_\mu$ and can be eliminateda
- another line was to cancel the quadratic divergence in correspondence to a specific value of the angle, i.e. "computing" the Cabibbo angle (Gatto, Sartori, Tonin(1968); Cabibbo, Maiani (1969));
- ...but the small cutoff in the $G(G\Lambda^2)$ terms still called for an explanation.

PHYSICAL REVIEW D VOLUME 2, NUMBER 7 1 OCTOBER 1970

Weak Interactions with Lepton-Hadron Symmetry*

S. L. Glashow, J. Iliopoulos, and L. Maiani†
Lyman Laboratory of Physics, Harvard University, Cambridge, Massachusetts 02139
(Received 5 March 1970)

We propose a model of weak interactions in which the currents are constructed out of four basic quark fields and interact with a charged massive vector boson. We show, to all orders in perturbation theory, that the leading divergences do not violate any strong-interaction symmetry and the next to the leading divergences respect all observed weak-interaction selection rules. The model features a remarkable symmetry between leptons and quarks. The extension of our model to a complete Yang-Mills theory is discussed.



GIM proposal

$$\begin{pmatrix} u \\ d_C \end{pmatrix}_L ; \begin{pmatrix} c \\ s_C \end{pmatrix}_L ; (d_C)_R; (s_C)_R; u_R; c_R$$

$$\begin{pmatrix} \nu_e \\ e \end{pmatrix}_L ; \begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix}_L ; e_R; \mu_R$$

$$J_\mu^W(quark) = \bar{q} C \gamma_\mu q_L$$

$$C = \begin{pmatrix} 0 & 0 & +\cos\theta & +\sin\theta \\ 0 & 0 & -\sin\theta & +\cos\theta \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$ =quark ***mixing matrix***

divergent amplitude for $m_u = m_c = 0$:
$\propto G(G\Lambda^2)[C, C^\dagger]$
= flavor diagonal!

GIM in short

1 *...it appears necessary to depart from the original phenomenological model of weak interactions*

New weak interactions

2 *The weak hadronic current is constructed in precise analogy with the weak lepton current, thereby revealing suggestive lepton-quark symmetry*

Quark-lepton symmetry

3 *In contradistinction to the conventional (three-quark) model, the couplings of the neutral intermediary - now hypercharge conserving -cause no embarrassment. The possibility of a synthesis of weak and electromagnetic interactions is also discussed.*

Electroweak unification is possible

4 *...suitable redefinitions of the relative phases of the quarks may be performed in order to make U* [i.e. the weak interaction, matrix, note added] *real and orthogonal*

No CP violation

5 *Why have none of these charmed particles been seen? Suppose they are all relatively heavy, say 2 GeV. Although some of the states must be stable under strong (charm-conserving) interactions, these will decay rapidly (* 10^{-13} *sec) by weak interactions into a very wide variety of uncharmed final states (there are about a hundred distinct decay channels).*

Since the charmed particles are copiously produced only in associated production, such events will necessarily be of very complex topology, involving the plentiful decay products of both charmed states. Charmed particles could easily have escaped notice.

Charmed particles have to be found

January 1970….

Eur. Phys. J. H **42**, 611 (2017)
arXiv:1707.01833 [physics.hist-ph]

- By the end of January 1970, I think we had understood all the essentials and we were very happy.
- I remember one day going to the Legal Sea Food for lunch where my wife Pucci joined us. Pucci told to Shelly how happy and excited I was about the new result and the work we were doing. He replied: *..right, this paper is going to be on all school books*
- Shelly was fantastic. . .I remember another occasion, a seminar which Shelly gave to the experimentalists of Harvard, working at the CEA (Cambridge Electron Accelerator), and he said: *Look, with charm we have essentially solved particle physics*.
- *Except*, he added, *for CP violation*.

Unified theory for quarks

- The matrices:

$$\mathcal{C},\ \mathcal{C}^{\dagger},\ [\mathcal{C},\ \mathcal{C}^{\dagger}] = 2\mathcal{C}_3,\ \text{and } Q$$

$$\mathcal{C}_3 = \begin{pmatrix} 1/2 & 0 & 0 & 0 \\ 0 & 1/2 & 0 & 0 \\ 0 & 0 & -1/2 & 0 \\ 0 & 0 & 0 & -1/2 \end{pmatrix};Q = \begin{pmatrix} 2/3 & 0 & 0 & 0 \\ 0 & 2/3 & 0 & 0 \\ 0 & 0 & -1/3 & 0 \\ 0 & 0 & 0 & -1/3 \end{pmatrix}$$

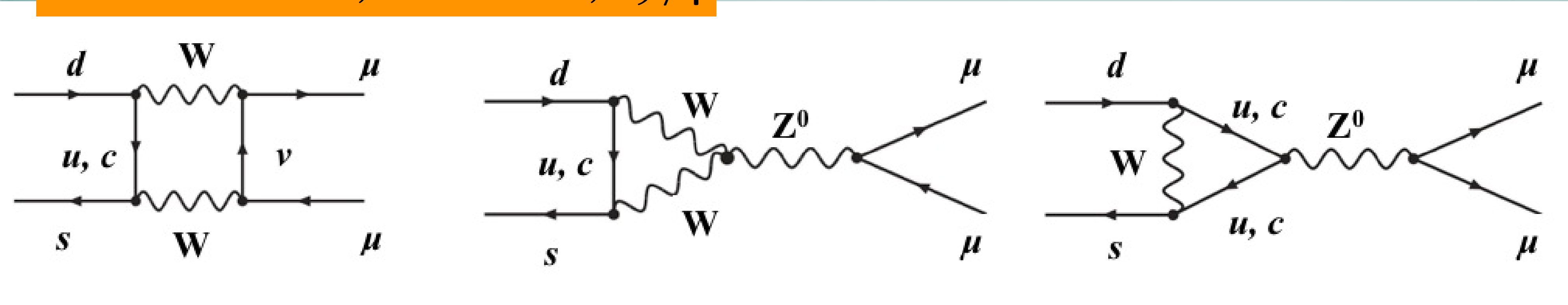
make an SU(2) ⊗ U(1) algebra without Flavor Changing Neutral Currents (FCNC) and can be taken as the generators of the unified Glashow-Weinberg-Salam theory of the Electroweak Interactions

- GIM: FCNC processes arise to order G^2

- UV divergences are cut-offed by heavy quark exchange; if there are no additional, long distance, contributions, amplitudes may be reliably computed, due to asymptotic freedom

heavy quarks in FCNC provide a tool to search for new physics at high energy

M. K. Gaillard, B. W. Lee, 1974



need to compute the long-distance contribution from: $K_L \to \gamma\gamma \to \mu^+\mu^-$

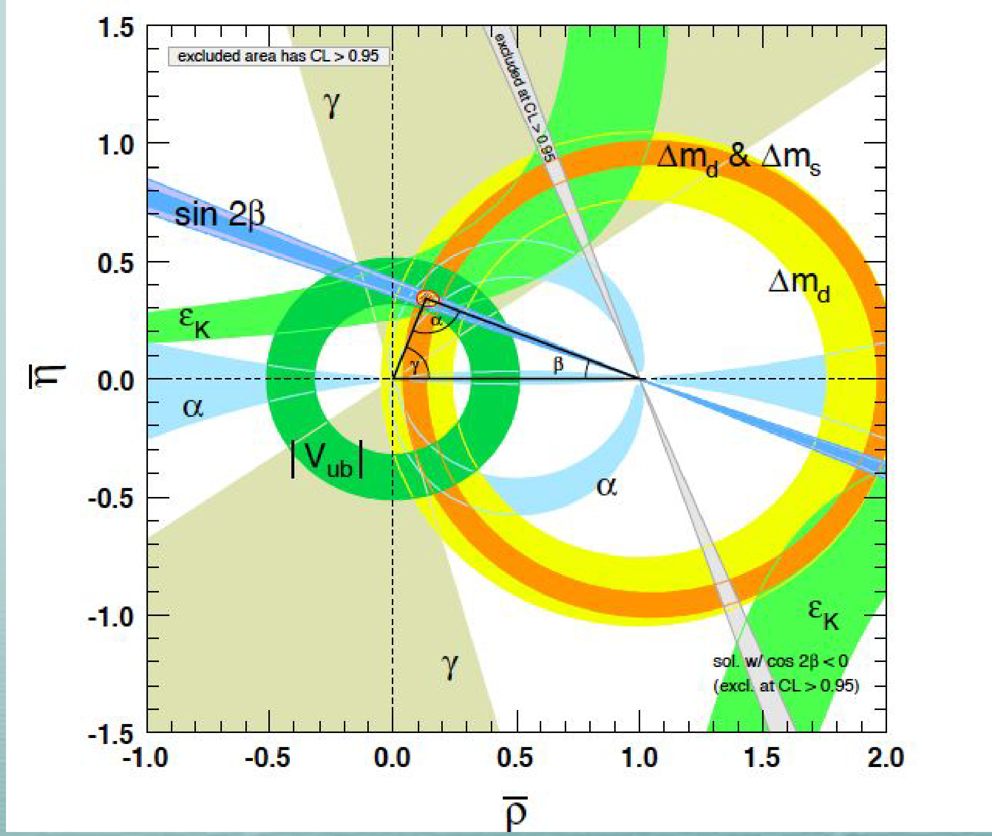
2. CP violation, in brief

- 1973, Kobayashi and Maskawa: three left-handed quark doublets allow for one CP violating phase in the quark mixing matrix, since known as the Cabibbo-Kobayashi-Maskawa matrix;
- the phase could agree with the observed CP violation in K decays and led to neutron electric dipole vanishing at one loop (Pakvasa & Sugawara, Maiani, 1976);
- 1986, I. Bigi and A. Sanda predict direct CP violation in B decay;
- 2001, Belle and BaBar discover CP violating mixing effects in B-decays.

Wolfenstein's parametrization

$$U_{CKM} = \begin{pmatrix} 1-\frac{1}{2}\lambda^2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\frac{1}{2}\lambda^2 & A\lambda^2 \\ A\lambda^3[1-(\rho+i\eta)] & -A\lambda^2 & 1 \end{pmatrix}$$

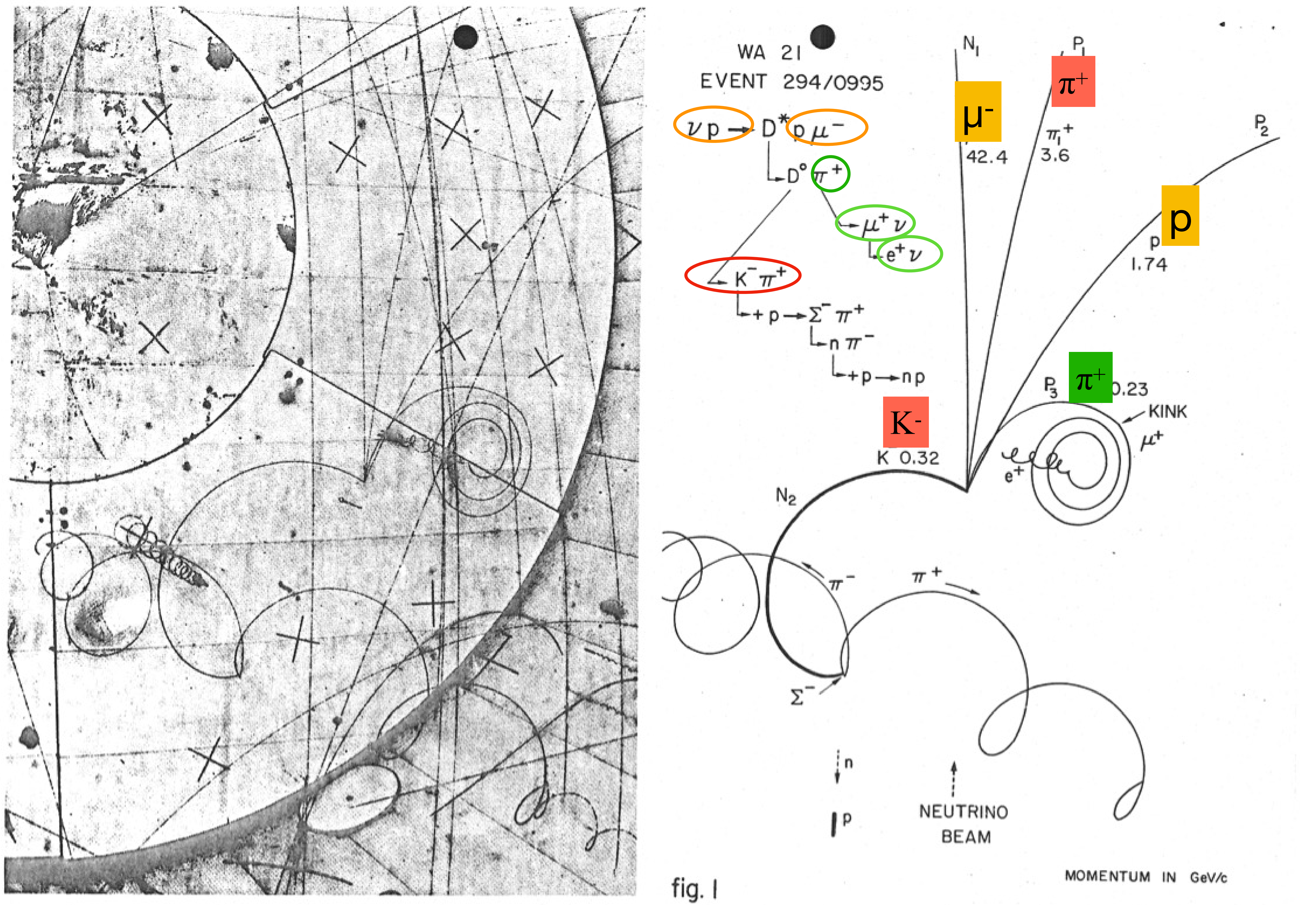
U_{CKM} is in an extraordinary agreement with data



C.-Amsler *et al.* [Particle Data Group Collaboration], Phys. Lett. B**667**, (2008)

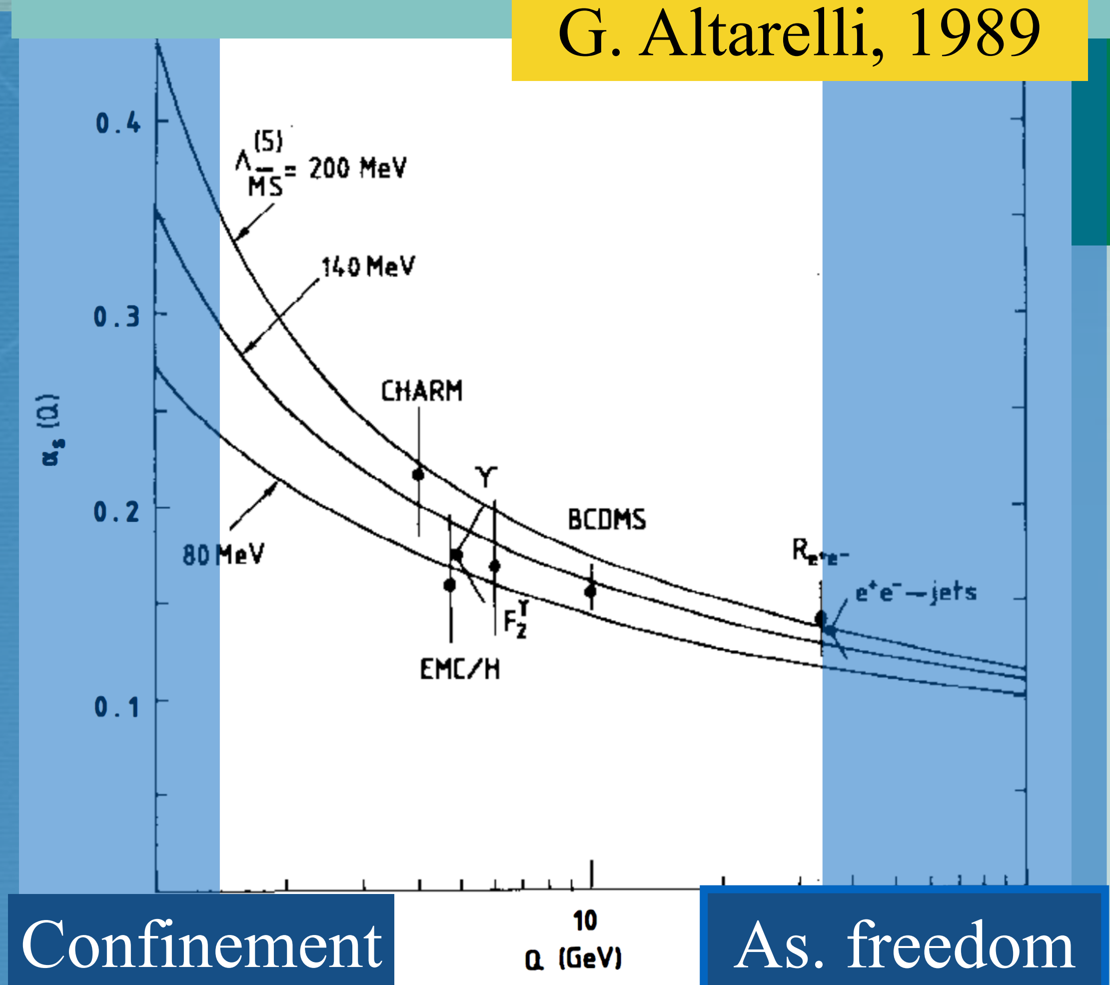
: …charmed particles found as advertised

Neutrino production of an excited charmed meson, D*, is captured by this spectacular picture taken at the CERN Hydrogen Bubble Chamber, with the decay chain of D* fully reconstructed (J.Blietschau~ et al. [Aachen-Bonn-CERN-Munich-Oxford Collaboration]. Physics. Lett. **86B** (1979) 108.

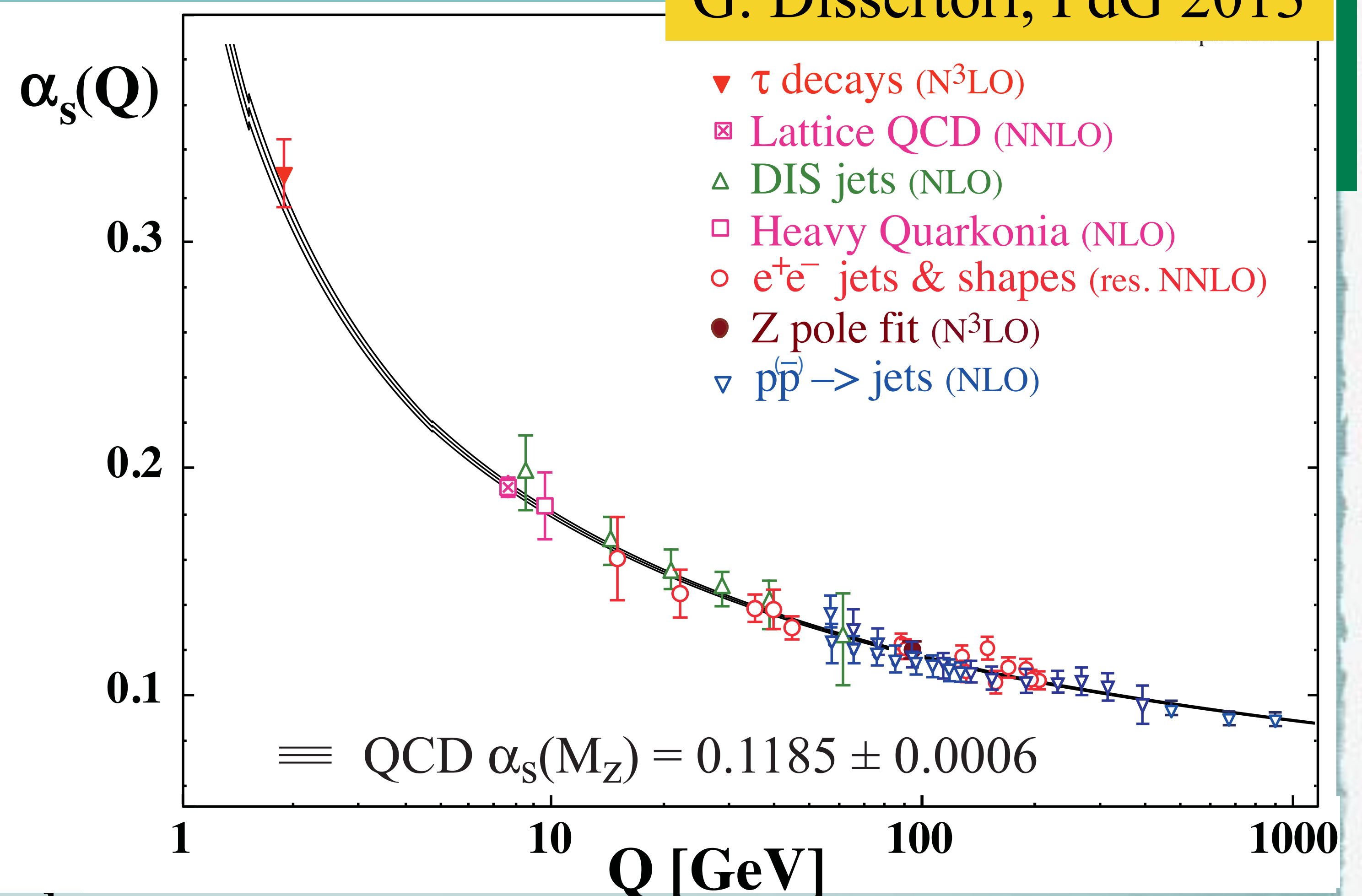


3. QCD IS THE ANSWER TO (ALMOST) ANY QUESTION

G. Altarelli, 1989



G. Dissertori, PdG 2013



- QCD is asymptotically free and confined:
- *Critical momentum:* Λ_{QCD}~300 MeV
- quarks carry color and are confined inside color singlet hadrons of radius ~$1/\Lambda_{QCD}$
- only one way to make a color singlet with tree quarks: $\Delta^{++} = u_\alpha^\uparrow u_\beta^\uparrow u_\gamma^\uparrow \ \epsilon^{\alpha\beta\gamma}$
- Fermi statistics is obeyed
- proton at rest: only 3 quarks, dressed by strong QCD

Constituent Quarks

- increasing $q^2 > (\Lambda_{QCD})^2$ quarks radiate gluons (the Altarelli-Parisi picture of scaling violations)
- at large $q^2 >> (\Lambda_{QCD})^2$ we see quarks and neutral gluons as almost free partons.

QCD Partons

4. Hidden charm and beauty hadrons reveal *tetraquarks* and *pentaquarks*

M. Gell-Mann, A Schematic Model of Baryons and Mesons, PL **8**, 214, 1964

Baryons can now be constructed from quarks by using the combinations $(q\,q\,q)$, $(q\,q\,q\,q\,\bar{q})$, etc., while mesons are made out of $(q\,\bar{q})$, $(q\,q\,\bar{q}\,\bar{q})$, etc.

•Heavy quark pairs inside hadrons ***are difficult to be created or destroyed by QCD forces***.

• Hadrons electrically charged ***and*** featuring a $c\bar{c}$ or $b\bar{b}$ pair ***must*** contain additional light quarks, ***realising the hypothesis advanced by Gell-Mann***

•These are the exotic X, Y, Z mesons and the pentaquarks discovered over the last decades

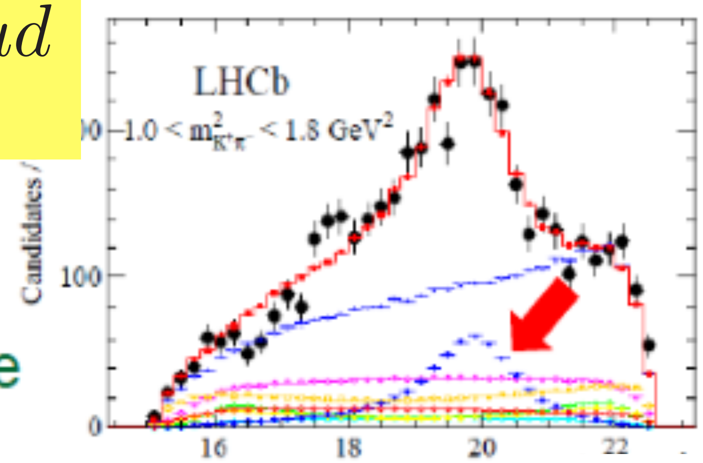
There are, indeed, new valence quark configurations !!

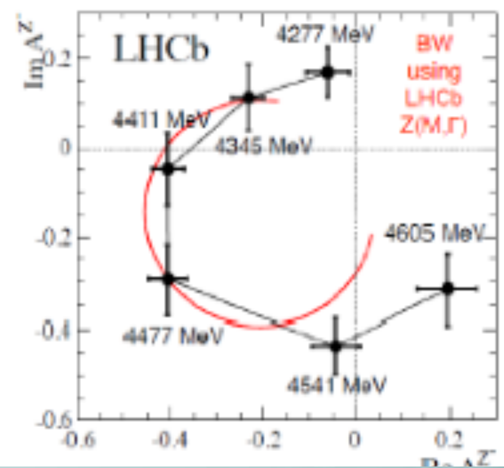
- First hypothesis of tetraquarks by R. Jaffe, as a model of the lightest scalar mesons
- Tetraquarks are more easy to identify at the increase of quark mass
- Hidden heavy flavors have been the first,
- hidden charm and open strangeness ***discovered now !!***
- The first, *unexpected charmonium* was the still controversial X(3872)
- Nearness to heavy pair threshold is to be expected, but the X(3872) is exceptionally close, we do not know yet if it is above or below the $D^0\bar{D}^{*0}$ threshold, within some 80 keV.

$Z_c(4430)^{\pm} \to J/\Psi + \pi$ (BELLE, 2007)
valence quark composition: $c\bar{c}u\bar{d}$
LHCb, 2014:

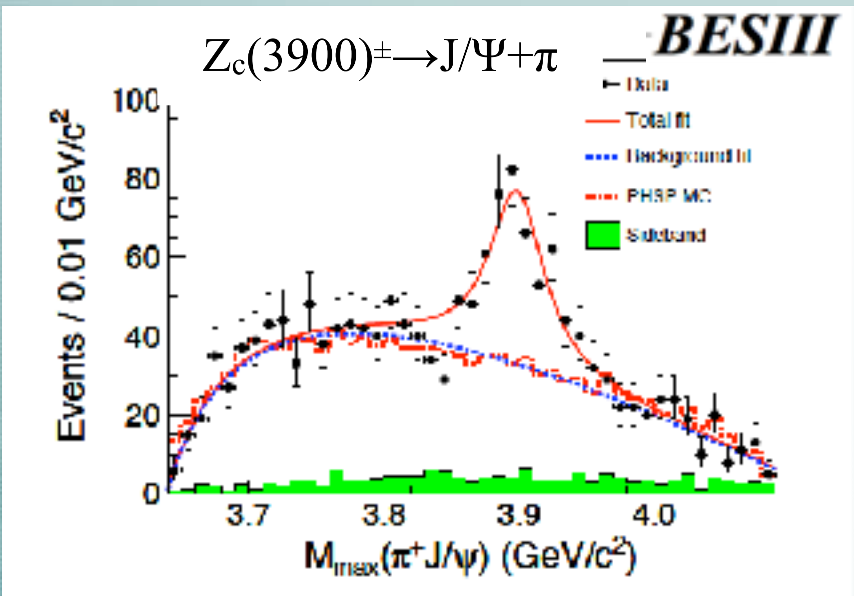
1. Confirm Belle's observation of 'bump'
2. Can NOT be built from standard states
3. Textbook phase variation of a resonance

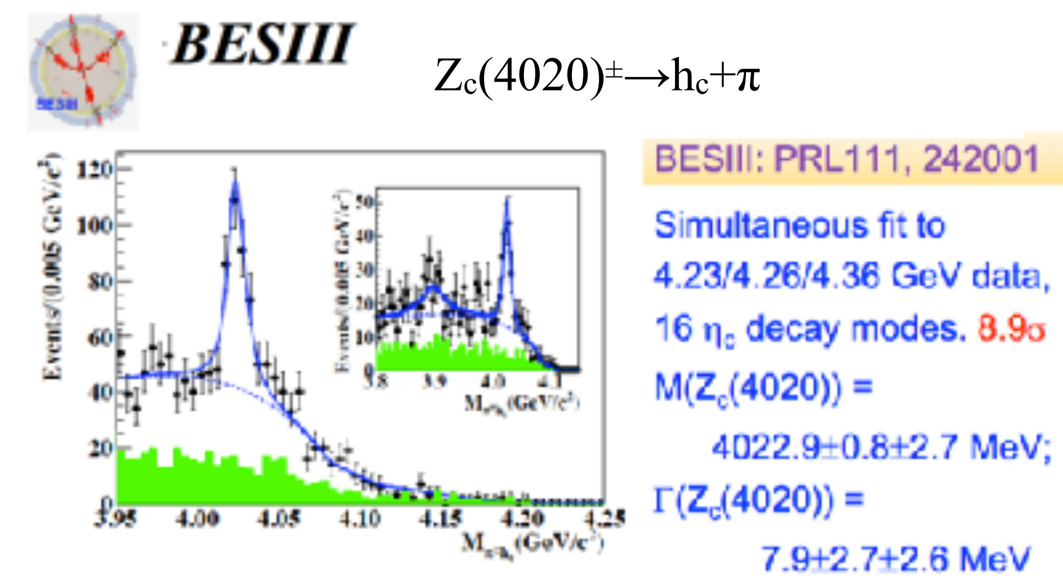
[PRL 112 (2014) 222002]



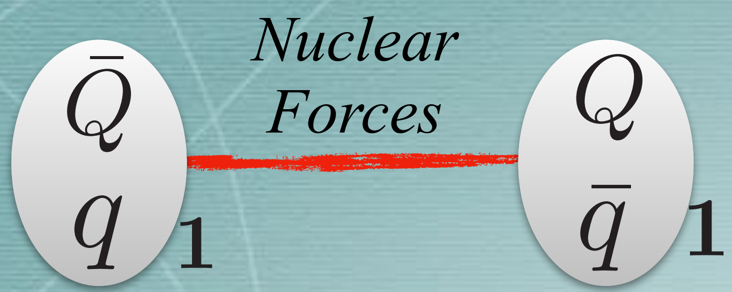


$Z_c(4430)$ is far from any threshold.
The mass difference $M(Z_c(4430)) - M(X(3872)) \sim 560$ MeV calls for it being the radial excitation of a similar resonance with $J^{PC} = 1^{+-}$ close to X(3872). This resonance was found by BES III in 2013, $Z_c(3900)$, together with $Z_c(4020)$:



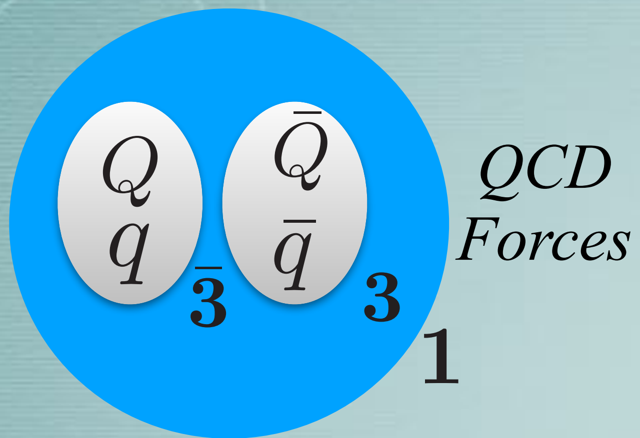


No consensus, yet



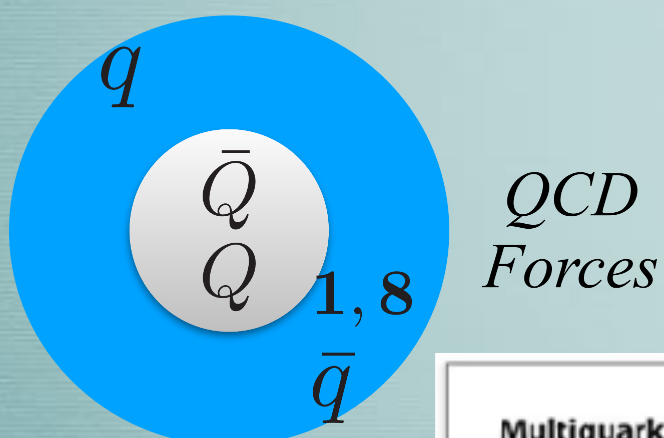
Hadron Molecule

F-K. Guo, C. Hanhart, U-G Meißner, Q. Wang, Q. Zhao, and B-S Zou, arXiv 1705.00141 (2017)



Compact Diquark-Antidiquark

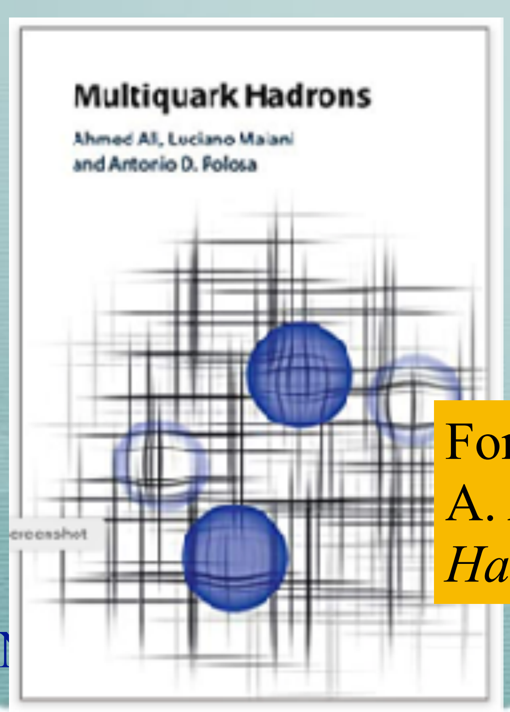
L. Maiani, F. Piccinini, A. D. Polosa and V. Riquer, Phys. Rev. D 71 (2005) 014028; **D 89** (2014) 114010.



HadroCharmonium (1)
Quarkonium Adjoint Meson (8)

S. Dubynskiy, S. and M. B.Voloshin, Phys. Lett. **B 666**,(2008) 344.

E. Braaten, C. Langmack and D. H. Smith, Phys. Rev. **D 90** (2014) 01404



For a review, see:
A. Ali, L. Maiani and A.D. Polosa, *Multiquark Hadrons*, Cambridge University Press (2019)

5. Exotic mesons: the *New Wave*

• Starting from 2016, new kinds of exotic hadrons have been discovered:

$J/\Psi\ \phi$ resonances, $di - J/\Psi$ resonances, open strangeness Exotics: $Z_{cs}(3082)$ and $Z_{cs}(4003)$

•Pion exchange forces cannot bind them as hadron molecules made by color singlet mesons: molecular models have to stand on the existence of "phenomenological forces" with undetermined parameters

• Not necessarily "just on threshold", no cusp behaviour..

• The New Exotics arise very naturally as $([cq]^{\bar{\mathbf{3}}}[\bar{c}\bar{q}']^{\mathbf{3}})_{\mathbf{1}}$ bound by QCD in color singlets

•A firm prediction: ***hidden charm tetraquarks must form complete multiplets of flavor SU(3)***

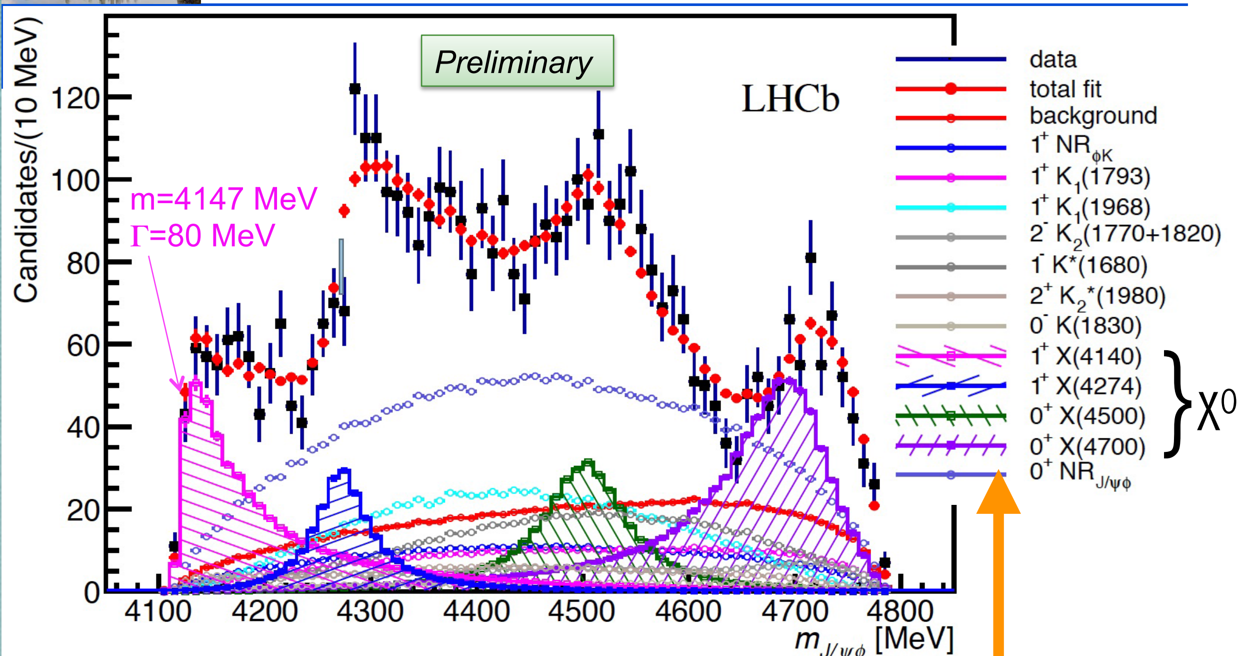
•with mass differences determined by:

$$m_s - m_u = 120 - 150\ \text{MeV}.$$

• ***with $Z_{cs}(3082)$ and $Z_{cs}(4003)$ we can almost fill two tetraquark nonets with the expected scale of mass differences.***

The New Wave started with the discovery of $J/\Psi\ \phi$ resonances, LHCb 2016

Results of fit: m(J/ψϕ)



- 4 visible structures fit with BW amplitudes

28 Recontres de Blois, June 2, 2016

Results of fit

- J^P also measured all with >4σ significances

Particle	J^P	Significance	Mass (MeV)	Γ (MeV)	Fit Fraction (%)
X(4140)	1^+	8.4 σ	$4146.5 \pm 4.5^{+4.6}_{-2.8}$	$83 \pm 21^{+21}_{-14}$	$13.0 \pm 3.2^{+4.8}_{-2.0}$
X(4274)	1^+	6.0 σ	$4273.3 \pm 8.3^{+17.2}_{-3.6}$	$56 \pm 11^{+8}_{-11}$	$7.1 \pm 2.5^{+3.5}_{-2.4}$
X(4500)	0^+	6.1 σ	$4506 \pm 11^{+12}_{-15}$	$92 \pm 21^{+21}_{-20}$	$6.6 \pm 2.4^{+3.5}_{-2.3}$
X(4700)	0^+	5.6 σ	$4704 \pm 10^{+14}_{-24}$	$120 \pm 31^{+42}_{-33}$	$12 \pm 5^{+9}_{-5}$
NR	0^+	6.4 σ			$46 \pm 11^{+11}_{-21}$

28 Recontres de Blois, June 2, 2016

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- Meson-Meson molecule: no way

- J/Ψ φ mass distribution: four structures
- positive parity, J=0 and 1, positive charge conjugation
- X(4140) seen previously by CMS and by BELLE
- interpreted as $[cs][\bar{c}\bar{s}]$ tetraquarks

- Baryon-antibaryon molecules? $\Xi_c^+ = [csu]$

$2M_{\Xi_c} \sim 4930$ MeV!!!

L. Maiani, A. Polosa, V. Riquer, Phys. Rev. D **94**, 054026 (2016)

Similarly

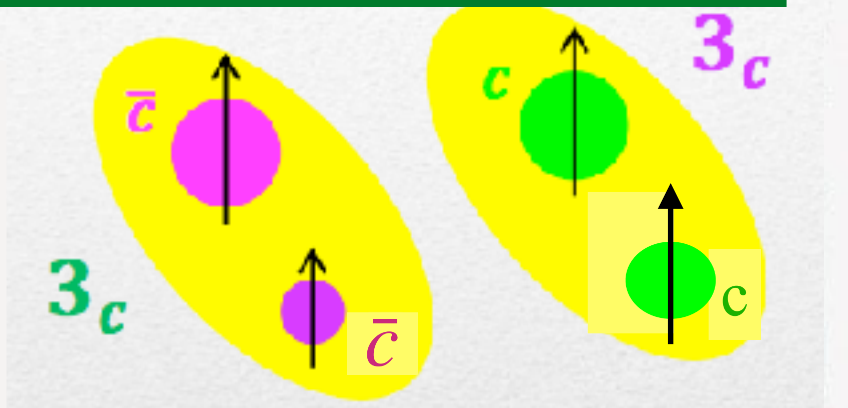
- $di - J/\Psi$ resonances, M=6920 MeV and higher:
  - -Meson-Meson molecule: no way
  - -Baryon-antibaryon molecule? $\Xi_{cc} = [ccu]$; $2M_{\Xi_{cc}} \sim 7242$ MeV!!!
- $Z_{cs}(3985)$, BESIII:
  - -Meson-Meson molecule: no way
  - -Baryon-antibaryon molecule? $M_{\Xi_c} + M_{\bar{\Sigma}_c} \sim 4923$ MeV;

    $M_{\Xi_c} + M_{\bar{\Sigma}_c} \sim 4923$ MeV !!
- $Z_{cs}(4003)$, LHCb:
  - -Baryon-antibaryon molecule? as before
- Important!!!!: LHCb and BESIII see two DIFFERENT particles (widths are NOT the same)

Tetraquark constituent picture of di-J/Ψ resonances

L.Maiani, F.Piccinini, A.D.Polosa and V.Riquer, Phys. Rev. D 71 (2005) 014028

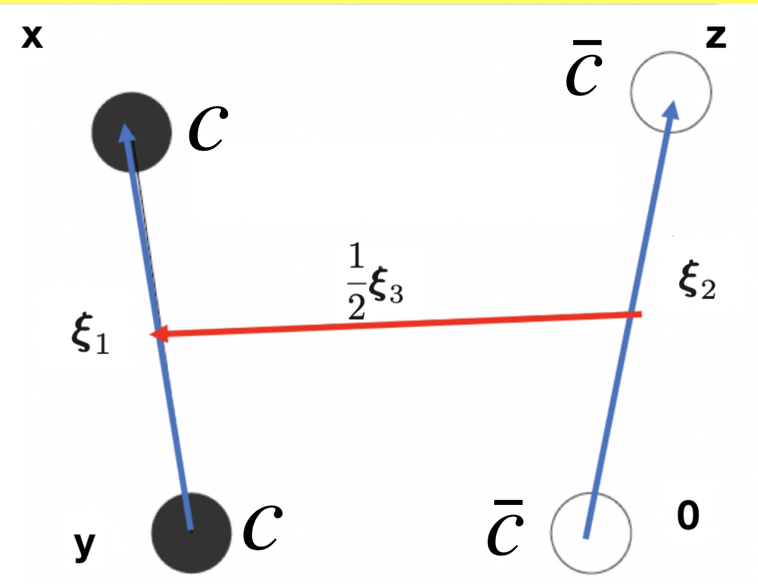
$$[cc]_{S=1}[\bar{c}\bar{c}]_{S=1}$$



S-wave, fully charm tetraquarks

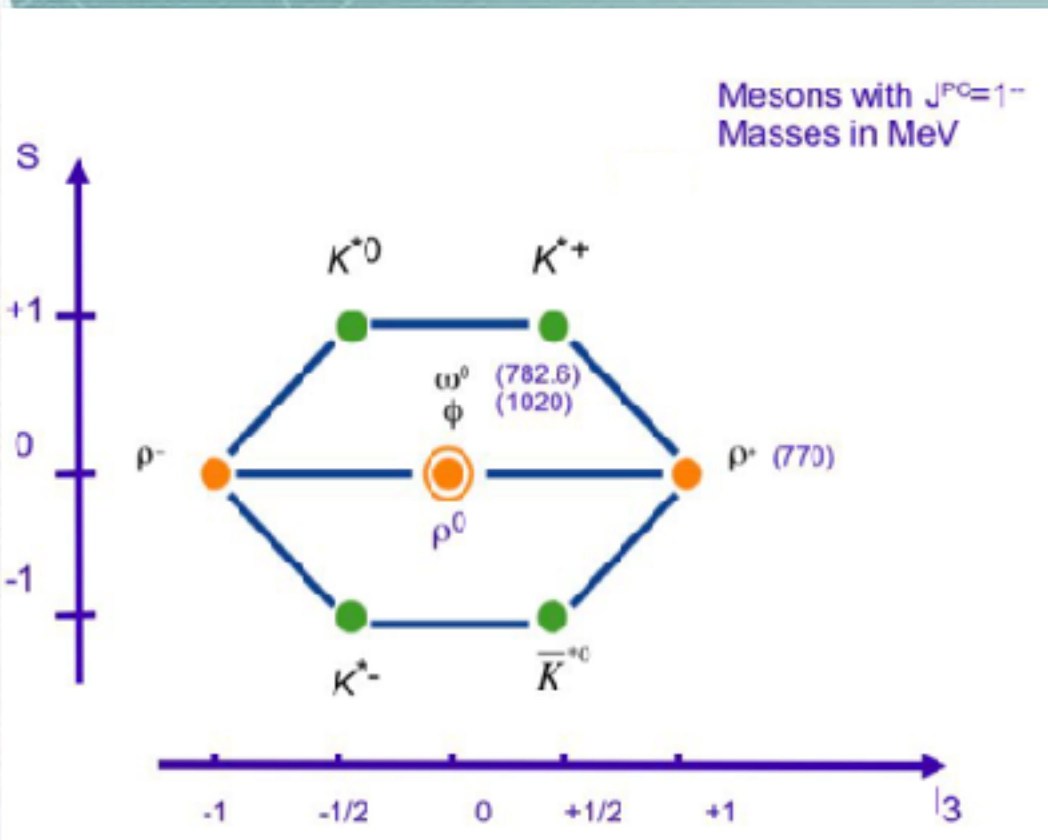
- [cc] in color $\bar{\mathbf{3}}$
- total spin of each diquark, S=1 (color antisymmetry and Fermi statistics)
- S-wave: positive parity
- C=+1 states: $J^{PC} = 0^{++}$, 2^{++}, $\rightarrow$ 2 J/ψ, 2 η_c, 2 $\chi_{c0}(1P)$, 2 $\chi_{c1,2}(1P)$, $D\bar{D}$
- C= -1 state: $J^{PC} = 1^{+-}$, $\rightarrow J/\psi + \eta_c$, $D\bar{D}$
- mass spectrum can be computed (see e.g. M.A.Bedolla, J.Ferretti, C.D.Roberts and E.Santopinto, arXiv:1911.00960 [hep-ph]:
- QCD inspired potential (Coulomb+linear potential), gaussian wave functions in the three Jacobi coordinates, ξ_1, ξ_2, ξ_3
- Urgent: measure Spin-Parity of the resonances

Jacobi coordinates in the tetraquark

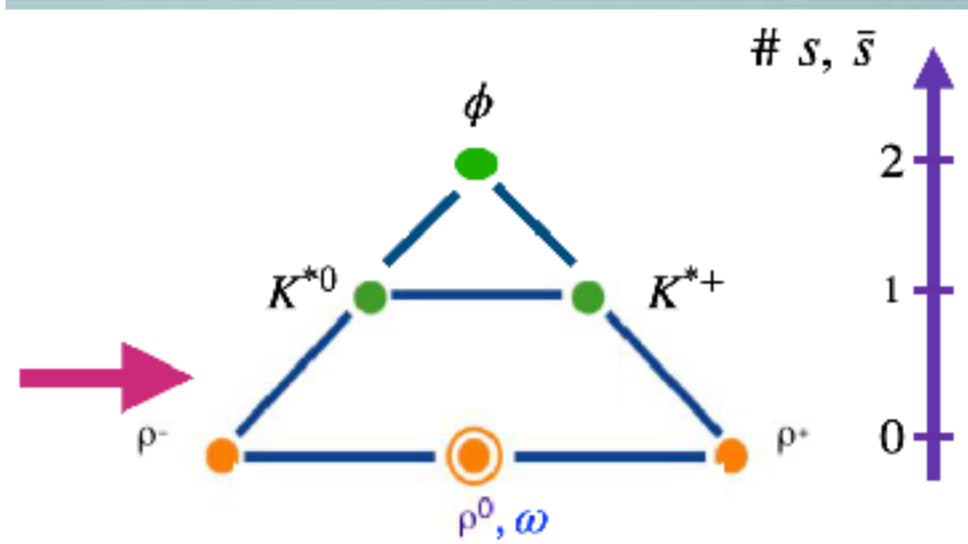


5. Hidden charm tetraquarks make Flavour SU(3) nonets

classical SU(3) nonet



can be plotted v.s. number of strange quarks/antiquarks



Nonet Mixing: mass differences $\propto (n_s + n_{\bar{s}})(m_s - m_u)$

Equal Spacing rule:

$$\frac{\rho(775) + \phi(1020)}{2} - K^*(892) \sim 6 \text{ MeV}$$

$$\phi(1020) - \rho(775) \sim 244 \text{ MeV}$$

The lightest $J^P = 1^+$ tetraquarks fall into two different nonets

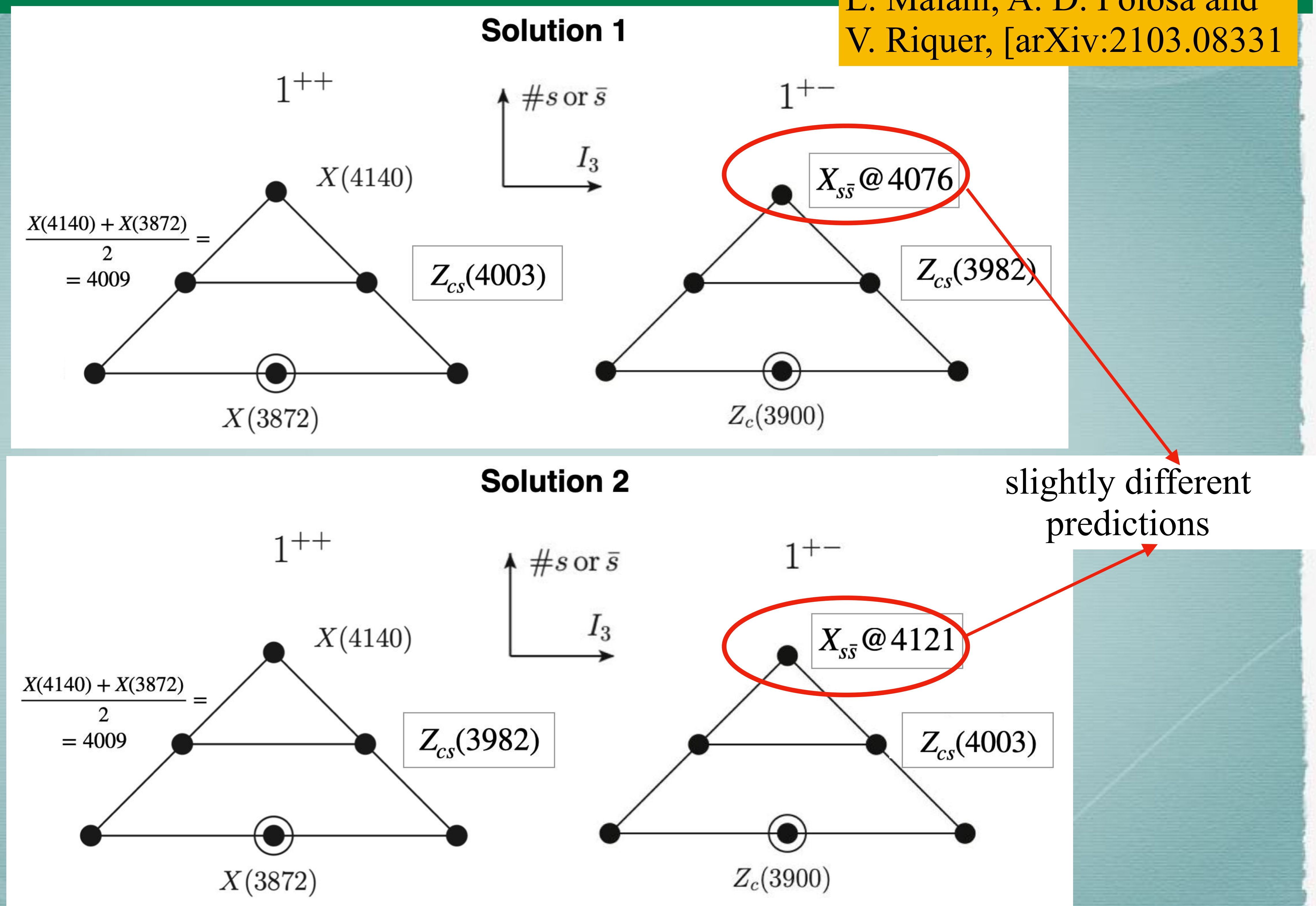
- X(3872) & X(4140) (the lowest $J/\Psi\ \phi$ resonance) belong to one ($J^{PC} = 1^{++}$) nonet, as indicated by the mass difference:

$$Z_c(4140) - X(3872) = 275 \text{ MeV}$$

- $Z_c(3900)$ needs a second ($J^{PC} = 1^{+-}$) nonet
- $Z_{cs}(3985)$ and $Z_{cs}(4003)$ almost completely fill the two nonets…

Two solutions

L. Maiani, A. D. Polosa and V. Riquer, [arXiv:2103.08331



Solution 1 is favoured. The scale of the mass differences is in line for both

a well identified shopping list

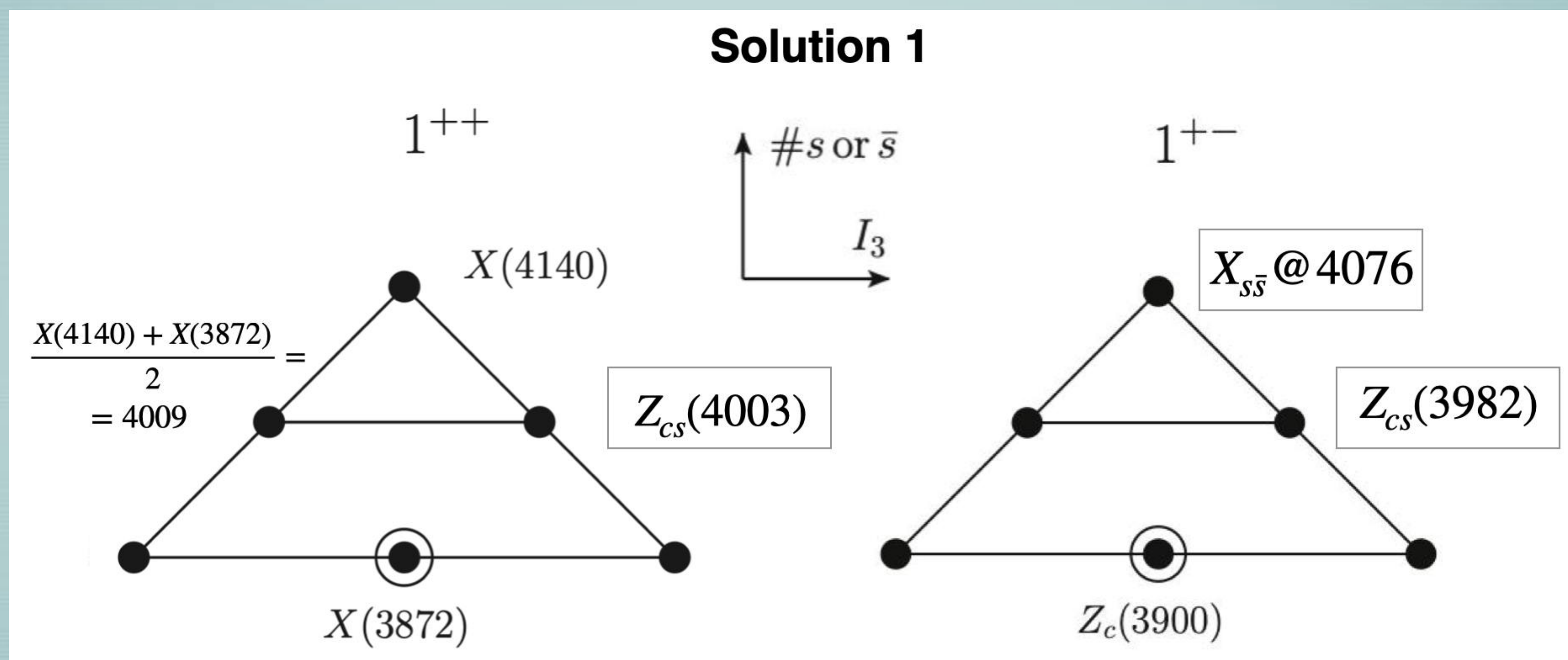
- $X_{s\bar{s}}$, $M = 4076$ or 4121, decays: $\eta_c\ \phi$, $D_s^* \bar{D}_s$:
- the I=1 partners of X(3872), decaying into $J/\Psi + \rho^{\pm}$ (see later)
- the I=0 partners of $Z_c(3900)$ and $Z_c(4020)$, possibly decaying into $J/\Psi + f_0(500)$(aka $\sigma(500)$)

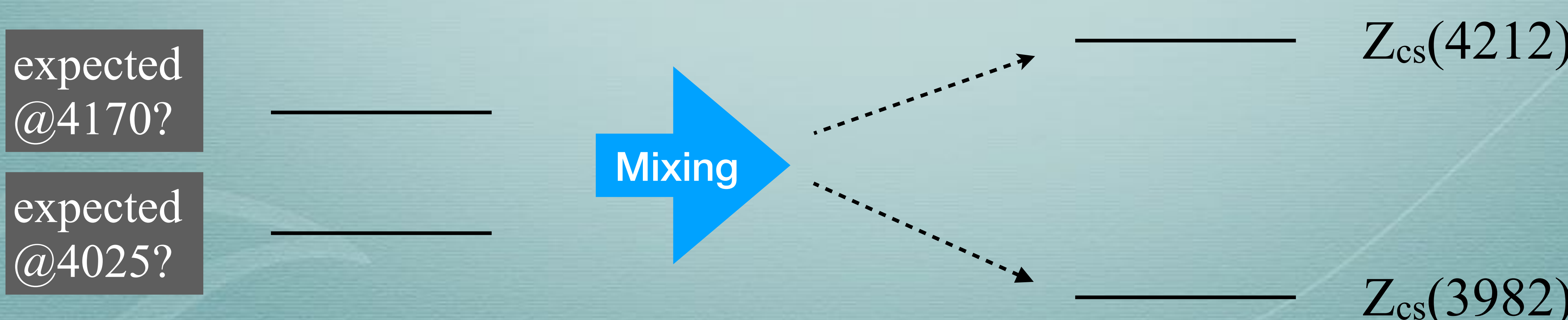
- ***There is a third nonet*** associated to $Z_c(4020)$, $J^{PC} = 1^{+-}$,
- a third Z_{cs} is required, Mass=4150 - 4170 MeV;
- ***LHCb sees indeed a $Z_{cs}(4220)$***, $J^P = 1^+$or 1^-, ***is it too heavy ?????***

A bold suggestion

- Zcs(3982) is a bit too low (Z(3900) is heavier than X(3872))
- if there is a mixing between the two $J^{PC} = 1^{+-}$ nonets:

levels repel eah other, one Z_{cs} would go down, the other would go up



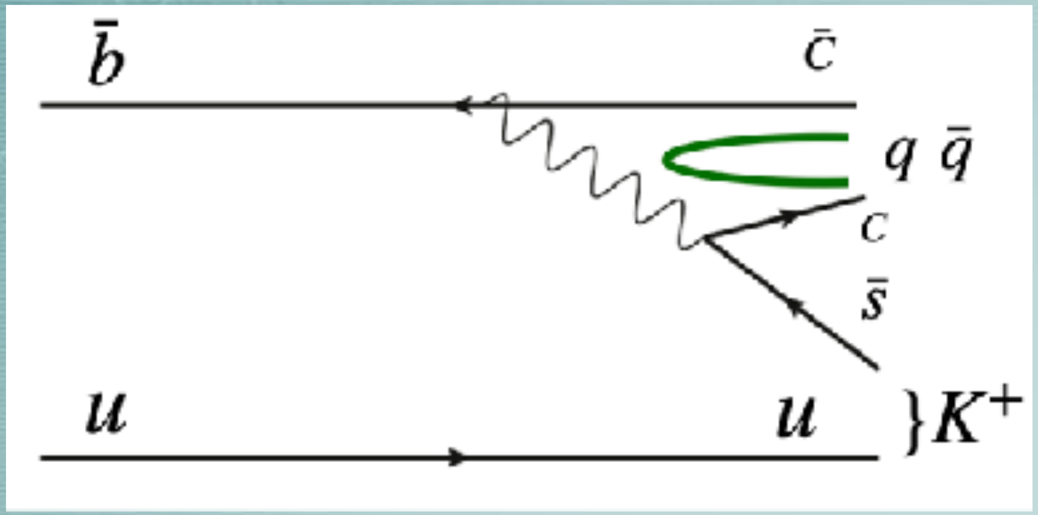


6. The doubly charmed Tetraquark, T_{cc}^{+}

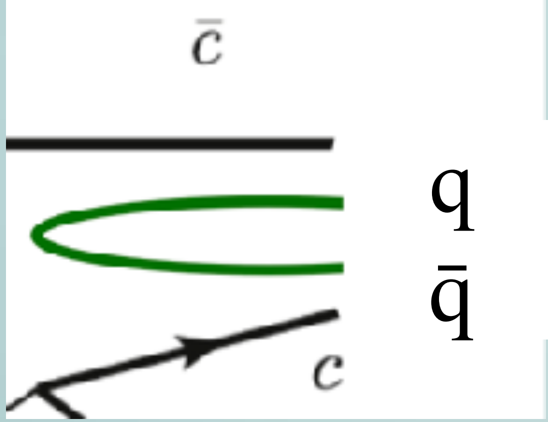
- The existence of doubly charmed tetraquarks, $[QQ\bar{q}\bar{q}]$, was considered in 2013 by Esposito et al. Esposito *et al*, PRD **88**(2013) 054029)
- Starting from the mass of the doubly charmed baryon, Karliner and Rosner estimated of the mass of the lowest lying, I=0 state at $M(T_{cc}^{+}) = 3882 \pm 12$ MeV, 7 MeV above the $D^0 D^{*+}$ threshold. M. Karliner and J. L. Rosner, PRL **119**(2017) 202001.
- A similar value was obtained by Eichten and Quigg E. J. Eichten and C. Quigg, PRL **119** (2017) 202002
- A value close to the $D^0 D^* \gamma$ threshold is obtained in the Born Approximation, using constituent quark masses derived from the meson spectrum L. Maiani et al., PRD **100** (2019) 074002
- The value $M(T_{cc}^{+}) - M(D^0 D^{*+}) = -23 \pm 11$ MeV is obtained in lattice QCD calculation P. Junnarkar *et al*, PRD **99**(2019) 034507
- The closeness to the $D^0 D^{*+}$ threshold has nonetheless invited speculations about a molecular, $D^0 D^{*+}$, nature of T_{cc}^{+}.

6. Molecule or compact? Back to the fundamentals

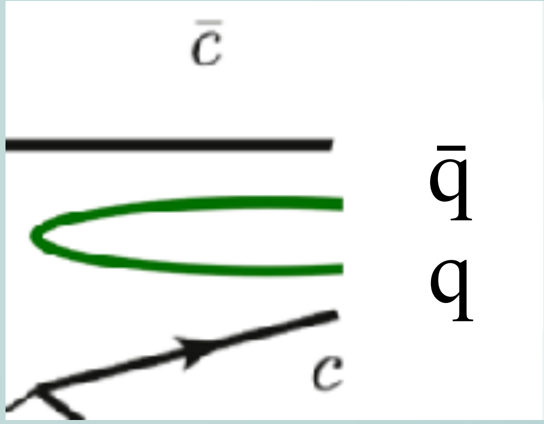
- Consider $B^+ \to K^+X$, $X \to D^*\bar{D} + \bar{D}^*D$



- The interesting part is the upper left corner, which we can specify in two ways



or



or both?

$$[\bar{C}q]^1[C\bar{q}]^1 \to D^*\bar{D} + \bar{D}^*D \qquad [Cq]^{\bar{3}}\,[\bar{C}\bar{q}]_3 \to D^*\bar{D} + \bar{D}^*D$$

- for a neutron-proton pair, the question posed by Weinberg was;
is the deuteron a bound state or is there an "elementary" dibaryon ?

Molecule or compact? the QCD framework

- We know for sure that QCD produces hidden charm, confined hadron states: charmonia, $D^*\bar{D} + \bar{D}^*D$: do confined tetraquarks exist??
- Suppose we switch off the interactions between confined hadrons. The space of possible hidden charm states is made by two components
  - -discrete energy states: charmonia and possibly tetraquarks: $|C><C| + |T><T|$
  - _continuum charmed meson pairs: $\int d\alpha |D^*\bar{D}(\alpha)><D^*\bar{D}(\alpha)|$

$$\text{In this limit: } |<X|X>|^2 = 1 = Z + \int d\alpha\ |<X|D^*\bar{D}(\alpha)>|^2, \text{ where,}$$

$$Z = |<X|C>|^2 + |<X|T>|^2$$

- Z=0: corresponds to a pure molecular state: X results from $D^* - \bar{D}$ interactions only (like the deuteron)
- $Z \neq 0$: some compact, discrete state ***must*** exist
- unlike for charmonium states, X decays violate isospin: $\Gamma(\Psi\rho) \sim \Gamma(\Psi\omega)$
- ➜***Tetraquark most likely exists***.

How can we know?

- The key is the $D^*\bar{D}$ scattering amplitude, *f*, that near threshold (k=center of mass momentum~0) can be written as

$$1/f = k \cot \delta(k) - ik = -\kappa_0 + \frac{1}{2} r_0 k^2 - ik + \ldots$$

- With Weinberg, we find

$$\kappa_0^{-1} = 2\frac{1-Z}{2-Z}\kappa^{-1} + O(1/m_\pi);\ \ r_0 = -\frac{Z}{1-Z}\kappa^{-1} + O(1/m_\pi)$$

$\kappa^{-1} = \sqrt{2\mu B}$, $B = M(D^*) + M(D) - M(X)$ (the "binding energy")

- It turns out that ***the parameters*** κ_0, r_0 ***can be determined from the X(3872) (or*** T^+_{cc}***) line-shape***

R. Aaij *et al*. (LHCb), PRD **102**, 092005 (2020

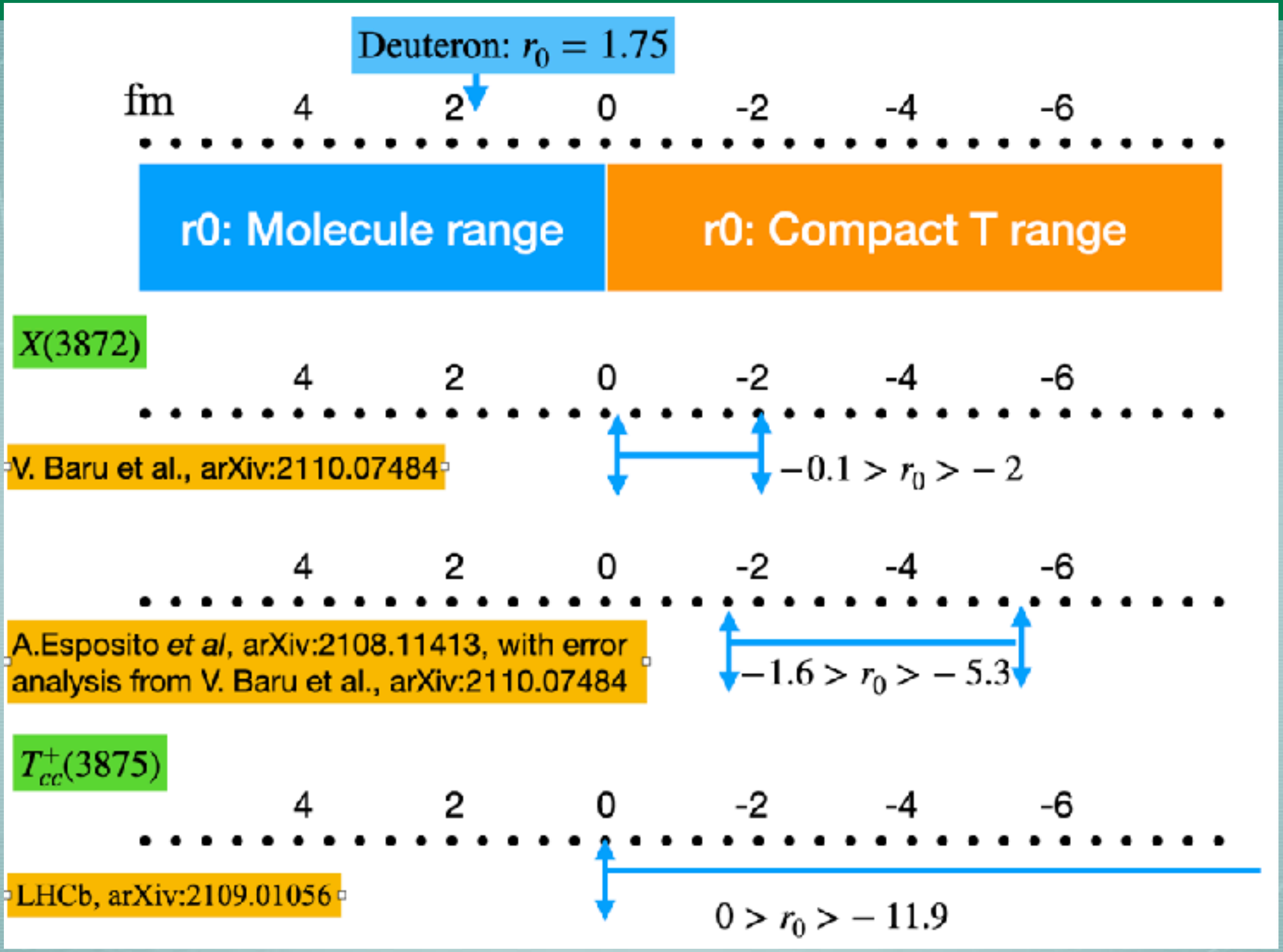
- in the molecular case (Z=0) one has $r_0 = O(1/m_\pi)$
- to this we added a little theorem by Landau and Smorodinsky that says that:

A.Esposito et al., arXiv:2108.11413

for a bound state with an attractive potential, $r_0 > 0$

- reassuringly: $r_0(\text{deuteron}) = +1.75\ \text{fm}$

r_0 : My Summary



A new analysis by the Valencia group claims $r_0 \sim +1$ fm for T_{cc}^{+}.

No consensus yet, but it seems we are on a very promising road. Stay tuned!!