Stability properties of rotating black holes with and without a positive cosmological constant

Marc Casals

Centro Brasileiro de Pesquisas Físicas (CBPF, Brazil)

University College Dublin (Ireland)

IPhT CEA-Saclay, 9/12/2021

Outline

1. Stability of Kerr

2. Stability of Kerr-de Sitter

3. Stability of the Cauchy horizon of Kerr-Newman-de Sitter

4. Conclusion

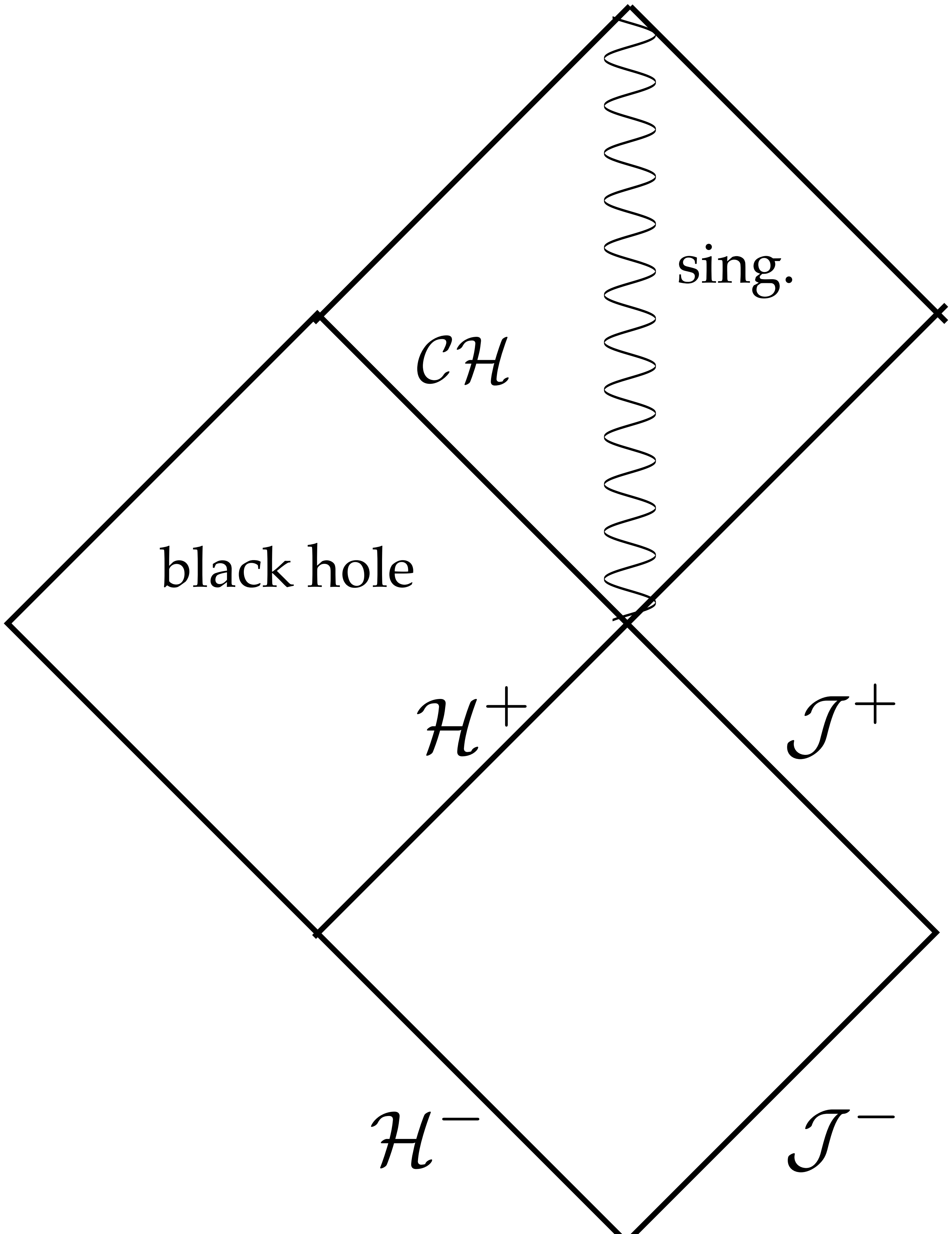
1. Stability of Kerr

2. Stability of Kerr-de Sitter

3. Stability of the Cauchy horizon of Kerr-Newman-de Sitter

4. Conclusion

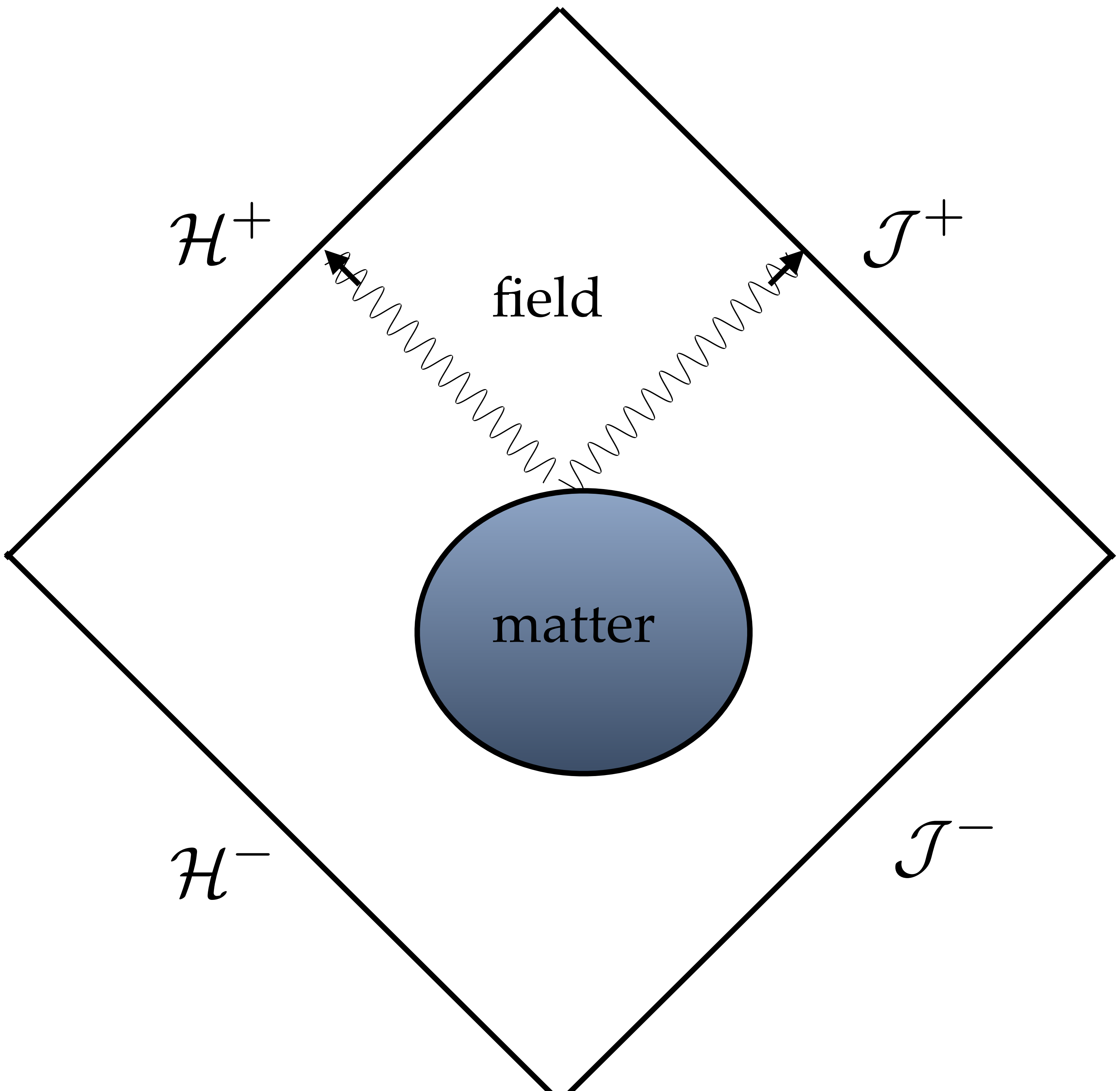
Kerr Black Hole



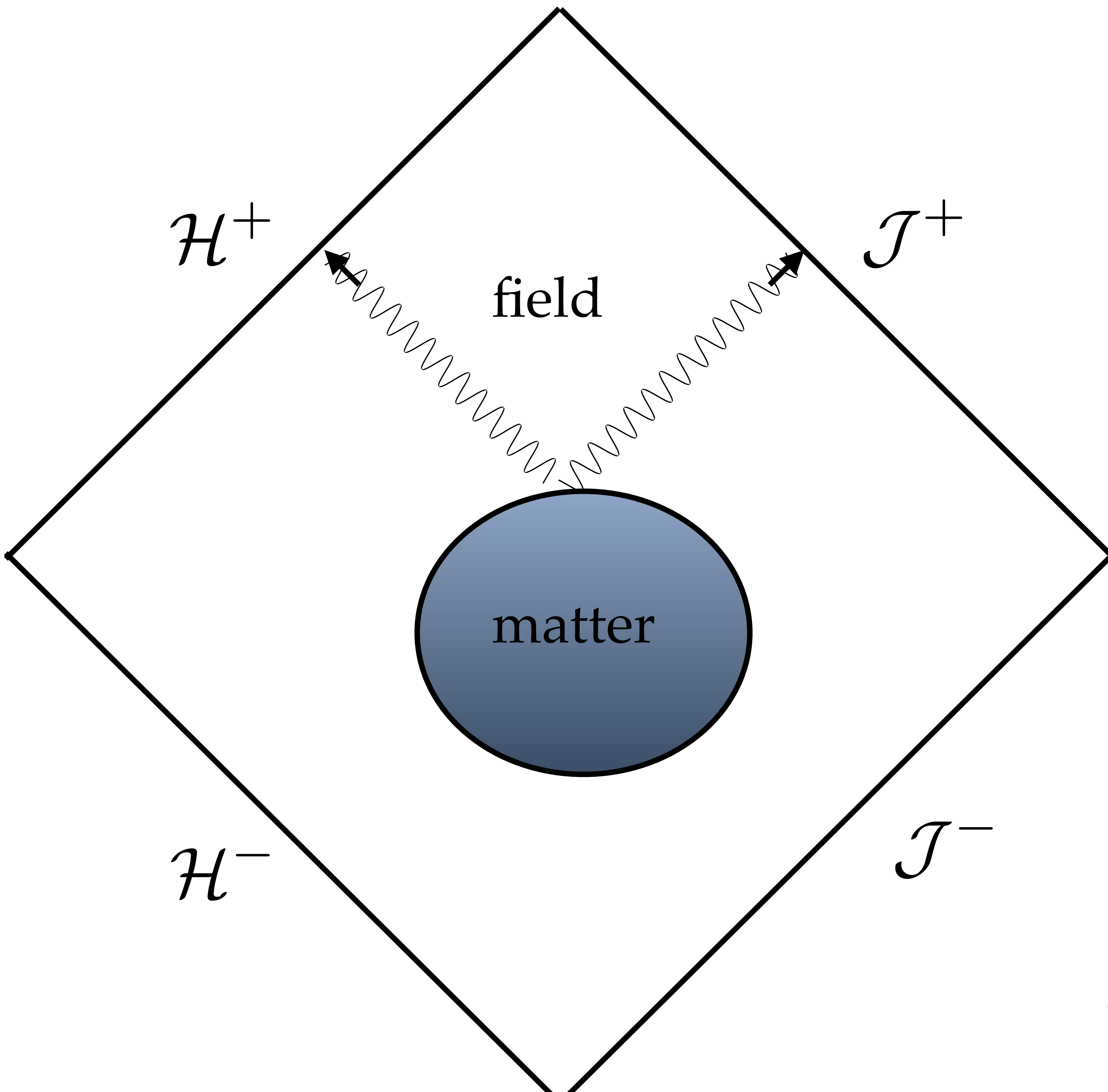
Penrose diagram of Kerr

(M, a)

BHs are not in isolation but are ‘perturbed’ by fields (scalar, fermion, electromagnetic, gravitational…) due to neighbouring matter (eg, accretion disk, another compact object, etc)



BHs are not in isolation but are ‘perturbed’ by fields (scalar, fermion, electromagnetic, gravitational…) due to neighbouring matter (eg, accretion disk, another compact object, etc)



Q: Is Kerr spacetime stable under field perturbations?

Wave Equation

We consider linear field perturbations of a fixed BH -> the fields propagate on a BH *background* $g_{\mu\nu}$

Teukolsky'73: scalar combinations of components & derivatives of the various fields (*spin* |s|=0 scalar, =1/2 neutrino, =1 emag for Faraday tensor, =2 grav for Weyl tensor) obey a wave-like eq.:

$$\hat{\mathcal{O}}\,\psi(x) = T(x)$$

field scalar

source of field

Green Function

A crucial object is the retarded Green function

$$\hat{\mathcal{O}}\, G_{ret}(x, x') = \delta_4(x, x') \qquad \text{with causal b.c.}$$

Green Function

A crucial object is the retarded Green function

$$\hat{\mathcal{O}}\, G_{ret}(x,x') = \delta_4(x,x') \qquad \text{with causal b.c.}$$

GF determines evolution in time of any initial field configuration

$$\psi(x) = \int_{t=0} \left[G_{ret}(x,x')\dot{\psi}^{ic}\,(\vec{x}') + \psi^{ic}\,(\vec{x}')\,\partial_t G_{ret}(x,x') \right] d^3\vec{x}'$$

Green Function

A crucial object is the retarded Green function

$$\hat{\mathcal{O}}\, G_{ret}(x,x') = \delta_4(x,x') \qquad \text{with causal b.c.}$$

GF determines evolution in time of any initial field configuration

$$\psi(x) = \int_{t=0} \left[G_{ret}(x,x')\dot{\psi}^{ic}\left(\vec{x}'\right) + \psi^{ic}\left(\vec{x}'\right)\partial_t G_{ret}(x,x') \right] d^3\vec{x}'$$

GF can be calculated by decomposing into spheroidal harmonics and Fourier modes:

$$G_{ret} = \sum_{\ell,m} \int_{-\infty}^{\infty} d\omega\ e^{im\varphi - i\omega t} S_{\ell m\omega}(\theta) S_{\ell m\omega}(\theta') G_{\ell m\omega}(r,r')$$

the Fourier modes satisfy a radial ODE

Mode solutions

Mode slns. correspond to frequencies $\omega_{\ell mn} \in \mathbb{C}$ which are *poles* of the GF Fourier modes:

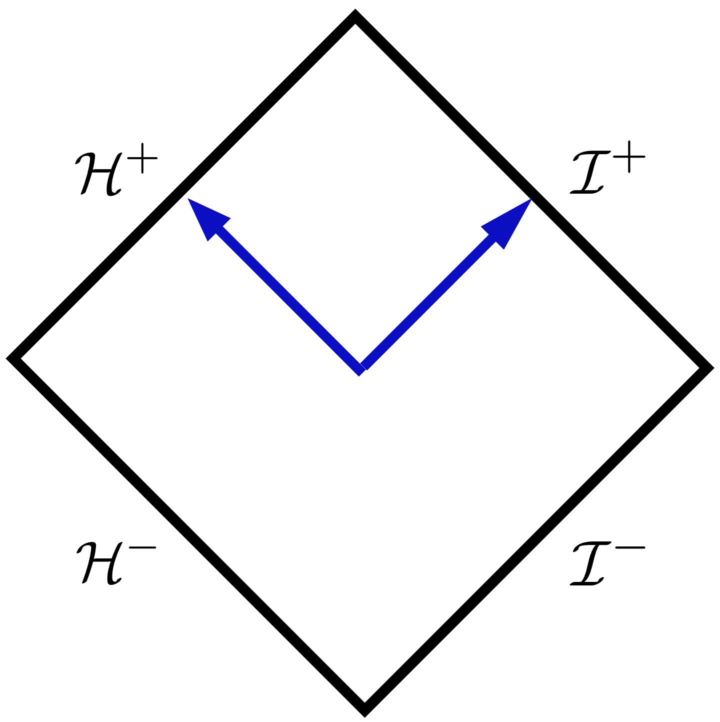
$$G_{\ell m\omega}\big|_{\omega=\omega_{\ell mn}} = \infty$$

Mode solutions

Mode slns. correspond to frequencies $\omega_{\ell m n} \in \mathbb{C}$ which are *poles* of the GF Fourier modes:

$$G_{\ell m \omega}\big|_{\omega=\omega_{\ell m n}} = \infty$$

They're waves that are purely ingoing into the horizon and purely ougoing at infinity:

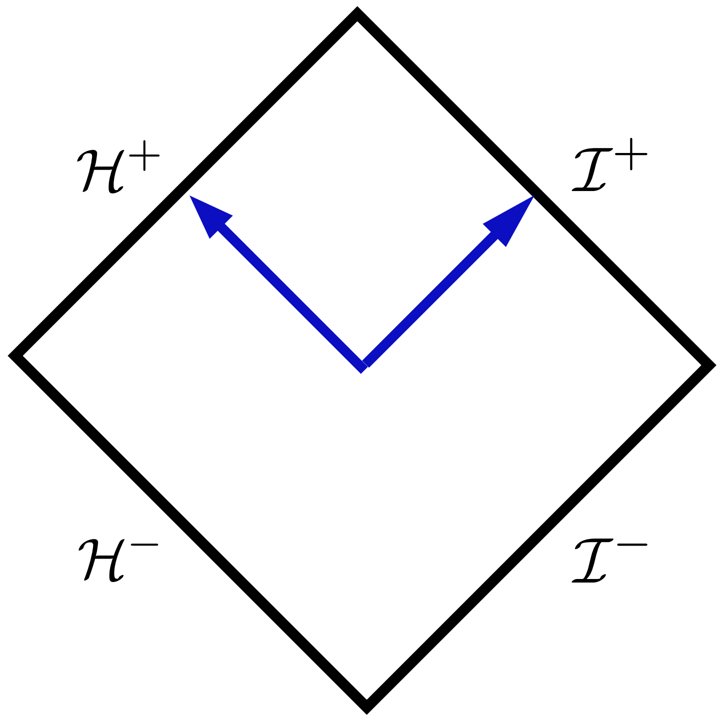


Mode solutions

Mode slns. correspond to frequencies $\omega_{\ell mn} \in \mathbb{C}$ which are *poles* of the GF Fourier modes:

$$G_{\ell m\omega}\big|_{\omega=\omega_{\ell mn}} = \infty$$

They're waves that are purely ingoing into the horizon and purely ougoing at infinity:



Calculate them by solving a continued fraction eq. on $\omega = \omega_{\ell mn}$

$$0 = \alpha_0^{(0)} + \cfrac{-\alpha_0^{(+1)}\alpha_1^{(-1)}}{\alpha_1^{(0)} + \cfrac{-\alpha_1^{(+1)}\alpha_2^{(-1)}}{\alpha_2^{(0)} + \cfrac{-\alpha_2^{(+1)}\alpha_3^{(-1)}}{\cdots}}}$$

$\alpha_n^{(\pm 1,0)}$ depend on M, a, ω, ℓ, m

Mode Stability of Subextremal Kerr

Time dependence: $e^{-i\omega_{\ell m n} t}$

$$\Big\downarrow \; t \to +\infty$$

$$\begin{cases} \text{If } Im(\omega_{\ell mn}) < 0 \text{: exponentially damped (quasinormal modes, QNMs)} \\ \text{If } Im(\omega_{\ell mn}) > 0 \text{: exponentially growing (unstable modes)} \end{cases}$$

Mode Stability of Subextremal Kerr

Time dependence: $e^{-i\omega_{\ell mn}t}$

$$\Big\downarrow \; t \to +\infty$$

$$\begin{cases} \text{If } Im(\omega_{\ell mn}) < 0 \text{: exponentially damped (quasinormal modes, QNMs)} \\ \text{If } Im(\omega_{\ell mn}) > 0 \text{: exponentially growing (unstable modes)} \end{cases}$$

Q: Are there any unstable modes ($Im(\omega_{\ell mn}) > 0$) in Kerr?

Mode Stability of Subextremal Kerr

Time dependence: $e^{-i\omega_{\ell mn}t}$

$$\Big\downarrow \; t \to +\infty$$

$$\begin{cases} \text{If } Im(\omega_{\ell mn}) < 0\text{: exponentially damped (quasinormal modes, QNMs)} \\ \text{If } Im(\omega_{\ell mn}) > 0\text{: exponentially growing (unstable modes)} \end{cases}$$

Q: Are there any unstable modes ($Im(\omega_{\ell mn}) > 0$) in Kerr?

No, for massless general-spin fields in Kerr => Kerr is mode-stable

(Easy proof for superradiant frequencies $0 < \mathrm{Re}(\omega) < |m|\Omega_+$ and Whiting'89 proved it for arbitrary ω)

Instabilities in Other Settings

- Kerr BH with event horizon removed by a "mirror" (may model wormholes) has unstable modes (Friedman'78)

Instability timescale ~ secs for supermassive wormholes (eg, Cardoso et al'08)

- All so far has been for *massless* fields. But Kerr is unstable under massive field perturbations (Damour et al'76)

Instability timescale has been used to constrain masses of fields, eg, mass of Proca field $\lesssim 4 \times 10^{-22}$ eV (Pani et al'12)

Stability Properties of *Extremal* Kerr

All results so far were for *subextremal* Kerr

In extremal Kerr ($a = M$) :

Field (& derivatives) off the horizon $\mathcal{H}$ decays

and

There're no exponentially-growing modes for massless general-spin fields in extremal Kerr (Teixeira da Costa'20, Casals&Longo´19)

Stability Properties of *Extremal* Kerr

All results so far were for *subextremal* Kerr

In extremal Kerr ($a = M$) :

Field (& derivatives) off the horizon $\mathcal{H}$ decays

and

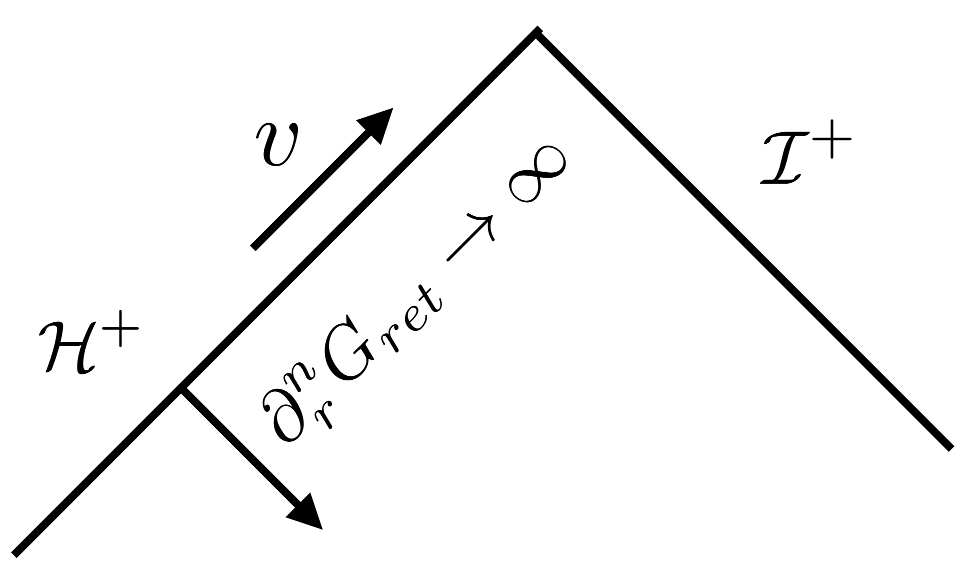
There're no exponentially-growing modes for massless general-spin fields in extremal Kerr (Teixeira da Costa'20, Casals&Longo´19)

And yet…

Transverse nth-derivative on horizon grows as

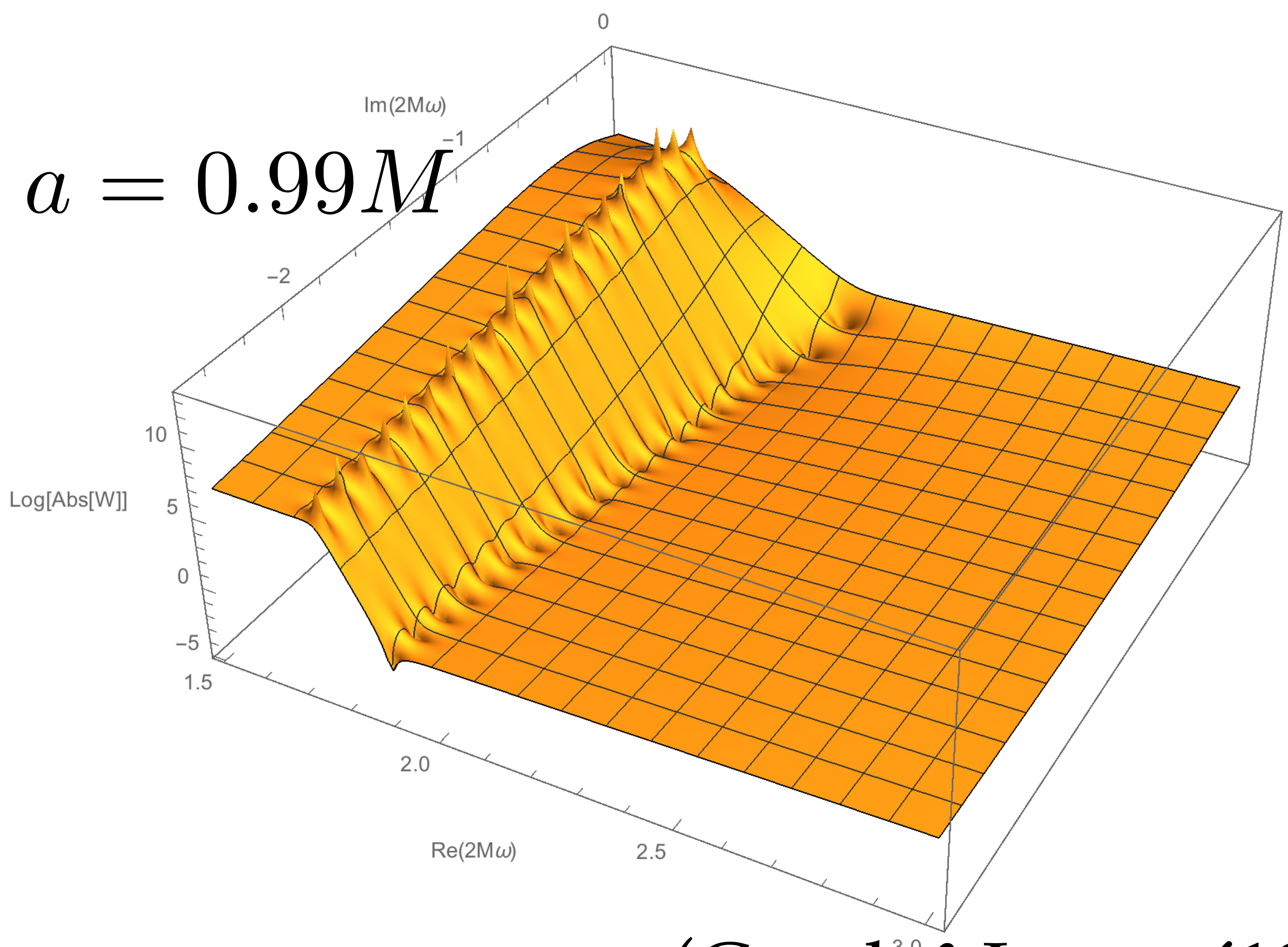
$$\left(\partial_r^n G_{ret}\right)\big|_{\mathcal{H}} \sim v^{n-s-1/2} \quad \text{as} \quad v \to \infty$$

(Casals,Gralla&Zimmerman'16, Aretakis'10)



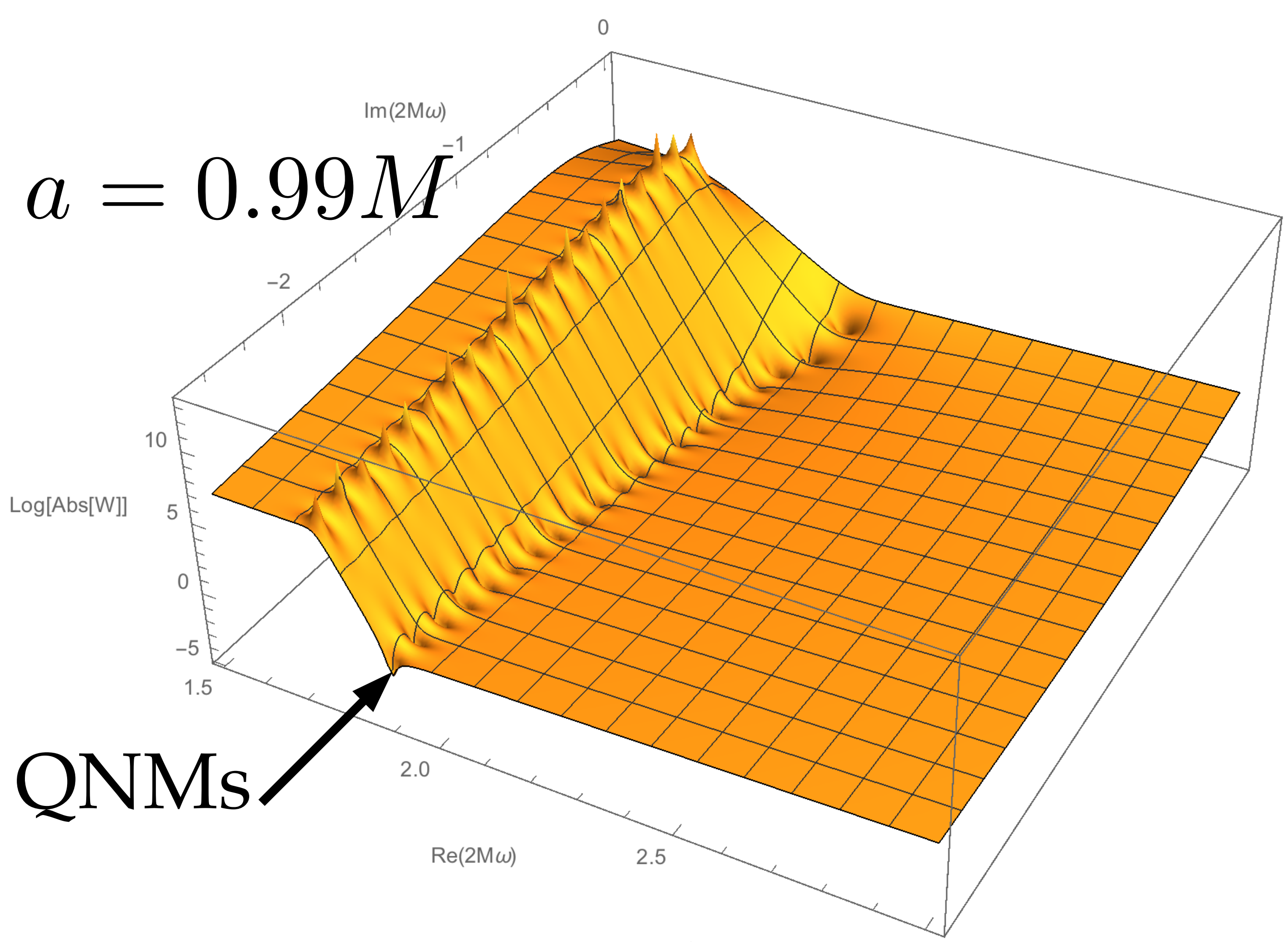
It’s due to a branch cut that forms at the superradiant-bound frequency $\omega = m\Omega_+$ by accumulation of QNMs as $a \to M$ (Detweiler’80)

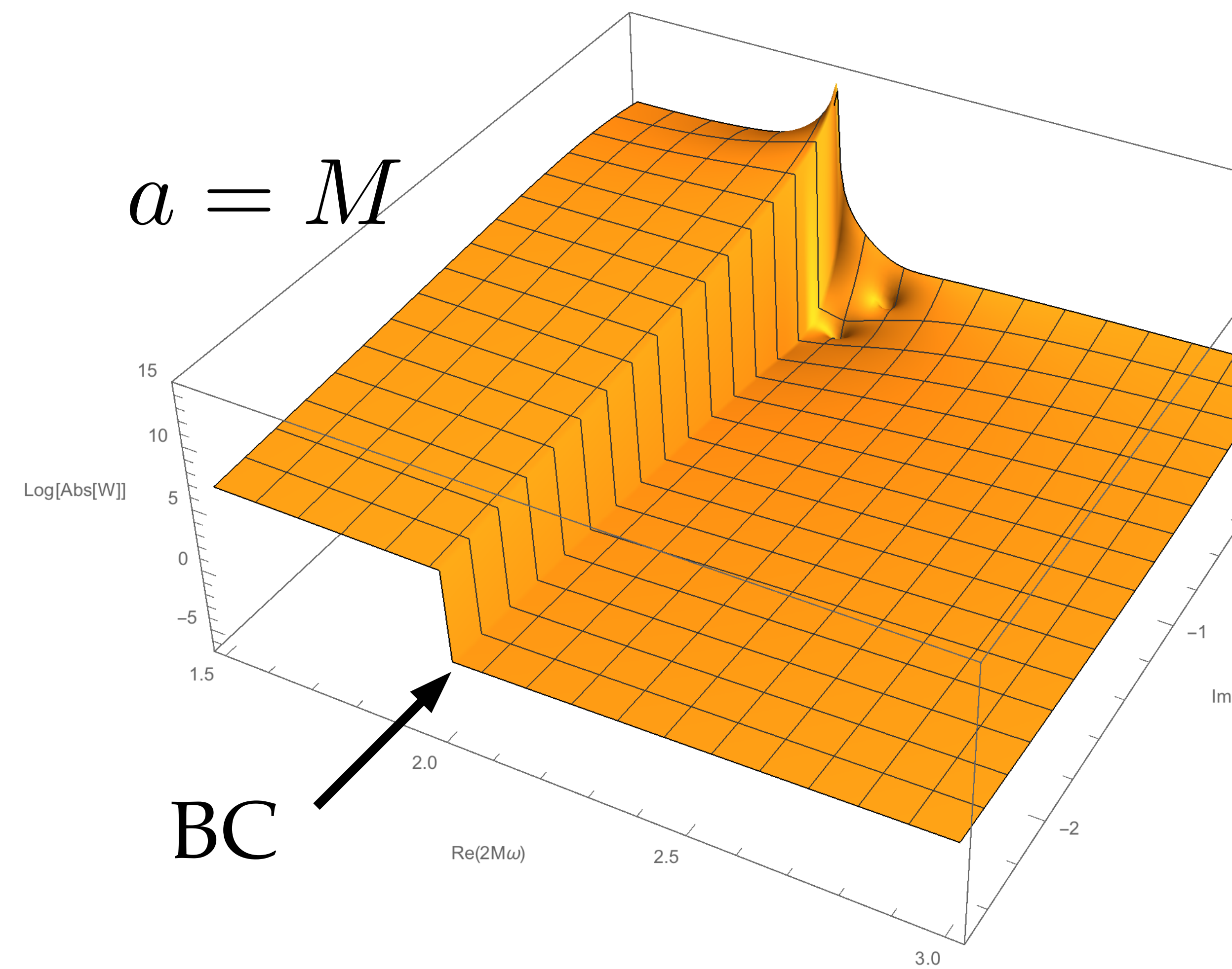
$a = 0.99M$



(Casals&Longo´19)

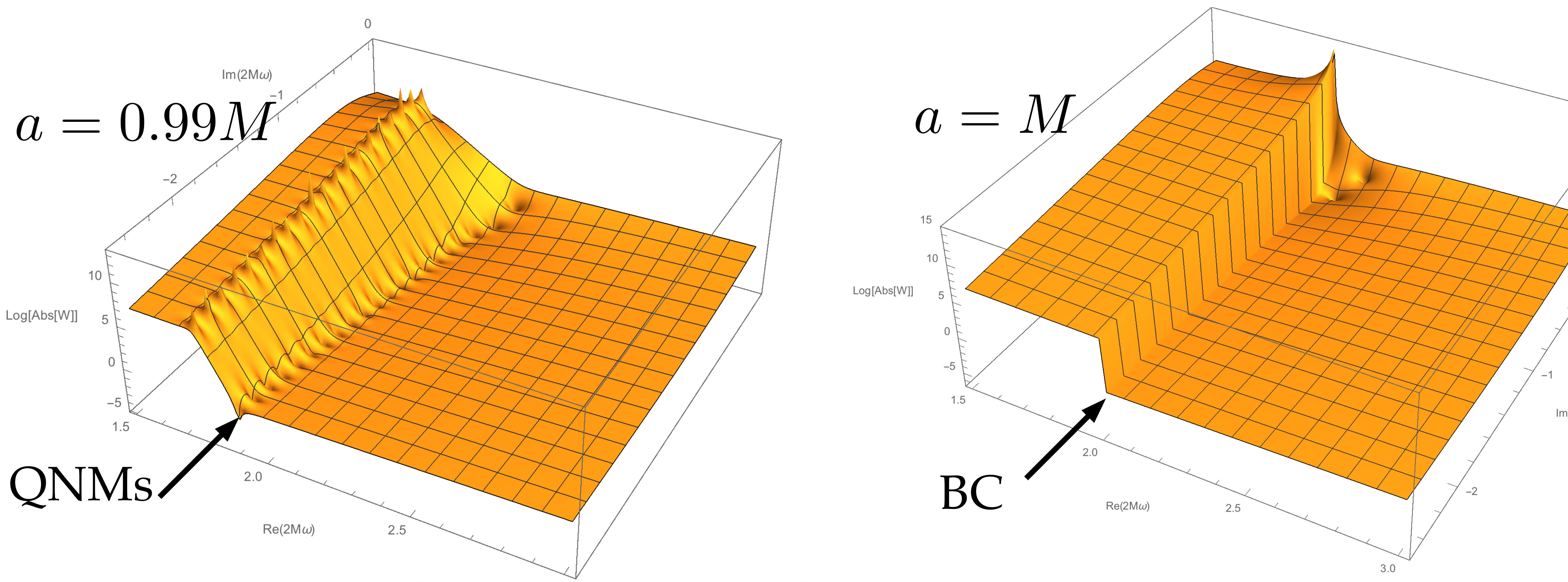
It’s due to a branch cut that forms at the superradiant-bound frequency $\omega = m\Omega_+$ by accumulation of QNMs as $a \to M$ (Detweiler’80)





(Casals&Longo´19)

It's due to a branch cut that forms at the superradiant-bound frequency $\omega = m\Omega_+$ by accumulation of QNMs as $a \to M$ (Detweiler'80)



(Casals&Longo´19)

However, Observer-independent scalars (such as $(\nabla_\alpha G_{ret} \cdot \nabla^\alpha G_{ret})|_{\mathcal{H}}$) all decay (Burko&Khanna'17)

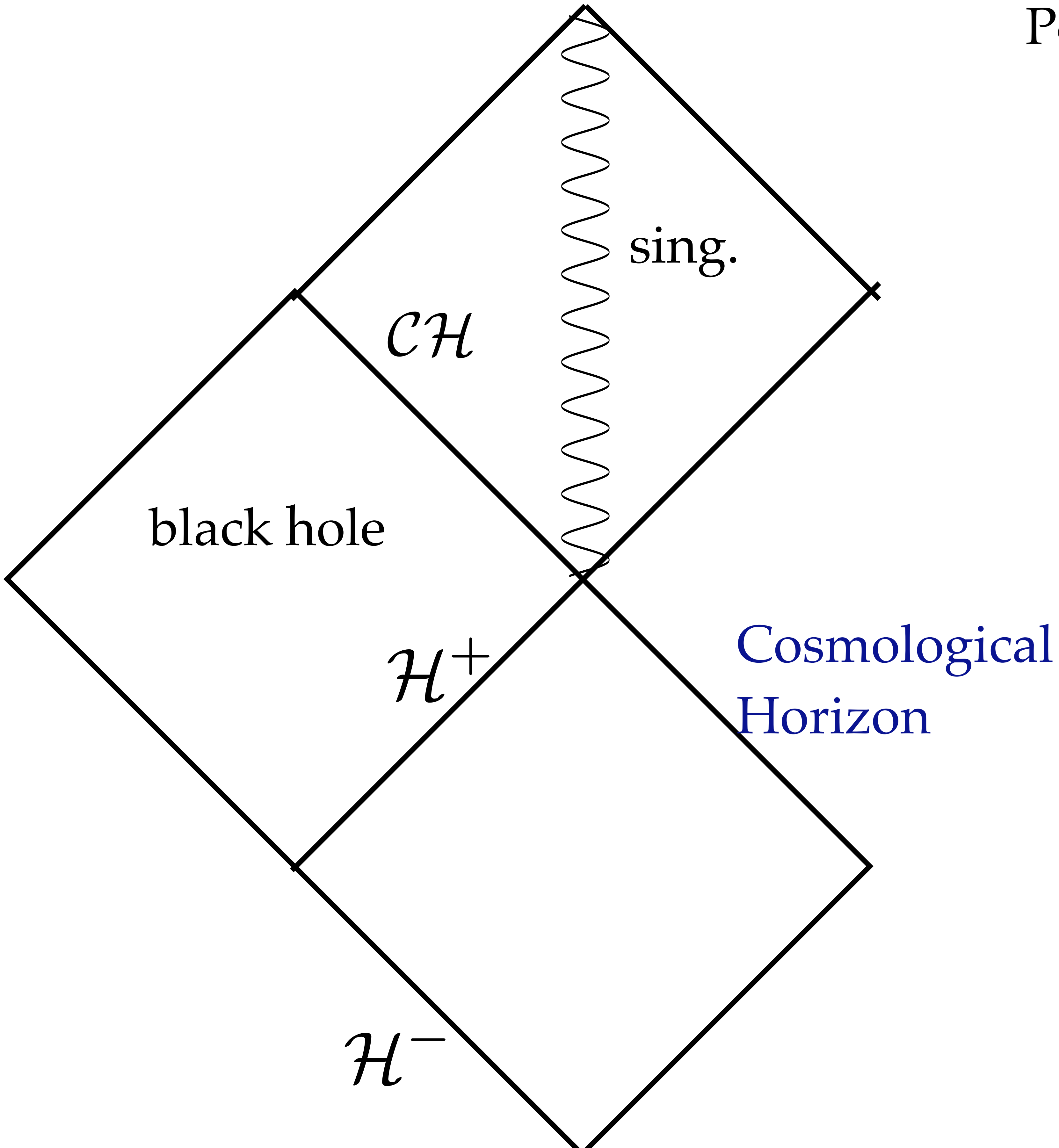
1. Stability of Kerr

2. Stability of Kerr-de Sitter

3. Stability of the Cauchy horizon of Kerr-Newman-de Sitter

4. Conclusion

Kerr-de Sitter Black Hole



Penrose diagram of Kerr-dS

(M, a, Λ)

Cosmological
const.

Casals&Teixeira da Costa'21: continued fraction eq. for mode freqs. of Kerr-dS

$$0 = \alpha_0^{(0)} + \cfrac{-\alpha_0^{(+1)}\alpha_1^{(-1)}}{\alpha_1^{(0)} + \cfrac{-\alpha_1^{(+1)}\alpha_2^{(-1)}}{\alpha_2^{(0)} + \cfrac{-\alpha_2^{(+1)}\alpha_3^{(-1)}}{\cdots}}}$$

is symmetric under the exchange of any pair of (SQCD) "masses":

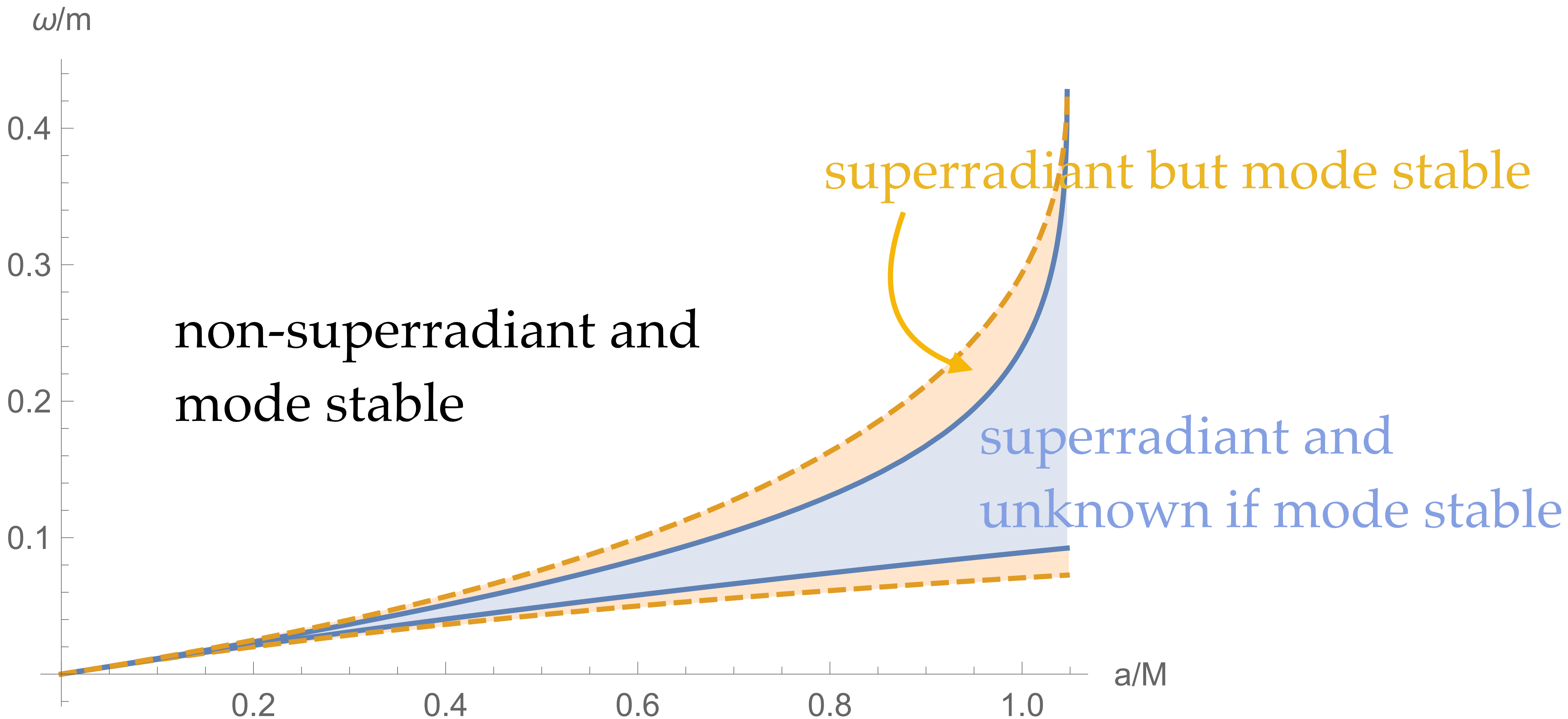
$$m_1 = s - \xi_- + \xi_+ \qquad m_3 = -s - \xi_- + \xi_+$$

$$m_2 = \xi_- + \xi_+ \qquad m_4 = \xi_- - \xi_+ + 2\xi_c$$

where $\xi_j \equiv i\,\dfrac{\omega - m\,\Omega_j}{2\,\kappa_j}$

Therefore, the mode freqs. $\omega_{\ell m n}$ are also symmetric under $m_i \leftrightarrow m_j$

$m_2 \leftrightarrow m_3$ allowed us to exclude unstable modes from a subregion of the superradiant frequency regime



Open question: exclude unstable modes from the blue region so as to prove the mode stability of Kerr-dS

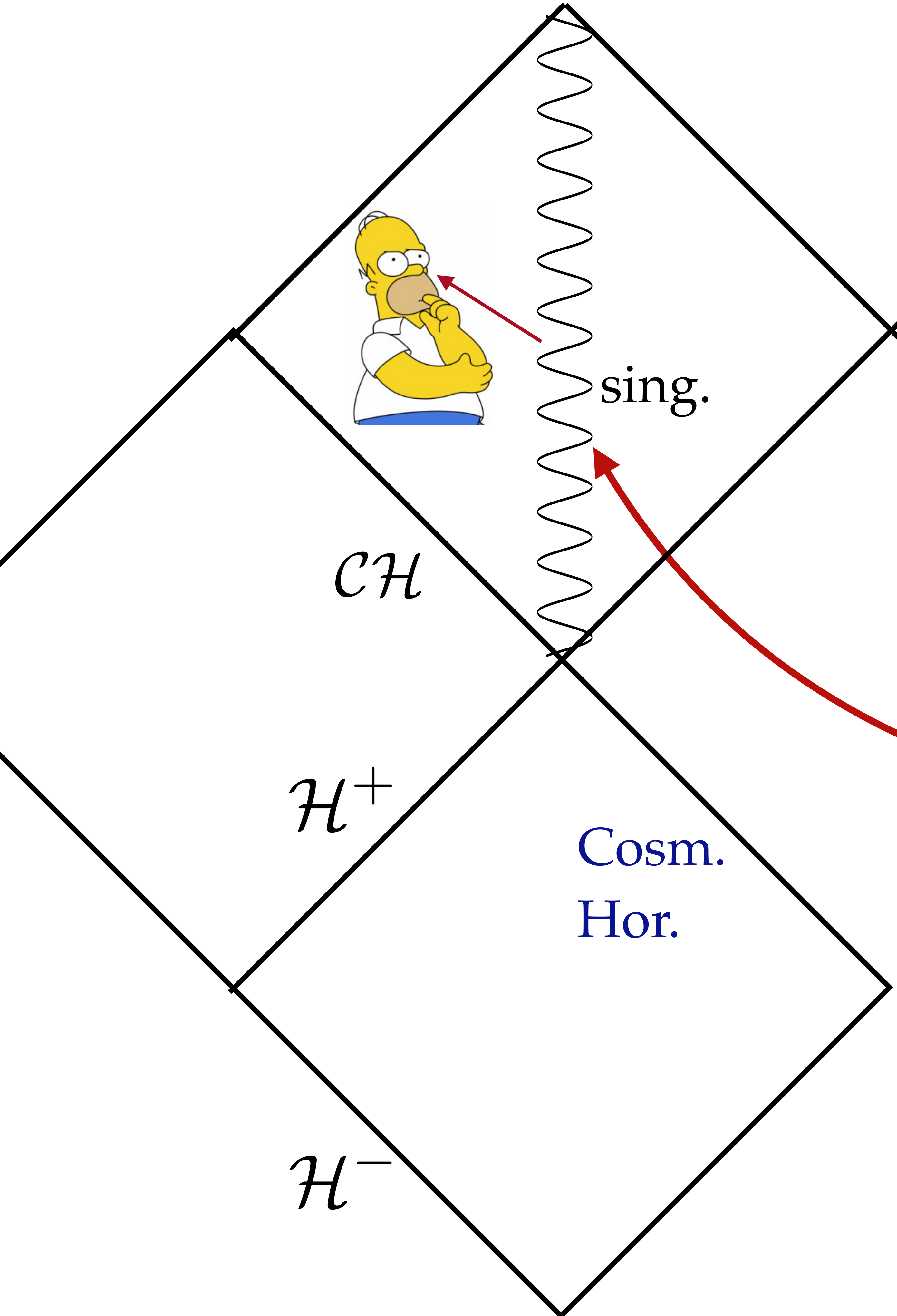
1. Stability of Kerr

2. Mode stability of Kerr-de Sitter

3. Mode stability of the CH of Kerr-Newman-dS

4. Conclusion

Loss of predicability inside the BH



Penrose diagram of Kerr-Newman-dS is like Kerr-dS's

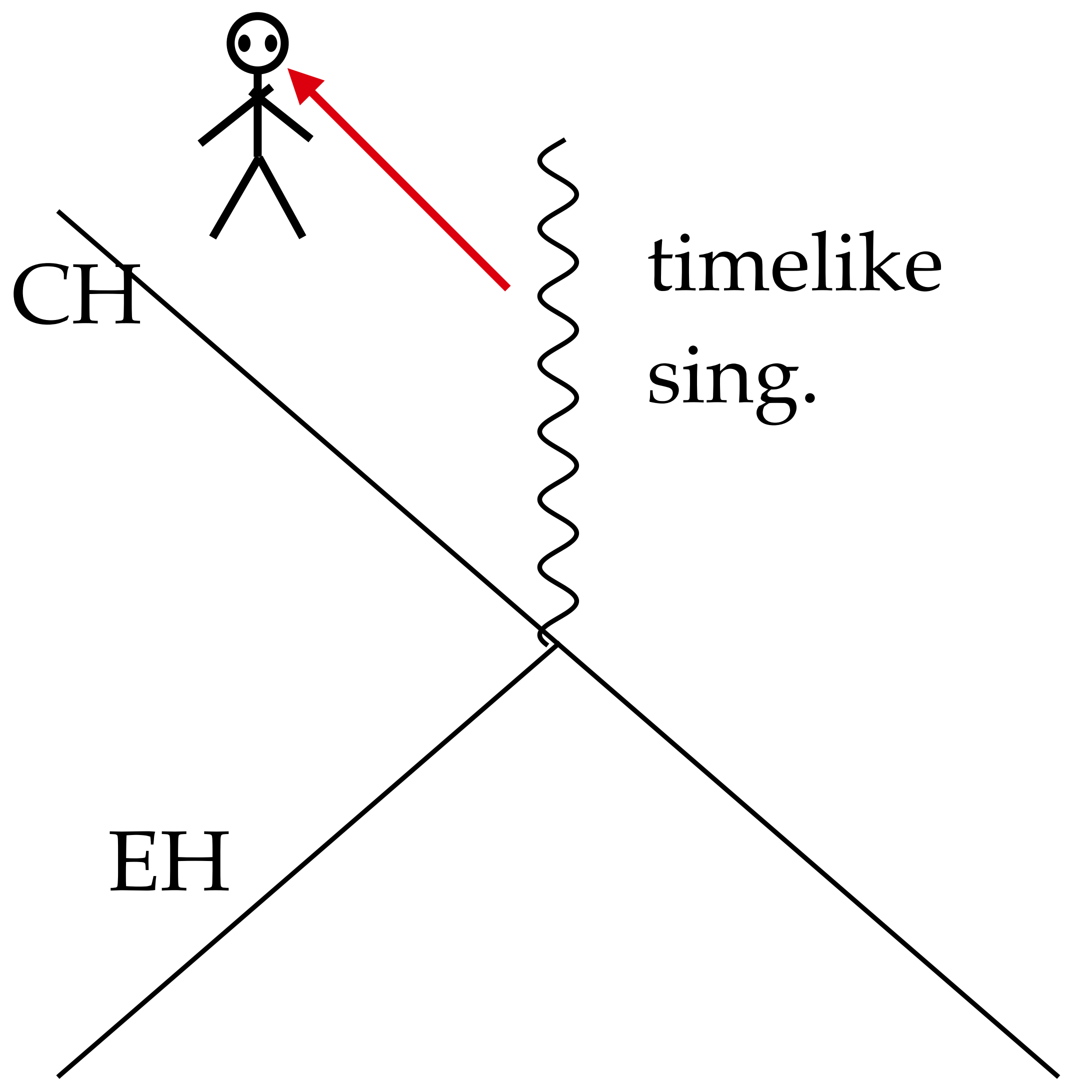
this is a *timelike* singularity, and so it's *visible* to an observer going into the BH

Unpredictability: the Initial Value Problem is not well posed

Strong Cosmic Censorship Hypothesis

SCC hypothesis by Penrose'72: if singularities exist inside BHs that exist in Nature, they're not visible even to observers inside (i.e., not timelike)

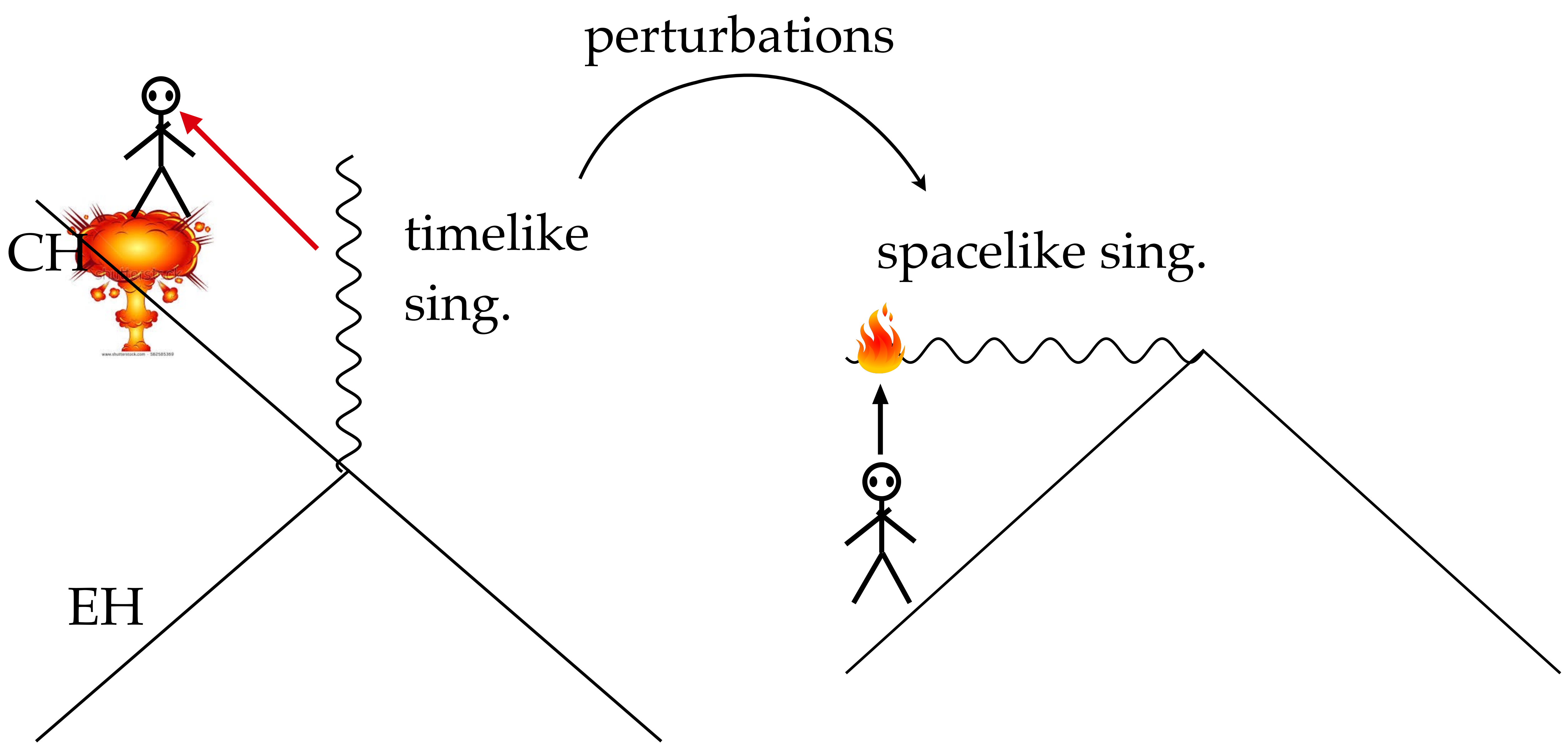
SCC is upheld if, eg, CH is "destroyed" by field perturbations:



Strong Cosmic Censorship Hypothesis

SCC hypothesis by Penrose'72: if singularities exist inside BHs that exist in Nature, they're not visible even to observers inside (i.e., not timelike)

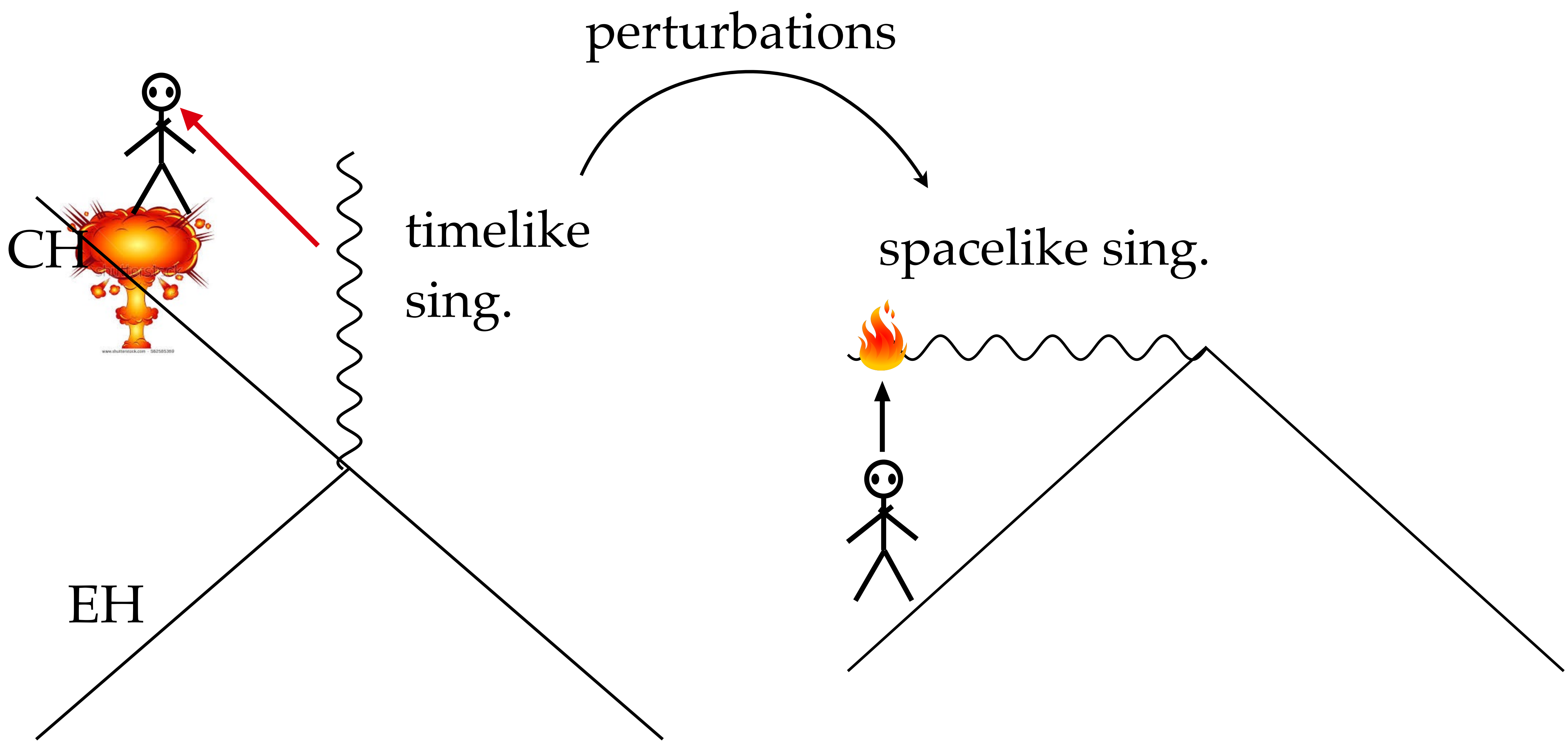
SCC is upheld if, eg, CH is "destroyed" by field perturbations:



Strong Cosmic Censorship Hypothesis

SCC hypothesis by Penrose'72: if singularities exist inside BHs that exist in Nature, they're not visible even to observers inside (i.e., not timelike)

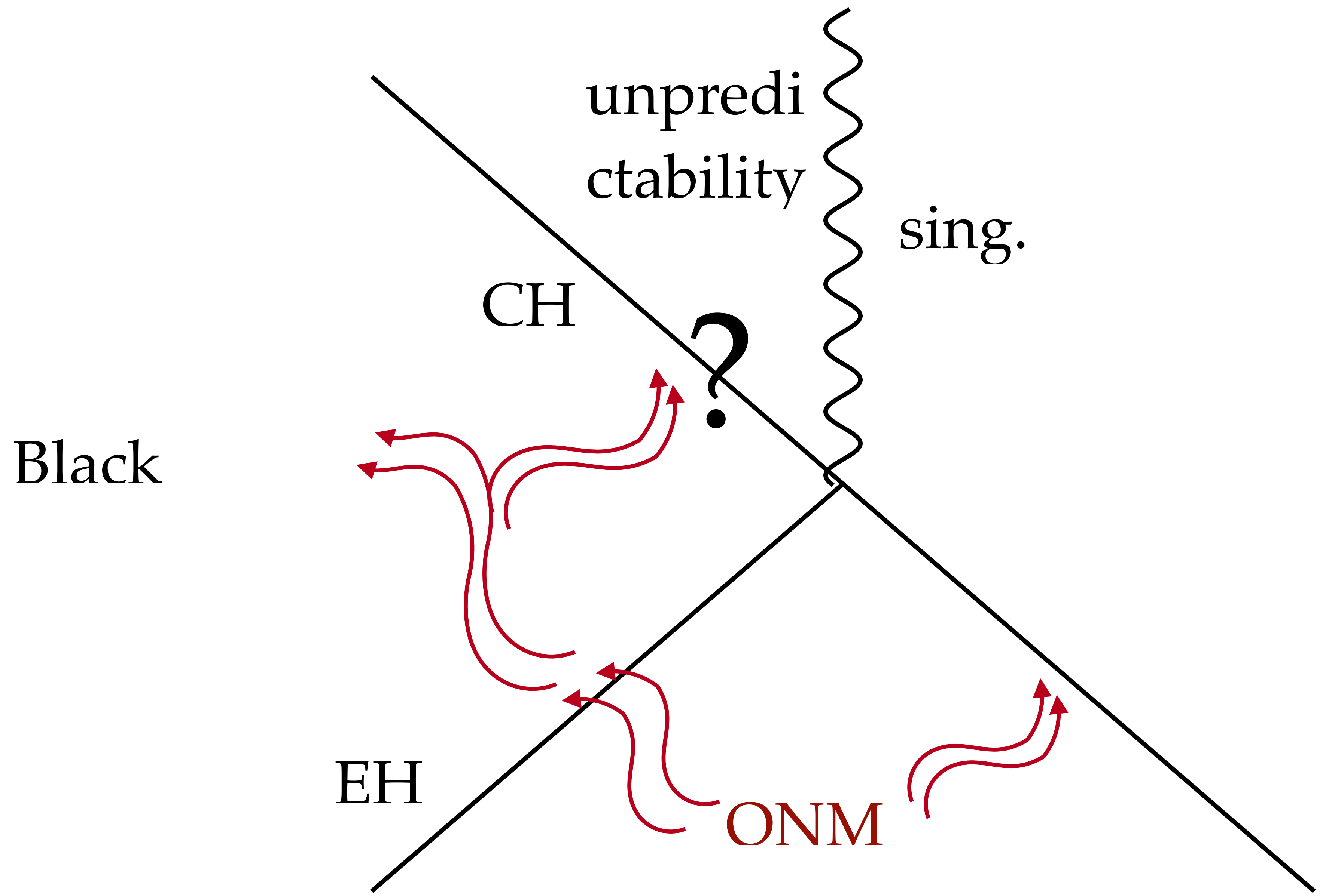
SCC is upheld if, eg, CH is "destroyed" by field perturbations:



But it's a hypothesis - it needs to be verified!

Stability of Cauchy Horizon?

Even if (exponentially-decaying) QNMs do not destabilize the outside of the BH, are they strong enough to destroy the CH as they reach it?



If $\beta \equiv \min\left(-\mathrm{Im}\left(\omega_{\ell m n}\right)\right)/\kappa_{-} > 1/2$, then QNM waves are too weak

to "destroy" the CH => violation of SCC (Hintz&Vasy´17 & others)

Stability of CH - previous results

CH is "destroyed" by the perturbation (SCC holds) in: Kerr (Dafermos et al'17) and Kerr-dS (Dias et al.´18)

😀

Stability of CH - previous results

CH is "destroyed" by the perturbation (SCC holds) in: Kerr (Dafermos et al'17) and Kerr-dS (Dias et al.´18)

CH is not "destroyed" (violation of SCC) in Reissner-Nordstrom-dS (Cardoso et al.'18)

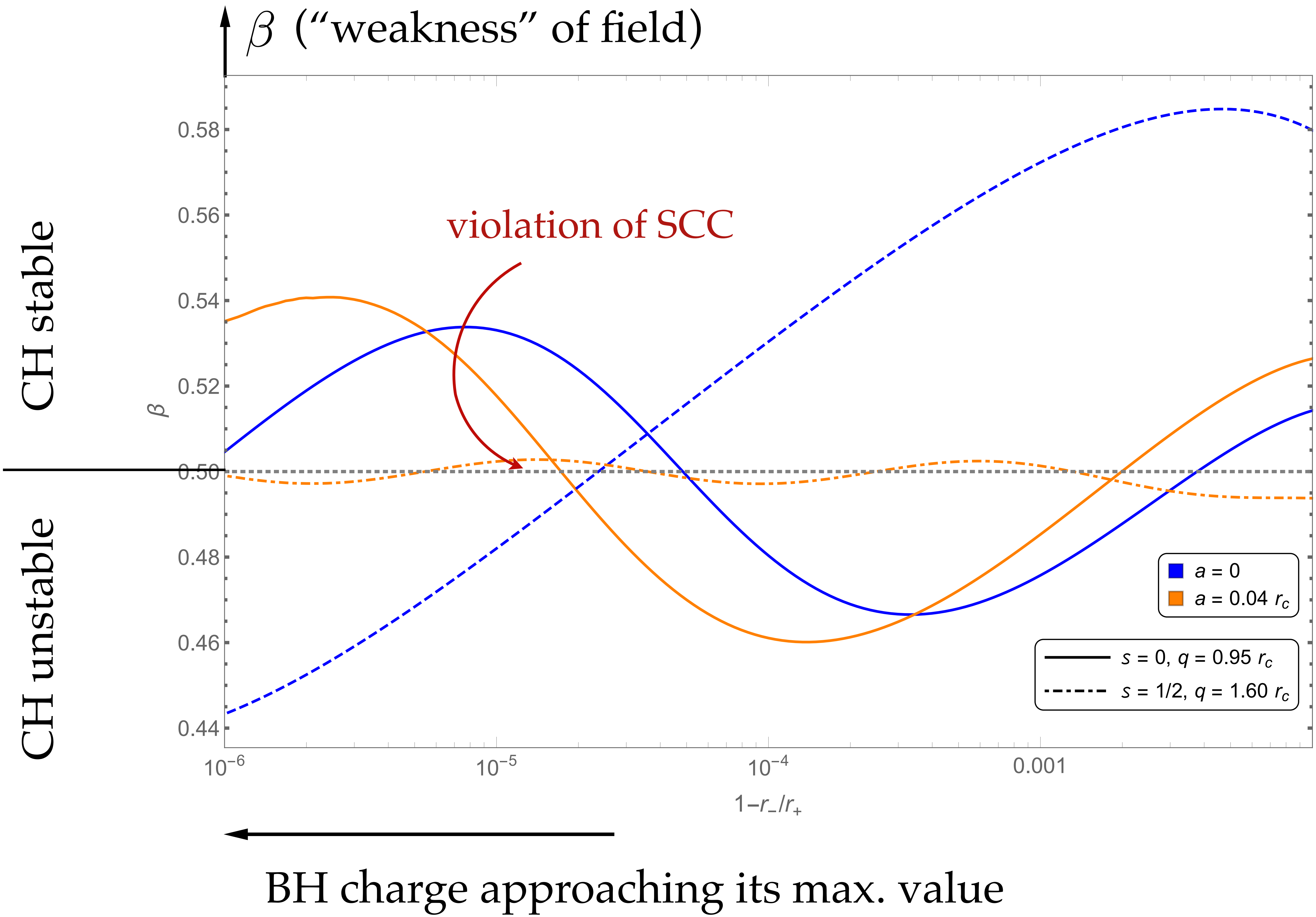
Stability of CH - previous results

CH is “destroyed” by the perturbation (SCC holds) in: Kerr (Dafermos et al’17) and Kerr-dS (Dias et al.´18)

CH is not “destroyed” (violation of SCC) in Reissner-Nordstrom-dS (Cardoso et al.’18)

What happens if we add *rotation* to it?

Casals&Marinho'20 find violation of SCC in Kerr-Newman-dS but for unphysical values of parameters (BH charge and Λ too large)



1. Stability of Kerr

2. Mode stability of Kerr-de Sitter

3. Mode stability of the Cauchy horizon of Kerr-Newman-de Sitter

4. Conclusion

Conclusion

- Kerr: mode stability proven

  Open question: prove *full* linear stability under grav. perturbations

- Kerr-dS: only *partial* mode stability proven

  Open question: *complete* proof of mode stability

- SCC: violated for unphysical parameters in Kerr-Newman-dS

Open questions: can SCC be saved in Kerr-Newman-dS (e.g., by nonlinearities or quantum effects)? Or better - can SCC be generally proven?

Merci bien!