# Scalar fields and gravitational molecules

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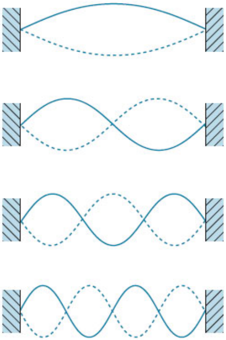
# Outline

# Spectroscopy

## String vibrations



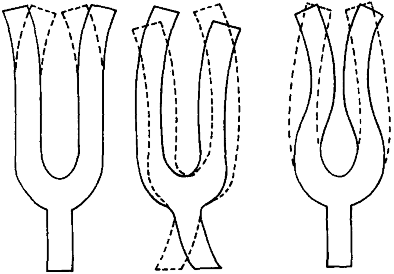
| mode | wavelength | frequency |
|---|---|---|
| first | $2L$ | $\frac{v}{2L}$ |
| second | $L$ | $\frac{v}{L}$ |
| third | $\frac{2L}{3}$ | $\frac{3v}{2L}$ |
| fourth | $\frac{L}{2}$ | $\frac{2v}{L}$ |

(Open University, CC)

$$f_n = \frac{nv}{2L}, \qquad n = 1, 2, 3, \ldots$$

# Spectroscopy

Tuning fork



$$f \approx \frac{\pi K}{8L^2}\sqrt{\frac{E}{\rho}}[1.2^2, 3^2, 5^2, \ldots] \quad \text{cases (a) and (}$$

$$f \approx \frac{\pi K}{8L^2}\sqrt{\frac{E}{\rho}}[3^2, 5^2, 7^2, \ldots] \quad \text{case (b)}$$

(Rossing++92, "On the acoustics of tuning forks")

By measuring the modes we can discover if it is a vibrating string, or something else . . .

# Black Holes (BHs)

Kerr metric

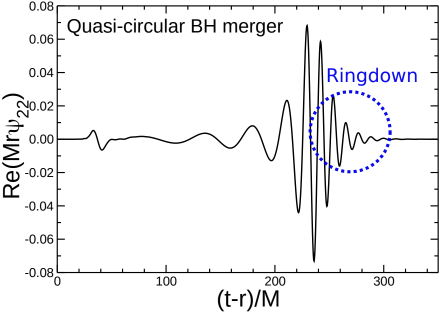
$$ds^2 = -\left(1 - \frac{r_s r}{\Sigma}\right) c^2 dt^2 + \frac{\Sigma}{\Delta} dr^2 + \Sigma d\theta^2$$
$$+ \left(r^2 + a^2 + \frac{r_s r\, a^2}{\Sigma} \sin^2\theta\right) \sin^2\theta\, d\phi^2 - \frac{2 r_s r\, a \sin^2\theta}{\Sigma}\, c\, dt\, d\phi$$

$$r_s = \frac{2GM}{c^2}, \quad \Sigma = r^2 + a^2 \cos^2\theta, \quad \Delta = r^2 - r_S r + a^2$$

- Describes rotating BH with mass $M$ and angular momentum $J = aM$
- Lack of complex multipolar structure is crucial to perform strong-field tests of the theory, for example through the late time relaxation of BHs, as a superposition of quasinormal modes (QNMs)

# Quasi-normal modes (QNM)

$$\frac{d^2\Psi}{dr_*^2} + (\omega^2 - V)\Psi = I(\omega, r)$$




- same decay timescale and ringing for different initial conditions
  - ringdown is universal
- only depends on mass, rotation (and electric charge)
- different matter contents produce same BH

# Black Hole Binaries (BHBs): gravitational molecules?

- If BHs are gravitational atoms. . . what is a gravitational molecule?

- Do BH binaries have characteristic ringdown modes? Can they be excited?

- Do “quasibound” states of light scalars engulfing BH binaries exist?

# Outline

# Perturbative treatment

- Klein-Gordon equation

$$\Box\phi = \mu^2\phi$$

- Binary metric (to lowest order in a post-Newtonian expansion)

$$ds^2 = -\left(1 + 2\Phi_N\right) dt^2 + (1 - 2\Phi_N)\delta_{ij} dx^i dx^j$$

where

$$\Phi_N(t, x^i) = -\frac{M_1}{|\vec{r} - \vec{r}_1(t)|} - \frac{M_2}{|\vec{r} - \vec{r}_2(t)|}$$

# Non-relativistic limit

$$\phi = \frac{1}{\sqrt{2\mu}}\left(\Psi e^{-i\mu t} + \Psi^* e^{i\mu t}\right)$$

moving to the binary rest frame (corotating frame) $\bar{x}^\mu$, the Klein-Gordon equation takes the form

$$i\partial_{\bar{t}}\bar{\Psi}(\bar{t},\bar{x}^i) = H_0\bar{\Psi}(\bar{t},\bar{x}^i) + i\Omega\partial_{\bar{\varphi}}\bar{\Psi}(\bar{t},\bar{x}^i)$$

where

$$H_0 = -\frac{1}{2\mu}\bar{\nabla}^2 - \frac{\mu M_1}{r_1} - \frac{\mu M_2}{r_2}$$

# Unperturbed system

$$i\partial_{\bar{t}}\bar{\Psi} = H_0\bar{\Psi}; \qquad V = -\frac{\mu M_1}{r_1} - \frac{\mu M_2}{r_2}$$

note that the potential $V$ is time-independent.

$$\bar{\Psi}(\bar{t}, \bar{x}^i) = \bar{\psi}(\bar{x}^i)e^{-i\bar{E}\bar{t}}$$

we then have

$$\bar{E}\bar{\psi} = -\frac{1}{2\mu}\bar{\nabla}^2\bar{\psi} + V\bar{\psi}$$

Klein-Gordon equation reduces to the Schrödinger equation in the ionized Di-Hydrogen molecule!

# Single black hole limit

At zero separation we are effectively dealing with one single BH:

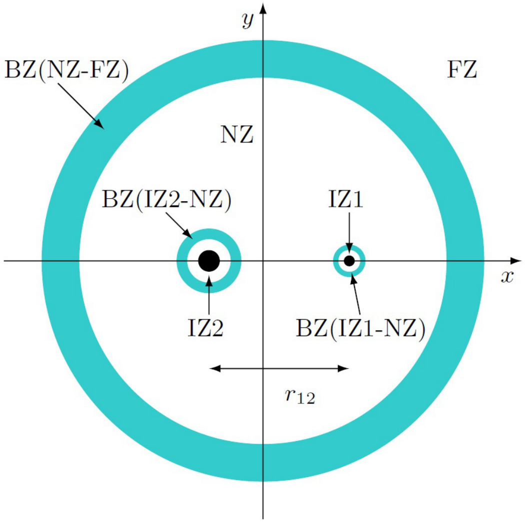
$$\bar{\psi}(\bar{r}) \sim e^{-M\mu^2\bar{r}}$$

Length-scale of the scalar field cloud: $\mathcal{S} \sim 1/(M\mu^2)$

# Outline

# Approximate BHB Metric

Mundim et al, 2014




- Inner Zones (IZ): close to BHs;
- Near Zone (NZ): "intermediate" region;
- Far Zone (FZ): gravitational wave region;
- Buffer Zones (BZ): transition regions.

# Gravitational molecules

Black hole binary with separation $D$

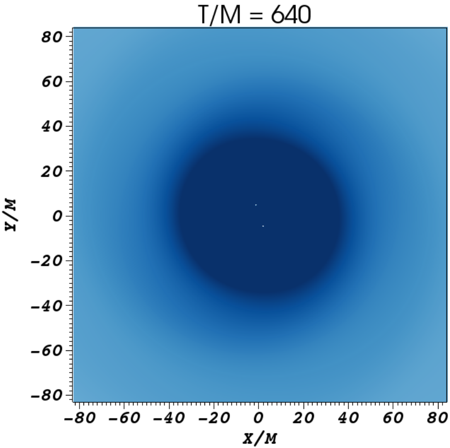
## Length scales

- isolated BH of mass $M_i$: $\mathcal{S}_i \sim 1/(M_i \mu^2)$
- BH binary with $M = M_1 + M_2$: $\mathcal{S}_{\rm BHB} \sim 1/(M\mu^2)$

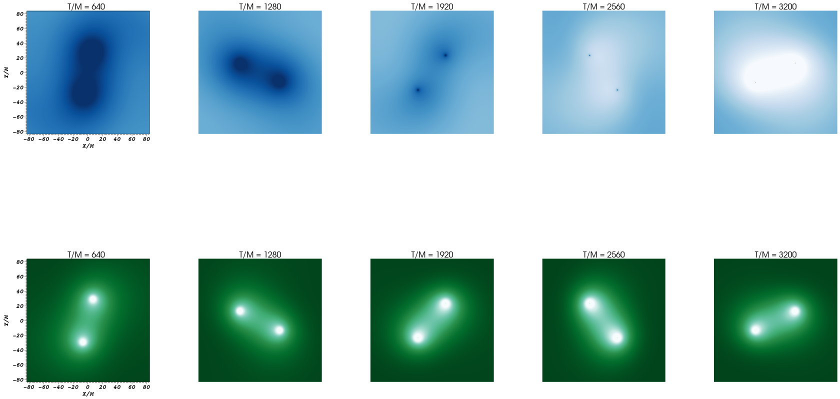
- if $\mathcal{S}_i \ll D$, a quasibound state can be formed around each BH, feeling a tidal force from the companion object
- if $\mathcal{S}_i \gg D$, the companion BH strongly disturbs such a state, destroying it.
  - However, we can expect that a quasibound state forms around the BHB.

# Initial data

$$D = 60M,\ \mu M = 0.2$$



# Monopole gravitational molecule



# Monopole gravitational molecule

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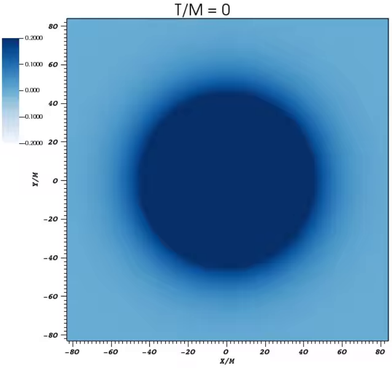
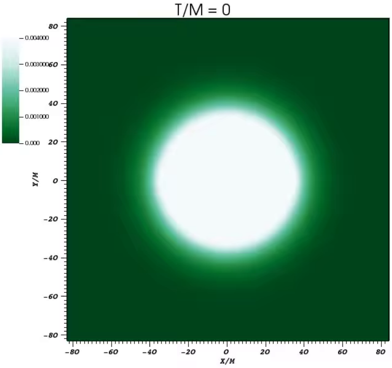
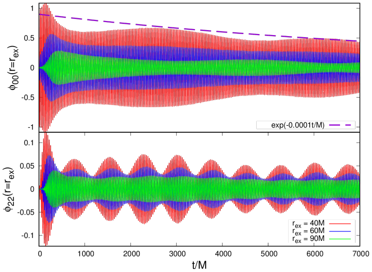


# Spectrum content




| $(l, m)$ | $M\omega$ | $M\,(\mu + E_{000} \pm m\Omega)$ |
|---|---|---|
| (0, 0) | 0.1976 | 0.1973 |
| (2, 2) | 0.2012 | 0.2016 |
| | 0.1930 | 0.1930 |

Note: values from last column come from solving the spectra for the Di-hydrogen molecule

# Outline

# Final remarks

- BHs in GR have been compared to the Hydrogen atom in quantum mechanics
  - Compelling to draw a parallel between BH binaries and the Hydrogen molecule ion
- Light scalar fields are interesting solutions to some of the most pressing problems in physics, such as the dark matter problem
- In the presence of a background scalar, its dynamics close to a BH binary parallels very closely that of an electron in Di-hydrogen molecule
  - Global geodesics for BH binaries seem to be connected to global QNMs
  - Possibility of doing spectroscopy of BH binaries?