Extreme Mass Ratio Inspirals as LISA probes of fundamental fields

@ Saclay, 9th Dec

In collaboration with
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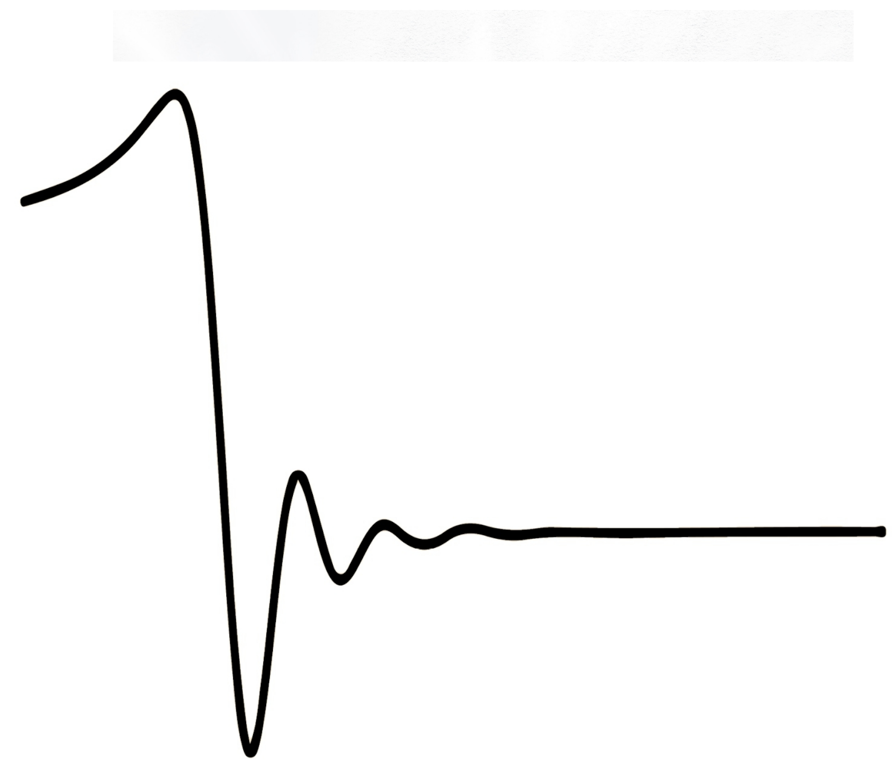

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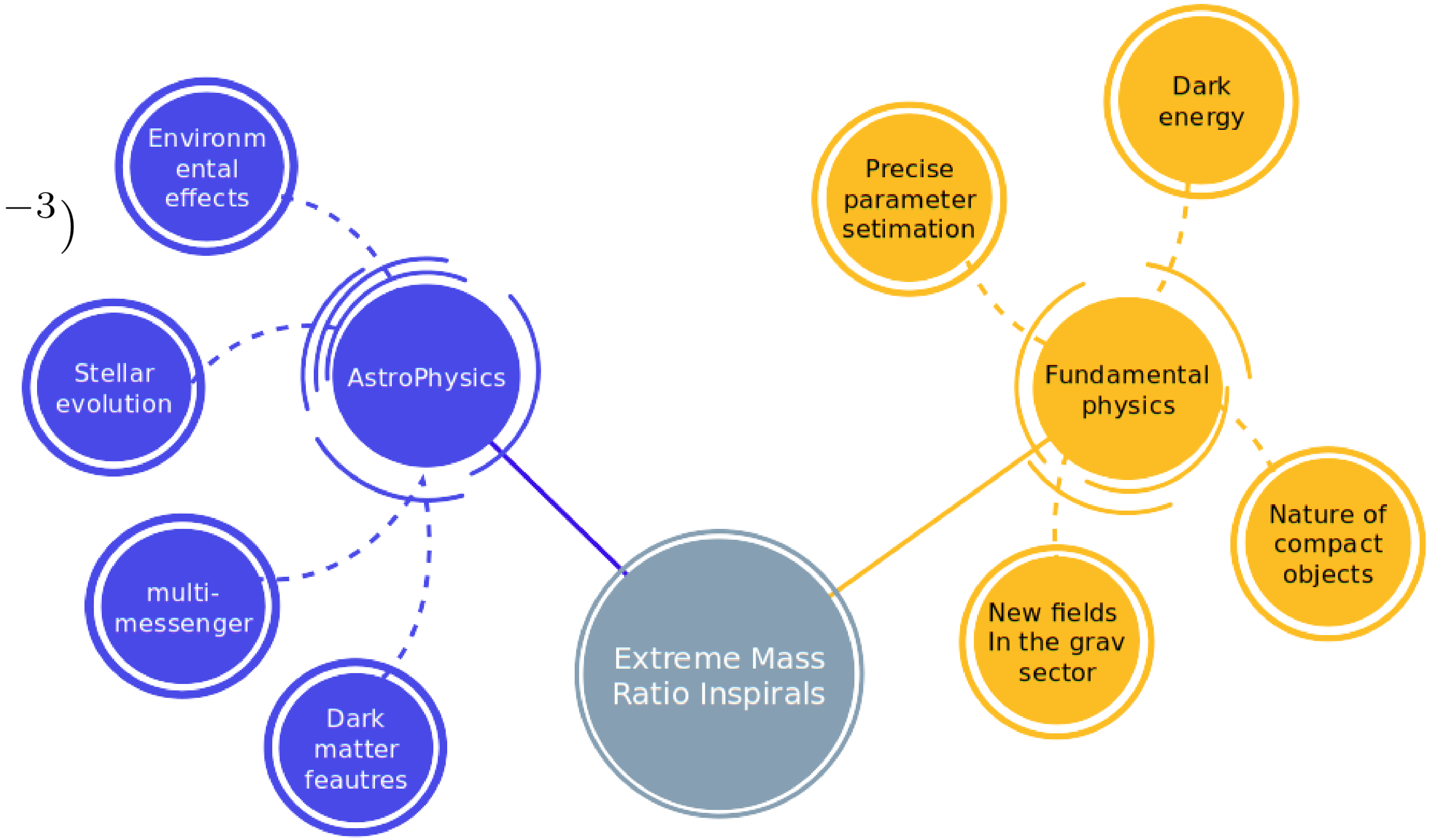
EMRI for a reason

- *Primary* $M \sim (10^4 - 10^7) M_\odot$
- *Secondary such* $q = m_p/M \sim (10^{-6} - 10^{-3})$
- *Emit GWs in the mHz, golden targets for LISA*
- *Rich phenomenology: non equatorial, eccentric orbits, resonances…*



Very appealing to test fundamental physics

- *Complete* $\sim (10^4 - 10^5)$ *cycles before the plunge: **bless** and **disguise***
  - *Precise space-time map and accurate binary parameters*
  - *Accurate templates to be compared against data*

Are EMRI sensitive to new fields?

Extra polarizations as generic features of modified theories of gravity

- *Tipically, proposed theories feature extra fields or can be reformulated in terms of them*

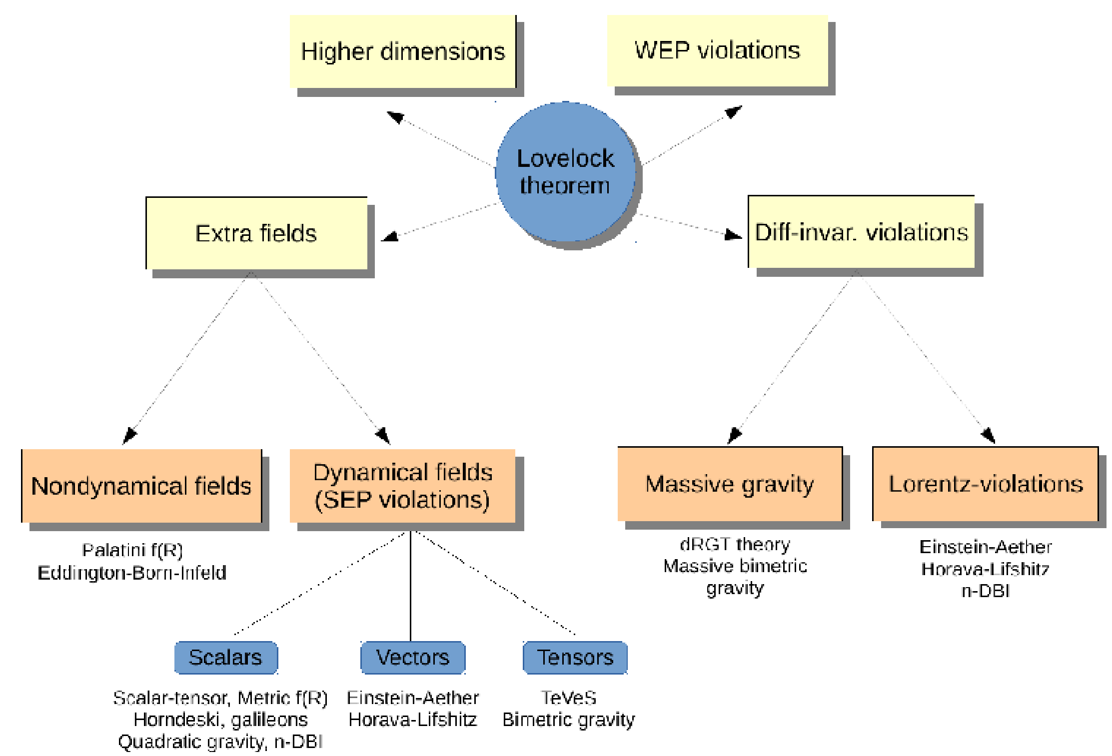
Compact binaries can probe the existence of such new fields

- *Comparable mass in the inspiral: dipole emission at -1PN*

  Barausse+, PRL 116, 241104 (2016)

- *Comparable mass in the merger*

  Okounkova+ PRD 100, 104026 (2019),
  Witek+, PRD 99, 064035 (2019)



What about very asymmetric binaries like EMRIs?

Testing gravity with EMRIs

It may be tempting to answer NOPE

- *In most scalar-tensor theories BHs are protected by no-hair theorems*
- *For hairy BHs, the scalar field generally couples with high-order curvature terms, aka dimensionful couplings*
- *Large suppression for massive objects, as deviations ~ M^{-n} (n>0), but….*
  - *Never forget of the little guys!*

Indeed

- *Scalar fields can leave a significant (detectable) imprint in the GW signal emitted by EMRIs*
- *For a vast class of theories, the (leading) GR deviations are **universal** and only controlled by the scalar **charge** of the **little** guy*

AM, Franchini, Gualtieri, Sotiriou, PRL 125 (2020)

The setup

Motivation: EMRIs beyond GR

- We need real waveforms to compare against data and GR predictions
- Depending on the theory complexity grows fast [very, very fast]

$$S[\mathbf{g},\varphi,\Psi] = S_0[\mathbf{g},\varphi] + \alpha S_c[\mathbf{g},\varphi] + S_{\mathrm{m}}[\mathbf{g},\varphi,\Psi]$$

$$S_0 = \int d^4x \frac{\sqrt{-g}}{16\pi}\left(R - \frac{1}{2}\partial_\mu\varphi\partial^\mu\varphi\right)$$

Non-minimal coupling

Matter fields

- Dimensionful coupling $[\alpha] = (mass)^n$

We assume that

- BH solutions are continuously connected to GR solutions $\alpha \to 0$
- S_c is analytic in φ

The setup

Key simplifications for the exterior space-time occur for

1) *Theories with no-hair theorems*

2) *Theories which evade no-hair but have dimensionful coupling* α *with* $n \geq 1$

- *Any correction depend on* $\zeta \equiv \frac{\alpha}{M^n} = q^n \frac{\alpha}{m_{\mathrm{p}}^n} \ll 1 \qquad q = \frac{m_p}{M} \ll 1$

The exterior space-time can be approximated by the Kerr metric

For the secondary, consider the skeletonized approach

Eardley, ApJ (1975)
Damour EF, PRD (1992)

$$S_{\mathrm{p}} = -\int m(\varphi) ds = -\int m(\varphi) \sqrt{g_{\mu\nu} \frac{dy_{\mathrm{p}}^{\mu}}{d\lambda} \frac{dy_{\mathrm{p}}^{\nu}}{d\lambda}} d\lambda$$

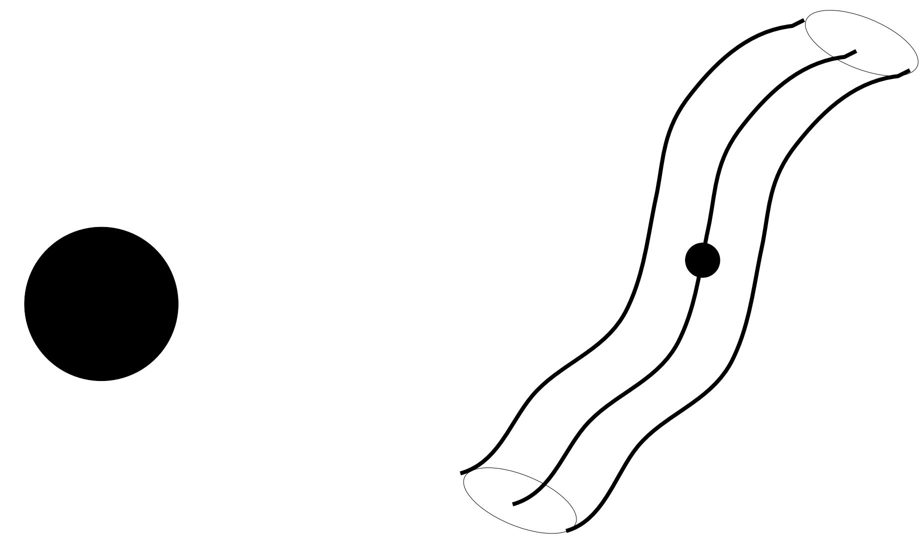
- *Extended body treated as point particle*
- $m(\varphi)$ *scalar function*

The setup

$l \ll L$

skeletonized body

exterior space-time



The orbital motion can be studied with perturbation theory in $q \ll 1$

- *GR modifications affect the motion of the particle but not the background*
- *The scalar field is a perturbation of a constant value* $\varphi = \varphi_0 + \varphi_1$

The field's equations

In our units $[S_0] = (mass)^2$ $[S_c] = (\text{mass})^{2-n}$ $\longrightarrow$ $S_c \sim M^{-n} S_0$ $(\zeta \sim q^n)$

$$G_{\mu\nu} = T^{\text{scal}}_{\mu\nu} + \alpha T^{c}_{\mu\nu} + T^{\text{p}}_{\mu\nu}$$

$$\boxed{G_{\mu\nu}} = \frac{1}{2}\partial_\mu\varphi\cancel{\partial_\nu\varphi - \frac{1}{4}g_{\mu\nu}}(\partial\varphi)^2 + \cancel{-\alpha\frac{16\pi}{\sqrt{-g}}\frac{\delta S_c}{\delta g^{\mu\nu}}} + \boxed{\int m(\varphi)\frac{\delta^{(4)}(x-y_p(\lambda))}{\sqrt{-g}}\frac{dy_p^\alpha}{d\lambda}\frac{dy_p^\beta}{d\lambda}d\lambda}$$

quadratic in φ

$\alpha T^c_{\mu\nu} \sim \zeta^2 G_{\mu\nu} \sim q^{2n} G_{\mu\nu}$

$$\boxed{\Box\varphi} + \cancel{\frac{8\pi\alpha}{\sqrt{-g}}\frac{\delta S_c}{\delta\varphi}} = \boxed{16\pi\int m'(\varphi)\frac{\delta^{(4)}(x-y_p(\lambda))}{\sqrt{-g}}d\lambda}$$

$\alpha\frac{\delta S_c}{\delta\varphi} \sim \zeta\Box\varphi \ll \Box\varphi$

$m(\varphi), m'(\varphi)$ *evaluated at the value of the exterior scalar field*

Almost as in GR

From the scalar field equation inside the world tube, but far way to be weak field. In the body's frame

$$\varphi = \varphi_0 + \frac{m_{\rm p}\, d}{\tilde{r}} + O\left(\frac{m_{\rm p}^2}{\tilde{r}^2}\right)$$

scalar charge

- *Matching with the scalar field equation outside the world tube*

$$m(\varphi_0) = m_p \qquad \frac{m'(\varphi_0)}{m_p} = -\frac{d}{4}$$

Change in the EMRI dynamics universally captured by the scalar charge

$$G_{\mu\nu} = T^{\rm p}_{\mu\nu} = 8\pi m_{\rm p} \int \frac{\delta^{(4)}(x - y_p(\lambda))}{\sqrt{-g}} \frac{dy_p^\alpha}{d\lambda} \frac{dy_p^\beta}{d\lambda} d\lambda$$

$$\Box\varphi = -4\pi d\, m_{\rm p} \int \frac{\delta^{(4)}(x - y_p(\lambda))}{\sqrt{-g}} d\lambda$$

grav-sector

$$g_{\alpha\beta} = g^0_{\alpha\beta} + h_{\alpha\beta}$$

scal-sector

$$\varphi = \varphi_0 + \varphi_1$$

The wave equation(s)

For Schwarzschild, 3 master equations for 3 perturbations

$$e^{-\lambda} = 1 - 2M/r$$
$$\Lambda = \ell(\ell+1)/2 - 1$$

GR

$$\frac{d^2 R_{\ell m}}{dr_\star^2} + \left[\omega^2 - e^{-\lambda}\left(\frac{\ell(\ell+1)}{r^2} - \frac{6M}{r^3}\right)\right] R_{\ell m} = J_{\rm ax}$$

***R**egge-**W**heeler*

$$\frac{d^2 Z_{\ell m}}{dr_\star^2} + \left[\omega^2 - \frac{18M^3 + 18M^2 r\Lambda + 6Mr^2\Lambda^2 + 2r^3\Lambda^2(1+\Lambda)}{r^3(3M + r\Lambda)}\right] Z_{\ell m} = J_{\rm pol}$$

***Z**erilli*

$$\frac{d^2 \delta\varphi_{\ell m}}{dr_\star^2} + \left[\omega^2 - e^{-\lambda}\left(\frac{\ell(\ell+1)}{r^2} + \frac{2M}{r^3}\right)\right] \delta\varphi_{\ell m} = J_\varphi$$

***S**calar field*

- *For circular equatorial orbits*

$$\omega_p = (M/r_p^3)^{1/2}$$

$$J_\varphi = \boxed{-d}\, m_{\rm p} \frac{4\pi P_{\ell m}(\frac{\pi}{2})}{r^{3/2} e^{\lambda}} \sqrt{r - 3M}\,\delta(r - r_{\rm p})\delta(\omega - m\omega_{\rm p})$$

Overall scale sets by the charge

orbit's radius

The GW energy flux

The full solutions at infinity/horizon are needed to compute the emitted gravitational wave fluxes

$$\dot{E}^{\pm}_{\rm grav}=\frac{1}{64\pi}\sum_{\ell=2}^{\infty}\sum_{m=-\ell}^{\ell}\frac{(\ell+2)!}{(\ell-2)!}(\omega^2|Z^{\pm}_{\ell m}|^2+4|R^{\pm}_{\ell m}|^2) \qquad \dot{E}^{\pm}_{\rm scal}=\frac{1}{32\pi}\sum_{\ell=1}^{\infty}\sum_{m=-\ell}^{\ell}\omega^2|\delta\varphi^{\pm}_{\ell m}|^2$$

- *The total contribution*

$$\boxed{\dot{E}=\dot{E}^{+}_{\rm grav}+\dot{E}^{-}_{\rm grav}+\dot{E}^{+}_{\rm scal}+\dot{E}^{-}_{\rm scal}=\dot{E}_{\rm GR}+\delta\dot{E}_d}$$

- *The binary accelerates due to the extra leakage of energy given by the scalar field channel*

- $\delta\dot{E}_d$ *enters at the **same** order in **q** as the GR leading dissipative contribution*

How much dephasing?

Once we have the total flux emitted by the binary we can determine its adiabatic evolution

- *For the orbital phase*

$$\frac{dr}{dt} = -\dot{E}\frac{dr}{dE_{\rm orb}} \quad , \quad \frac{d\Phi}{dt} = \omega_p = \pm\frac{M^{1/2}}{r^{3/2} \pm \chi M^{3/2}}$$

- *The total phase can be written as*

$$\Phi_d(t) \sim \Phi_{\rm GR}(t) + \delta\Phi_d(t)$$

  - *Both contributions are of the same order* $\mathcal{O}(1/q)$
  - *The term* $q\delta\Phi_d(t)$ *depends only on the scalar charge*

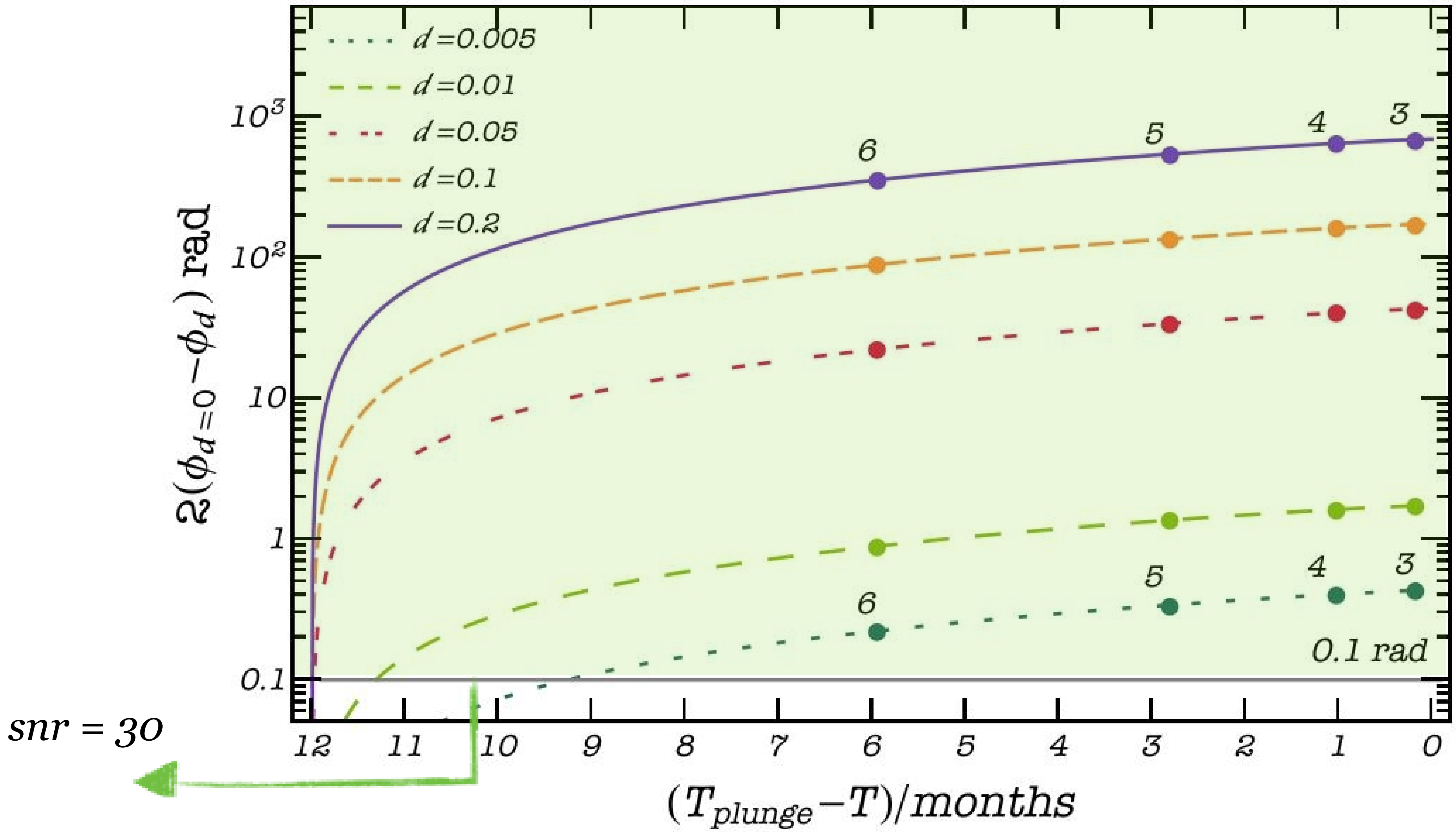
A first assessment of the charge impact is given by studying the ***dephasing*** *induced on the orbital phase*

$$\Phi_{d=0}(t) - \Phi_d(t)$$

How much dephasing?

*Difference between GR - GR**d** phase evolution during the inspiral (12 months the plunge)*

$(M, m_p) = (10^6, 10)M_\odot \quad \chi = 0.9$



gr-qc: 2106.11325

- *Potentially able to observe changes induced by scalar charges* $d \sim 0.005$

The waveform

The recipe to generate EMRI waveforms

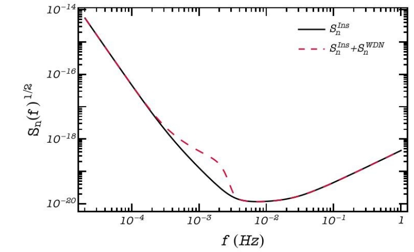
- *Compute the total energy flux emitted by the binary* $\dot{E} = \dot{E}_{\rm GR} + \delta\dot{E}_d$
- *The flux drives the binary orbital evolution*

$$\frac{dr(t)}{dt} = -\dot{E}\frac{dr}{dE_{\rm orb}} \quad , \quad \frac{d\Phi(t)}{dt} = \frac{M^{\frac{1}{2}}}{r_p^{3/2}}$$

- *Build the GW polarizations* $h_+[r(t),\Phi(t)] \quad , \quad h_\times[r(t),\Phi(t)]$
- *Given the source localization, construct the strain* $h(t) = \frac{\sqrt{3}}{2}[h_+F_+(\theta,\phi,\psi) + h_\times F_\times(\theta,\phi,\psi)]$

Everything as in GR but $\delta\dot{E}_d$, that only depends on the scalar charge

- *(rather) Universal family of waveforms to be tested against GR*



Detectability in nuce

Quadrupole approximation for h(t) ***[for laziness]***

$$h_{ij}^{\rm TT} = \frac{2}{D}(P_{il}P_{jm} - \frac{1}{2}P_{ij}P_{lm})\ddot{I}_{lm} \qquad I_{ij} = \int d^3x T^{tt}(t,x^i)x^ix^j = m_p x^i x^j$$

- *Scalar product for waveforms* $\langle h_1|h_2\rangle = 4\Re\int_{f_{\rm min}}^{f_{\max}} \frac{\tilde{h}_1(f)\tilde{h}_2^\star(f)}{S_n(f)}df$

  - *The* ***faithfulness*** $\mathcal{F}[h_1,h_2] = \max_{\{t_c,\phi_c\}} \frac{\langle h_1|h_2\rangle}{\sqrt{\langle h_1|h_1\rangle\langle h_2|h_2\rangle}}$

    1 perfect match

    0 what a sh.. template

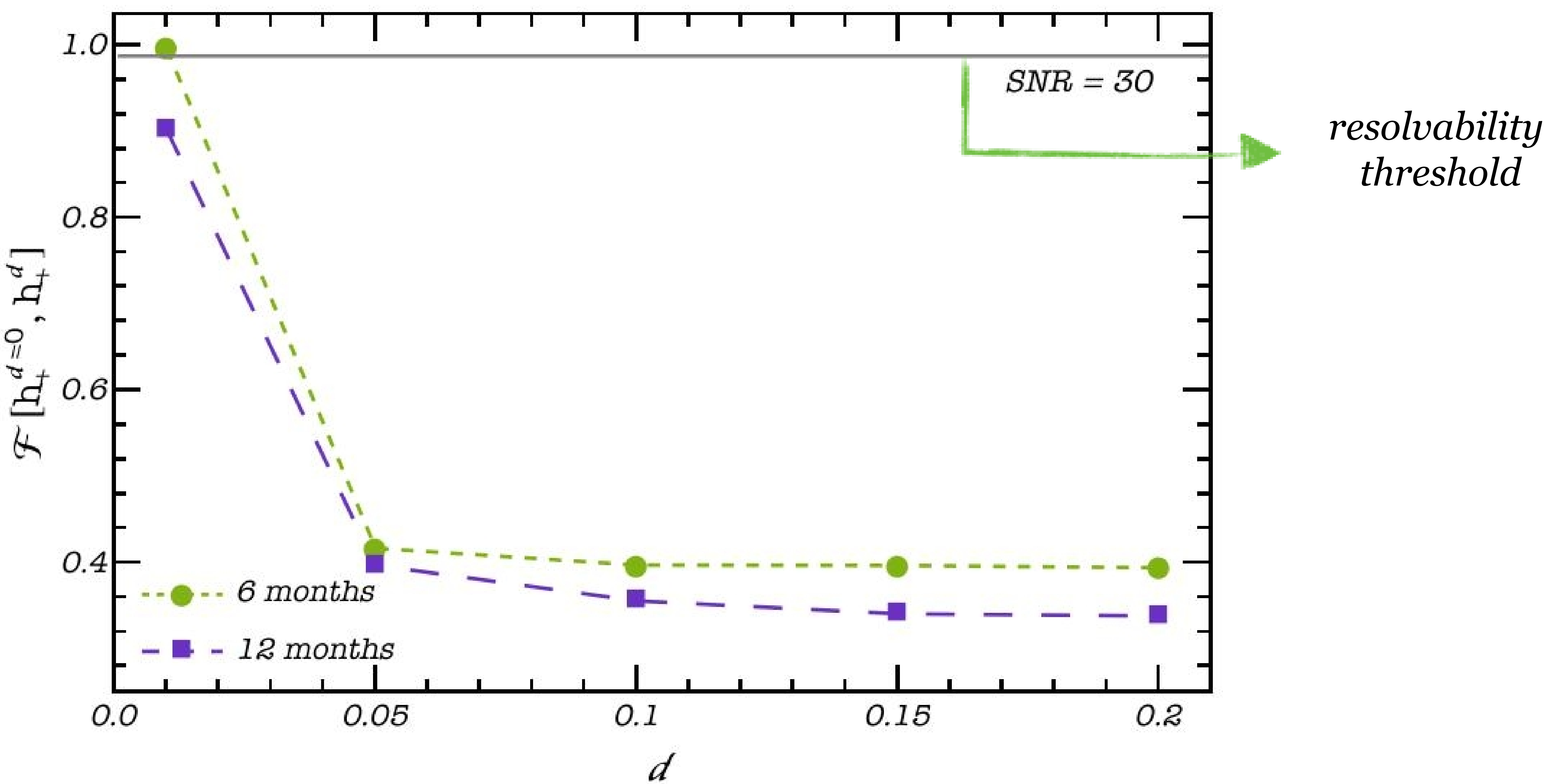
- *For large **S**ignal-to-**N**oise **R**atio, the parameters probability* $p(\vec{x}|s) \propto e^{-\frac{1}{2}\Gamma_{ab}\Delta x^a \Delta x^b}$

  parameters (M,...,d)

$$\Gamma_{ab} = 4\Re\int \frac{\partial\tilde{h}(f)}{\partial x_a}\frac{\partial\tilde{h}^\star(f)}{\partial x_b}\frac{df}{S_n(f)}$$

  - *The parameter* ***errors*** $\Sigma_{ab} = (\Gamma^{-1})_{ab} \longrightarrow \sigma_a = \sqrt{\Sigma_{aa}}$

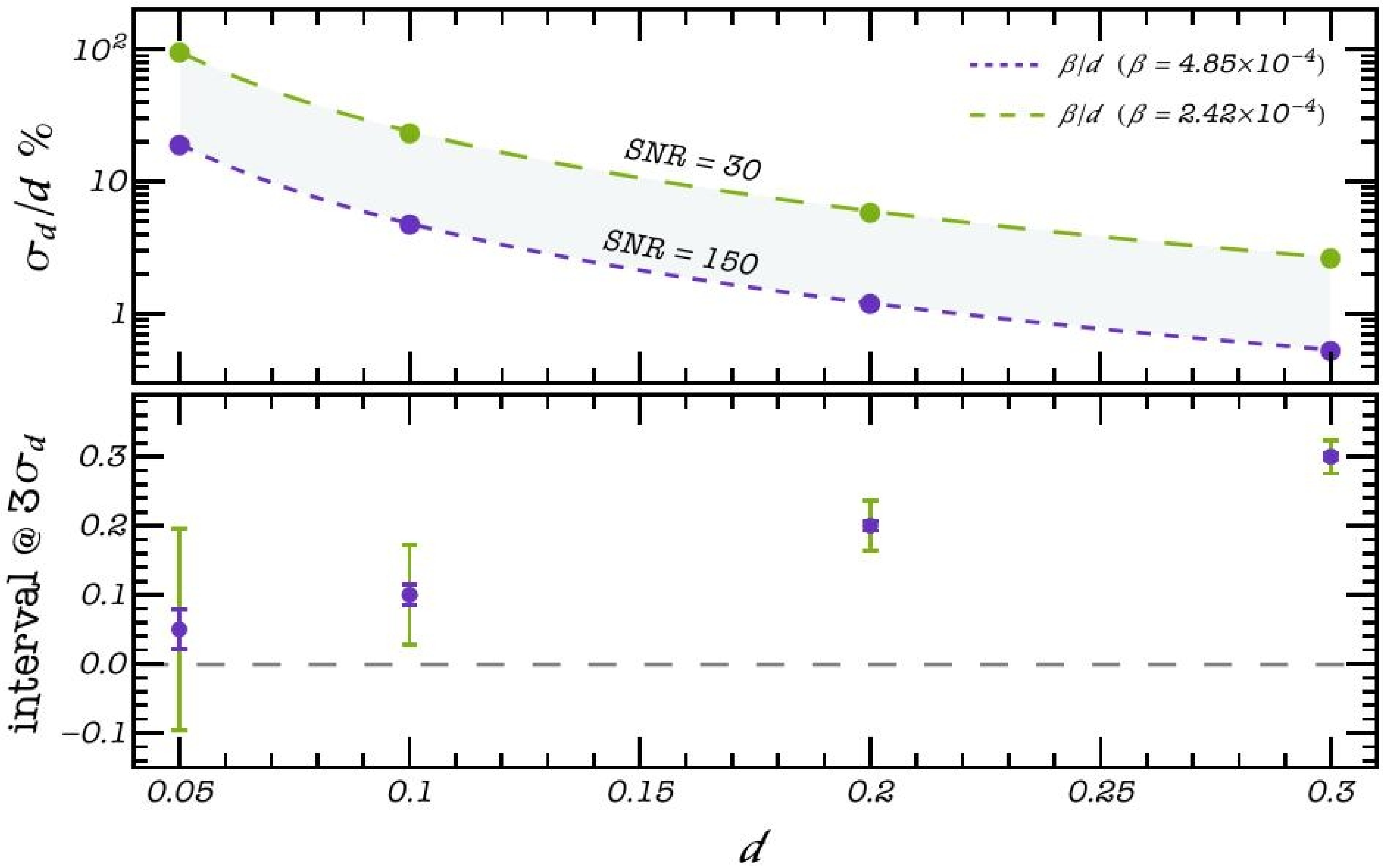
Overlap & Faithfulness



- *Potentially able to observe changes induced by scalar charges* $d \sim 0.005 - 0.01$

Forecast on LISA bounds

Constraints on the scalar charge for prototype EMRIs with SNR = (30,150)



- *LISA potentially able to measure* ***d*** *with % accuracy and better*
- *LISA potentially able to constrain* ***d*** *~ 0.05 to be inconsistent with zero @ at 3-* σ

Tracing back the couplings

Joulie & Berti, PRD 100 (2019)

A notable example: scalar Gauss-Bonnet (sGB) gravity

$$\alpha S_c = \frac{\alpha}{4}\int d^4x \frac{\sqrt{-g}}{16\pi} f(\varphi)\mathcal{G}$$

- *n =2,* $[\alpha] = mass^2$ $\longrightarrow$ $\zeta \equiv \frac{\alpha}{M^2} = q^2 \frac{\alpha}{m_2^n}$
- $f(\varphi)$ *generic function of the scalar field*
- $\mathcal{G} = R^2 - 4R_{\mu\nu}R^{\mu\nu} + R_{\alpha\beta\gamma\delta}R^{\alpha\beta\gamma\delta}$ *Gauss Bonnet invariant*
- *The scalar charge is proportional to the dimensionless coupling constant* $\beta = \frac{\alpha}{m_p^2}$

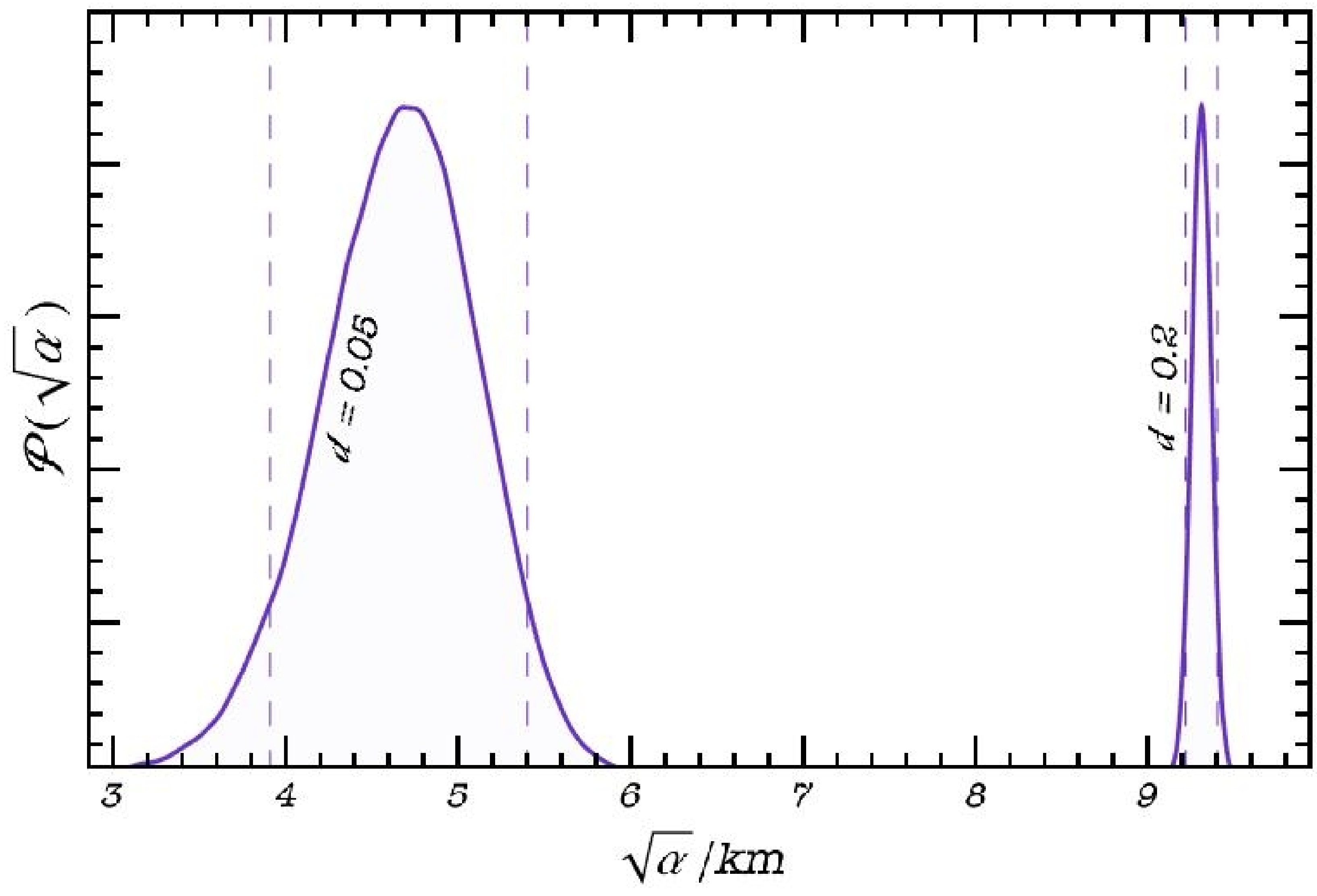
$f(\varphi) = e^{\varphi}$	$f(\varphi) = \varphi$
(exponential)	*(shift-symmetric)*
$d = 2\beta + \frac{73}{30}\beta^2 + \frac{15577}{2520}\beta^3$	$d = 2\beta + \frac{73}{60}\beta^3$

For hairy BHs bounds on ***d*** *can be mapped to bounds on* ***couplings***

Forecast on LISA bounds: couplings

Map constraints on the charge to constraints on the coupling $d(\beta) \leftrightarrow \beta(d)$

- *Shift symmetric sGB* $f(\varphi) = \varphi$



Summary

EMRIs are golden binaries to test fundamental physics/astrophysics

For a vast class of theories, waweform changes are driven by the BH scalar charge only

- *Universal behavior in terms of* d *. Ready-to-use waveforms*
- *Constraints on the scalar charge can be traced back to the theory's fundamental couplings*
- *Easy to extend to vector modes, multiple couplings (with different dimensions) and fields*

But

- *Valid at leading order. What about SF? and massive fields? (Susanna Barsanti)*
- *Correlation with astrophysical effects*
- *More sophisticate analysis with MCMC, FEW (Speri, Franchini underway)*

Back up

The perturbation scheme

EMRI small mass ratio naturally leads to use relativistic perturbation theory to describe their evolution

- *Consider linear perturbations of a Schwarzschild background induced by the small body*

grav-sector

$$g_{\alpha\beta} = g^0_{\alpha\beta} + h_{\alpha\beta}$$

scal-sector

$$\varphi = \varphi_0 + \varphi_1$$

- *Decompose* $h_{\alpha\beta}$ *and* φ_1 *in tensor and scalar spherical harmonics*
  - *For the scalar field*

$$\varphi_1(t,r,\theta,\phi) = \sum_{\ell=0}^{\infty} \sum_{m=-\ell}^{\ell} \frac{\delta\varphi_{\ell m}(t,r)}{r} Y_{\ell m}(\theta,\phi)$$

- *Go to the Fourier space, replace into the field's equation and solve for* $\delta\varphi_{\ell m}$

The perturbation scheme

For the gravitational sector

$$h_{\alpha\beta} = h^{\rm pol}_{\alpha\beta} + h^{\rm ax}_{\alpha\beta}$$

$$(-1)^{\ell} \qquad\qquad (-1)^{\ell+1}$$

$$\mathbf{h} = \sum_{\ell=0}^{\infty}\sum_{m=-\ell}^{\ell} \boxed{\mathcal{A}^{(0)}_{\ell m}\mathbf{a}^{(0)}_{\ell m} + \mathcal{A}^{(1)}_{\ell m}\ \mathbf{a}^{(1)}_{\ell m} + \mathcal{A}_{\ell m}\mathbf{a}_{\ell m} + \mathcal{B}^{(0)}_{\ell m}\ \mathbf{b}^{(0)}_{\ell m} + \mathcal{B}_{\ell m}\ \mathbf{b}_{\ell m}} + \boxed{\mathcal{Q}^{(0)}_{\ell m}\ \mathbf{c}^{(0)}_{\ell m} + \mathcal{Q}_{\ell m}\ \mathbf{c}_{\ell m}}$$
$$\boxed{+\mathcal{D}_{\ell m}\ \mathbf{d}_{\ell m}} + \boxed{\mathcal{G}_{\ell m}\ \mathbf{g}_{\ell m} + \mathcal{F}_{\ell m}\ \mathbf{f}_{\ell m}}$$

$$\mathbf{b}_{\ell m} = \frac{n_\ell r}{\sqrt{2}}\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & Y^{\ell m}_{,\theta} & Y^{\ell m}_{,\phi} \\ 0 & Y^{\ell m}_{,\theta} & 0 & 0 \\ 0 & Y^{\ell m}_{,\phi} & 0 & 0 \end{pmatrix}$$

- *7 **polar** components + 3 **axial** harmonics*
  - *For a spherically symmetric background the 2 families decouple*
- *Go to the Fourier space, and solve for each sector independently*
- *In the Regge-Wheeler-Zerilli gauge the components reduce to **1** axial and **1** polar functions*

Regge, Wheeler +, PRD 108, 1063 (1957)
Zerilli, PR D2, 2141 (1970)

What about the spin?

Field equations remain unchanged, but perturbations are on top of a Kerr background with spin ***a***

Barsanti, + in preparation

- *Decomposition in spheroidal harmonics* $\varphi_1(t,r,\theta,\phi) = \sum_{\ell=0}^{\infty}\sum_{m=-\ell}^{\ell} \tilde{R}_{\ell m}(t,r) S_{\ell m}(\theta) e^{im\phi}$
  - *Angular equation*

$$\frac{1}{\sin\theta}\frac{d}{d\theta}\left(\sin\theta\frac{dS_{\ell m}}{d\theta}\right) + \left(a^2\omega^2\cos^2\theta - \frac{m^2}{\sin^2\theta} + \lambda\right) S_{\ell m} = 0$$

  - *Radial equation*

$$\delta\varphi_{\ell m} = \sqrt{r^2+a^2}\,\tilde{R}_{\ell m} \qquad \frac{d^2\delta\varphi_{\ell m}}{dr_\star^2} + \left[\omega^2 - V(\omega)\right]\delta\varphi_{\ell m} = J_\varphi$$

$$\boxed{V(\omega) = \frac{4Mrma\omega - a^2m^2 + \Delta(\lambda + \omega^2 a^2)}{(r^2+a^2)^2} + \frac{\Delta(3r^2 - 4Mr + a^2)}{(r^2+a^2)^3} - \frac{3\Delta^2 r^2}{(r^2+a^2)^4}}$$

- *Decomposition in spheroidal harmonics* $J_\varphi \propto \delta(r - r_p)\delta(\omega - m\omega_p)$ $\qquad \omega_p = \dfrac{M^{1/2}}{r^{3/2} + aM^{1/2}}$

The recipe for the solution

Solutions for a scattering problem for $\psi_{\ell m} = (R_{\ell m}, Z_{\ell m}, \delta\varphi_{\ell m})$

$$\frac{d^2\psi_{\ell m}}{dr_\star^2} + \left[\omega^2 - V\right]\psi_{\ell m} = S$$

- *Solve for the homogeneous part with suitably boundary conditions*

horizon

$\psi_{\ell m}^{(-)} \sim e^{-i\omega r_\star}$

(ingoing)

infinity

$\psi_{\ell m}^{(+)} \sim e^{+i\omega r_\star}$

(outgoing)

- *Integrate over the source term for the full solution*

$$\psi_{\ell m}^{\pm} \equiv \lim_{r_\star \to \pm\infty} \psi_{\ell m}(r_*) = e^{\pm i\omega r_\star} \int_{-\infty}^{\infty} \frac{\psi_{\ell m}^{(\mp)} J}{W} dr_\star$$

(Wronskian $W = \psi_{\ell m}^{\prime(+)}\psi_{\ell m}^{(-)} - \psi_{\ell m}^{\prime(-)}\psi_{\ell m}^{(+)}$ *)*

- *For circular orbits the integral greatly simplifies because of* $\delta(r - r_p)$

Some power counting

The system phase evolution

$$\Phi(t) = \Phi_{\text{diss-1}} + \Phi_{\text{cons-1}} + \Phi_{\text{diss-2}} + \ldots$$

$$\mathcal{O}(1/q) \qquad \mathcal{O}(1) \qquad \mathcal{O}(1)$$

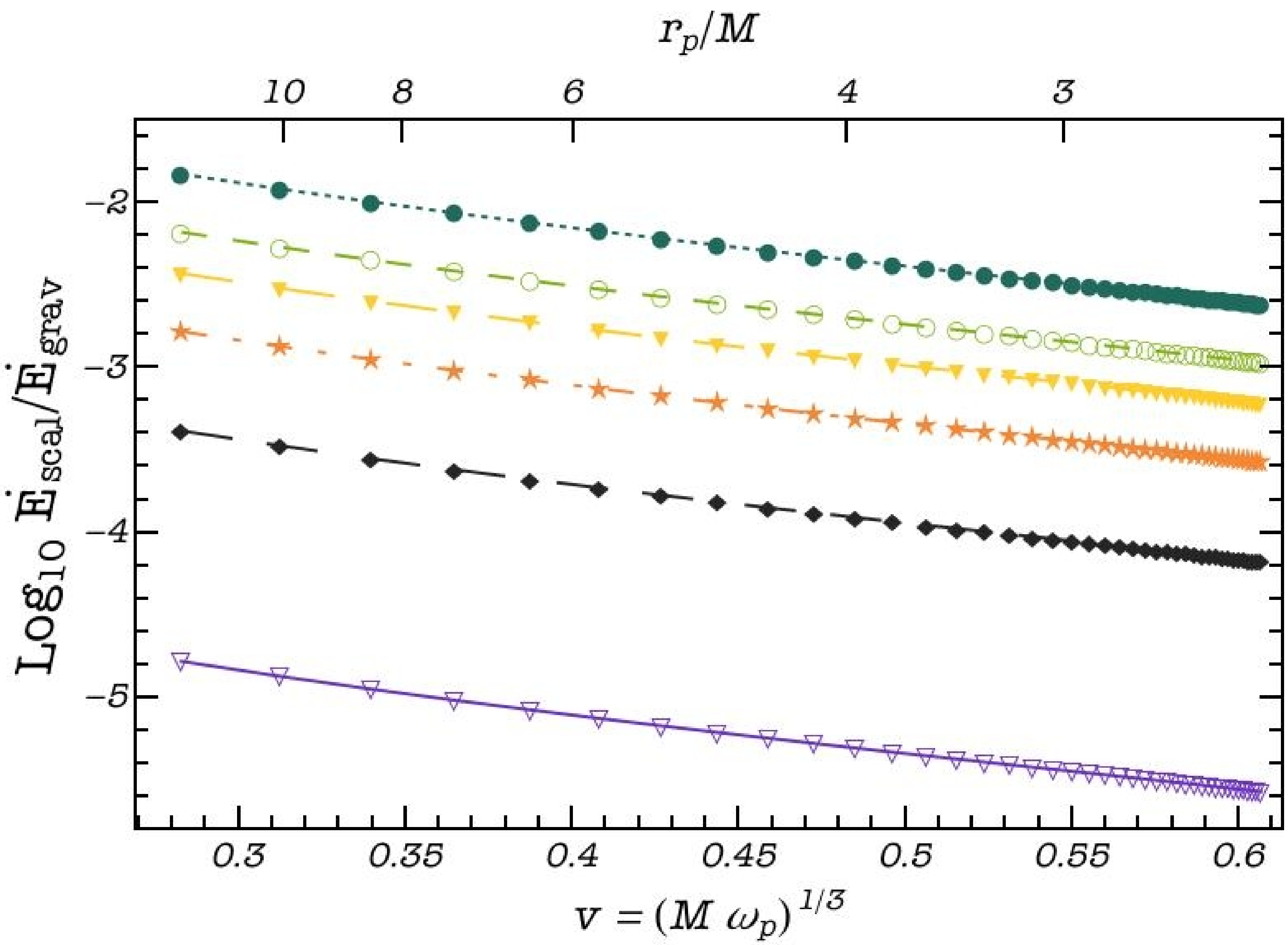
charge of secondary

Primary contributes $\mathcal{O}(q^n)$

- *Sort of re-shaping of the LISA-testing-background-spacetime-idea*
- *All of this generalizes immediately for spinning BH* $\longrightarrow$ *Kerr as a background*

The GW flux (Kerr background)

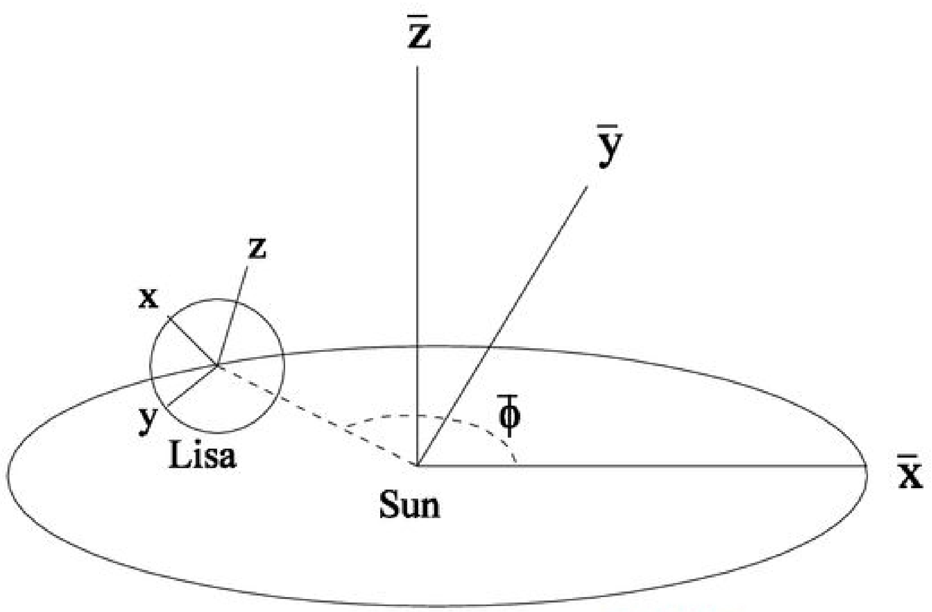
Scalar-to-tensor flux as v.s. orbital velocity for a spinning primary $\chi = 0.9$



The waveform

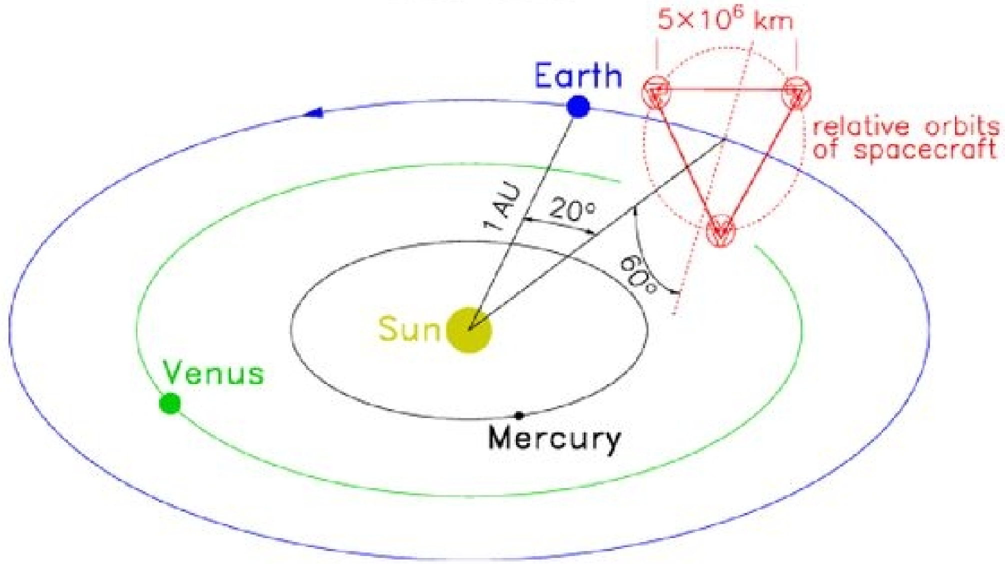
Pattern functions depend on time as LISA orbits around the Sun

$$F_+ = \frac{1+\cos^2\theta(t)}{2}\cos 2\phi(t)\cos 2\psi(t) - \cos\theta(t)\sin 2\phi(t)\sin 2\psi(t)$$
$$F_\times = \frac{1+\cos^2\theta(t)}{2}\cos 2\phi(t)\sin \psi(t) + \cos\theta(t)\sin 2\phi(t)\cos 2\psi(t)$$



- *Express all the angles in a Solar system fixed reference frame*

$$\cos\theta(t) = \frac{1}{2}\cos\theta_{\rm s} - \frac{\sqrt{3}}{2}\sin\theta_{\rm s}\cos[\phi_t - \phi_{\rm s}]$$
$$\phi(t) = \alpha_0 + \phi_t + \tan^{-1}\left[\frac{\sqrt{3}\cos\theta_{\rm s} + \sin\theta_{\rm s}\cos[\phi_t - \phi_{\rm s}]}{2\sin\theta_{\rm s}\sin[\phi_t - \phi_{\rm s}]}\right]$$

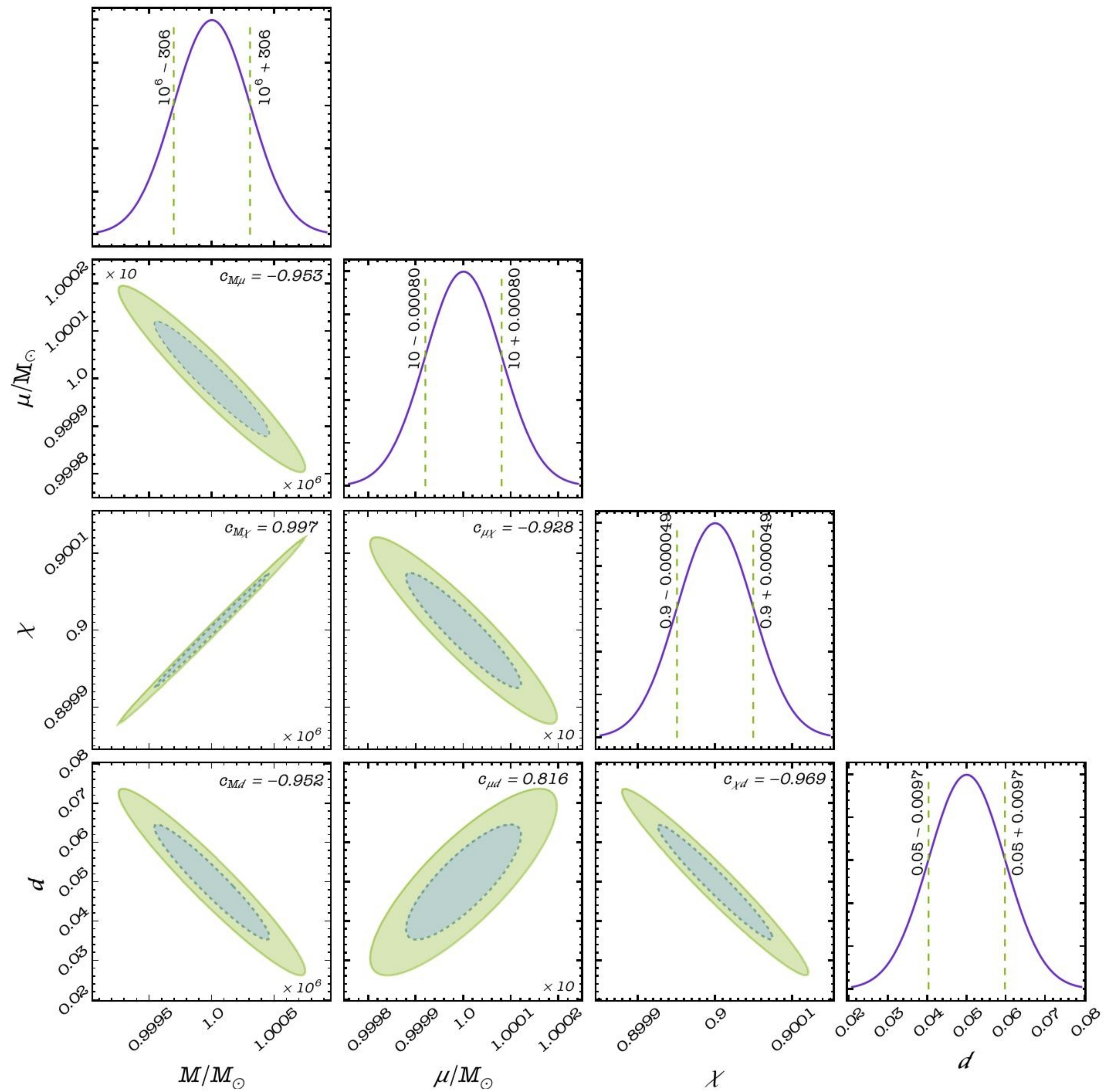


- *Add Doppler shift*

$$\Phi \to \Phi + 2\pi R_\odot \sin\theta_s \cos\left(\frac{2\pi t}{T} - \phi_s\right)$$

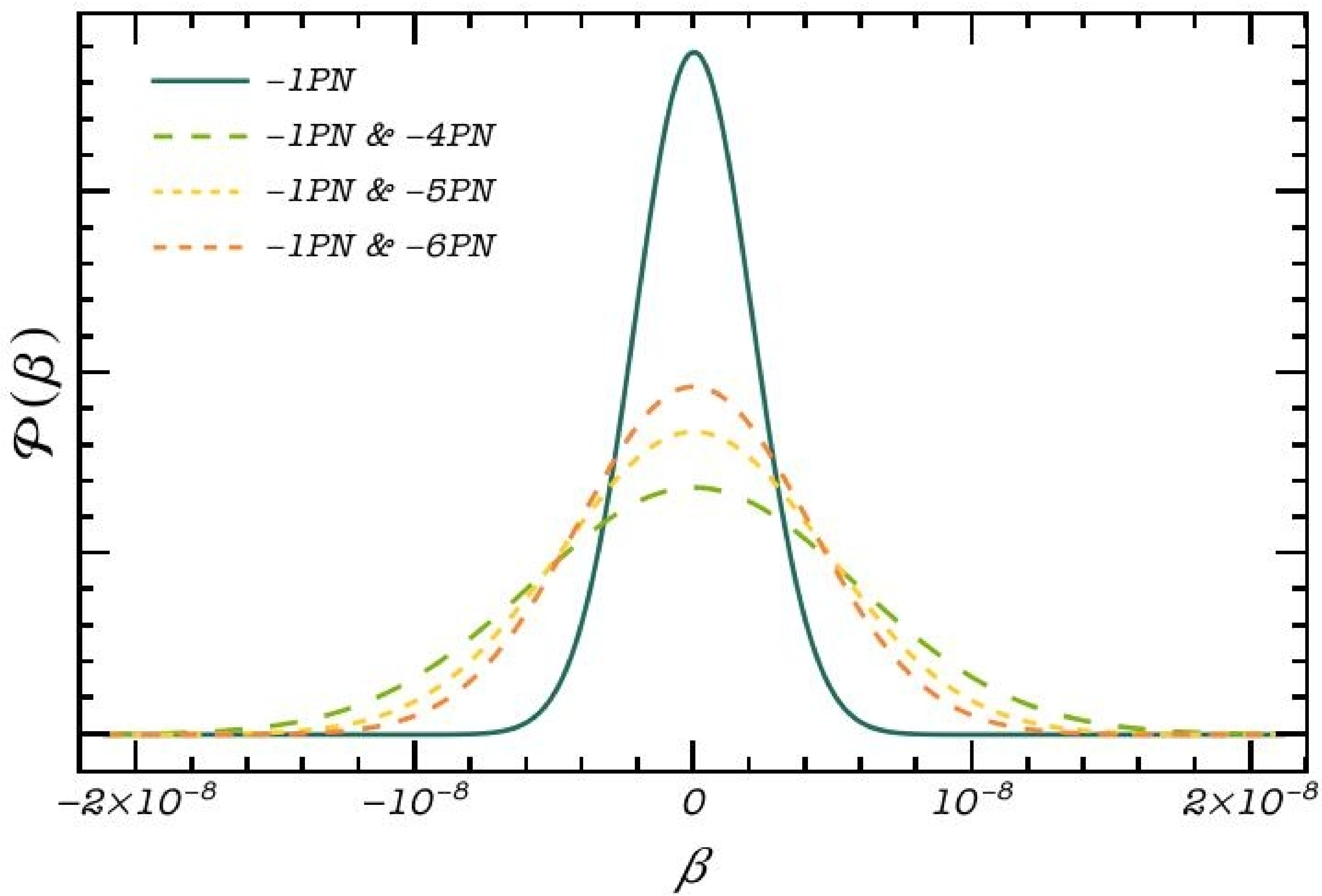
Berti +, PRD 71 (1995)

Correlation with the environment



Correlation with the environment

What about multiple competing effects? $h = \mathcal{A}e^{i\psi_{\rm GW}}$ $\psi_{\rm GW} = \psi_{\rm BH} + \beta(M\pi f)^b$



The rotating dephasing

Barsanti, + in preparation