Black hole sensitivities in Einstein-scalar-Gauss-Bonnet gravity

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GdR Ondes Gravitationnelles

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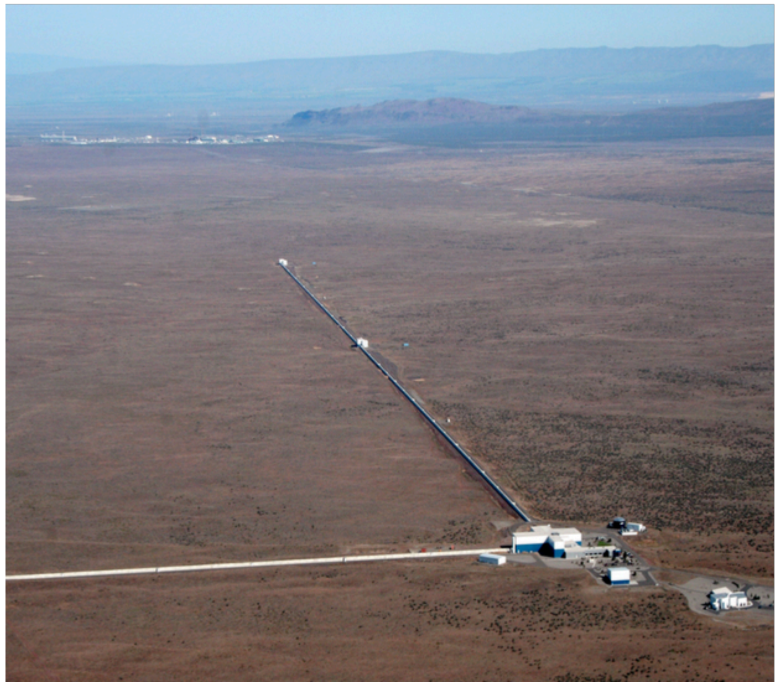
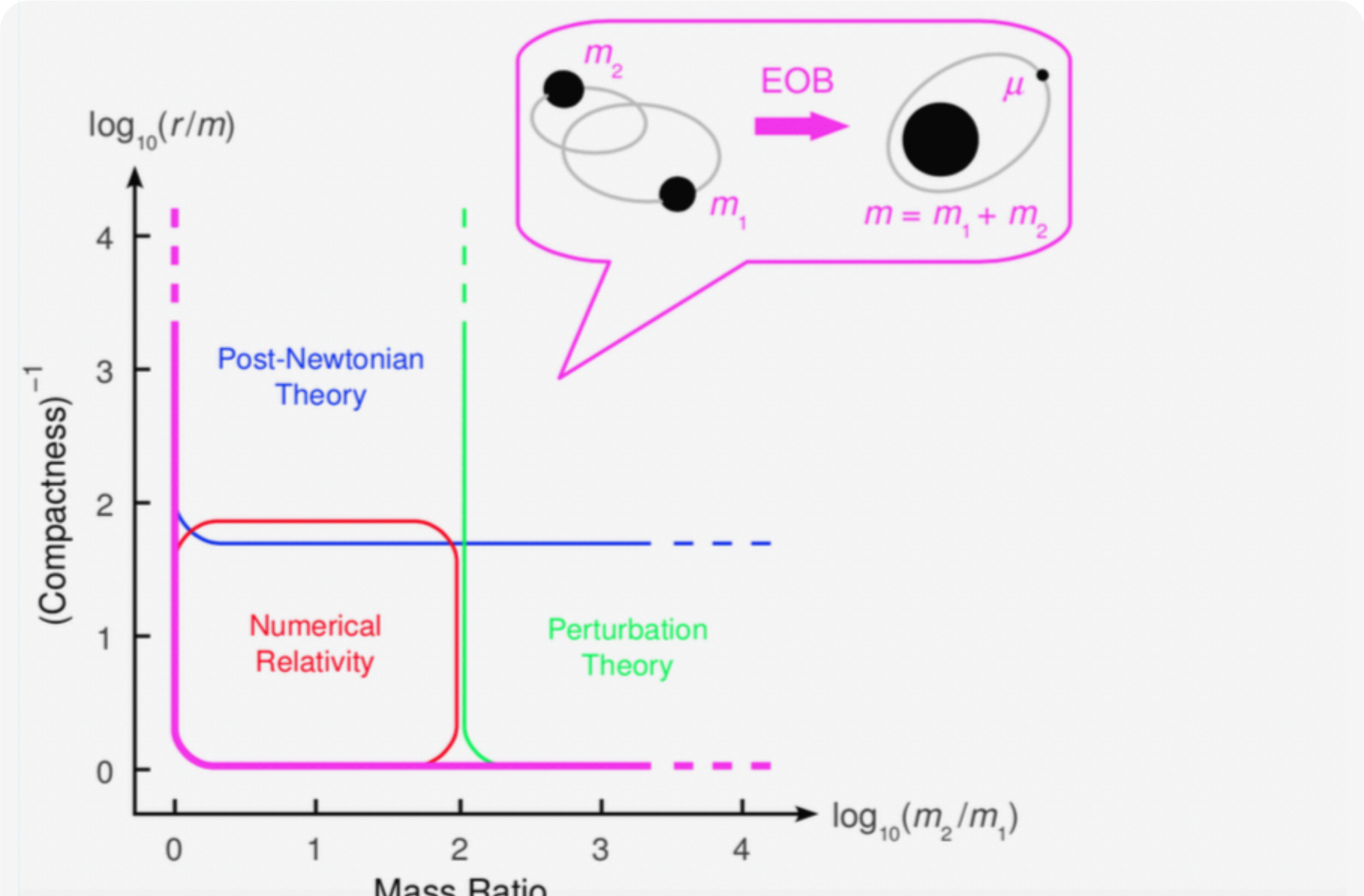
The era of gravitational wave astronomy

- GW150914: first observation of a BBH coalescence by LIGO-Virgo
- GW170817: first BNS with EM counterparts (multimessenger astronomy)
- O3: 56 gravitational-wave detections between April 2019 and March 2020
- Since March 2020: O4 in preparation, possibly with KAGRA...



Opportunity of **new tests of general relativity and modified gravities**, in the strong-field regime of a compact binary coalescence.

"Knowing the chirp to hear it"...



[courtesy Alexandre Le Tiec]

In general relativity: PN theory, self-force calculations, EOB framework, numerical relativity...

To be generalized to modified gravities, such as Einstein-scalar-Gauss-Bonnet theory.

Einstein-Scalar-Gauss-Bonnet gravity

ESGB vacuum action ($G = c = 1$)

$$I_{\text{ESGB}} = \frac{1}{16\pi}\int d^4x\sqrt{-g}\left(R - 2g^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi + \alpha f(\varphi)\mathscr{R}^2_{\text{GB}}\right)$$

- Massless scalar field φ
- Gauss-Bonnet scalar $\mathscr{R}^2_{\text{GB}} = R^{\mu\nu\rho\sigma}R_{\mu\nu\rho\sigma} - 4R^{\mu\nu}R_{\mu\nu} + R^2$
- Fundamental coupling α with dimensions L^2 and $f(\varphi)$ defines the ESGB theory
- $\int d^Dx\sqrt{-g}\mathscr{R}^2_{\text{GB}}$ is a boundary term in $D \leqslant 4$ [see e.g. Myers 87]

Second order field equations

$$R_{\mu\nu} = 2\partial_\mu\varphi\partial_\nu\varphi - 4\alpha\left(P_{\mu\alpha\nu\beta} - \frac{g_{\mu\nu}}{2}P_{\alpha\beta}\right)\nabla^\alpha\nabla^\beta f(\varphi)$$

$$\Box\varphi = -\frac{1}{4}\alpha f'(\varphi)\mathscr{R}^2_{\text{GB}}$$

with $P_{\mu\nu\rho\sigma} = R_{\mu\nu\rho\sigma} - 2g_{\mu[\rho}R_{\sigma]\nu} + 2g_{\nu[\rho}R_{\sigma]\mu} + g_{\mu[\rho}g_{\sigma]\nu}R$

Hairy black holes in ESGB gravity

Analytical solutions in the small Gauss-Bonnet coupling α limit

- **Einstein-dilaton-Gauss-Bonnet,** $f(\varphi) = e^{\varphi}$

  Mignemi-Stewart 93 at $\mathcal{O}(\alpha^2)$, *Maeda at al. 97* at $\mathcal{O}(\alpha)$, *Yunes-Stein 11* at $\mathcal{O}(\alpha)$

  Ayzenberg-Yunes 14 at $\mathcal{O}(\alpha^2, S^2)$, *Pani et al. 11* at $\mathcal{O}(\alpha^2, S^2)$, *Maselli et al. 15* at $\mathcal{O}(\alpha^7, S^5)$

- **Shift-symmetric theories,** $f(\varphi) = \varphi$

  Sotiriou-Zhou 14 at $\mathcal{O}(\alpha^2)$

- **Generic ESGB theories**

  Julié-Berti 19 at $\mathcal{O}(\alpha^4)$

Numerical solutions

- **Einstein-dilaton-Gauss-Bonnet,** $f(\varphi) = e^{\varphi}$

  Kanti et al. 95, *Pani-Cardoso 09*, *Kleihaus 15* (includes spins)

- **Shift-symmetric theories,** $f(\varphi) = \varphi$

  Delgado et al. 20 (includes spin)

- **Generic ESGB theories**

  Antoniou et al. 18

- **Quadratic couplings,** $f(\varphi) = \varphi^2(1 + \lambda\varphi^2)$ and $f(\varphi) = -\, e^{-\lambda\varphi^2}$

  Doneva-Yazadjiev 17, Silva et al. 17, Minamitsuji-Ikeda 18, Macedo et al. 19,

  Dima-Barausse et al. 20, etc...

"Skeletonizing" an ESGB compact binary system

[in GR: Mathisson 1931, Infeld 1950,...]

$$I_{\rm ESGB} = \frac{1}{16\pi}\int d^4x\sqrt{-g}\left(R - 2g^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi + \alpha f(\varphi)\mathscr{R}^2_{\rm GB}\right) + I_{\rm pp}$$

Generic ansatz for compact bodies [FLJ-Berti, PRD100 (2019) 10, 104061]

$$I_{\rm pp}[g_{\mu\nu}, \varphi, \{x^\mu_A\}] = -\sum_A \int m_A(\varphi)\, ds_A$$

with $ds_A = \sqrt{-g_{\mu\nu}dx^\mu_A dx^\nu_A}$.

- $m_A(\varphi)$ is a function of the local value of φ to encompass the effect of the background scalar field on the equilibrium of body A [Eardley 75, Damour-Esposito-Farèse 92].
- Starting point for post-newtonian calculations: $\mathcal{O}\left(\frac{v}{c}\right)^{2n} \sim \mathcal{O}\left(\frac{GM}{r}\right)^n$ corrections to Newton

$$\ln m_A(\varphi) = \ln m^0_A + \alpha^0_A\,\varphi + \frac{1}{2}\beta^0_A\,\varphi^2 + \cdots$$

$$I^A_{\rm pp}[g_{\mu\nu},\varphi,x^\mu_A] = -\int m_A(\varphi)\, ds_A$$

Question: How to derive $m_A(\varphi)$ for an ESGB black hole?

Answer: by identifying the BH's fields to those sourced by the particle.

$$R_{\mu\nu} = 2\partial_\mu\varphi\partial_\nu\varphi - 4\alpha\left(P_{\mu\alpha\nu\beta} - \frac{1}{2}g_{\mu\nu}P_{\alpha\beta}\right)\nabla^\alpha\nabla^\beta f(\varphi) + 8\pi\left(T^A_{\mu\nu} - \frac{1}{2}g_{\mu\nu}T^A\right)$$

$$\square\varphi = -\frac{1}{4}\alpha f'(\varphi)\mathscr{R}^2_{\rm GB} + 4\pi\frac{ds_A}{dt}\frac{dm_A}{d\varphi}\frac{\delta^{(3)}(\mathbf{x}-\mathbf{x}_A(t))}{\sqrt{-g}}$$

with $T^{\mu\nu}_A = m_A(\varphi)\dfrac{\delta^{(3)}(\mathbf{x}-\mathbf{x}_A(t))}{\sqrt{g g_{\alpha\beta}\frac{dx^\alpha_A}{dt}\frac{dx^\beta_A}{dt}}}\dfrac{dx^\mu_A}{dt}\dfrac{dx^\nu_A}{dt}$

Fields of the particle A in its rest frame

$x^i_A = 0$, harmonic gauge $\partial_\mu(\sqrt{-g}g^{\mu\nu}) = 0$

$$g_{\mu\nu} = \eta_{\mu\nu} + \delta_{\mu\nu}\left(\frac{2m_A(\varphi_\infty)}{\tilde r}\right) + \mathcal{O}\left(\frac{1}{\tilde r^2}\right)$$

$$\varphi = \varphi_\infty - \frac{1}{\tilde r}\frac{dm_A}{d\varphi}(\varphi_\infty) + \mathcal{O}\left(\frac{1}{\tilde r^2}\right)$$

Fields of the ESGB black hole

Isotropic coordinates

$$g_{\mu\nu} = \eta_{\mu\nu} + \delta_{\mu\nu}\left(\frac{2M_{\rm ADM}}{\tilde r}\right) + \mathcal{O}\left(\frac{1}{\tilde r^2}\right)$$

$$\varphi = \varphi_\infty + \frac{D}{\tilde r} + \mathcal{O}\left(\frac{1}{\tilde r^2}\right)$$

Matching conditions

(a) $m_A(\varphi_\infty) = M_{\rm ADM}$

(b) $m'_A(\varphi_\infty) = -D$

[FLJ JCAP 01 (2018) 026]

[FLJ-Berti, PRD100 (2019) 10, 104061]

ESGB black hole thermodynamics

- **Temperature:**

$$\boxed{T = \frac{\kappa}{4\pi}}$$ where $\kappa^2 = -\frac{1}{2}(\nabla_\mu \xi_\nu \nabla^\mu \xi^\nu)_{r_H}$ is the surface gravity

- **Wald entropy:**

$$S_w = -8\pi \int_{r_H} d\theta d\phi \sqrt{\sigma} \frac{\partial \mathscr{L}}{\partial R_{\mu\nu\rho\sigma}} \epsilon_{\mu\nu}\epsilon_{\rho\sigma}$$ with $\epsilon_{\mu\nu} = n_{[\mu} l_{\nu]}$

$$\boxed{S_w = \frac{\mathscr{A}_H}{4} + 4\alpha\pi f(\varphi_H)}$$ in ESGB gravity.

- **Mass as a global charge:**

$$\boxed{M = M_{ADM} + \int D\, d\varphi_\infty}$$

[Henneaux et al. 02, Cardenas et al. 16, Anabalon-Deruelle-FLJ 16,...]

The variations of S_w and M with respect to the BH's integration constants satisfy:

$$T\delta S_w = \delta M$$

Reminder: matching conditions

(a) $m_A(\varphi_\infty) = M_{ADM}$

(b) $m_A'(\varphi_\infty) = -D$

(a) and (b) $\Rightarrow$ $\delta M = \delta M_{ADM} + D\delta\varphi_\infty = 0$

As a consequence, $\delta S_w = 0$

A skeletonized black hole is described by a sequence of constant Wald entropy equilibrium configurations.

Example: numerical sensitivity of an Einstein-dilaton-Gauss-Bonnet black hole

$$I_{\rm EDGB} = \frac{1}{16\pi}\int d^4x\sqrt{-g}\left(R - 2g^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi + \frac{1}{4}\alpha\, e^{2\varphi}\mathscr{R}^2_{\rm GB}\right)$$

- Static, spherically symmetric spacetime in Schwarzschild-Droste coordinates

$$ds^2 = -A(r)dt^2 + B(r)^{-1}dr^2 + r^2 d\phi^2\ , \qquad \varphi = \varphi(r)$$

- The Klein-Gordon, and (tt) and (rr) components of the Einstein equations yield a system of coupled 2nd order differential equations on A and φ

$$\boxed{A'' = A''(A, A', \varphi, \varphi')\ , \quad \varphi'' = \varphi''(A, A', \varphi, \varphi')} \qquad \text{together with} \quad B(A, A', \varphi, \varphi').$$

- Four initial conditions for A, φ and their first derivatives. On the assumed horizon radius $r_{\rm H}$, $A = 0$:

$$\boxed{\begin{aligned} A(r_*) &= A_1^{\rm H}(r_* - 1) + \ldots \\ \varphi(r_*) &= \varphi_{\rm H} + \varphi_1^{\rm H}(r_* - 1) + \ldots \end{aligned}} \qquad \text{with} \quad r_* \equiv r/r_{\rm H} \ \text{ and } \ a' \equiv da/dr_*$$

- Since $A_1^{\rm H}$ is pure gauge, black holes depend on two integration constants only.

- Choose $\varphi_{\rm H}$ and the irreducible mass $\mu_A = \sqrt{S_{\rm w}/4\pi}$

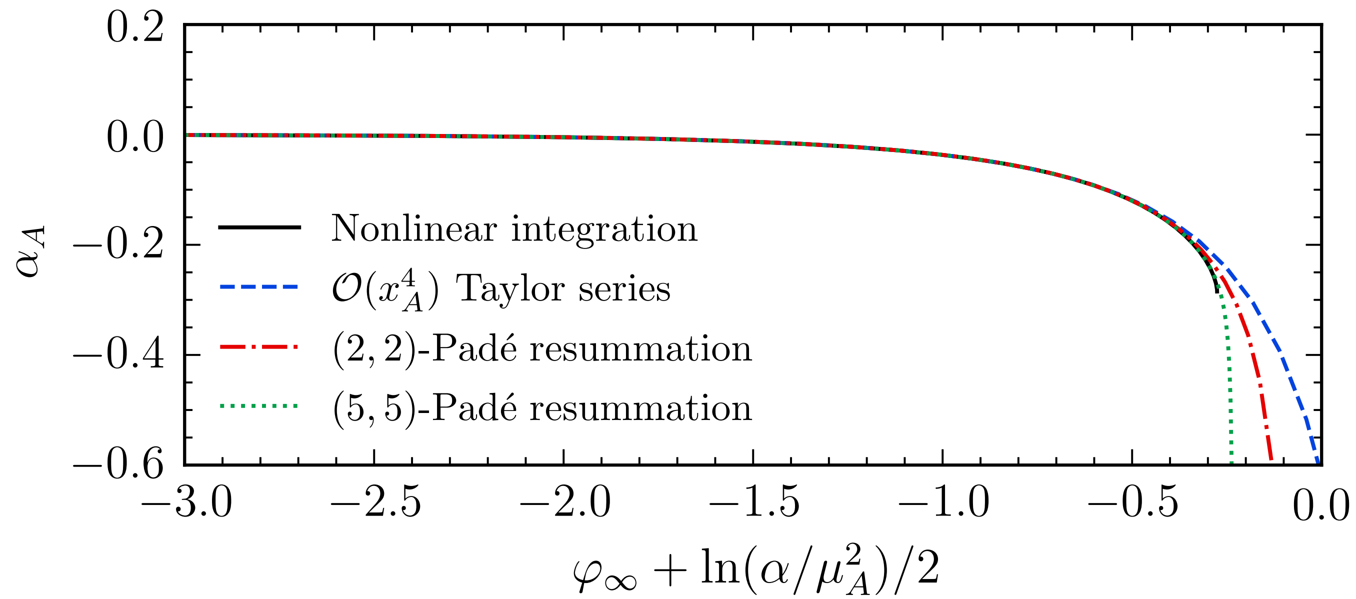
$$\boxed{\varphi_1^{\rm H} = \frac{-1 + \sqrt{1 - 6\alpha_*^2 e^{4\varphi_{\rm H}}}}{2\alpha_*\, e^{2\varphi_{\rm H}}}} \qquad \text{with} \quad \alpha_* \equiv \alpha/r_{\rm H}^2 = \frac{1}{4\mu_A^2/\alpha - e^{2\varphi_{\rm H}}}$$

- Extract $M_{\rm ADM}$, D and φ_∞ from the asymptotic $\mathcal{O}(1/r)$ fall off of the fields

$$B = 1 - \frac{2M_{\rm ADM}}{r} + \mathcal{O}(1/r^2) \qquad \varphi = \varphi_\infty + \frac{D}{r} + \mathcal{O}(1/r^2)$$

The sensitivity of an Einstein-dilaton-Gauss-Bonnet black hole

$$\alpha_A = \frac{d \ln M_{\rm ADM}}{d\varphi_\infty} = -\frac{D}{M_{\rm ADM}}$$



- α_A is invariant under the fundamental symmetry $\varphi_\infty \to \varphi_\infty + \Delta\varphi$ and $\alpha \to \alpha e^{-2\Delta\varphi}$
- A BH with fixed irreducible mass $\mu_A = \sqrt{S_{\rm w}/4\pi}$ does not exist when φ_∞ violates

$$\varphi_\infty + \frac{1}{2}\ln\left(\frac{\alpha}{\mu_A^2}\right) < -0.276$$

- The endpoint corresponds to the saturation of the horizon bound [see also Kanti at al. 95, Doneva-Yazadjiev 17]

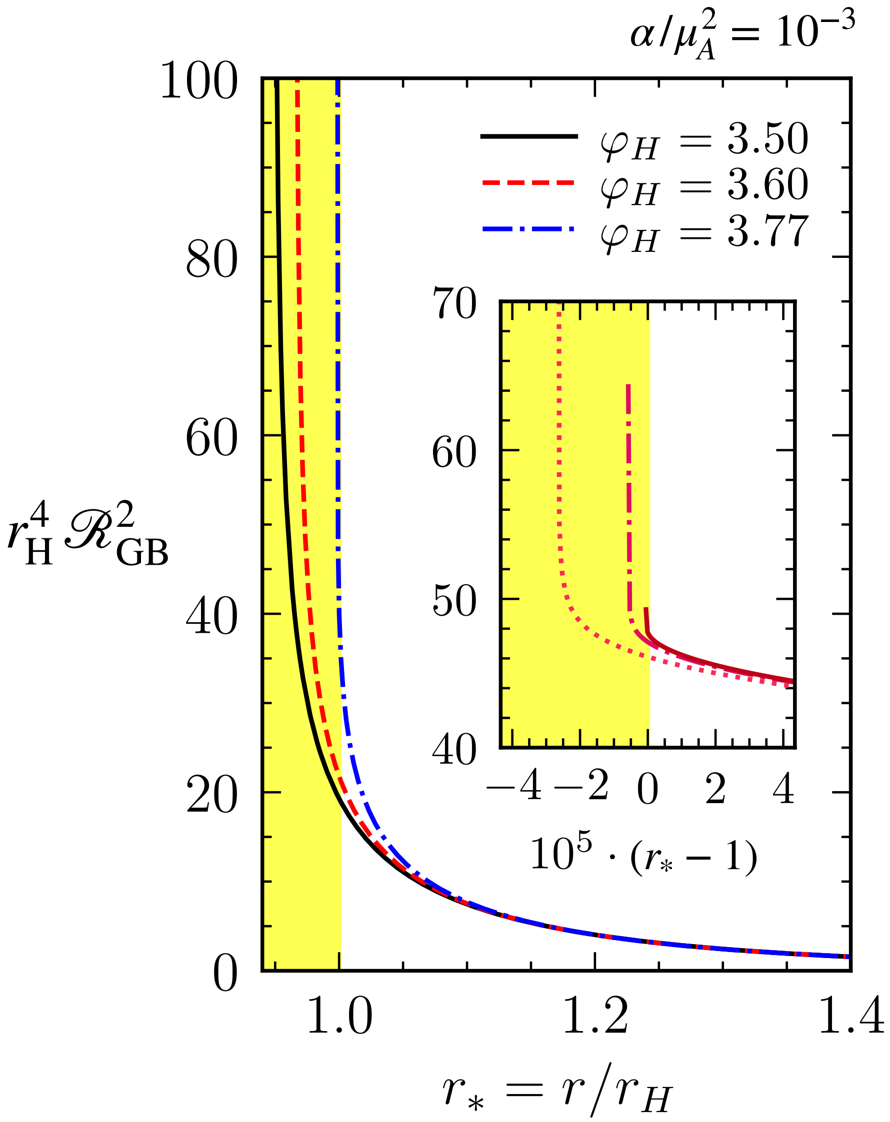
$$1 - 6\alpha_*^2 e^{4\varphi_H} > 0 \quad \Rightarrow \quad \varphi_H + \frac{1}{2}\ln\left(\frac{\alpha}{\mu_A^2}\right) < \frac{1}{2}\ln\left(\frac{4}{1+\sqrt{6}}\right)$$

for a black hole with fixed Wald entropy.

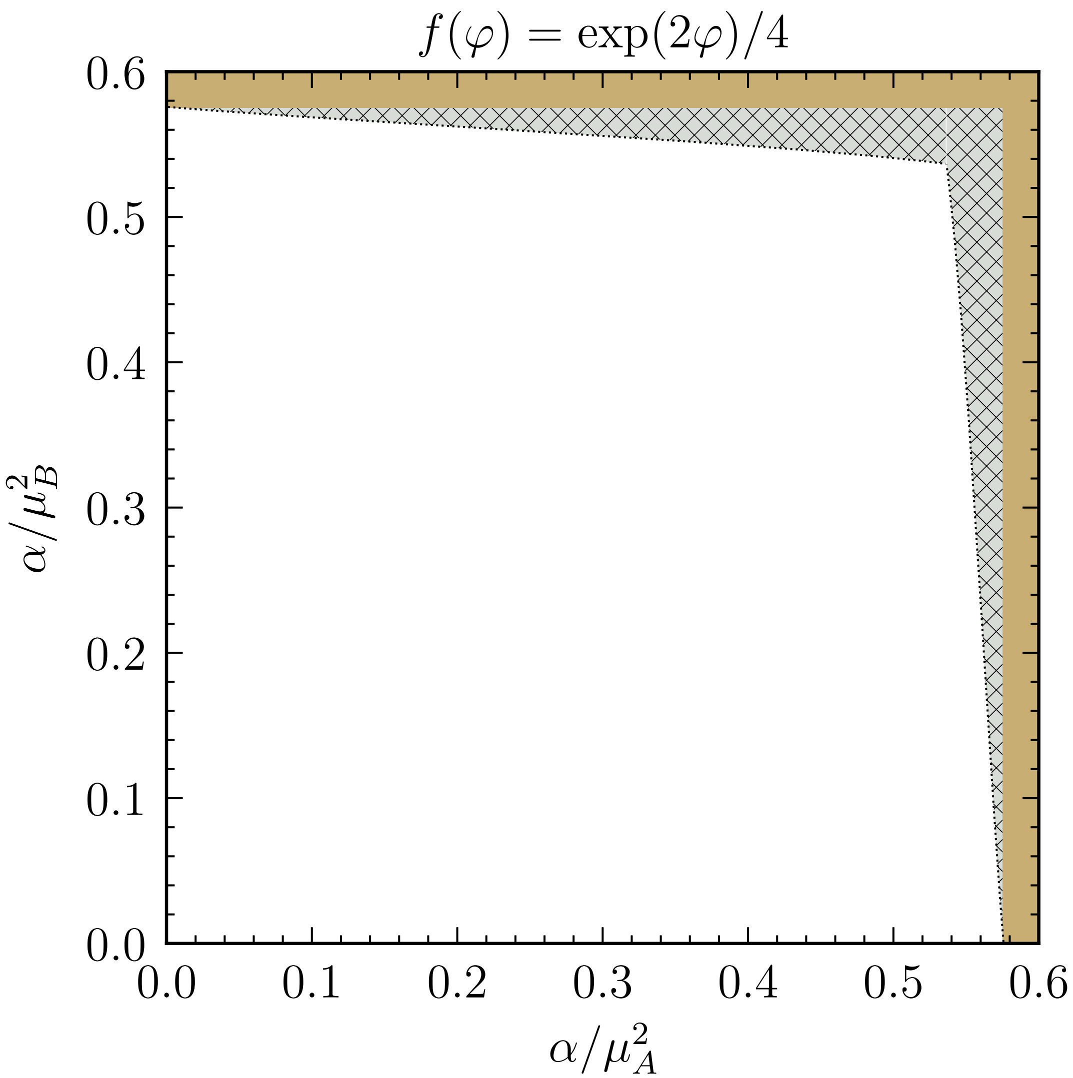
- Setting $\epsilon \equiv 1 - 6\alpha_*^2 e^{4\varphi_H} \ll 1$ we find

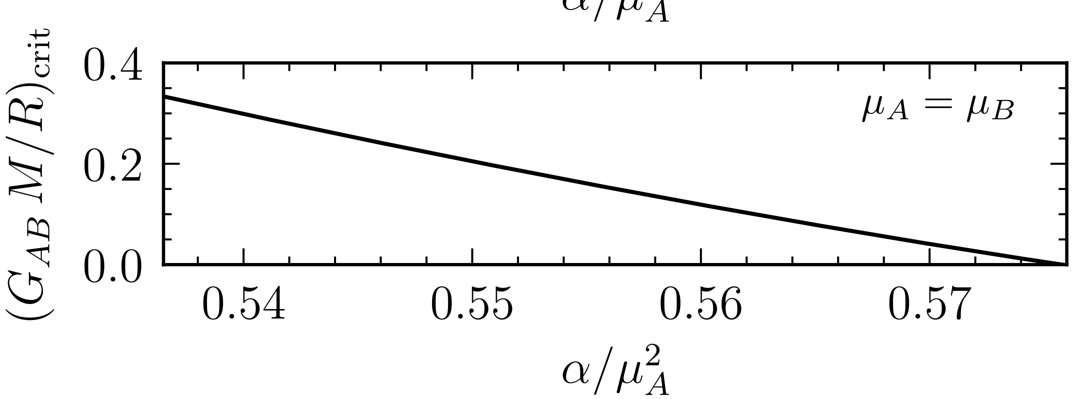
$$r_{\rm H}^4 \mathscr{R}_{\rm GB}^2 = 48 + g_1^{\rm H}(r_* - 1) + \mathcal{O}(r_* - 1)^2$$

$$\text{with} \quad g_1^{\rm H} = -\frac{648 + 864\alpha_* e^{2\varphi_H}}{\epsilon} + \mathcal{O}(\epsilon^0)$$



The fate of black hole binaries





$$\frac{G_{AB}M}{R_{\rm crit}} = (G_{AB}M\dot{\phi}_{\rm crit})^{2/3} + \mathcal{O}(v^4)$$

- A black hole binary $(\mu_A^2/\alpha,\ \mu_B^2/\alpha)$ must simultaneously satisfy

$$\text{(a)}\quad \varphi_\infty^A + \frac{1}{2}\ln\left(\frac{\alpha}{\mu_A^2}\right) < -0.276$$

$$\text{(b)}\quad \varphi_\infty^B + \frac{1}{2}\ln\left(\frac{\alpha}{\mu_B^2}\right) < -0.276$$

- We estimate φ_∞^A and φ_∞^B perturbatively around $\varphi_\infty^{A/B} = 0$ in the PN framework. At Newtonian order ($R = |\mathbf{x}_A - \mathbf{x}_B|$):

$$\varphi_\infty^A = \varphi(t, \mathbf{x}_A) = -\frac{m_B^0\alpha_B^0}{R} + \mathcal{O}(v^4)\ ,\quad \varphi_\infty^B = \varphi(t, \mathbf{x}_B) = -\frac{m_A^0\alpha_A^0}{R} + \mathcal{O}(v^4)$$

- (a) and (b) depend only on R, α/μ_A^2 and α/μ_B^2.

- When $R \to \infty$ (far inspiral), $\varphi_\infty^A = \varphi_\infty^B = 0$ and

$$\alpha/\mu_A^2 < 0.576\ ,\quad \alpha/\mu_B^2 < 0.576$$

see also [Witek et al. 2019]

- When R is finite, φ_∞^A and φ_∞^B are positive and tighten (a) and (b); the latter can saturate before the system reaches its light-ring

$$R_{\rm LR} = 3G_{AB}M\ ,$$

where $G_{AB} = 1 + \alpha_A^0\alpha_B^0$ and $M = m_A^0 + m_B^0$.

Conclusion

- The ESGB two-body Lagrangian is the **same as that of scalar-tensor (ST) theories at 3PN**, modulo a **finite Gauss-Bonnet 3PN contribution** given in [*FLJ-Berti 19*].
- The ST two-body Lagrangian is known at 2PN [*Mirshekari-Will 13*] and **3PN** [*Bernard 19*].
- The energy fluxes are **fully known** at -1PN, 0PN [*Damour-Esposito-Farèse 92*] and **1PN** [*Lang 14*] as they only differ from those of ST from 2PN on [*Yagi et al. 12, see also Shiralilou et al. 20-21*].
- The "EOBization" of ST theories at 2PN in [*FLJ-Deruelle 17*] includes ESGB gravity.
- The quantities above can now be fully **specified for hairy binary BHs in ESGB theories** with dilatonic couplings $f(\varphi) = e^{2\varphi}/4$, but also shift symmetric couplings $f(\varphi) = 2\varphi$.

Ongoing and future developments

- Include **"spontaneously scalarized" black holes in ESGB models with quadratic couplings** $f(\varphi) = e^{\varphi^2}$ [*Silva et al. 17*].
- Generalize our work to **spinning black holes.**
- Further explore the highlighted BBH parameter space using **higher PN orders** or **numerical relativity**? *[see also Witek et al. 19, Okounkova 20, East-Ripley 20 for numerical waveforms in the small α limit or East-Ripley 21 for numerical head-on collisions with large α].*

Thank you for your attention.