Status and prospects on FeynArts, FormCalc, and LoopTools

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Automated Diagram Evaluation

FeynArts

Amplitudes

FormCalc

Fortran Code

LoopTools

$|\mathcal{M}|^2 \longrightarrow$ **Cross-sections, Decay rates, . . .**

Diagram Generation:

- Create the topologies
- Insert fields
- Apply the Feynman rules
- Paint the diagrams

Algebraic Simplification:

- Contract indices
- Calculate traces
- Reduce tensor integrals
- Introduce abbreviations

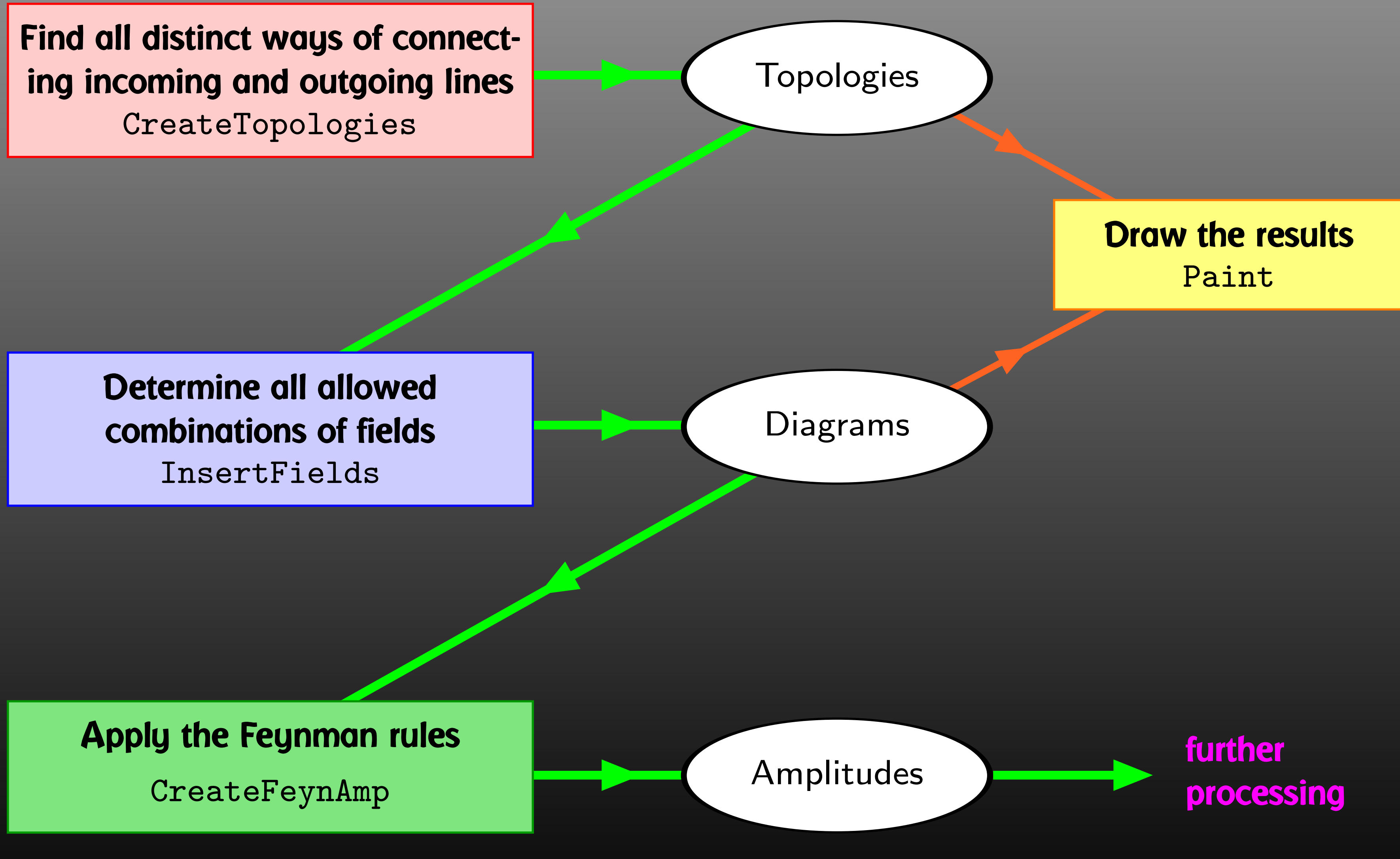
Numerical Evaluation:

- Convert Mathematica output to Fortran code
- Supply a driver program
- Implementation of the integrals

Symbolic manipulation (Computer Algebra) for the structural and algebraic operations.

Compiled high-level language (Fortran) for the numerical evaluation.

FeynArts



Three Levels of Fields

Generic level, e.g. `F, F, S`

$$C(F_1, F_2, S) = G_-\omega_- + G_+\omega_+$$

Kinematical structure completely fixed, most algebraic simplifications (e.g. tensor reduction) can be carried out.

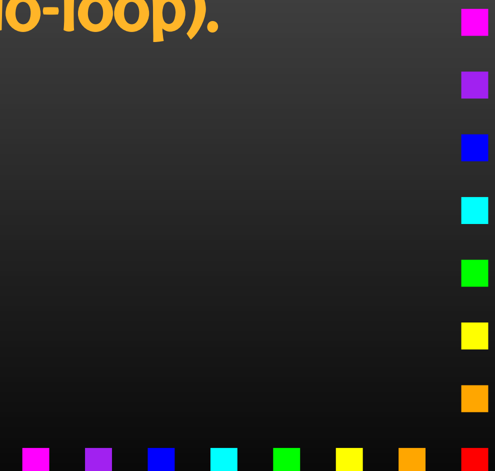
Classes level, e.g. `-F[2], F[1], S[3]`

$$\bar{\ell}_i \nu_j G: \quad G_- = -\frac{\mathrm{i}\, e\, m_{\ell,i}}{\sqrt{2}\sin\theta_w M_W}\delta_{ij}\,, \quad G_+ = 0$$

Coupling fixed except for i, j (can be summed in do-loop).

Particles level, e.g. `-F[2,{1}], F[1,{1}], S[3]`

insert fermion generation (1, 2, 3) for i and j



The Model Files

One has to set up, once and for all, a

- **Generic Model File** (seldomly changed) containing the generic part of the couplings,

Example: the FFS coupling

$$C(F, F, S) = G_{-}\omega_{-} + G_{+}\omega_{+} = \vec{G} \cdot \begin{pmatrix} \omega_{-} \\ \omega_{+} \end{pmatrix}$$

```
AnalyticalCoupling[s1 F[j1, p1], s2 F[j2, p2], s3 S[j3, p3]]
== G[1][s1 F[j1], s2 F[j2], s3 S[j3]] .
     { NonCommutative[ ChiralityProjector[-1] ],
       NonCommutative[ ChiralityProjector[+1] ] }
```

The Model Files

One has to set up, once and for all, a

- **Classes Model File** (for each model)
  declaring the particles and the allowed couplings

Example: the $\bar{\ell}_i \nu_j G$ coupling in the Standard Model

$$\vec{G}(\bar{\ell}_i, \nu_j, G) = \begin{pmatrix} G_- \\ G_+ \end{pmatrix} = \begin{pmatrix} -\frac{\mathrm{i}\, e\, m_{\ell,i}}{\sqrt{2}\sin\theta_w M_W}\delta_{ij} \\ 0 \end{pmatrix}$$

```
C[ -F[2,{i}], F[1,{j}], S[3] ]
== { {-I EL Mass[F[2,{i}]]/(Sqrt[2] SW MW) IndexDelta[i, j]},
     {0} }
```

Current Status of Model Files

Model Files presently available for FeynArts:

- SM [w/QCD], normal and background-field version.
  All one-loop counter terms included.
- MSSM [w/QCD].
  Counter terms by T. Fritzsche.
- Two-Higgs-Doublet Model.
  Counter terms not included yet.
- ModelMaker utility generates Model Files from the Lagrangian.
- “3rd-party packages” FeynRules and LanHEP generate Model Files for FeynArts and others.
- SARAH package derives SUSY Models.

Partial (Add-On) Model Files

FeynArts distinguishes

- **Basic Model Files** and
- **Partial (Add-On) Model Files.**

Basic Model Files, e.g. `SM.mod`, `MSSM.mod`, can be modified by Add-On Model Files. For example,

```
InsertFields[..., Model -> {"MSSMQCD", "FV"}]
```

This loads the Basic Model File `MSSMQCD.mod` and modifies it through the Add-On `FV.mod` (non-minimal flavour violation).

Model files can thus be built up from several parts.

Tweaking Model Files

Or, How to efficiently make changes in an existing model file.

Bad: Copy the model file, modify the copy. – Why?

- It is typically not very transparent what has changed.
- If the original model file changes (e.g. bug fixes), these do not automatically propagate into the derivative model file.

Better: Create an add-on model file which modifies the particles and coupling tables.

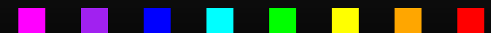
- `M$ClassesDescription` = list of particle definitions,
- `M$CouplingMatrices` = list of couplings.

Tweaking Model Files

Example: Introduce enhancement factors for the b-$\bar{b}$-h_0 and b-$\bar{b}$-H_0 Yukawa couplings in the MSSM.

```
EnhCoup[(lhs:C[F[4,{g_,_}], -F[4,_], S[h:1|2]]) == rhs_] :=
  lhs == Hff[h,g] rhs
EnhCoup[other_] = other

M$CouplingMatrices = EnhCoup/@ M$CouplingMatrices
```



Linear Combinations of Fields

FeynArts can automatically linear-combine fields, i.e. one can specify the couplings in terms of gauge rather than mass eigenstates. For example:

```
M$ClassesDescription = { ...,
  F[11] = {...,
    Indices -> {Index[Neutralino]},
    Mixture -> ZNeu[Index[Neutralino],1] F[111] +
               ZNeu[Index[Neutralino],2] F[112] +
               ZNeu[Index[Neutralino],3] F[113] +
               ZNeu[Index[Neutralino],4] F[114]} }
```

Since `F[111]...F[114]` **are not listed in** `M$CouplingMatrices`**, they drop out of the model completely.**

Linear Combinations of Fields

Higher-order mixings can be added, too:

```
M$ClassesDescription = { ...,
  S[1] = {...},
  S[2] = {...},
  S[10] == {...,
    Indices -> {Index[Higgs]},
    Mixture -> UHiggs[Index[Higgs],1] S[1] +
               UHiggs[Index[Higgs],2] S[2],
    InsertOnly -> {External, Internal}} }
```

This time, `S[10]` ***and*** `S[1]`, `S[2]` appear in the coupling list (including all mixing couplings) because all three are listed in `M$CouplingMatrices`.

Due to the `InsertOnly`, `S[10]` is **inserted only on tree-level parts of the diagram, not in loops.**

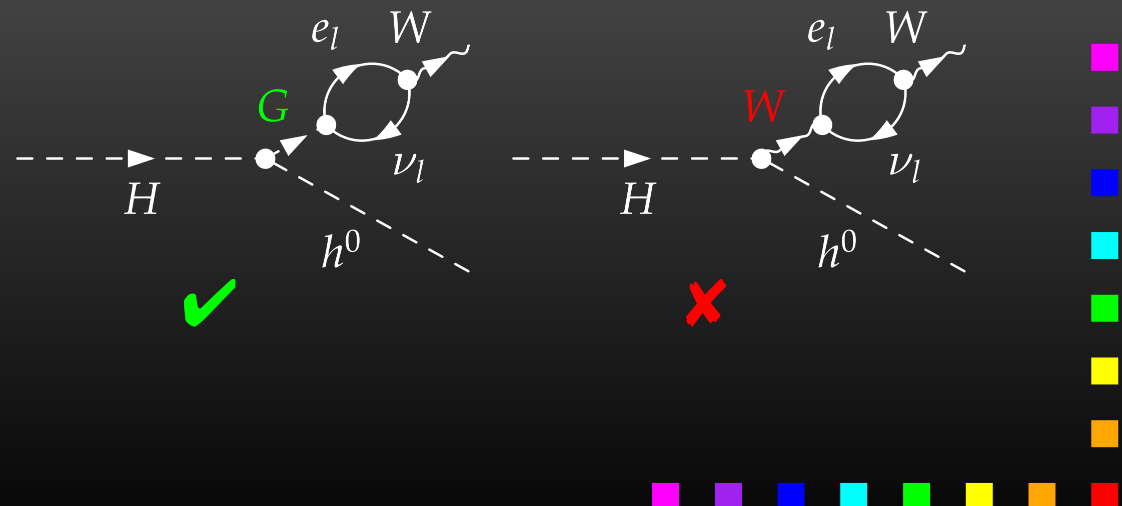
Enhanced Diagram Selection

FeynArts has easier way to pick wave-function corrections:

```
CreateTopologies[ ...
  ExcludeTopologies -> WFCorrections[1|3] ]
```

Select only those where the in- and out-fields of the self-energy are not the same:

```
DiagramSelect[ ...,
  UnsameQ@@ WFCorrectionFields[##] & ]
```

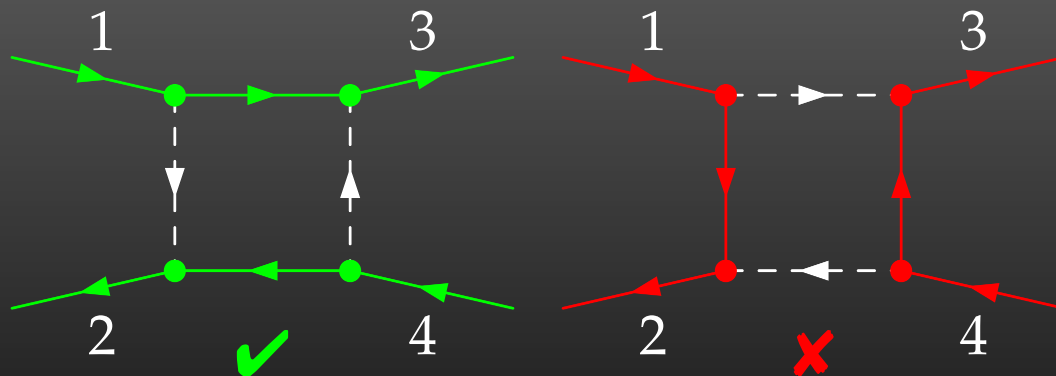


Enhanced Diagram Selection

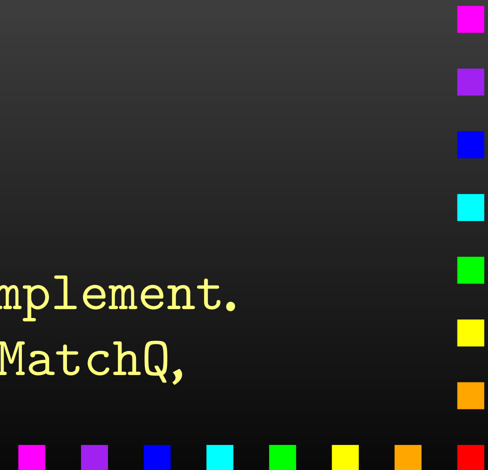
The new FeynArts function FermionRouting can be used to select diagrams according to their fermion structure, e.g.

```
DiagramSelect[...,
   FermionRouting[##] === {1,3, 2,4} & ]
```

selects only diagrams where external legs 1–3 and 2–4 are connected through fermion lines.



More Functions: DiagramGrouping, DiagramMap, DiagramComplement.
More Filters: Vertices, FieldPoints, FeynAmpCases, FieldMatchQ, FieldMemberQ, FieldPointMatchQ, FieldPointMemberQ.



Programming Diagram Filters

Or, What if FeynArts' selection functions are not enough.

Observe the structure of inserted topologies:

TopologyList[__][t_1, t_2, ...]

t_i: Topology[_][__] -> Insertions[Generic][g_1, g_2, ...]

g_i: Graph[__][__] -> Insertion[Classes][c_1, c_2, ...]

c_i: Graph[__][__] -> Insertion[Particles][p_1, p_2, ...]

Example: Select the diagrams with only fermion loops.

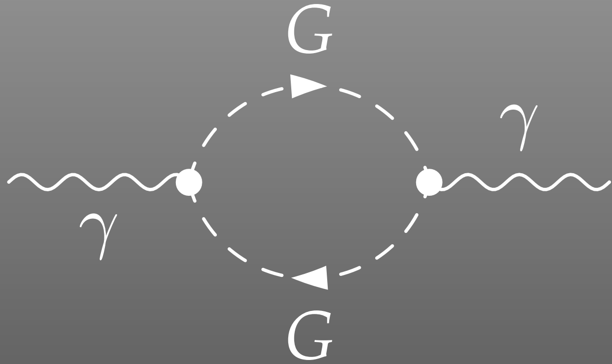
```
FermionLoop[t:TopologyList[___][__]] := FermionLoop/@ t

FermionLoop[(top:Topology[_][__]) -> ins:Insertions[Generic][__]] :=
  top -> TestLoops[top]/@ ins

TestLoops[top_][gi_ -> ci_] := (gi -> ci) /;
  MatchQ[Cases[top /. List@@ gi,
    Propagator[Loop[_]][v1_, v2_, field_] -> field]], F..]

TestLoops[_][_] := Sequence[]
```

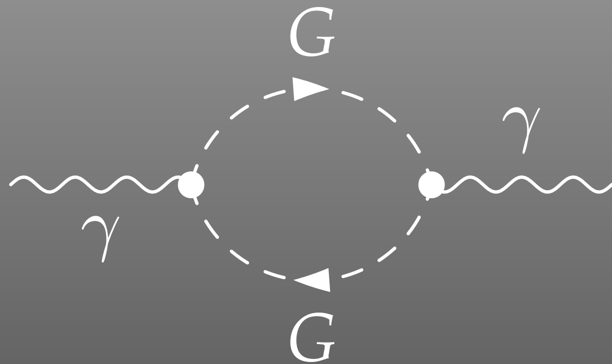
Sample CreateFeynAmp output



= FeynAmp[*identifier* ,
loop momenta ,
generic amplitude ,
insertions]

```
GraphID[Topology == 1, Generic == 1]
```

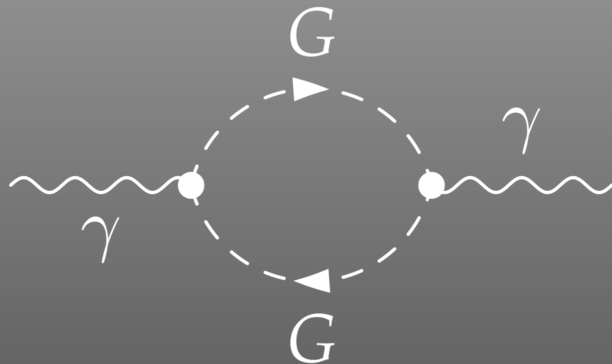
Sample CreateFeynAmp output



= FeynAmp[*identifier* ,
loop momenta ,
generic amplitude ,
insertions]

`Integral[q1]`

Sample CreateFeynAmp output



= FeynAmp[*identifier*,
loop momenta,
generic amplitude,
insertions]

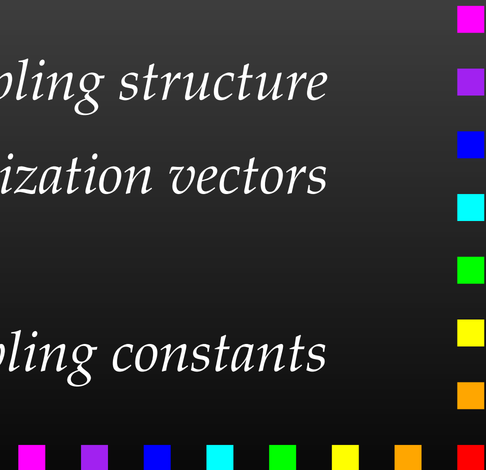
$\frac{\mathtt{I}}{\mathtt{32\ Pi}^4}$ RelativeCF . . . . *prefactor*

FeynAmpDenominator[$\frac{1}{\mathtt{q1}^2 - \mathtt{Mass[S[Gen3]]}^2}$,
$\frac{1}{(\mathtt{-p1 + q1})^2 - \mathtt{Mass[S[Gen4]]}^2}$] . . . . *loop denominators*

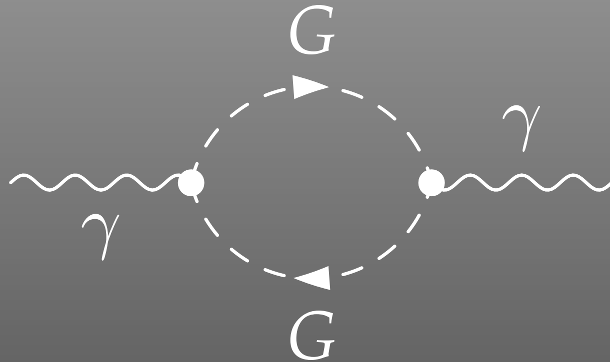
(p1 - 2 q1)[Lor1] (-p1 + 2 q1)[Lor2] . . . . *kin. coupling structure*

ep[V[1], p1, Lor1] ep*[V[1], k1, Lor2] . . . . *polarization vectors*

$\mathtt{G}^{(0)}_{\mathtt{SSV}}$[(Mom[1] - Mom[2])[KI1[3]]]
$\mathtt{G}^{(0)}_{\mathtt{SSV}}$[(Mom[1] - Mom[2])[KI1[3]]] . . . . *coupling constants*

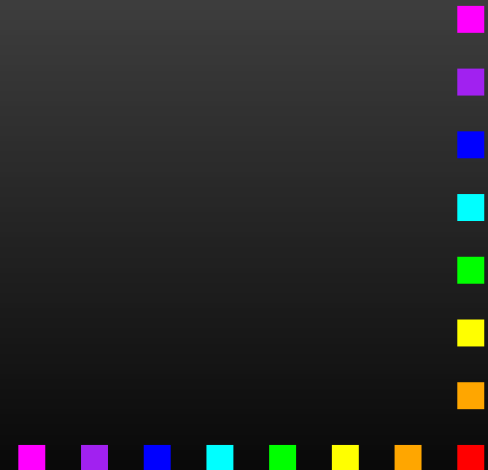


Sample CreateFeynAmp output



= FeynAmp[*identifier* ,
loop momenta ,
generic amplitude ,
insertions]

```
{ Mass[S[Gen3]],
  Mass[S[Gen4]],
```
$G^{(0)}_{\mathrm{SSV}}$ [(Mom[1] - Mom[2])[KI1[3]]],
$G^{(0)}_{\mathrm{SSV}}$ [(Mom[1] - Mom[2])[KI1[3]]],
```
  RelativeCF } ->
Insertions[Classes][{MW, MW, I EL, -I EL, 2}]
```



Algebraic Simplification

The amplitudes of `CreateFeynAmp` are in no good shape for direct numerical evaluation.

A number of steps have to be done analytically:

- contract indices as far as possible,
- evaluate fermion traces,
- perform the tensor reduction,
- add local terms arising from D·(divergent integral) (dim reg + dim red),
- simplify open fermion chains,
- simplify and compute the square of SU(N) structures,
- "compactify" the results as much as possible.

FormCalc Internals

FormCalc

Mathematica

FORM

FeynArts amplitudes

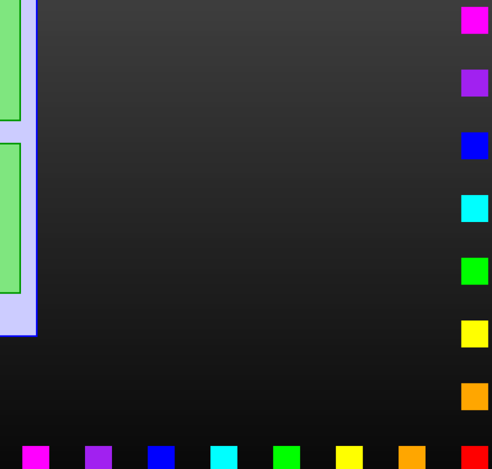
Analytical results

Generated Code
SquaredME
RenConst

Fortran

Driver programs

Utilities library



FormCalc Output

A typical term in the output looks like

```
C0i[cc12, MW2, MW2, S, MW2, MZ2, MW2] *
  ( -4 Alfa2 MW2 CW2/SW2 S AbbSum16 +
    32 Alfa2 CW2/SW2 S^2 AbbSum28 +
    4 Alfa2 CW2/SW2 S^2 AbbSum30 -
    8 Alfa2 CW2/SW2 S^2 AbbSum7 +
    Alfa2 CW2/SW2 S (T - U) Abb1 +
    8 Alfa2 CW2/SW2 S (T - U) AbbSum29 )
```

= loop integral

= kinematical variables

= constants

= automatically introduced abbreviations

Abbreviations

Outright factorization is usually out of question.
Abbreviations are necessary to reduce size of expressions.

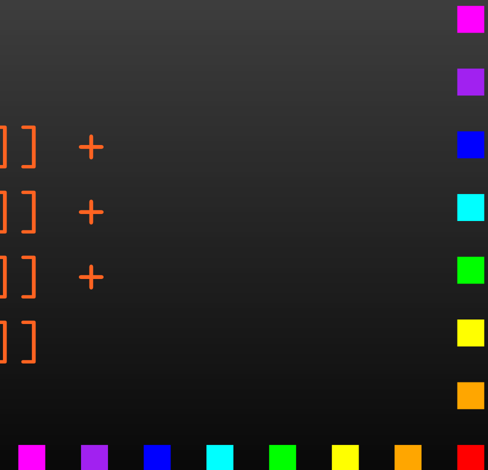
```
AbbSum29 = Abb2 + Abb22 + Abb23 + Abb3

    Abb22 = Pair1 Pair3 Pair6

        Pair3 = Pair[e[3],k[1]]
```

The full expression corresponding to `AbbSum29` is

```
Pair[e[1],e[2]] Pair[e[3],k[1]] Pair[e[4],k[1]] +
Pair[e[1],e[2]] Pair[e[3],k[2]] Pair[e[4],k[1]] +
Pair[e[1],e[2]] Pair[e[3],k[1]] Pair[e[4],k[2]] +
Pair[e[1],e[2]] Pair[e[3],k[2]] Pair[e[4],k[2]]
```



More Abbreviations

The `Abbreviate` **Function** allows to introduce abbreviations for arbitrary (sub-)expressions and extends the advantage of categorized evaluation.

The subexpressions are **retrieved with** `Subexpr[]`.

Abbreviations were **so far restricted to one FormCalc session**, e.g. one **could not save intermediate results involving abbreviations** and resume computation in a new session.

FormCalc 6 adds two **functions to 'register' abbreviations and subexpressions** from an earlier session:

```
RegisterAbbr[abbr]
RegisterSubexpr[subexpr]
```

Categories of Abbreviations

- Abbreviations are recursively defined in several levels.
- When generating Fortran code, FormCalc introduces another set of abbreviations for the loop integrals.

In general, the abbreviations are thus costly in CPU time.
It is key to a decent performance that the abbreviations are separated into different Categories:

- Abbreviations that depend on the helicities,
- Abbreviations that depend on angular variables,
- Abbreviations that depend only on $\sqrt{s}$.

Correct execution of the categories guarantees that almost no redundant evaluations are made and makes the generated code essentially as fast as hand-tuned code.

External Fermion Lines

An amplitude containing **external fermions** has the form

$$\mathcal{M} = \sum_{i=1}^{n_F} c_i\, F_i \quad \textbf{where} \quad F_i = \textbf{(Product of)}\, \langle u|\, \Gamma_i\, |v\rangle\,.$$

n_F = number of fermionic structures.

Textbook procedure: **Trace Technique**

$$|\mathcal{M}|^2 = \sum_{i,j=1}^{n_F} c_i^*\, c_j\, F_i^* F_j$$

where $F_i^* F_j = \langle v|\, \bar\Gamma_i\, |u\rangle\, \langle u|\, \Gamma_j\, |v\rangle = \mathrm{Tr}\big(\bar\Gamma_i\, |u\rangle\langle u|\, \Gamma_j\, |v\rangle\langle v|\big)\,.$

Problems with the Trace Technique

PRO: Trace technique is independent of any representation.

CON: For n_F F_i's there are n_F^2 $F_i^* F_j$'s.

Things get worse the more vectors are in the game: multi-particle final states, polarization effects . . .

Essentially $n_F \sim$ (# of vectors)! because all combinations of vectors can appear in the Γ_i.

Solution: Use Weyl-van der Waerden spinor formalism to compute the F_i's directly.

Sigma Chains

Define **Sigma matrices** and **2-dim. Spinors** as

$$\sigma_\mu = (\mathbb{1}, -\vec\sigma)\,, \qquad \langle u|_{4\mathrm{d}} \equiv \big(\langle u_+|_{2\mathrm{d}}\,, \langle u_-|_{2\mathrm{d}}\big)\,,$$

$$\overline\sigma_\mu = (\mathbb{1}, +\vec\sigma)\,, \qquad |v\rangle_{4\mathrm{d}} \equiv \begin{pmatrix} |v_-\rangle_{2\mathrm{d}} \\ |v_+\rangle_{2\mathrm{d}} \end{pmatrix}.$$

Using the chiral representation it is easy to show that **every chiral 4-dim. Dirac chain** can be converted to a ***single*** **2-dim. sigma chain:**

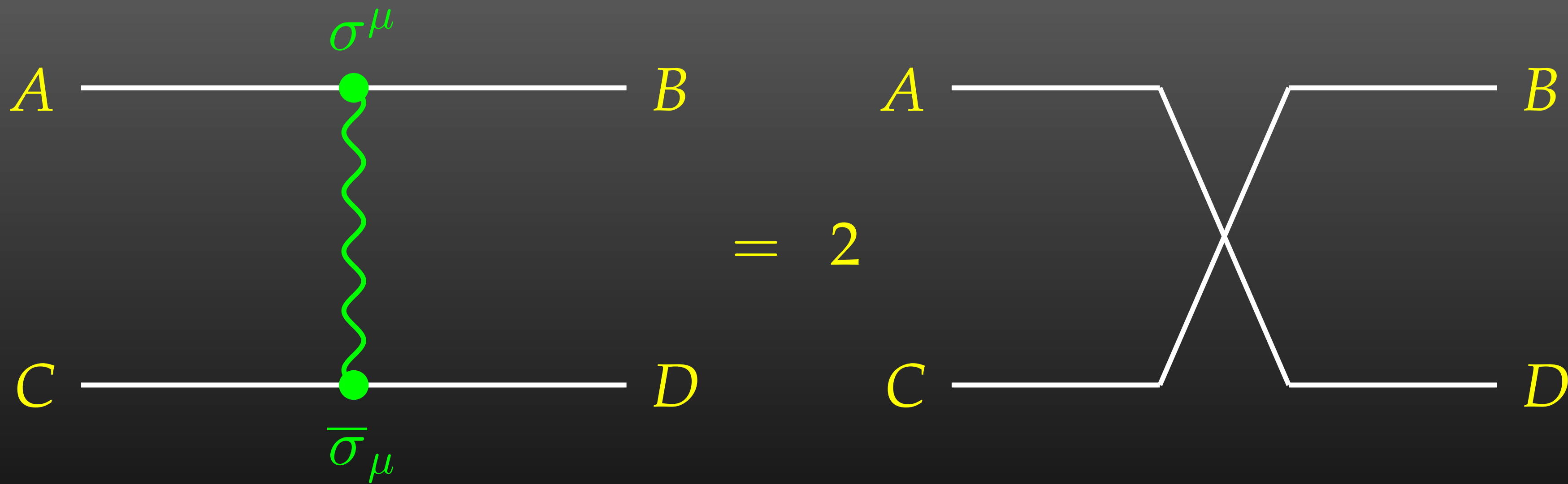
$$\langle u|\,\omega_-\gamma_\mu\gamma_\nu\cdots|v\rangle = \langle u_-|\,\overline\sigma_\mu\sigma_\nu\cdots|v_\pm\rangle\,,$$

$$\langle u|\,\omega_+\gamma_\mu\gamma_\nu\cdots|v\rangle = \langle u_+|\,\sigma_\mu\overline\sigma_\nu\cdots|v_\mp\rangle\,.$$

Fierz Identities

With the Fierz identities for sigma matrices it is possible to **remove all Lorentz contractions** between sigma chains, e.g.

$$\langle A|\,\sigma_\mu\,|B\rangle\ \langle C|\,\overline{\sigma}^\mu\,|D\rangle \;=\; 2\,\langle A|D\rangle\ \langle C|B\rangle$$



Implementation

- **Objects (arrays):** $|u_\pm\rangle \sim \begin{pmatrix} u_1 \\ u_2 \end{pmatrix}, \quad (\sigma \cdot k) \sim \begin{pmatrix} a & b \\ c & d \end{pmatrix}$
- **Operations (functions):**

$$\langle u|v\rangle \sim (u_1 \;\; u_2) \cdot \begin{pmatrix} v_1 \\ v_2 \end{pmatrix} \qquad \texttt{SxS}$$

$$(\overset{(-)}{\sigma} \cdot k)\,|v\rangle \sim \begin{pmatrix} a & b \\ c & d \end{pmatrix} \cdot \begin{pmatrix} v_1 \\ v_2 \end{pmatrix} \qquad \texttt{VxS},\ \texttt{BxS}$$

Sufficient to compute any sigma chain:

$$\langle u|\,\sigma_\mu \overline{\sigma}_\nu \sigma_\rho\,|v\rangle\, k_1^\mu k_2^\nu k_3^\rho = \texttt{SxS}(u,\ \texttt{VxS}(k_1,\ \texttt{BxS}(k_2,\ \texttt{VxS}(k_3,\ v))))$$

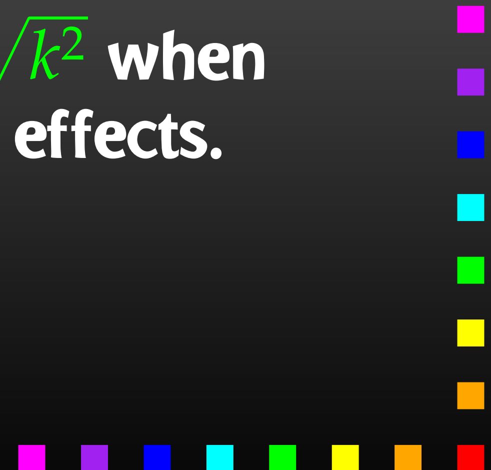
More Freebies

- **Polarization does not ‘cost’ extra**
  = Get spin physics for free.
- **Better numerical stability** because components of k^μ are arranged as ‘small’ and ‘large’ matrix entries, viz.

$$\sigma_\mu k^\mu = \begin{pmatrix} k_0 + k_3 & k_1 - \mathrm{i}k_2 \\ k_1 + \mathrm{i}k_2 & k_0 - k_3 \end{pmatrix}$$

$$\downarrow$$

Large cancellations of the form $\sqrt{k^2 + m^2} - \sqrt{k^2}$ when $m \ll k$ are avoided: better precision for mass effects.



Dirac Chains in 4D

As numerical calculations are done mostly using Weyl-spinor chains, there has been a paradigm shift for Dirac chains to make them better suited for analytical purposes, e.g. the extraction of Wilson coefficients.

- Already in Version 5, Fierz methods have been implemented for Dirac chains, thus allowing the user to force the fermion chains into almost any desired order.
- Version 6 further adds the `Colour` method to the `FermionOrder` option of `CalcFeynAmp`, which brings the spinors into the same order as the external colour indices.
- Also new in Version 6: completely antisymmetrized Dirac chains, i.e. `DiracChain[-1, μ, ν]` $= \sigma_{\mu\nu}$.

Alternate Link between FORM and Mathematica

FORM is able to handle very large expressions. To produce (pre-)simplified expressions, however, terms have to be wrapped in functions, to avoid immediate expansion:

```
a*(b + c)    →  a*b + a*c
a*f(b + c)   →  a*f(b + c)
```

The number of terms in a function is rather limited in FORM: on 32-bit systems to 32767.

Dilemma: FormCalc gets more sophisticated in pre-simplifying amplitudes while users want to compute larger amplitudes. Thus, recently many 'overflow' messages from FORM.

Solution: Send pre-simplified generic amplitude via external channel to Mathematica for introducing abbreviations. Significant reduction in size of intermediate expressions.

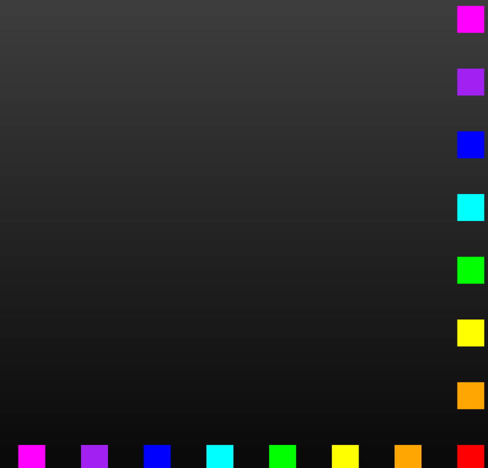
Tentukov, Vermaseren 2006

Effect on Intermediate Amplitudes

FORM $\longrightarrow$ Mathematica:

part of $uu \to gg$ @ tree level

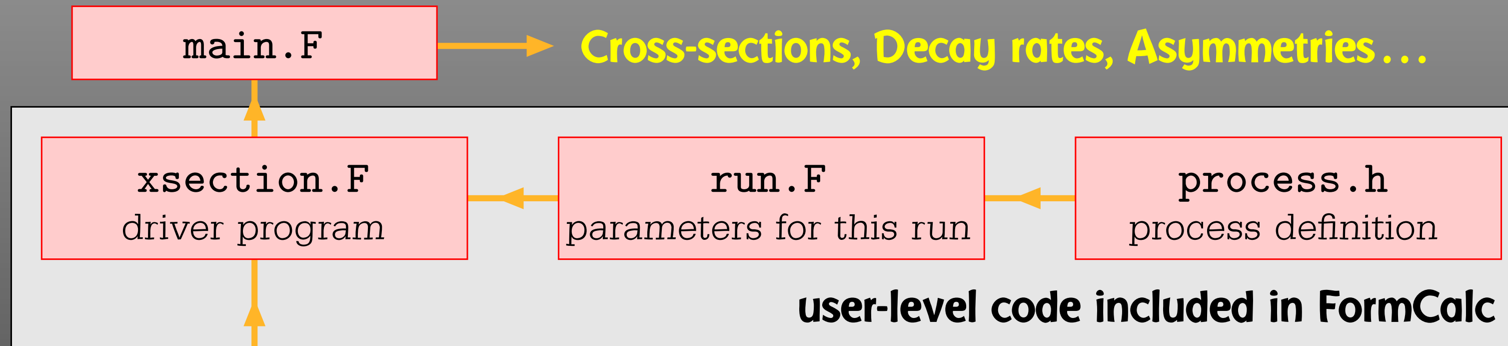
```
+Den[U,MU2]*(
  -8*SUNSum[Col5,3]*SUNT[Glu3,Col5,Col2]*SUNT[Glu4,Col1,Col5]*mul[Alfas*Pi]*
  abb[fme[WeylChain[DottedSpinor[k1,MU,-1],6,Spinor[k2,MU,1]]]*ec3.ec4
    -1/2*fme[WeylChain[DottedSpinor[k1,MU,-1],6,ec3,ec4,Spinor[k2,MU,1]]]
    +fme[WeylChain[DottedSpinor[k1,MU,-1],7,Spinor[k2,MU,1]]]*ec3.ec4
    -1/2*fme[WeylChain[DottedSpinor[k1,MU,-1],7,ec3,ec4,Spinor[k2,MU,1]]]]*MU
  -4*SUNSum[Col5,3]*SUNT[Glu3,Col5,Col2]*SUNT[Glu4,Col1,Col5]*mul[Alfas*Pi]*
  abb[fme[WeylChain[DottedSpinor[k1,MU,-1],6,ec3,ec4,k3,Spinor[k2,MU,1]]]
    -2*fme[WeylChain[DottedSpinor[k1,MU,-1],6,ec4,Spinor[k2,MU,1]]]*ec3.k2
    -2*fme[WeylChain[DottedSpinor[k1,MU,-1],6,k3,Spinor[k2,MU,1]]]*ec3.ec4
    +fme[WeylChain[DottedSpinor[k1,MU,-1],7,ec3,ec4,k3,Spinor[k2,MU,1]]]
    -2*fme[WeylChain[DottedSpinor[k1,MU,-1],7,ec4,Spinor[k2,MU,1]]]*ec3.k2
    -2*fme[WeylChain[DottedSpinor[k1,MU,-1],7,k3,Spinor[k2,MU,1]]]*ec3.ec4]
  +8*SUNSum[Col5,3]*SUNT[Glu3,Col5,Col2]*SUNT[Glu4,Col1,Col5]*mul[Alfas*MU*Pi]*
  abb[fme[WeylChain[DottedSpinor[k1,MU,-1],6,Spinor[k2,MU,1]]]*ec3.ec4
    -1/2*fme[WeylChain[DottedSpinor[k1,MU,-1],6,ec3,ec4,Spinor[k2,MU,1]]]
    +fme[WeylChain[DottedSpinor[k1,MU,-1],7,Spinor[k2,MU,1]]]*ec3.ec4
    -1/2*fme[WeylChain[DottedSpinor[k1,MU,-1],7,ec3,ec4,Spinor[k2,MU,1]]]] )
```

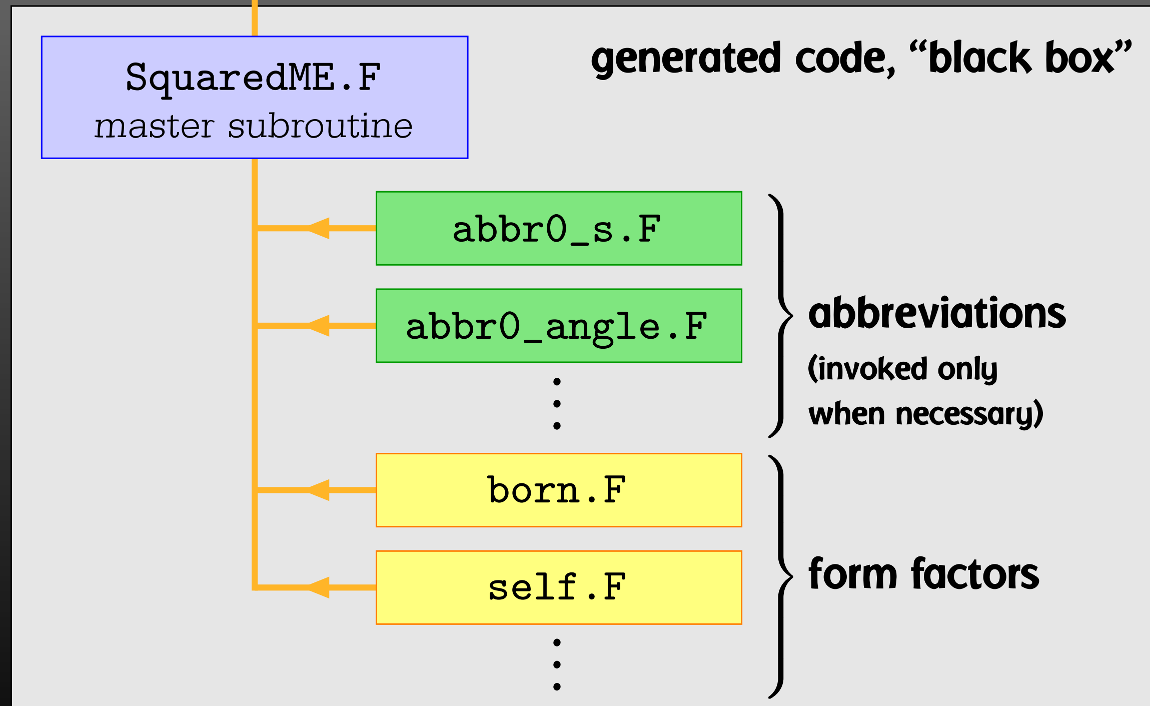


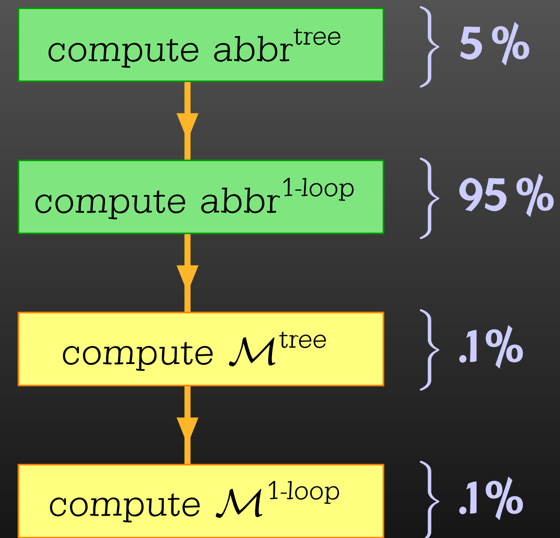
Mathematica $\longrightarrow$ FORM:

```
-4*Den(U,MU2)*SUNSum(Col5,3)*SUNT(Glu3,Col5,Col2)*SUNT(Glu4,Col1,Col5)*
  AbbSum5*Alfas*Pi
```

Numerical Evaluation in Fortran 77







Features of the Generated Code

- **Modular:** largely autonomous pieces of code provide
  - kinematics,
  - model initialization,
  - convolution with PDFs.
- **Extensible:** default code serves (only) as an example. Other ‘Frontends’ can be supplied, e.g. HadCalc, sofox.
- **Re-usable:** external program need only call `ProcessIni` (to set up the process) and `ParameterScan` (to set off the calculation).
- **Interactive:** Mathematica interface provides Mathematica function for cross-section/decay rate.
- **Parallel:** built-in distribution of parameter scans.

Choice of Language

Mentioning Fortran 77 as the programming language in many circles draws a “Not that dinosaur again” response.

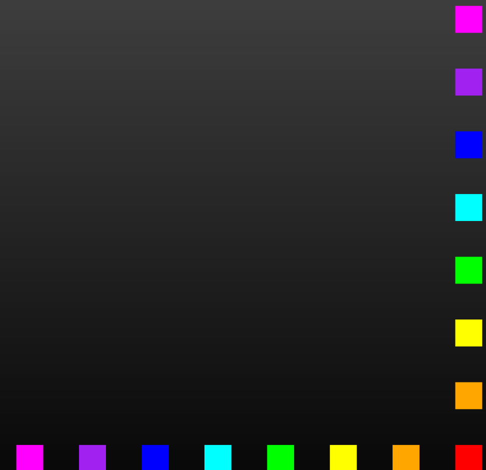
But consider:

- Fortran was designed for ‘number crunching,’ i.e. efficient evaluation of large formulas.
- Good and free compilers are available.
- Fortran is still widely used in theoretical physics.
- The code is generated, so largely ‘invisible’ for the user.
- Linking Fortran 77 to C/C++ is pretty straightforward (particularly inside gcc), so is in some sense a ‘smallest common denominator.’

Code-generation Functions

FormCalc's code-generation functions are now public and disentangled from the rest of the code. They can be used to **write out an arbitrary Mathematica expression as optimized Fortran code:**

- *handle* `= OpenFortran["`*file.F*`"]`
  opens *file.F* as a Fortran file for writing,
- `WriteExpr[`*handle*`, {`*var*` -> `*expr*`, ...}]`
  writes out Fortran code which calculates *expr* and stores the result in *var*,
- `Close[`*handle*`]`
  closes the file again.



Code generation

- **Expressions too large** for Fortran are split into parts, as in

```
var = part1
var = var + part2
...
```

- **High level of optimization,** e.g. common subexpressions are pulled out and computed in temporary variables.
- **Many ancillary functions,** e.g.

  `PrepareExpr`, `OnePassOrder`, `SplitSums`,
  `$SymbolPrefix`, `CommonDecl`, `SubroutineDecl`,
  etc.

  make code generation versatile and highly automatable.
  Resulting code needs few or no changes by hand.

Not the Cross-Section

Or, How to get things the Standard Setup won't give you.

Example: extract the Wilson coefficients for $b \to s\gamma$.

```
tops = CreateTopologies[1, 1 -> 2]
ins = InsertFields[tops, F[4,{3}] -> {F[4,{2}], V[1]}]
vert = CalcFeynAmp[CreateFeynAmp[ins], FermionChains -> Chiral]

mat[p_Plus] := mat/@ p

mat[r_. DiracChain[s2_Spinor, om_, mu_, s1:Spinor[p1_, m1_, _]]] :=
  I/(2 m1) mat[r DiracChain[sigmunu[om]]] +
  2/m1 r Pair[mu, p1] DiracChain[s2, om, s1]

mat[r_. DiracChain[sigmunu[om_]], SUNT[Col1, Col2]] :=
  r O7[om]/(EL MB/(16 Pi^2))

mat[r_. DiracChain[sigmunu[om_]], SUNT[Glu1, Col2, Col1]] :=
  r O8[om]/(GS MB/(16 Pi^2))

coeff = Plus@@ vert //. abbr /. Mat -> mat

c7 = Coefficient[coeff, O7[6]]
c8 = Coefficient[coeff, O8[6]]
```

Not the Cross-Section

Using FormCalc's output functions it is also pretty straightforward to **generate your own Fortran code:**

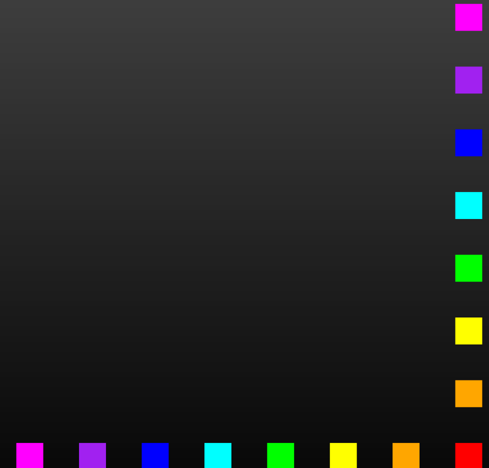
```
file = OpenFortran["bsgamma.F"]

WriteString[file,
  SubroutineDecl["bsgamma(C7,C8)"] <>
  "\tdouble complex C7, C8\n" <>
  "#include \"looptools.h\"\n"]

WriteExpr[file, {C7 -> c7, C8 -> c8}]

WriteString[file, "\tend\n"]

Close[file]
```



Scripting Mathematica

Or, How to do efficient batch processing with Mathematica.

Put everything into a script, using **sh's Here documents:**

```
#! /bin/sh ................. Shell Magic
math << \_EOF_ ........... start Here document (note the \)
  << FeynArts`
  << FormCalc`
  top = CreateTopologies[...];
  ...
_EOF_ ..................... end Here document
```

Everything between "<< \\tag" and "tag" goes to Mathematica as if it were typed from the keyboard.

Note the "\\" before tag, it makes the shell pass everything literally to Mathematica, without shell substitutions.

Scripting Mathematica

- Everything contained in **one compact shell script,** even if it involves several Mathematica sessions.
- Can combine with arbitrary shell programming, e.g. can use **command-line arguments** efficiently:

```
#! /bin/sh
math -run "arg1=$1" -run "arg2=$2" ... << \END
  ...
END
```

- Can easily be **run in the background,** or combined with utilities such as **make.**

Debugging hint: `-x` **flag** makes shell echo every statement,

```
#! /bin/sh -x
```

LoopTools

LoopTools is a library for the one-loop integrals. It is based on FF and has a Fortran, C/C++, and Mathematica interface.

- **D0 for complex masses added.**

  Le, Dao 2009

- **Dim.reg. IR/collinear cases (QCDLoop) added.**
  Scalar integrals only so far (as in QCDLoop).

$\lambda^2 > 0$	regularization with photon 'mass' λ,	$\lambda^2 = -2$	coefficient of $1/\varepsilon^2$ in dim.reg.,
		$\lambda^2 = -1$	coefficient of $1/\varepsilon$ in dim.reg.,
		$\lambda^2 = 0$	finite piece in dim.reg.

  Ellis, Zanderighi 2008

- **New dispatcher for IR and collinear divergences.**
  Construct bit pattern: 1 for zero argument, 0 otherwise, then a single table lookup leads to correct case.

LoopTools Environment Variables

Most LoopTools parameters can be set from the outside through environment variables:

`LTCMPBITS`	# of bits compared in cache lookups
`LTVERSION`	bit mask for alternate versions
`LTMAXDEV`	maximum allowed relative deviation in comparing to alternate versions
`LTDEBUG`	bit mask for debugging
`LTRANGE`	range of integrals to print out in debug mode
`LTWARN`	number of digits lost before warning
`LTERR`	number of digits lost before error
`LTDELTA`	'divergence' Δ
`LTMUDIM`	renormalization scale μ^2
`LTLAMBDA`	IR regulator parameter λ^2
`LTMINMASS`	threshold m^2_{min} below which particles are considered 'massless'

E.g. check finiteness without re-compilation by modifying `LTMUDIM`, `LTLAMBDA`.

Alternate Versions

For some functions **Alternate Versions** exist, most of which are based on an implementation by Denner. The user can choose **at run-time** which version to use, and whether checking is performed. This is determined by the **Version Key:**

0 * key	compute version 'a' (mostly FF),
1 * key	compute version 'b' (mostly Denner),
2 * key	compute both, compare, return 'a',
3 * key	compute both, compare, return 'b'.

Alternate versions are currently available for the following functions: `A0`, `Bget`, `C0`, `D0`, `D0C`, `Eget`, `EgetC`.

Example: `call setversionkey(2*KeyD0 + 3*KeyBget)`

Command-line Interface

The **Command-line Interface** is useful in particular for testing and debugging.

It lists the N-point scalar and tensor coefficients corresponding to the number of arguments, i.e. 3 arguments = B, 6 arguments = C, etc.

```
> lt 250000 6464.16 8315.38
====================================================
  FF 2.0, a package to evaluate one-loop integrals
written by G. J. van Oldenborgh, NIKHEF-H, Amsterdam
====================================================
for the algorithms used see preprint NIKHEF-H 89/17,
'New Algorithms for One-loop Integrals', by G.J. van
Oldenborgh and J.A.M. Vermaseren, published in
Zeitschrift fuer Physik C46(1990)425.
====================================================
  p     =   250000.000000000
  m1    =   6464.16000000000
  m2    =   8315.38000000000
bb0     = (-10.1569090105893,2.95011861955466)
bb1     = (5.07382021909957,-1.46413667259555)
bb00    = (165409.773493414,-54197.2752510472)
bb11    = (-3.31323987482202,0.943436559119877)
bb001   = (-81984.2510317697,26897.9754657431)
bb111   = (2.43370306005361,-0.683409066031936)
dbb0    = (-4.868613391538015E-006,7.903840552878544E-007)
dbb1    = (2.434818091584023E-006,-4.359562268294025E-007)
dbb00   = (0.880290172138776,-0.260350056737834)
dbb11   = (-1.624235905363170E-006,4.122913473051599E-007)
====================================================

total number of errors and warnings
===================================
fferr: no errors
```

CutTools

Tensor loop integrals have in FormCalc so far been treated by **Passarino-Veltman reduction** only, e.g.

$$\frac{q_\mu q_\nu}{D_0 D_1} = g_{\mu\nu}\,\texttt{B00}(p^2, m_1^2, m_2^2) + p_\mu p_\nu\,\texttt{B11}(p^2, m_1^2, m_2^2)$$

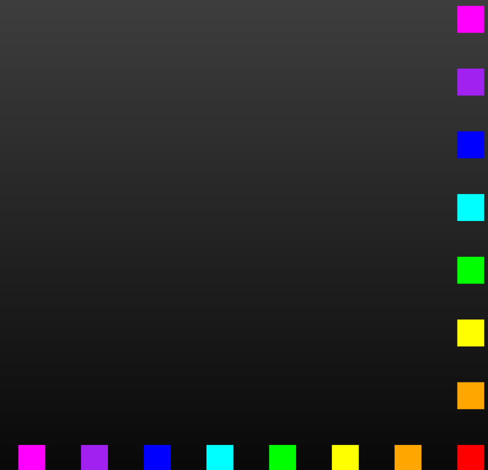
where `B00` and `B11` are **provided by LoopTools.**

CutTools implements the cutting-technique-inspired OPP (Ossola, Papadopoulos, Pittau) method. It needs the numerator as a function of q which it can sample:

$$\texttt{Bcut}(2, \texttt{num}, p, m_1^2, m_2^2)$$

where $\texttt{num} = q_\mu q_\nu$.

Independent way of checking LoopTools results.
Performance?



A Final Look

Using FeynArts, FormCalc, and LoopTools is a lot like driving a car:

- You have to decide where to go (this is often the hardest decision).
- You have to turn the ignition key, work gas and brakes, and steer.
- But you don't have to know, say, which valve has to open at which time to keep the motor running.
- On the other hand, you can only go where there are roads. You can't climb a mountain with your car.

feynarts.de