Screening mechanisms
-
K-mouflage

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IPhT - CEA Saclay

Atelier Dark Energy, October 15, 2021

Motivations

Accelerated expansion of the Universe →

- dark energy
- modified gravity

Most of the models involve one or more scalar fields, which experience self-interactions and may also interact with matter.

→ "Fifth force" that has not been seen in local gravity experiments !

- the scalar field does not interact with baryonic matter components
- there is a mechanism to suppress the fifth force in local environments

→ "Screening" mechanisms associated with non-linearities of the system.

Khoury (1011.5909)

Two approaches:

- Focus on the cosmological behavior and on low-order (linear) perturbation theory.

One may study specific models or build general frameworks (EFT) that apply to a large class of theories.

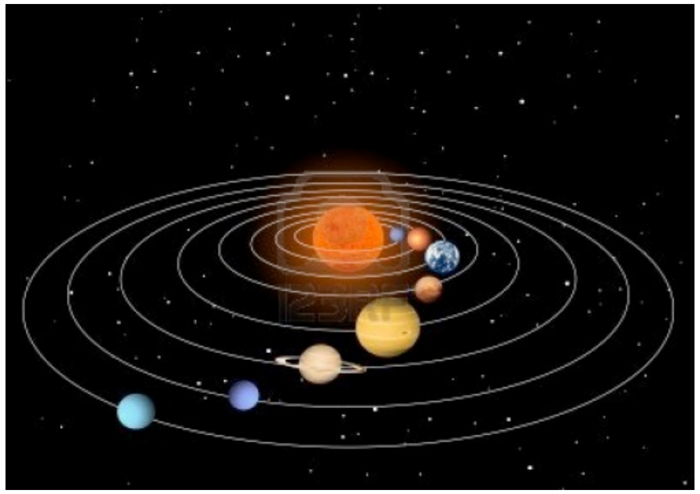
Gubitosi, Piazza & Vernizzi (JCAP 032, 2013)

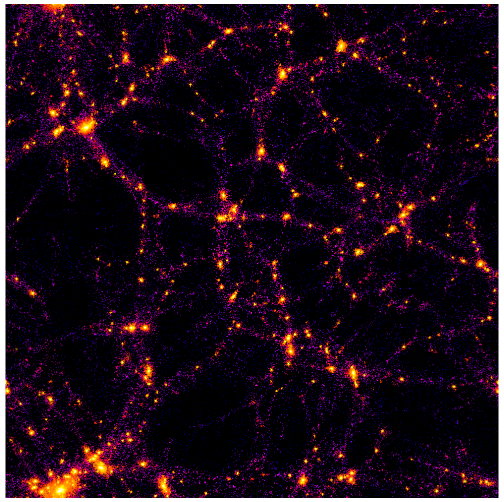
The cosmological regime may be decoupled from the small-scale regime.

- Look for explicit models that make sense from local to cosmological scales.

One needs to specify the model and its nonlinear screening mechanism. Combining Solar System and cosmological tests can provide strong constraints on the model.

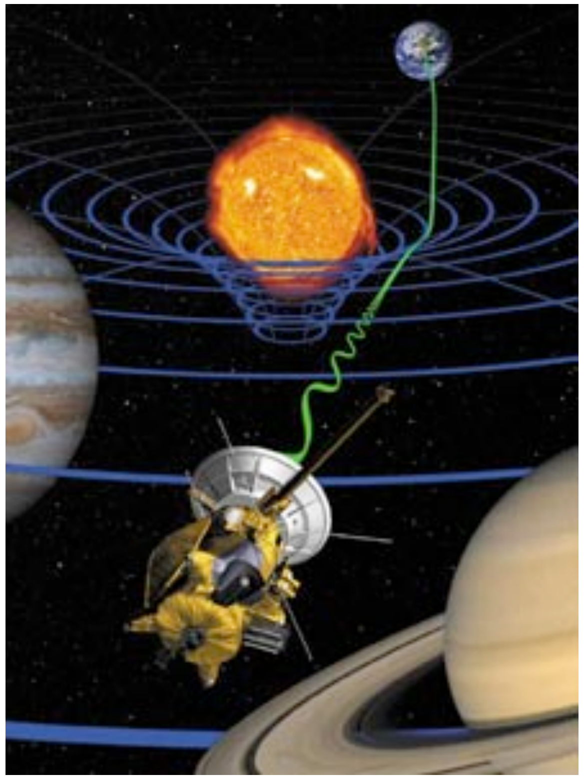
Gravity acts on all scales: it would be nice to have unified scenarios
(or at least to see how one can build unified models).





Deviations from Newton's law are parametrized by

$$\Phi_N = -\frac{G_N M}{r}(1 + 2\beta^2 e^{-r/\lambda})$$

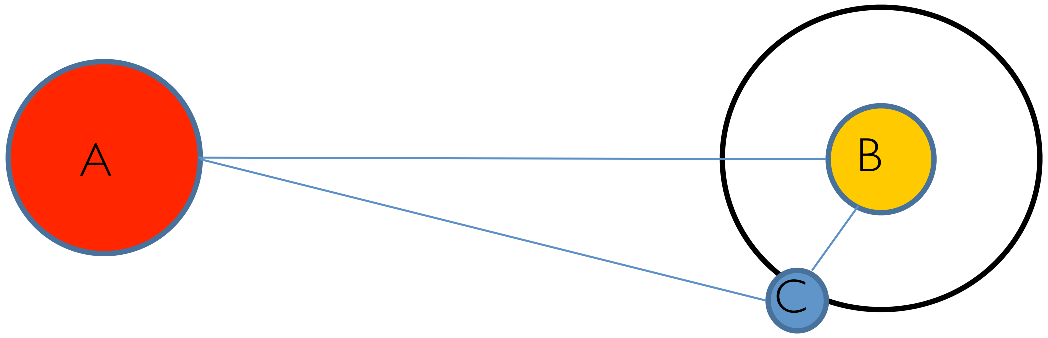


For long-range forces with large λ , the tightest constraint on the coupling β comes from the Cassini probe measuring the Shapiro effect (time delay):

Bertotti et al. (Nature 425, 374, 2003)

$$\beta^2 \leq 4 \times 10^{-5}$$

Violation of the equivalence principle



$$\eta_{BC} \equiv \left| \frac{a_C - a_B}{a_C + a_B} \right|$$

$$\eta_{\rm Moon-Earth} \leq 10^{-13}$$

Will (Liv. Rev. Relat. 17, 4, 2014)

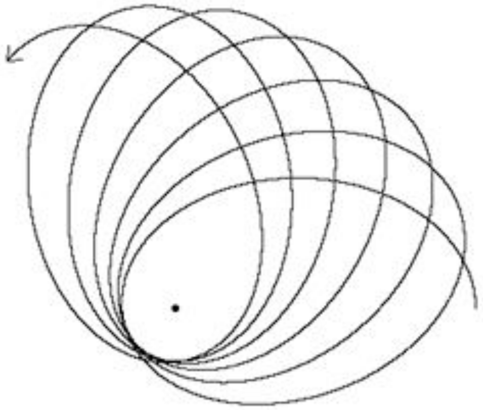
Lunar Laser Ranging experiment

This experiment also constrains the time variation of the local Newton's constant:

$$\left| \frac{d \ln G_N}{dt} \right| < 10^{-12} \ {\rm yr}^{-1}$$

Williams et al. (PRL 93, 261101, 2004)

It also constrains the anomalous perihelion of the Moon:



$$|\delta\theta| < 2 \times 10^{-11}$$

Williams et al. (Class. Quant. Grav. 29, 184004, 2012)

Scalar-tensor theories

I- DEFINITIONS

A simple way to modify GR is to introduce **2 metrics:**

- the first metric enters the Einstein-Hilbert action (gravitational part) $\tilde{g}_{\mu\nu}$
- the second metric enters the matter action (dynamical part) $g_{\mu\nu}$

$$S = \int d^4x \, \sqrt{-\tilde{g}} \frac{\tilde{M}_{\rm Pl}^2}{2} \tilde{R} + S_m(\psi_{\rm m}^{(i)}, g_{\mu\nu}) + \dots$$

The relationship between these two metrics is set by additional degrees of freedom, such as a scalar field:

$$g_{\mu\nu} = C(\varphi, X)\tilde{g}_{\mu\nu} + D(\varphi, X)\partial_\mu\varphi\partial_\nu\varphi \qquad X = -\frac{1}{2}\,\partial^\mu\varphi\,\partial_\mu\varphi$$

Bekenstein (1993)

Simple case of a conformal coupling:

$$S = \int d^4x \, \sqrt{-\tilde{g}} \left[\frac{\tilde{M}_{\rm Pl}^2}{2} \tilde{R} + \tilde{\mathcal{L}}_\varphi(\varphi) \right] + S_m(\psi_{\rm m}^{(i)}, g_{\mu\nu}) \qquad g_{\mu\nu} = A^2(\varphi)\,\tilde{g}_{\mu\nu}$$

Coupling matter -- scalar field through the
Jordan-metric conformal rescaling

$$\boxed{\beta \equiv \tilde{M}_{\rm Pl} \frac{d \ln A}{d\varphi}}$$

II- GENERAL FEATURES

Newton's constant becomes time dependent: $\nabla^2 \tilde{\Psi}_{\mathrm{N}} = 4\pi A^2(\bar{\varphi}(t))\tilde{\mathcal{G}}_{\mathrm{N}} \delta\rho_{\mathrm{m}}$

The gravitational potentials seen by matter receive an additional contribution:

$$ds^2 = -a^2(1+2\Phi)d\tau^2 + a^2(1-2\Psi)d\mathbf{x}^2 \qquad g_{\mu\nu} = A^2 \tilde{g}_{\mu\nu}$$

$$\Phi = \tilde{\Psi}_{\mathrm{N}} + \frac{\delta A}{A}, \quad \Psi = \tilde{\Psi}_{\mathrm{N}} - \frac{\delta A}{A} \quad \Longrightarrow \quad \Phi \neq \Psi \qquad \frac{\Phi+\Psi}{2} \neq \Phi$$

dynamical and lensing
masses are different

- If A, hence $\mathcal{G}_{\mathrm{N}}$ change too much with time, this can modify BBN and orbits of planets and stars (binary pulsars and Lunar Ranging exp. testing Equiv. princ.)

$\left|\frac{\Delta A}{A}\right| \leq 0.1$ since BBN, therefore $A \simeq 1$ in these models.

- Screening: we wish to suppress the gradients of the scalar field

Screening mechanisms

Theories with a single nearly massless scalar field on large scales, with second-order equations of motion.

Khoury (1011.5909)

Brax & PV (PRD 90, 023507, 2014)

→ Screening mechanisms may be classified in 3 categories:

Write the Lagrangian of the scalar fluctuations up to quadratic order as:

$$\mathcal{L} = -\frac{Z(\varphi_0)}{2}(\partial\delta\varphi)^2 - \frac{m^2(\varphi_0)}{2}(\delta\varphi)^2 - \beta(\varphi_0)\frac{\delta\varphi}{M_{\rm Pl}}\delta\rho_m$$

Vainshtein / K-mouflage

Chameleon

Damour-Polyakov

We can suppress the gradients of the scalar field (in dense environments) by:

- decreasing the coupling to matter → no fifth force
- increasing the mass of the scalar field
- increasing the inertia of the scalar field (prefactor of the kinetic term)

→ the field is frozen

These 3 mechanisms give rise to different behaviors.

$$\mathcal{L} = -\frac{Z(\varphi_0)}{2}(\partial\delta\varphi)^2 - \frac{m^2(\varphi_0)}{2}(\delta\varphi)^2 - \beta(\varphi_0)\frac{\delta\varphi}{M_{\rm Pl}}\delta\rho_m$$

Chameleon and Damour-Polyakov

$$Z(\varphi) = 1$$

linear order + quasi-static approximation

$$\frac{\delta\varphi}{M_{\rm Pl}} = -\frac{\beta(\varphi_0)\delta\rho_m}{M_{\rm Pl}^2\left(m^2(\varphi_0)+\frac{k^2}{a^2}\right)}$$

$$\Psi = \left[1 + \frac{2\beta^2(\varphi_0)}{1+m^2(\varphi_0)a^2/k^2}\right]\Psi_{\rm N}$$

GR is recovered on large (linear) scales, outside the Compton radius

Gravity is amplified on smaller scales by

$$1+2\beta^2$$

Vainshtein and K-mouflage mechanisms

$$m = 0$$

$$\frac{\delta\varphi}{M_{\rm Pl}} = -\frac{\beta(\varphi_0)a^2\delta\rho_m}{M_{\rm Pl}^2 Z(\varphi_0)k^2} = \frac{2\beta}{Z}\Psi_{\rm N}$$

$$\Psi = \left[1+\frac{2\beta^2(\varphi_0)}{Z(\varphi_0)}\right]\Psi_{\rm N}$$

GR is not recovered on large linear scales

Gravity is amplified by

$$1+2\beta^2/Z$$

When the linear approximation breaks down: ⇨ screening

Small-scale linear regime: $\delta\varphi/M_{\rm Pl} \simeq 2\beta\Psi_{\rm N}$

the condition for screening, $|\delta\varphi| \sim |\varphi_0|$, reads as a condition on the value of Newton's potential

$$Z(\varphi) = 1 + a(\varphi)\frac{(\partial\varphi)^2}{\mathcal{M}^4} + b(\varphi)L^2\frac{\Box\varphi}{M_{\rm Pl}} + \dots$$

K-mouflage

$$|\nabla\varphi| \gtrsim \mathcal{M}^2$$

gradient of Newton's potential

Vainshtein

$$\frac{|\nabla^2\varphi|}{M_{\rm Pl}} \gtrsim L^{-2}$$

curvature of Newton's potential

These 3 screening mechanisms appear at different scales and densities (different criteria).

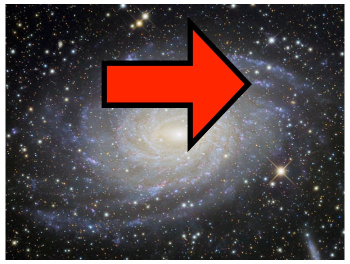
Their effects are different:

- recover GR at large scales (beyond Compton wavelenght) or not
- thin-shell effect or not
- time dependence of Newton’s constant or not

The 5th force is screened because it is:

short range

Chameleon:

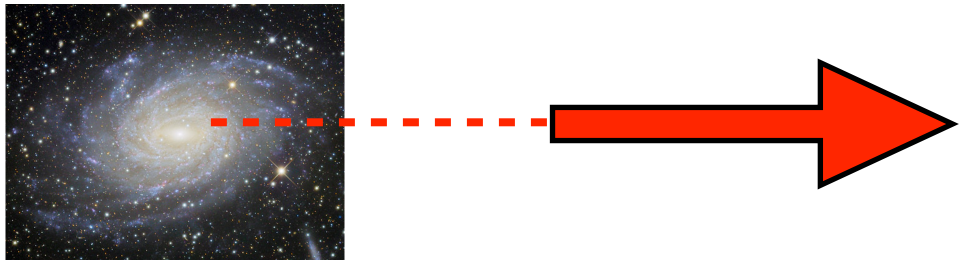


low amplitude

Damour-Polyakov
(dilaton/symmetron):

damped within a characteristic radius

K-mouflage/
Vainshtein:



I- CHAMELEON SCENARIO

f(R) theories: $S_{\rm grav} = \int d^4x \sqrt{-g} \frac{M_{\rm Pl}^2}{2} f(R)$ GR: $f(R) = R$

This is equivalent to a scalar-tensor theory:

$$S = \int d^4x \sqrt{-\tilde{g}} \left[\frac{\tilde{M}_{\rm Pl}^2}{2} \tilde{R} - \frac{1}{2} (\partial\varphi)^2 - V(\varphi) \right] + S_m(\psi_m^{(i)}, g_{\mu\nu})$$

$$e^{-2\varphi/\sqrt{6}\tilde{M}_{\rm Pl}} = f', \quad V(\varphi) = \tilde{M}_{\rm Pl}^2 \frac{Rf' - f}{2f'^2} \qquad g_{\mu\nu} = e^{2\varphi/\sqrt{6}\tilde{M}_{\rm Pl}} \tilde{g}_{\mu\nu}$$

Hu & Sawicki (2007)

Because of the conformal coupling, there is an explicit coupling between matter and the scalar field. The KG eq. for the scalar field involves the effective potential:

$$V_{\text{eff}}(\varphi) = V(\varphi) + \rho[A(\varphi) - 1]$$

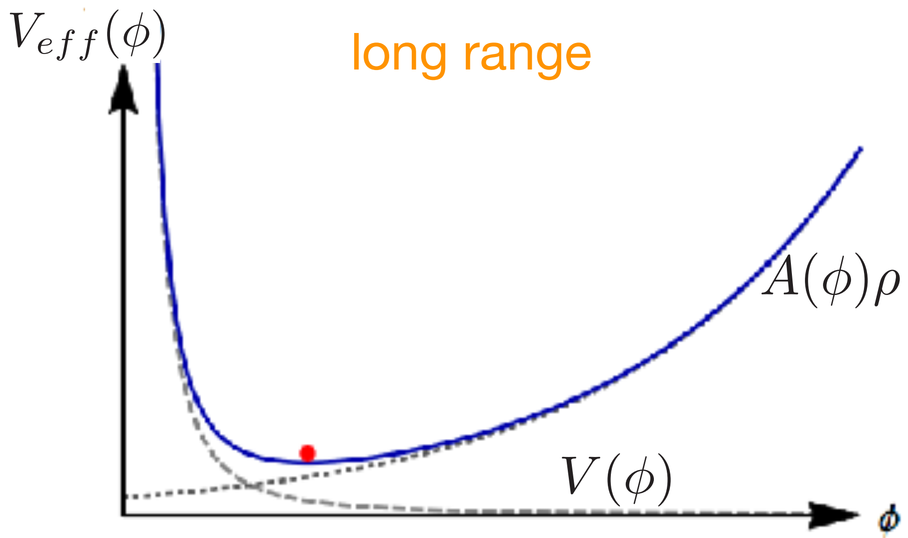
Khoury & Weltman (2004)

Brax et al. (2012)

Wang et al. (2012)

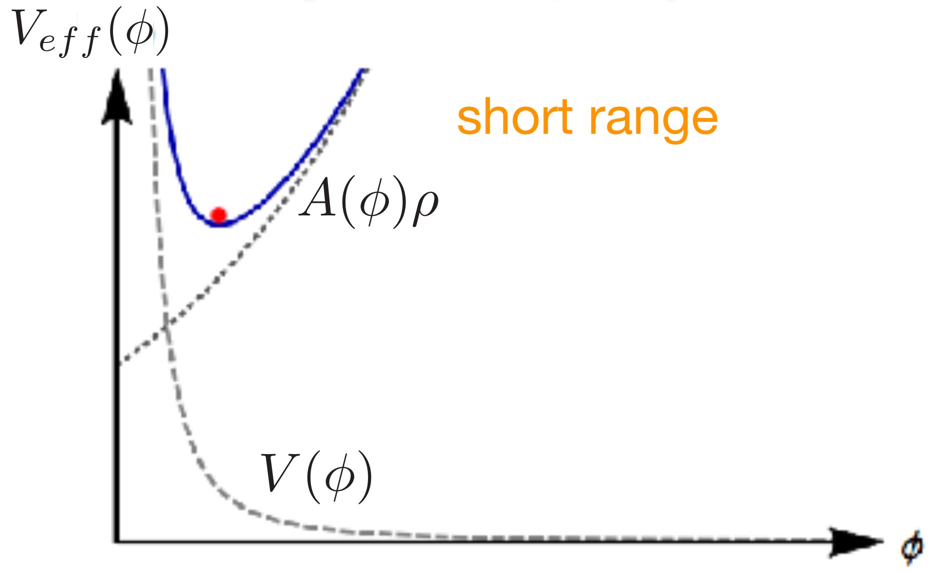
Typically: $V(\varphi) = \frac{M^{4+n}}{\varphi^n}$ $A(\varphi) = e^{\beta\varphi/\tilde{M}_{\text{Pl}}}$

Low density region



$V''(\phi) \ll 1 \rightarrow$ Unscreened

High density region

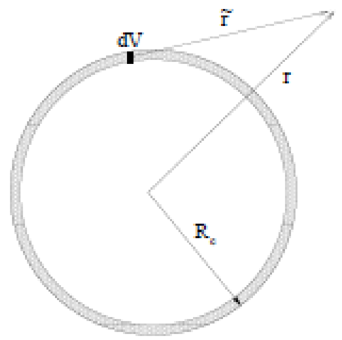


$V''(\phi) \gg 1 \rightarrow$ Screened

Mota (2016)

The minimum and curvature of the effective potential depend on the environment.

Thin-shell effect:



Khoury & Weltman (2004)

Brax (2016)

In a high-density object like a star, the scalar field becomes short-ranged. Only the surface of the object where the field has nonzero gradients contributes to the fifth force.

Screened and unscreened objects do not respond in the same fashion to a distant mass

→

violation of the strong equivalence principle

II- DAMOUR-POLYAKOV SCENARIO

Damour & Polyakov (1994)

A) Dilaton models

$$V_{\rm eff}(\varphi) = V(\varphi) + \rho[A(\varphi) - 1]$$

Typically: $V(\varphi) = V_0 e^{-\varphi/\tilde{M}_{\rm Pl}}$

$$A(\varphi) = 1 + \frac{A_2}{2\tilde{M}_{\rm Pl}^2}\varphi^2$$

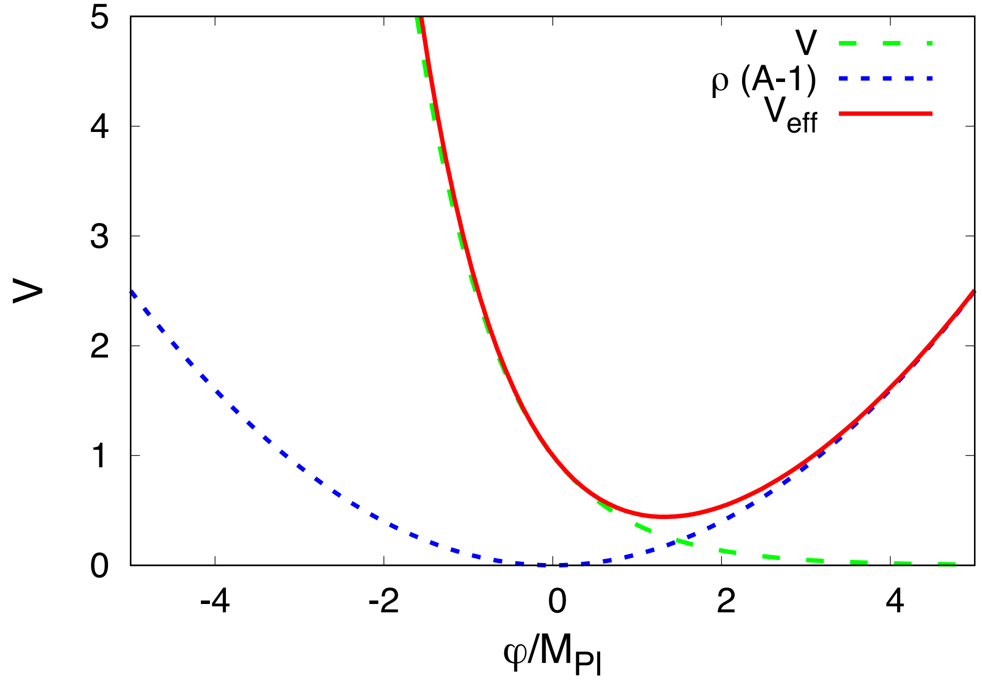
conformal function
has a minimum

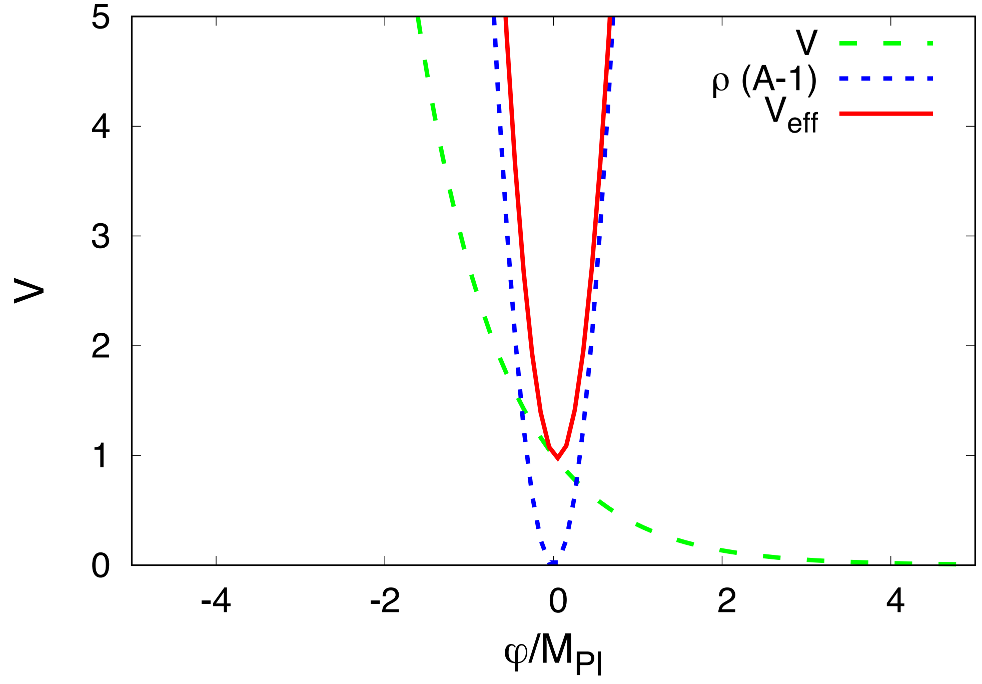
Low-density region

long range, large coupling

High-density region

short range, small coupling





$$\beta \equiv \tilde{M}_{\rm Pl}\frac{d\ln A}{d\varphi} \to 0$$

The coupling depends on the environment.

B) Symmetron models

$$V_{\rm eff}(\varphi) = V(\varphi) + \rho[A(\varphi) - 1]$$

Typically: $V(\varphi) = -\frac{\mu^2}{2}\varphi^2 + \frac{\lambda}{4}\varphi^4$

double well

$A(\varphi) = 1 + \frac{1}{2M^2}\varphi^2$

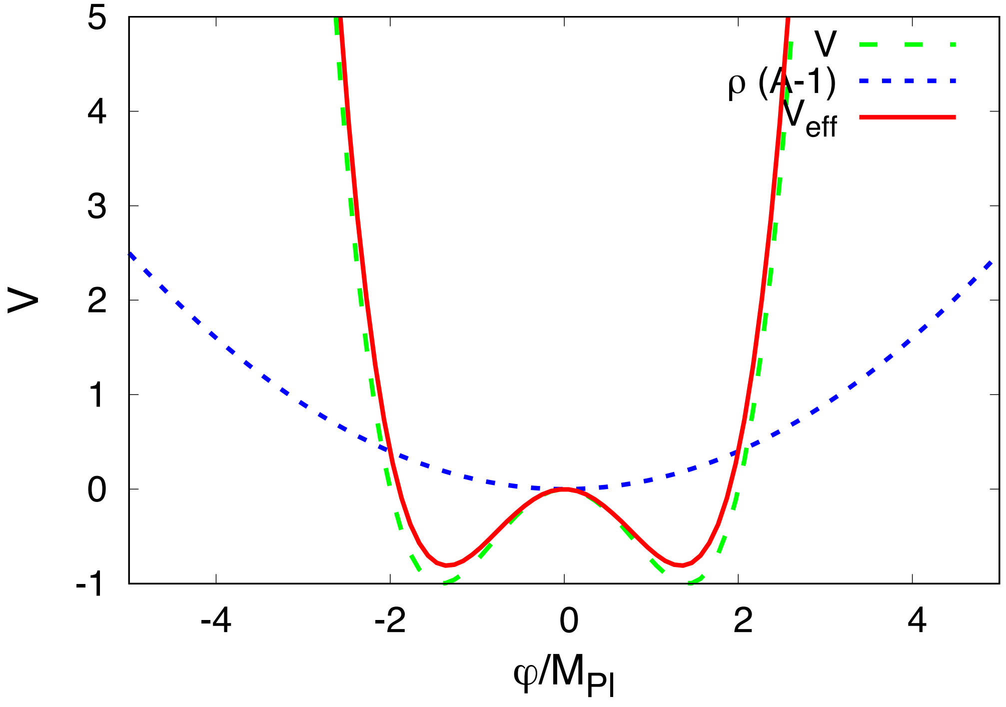
conformal function
has a minimum

$$V_{\rm eff}(\varphi) = \frac{1}{2}\left(\frac{\rho}{M^2} - \mu^2\right)\varphi^2 + \frac{\lambda}{4}\varphi^4$$

phase transition between low and high-density regions

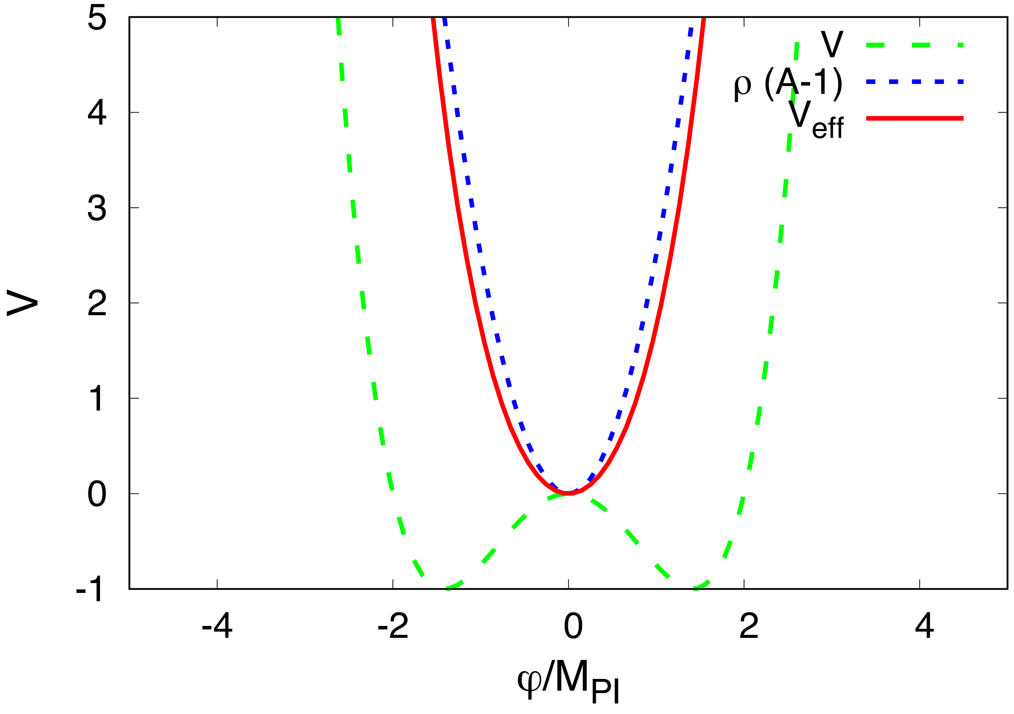
Low-density region

large coupling



High-density region

zero coupling



The coupling depends on the environment.

Hinterbichler & Khoury (2010)

Brax et al. (2012)

III- K-MOUFLAGE SCENARIO

Babichev et al. (2009)

Brax & V. (2014)

$$S = \int d^4x \sqrt{-\tilde{g}} \left[\frac{\tilde{M}_{\rm Pl}^2}{2} \tilde{R} + \mathcal{M}^4 K(\tilde{\chi}) \right] + S_m(\psi_{\rm m}^{(i)}, A^2(\varphi)\tilde{g}_{\mu\nu})$$

$$\tilde{\chi} = -\frac{1}{2\mathcal{M}^4} \partial^\mu \varphi \, \partial_\mu \varphi$$

In the linear regime the deviations from GR are set by: $\frac{2\beta^2}{\bar{K}'}$

Screening in the non-linear regime: $\bar{K}' \gg 1$

KG: $$\frac{d\varphi}{dr} K' \left(-\frac{1}{2\mathcal{M}^4} \left(\frac{d\varphi}{dr} \right)^2 \right) = \frac{\beta M(<r)}{\tilde{M}_{\rm Pl} 4\pi r^2}$$

$$\frac{\beta}{\tilde{M}_{\rm Pl}} \frac{d\varphi}{dr} = \frac{2\beta^2}{K'} \frac{d\Psi_{\rm N}}{dr}$$

- far from the compact object:

$$\frac{d\Psi_{\rm N}}{dr} \to 0, \quad \frac{d\varphi}{dr} \to 0, \quad K' \to 1$$

gravity amplified by $1 + 2\beta^2$

- close to the compact object:

$$\frac{d\Psi_{\rm N}}{dr} \to \infty, \quad \frac{d\varphi}{dr} \to \infty, \quad K' \to \infty$$

5th force is negligible

Brax & V. (2014)

K-mouflage radius:

$$R_K = \left(\frac{\beta M}{4\pi \tilde{M}_{\rm Pl}\mathcal{M}^2}\right)^{1/2}$$

Inside R_K → we recover GR

Outside R_K → deviation from GR, gravity is amplified

No thin-shell effect !

$$\frac{d\varphi}{dr}\, K'\left(-\frac{1}{2\mathcal{M}^4}\left(\frac{d\varphi)}{dr}\right)^2\right) = \frac{\beta M(< r)}{\tilde{M}_{\rm Pl}4\pi r^2}$$

IV- VAINSHTEIN SCENARIO

The mechanism is similar to the K-mouflage case, except that it relies on the curvature rather than the gradient.

Cubic Galileon model:

$$\mathcal{L}(\varphi) = -\frac{1}{2}(\partial\varphi)^2 - \frac{\partial^2\varphi}{2\Lambda^3}(\partial\varphi)^2 + \frac{\beta}{\tilde{M}_{\rm Pl}}\varphi T$$

We recover GR inside the Vainshtein radius:

$$R_V = \left(\frac{3\beta M}{4\pi\tilde{M}_{\rm Pl}\Lambda^3}\right)^{1/3}$$

Vainshtein (1972)

Deffayet et al. (2011)

Nicolis, Rattazzi, Trincherini (2009)

K-mouflage

I- DEFINITION OF THE MODEL

(K-essence model with universal conformal coupling to matter)

$$S = \int d^4x \, \sqrt{-g} \left[\frac{M_{\rm Pl}^2}{2} R + \mathcal{M}^4 K(\chi) \right] + S_m(\psi_{\rm m}^{(i)}, A^2(\varphi) g_{\mu\nu})$$

$$\chi = - \frac{1}{2\mathcal{M}^4} \, \partial^\mu \varphi \, \partial_\mu \varphi$$

non-standard non-linear
kinetic function

Coupling matter -- scalar field
through the Jordan-metric
conformal rescaling

$$\beta \equiv M_{\rm Pl} \frac{d \ln A}{d\varphi}$$

$$\tilde{g}_{\mu\nu} = A^2(\varphi) \, g_{\mu\nu}$$

Babichev et al. (Int. J. Mod. Phys. D, 18, 2147, 2009)
Brax & PV (PRD 90, 023507, 2014)

- We recover a cosmological-constant behavior at late times if: $\chi \to 0 : \quad K(\chi) \simeq -1 + \chi + ...$

  $\mathcal{M}^4 = \rho_\Lambda$

- Deviations from LCDM and GR are set by: $\frac{2\beta^2}{K'}$

  → coupling

  → inertia

- Positive and negative tails: - uniform time-dependent configurations (i.e., background): $\chi > 0$

  The cosmological background and cosmological structures only probe the positive tail.

  Good or Bad ??

  - quasi-static configurations (i.e., small-scale nonlinear structures): negative tail $\chi < 0$

- In Jordan frame the Planck mass depends on time: $\mathcal{G}_{\rm N} \propto \bar{A}^2$

II- COSMOLOGICAL AND SOLAR SYSTEM CONSTRAINTS

A) Cosmological constraints $\chi > 0$

$K' > 0, \quad K' + 2\chi K'' > 0$ no ghosts, no small-scale instabilities around cosmological background

$\sqrt{\chi} K'(\chi) \rightarrow +\infty \;\; \text{for} \;\; \chi \rightarrow +\infty$ well-defined cosmology up to high redshift

$K' \gg 1 \;\; \text{for} \;\; \chi \gg 1$ dark energy is subdominant at high z

$\beta \lesssim 0.1$

< 1-10% deviation for large-scale structures

< 10% deviation of Newton's constant since BBN

B) Small-scale constraints $\chi < 0$

$K' > 0, \quad K' + 2\chi K'' > 0$ no small-scale instabilities, well-defined static profile and Cauchy problem

$\sqrt{-\chi} K'(\chi) \rightarrow +\infty \;\; \text{for} \;\; \chi \rightarrow -\infty$ well-defined profile up to high densities

C) Solar System constraints

$$R_K(M) = \sqrt{\frac{\beta M}{M_\odot}}\, 3470\ \mathrm{AU}$$

The Solar System is screened

$$\frac{\beta^2}{K'} \leq 10^{-5}$$

Cassini bound on the amplitude of the fifth force

$\beta \leq 0.1$ Lunar Laser Ranging upper bound on the local rate of change of Newton's constant

→ This gives a direct constraint on cosmological structure formation !

Deviations of the linear matter power spectrum cannot be more than few percents.

A very tight constraint comes from the bound on the anomalous perihelion of the Moon:

$$\delta\theta = \pi r \frac{d}{dr}\left[r^2 \frac{d}{dr}\left(\frac{\epsilon}{r}\right)\right] \leq 2 \times 10^{-11}$$

where $\epsilon = \frac{\delta\Psi}{\Psi_{\mathrm{N}}} = \frac{\beta c^2 \varphi}{M_{\mathrm{Pl}}\Psi_{\mathrm{N}}}$ is the ratio between the fifth-force potential and the Newtonian potential

We obtain: $$\delta\theta = -8\pi \frac{\beta^2}{K'} \frac{\chi K''}{K' + 2\chi K''} \leq 2 \times 10^{-11}$$

→ The only way of satisfying the perihelion bound is to suppress K" in the Solar System.

D) Laboratory constraints

measures of the Newtonian force

$$\frac{\beta^2}{K'} \leq 10^{-4}$$

less stringent than Cassini but further in the non-linear regime

E) Models

A family of models that pass all constraints:

$$K' = 1 + K_* \frac{\chi^n}{\chi_*^n + \chi^n}$$

with $\beta \leq 0.1, \quad K_* \geq 10^3, \quad \chi_* < \left(\frac{K_*}{n} 10^{-10} \right)^{1/n} \frac{10^{12}}{K_*^2}$

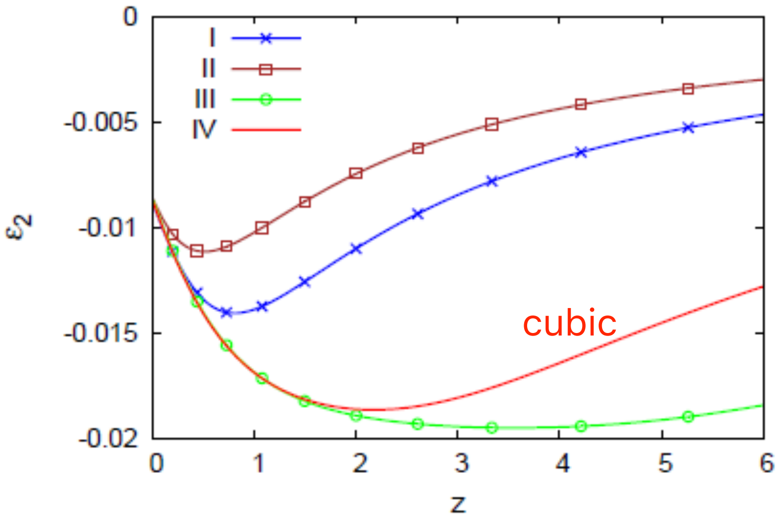
In particular, the field can behave like a canonically normalized field up to high redshift (K'=1), giving a maximal deviation from LCDM.

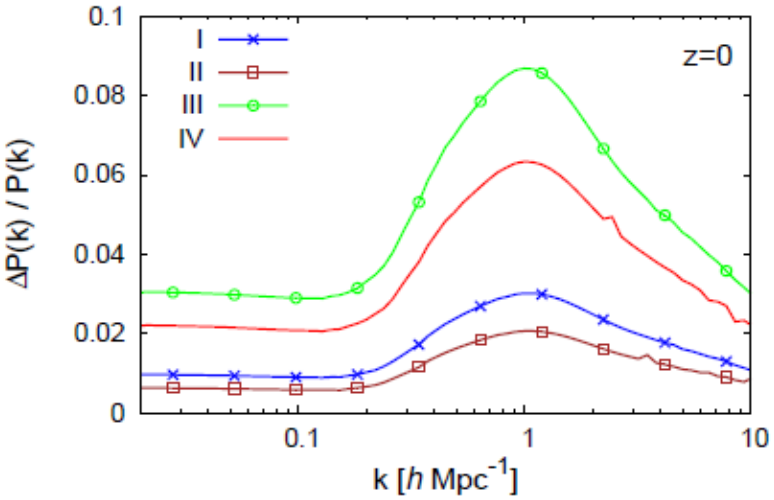
Three models with n=2: $(\chi_*, K_*) = (1, 10^3),\ (1, 10^4),\ (10^2, 10^3)$

In the next slides we show:

cubic model: $K(\chi) = -1 + \chi + \chi^3$

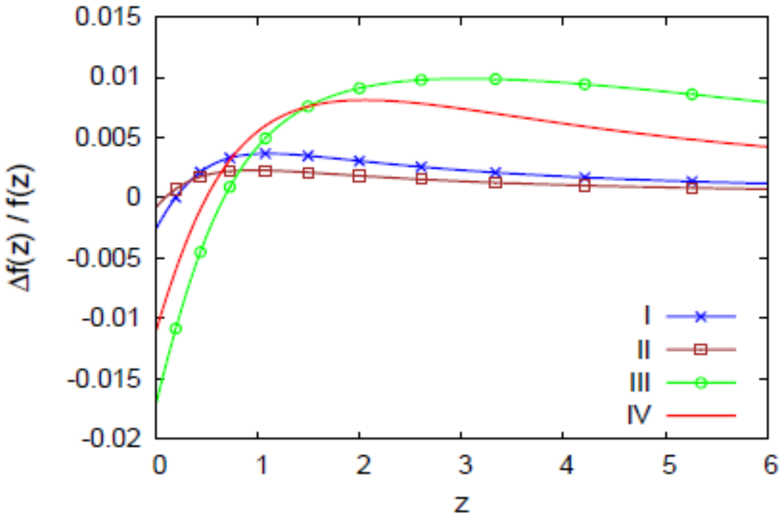
$$\epsilon_2 = \frac{d\ln \bar{A}}{d\ln a} \sim -\frac{2\beta^2}{K'}$$

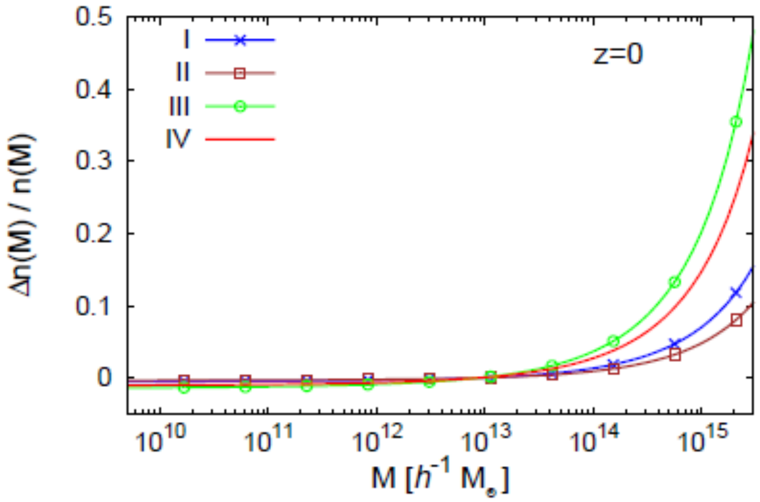




power spectrum

$$f(z) = \frac{d\ln D_+}{d\ln a}$$





halo mass function

$$(\chi_\star, K_\star) = \boxed{(1, 10^3)}, \boxed{(1, 10^4)}, \boxed{(10^2, 10^3)}$$

K-mouflage models can reach a 10% deviation in the power spectrum on non-linear scales and few percents on linear scales.

The large-mass tail of the halo mass function shows large deviations. This is expected as K-mouflage does not screen clusters.

These properties are different from what happens for the Vainshtein mechanism (large clusters are screened) and for chameleons such as f(R) (where GR is recovered on large scales).

III- COSMOLOGICAL CONSTRAINTS

Benevento et al. (2019)

A) Effects on the Background

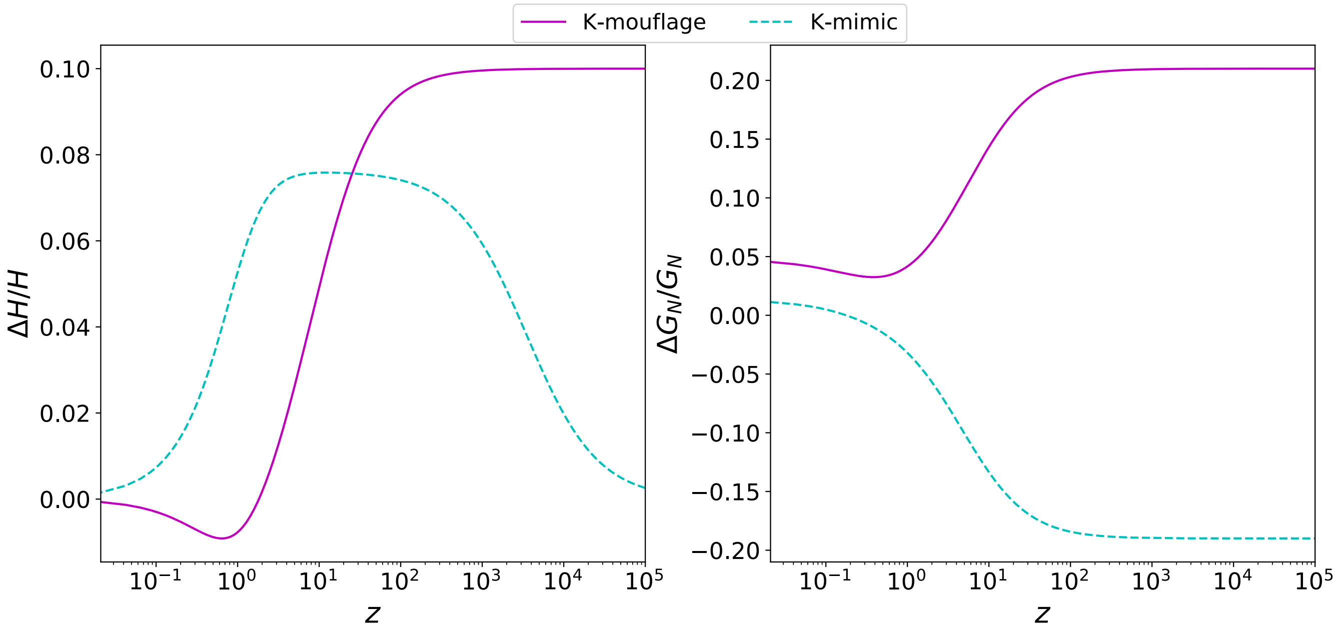
In the Jordan frame the Planck mass becomes time dependent (the field is not frozen to a fixed value)

→ drift with redshift of Newton's constant

deviations from LCDM at the background level (unless tuning)

$|\epsilon_2| = 2 \times 10^{-2}$

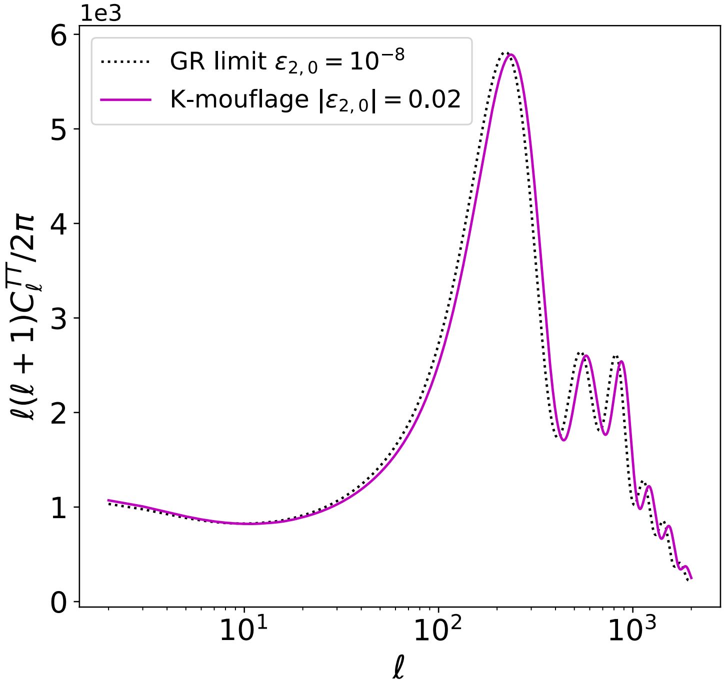
$\Delta H/H$

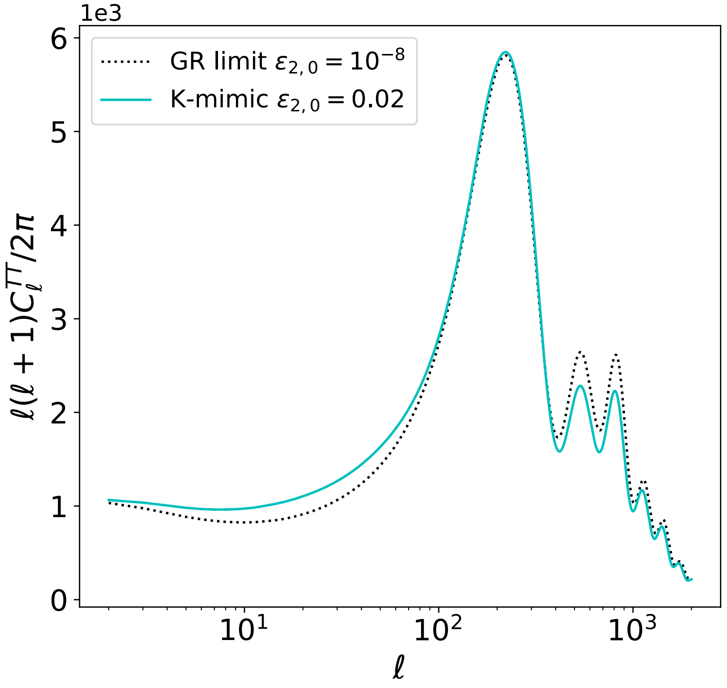


$\Delta G_N/G_N$

K-mouflage

shift of the CMB peaks





K-mimic

deviation of the amplitude of the CMB peaks

Equation (2.23) shows that even within K-mimic models, the background evolution cannot be completely degenerate with ΛCDM. Indeed, given a set of cosmological parameters $\{\Omega_{b0},\ \Omega_{c0},\ \Omega_{\gamma0},\ H_0\}$ K-mimic models reproduce the same $H(a)$ of a ΛCDM model with a slightly higher matter density.

B) Large-scale power spectra

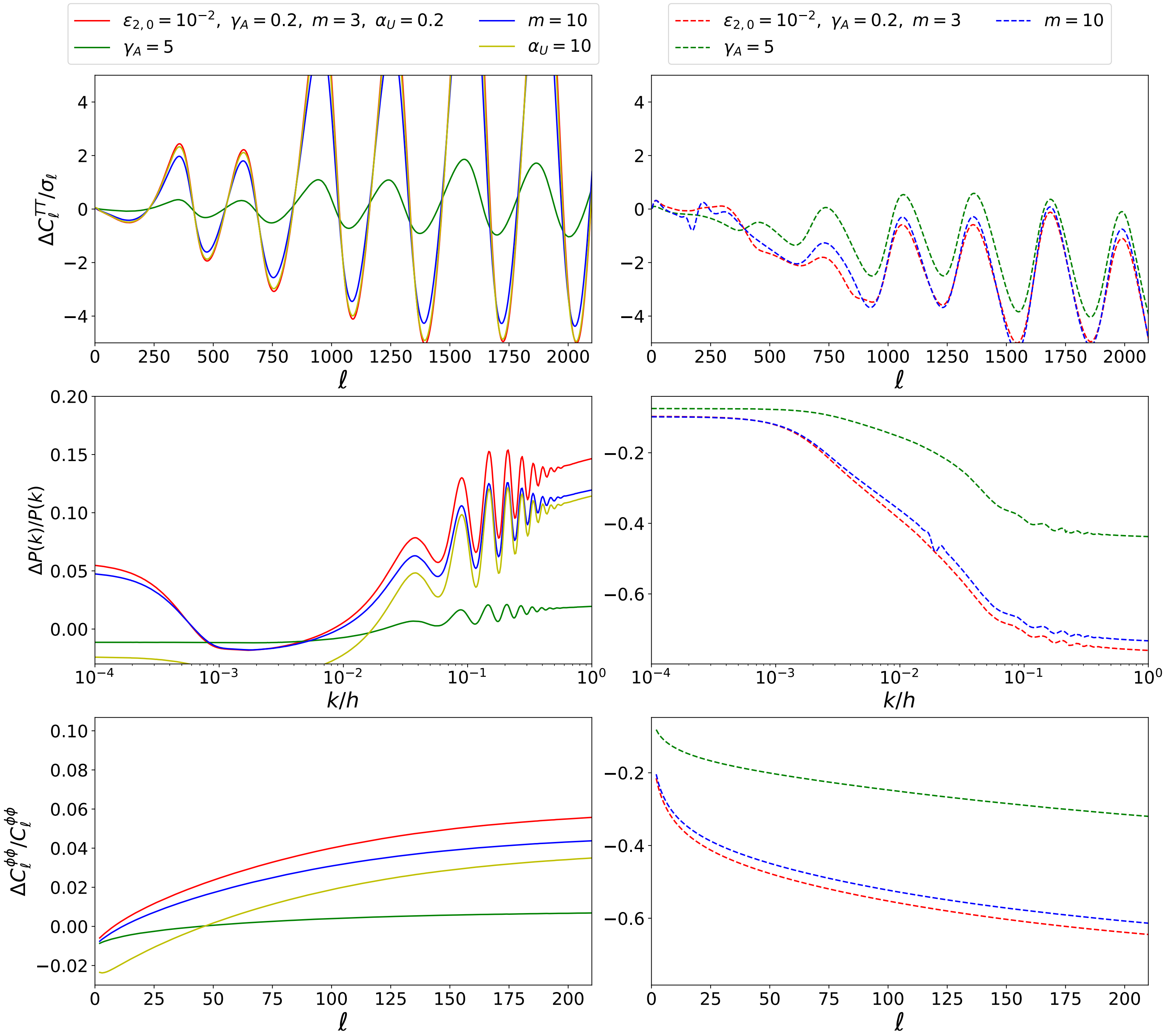
K-mouflage

K-mimic

CMB temp. anisotropies

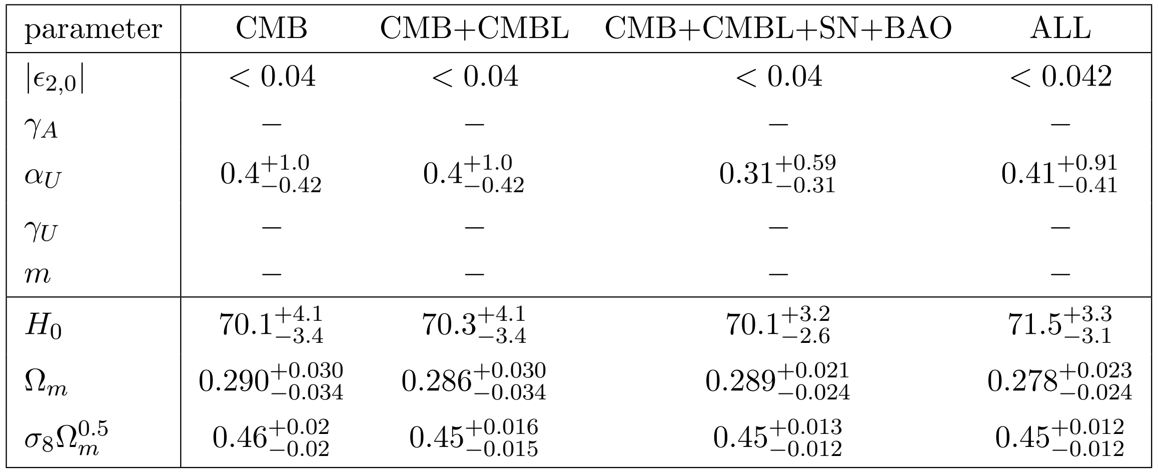
matter power spectrum

lensing potential

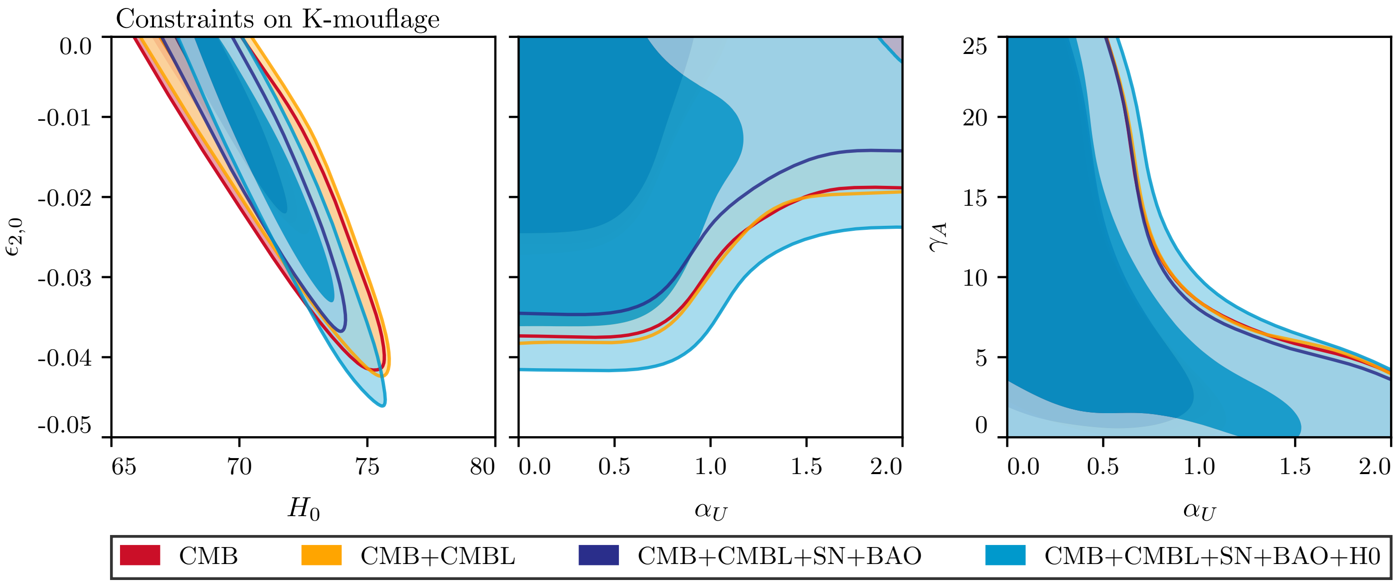


C) Parameter constraints

K-mouflage

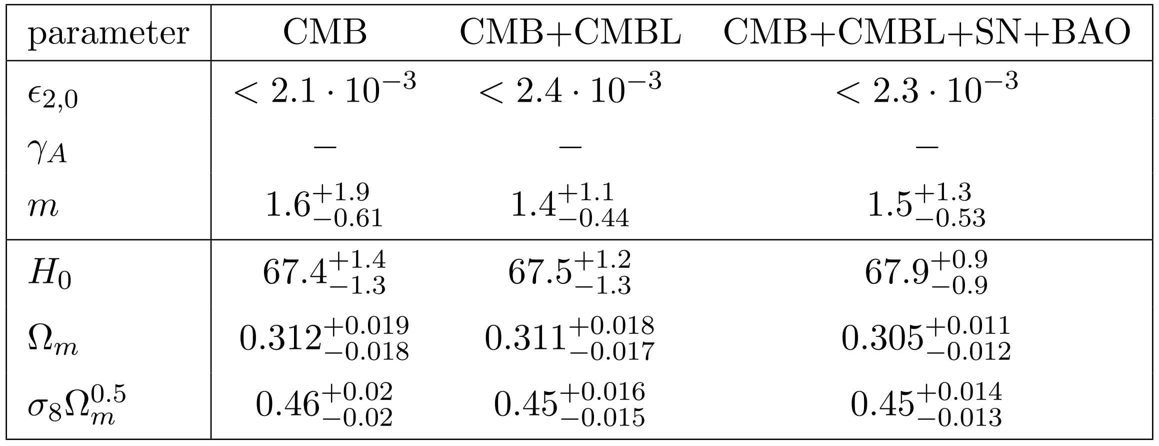


parameter	CMB	CMB+CMBL	CMB+CMBL+SN+BAO	ALL
$\lvert\epsilon_{2,0}\rvert$	< 0.04	< 0.04	< 0.04	< 0.042
γ_A	$-$	$-$	$-$	$-$
α_U	$0.4^{+1.0}_{-0.42}$	$0.4^{+1.0}_{-0.42}$	$0.31^{+0.59}_{-0.31}$	$0.41^{+0.91}_{-0.41}$
γ_U	$-$	$-$	$-$	$-$
m	$-$	$-$	$-$	$-$
H_0	$70.1^{+4.1}_{-3.4}$	$70.3^{+4.1}_{-3.4}$	$70.1^{+3.2}_{-2.6}$	$71.5^{+3.3}_{-3.1}$
Ω_m	$0.290^{+0.030}_{-0.034}$	$0.286^{+0.030}_{-0.034}$	$0.289^{+0.021}_{-0.024}$	$0.278^{+0.023}_{-0.024}$
$\sigma_8\Omega_m^{0.5}$	$0.46^{+0.02}_{-0.02}$	$0.45^{+0.016}_{-0.015}$	$0.45^{+0.013}_{-0.012}$	$0.45^{+0.012}_{-0.012}$



the best-fit value for the K-mouflage H_0 is higher than the one estimated assuming ΛCDM. This means K-mouflage models can mitigate the tension between CMB estimates and direct measurements of H_0 via distance ladder, that is found at about 3σ in ΛCDM. CMB and BAO

K-mimic



parameter	CMB	CMB+CMBL	CMB+CMBL+SN+BAO
$\epsilon_{2,0}$	$< 2.1 \cdot 10^{-3}$	$< 2.4 \cdot 10^{-3}$	$< 2.3 \cdot 10^{-3}$
γ_A	$-$	$-$	$-$
m	$1.6^{+1.9}_{-0.61}$	$1.4^{+1.1}_{-0.44}$	$1.5^{+1.3}_{-0.53}$
H_0	$67.4^{+1.4}_{-1.3}$	$67.5^{+1.2}_{-1.3}$	$67.9^{+0.9}_{-0.9}$
Ω_m	$0.312^{+0.019}_{-0.018}$	$0.311^{+0.018}_{-0.017}$	$0.305^{+0.011}_{-0.012}$
$\sigma_8\Omega_m^{0.5}$	$0.46^{+0.02}_{-0.02}$	$0.45^{+0.016}_{-0.015}$	$0.45^{+0.014}_{-0.013}$

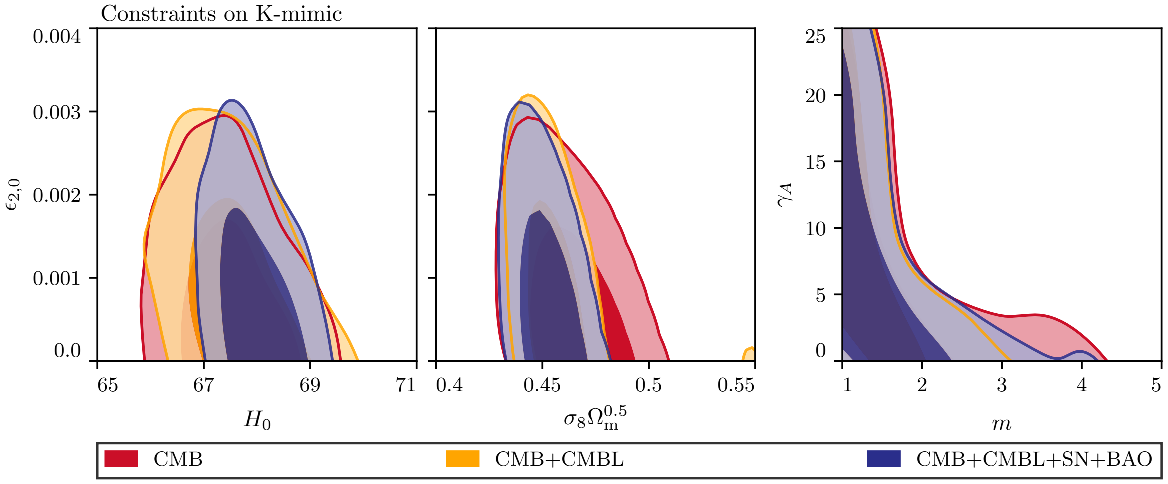


Figure 7. The marginalized joint posterior for a subset of parameters of the K-mimic model, the Hubble constant and $\sigma_8\Omega_m^{0.5}$. In all three panels different colors correspond to different combination of experiments, as shown in legend. The darker and lighter shades correspond respectively to the 68% C.L. and the 95% C.L. regions.

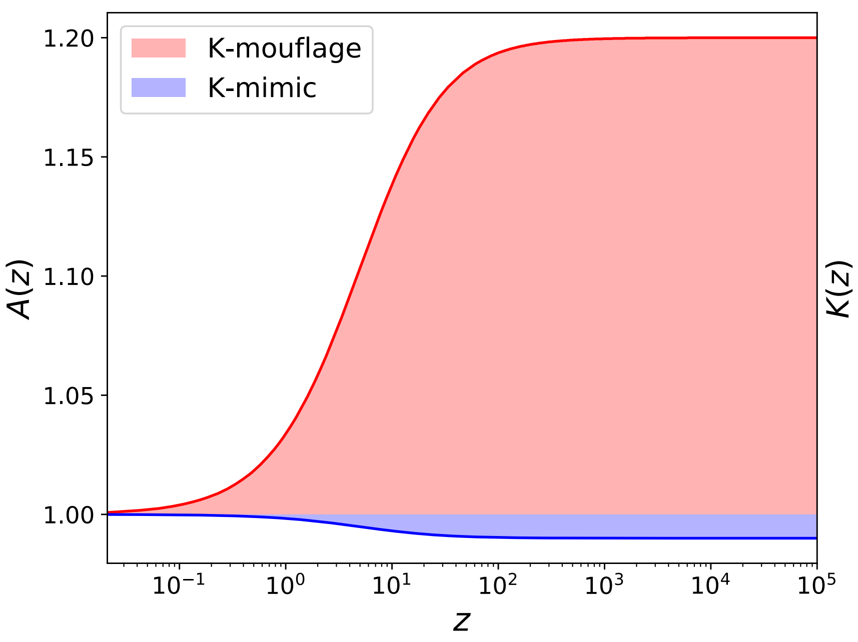
there is now no degeneracy between $\epsilon_{2,0}$ and the Hubble constant, as can be clearly seen from figure 7. The K-mimic model cannot be used to solve the tension between Planck measurements and distance ladder measurements.

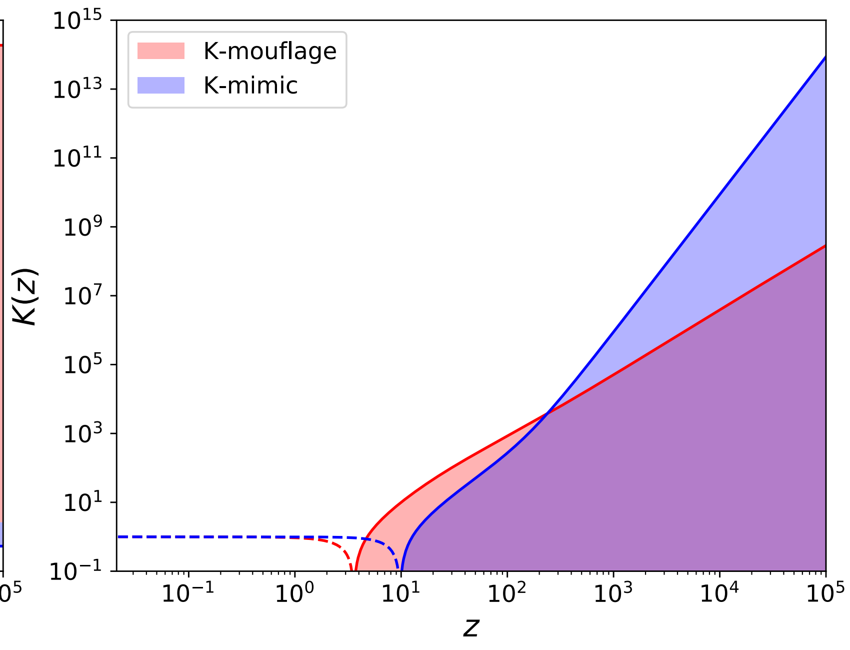
D) Viability regions for the model functions

$$S = S_m(\psi_m^{(i)}, A^2(\varphi)g_{\mu\nu} + \int d^4x\sqrt{-g}\left[\frac{M_{\rm Pl}^2}{2}R + \mathcal{M}^4 K(\chi)\right]$$

conformal
coupling function

kinetic function





IV- SPHERICAL COLLAPSE

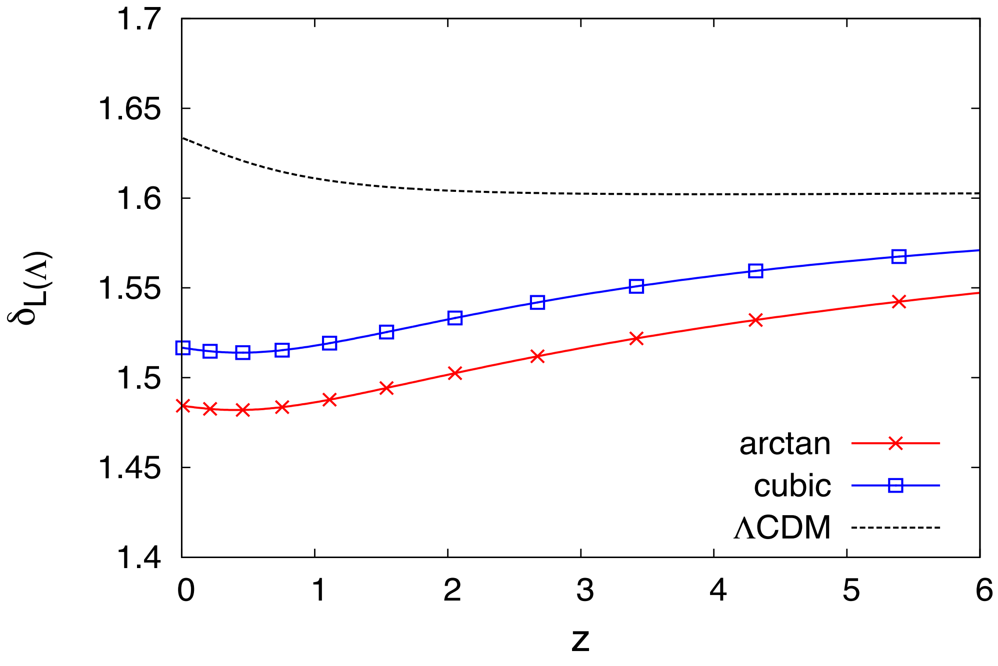
Define the normalized Lagrangian radius of a mass shell:

$$y(t) = \frac{r(t)}{a(t)q} \quad \text{with} \quad q = \left(\frac{3M}{4\pi\bar{\rho}_0} \right)^{1/3} , \quad y(t=0) = 1.$$

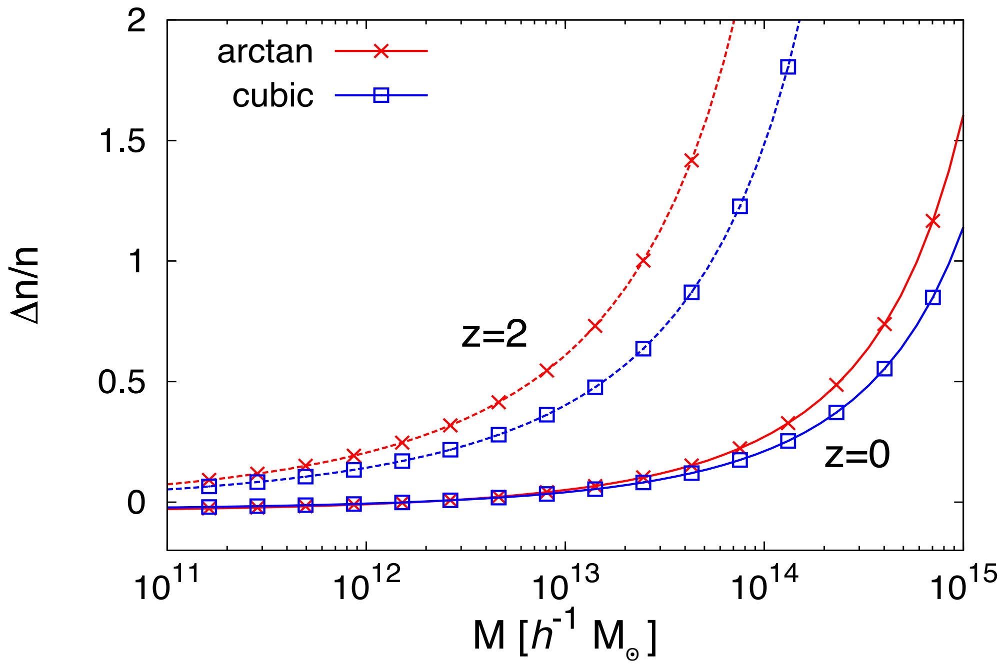
$$\frac{d^2 y}{(d\ln a)^2} + \left(2 + \frac{1}{H^2}\frac{dH}{dt} \right) \frac{dy}{d\ln a} + \frac{\Omega_{\rm m}}{2}(1+\epsilon_1)(y^{-3} - 1) y = 0$$

no scale dependence in the unscreened regime,
mass shells are not coupled

Linear density threshold required to reach $\Delta = 200$



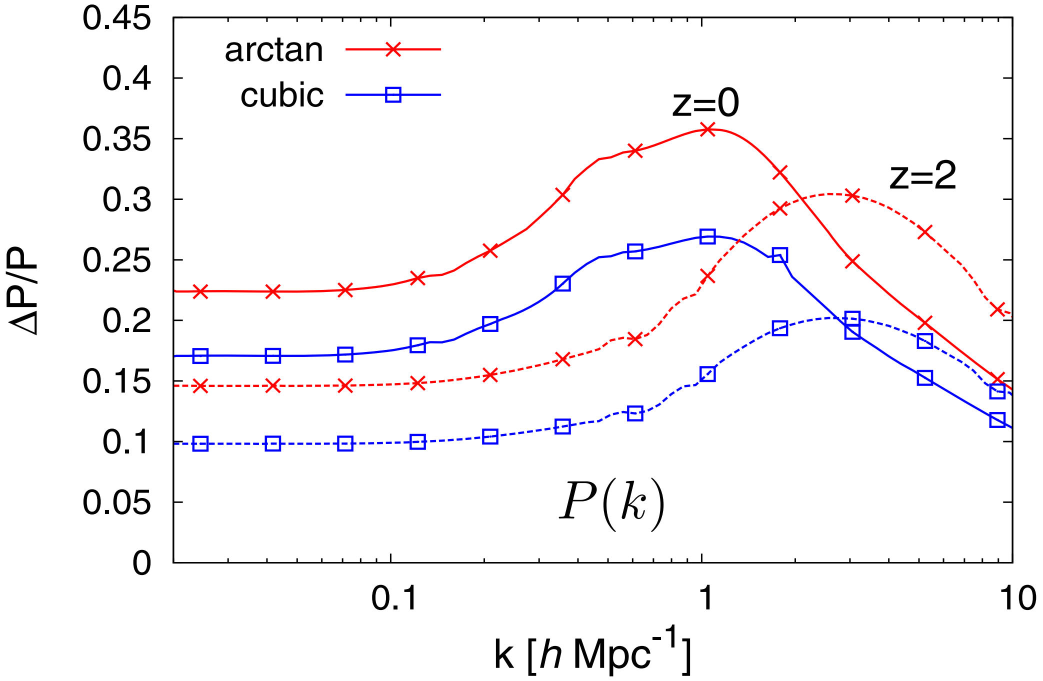
relative deviation from LCDM of the halo mass function

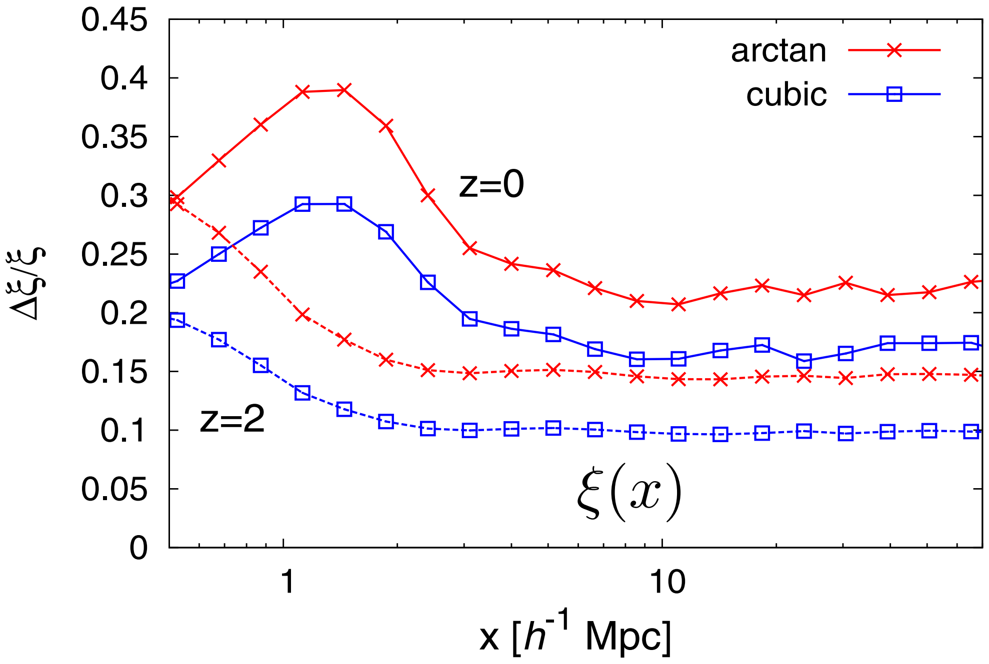


It is easier to collapse → More massive halos

V- NON-LINEAR MATTER POWER SPECTRUM

Combining 1-loop perturbation theory and a halo model, we can estimate the density power spectrum up to nonlinear scales.





- Amplification grows with time

- It does not vanish on very large scales (massless scalar field)

- It peaks around the non-linear scale (small scales probe low mass halo and inner profiles)

- The relative deviations are significantly greater (x10) than for background quantities such as H(z)

- The deviations from LCDM decrease rather slowly at higher z

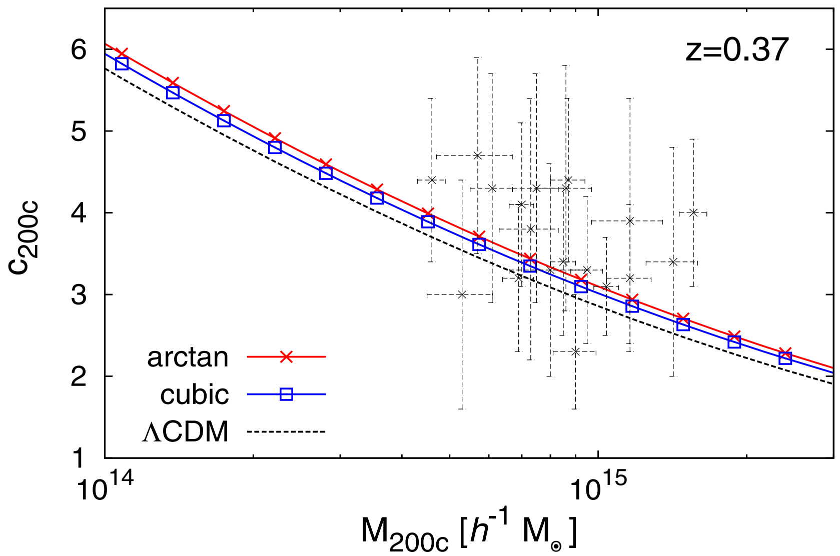
VI- EFFECTS ON CLUSTERS OF GALAXIES

Clusters are not screened: they feel the fifth force.

Brax, Rizzo & V
(PRD 92, 043519, 2015)

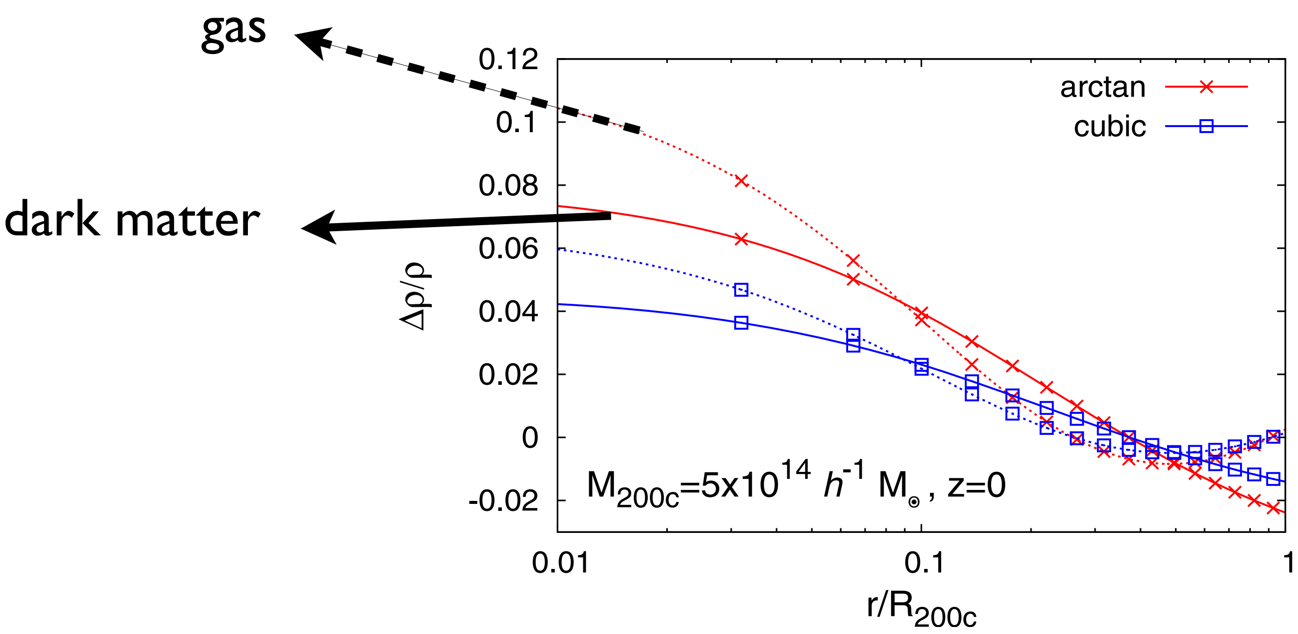
Small effect on halo concentration parameter

$$\rho_s(M) = \Delta_{\rm f}\, \rho_{\rm crit}(z_{\rm f})$$



Hydrostatic equilibrium with the fifth force:

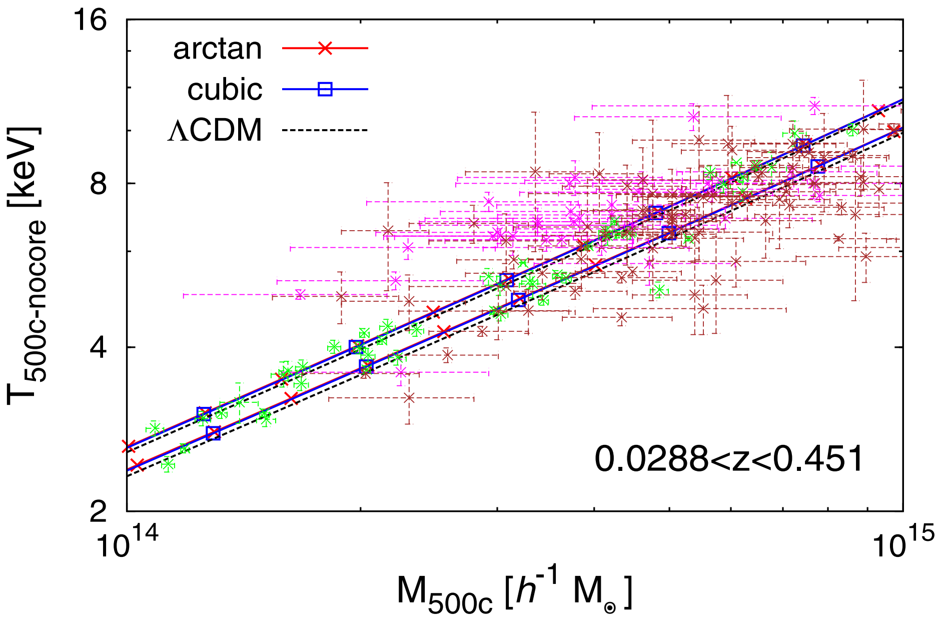
$$\nabla\Phi = \nabla\left(\Psi_{\rm N} + \frac{\beta c^2 \varphi}{\tilde{M}_{pl}}\right) = -\frac{\nabla p_g}{\rho_g}$$



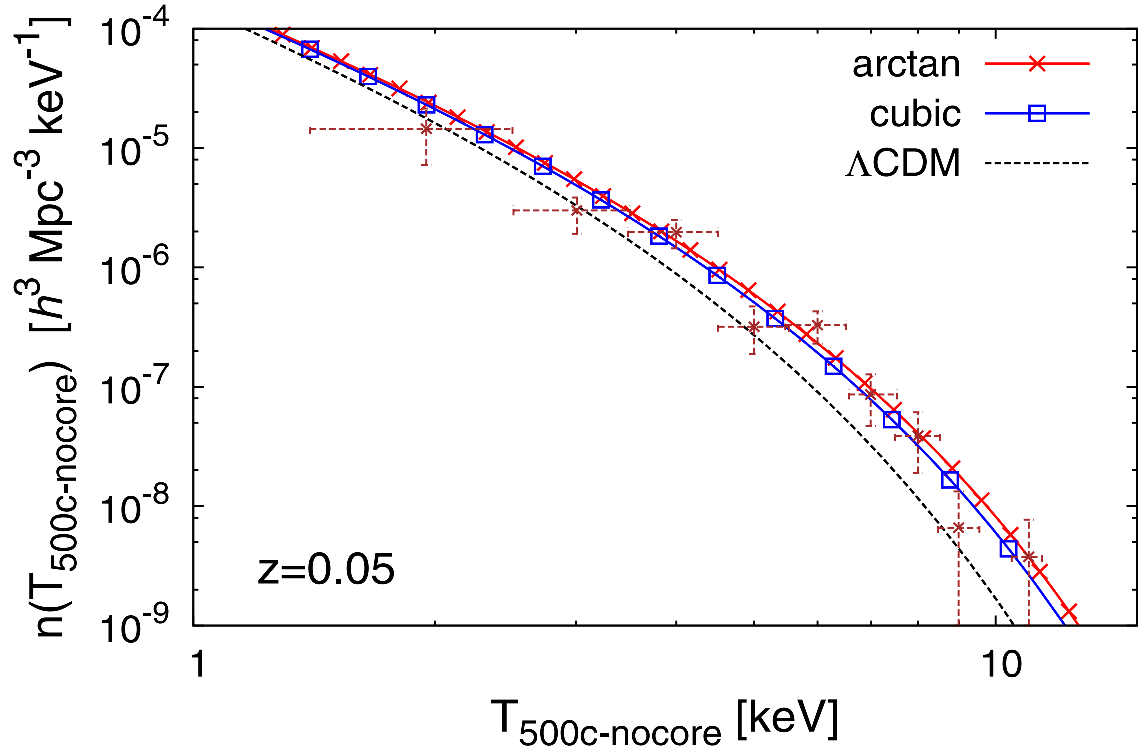
Relative deviation from LCDM for dark matter and gas profiles

➡ Small effect on dark matter and gas profiles

Small effect on
Mass - Temperature relation



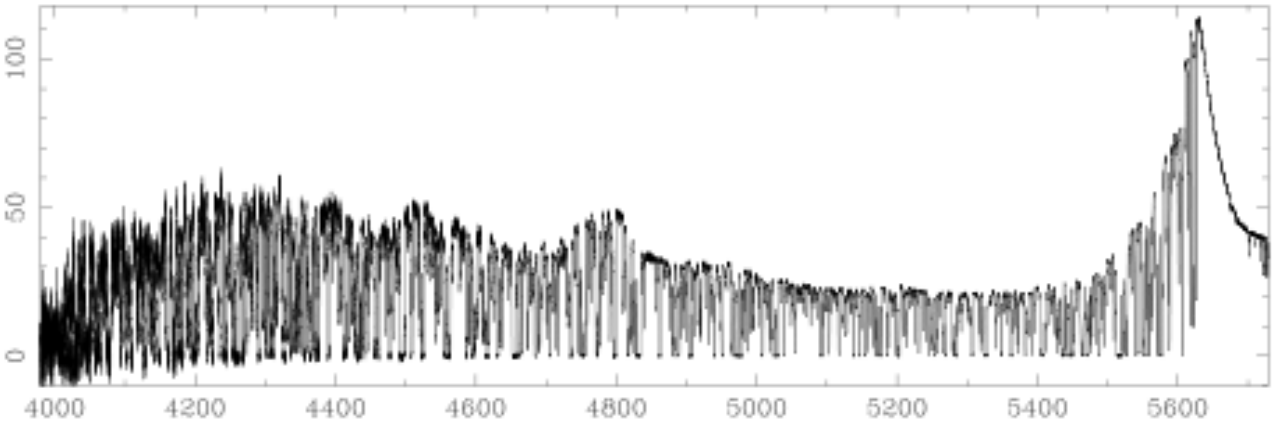
The tail of the temperature multiplicity function is amplified, mostly because of the enhanced formation of large-scale structures.



VII- LYMAN-ALPHA POWER SPECTRUM

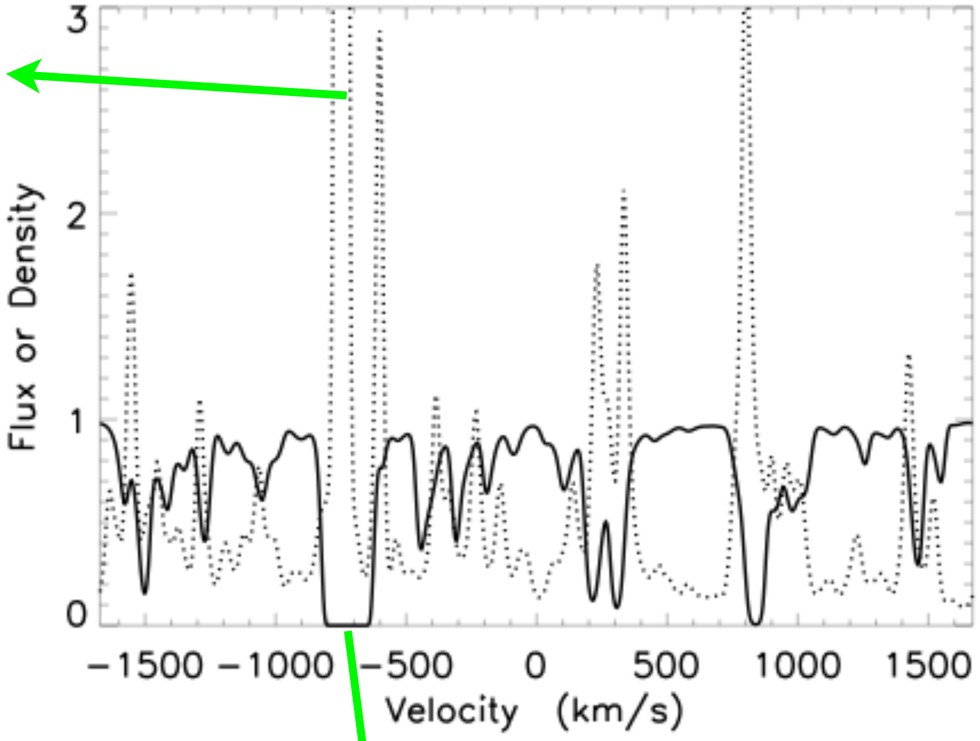
M.White

Spectrum of the light received
from a distant quasar



QSO 1422+23

Spectrum '=' density



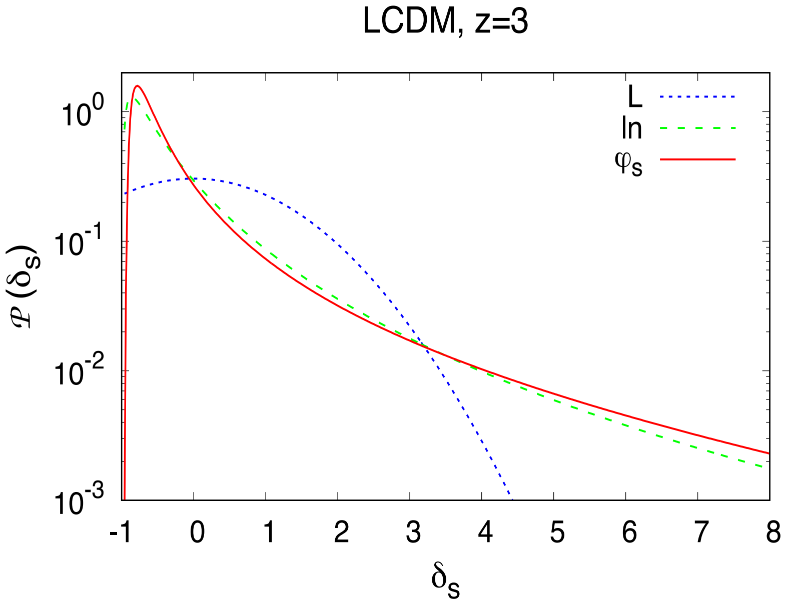
density peak

strong
absorption → zero transmitted flux F

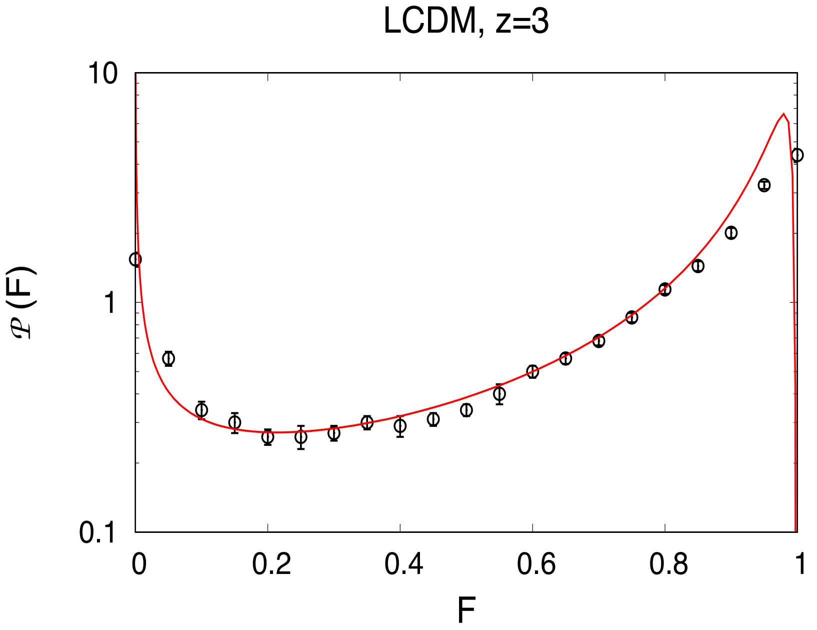
A- PDFs of the density and of the flux

LCDM:

density contrast

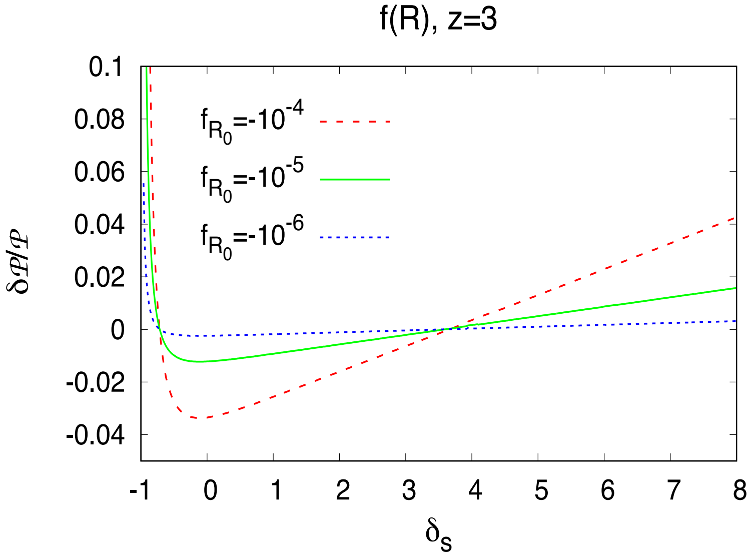


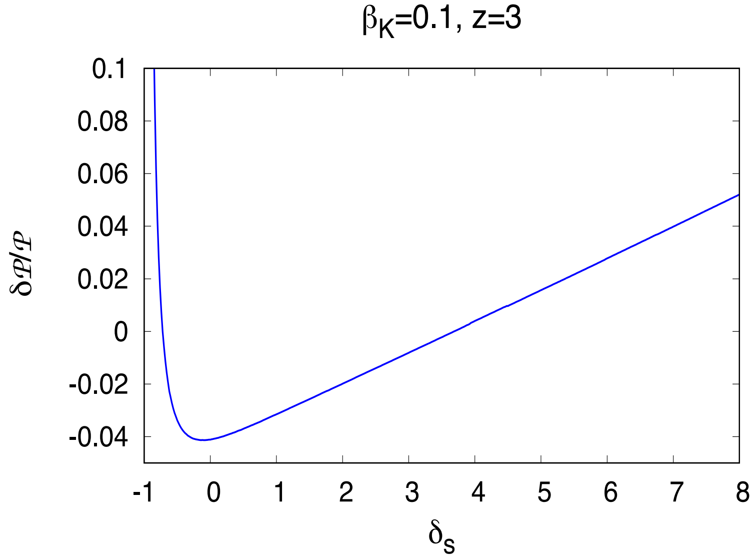
Flux



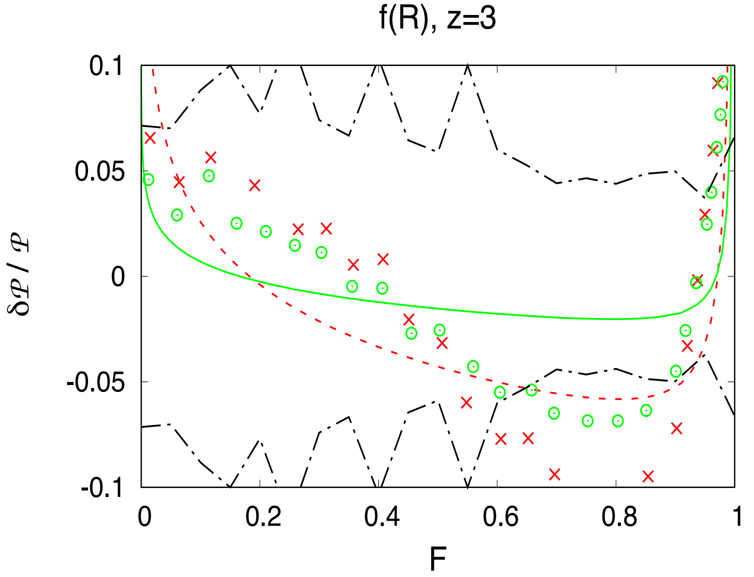
Modified-gravity: f(R) and K-mouflage

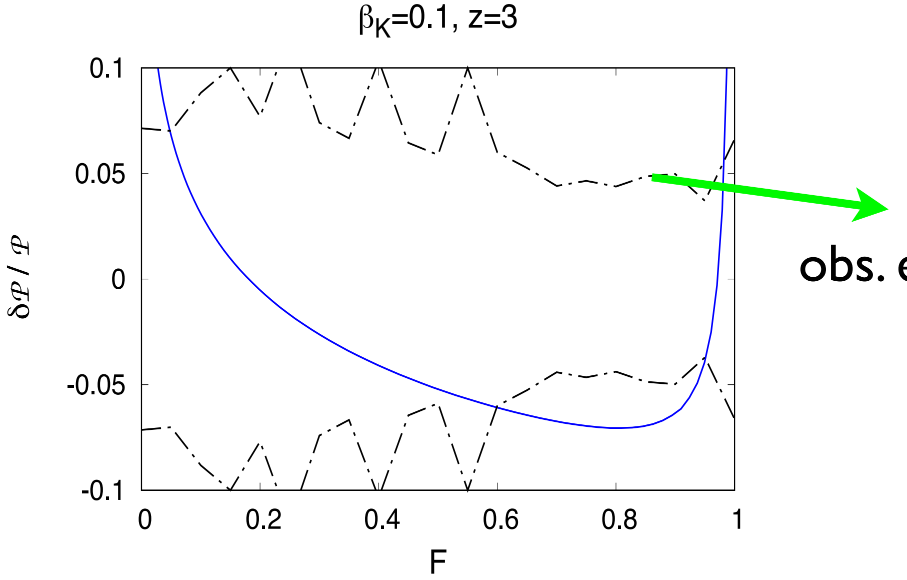
relative deviation for PDF of the density contrast





relative deviation for PDF of the flux

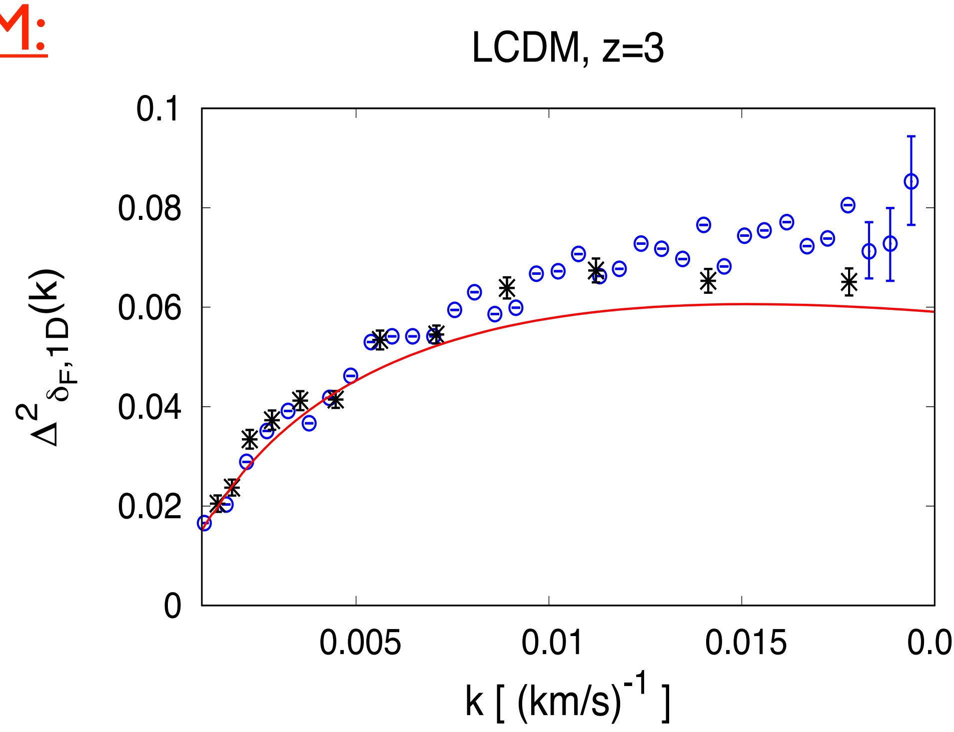




1σ
obs. errorbar

B- 1D power spectrum

LCDM:

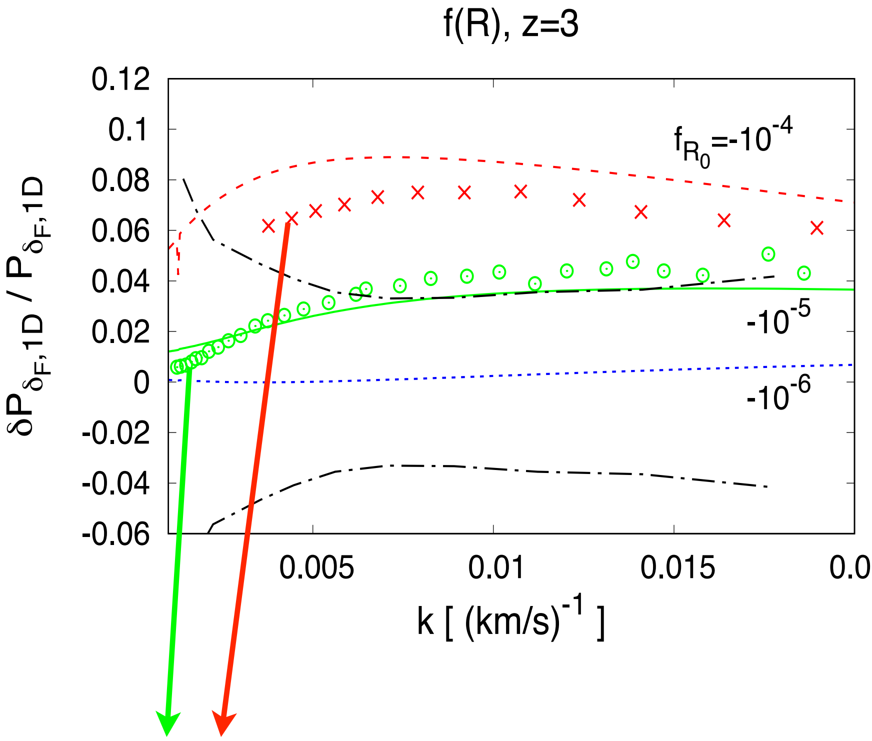


1D Lyman-alpha power spectrum along the line of sight

$$P_{\delta_F,1\mathrm{D}}(k_z) = \int_{-\infty}^{\infty} dk_x dk_y P_{\delta_F}(\mathbf{k}) = 2\pi \int_{k_z}^{\infty} dk\, k P_{\delta_F}(k, \mu = k_z/k).$$

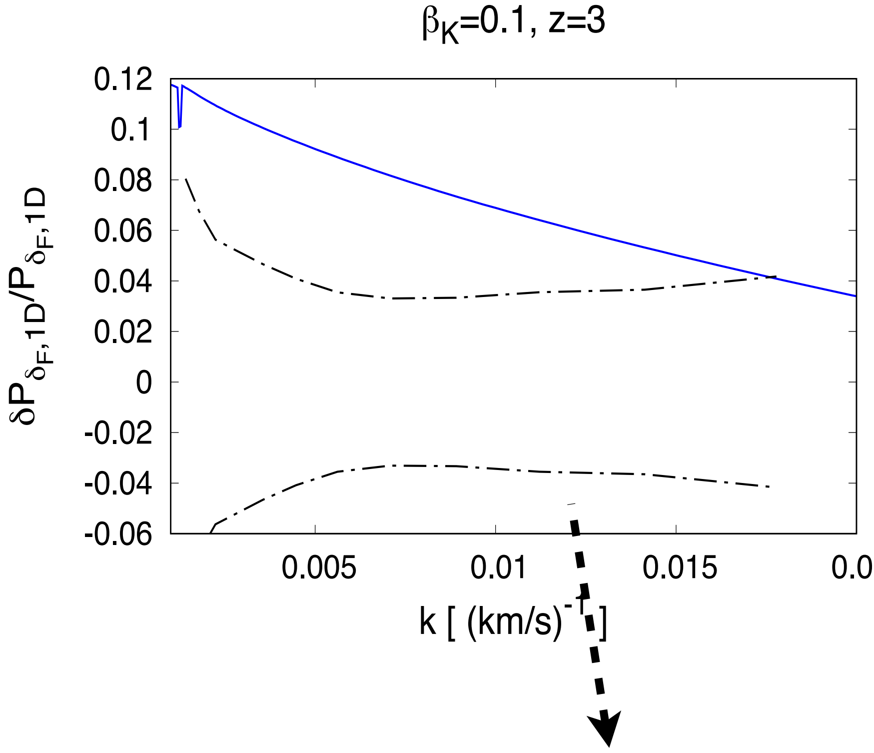
Modified-gravity:

Relative deviation of the 1D Lyman-alpha power spectrum



simulations

Arnold et al. (2015)



1σ

obs. errorbar

CONCLUSIONS

Light scalar fields involved in modified-gravity theories must be screened in the Solar System to satisfy very tight observational bounds.

There are **3 main mechanisms:**

- chameleon
- Damour-Polyakov
- Kmouflage/Vainshtein

They operate in different manners, so that the screening transition appears at different scales and densities and behaves in different ways.

Observational probes can put constraints on these models and distinguish between the screening mechanisms.

- formation of cosmological structures (amplification/decrease of gravity)
- impact on velocity fields
- difference between dynamical and lensing mass (look for clusters of galaxies)
- violations of the equivalence principle
- non-universal coupling (baryons - dark matter)

Screening does not remove all modifications to gravity:

- speed of gravitational waves
- time dependence of Newton's constant (and of the Hubble expansion rate)
- scalar waves generated by catastrophic events (supernovae) could make screening unefficient and be detected ?

ADDITIONAL ITEMS

I- QUANTUM CORRECTIONS

Quantum corrections are negligible in practical cases

There exists a classical regime where quantum corrections are small.
Moreover, the operators of the classical Lagrangian are not renormalized.

Scalar field action = classical K-mouflage Lagrangian + counterterms

$$S_{\rm bare}[\varphi] = \int d^4x \, \mathcal{L}_{\rm bare}(\varphi) = \int d^4x \, [\mathcal{L}_{\rm classical}(\varphi) + \Delta\mathcal{L}(\varphi)]$$

$$\mathcal{L}_{\rm classical}(\varphi) = \mathcal{M}^4 K(\chi)$$

Effective action: $\Gamma_{\rm renorm}[\varphi] = \int d^4x \, \mathscr{L}_{\rm renorm}(\varphi) = \int d^4x \, [\mathcal{L}_{\rm classical}(\varphi) + \Delta\mathscr{L}(\varphi)]$

Classical regime: $\Delta\mathscr{L} \ll \mathcal{L}_{\rm classical}$ if: $\bar{p} \ll \mathcal{M} \, (\bar{K}'\bar{\chi})^{1/4} \left(\frac{\bar{K}'}{\bar{K}''\bar{\chi}} \right)^{1/2}$

cosmological background: $H \ll 2.3 \times 10^{-12} \, (\bar{K}'\bar{\chi})^{1/4} \left(\frac{\bar{K}'}{\bar{K}''\bar{\chi}} \right)^{1/2} \; \mathrm{GeV}$ $\quad (H_{\rm BBN} \lesssim 10^{-23} \, \mathrm{GeV})$

astrophysical background: $\frac{r}{R_K} \gg 4.7 \times 10^{-20} \left(\frac{M_\odot}{\beta M} \right)^{1/2} (\bar{K}'\bar{\chi})^{-1/4} \left(\frac{\bar{K}''\bar{\chi}}{\bar{K}'} \right)^{1/2}$ $\quad (R_{K-{\rm Sun}} \sim 1000 \; \mathrm{AU})$

➡ The quantum corrections are negligible in practical cases.

II- OBSERVATIONAL PROBES

A) Deviations from LCDM on cosmological scales

Cosmological structures may probe the transition to the screening domain.

Deviation of the matter power spectrum on cosmological scales, for f(R) models.

Deviation of the halo mass function, for K-mouflage models.

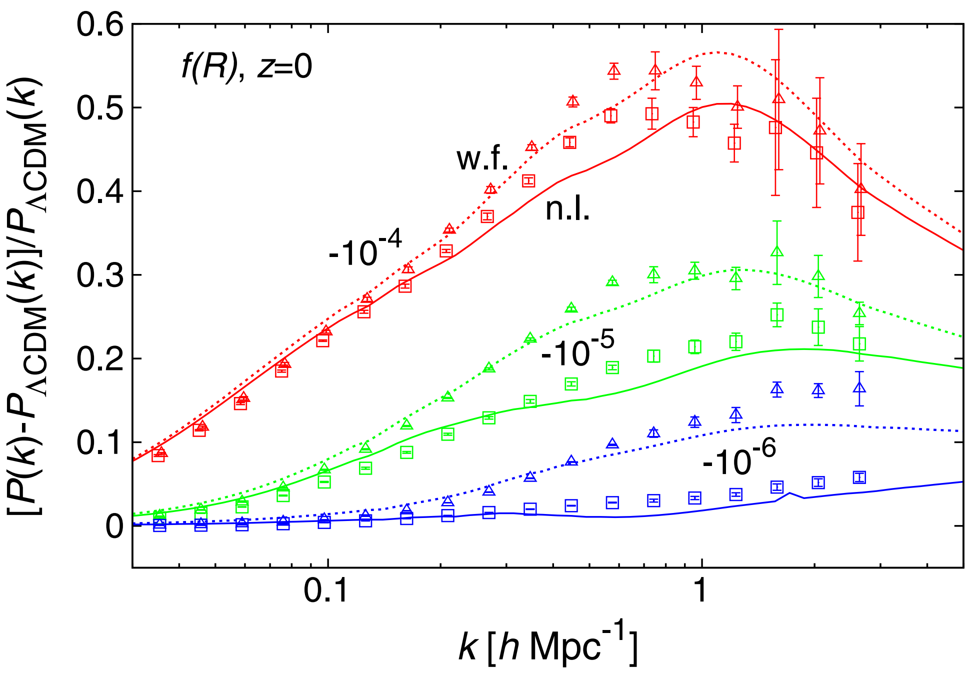
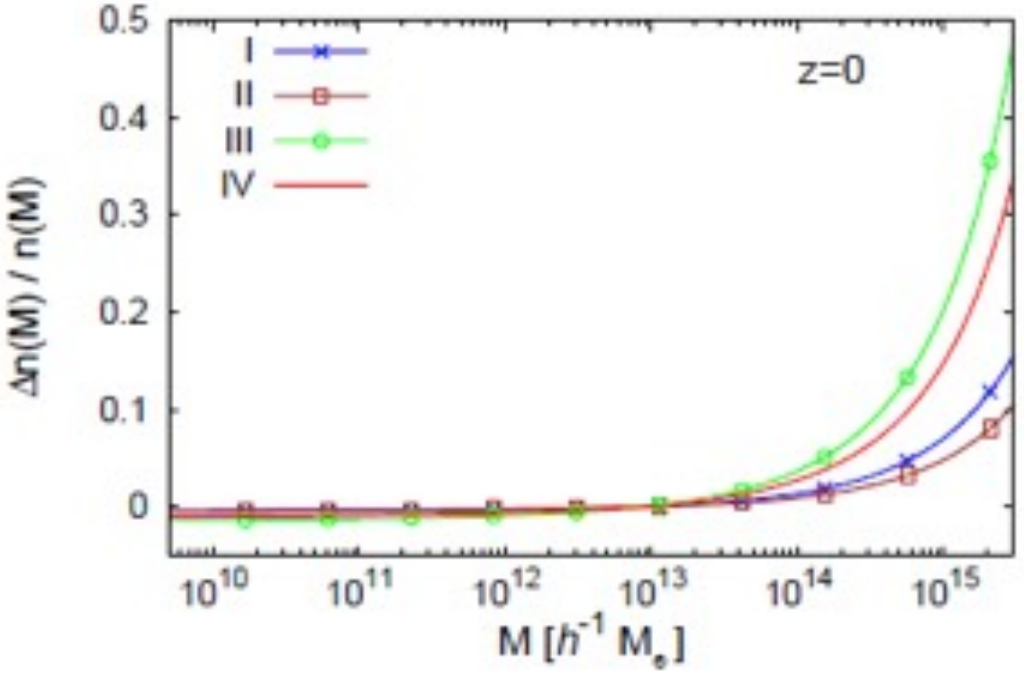


FIG. 13 (color online). Relative deviation from ΛCDM of the power spectrum in $f(R)$ theories, at redshift $z = 0$, for $n = 1$ and $f_{R_0} = -10^{-4}$, -10^{-5}, and -10^{-6}. In each case, the triangles and the squares are the results of the "no-chameleon" and "with-chameleon" simulations from Ref. [25], respectively. We plot the relative deviation of the nonlinear power spectrum without the chameleon effect (w.f., dotted lines) and with the chameleon effect (n.l., solid lines).



B) Deviations from GR on small scales

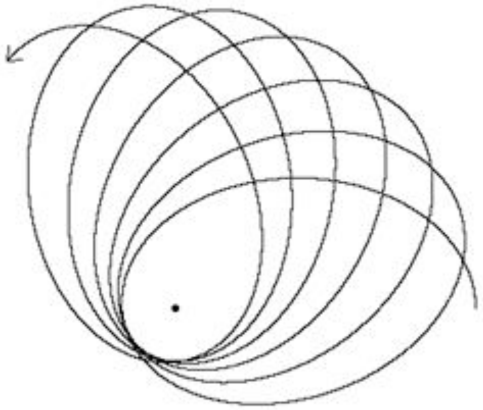
Screening ensures that the 5th force is much smaller than Newtonian gravity.

However, small deviations can still produce non-negligible effects,
for instance for the K-mouflage model:

anomalous perihelion of the Moon around the Earth:

$$\delta\theta = \pi r \frac{d}{dr}\left[r^2 \frac{d}{dr}\left(\frac{\epsilon}{r}\right)\right]$$

$$\epsilon = \frac{\delta \ln A}{\Psi_{\rm N}}$$



$$|\delta\theta| < 2 \times 10^{-11}$$

Williams et al. (Class. Quant. Grav. 29, 184004, 2012)

small force that does not behave as 1/r^2 → orbit does not close

One obtains: $$\delta\theta = -8\pi \frac{\beta^2}{K'} \frac{\chi K''}{K' + 2\chi K''} \leq 2 \times 10^{-11}$$

Brax & V. (2015)

→ The only way of satisfying the perihelion bound is to suppress K” in the Solar System.

C) Speed of gravitational waves

Many more complex models (e.g. galileons) give a speed c_T for gravitational waves that is different from the speed of light c.

If $c_T < c$ observed cosmic rays should have decayed away into gravitons by Cherenkov-like emission.

Detections of optical counterparts to gravitational waves sources would rule out models that give $c_T \neq c$

A multi-messenger event gives:

$$\Delta t \sim \left(\frac{c_T}{c} - 1\right) \frac{d}{200\mathrm{Mpc}} 10^{17}\mathrm{seconds}$$

Will (2014)