Building a matter power spectrum emulator from N-body simulations in $f(R)$CDM cosmology

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Work in progress with Yann Rasera

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Accelerated expansion of the Universe

Accelerated expansion of the Universe discovered in 1998 (SNe).
Acceleration condition, fluid with an equation of state:

$$w = P/\rho c^2 < -1/3$$

$\longrightarrow$Concept of **Dark Energy**.

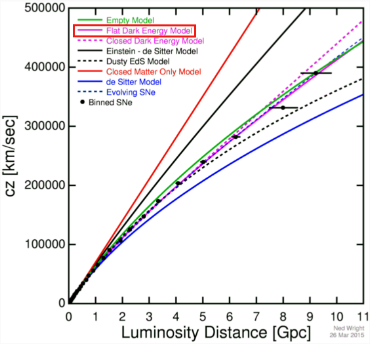

Figure 1: Hubble diagram, credits : N. Wright.

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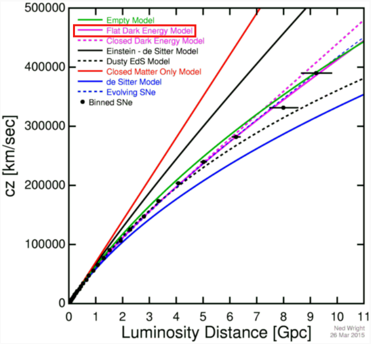

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Question : What is the nature of DE ?

$\longrightarrow$Standard cosmological model: Cosmological Constant Λ
$\longrightarrow$DE fluid ?
$\longrightarrow$Modified gravity?

Models beyond ΛCDM

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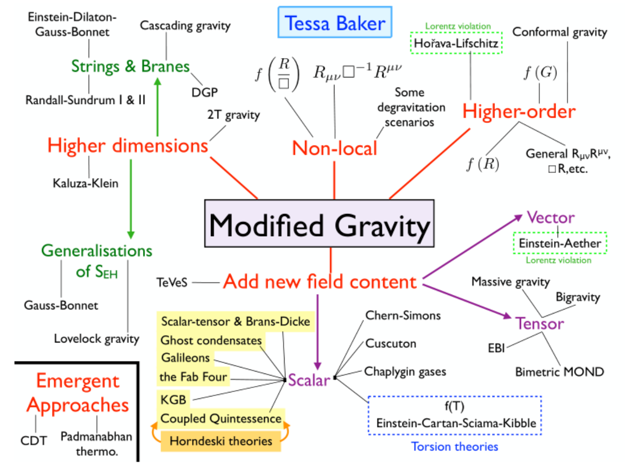
⟶**Modified gravity**: lift assumptions from General Relativity.



Measuring the structure of the Universe

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$\longrightarrow$One of the objectives of the ongoing (DESI) and future (Euclid, LSST $\sim$ 2022) large surveys.

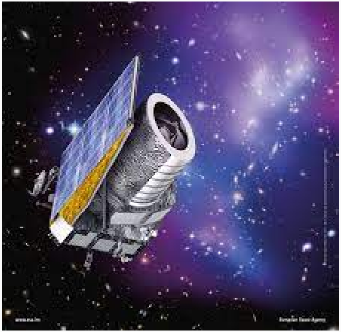

Figure 2: EUCLID satellite.

Figure 3: LSST telescope.

N-body simulations

⟶In **the non-linear regime** ($k \gtrsim 0.1/\mathrm{Mpc}$) we can discriminate between **dark energy** et **modified gravity**.
⟶**N-body simulations** needed to obtain precise predictions deep into the NL regime (but very slow...).

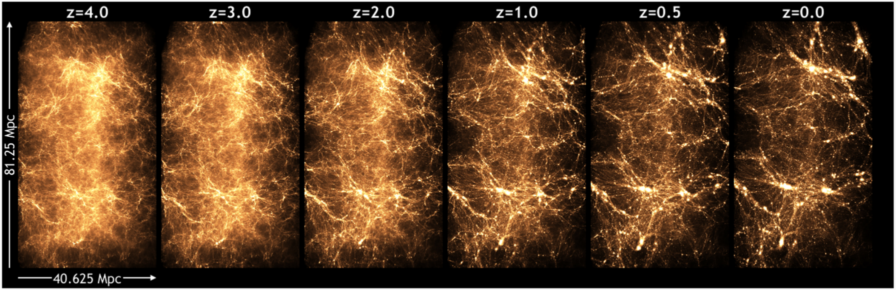

Figure 4: N-body cosmological simulation, snapshots at different redshifts. Credits : Heitmann et al. (2015).

Objectives

Study a simple MG model $f(R)$, belonging to the scalar-tensor family.

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- Impact of MG on the distribution of matter.
- Explore the space of cosmological parameters.
- Build an **emulator** for $P(k)$ calibrated on N-body simulations: obtain fast predictions for the non-linear matter power spectrum.

The $f(R)$ model

$$S_{\mathrm{EH}} = \frac{c^4}{16\pi G}\int \mathrm{d}^4x\sqrt{-g}\,[R - 2\Lambda] \longrightarrow \frac{c^4}{16\pi G}\int \mathrm{d}^4x\sqrt{-g}\,[R + f(R)]$$

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$\longrightarrow$New dynamical degree of freedom, scalar field $f_R = \frac{\mathrm{d}f}{\mathrm{d}R}$.

$\longrightarrow$Depending on the form of $f(R)$ this model can:

- produce cosmic acceleration,
- satisfy local constraints (”chameleon” screening mechanism) where gravity is well constrained (laboratory tests, solar system...).

$f(R)$ gravity - The Hu & Sawicki model (2007)

The Hu & Sawicki model:

$$f(R) = -m^2 \frac{c_1 (R/m^2)^n}{c_2 (R/m^2)^n + 1} \quad \text{with} \quad m^2 \equiv \Omega_\mathrm{m} H_0^2$$

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In the high curvature limit (i.e. high density) :

$$\lim_{m^2/R \to 0} f(R) \simeq -\frac{c_1}{c_2} m^2 + \frac{c_1}{c_2^2} m^2 \left(\frac{m^2}{R} \right)^n \xrightarrow[m^2/R \to 0]{} -2\Lambda$$

We fix $c_1/c_2 = 6\Omega_\Lambda/\Omega_\mathrm{m}$ to match ΛCDM at high redshift ($R \to \infty$).

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$\longrightarrow$Two remaining free parameters: $c_1/c_2^2 \sim f_{R_0}$ and n.

N-body simulations in $f(R)$ gravity

N-body cosmological simulations for the Hu & Sawicki model with ECOSMOG [Li et al. 2012, Bose et al. 2017], modified version of RAMSES [Teyssier 2002].
$\longrightarrow$fast $f(R)$ solver but limited to $n = 1$, only free parameter: f_{R_0}.
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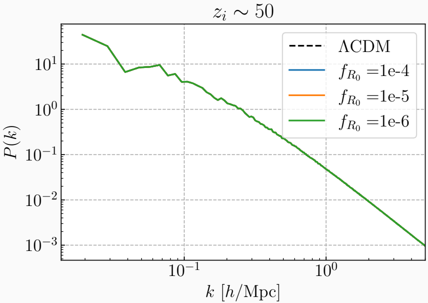
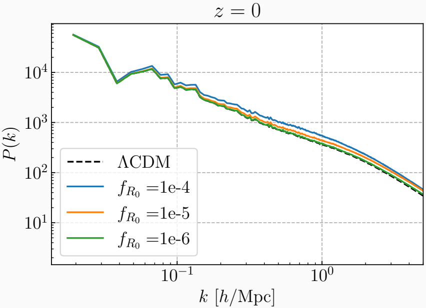
Simulations:

- BoxSize: $(325\,\mathrm{Mpc}/h)^3$
- # particles: $512^3 \sim 135$ million
- Mass resolution: $m_\mathrm{p} \sim 10^{10}\mathrm{M_{sun}}h^{-1}$
- In ΛCDM, at $z = 0$, $P(k)$ at 1% for $k \lesssim 2h^{-1}$Mpc.
- CPU time: $5 - 20$ hours on 512 cores ($\sim 2 - 10$ times slower than ΛCDM)

Influence of $f(R)$ on the matter distribution

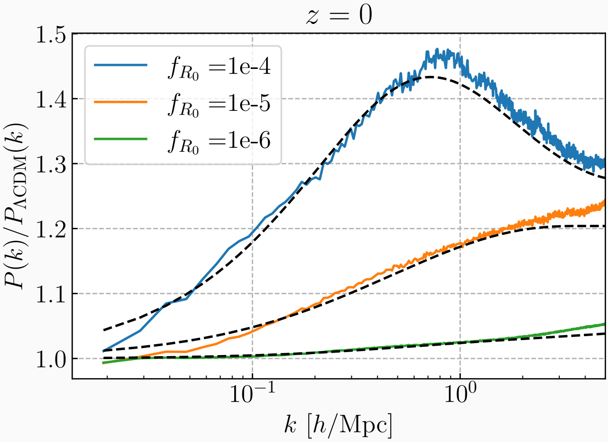




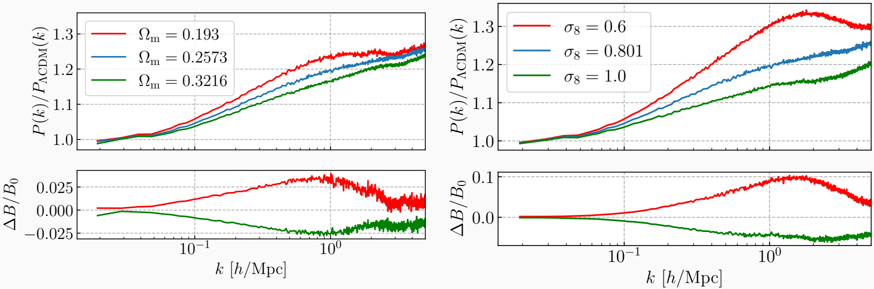
Left : Initial spectrum (beginning of the simulation).
Right : Final spectrum (end of the simulation).

$\longrightarrow$The spectrum is amplified by $f(R)$ gravity.
$\longrightarrow$This amplification is stronger at non-linear scales ($k > 0.1$).

Power spectrum boost - comparison with Winther 2020



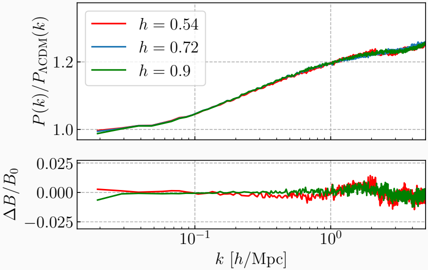
Influence of the cosmology on the boost due to $f(R)$

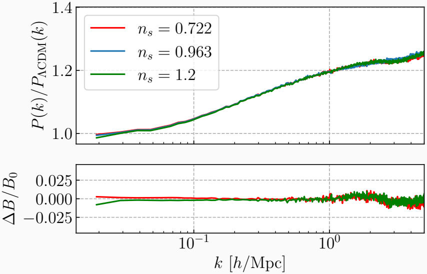


$\longrightarrow$Increasing the amount of matter (Ω_m) or the normalisation of the initial spectrum (σ_8) increases the density.

$\Rightarrow$ Screening of $f(R)$ more efficient $\Rightarrow$ weaker boost.

Influence of the cosmology on the boost due to $f(R)$





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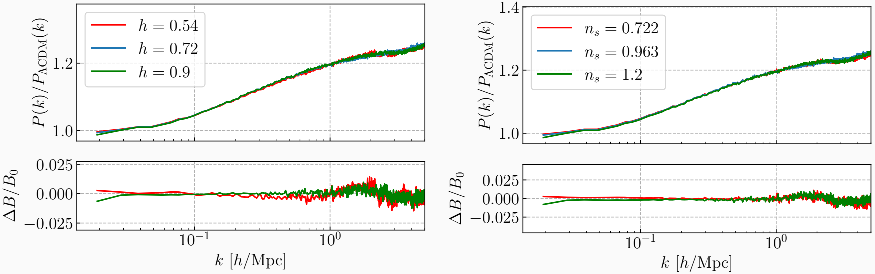

$\longrightarrow$The influence of h et n_s (and $\Omega_b h^2$) on the power spectrum boost is negligible ($< 1\%$).

$\Rightarrow$ Not necessary to take these parameters into account if we build an emulator for the boost !

Building an emulator to predict the effect of $f(R)$

Emulator: Interpolates between the results of simulations performed for different cosmological models. An emulator allows us to obtain fast predictions in the non-linear regime, unlike N-body simulations which are very slow.

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Current emulators:

- [Winther 2020]: doesn't take into account the variation of the cosmology.
- [Ramachandra 2021]: takes into account the cosmology but calibrated using COLA simulations, less precise than N-body methods.
- [Arnold 2021]: very recent (few weeks ago) more accurate emulator, complementary to our work.

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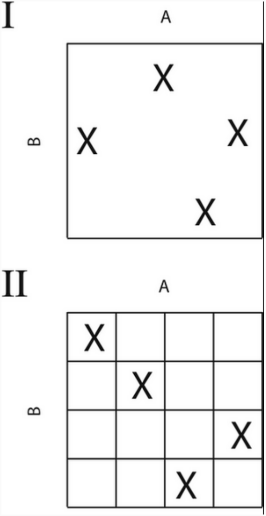
Objectives :

- Explore the parameter space: $\{\Omega\mathrm{m}, \sigma_8, f_{R_0}\}$.
- Build an emulator that predicts the boost $P(k)/P_{\Lambda\mathrm{CDM}}(k)$ as a function of these parameters.

Latin Hypercube Sampling

⟶**Example I:** each point is randomly sampled without taking into account the positions of the other points.

⟶**Example II:** each point is sampled so that it has no common coordinates with the already chosen points.



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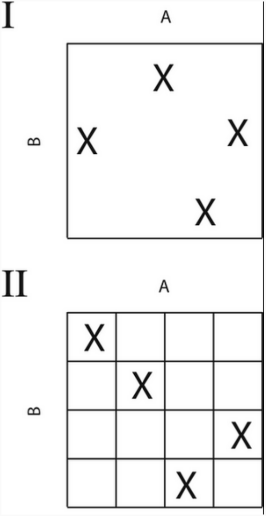
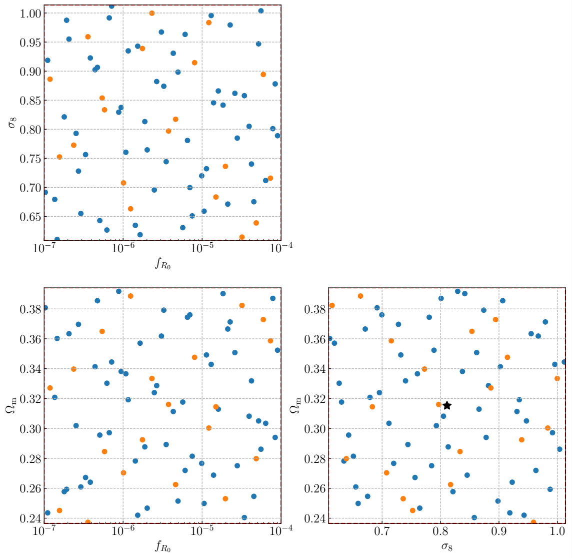
⇒ efficiently samples the parameter space
⟶important in order to minimise the number of simulations required to build an accurate emulator.



Sliced Latin Hypercube Sampling - Ba et. al 2015

⟶Modified algorithm of [Ba 2015]:

- maximises the distances between points,
- points distributed in sub-samples.



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$$p(\mathbf{y}) = \mathcal{N}(\mathbf{0}, \mathbf{K})$$

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$\longrightarrow$The kernel function describes how similar is the data at different locations. Simple example:

$$k_{\mathrm{RBF}}(x_i, x_j) = \exp\left(-\frac{(x_i - x_j)^2}{l^2}\right)$$

$\longrightarrow l$ is a free hyperparameter (inferred by likelihood maximization).

Gaussian Process Regression

We want to predict new data $\mathbf{y}^*$ at new locations $\mathbf{x}^*$. We can extract it from the joint probability distribution:

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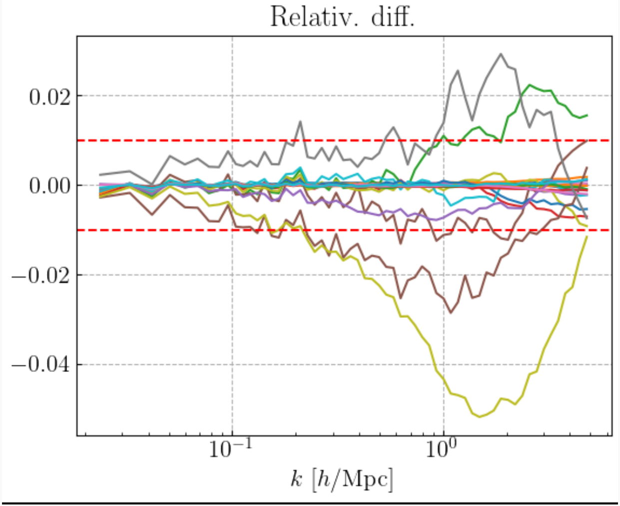
$\longrightarrow$We use the scikit-learn python library.

Testing the emulator



- Small average relative error: $<\epsilon>= 0.3\%$
- But a few models fail with large errors ($\sim 1 - 5\%$)

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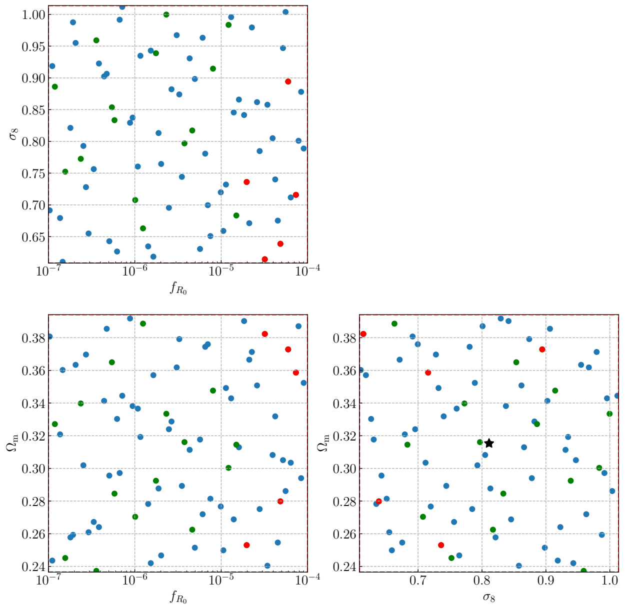
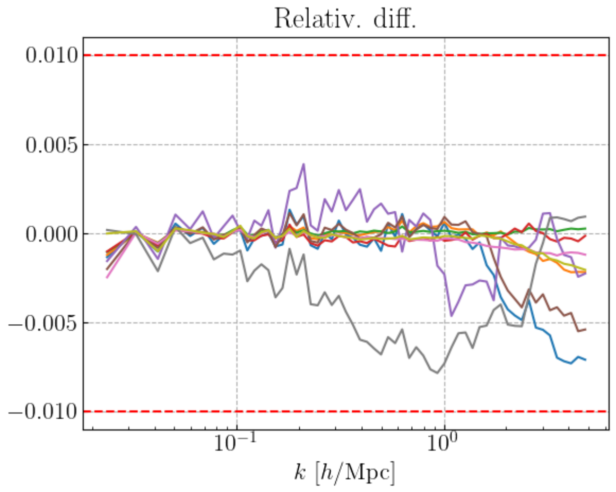


Reduced testing parameter space



- Very small average relative error: $<\epsilon>= 0.1\%$
- All models within 1% error bars !

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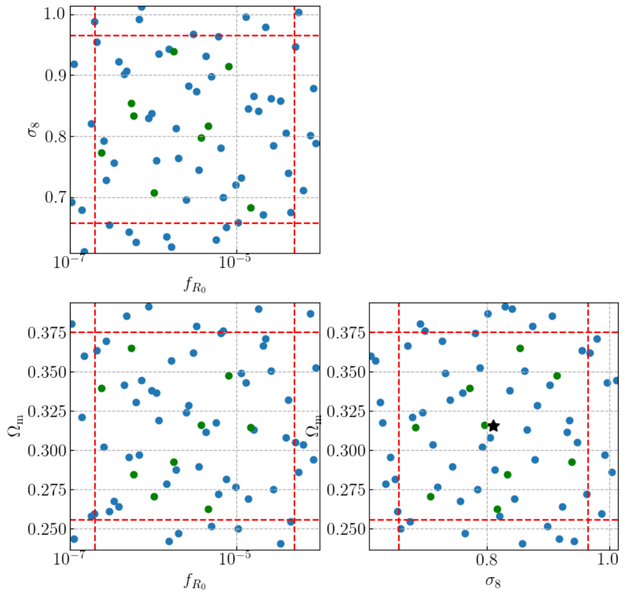

Conclusion

⟶We have studied the effect of the different cosmological parameters on the PS boost due to $f(R)$.
⟶We are in the process of building an emulator for the PS boost.
⟶We are also studying other quantities: we build a FoF halo catalogue at each snapshots.

- halo mass function
- halo power spectrum
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- halo bias

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Long term perspectives:

- Study the effect of n.
- Can we consider and simulate more general MG models ?