UV behaviour of Higgs inflation models

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Introduction : Higgs inflation

Simplest models of inflation require a scalar field $\rightarrow$ **can it be the Higgs ?**
Yes, if we add:

$$\mathcal{L} \supset \xi |H|^2 R \quad \text{with} \quad \xi \sim 10^5 \sqrt{\lambda}$$

[Bezrukov, Shaposhnikov '07]

Problem: this seems to introduce a **cutoff** at a scale $\Lambda \sim M_p/\xi$ (in the metric formalism), to which the Hubble rate is dangerously close during inflation.

[Burgess, Lee, Trott '09], [Barbon, Espinosa '09]

But in fact, the cutoff depends on the background field, and during inflation:

$$\Lambda_{\text{inf}} \sim M_p/\sqrt{\xi}$$

[Bezrukov, Magnin, Shaposhnikov, Sibiryakov '10]

as can be seen by power counting or using some perturbative unitarity bound.

[Ren, Xianyu, He '14]

Metric v.s. Palatini formulation

There are choices in the formulation of GR (c.f. previous talk); for instance:

- impose $\Gamma^{\rho}_{\mu\nu} = \Gamma^{\rho}_{(\mu\nu)}$ as a constraint (second order, or **metric** formalism)
- let it free to follow its -algebraic- equation of motion (**Palatini** formalism)

For pure gravity, the two formalisms are equivalent; but for instance:

$$\frac{{M_p}^2}{2}\Omega^2(H)R(g,\Gamma) \quad \rightarrow \quad \frac{{M_p}^2}{2}\Omega^2(H)R(g) + \frac{3{M_p}^2}{4}\frac{(\partial\Omega^2)^2}{\Omega^2}$$

after integrating out the affine connection. For Higgs inflation we have:

$$\Omega^2 = 1 + 2\xi|H|^2/M_p^2$$

The inflationary predictions depend on this choice:

$$\begin{array}{rlll} \text{metric} & 1 - n_s \simeq 2/N, & r \simeq 12/N^2, & \xi \simeq 10^3 N\sqrt{\lambda} \\ \text{Palatini} & 1 - n_s \simeq 2/N, & r \simeq 12/(\xi N^2), & \xi \simeq 10^6 N^2 \lambda \end{array}$$

but the cutoff scale is the same for both formulations.

Jordan and Einstein frames

The cutoff is visible the scattering amplitude of longitudinal gauge bosons. But it is simpler to work with the would-be Goldstones. We consider:

$$H = \frac{1}{\sqrt{2}} \begin{pmatrix} \phi_1 + i\phi_2 \\ \phi_3 + i\phi_4 \end{pmatrix}$$

such that $\phi_1 = \bar{\phi}_1 + \phi_1'$. Also: $\bar{\Omega}^2 = 1 + \xi\bar{\phi}_1^2/M_p^2$ and $x^2 = \xi\bar{\phi}_1^2/(M_p^2\bar{\Omega}^2)$.

The Higgs-gravity action is (the potential is flat during inflation):

$$\mathcal{S}_{\text{Jordan}} = \int d^4x\sqrt{-g}\left(\frac{1}{2}M_p^2\Omega^2 R - |\partial H|^2 + \cdots\right)$$

after a Weyl transformation $g_{\mu\nu} \to \Omega^{-2} g_{\mu\nu}$ to remove the coupling:

$$\mathcal{S}_{\text{Einstein}} = \int d^4x\sqrt{-g}\left(\frac{1}{2}M_p^2 R - \frac{1}{\Omega^2}|\partial H|^2 - \underline{\frac{3M_p^2}{4}\frac{(\partial\Omega^2)^2}{\Omega^4}}\right)$$

where the underlined term is only present in the metric formalism. Both frames are equivalent, but it is quite simpler to work in the Einstein frame.

Cutoff in the Einstein frame

After expanding around the v.e.v. $\bar{\phi}_1$, computing the interaction vertices, etc:

Metric:

$$\mathcal{M}(\phi_1'\phi_2 \to \phi_1'\phi_2) = \frac{2((1-x^2)+3\xi(1-x^4))}{(1+6\xi x^2)^2}\frac{\xi t}{M_p^2}$$

$$\mathcal{M}(\phi_2\phi_3 \to \phi_2\phi_3) = \frac{2-x^2+6\xi}{1+6\xi x^2}\frac{\xi t}{M_p^2}$$

Palatini:

$$\mathcal{M}(\phi_1'\phi_2 \to \phi_1'\phi_2) = 2(1-x^2)\frac{\xi t}{M_p^2}, \quad \mathcal{M}(\phi_2\phi_3 \to \phi_2\phi_3) = (2-x^2)\frac{\xi t}{M_p^2}$$

[Antoniadis, AG, Tamvakis '22]

If $x \to 0$ (vacuum), we obtain $\Lambda \sim M_p/\xi$ (metric) and $\Lambda \sim M_p/\sqrt{\xi}$ (Palatini).

If $x \to 1$ (inflation), we obtain $\Lambda \sim M_p/\sqrt{\xi}$ in both formalisms.
Note that in this limit both Higgs-Goldstone amplitude vanish.

Cutoff in the Jordan frame

The amplitudes must be the same in the Jordan frame, up to $E^{(E)} = \bar{\Omega} E^{(J)}$.
How to do the computation ? Problem; after expanding $h_{\mu\nu} = \eta_{\mu\nu} + \kappa h_{\mu\nu}$:

$$\mathcal{L} \supset \mathcal{L}_{\text{mix}} = \kappa\xi\bar{\phi}_1\phi_1'(\partial^\mu\partial^\nu h_{\mu\nu} - \Box h)$$

gauge fix $\partial^\mu h_{\mu\nu} = \partial_\nu h$? No, this condition is gauge invariant... Rather we can shift the graviton:

$$h_{\mu\nu} \to h_{\mu\nu} - \sqrt{\xi}\phi_1'\eta_{\mu\nu}$$

and then expansion of $\sqrt{-g}R$ to third order around Minkowski yields $\tilde{h}\phi_1'^2$ and $\phi_1'^3$ interactions. But beware, the "textbook" development of $\sqrt{-g}R|_{h^2}$:

$$\sqrt{-g}R|_{h^2} = \kappa^2/4(\partial^\mu h\partial_\mu h - \partial^\mu h^{\rho\sigma}\partial_\mu h_{\rho\sigma} - 2\partial^\mu h\partial^\nu h_{\mu\nu} + \partial^\mu h^{\rho\sigma}\partial_\rho h_{\mu\sigma})$$

uses integration by parts, that cannot be done with the coupling. Instead use:

$$\begin{aligned}\sqrt{-g}R|_{h^2} = \kappa^2/4(&2h\partial^\mu\partial^\nu h_{\mu\nu} - 2h\Box h - 8h^{\mu\nu}\partial_\mu\partial^\rho h_{\nu\rho} + 4h^{\mu\nu}\Box h_{\mu\nu}\\ &+4h^{\mu\nu}\partial_\mu\partial_\nu h - 4\partial^\mu h_{\mu\rho}\partial_\nu h^{\nu\rho} + 4\partial^\mu h\partial^\nu h_{\mu\nu}\\ &+3\partial^\rho h^{\mu\nu}\partial_\rho h_{\mu\nu} - \partial^\mu h\partial_\mu h - 2\partial^\mu h^{\rho\sigma}\partial_\rho h_{\mu\sigma})\end{aligned}$$

From there, this is straightforward and we obtain the expected amplitudes.

Adding a R^2 term to the action

What if we add a term in R^2 ? Standard trick: embed it in an auxiliary scalar:

$$\frac{1}{2}M_p^2\Omega^2 R + \frac{\alpha}{4}R^2 + \cdots \quad \rightarrow \quad \frac{1}{2}M_p^2\tilde{\Omega}^2 R - \frac{\alpha}{4}\chi^2 + \cdots$$

here:

$$\tilde{\Omega}^2 = 1 + 2\xi|H|^2/M_p^2 + \alpha\chi^2/M_p^2$$

then go to the Einstein frame as before. Differences in metric v.s. Palatini:

- metric: χ is a propagating d.o.f., can be the inflaton (Starobinsky)
- Palatini: χ is an auxiliary degree of freedom, can be integrated out. This provides a mechanism to further flatten the potential of any model

[Enckell et. al.],[Antoniadis et. al.]

Model with a R^2 term in the Palatini formalism

Let us restore the potential $V(H) = \lambda/4(h^2 - v^2)^2$; in the Einstein frame, after integrating out χ, and in the unitary gauge where h is the Higgs:

$$\mathcal{S} = \int d^4x\sqrt{-g}\left(\frac{1}{2}M_p^2 R - \frac{\xi h^2 M_p^2}{\xi^2 h^4 + 4\alpha V}\frac{1}{2}|\partial H|^2 - \frac{V}{\xi^2 h^4 + 4\alpha V} + \cdots\right) \quad (1)$$

the canonical scalar is (in the inflationary background $\xi h^2 \gg M_p^2$):

$$\zeta \simeq M_p\sqrt{\frac{\xi}{\xi^2 + \alpha\lambda}}\sinh^{-1}\left(\frac{h\sqrt{\xi}}{M_p}\sqrt{\frac{\xi^2 + \alpha\lambda}{\xi^2 - \alpha\lambda}}\right) \quad (2)$$

the effective potential of this canonical scalar reads:

$$U(\zeta) \simeq \frac{\lambda}{4}\frac{M_p^2}{\xi^2 + \alpha\lambda}\frac{\sinh^2((\xi + \alpha\lambda/\xi)^{1/2}\zeta/M_p)}{2\xi^2/(\xi^2 - \alpha\lambda) + \sinh^2((\xi + \alpha\lambda/\xi)^{1/2}\zeta/M_p)} \quad (3)$$

we see that both ξ and α play a role in flattening the potential at large ζ. So including αR^2 term in the action can help by lowering the value of ξ.

Does it affect the cutoff ?

We can compute the amplitudes as before in the presence of the R^2 term. Here we give them in the limit $x \to 1$.

Metric: the αR^2 term unlocks a new (massive) scalar degree of freedom:

$$\mathcal{M}^{(\alpha)}(\phi_1'\phi_2 \to \phi_1'\phi_2) = \frac{2\alpha(1-6\xi)}{1+6\alpha t/M_p^2}\frac{t^2}{M_p^4} \quad \to \quad \Lambda \sim \frac{M_p}{\sqrt{\xi}}$$

Palatini: the αR^2 term adds a new $(\partial \cdot)^4$ interaction between existing d.o.f:

$$\mathcal{M}^{(\alpha)}(\phi_1'\phi_2 \to \phi_1'\phi_2) = \frac{2\alpha t^2}{M_p^4} \quad \to \quad \Lambda \sim \frac{M_p}{\alpha^{1/4}}$$

so if $\alpha \gg \xi^2$ in the Palatini formalism, the cutoff is lower than without αR^2.

Conclusion

Higgs inflation: new coupling $\xi|H|^2R$ to the SM, with $\xi \sim 10^4 - 10^9$ (as long as $\xi \ll 10^{17}$, it does not contribute to the physics in collider experiments).

Cutoff $\Lambda \sim M_p/\sqrt{\xi}$ in the inflationary background, in metric and Palatini.

Also, we looked at the influence of a R^2 term, which can be used to flatten the potential in the Palatini formalism. New cutoff at $\Lambda \sim M_p/\alpha^{1/4}$ in Palatini.

Thank you for your attention. Any questions ?