UNIVERSITY OF LIVERPOOL

PLANCK 2022, 2nd June 2022

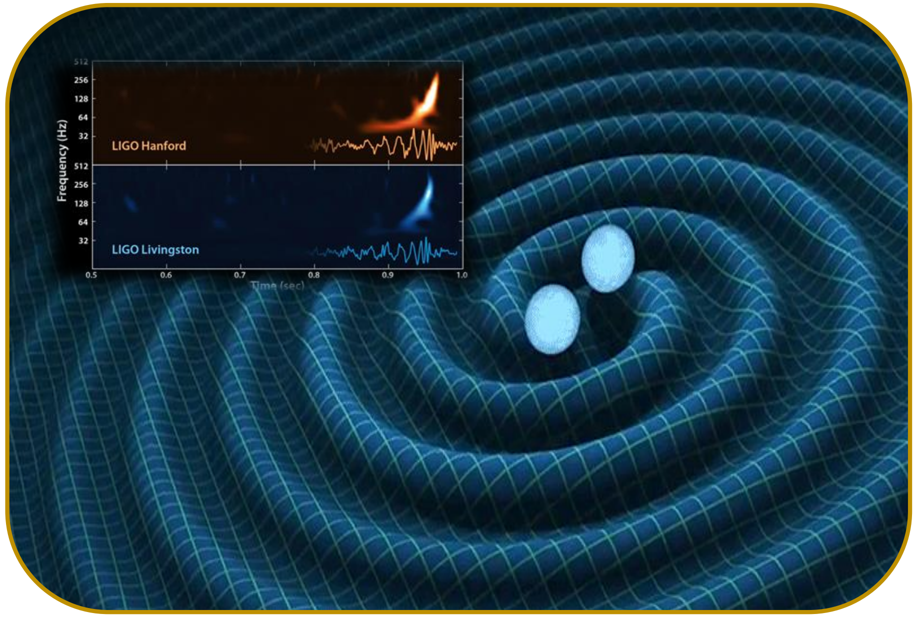
Gravity at the Tip of the Throat

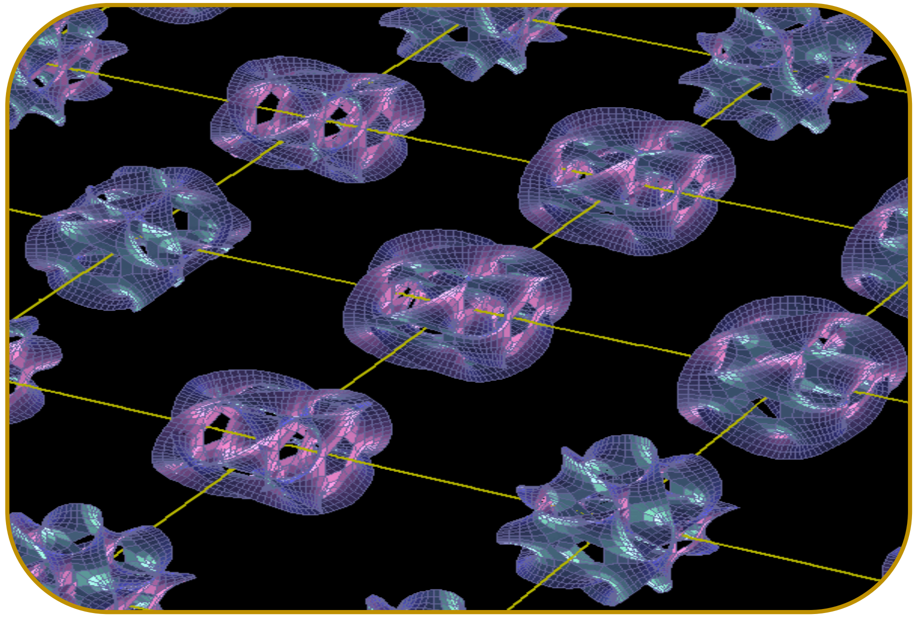
Bruno Valeixo Bento

Based on [2204.02086] in collaboration with:

Dibya Chakraborty, Susha Parameswaran, Ivonne Zavala

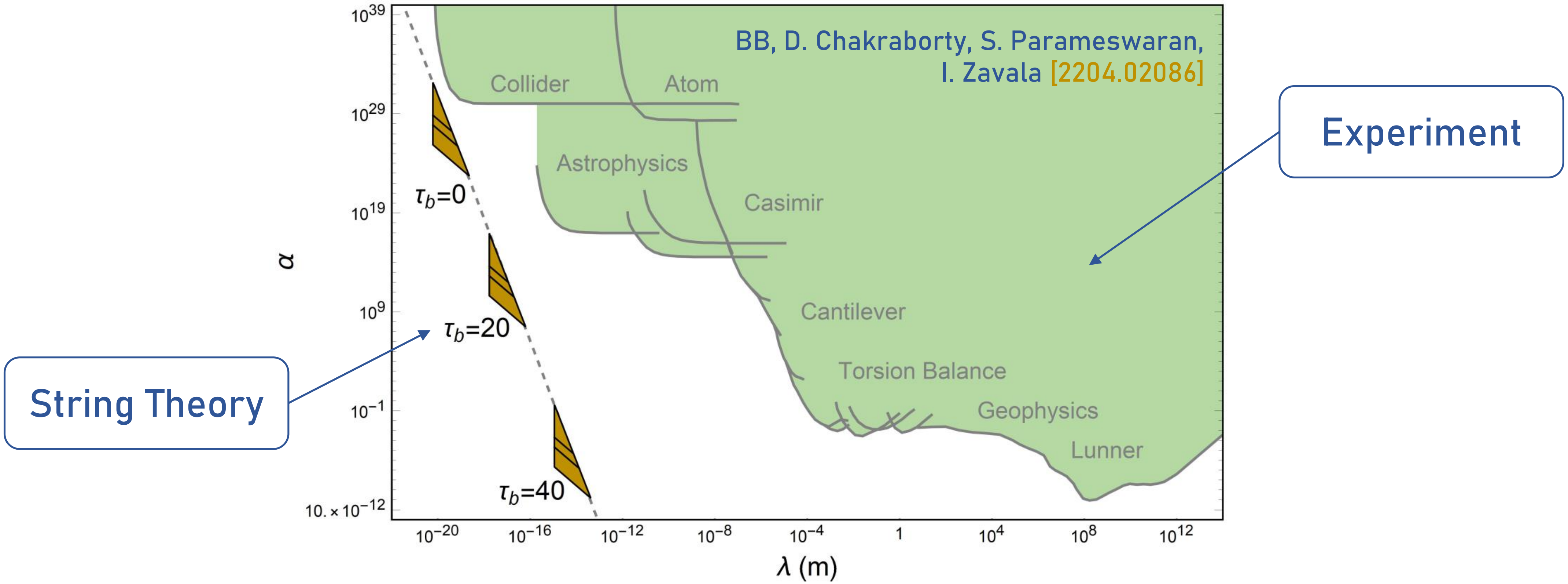
The Question





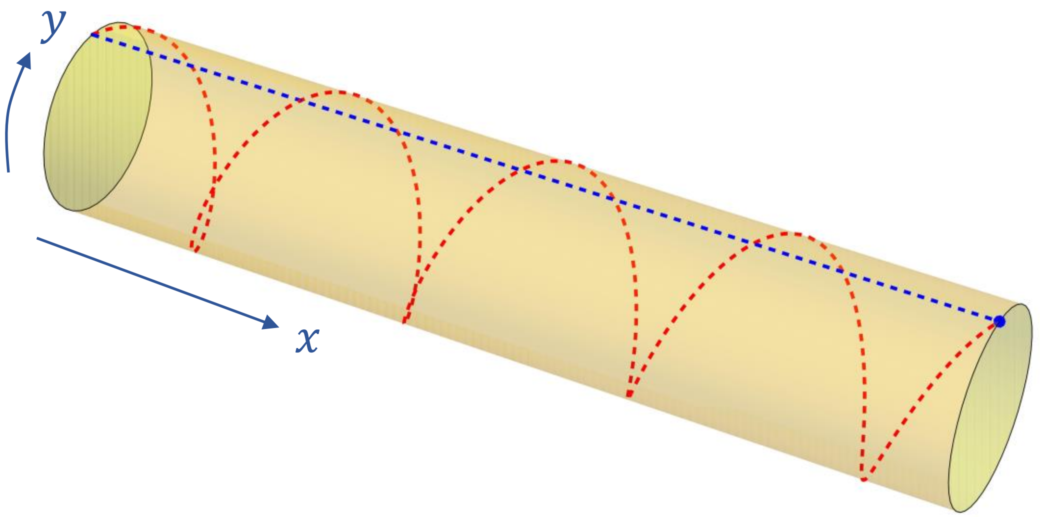
Can GWs tell us about (warped) extra dimensions?

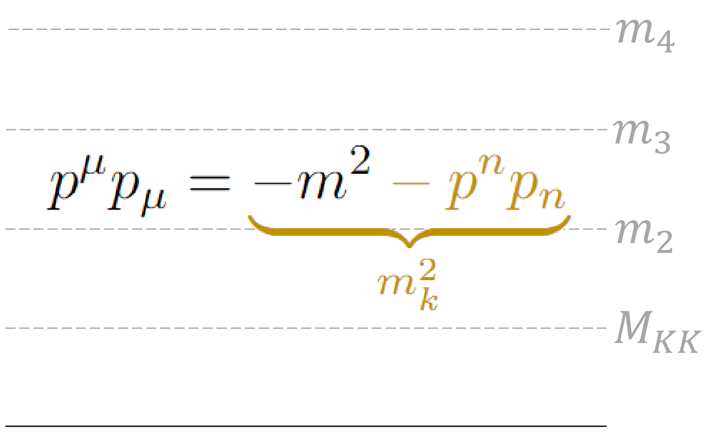
Spoiler



BB, D. Chakraborty, S. Parameswaran,
I. Zavala [2204.02086]
Collider
Atom
Astrophysics
Casimir
Cantilever
Torsion Balance
Geophysics
Lunner
τ_b=0
τ_b=20
τ_b=40
α
λ (m)
10^{39}
10^{29}
10^{19}
10^{9}
10^{-1}
$10. \times 10^{-12}$
10^{-20}
10^{-16}
10^{-12}
10^{-8}
10^{-4}
1
10^{4}
10^{8}
10^{12}
Experiment
String Theory

Kaluza-Klein modes

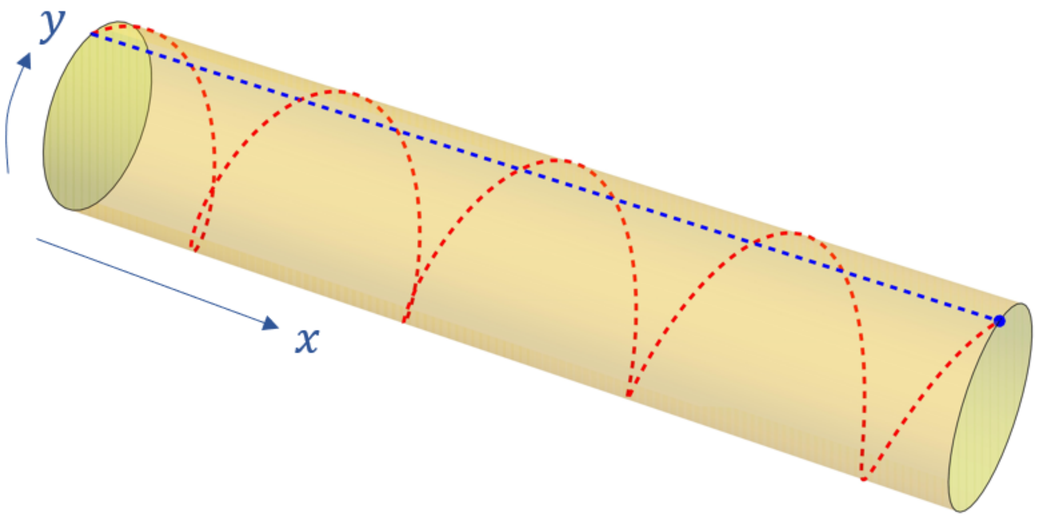




$$h_*(x, y) = \sum_k h_*^k(x)\phi_*^k(y)$$

The tower is the signature of extra dimensions.

Kaluza-Klein modes



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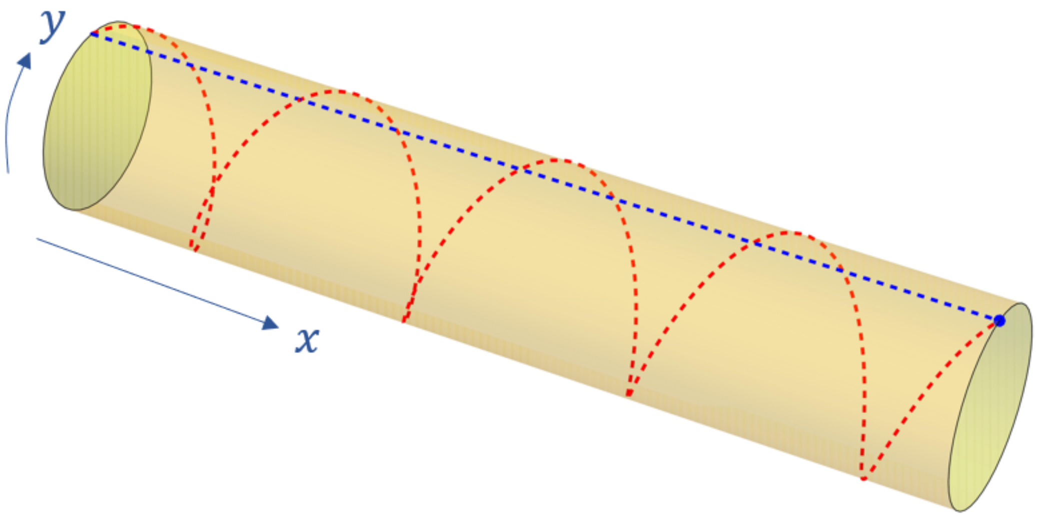


$$h_*(x, y) = \sum_k h_*^k(x)\phi_*^k(y)$$

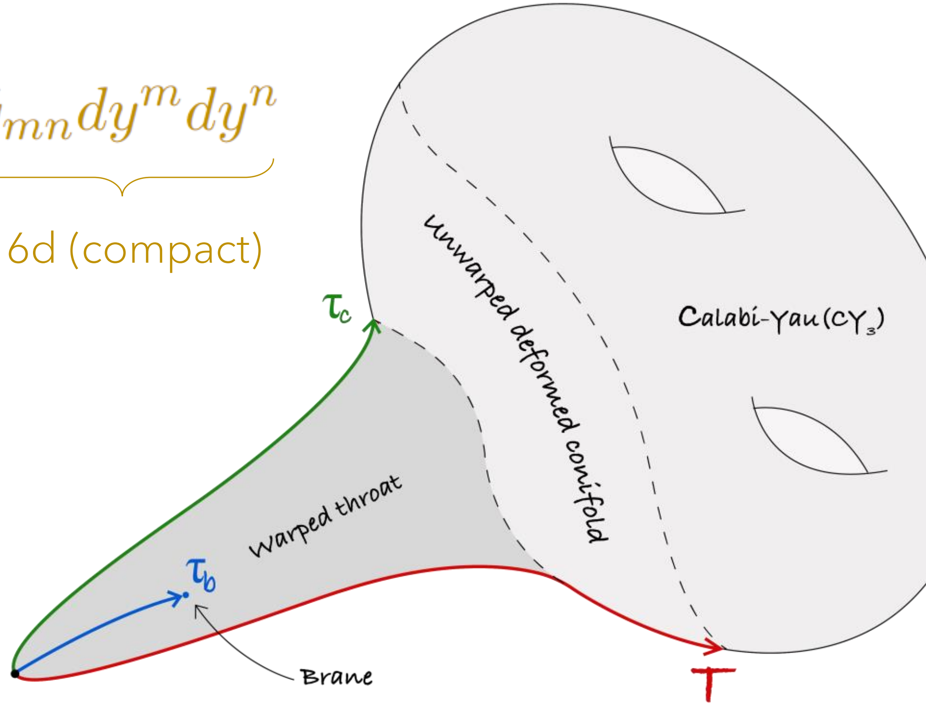
The tower is the signature of extra dimensions.

Warped compactification

String Theory: 10d → 4d

$$ds^2 = H(y)^{-1/2} \underbrace{g_{\mu\nu} dx^\mu dx^\nu}_{\text{4d}} + H(y)^{1/2} \underbrace{\tilde{g}_{mn} dy^m dy^n}_{\text{6d (compact)}}$$

Warp factor



The warping affects all energy scales

$$H(y)^{1/2} p_\mu p^\mu = -m^2 \implies m^w = H(y)^{-1/4} m$$

What does it do to Gravitational Waves?

KK Gravitational Waves

$$S_{\rm IIB} = \frac{1}{2\kappa^2}\int d^{10}x\sqrt{-G}\,(R + \mathcal{L}_{\rm IIB}) + S_{\rm brane}$$

10d gravity (e.g. Type IIB)

$$\Box_{10} h_{MN} + 2\bar{R}_{MPNQ} h^{PQ} = T^{(1)}_{MN}$$

10d GWs

$$\Box_4 h^k_{\mu\nu} - m_k^2 h^k_{\mu\nu} = \mathrm{T}_{\mu\nu}$$

4d GWs (tower)

KK Gravitational Waves

(e.g. Type IIB)

$$h_{\mu\nu}(x,y) = \sum_k h^k_{\mu\nu}(x)\,\phi_k(y)$$

wavefunctions

$$\Box_4 h^k_{\mu\nu} - m_k^2 h^k_{\mu\nu} = \mathrm{T}_{\mu\nu}$$

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KK Gravitational Waves

$S_{\text{IIB}} = \frac{1}{2\kappa^2} \int$ (e.g. Type IIB)

$\Box_{10} h_{M}$ GWs

$$h_{\mu\nu}(x, y) = \sum_k h^k_{\mu\nu}(x) \phi_k(y)$$

wavefunctions

$$\Box_4 h^k_{\mu\nu} - m_k^2 h^k_{\mu\nu} = \mathrm{T}_{\mu\nu}$$

$$\Box_4 h^k_{\mu\nu} - m_k^2 h^k_{\mu\nu} = \mathrm{T}_{\mu\nu}$$

4d GWs (tower)

$$h_{MN} \to h_{\mu\nu},\ h_{\mu n},\ h_{mn}$$

KK Gravitational Waves

$$\Box_4 h^k_{\mu\nu} - m_k^2 h^k_{\mu\nu} = \mathrm{T}_{\mu\nu}$$

→ Ladder of GWs $(\omega_k \sim m_k)$

OBSERVATION

$\omega_k \sim m_k$ cannot* be too high (10^4 Hz $\sim 10^{-23}$ eV)

QUESTION

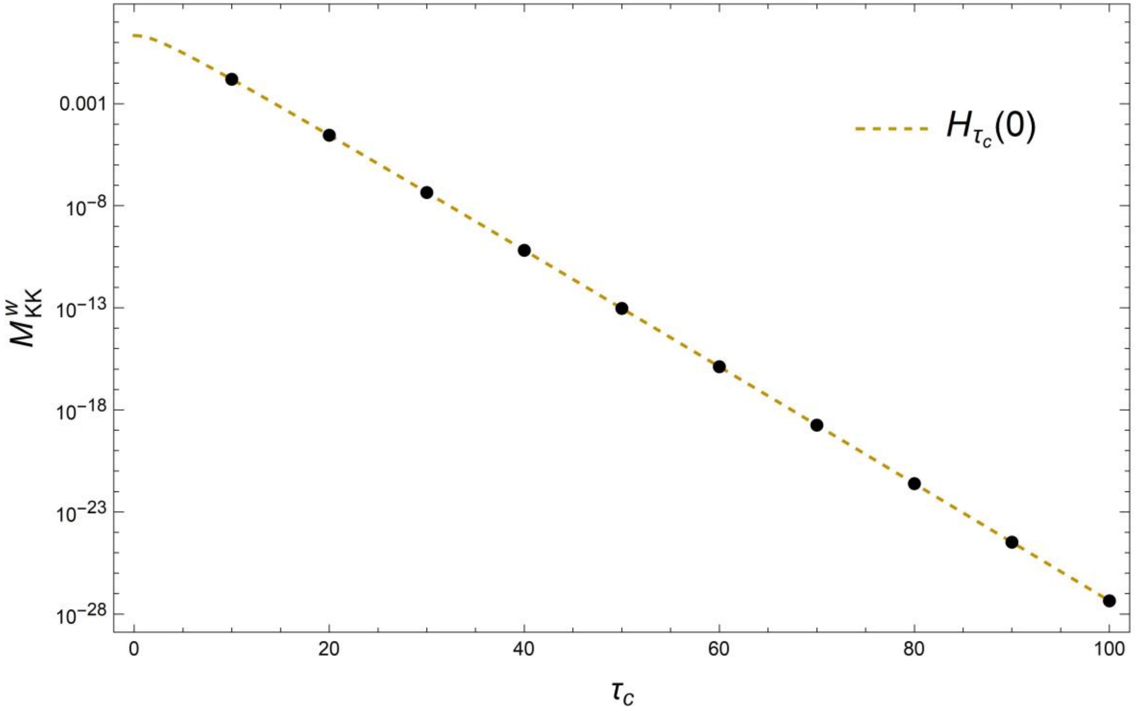
Wouldn't we have seen $h^k_{\mu\nu}$ already? 🤨

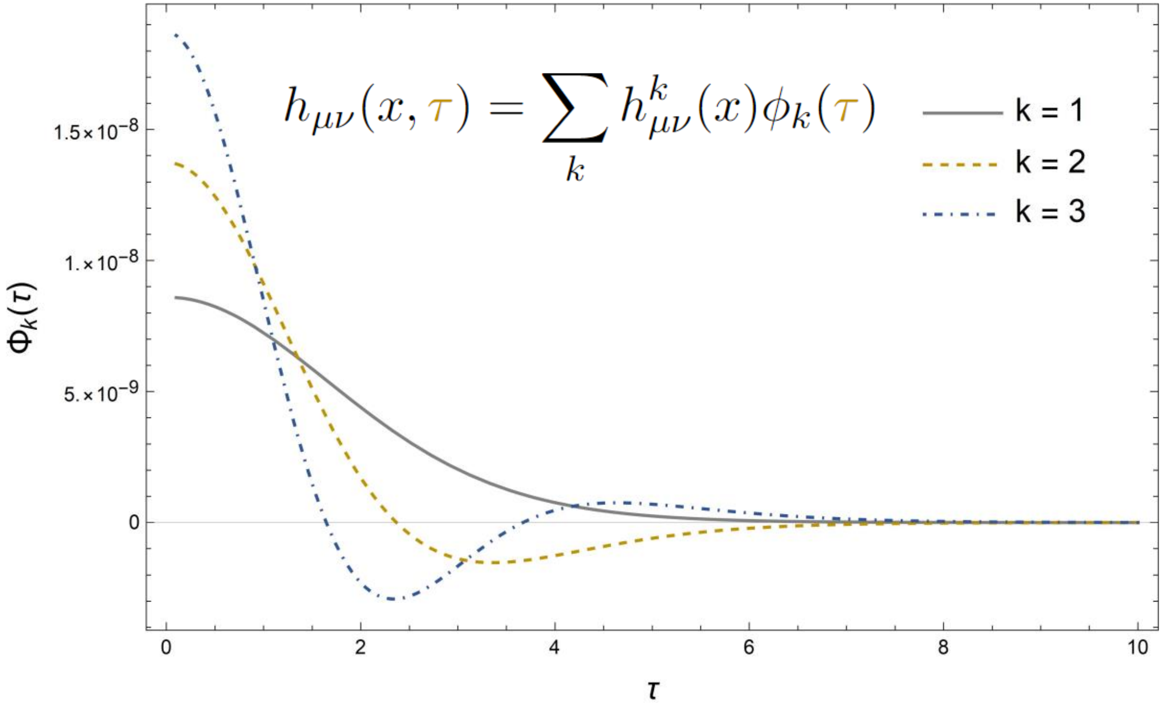
H. Firouzjahi and S. H. H. Tye [hep-th/0512076]

G. Shiu, B. Underwood, K. M. Zurek, and D. G. E. Walker [0705.4097]

KK Gravitons

Fully warped limit (τ_c = T)





Masses ⟷ Energy

Wavefunctions ⟷ Couplings

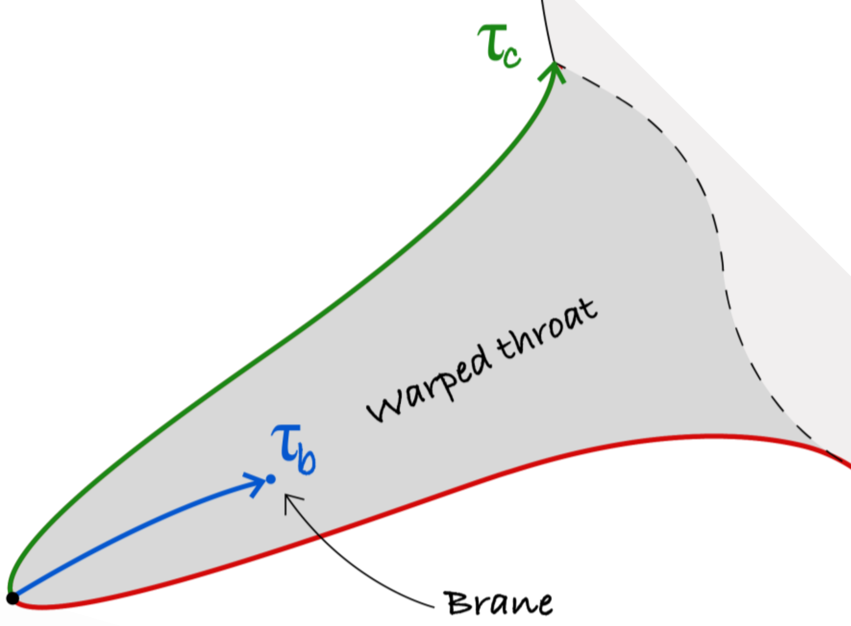
Braneworld

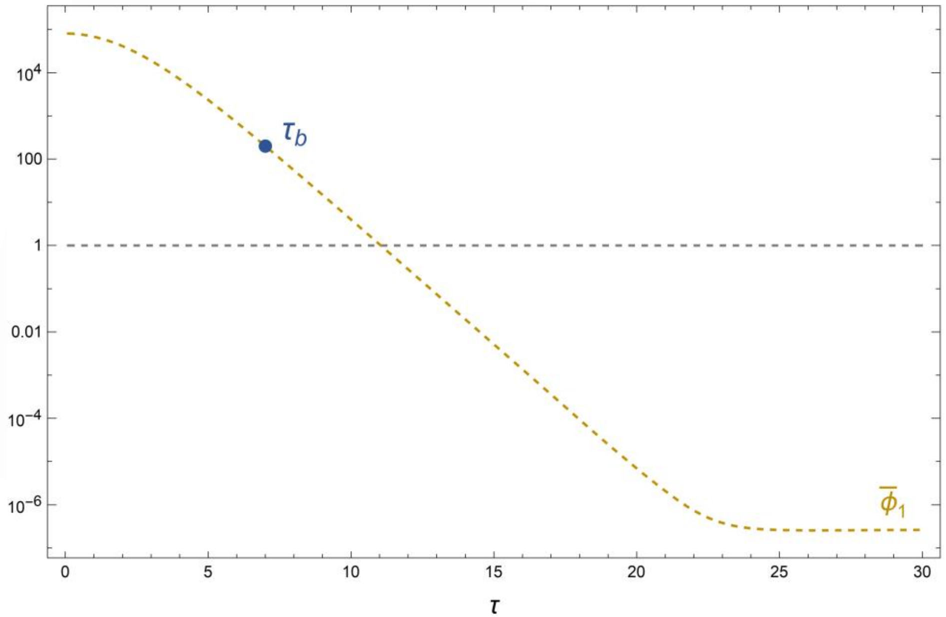
Lets imagine we live on a brane located somewhere in the throat

$$\mathcal{L}_{\text{brane}} \sim \boxed{\frac{\overline{\phi}_k(\tau_b)}{M_P}} h^k_{\mu\nu} T^{\mu\nu}$$

Couplings

Gravitational interactions on the brane include the whole tower



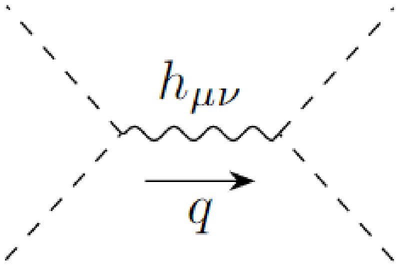


P. Callin and F. Ravndal [hep-ph/0403302]

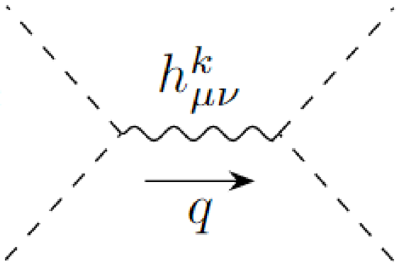
Do we see the extra dimensions?

Gravitational interactions on the brane include the whole tower

$$V(q) = \lim_{q^0 \to 0}$$



$$= \lim_{q^0 \to 0} \sum_k |\Phi_k(y)|^2$$



$$h_{\mu\nu}(x,y) = \sum_k h^k_{\mu\nu}(x)\phi_k(y)$$

To compare with experiments we express it as

$$V(r) = G\frac{m_1 m_2}{r}\left(1 + \alpha e^{-r/\lambda}\right)$$

Strength (α) Range (λ)

Do we see the extra dimensions?

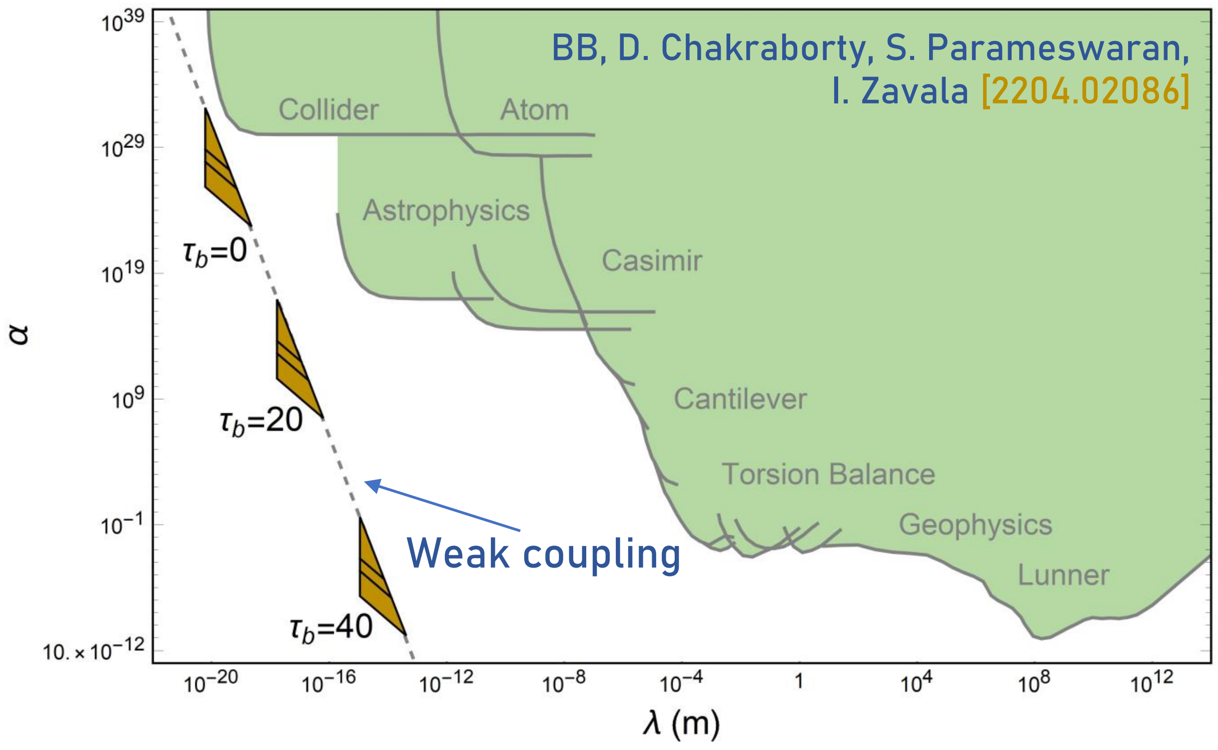
Compare predictions with experimental constraints

$$|\bar{\phi}_k|^2 \sim \alpha \approx \frac{(2\pi)^2}{(g_s M)^3} \frac{2|\phi_1(\tau_b)|^2}{I(\tau_b)^{1/2}} \frac{g_s^2}{\mathcal{H}^2}$$

$$m_k \sim \lambda^{-1} \approx \frac{\mathcal{H}}{2^{1/6}} \frac{2\pi}{\sqrt{g_s M}} I(\tau_b)^{1/4}$$

Triangle regions:

$g_s M > 1$	Supergravity (α')
$g_s < 1$	String loop expansion
$M < M_{max}$	D3 Tadpole



Observational constraints from J. Murata, S. Tanaka [1408.3588]
J. A. R. Cembranos, A. L. Maroto, and H. Villarrubia-Rojo [1706.07818]

Conclusions

- (compact) Extra dimensions $\longrightarrow$ Tower of gravitons in 4d
- Phenomenological parameters $(\alpha, \lambda) \leftrightarrow$ String theory parameters
- No conflict with other experiments
- Wavefunction profiles are important! What's the effect on GWs?

Conclusions

- (compact) Extra dimensions $\longrightarrow$ Tower of gravitons in 4d
- Phenomenological parameters $(\alpha, \lambda) \leftrightarrow$ String theory parameters
- No conflict with other experiments
- Wavefunction profiles are important! What's the effect on GWs?

Thank You

BACKUP SLIDES

KK Gravitational Waves

Higher dimensional GWs

$$\Box_{10} h_{MN} - 2\bar{R}^{S}{}_{MNP} g^{PQ} h_{QS} = T^{(1)}_{MN}$$

give rise 4d GWs

$$\Box_4 h_{\mu\nu} + \underbrace{\left(\frac{\Delta_6}{c^{1/2} H} - \frac{2}{3}\Lambda_4\right)}_{\text{6d (compact)}} h_{\mu\nu} = T^{(1)}_{\mu\nu}$$

$$h_{\mu\nu}(x, y) = \sum_k h^k_{\mu\nu}(x)\phi^k(y)$$

KK Gravitational Waves

Higher dimensional GWs

$$\Box_{10} h_{MN} - 2$$

give rise 4d GWs

$$\Box_4 h_{\mu\nu} + \Big(\frac{}{c}$$

Define the functions as

$$\Big(\frac{\Delta_6}{c^{1/2}H} - \frac{2}{3}\Lambda_4\Big)\phi^k(y) = -m_k^2\phi^k(y)$$

$\Rightarrow$ Infinitely many orthogonal modes.

6d (compact)

$$h_{\mu\nu}(x,y) = \sum_k h^k_{\mu\nu}(x)\phi^k(y)$$

KK Gravitational Waves

The decomposition is defined by

$$\left(\frac{\Delta_6}{c^{1/2}H} - \frac{2}{3}\Lambda_4\right)\phi^k(y) = -m_k^2\phi^k(y)$$

i.e. the functions $\phi^k(y)$ are eigenfunctions with eigenvalue $-m_k^2$ and orthogonal

$$\int d^6y\sqrt{g_6}\; H(y)\phi_k(y)\phi_{k'}(y) = \mathcal{N}^2_{(k)}\delta_{kk'}$$

Infinitely many solutions (discrete) ⇒ Tower of states

KK Gravitational Waves

$$\Box_4 h_{\mu\nu} + \left(\frac{\Delta_6}{c^{1/2}H} - \frac{2}{3}\Lambda_4\right) h_{\mu\nu} = 2\tilde{T}^{(1)}_{\mu\nu}$$

$$h_{\mu\nu}(x,y) = \sum_{k'} h^{k'}_{\mu\nu}(x)\phi_{k'}(y)$$

$$\sum_{k'} \left(\Box_4 h^{k'}_{\mu\nu} - m^2_{k'} h^{k'}_{\mu\nu}\right)\phi_{k'}(y) = 2\tilde{T}^{(1)}_{\mu\nu}(x,y)$$

$$\int d^6y \sqrt{g_6}\; H(y)\phi_k(y)$$

$$\Box_4 h^k_{\mu\nu} - m^2_k h^k_{\mu\nu} = \mathrm{T}_{\mu\nu}$$

$$\mathrm{T}_{\mu\nu} \equiv 2\int d^6y \sqrt{g_6}\; H(y)\phi_k(y)\tilde{T}^{(1)}_{\mu\nu}(x,y)$$

KK Gravitational Waves

$$\Box_4 h^k_{\mu\nu} - m_k^2 h^k_{\mu\nu} = \mathrm{T}_{\mu\nu}$$

$$\mathrm{T}_{\mu\nu} \equiv 2 \int d^6y \sqrt{g_6}\, H(y) \phi_k(y) \tilde{T}^{(1)}_{\mu\nu}(x, y)$$

The warped extra dimensions contribute with

→ Ladder of GWs ($k > 0$) (high frequencies $\omega_k \sim m_k$)

→ More sources

→ Bulk (e.g. h_{mn})

→ Localised (e.g. brane fields)

→ Warped energy-momentum

Warped deformed conifold

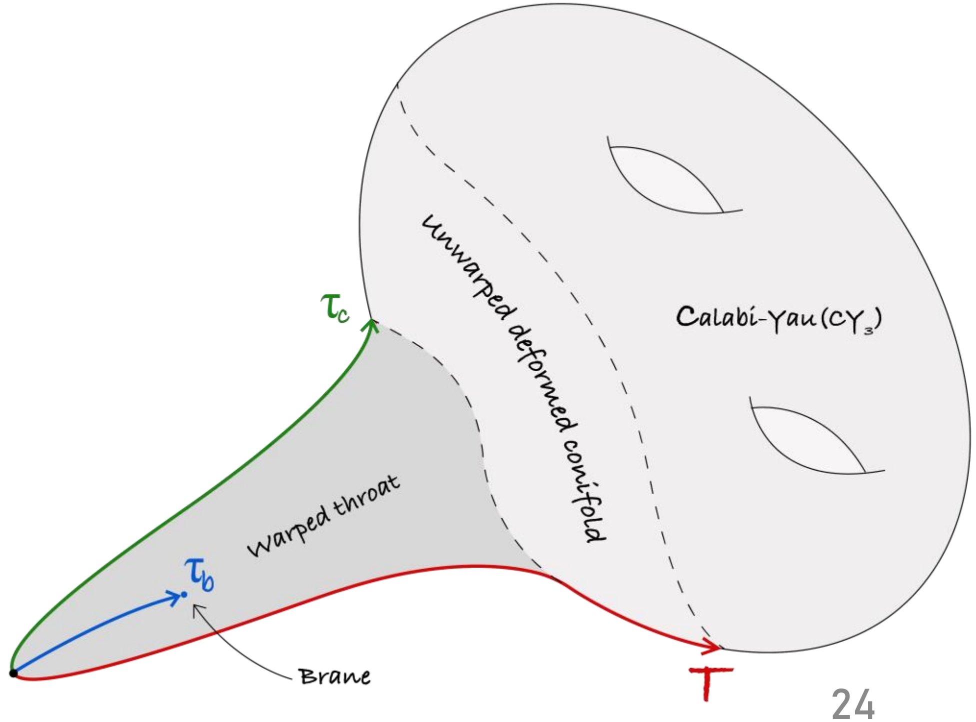
Klebanov-Strassler (KS) solution

$$ds^2 = H(y)^{-1/2} g_{\mu\nu} dx^\mu dx^\nu + H(y)^{1/2} c^{1/2} g_{mn} dy^m dy^n$$

Compact space = CY3 + warped throat

$$g_{mn} = \begin{cases} g_{mn}^{(\mathrm{KS})} & H(y) \gg 1 \\ g_{mn}^{(\mathrm{CY})} & H(y) \sim 1 \end{cases}$$

$$g_{mn}^{(KS)} = \frac{\epsilon^{4/3}}{2} \mathcal{K}(\tau) \begin{pmatrix} \frac{1}{3\mathcal{K}^3(\tau)} \mathbb{1}_2 & 0 & 0 \\ 0 & \sinh^2(\tau/2) \mathbb{1}_2 & 0 \\ 0 & 0 & \cosh^2(\tau/2) \mathbb{1}_2 \end{pmatrix}$$



Warped deformed conifold

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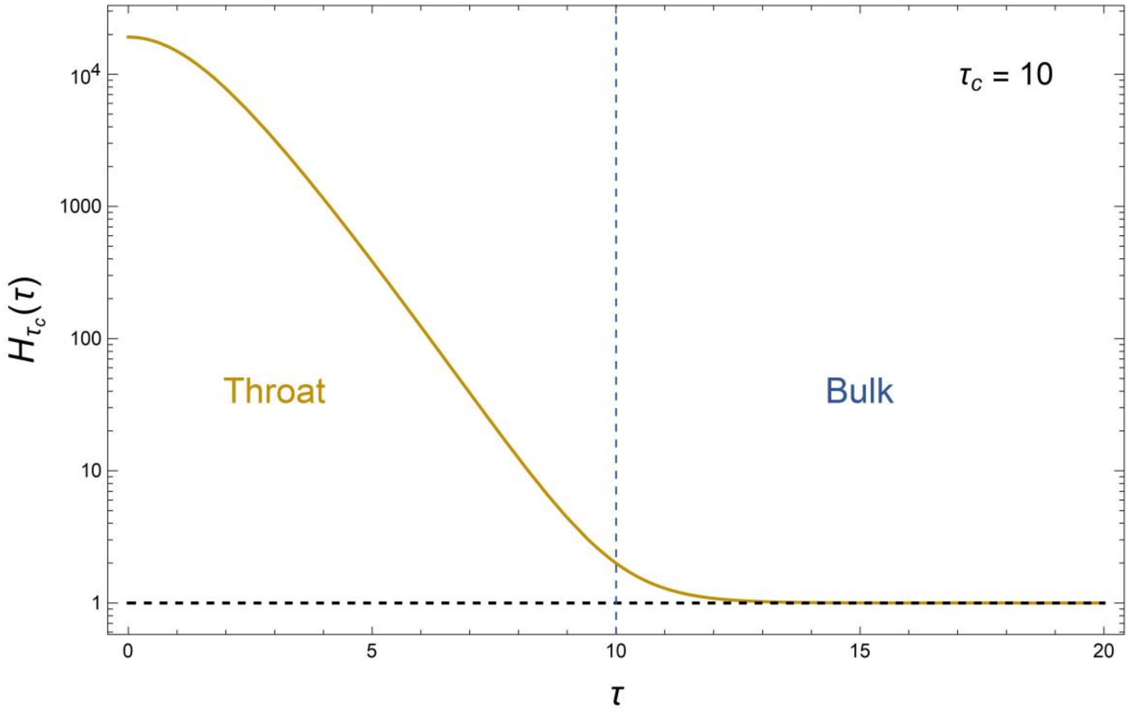
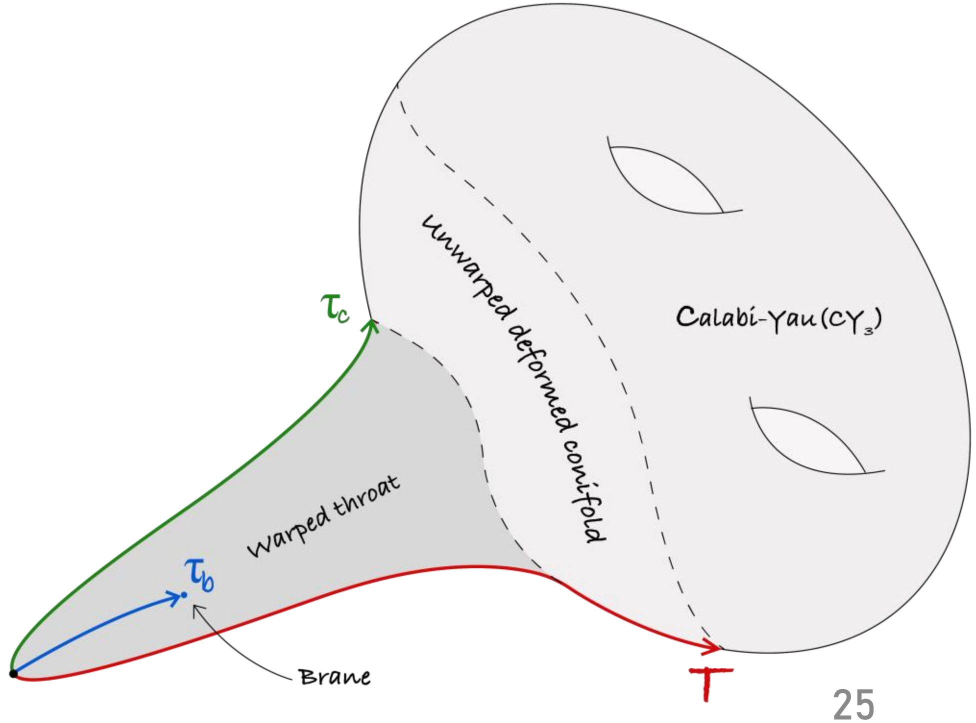
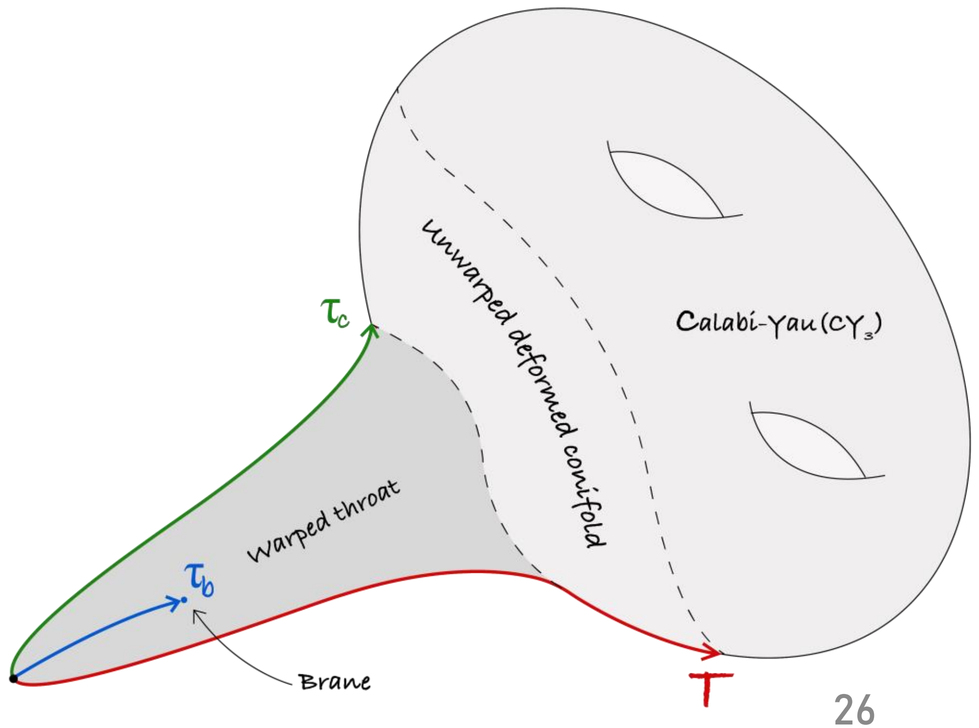




Our paper [2204.02086]

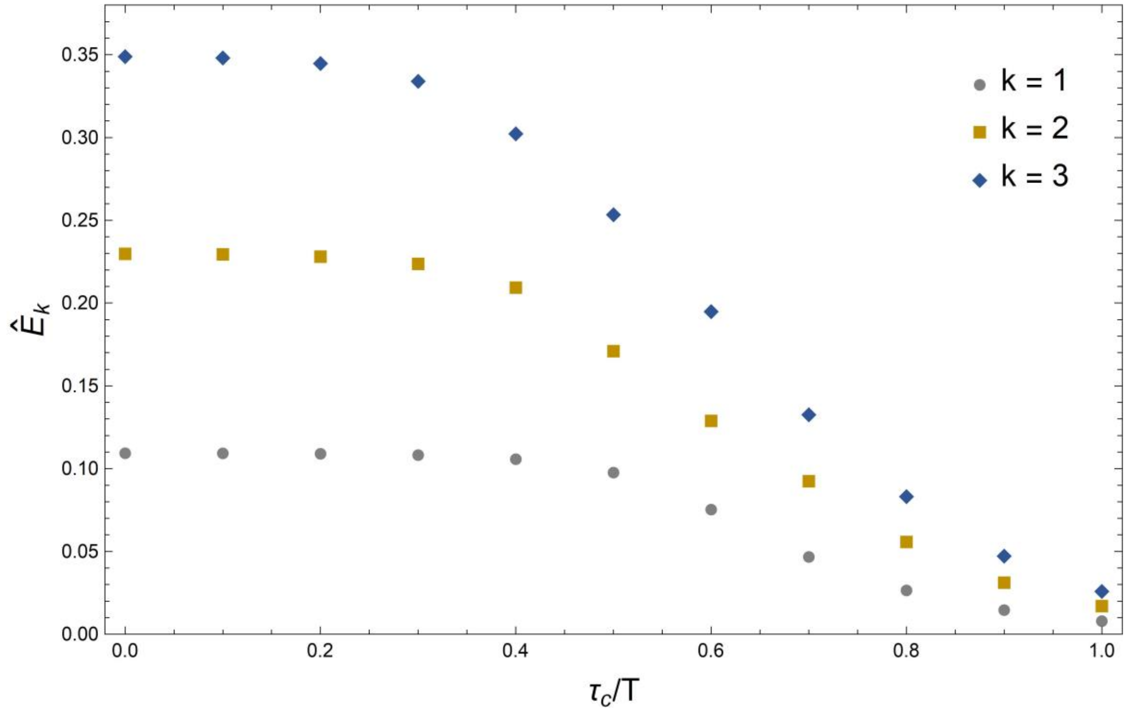
1. 4d Minkowski
2. Trivial angular solutions ($\phi_k = \phi_k(\tau)$)
3. Unwarped conifold region ($\tau_c < \tau < \mathrm{T}$)
4. Vanishing boundary conditions on CY3 ($\tau = \mathrm{T}$)
5. (3+1)-brane somewhere in the conifold ($\tau_b < \mathrm{T}$)



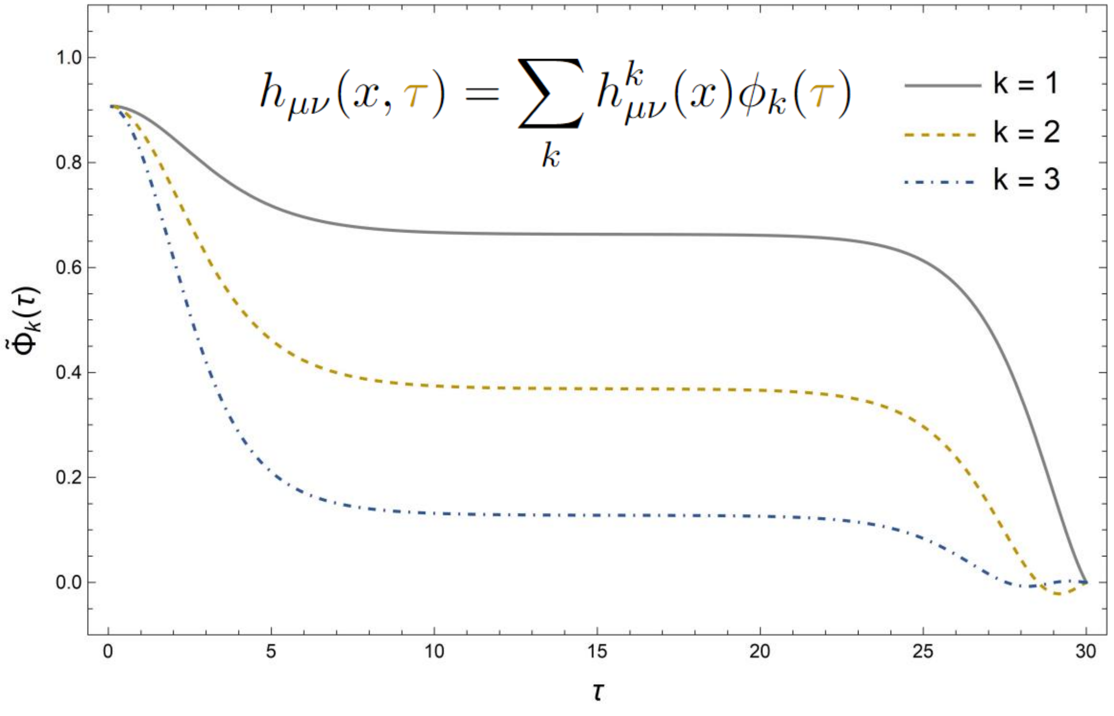
Throat vs Bulk

There is a competition between warping and bulk size.

cf. H. Firouzjahi and S. H. H. Tye [hep-th/0512076]



$\tau_c/T = 0.5$



Throat vs Bulk

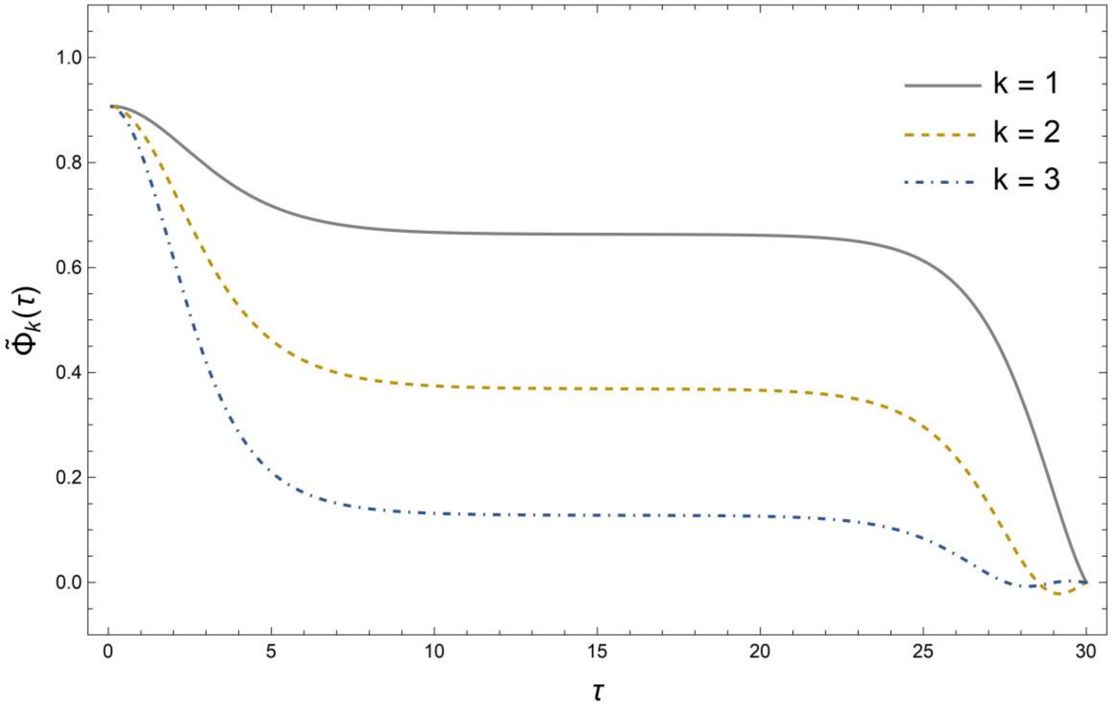
There is a competition between warping and bulk size.

$$r \sim e^{\tau/3} \implies r_c \gtrsim \sqrt{R}$$

Actually in terms of the volume

$$\frac{V_{\text{throat}}}{V_{\text{bulk}}} \gtrsim \frac{g_s M K}{R^5}$$

$\tau_c/T = 0.5$



Gravitational Waves

Each mode in the tower has its own wave equation

$$\Box_4 h^k_{\mu\nu} - m_k^2 h^k_{\mu\nu} = \mathrm{T}_{\mu\nu}$$

Tower of frequencies $\omega_k \sim m_k \quad (\Delta\omega = M^w_{KK})$

	f_{GW} (Hz)	M^w_{KK} (eV)	τ_b	τ_c	$z^{1/3}$	r_{UV}	$\mathcal{V}_{th}$	MK
LISA	10^0	10^{-27}	195	239	1.51×10^{-47}	1.70	290	3259
LIGO-Virgo/ET	10^4	10^{-23}	168	211	1.51×10^{-43}	1.64	240	2906
UHF	10^9	10^{-18}	133	176	1.51×10^{-38}	1.57	183	2464

cf. D. Andriot and G. Lucena Gómez [1704.07392]
cf. A. K. Mishra, A. Ghosh, S. Chakraborty [2106.05558]