How Special Are Black Holes?

Correspondence with Saturons in Generic Theories

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Based on: **Phys. Rev. D 105, 056013 (2022)**

How Special Are Black Holes?

- Their entropy satisfies the *area law*: [1]

$$S \sim \frac{Area}{G_N} \sim \frac{Area}{M_p^{-2}}$$

- Decay rate is thermal and they have *temperature*

$$T \sim \frac{1}{R}$$

- They exhibit a (semiclassical) *information horizon.*

- Time-scale required for beginning of the *information retrieval* is [2]

$$t_{min} = SR \sim \frac{R^3}{M_p^{-2}} \sim \frac{Volume}{M_p^{-2}}$$

[1] S. W. Hawking, Particle creation by black holes, Commun. Math. Phys. 43, 199 (1975).
[2] D. N. Page, Information in Black Hole Radiation, Phys. Rev. Lett. 71, 3743 (1993).

How Special Are Black Holes?

- Does the *area-law* entropy bound extend beyond gravity?
  - What is its underlying meaning?

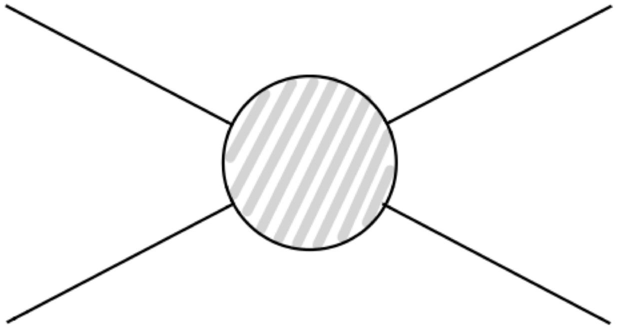
$$S \leqslant \frac{Area}{G_{Gold}}$$

The entropy bound is imposed by unitarity

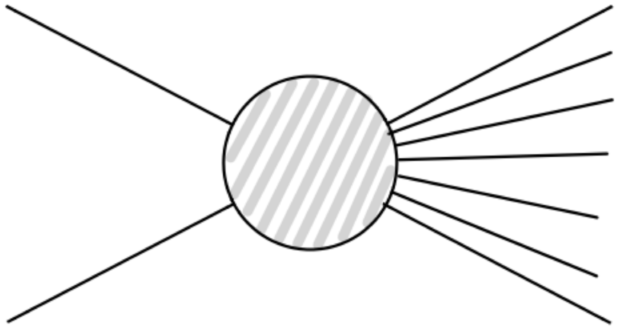
- G. Dvali, Entropy Bound and Unitarity of Scattering Amplitudes, JHEP03(2021) 126, arXiv:2003.05546.
- G. Dvali, Bounds on Quantum Information Storage and Retrieval, PTRS A, arXiv:2107.10616

Unitarity

Cross-section:
$\boldsymbol{\sigma_{2\to 2} \sim \alpha}$



Cross–section:
$\boldsymbol{\sigma_{2\to N}\, n_{st} \sim \alpha^N N!\, n_{st}}$



The *bound* reads

$$S \leqslant \frac{1}{\alpha} \sim N$$

For self-sustained objects of size R, $\alpha = \alpha(q)$ is as an effective running coupling evaluated at the scale $q \sim 1/R$, and $\alpha N \sim O(1)$

Entropy Bound Imposed by Unitarity

- E.g. consider bound states of Goldstone bosons of de Broglie wavelength R

$$\alpha = \frac{q^2}{f^2} = \frac{1}{(fR)^2}$$

f is the canonically normalized *Goldstone decay constant*.

- Thus

$$S \leqslant \frac{1}{\alpha} = \frac{Area}{G_{Gold}}$$

- $Area \sim R^2$
- $G_{Gold} \equiv f^2$ is the Goldstone coupling

Saturons

- We refer to the objects saturating the entropy bounds as ***Saturons***.
- Different saturons are discussed in [1-3]. These include:
  - ***Black Holes***,
  - ***Vacuum Bubbles.***
  - *Classical lumps, Instantons, Monopoles…*
  - *Color Glass Condensates*
- *Area law*: $$S_{\text{Max}} = \frac{1}{\alpha} = \frac{Area}{f^{-2}}$$
- *Temperature:* $$T \sim \frac{1}{R},$$
- *Information retrieval* $$t_{min} = \frac{Volume}{G_{Gold}} = \frac{R}{\alpha} = S_{\max}$$
- *Information horizon.*

1. G. Dvali, JHEP03(2021) 126, arXiv:2003.05546, arXiv:2107.10616
2. G. Dvali and O. Sakhelashvili, Black-Hole-Like Saturons in Gross-Neveu, PRD 105 (2022) 6, 065014arXiv:2111.03620.
3. G. Dvali, R. Venugopalan, Classicalization and unitarization of wee partons in QCD and gravity: The CGC-black hole correspondence PRD 105 (2022) 5, 056026

Saturon as a Vacuum Bubble

A Model

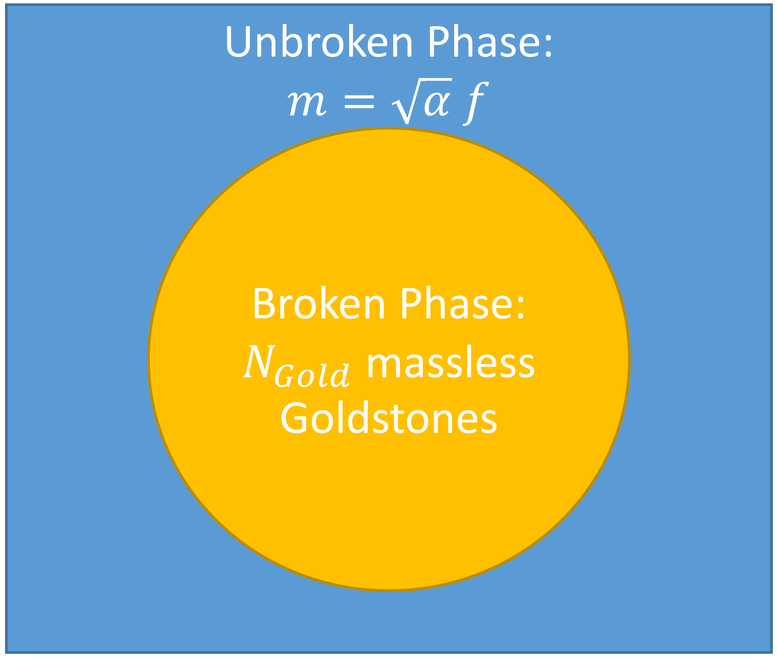
Model of a Saturon as a Vacuum Bubble

- We consider $d = 4$ model of a scalar field ϕ in the *adjoint rep.* of $SU(N)$, and $N \gg 1$

$$\mathcal{L} = \frac{1}{2}\,\mathrm{tr}\left[(\partial_\mu \phi)(\partial^\mu \phi)\right] - V[\phi]$$

- *Unitarity requires $\alpha \leq \frac{1}{N}$*

$$V[\phi] = \frac{\alpha}{2}\,\mathrm{tr}\left[\left(f\phi - \phi^2 + \frac{I}{N}\,\mathrm{tr}\,[\phi^2]\right)^2\right]$$



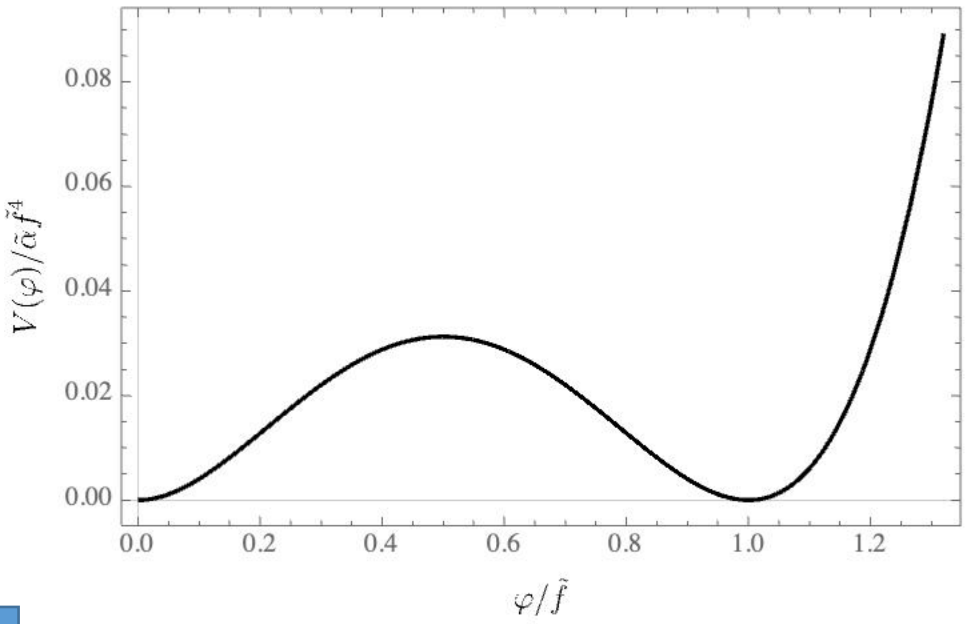
$$SU(N) \to SU(N-K) \times SU(K) \times U(1),$$

Model of a Saturon as a Vacuum Bubble

- We consider the breaking with $K = 1$, and the ansatz

$$\phi^{\alpha}_{\beta} = \frac{\varphi(x)}{\sqrt{N(N-1)}} \mathrm{diag}\left((N-1), -1, \ldots, -1\right).$$

$$\varphi^2 \equiv \mathrm{tr}[\phi^2]$$



$$\langle\varphi\rangle = \begin{cases} 0 & \rightarrow \text{Unbroken Phase: } SU(N) \\ f & \rightarrow \text{Broken Phase: } SU(N-1)\times U(1) \end{cases}$$

$$V(\varphi) \sim \frac{\alpha}{2}\varphi^2(f-\varphi)^2$$

Vacuum Bubbles Stabilization

A Memory Burden Effect

Vacuum Bubbles Stabilization

- Rotate in internal space the ansatz

$$\Phi_{\rm D} = \frac{\varphi(r)}{\sqrt{N(N-1)}}\,\mathrm{diag}\left((N-1), -1, \ldots, -1\right)$$

as

$$\phi = U^{\dagger}\Phi_{\rm D} U$$

$$U = \exp[-i\theta T]\,.$$

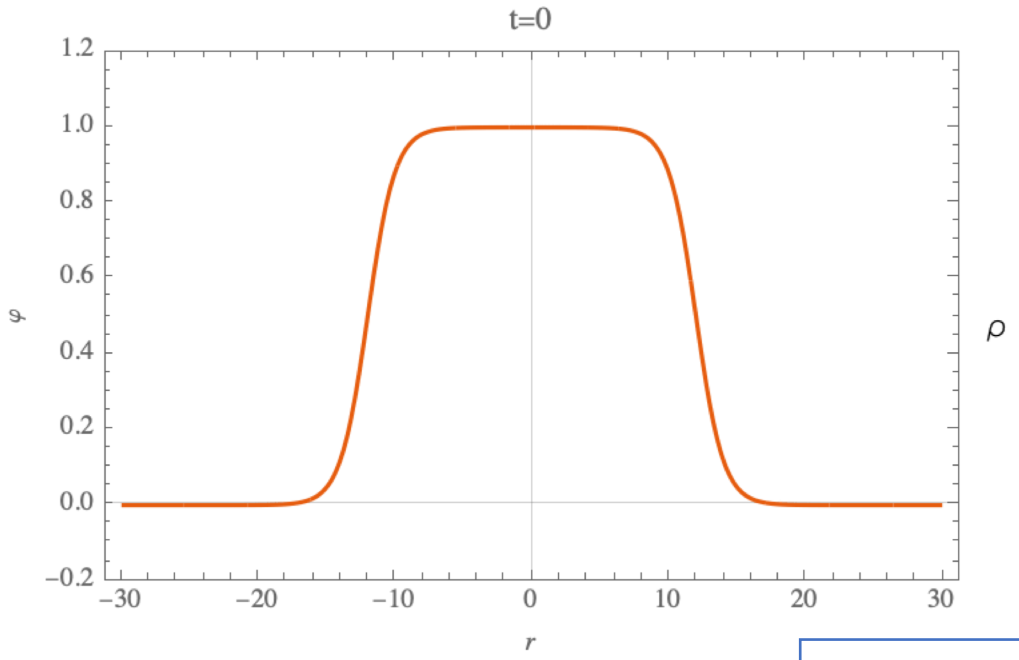
- Here T corresponds to the respective broken generators of $SU(N)$

$$\theta = \omega t$$

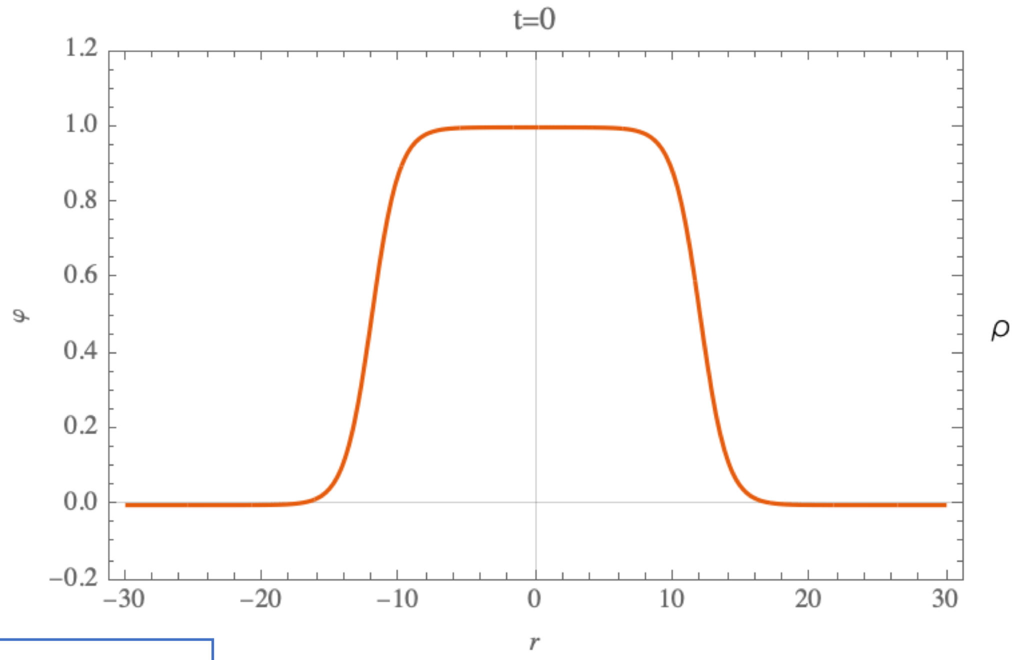
- G. Dvali, O. Kaikov, and J.S. Valbuena B, (2021), PRD 105, 056013 (2022); 2112.00551 [hep-th]

Vacuum Bubbles Stabilization:

$\dot{\boldsymbol{\theta}} = \mathbf{0}$



$\dot{\boldsymbol{\theta}} = \boldsymbol{\omega}$



Thin wall approximation for:
$\omega \approx 0.24\, m,$
$R_\omega \sim \frac{m}{\omega^2} \approx \frac{12}{m}$

Vacuum Bubbles Stabilization:

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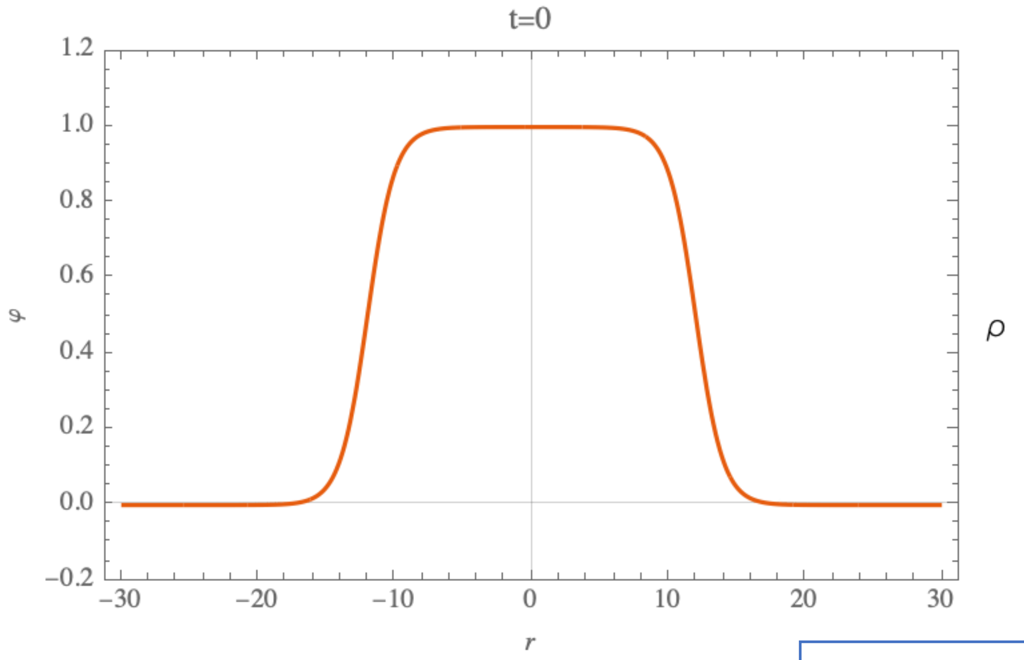

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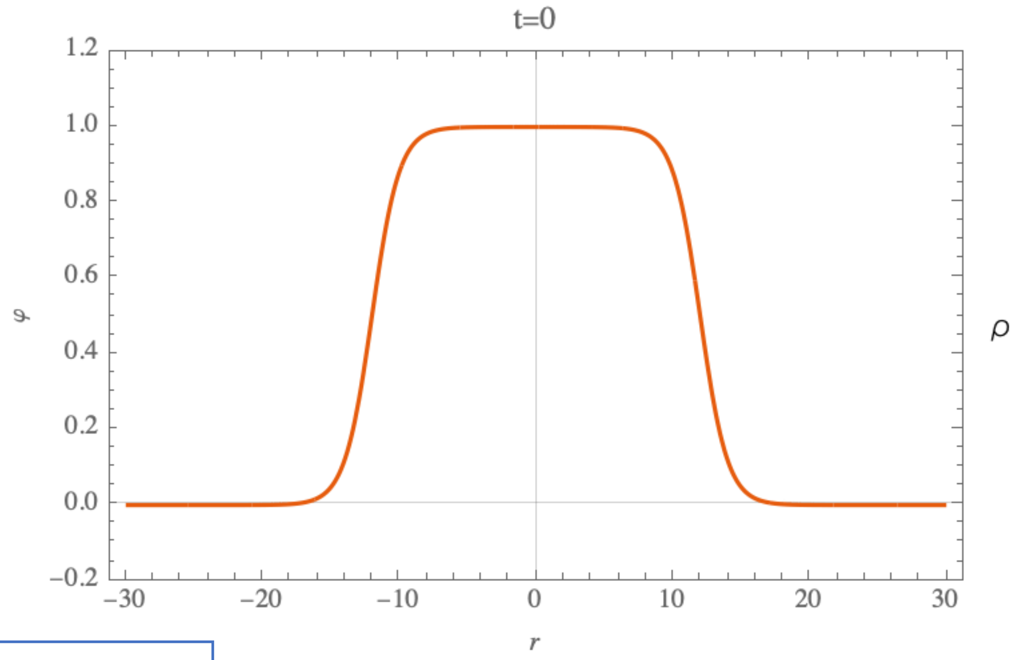


Thin wall approximation for:

$\omega \approx 0.24\, m,$

$$R_\omega \sim \frac{m}{\omega^2} \approx \frac{12}{m}$$

Vacuum Bubbles Stabilization: Quantum picture of classical stability

$$E_{\text{Bubble}} = E_{\text{int}} + E_{\text{wall}} = \frac{\omega}{\alpha}\frac{m^5}{\omega^5}\left(\frac{40\pi}{81}\right),$$

- In terms of the *occupation numbers* of the corresponding quanta, the energies are:

$$E_{\text{int}} = \omega N_G, \qquad \text{where,} \qquad N_G \equiv \frac{1}{\alpha}\frac{m^5}{\omega^5}\left(\frac{16\pi}{81}\right),$$

$$E_{\text{wall}} = m N_\varphi, \qquad \text{where,} \qquad N_\varphi \equiv \frac{1}{\alpha}\frac{m^4}{\omega^4}\left(\frac{8\pi}{27}\right).$$

Vacuum Bubbles Stabilization: Quantum picture of classical stability

- *A stationary bubble is obtained thanks to the excitations of the Goldstone mode(s).*
- The bubble is stable because of two factors:
  1) The fact that the Goldstone $SU(N)$ charge is conserved; and
  2) The fact that the same amount of charge in the exterior vacuum would cost higher energy.

- G. Dvali, Entropy Bound and Unitarity of Scattering Amplitudes, JHEP03 (2021) 126, arXiv:2003.05546.
- G. Dvali, A Microscopic Model of Holography: Survival by the Burden of Memory, arXiv:1810.02336.
- G. Dvali, L. Eisemann, M. Michel, and S. Zell, Universe's Primordial Quantum Memories, JCAP03 (2019) 010, arXiv:1812.08749.

Vacuum Bubbles Microstates

Towards Entropy Saturation

Vacuum Bubbles Microstates

- $N_G = E_{int}/\omega$ is the total mean *occupation number*.
- $N_{Gold} = 2(N-1)$ Goldstone modes (flavors).
- N_G can be arbitrarily *redistributed* among the N_{Gold} modes.

$$\sum_{a=1}^{2N-1} n^a = N_G$$

- Each sequence represents a *memory pattern*

$$|Pattern\rangle = |\, n_\omega^1, n_\omega^2, \dots \rangle$$

- The number of degenerate micro-states, n_{st}, is the number of *patterns* satisfying the *constraint* above, and the ***entropy*** is

$$\mathrm{S} = \ln n_{st} \approx 2N \ln\left[\left(1+\frac{2N}{N_G}\right)^{\frac{N_G}{2N}}\left(1+\frac{N_G}{2N}\right)\right]$$

Vacuum Bubbles Microstates

- Thick wall (Small) Bubbles correspond to Saturons

$$\omega \sim m$$

$$m^{-1} \sim R$$

$$N_G \sim \frac{1}{\alpha} \sim N \sim N_{Gold}$$

$$\mathrm{S} \sim S_{max} \sim \frac{1}{\alpha} \sim E_{\mathrm{Bubble}} R$$

- J. D. Bekenstein,Universal Upper Bound on the Entropy-to-Energy Ratio for BoundedSystems, Phys. Rev. D23no. 2 (1981), 287-298.47

Information Horizon

Saturons in semiclassical limit

Semi-classical Limit

- The limit in which the classical bubble solution experiences no back reaction from quantum fluctuations

$$\alpha \to 0, \qquad R = \text{finite}, \qquad \omega = \text{finite}, \qquad \alpha N = \text{finite}$$

- Simultaneously

$$f \to \infty, \qquad m = \text{finite}, \qquad N \to \infty$$

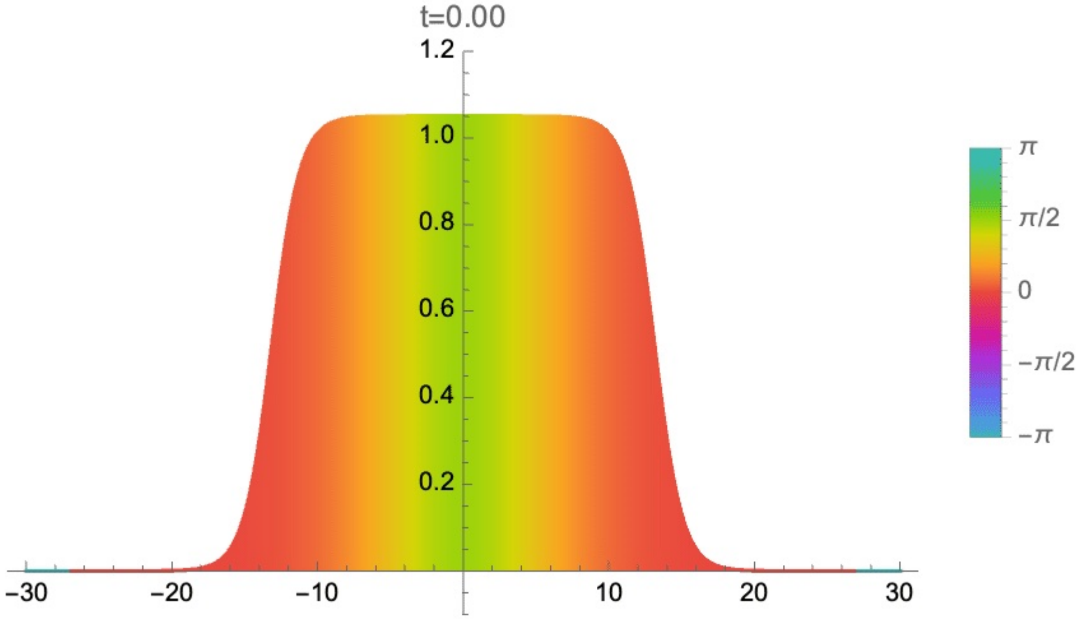
- In this limit, saturons possess a strict *information horizon*.
- Recall: For BH $f \sim M_p$

Goldstone Horizon: An Example

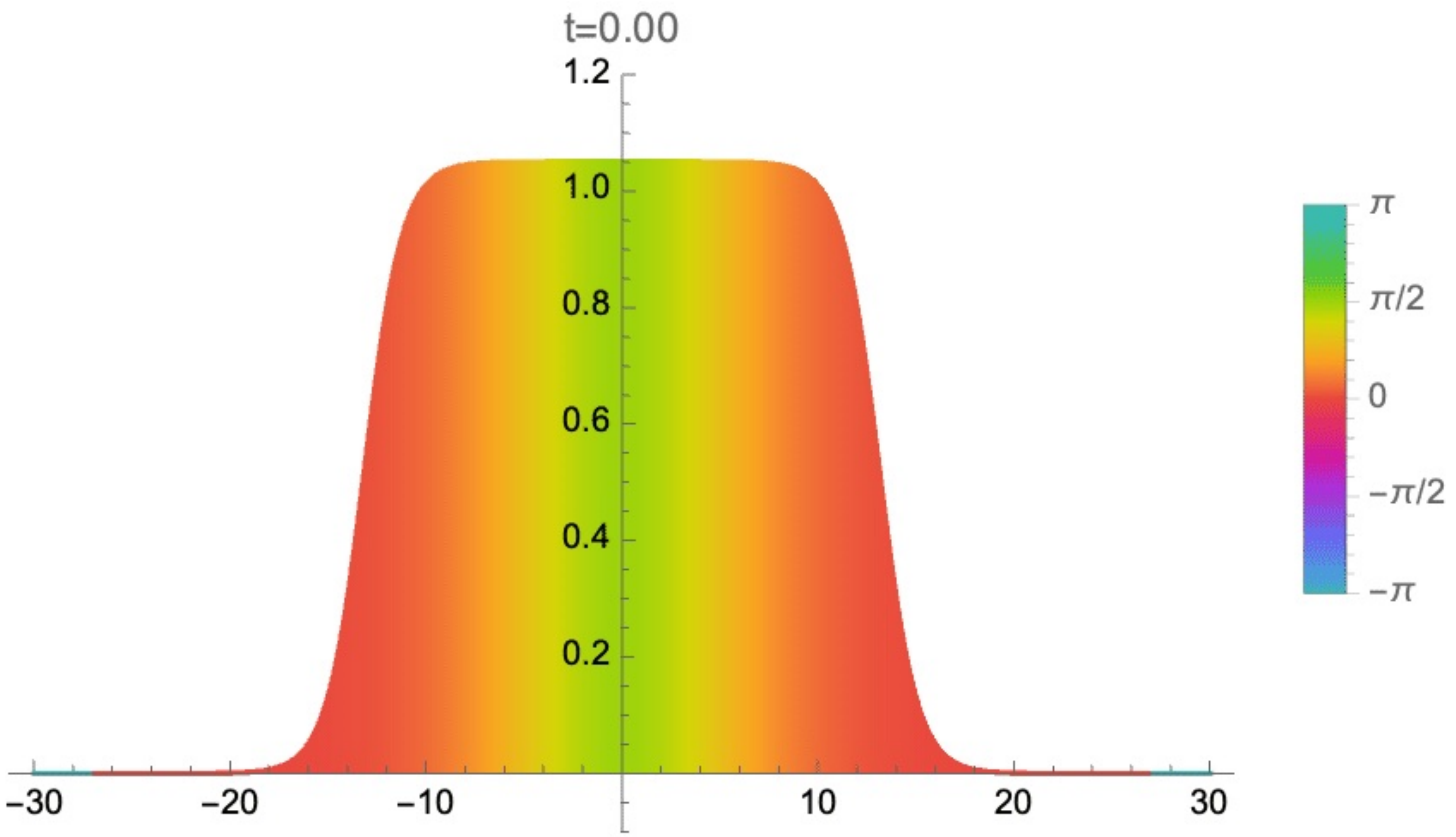
- Lets consider a perturbation on a stable vacuum bubble, ϕ_{VB},

$$\phi\Big|_{t=0} = p(r)\phi_{VB}(r)$$

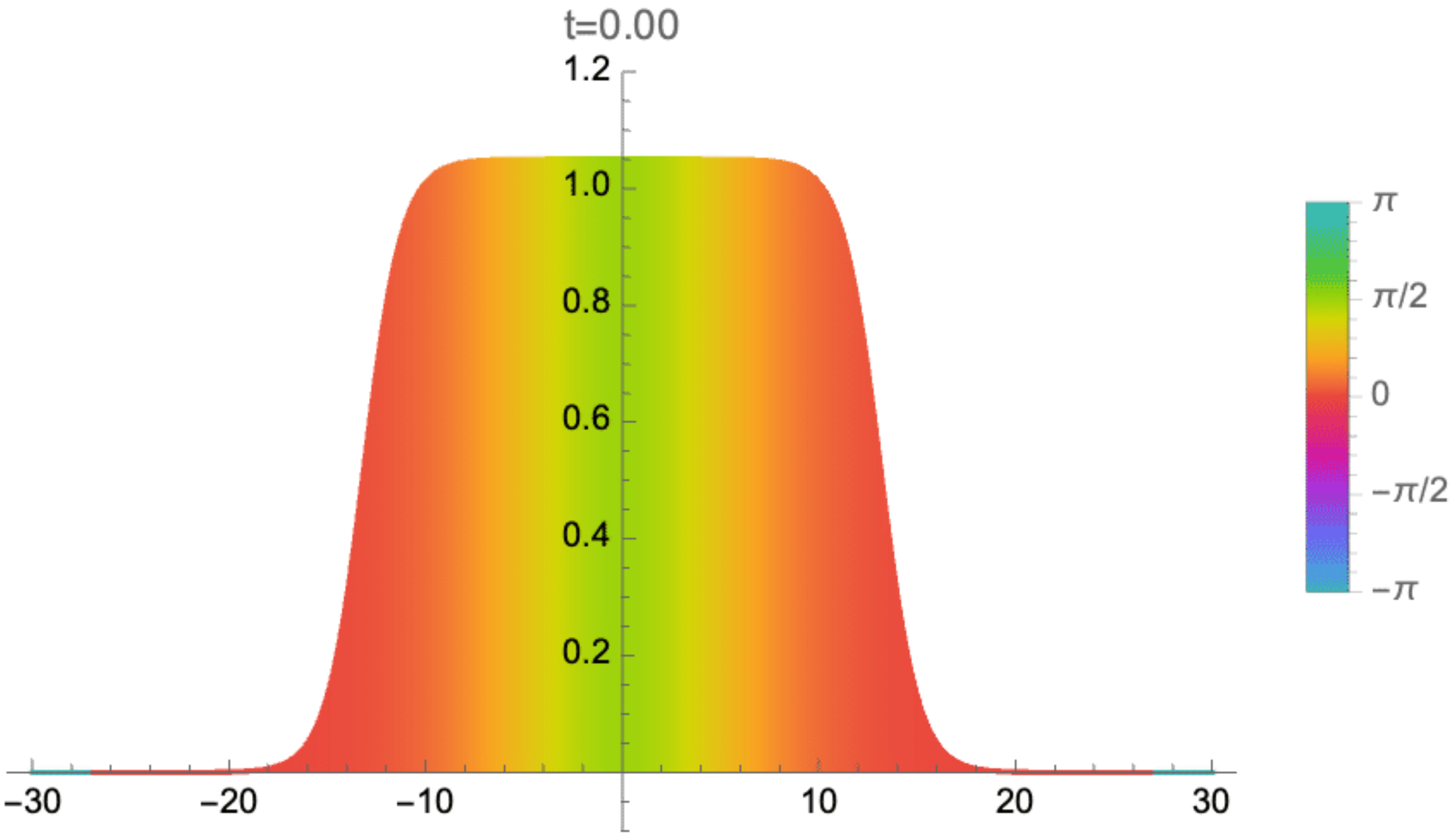
$$p(r) = \exp\left[\frac{i\pi}{2}\mathrm{e}^{-\frac{r^2}{2r_0^2}T}\right]$$



Goldstone Horizon: An Example



Goldstone Horizon: An Example



Correspondence to Black Holes

Correspondence to Black Holes

Saturons

- $S = (fR)^2 = \alpha^{-1}$
- $T = R^{-1}$
- $t_{\min} = R^3\, f^2 = SR$
- Information/Goldstone Horizon

Black Holes

- $S = (M_P R)^2$
- $T = R^{-1}$
- $t_{\min} = R^3\, M_P^2 = SR$
- Information Horizon

Conclusions and outlook

- **Black Holes are certainly special but not unique.**
- BH belong to a larger class of objects: Saturons, which saturate the entropy bound.
- We have shown an explicit example of a Saturon as a Vacuum Bubble.
- Vacuum Bubbles exhibit a goldstone horizon, analog to the information horizon of saturons.
- A large (macroscopic) occupation number of the Goldstone modes stabilizes the Vacuum Bubbles. This phenomenon is due to the memory burden effect

[7] G. Dvali, F. Kühnel and M. Zantedeschi, Vortexes in Black Holes, arXiv:hep-th/2112.08354.

Conclusions and outlook

- Departures from semi-classical behavior can become observable for BH that are relatively old and close to their half-decay time.

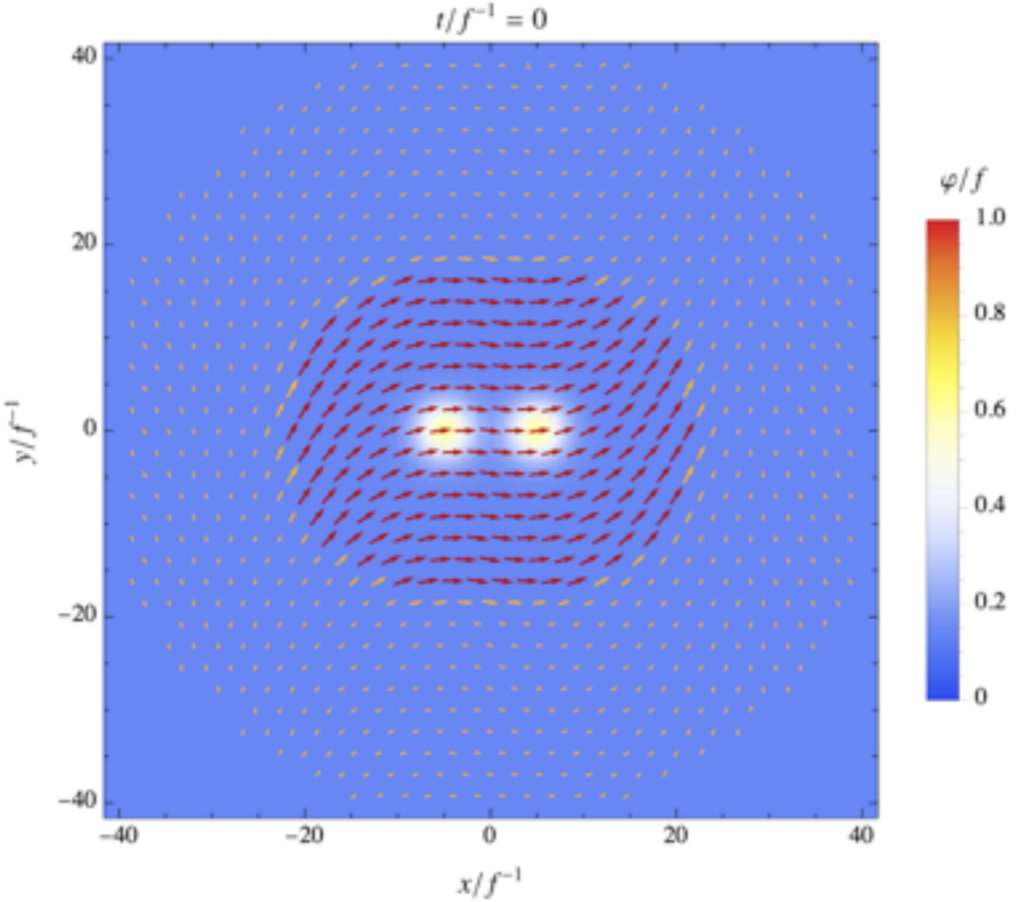
- The light Primordial Black Holes, provided they exist, can be within a potentially interesting window.

- Other possible observational consequences for rotating black holes are discussed in [7]

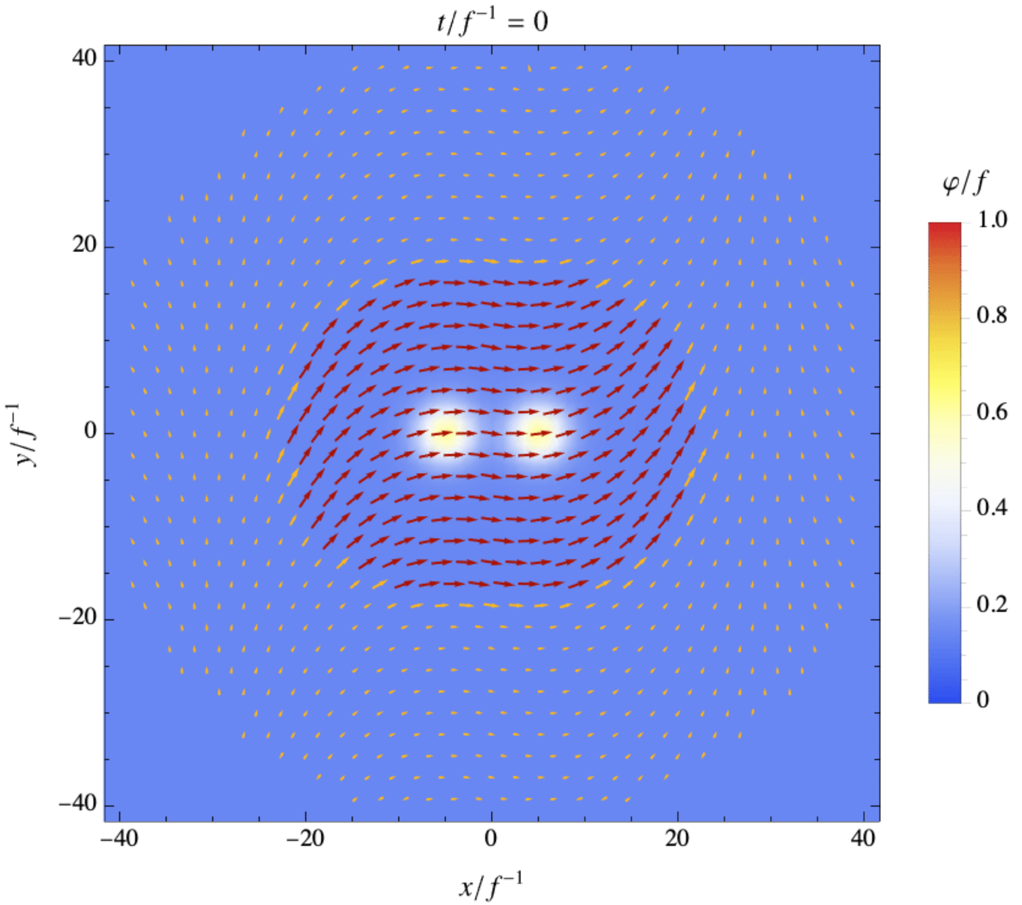
[7] G. Dvali, F. Kühnel and M. Zantedeschi, Vortexes in Black Holes, arXiv:hep-th/2112.08354.

Thank you

Outlook



Outlook

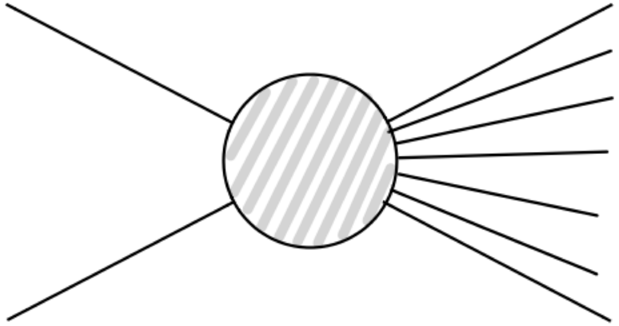


Saturation of Unitarity

If $N \sim \frac{1}{\alpha} \gg 1$

$$\boldsymbol{\sigma_{2\to N}\, n_{st} \sim \alpha^N N!\, n_{st}}$$

$$\sigma_{2\to N}\, n_{st} \sim \alpha^N \left(\frac{1}{\alpha}\right)^N e^{-\frac{1}{\alpha}}\, e^S$$



- Exponential suppression of high occupancy state (*classical lumps*), unless

$$S \sim \frac{1}{\alpha} \sim N$$

Vacuum Bubbles Micro-state Entropy

Large Bubbles

$$\omega \ll m$$
$$m^{-1} \ll R$$

- $N_G \gg \frac{1}{\alpha} \sim N \sim N_{Gold}$
- $1 \gg \lambda$
- $S \approx 2N \ln\left(\frac{e}{\lambda}\right) \sim \frac{1}{\alpha} \ln\left(\frac{m^{10}}{\omega^{10}}\right)$
- $S \ll S_{\max} \sim \frac{1}{\alpha}\frac{m^6}{\omega^6}$

Small Bubbles -> Saturons

$$\omega \sim m$$
$$m^{-1} \sim R$$

- $N_G \sim \frac{1}{\alpha} \sim N \sim N_{Gold}$
- $1 \sim \lambda$
- $S \sim \frac{1}{\alpha} \sim E_{\text{Bubble}} R$
- $S \sim S_{max}$

[8] J. D. Bekenstein,Universal Upper Bound on the Entropy-to-Energy Ratio for BoundedSystems, Phys. Rev. D23no. 2 (1981), 287-298.47

Memory Burden Effect

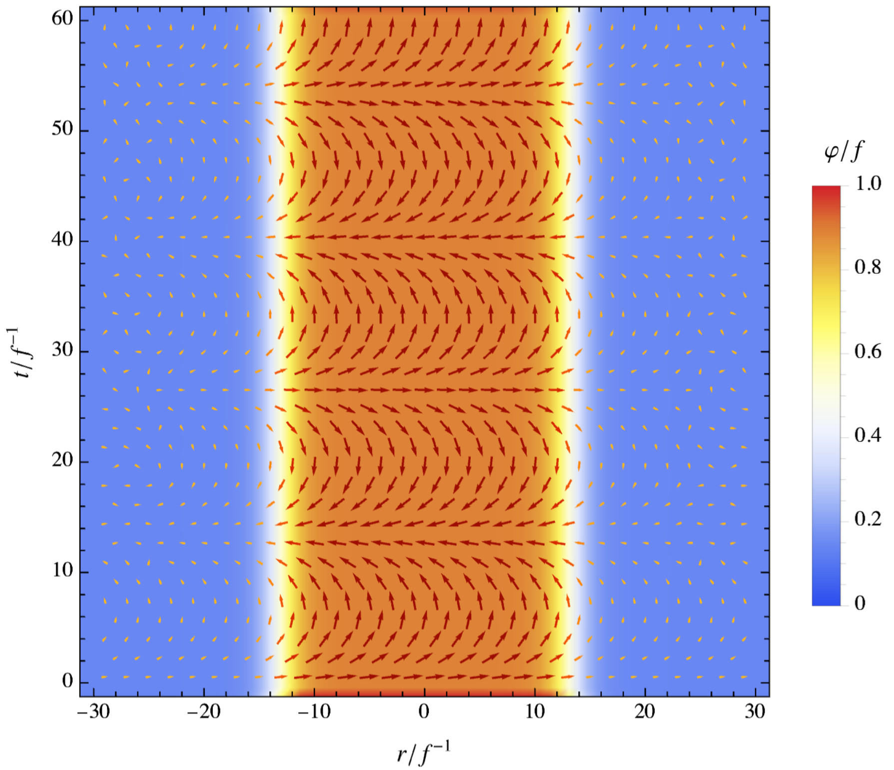
- Large amount of Memory patterns
- Stored quantum information.
- Slowdown of the system's evolution

Vaccum Bubbles Stabilization

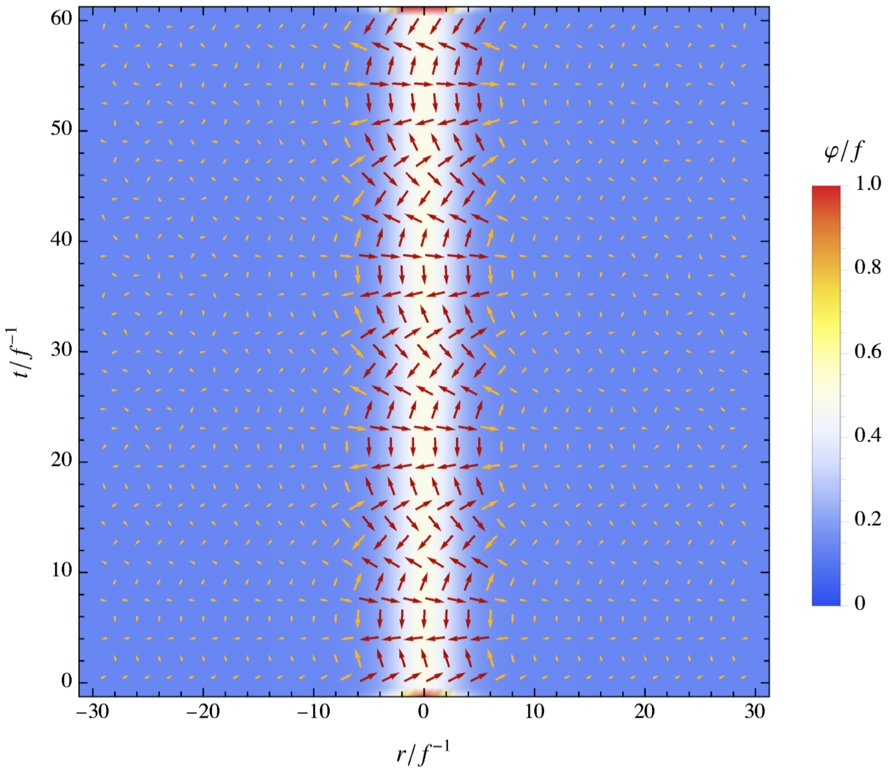
- Large amount of Bubble micro-states
- Excitations of the Goldstone modes
- Slowdown of bubble's decay

Vacuum Bubbles Stabilization

Large Bubbles
$R \sim 12m^{-1}$



Small Bubbles
$R \sim 1.02m^{-1}$



Semi-classical Limit

- In Semi-classical limit, the effective coupling of a Goldstone mode of frequency ε

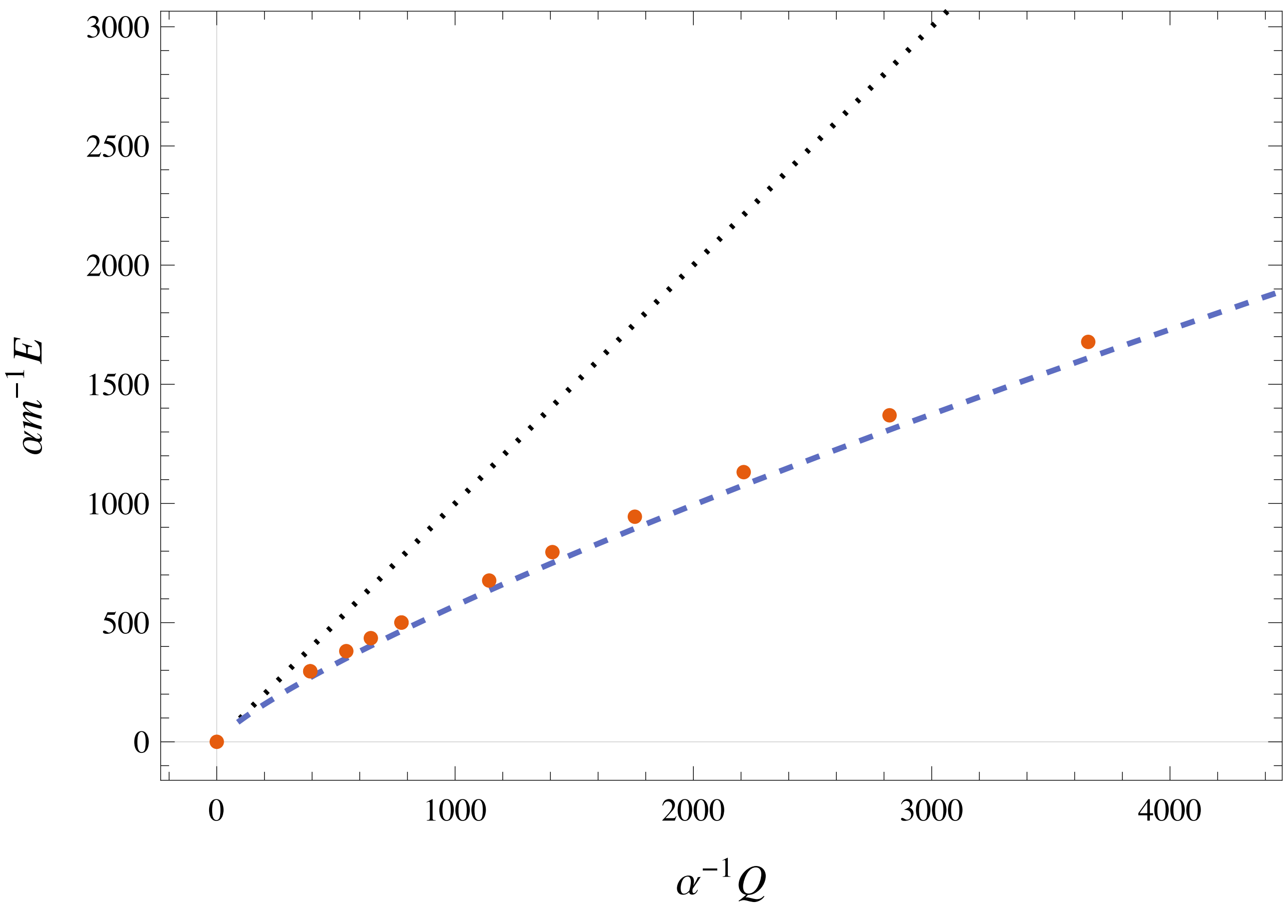
$$\alpha_G = \frac{\varepsilon^2}{f^2} \to 0$$

- **At finite $\boldsymbol{f}$**, and $\varepsilon \ll m$ cannot propagate outside the bubble, even though the coupling α_G is finite.
  - The energy ε is such propagation is impossible due to the finite energy gap.
  - $\varepsilon \ll m$, the perturbation energy can exceed the mass gap at the expense of a large occupation number n_ε of Goldstone quanta.

$$n_\varepsilon \to 1,$$

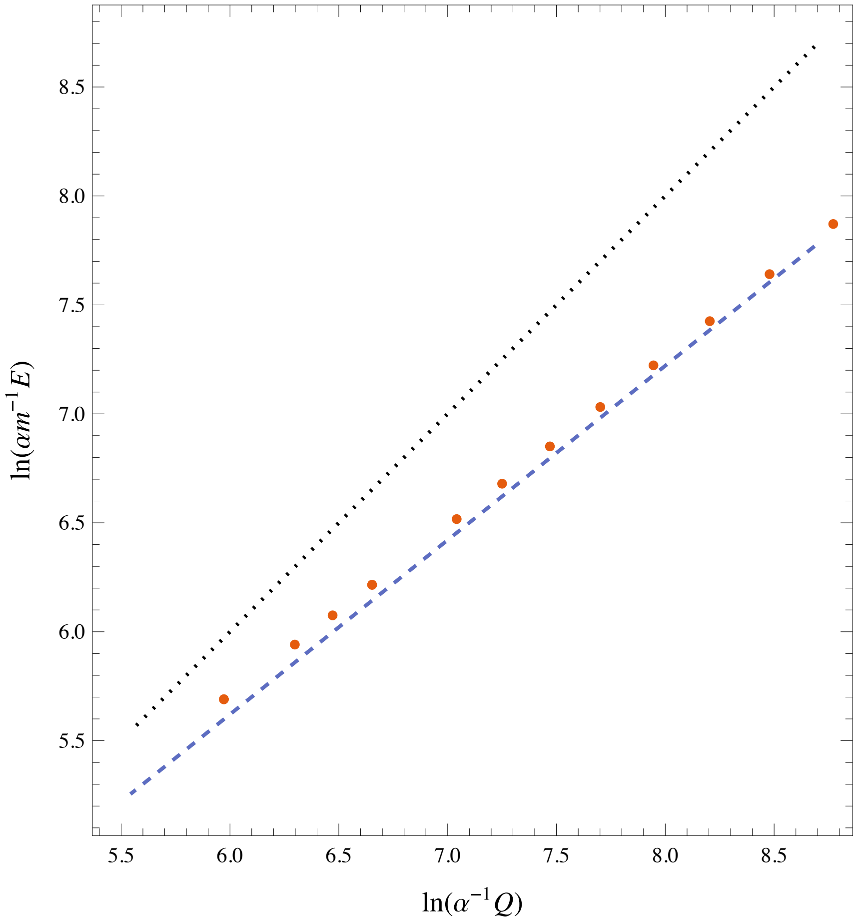
  such a process is exponentially suppressed by afactor $\mathrm{e}^{-n_\varepsilon}$

Spectrum

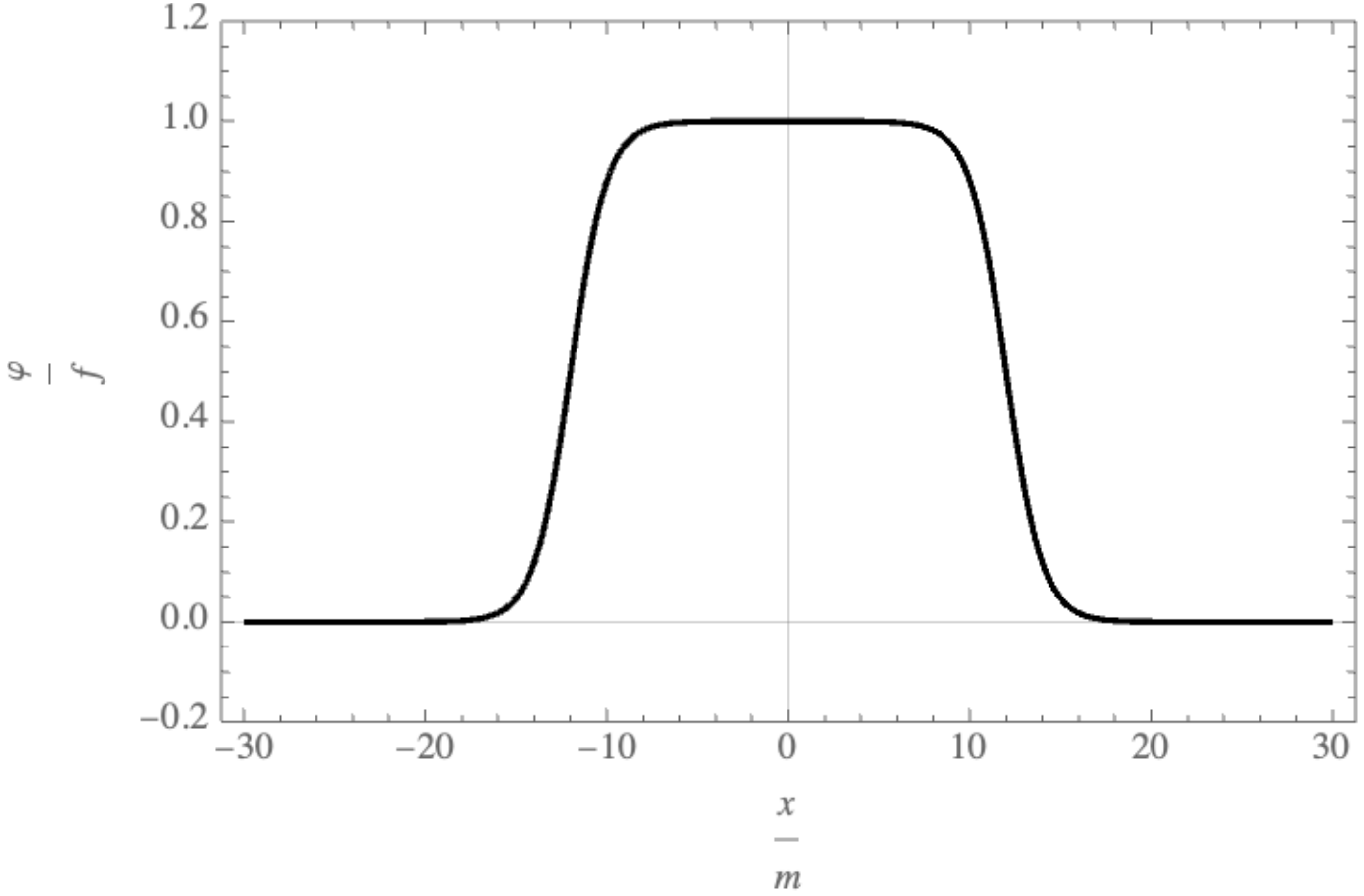


3000
2500
2000
1500
1000
500
0
$\alpha m^{-1} E$
0
1000
2000
3000
4000
$\alpha^{-1} Q$

Spectrum



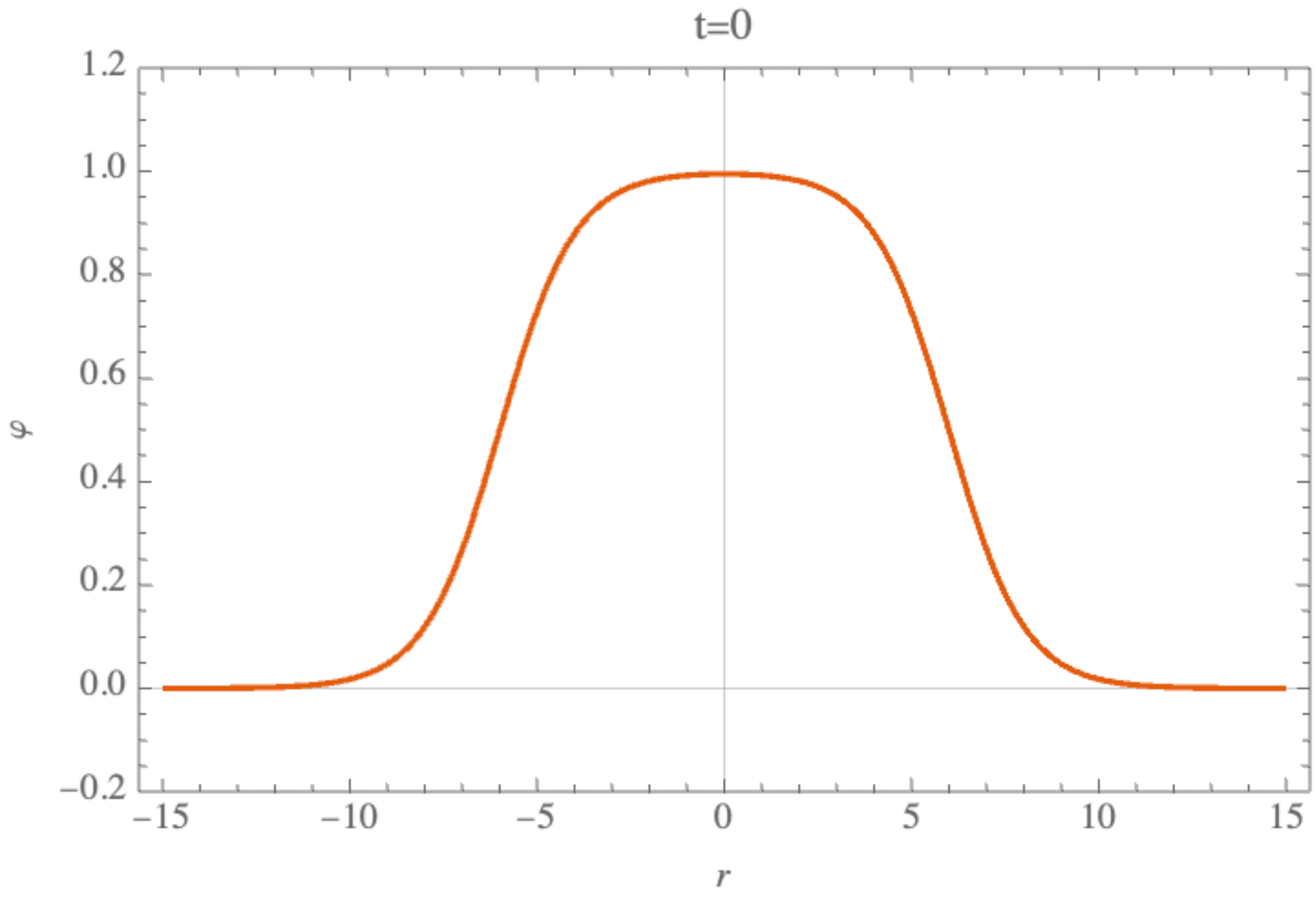
Vacuum Bubbles: Thin Wall Approximation ($R \gg m^{-1}$)



$m = \sqrt{\alpha}\, f = 1$
$R = 12$

$$\varphi(r) = \frac{f}{2}\left(1 + \tanh\left(\frac{m(R-r)}{2}\right)\right)$$

1+1



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