Ending inflation with a bang: Higgs vacuum metastability in $R + R^2$ gravity

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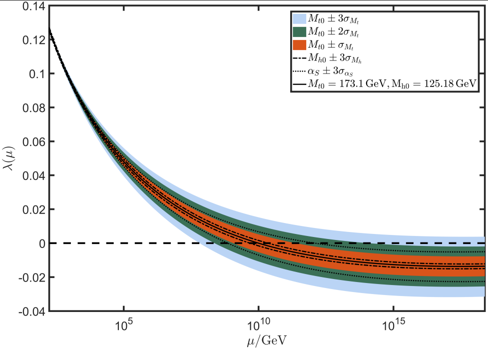
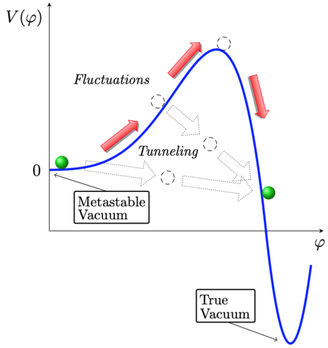
The EW vacuum metastability

Experimental values of SM particle masses m_h, m_t indicate that:

- SM may be valid up to $\mu_{\rm QG}$; early Universe consistent minimal model.
- currently in metastable EW vacuum $\rightarrow$ constrain fundamental physics.

$$V_{\rm H}(h,\mu,R) = \frac{\xi(\mu)}{2}Rh^2 + \frac{\lambda(\mu)}{4}h^4$$





Markkanen *et al*, "Cosmological Aspects of Higgs Vacuum Metastability", 2018.

Bubble nucleation from vacuum decay

- Decay expands at c with singularity within $\rightarrow$ true vacuum bubbles:

$$d\langle\mathcal{N}\rangle = \Gamma d\mathcal{V} \;\Rightarrow\; \langle\mathcal{N}\rangle = \int_{\text{past}} d^4x\sqrt{-g}\,\Gamma(x)$$

- Universe still in metastable vacuum $\rightarrow$ no bubbles in past light-cone:

$$P(\mathcal{N}=0) \propto e^{-\langle\mathcal{N}\rangle} \sim \mathcal{O}(1) \;\Rightarrow\; \langle\mathcal{N}\rangle \lesssim 1$$

- Low decay rate Γ today, but higher rates in the early Universe.

Vacuum bubbles expectation value (during inflation)

$$\langle\mathcal{N}\rangle = \frac{4\pi}{3}\int_0^{N_{\text{start}}} dN \left(\frac{a_{\text{inf}}\left(\eta_0 - \eta\left(N\right)\right)}{e^N}\right)^3 \frac{\Gamma(N)}{H(N)} \leq 1$$

Decay rate from Hawking-Moss instanton

- Classical solutions to the tunneling process from false to true vacuum.

- High H's during inflation, CdL$\to$HM instanton with action difference

$$B_{\rm HM}(R) \approx \frac{384\pi^2 \Delta V_{\rm H}}{R^2}$$

where $\Delta V_{\rm H} = V_{\rm H}(h_{\rm bar}) - V_{\rm H}(h_{\rm fv})$: barrier height $\to$ decay rate

$$\Gamma_{\rm HM}(R) \approx \left(\frac{R}{12}\right)^2 e^{-B_{\rm HM}(R)}$$

- Curvature effects enter at tree level via non-minimal coupling ξ:

$$V_{\rm H}(h, \mu, R) = \frac{\xi(\mu)}{2} R h^2 + \frac{\lambda(\mu)}{4} h^4$$

Coleman (1977), Coleman and De Lucia (1980), Hawking and Moss (1987).

Higgs potential in curved space-time

- Minkowski terms to 3-loops, curvature corrections in dS at 1-loop:

$$V_{\rm H}(h,\mu,R) = V_\Lambda - \kappa R - \frac{m^2}{2}h^2 + \frac{\xi}{2}Rh^2 + \frac{\lambda}{4}h^4 + \frac{\alpha}{144}R^2 + \Delta V_{\rm loops}\,,$$

where the loop contribution can be parametrized as

$$\Delta V_{\rm loops} = \frac{1}{64\pi^2}\sum_{i=1}^{31}\left\{n_i\mathcal{M}_i^4\left[\log\left(\frac{|\mathcal{M}_i^2|}{\mu^2}\right) - d_i\right] + \frac{n_i' R^2}{144}\log\left(\frac{|\mathcal{M}_i^2|}{\mu^2}\right)\right\}$$

- RGI: choose $\mu = \mu_*(h,R)$ such that $\Delta V_{\rm loops}(h,\mu_*,R) = 0 \rightarrow$

Renormalization Group Improved effective Higgs potential

$$V_{\rm H}^{\rm RGI}(h,R) = \frac{\xi(\mu_*(h,R))}{2}Rh^2 + \frac{\lambda(\mu_*(h,R))}{4}h^4 + \frac{\alpha(\mu_*(h,R))}{144}R^2$$

Markkanen *et al*, "The 1-loop effective potential for the Standard Model in curved spacetime", 2018.

Overview of computation

1. Calculate $\Delta V_{\rm H}$ and plug it in $\Gamma \approx \left(\frac{R}{12}\right)^2 e^{-\frac{384\pi^2 \Delta V_{\rm H}}{R^2}}$.

2. Cosmological quantities according to the inflationary model $V_{\rm I}(\phi)$.

3. Complete calculation of $\langle \mathcal{N} \rangle$ imposing the condition $\langle \mathcal{N} \rangle \leq 1$.

$$\langle \mathcal{N} \rangle = \frac{4\pi}{3} \int_0^{N_{\rm start}} dN \left(\frac{a_{\rm inf} \left(\eta_0 - \eta\left(N\right)\right)}{e^N} \right)^3 \frac{\Gamma(N)}{H(N)} \leq 1$$

4. Result: constraints on $\xi \geq \xi_{\langle \mathcal{N} \rangle = 1}$ and cosmological implications from the time of predominant bubble nucleation.

$$S = \int d^4x\sqrt{-g_J}\left[\frac{M_P^2}{2}\left(1 - \frac{\xi h^2}{M_P^2}\right)R_J + \frac{1}{12M^2}R_J^2 + \frac{1}{2}g_J^{\mu\nu}\partial_\mu h \partial_\nu h - \frac{\lambda}{4}h^4\right]$$

$$\Rightarrow \ldots \Rightarrow \mathcal{L} \approx \frac{M_P^2}{2}R + \frac{1}{2}\partial_\mu\tilde{\phi}\partial^\mu\tilde{\phi} + \frac{1}{2}\partial_\mu\rho\partial^\mu\rho - U(\tilde{\phi}, \rho)\,,$$

$$U(\tilde{\phi}, \rho) = V_{\rm I}(\tilde{\phi}) + m_{\rm eff}^2(\tilde{\phi}, \mu_*)\frac{\rho^2}{2} + \lambda_{\rm eff}(\tilde{\phi}, \mu_*)\frac{\rho^4}{4} + \frac{\alpha(\mu_*)}{144}R^2(\tilde{\phi}) + \mathcal{O}\left(\frac{\rho^6}{M_P^2}\right),$$

where $\Xi(\mu_*) = \xi(\mu_*) - \frac{1}{6}$ and

$$V_{\rm I}(\tilde{\phi}) = \frac{3M^2M_P^4}{4}\left(1 - e^{-\sqrt{\frac{2}{3}}\frac{\tilde{\phi}}{M_P}}\right)^2,$$

$$m_{\rm eff}^2 = \xi R + 3M^2M_P^2\Xi\left(1 - e^{-\sqrt{\frac{2}{3}}\frac{\tilde{\phi}}{M_P}}\right)e^{-\sqrt{\frac{2}{3}}\frac{\tilde{\phi}}{M_P}} + \frac{\Xi}{M_P^2}\partial_\mu\tilde{\phi}\partial^\mu\tilde{\phi}\,,$$

$$\lambda_{\rm eff} = \lambda + 3M^2\Xi^2e^{-2\sqrt{\frac{2}{3}}\frac{\tilde{\phi}}{M_P}} + \frac{4\left[\xi R + \Delta m_1^2\right]\Xi^2}{M_P^2} + \frac{4\Xi^3}{M_P^4}\partial_\mu\tilde{\phi}\partial^\mu\tilde{\phi}\,.$$

Overview of computation

1. Calculate ΔV_{H} and plug it in $\Gamma \approx \left(\frac{R}{12}\right)^2 e^{-\frac{384\pi^2 \Delta V_{\mathrm{H}}}{R^2}}$.

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System of coupled differential equations

Solve the system of coupled differential equations

$$\frac{d^2\tilde{\phi}}{dN^2} = \frac{V(\tilde{\phi})}{M_P^2 H^2}\left(\frac{d\tilde{\phi}}{dN} - M_P^2 \frac{V'(\tilde{\phi})}{V(\tilde{\phi})}\right)$$

$$\frac{d\tilde{\eta}}{dN} = -\tilde{\eta}(N) - \frac{1}{a_{\text{inf}} H(N)}$$

$$\frac{d\langle\mathcal{N}\rangle}{dN} = \gamma(N) = \frac{4\pi}{3}\left[a_{\text{inf}}\left(\frac{3.21e^{-N}}{a_0 H_0} - \tilde{\eta}(N)\right)\right]^3 \frac{\Gamma(N)}{H(N)}$$

where $\tilde{\eta} = e^{-N}\eta$ and η: conformal time and we set the end of inflation at

$$\left.\frac{\ddot{a}}{a}\right|_{\tilde{\phi}=\tilde{\phi}_{\text{inf}}} = H^2\left[1 - \frac{1}{2M_P^2}\left(\frac{d\tilde{\phi}}{dN}\right)^2\right]\Bigg|_{\tilde{\phi}=\tilde{\phi}_{\text{inf}}} = 0$$

Overview of computation

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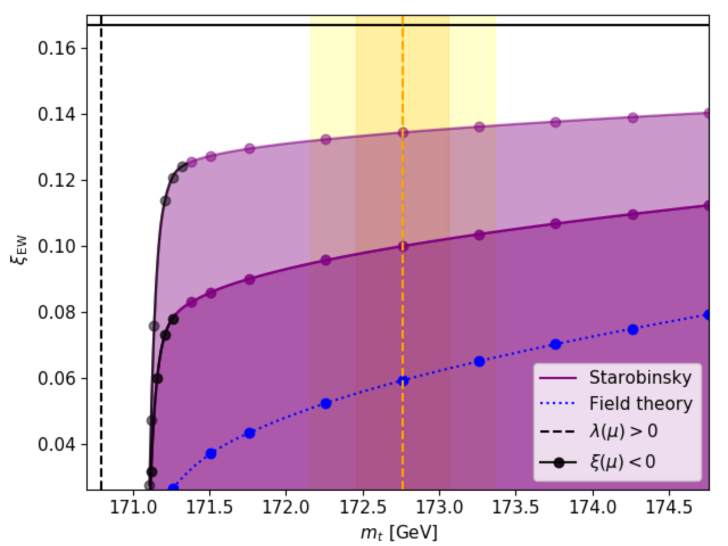
Results: Lower ξ-bounds for varying top quark mass



Conclusions

- Consistent inclusion of 1-loop curvature corrections beyond dS in $R + R^2$ gravity $\rightarrow$ stricter ξ-bounds from extra negative terms in V_H:

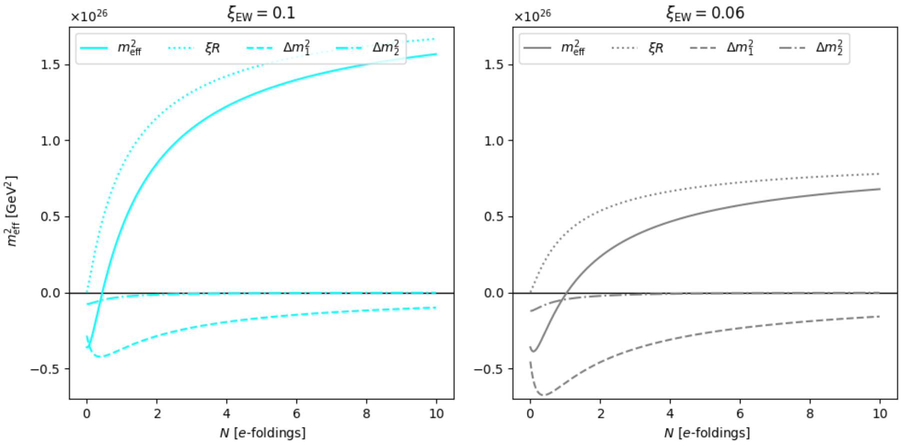
$$\xi_{\rm EW} \gtrsim 0.1 > 0.06$$

- Bubble nucleation in the last moments of inflation: breakdown of dS approximations and necessity to consider the dynamics of reheating.

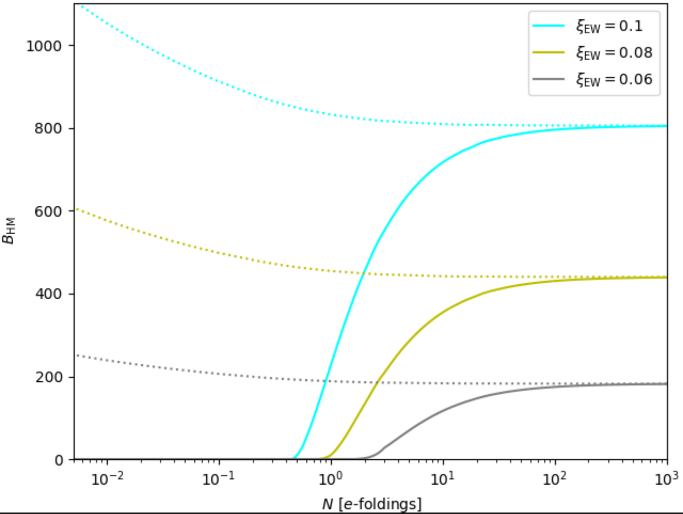
- Approaching the conformal point $\Rightarrow$ HM validity questionable.
- Possibly hints against eternal inflation (again).

Additional slides

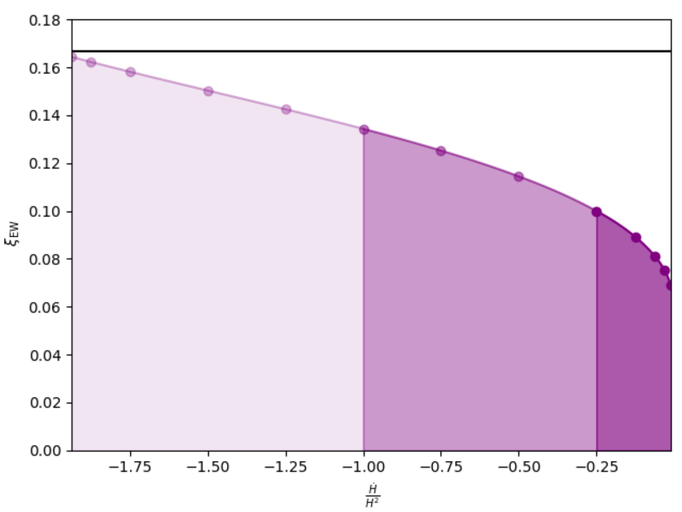
Factors of the potential's quadratic term



HM bounce in Starobinsky Inflation and Field Theory



ξ-bounds with varying definition for the end of inflation



$\langle \mathcal{N} \rangle$ integrands in Starobinsky Inflation and Field Theory