# Unified Emergence of Energy Scales and Cosmic Inflation

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#### Introduction

# Scale invariance

Classical action invariant under rescalings

$$g_{\mu\nu} \to e^{2\sigma} g_{\mu\nu} \,, \quad \Phi \to e^{-w_\Phi \sigma} \Phi$$

• Symmetry of SM at high energies:  $E \gg m_i$ 

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• If scale invariance broken by scale anomaly,  $M_{PI}$  and  $v_{EW}$  exponentially separated and radiatively stable if: no intermediate scales [Bardeen '95] [Meissner, Nicolai, hep-th/0612165]

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The scale-invariant model and symmetry breaking

### Inflation





### The Model

$$\begin{split} \frac{\mathcal{L}_{\rm CW}}{\sqrt{-g}} &= \frac{1}{2} g^{\mu\nu} \partial_{\mu} S \partial_{\nu} S + \frac{1}{2} g^{\mu\nu} \partial_{\mu} \sigma \partial_{\nu} \sigma - \frac{1}{4} \lambda_S S^4 - \frac{1}{4} \lambda_\sigma \sigma^4 - \frac{1}{4} \lambda_{s\sigma} S^2 \sigma^2 \\ \frac{\mathcal{L}_{\rm GR}}{\sqrt{-g}} &= -\frac{1}{2} (\beta_S S^2 + \beta_\sigma \sigma^2 + \beta_H H^{\dagger} H) R + \gamma R^2 + \kappa W_{\mu\nu\alpha\beta} W^{\mu\nu\alpha\beta} \\ \frac{\mathcal{L}_{\rm SM}}{\sqrt{-g}} &= \mathcal{L}_{\rm SM}|_{\mu_H=0} - \frac{1}{4} (\lambda_{HS} S^2 + \lambda_{H\sigma} \sigma^2) H^{\dagger} H \\ \frac{\mathcal{L}_N}{\sqrt{-g}} &= \frac{i}{2} \overline{N_R} \not{\partial} N_R - \left( \frac{1}{2} y_M S \overline{N_R} (N_R)^c + y_\nu \bar{L} \tilde{H} N_R + \text{h.c.} \right) \end{split}$$

**(** Additional scalar sector for Coleman-Weinberg mechanism ( $\langle S \rangle = v_S$ )

- **②** Gravity with global scale invariance (identifcation of  $M_{\rm Pl}$  and inflation)
- SM interactions + Higgs portals
- type-I seesaw (also inducing Higgs mass)

# Dimensional transmutation

Coleman-Weinberg mechanism [Coleman, Weinberg '73]

e.g. massless sQED 
$$V_{\text{eff}}(\varphi) = \frac{\lambda}{4!}\varphi^4 + 3\frac{(g\varphi)^4}{64\pi^2} \left[ \log\left(\frac{(g\varphi)^2}{\mu^2}\right) - \frac{5}{6} \right]$$
  
 $\mathcal{O}(\lambda) \sim \mathcal{O}(g^4) \rightarrow \langle \varphi \rangle \neq 0$ 

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Approximation tool for multi-scalar potential: Gildener-Weinberg approach [Gildener, Weinberg '76]



$$V_{\text{tree}}(S,\sigma) = \frac{1}{4} \left( \lambda_S S^4 + \lambda_\sigma \sigma^4 + \lambda_{s\sigma} S^2 \sigma^2 \right) + \frac{1}{4} (\lambda_{HS} S^2 + \lambda_{H\sigma} \sigma^2) H^{\dagger} H$$

Desired flat direction  $(S \neq 0, \sigma = 0)$  for  $\lambda_S \ll \lambda_{S\sigma}$  and  $\lambda_S \ll \lambda_{\sigma}$ 

# SSB of scale invariance

$$\frac{\mathcal{L}_{\rm GR}}{\sqrt{-g}} = -\frac{1}{2} (\beta_S S^2 + \beta_\sigma \sigma^2 + \beta_H H^{\dagger} H) R + \gamma R^2 + \kappa W_{\mu\nu\alpha\beta} W^{\mu\nu\alpha\beta}$$

• Coleman-Weinberg potential in background  $\sigma=0,\ S\neq 0$  and  $R\neq 0:$ 

$$U_{\rm eff}(S,R,\sigma) = \frac{\lambda_S}{4}S^4 + \frac{\lambda_{\sigma}}{4}\sigma^4 + \frac{\lambda_{S\sigma}}{4}S^2\sigma^2 + \frac{1}{64\pi^2} \left(\tilde{m}_s^4 \ln[\tilde{m}_s^2/\mu^2] + \tilde{m}_{\sigma}^4 \ln[\tilde{m}_{\sigma}^2/\mu^2]\right)$$

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• During Inflation  $\sigma=0,\ \beta_S R<3\lambda_S S^2$  and  $\beta_\sigma R<(1/2)\lambda_{S\sigma}S^2$ 

$$\tilde{U}_{\text{eff}}(S,R) = U_{\text{eff}}(S,R,0) - U_0 = U_{\text{CW}}(S) + U_{(1)}(S)R + U_{(2)}(S)R^2 + \mathcal{O}(R^3)$$

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Subtracting induced cosmological constant

$$U_{\rm CW}(S=v_S)=0$$
,  $U_0=-\mu^4 \frac{\beta_{\lambda_S}}{16} \exp\left[-1-16C/\beta_{\lambda_S}\right]$ 

• Identification of Planck mass:  $M_{\rm Pl} = v_S \sqrt{\beta_S + 2U_{(1)}(v_S)/v_S^2}$ For inflation  $\beta_S \sim 10^{(2-3)} \Rightarrow v_S \sim 10^{(16-17)}$  GeV

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### Effective action for inflation

 $\begin{array}{l} \text{Jordan frame:} \ \frac{\mathcal{L}_{\text{eff}}}{\sqrt{-g_J}} = -\frac{1}{2}B(S)R_J + G(S)R_J^2 + \frac{1}{2}g_J^{\mu\nu}\partial_\mu S\partial_\nu S - U_{\text{CW}}(S) \\ \text{Einstein frame:} \ \frac{\mathcal{L}_{\text{eff}}^E}{\sqrt{-g}} = -\frac{M_{\text{Pl}}^2}{2}R + \frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi + \frac{1}{2}e^{-\sqrt{2/3}\frac{\phi}{M_{\text{Pl}}}}g^{\mu\nu}\partial_\mu S\partial_\nu S - V(S,\phi) \end{array}$ 

### Effective action for inflation





 $\text{Contour } \mathcal{C} = \{S, \tilde{\phi}(S)\} \rightarrow \frac{\mathcal{L}_{\text{eff}}^e}{\sqrt{-g}} = -\frac{1}{2} M_{\text{Pl}}^2 R + \frac{1}{2} F(S)^2 g^{\mu\nu} \partial_{\mu} S \partial_{\nu} S - V_{\text{inf}}(S)$ 

# Slow-roll approximation

• Potential slow-roll parameters

$$\varepsilon(S) = \frac{M_{\rm Pl}^2}{2 F^2(S)} \left(\frac{V_{\rm inf}'(S)}{V_{\rm inf}(S)}\right)^2$$
$$\eta(S) = \frac{M_{\rm Pl}^2}{F^2(S)} \left(\frac{V_{\rm inf}'(S)}{V_{\rm inf}(S)} - \frac{F'(S)}{F(S)}\frac{V_{\rm inf}'(S)}{V_{\rm inf}(S)}\right)$$



# Slow-roll approximation



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• CMB observables

$$A_s = \frac{V_{\inf}(S_*)}{24\pi^2 \,\varepsilon(S_*) \,M_{\rm Pl}^4} \,, \quad n_s = 1 + 2\,\eta(S_*) - 6\,\varepsilon(S_*) \,, \quad r = 16\,\varepsilon(S_*)$$

# Inflation results

• Free parameters in 
$$V_{inf}$$
 : 
$$\begin{cases} \lambda_{\sigma}, \lambda_{S} \\ \beta_{\sigma}, \beta_{S} \\ \gamma \end{cases}$$

 $\begin{cases} \lambda_S, \boldsymbol{\lambda_{S\sigma}} & \text{tree-level potential} \\ \beta_{\sigma}, \boldsymbol{\beta_S} & \text{non-minimal couplings} \\ \boldsymbol{\gamma} & R^2 - \text{coefficient} \end{cases}$ 

### Inflation results



• Scalar amplitude constraint [Planck 2018]



### Inflation results



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2 The scale-invariant model and symmetry breaking

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# How to connect the Planck and EW scale?



### • New approach to hierarchy problem: Neutrino Option

[Brivio, Trott, 1703.10924]

### How to connect the Planck and EW scale?



### How to connect the Planck and EW scale?



### How to connect the Planck and EW scale?



## Fine-tuning

$$\mathcal{L} \supset \frac{i}{2} \overline{N_R} \not \partial N_R - \left( \frac{1}{2} y_M S \overline{N_R} (N_R)^c + y_\nu \bar{L} \tilde{H} N_R + \text{h.c.} \right) - \frac{1}{4} (\lambda_{HS} S^2 + \lambda_{H\sigma} \sigma^2) H^{\dagger} H$$

Induced Majorana mass (Yukawa coupling fixed by Planck scale and inflation)

$$\begin{split} m_N &= y_M \, v_S \simeq 10^7 \, \text{GeV} \qquad \text{(neutrino option)} \\ y_M &= \frac{m_N \beta_S^{1/2}}{M_{\text{Pl}}} \simeq 10^{-10} \left(\frac{\beta_S}{10^3}\right)^{1/2} \end{split}$$

•  $y_M 
ightarrow 0$  technically natural (U(1)\_{B-L} restored) ['t Hooft '80]

### Fine-tuning

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$$m_N = y_M v_S \simeq 10^7 \text{ GeV}$$
(neutrino option)  
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• Another contribution to the Higgs mass

$$\begin{split} \lambda_{HS}S^2(\boldsymbol{H}^\dagger\boldsymbol{H}) &\to \lambda_{HS}v_S^2(\boldsymbol{H}^\dagger\boldsymbol{H}) \\ \Delta\lambda_{HS} &\sim y_\nu^2 y_M^2/16\pi^2 \end{split}$$



- $\lambda_{HS} \ll 1$  but not fine-tuned to special value
- $\{\lambda_{HS}, \lambda_{H\sigma}, y_M\} \sim 0$  stable under RG (in absence of gravity)

### Summary & conclusion

- Classically scale invariant model with dynamical generation of all scales
- VEV  $v_S = 10^{16-17}~{
  m GeV}$  generates Planck scale  $M_{
  m Pl} pprox eta_S^{1/2} v_S$
- Inflation predictions consistent with Planck observations
- Majorana mass scale  $M_N = y_M v_S \sim 10^7 \text{ GeV}$
- Higgs mass realized by neutrino option (+ light active neutrinos)

# Thank you!

# Inflation

$$U_{\text{eff}}(S,R,\sigma) = \frac{\lambda_S}{4}S^4 + \frac{\lambda_\sigma}{4}\sigma^4 + \frac{\lambda_{S\sigma}}{4}S^2\sigma^2 + \frac{1}{64\pi^2} \left(\tilde{m}_s^4 \ln[\tilde{m}_s^2/\mu^2] + \tilde{m}_\sigma^4 \ln[\tilde{m}_\sigma^2/\mu^2]\right)$$
$$\tilde{m}_s^2 = 3\lambda_S S^2 + \beta_S R$$
$$\tilde{m}_\sigma^2 = \frac{1}{2}\lambda_{S\sigma}S^2 + \beta_\sigma R$$

Inflaton potential

$$V(S,\phi) = e^{-2\sqrt{2/3}\frac{\phi}{M_{\rm Pl}}} \left[ U_{\rm CW}(S) + \frac{M_{\rm Pl}^4}{16\,G(S)} \left( B(S) - e^{\sqrt{2/3}\frac{\phi}{M_{\rm Pl}}} \right)^2 \right]$$
$$B(S) = \beta_S S^2 + 2U_{(1)}(S) \,, \quad G(S) = \gamma - U_{(2)}(S)$$

Contours

$$\begin{array}{c|c} \mathbf{C} = \{S, \tilde{\phi}(S)\} & \text{where} & \left. \frac{\partial V(S, \phi)}{\partial \phi} \right|_{\phi = \tilde{\phi}(S)} = 0 \,, \quad V_{\inf}(S) = V(S, \tilde{\phi}(S)) \\ \\ & \rightarrow \frac{\mathcal{L}_{\text{eff}}^E}{\sqrt{-g}} = -\frac{1}{2} \, M_{\text{Pl}}^2 R + \frac{1}{2} \, F(S)^2 \, g^{\mu\nu} \, \partial_\mu S \, \partial_\nu S - V_{\inf}(S) \\ \\ \mathbf{O} \quad \mathcal{C}' = \{\tilde{S}(\phi), \phi\} \,, \quad \text{where} \quad \left. \frac{\partial V(S, \phi)}{\partial S} \right|_{S = \tilde{S}(\phi)} = 0 \,, \quad V_{\inf}(\phi) = V(\tilde{S}(\phi), \phi) \end{array}$$

### Inflationary observables

Power spectrum of adiabatic and Gaussian scalar fluctuations

$$\langle \mathcal{RR} \rangle = (2\pi)^3 \delta(k+k') P_{\mathcal{R}}(k), \quad \Delta_s^2(k) = \frac{k^3}{2\pi^2} P_{\mathcal{R}}(k)$$

$$\Delta_s^2(k) = A_s(k_*) \left(\frac{k}{k_*}\right)^{n_s(k_*)-1} \quad \begin{cases} A_s \\ n_s \\ k_* \end{cases}$$

scalar power spectrum amplitude scalar spectral-tilt pivot scale

Tensor perturbations (sum of two polarizations:  $h_x, h_+$ )

$$\Delta_t^2(k) = A_t(k_*) \left(\frac{k}{k_*}\right)^{n_t(k_*)} \left\{ r = \frac{\Delta_t^2}{\Delta_s^2} \quad \text{tensor-to-scalar ratio} \right\}$$

# Slow-roll approximation

Potential slow-roll parameters

$$\varepsilon(S) = \frac{M_{\rm Pl}^2}{2 F^2(S)} \left(\frac{V_{\rm inf}'(S)}{V_{\rm inf}(S)}\right)^2 \eta(S) = \frac{M_{\rm Pl}^2}{F^2(S)} \left(\frac{V_{\rm inf}'(S)}{V_{\rm inf}(S)} - \frac{F'(S)}{F(S)} \frac{V_{\rm inf}'(S)}{V_{\rm inf}(S)}\right)$$



• End of inflation

$$\varepsilon(S = S_{end}) = 1$$
 or  $|\eta(S = S_{end})| = 1$ 

• Field value at horizon crossing  $S_{\ast}$  fixed by e-folds  $N_{e}=50-60$ 

$$N_e = \int_{t_*}^{t_{end}} H dt = \int_{S_*}^{S_{end}} \frac{dS}{\sqrt{2\varepsilon(S)}}$$

CMB observables

$$A_s = \frac{V_{\text{inf}}(S_*)}{24\pi^2 \,\varepsilon(S_*) \,M_{\text{Pl}}^4} \,, \quad n_s = 1 + 2\,\eta(S_*) - 6\,\varepsilon(S_*) \,, \quad r = 16\,\varepsilon(S_*)$$

# Inflation



# Inflation



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Figure 3: Values of the parameters  $\lambda$  (left) and  $\sqrt{m_{\pi}^2}$  (right) extrapolated at the scale  $\mu = m_{\pi}$  as a function of the two seesawe parameters M and  $\omega$  respectively, in the preliminary study of Ref. [1]. The dashed lines and surrounding bands indicate the values consistent with the measured Higgs mass within  $\pm 1\sigma$  [15]. Left panel: the red line assumes  $m_{\pi} = 173.2$  GeV and the orange band corresponds to varying  $m_{\pi}$  between 171 and 175 GeV. Right panel: the solid red line assumes  $M = 10^{-5}$  GeV. The grey region is disfavoured by the  $\Lambda$  CDM cosmology limit  $\sum m_{\pi} \leq 0.23$  eV. The three solid lines indicate, for reference, the sum of metriton masses predicted, in eV.

Figures taken from [Brivio, 1904.07029]

# Neutrino option



Figures taken from [Brdar et al, 1807.11490]