# Higgs Inflation and the Ambiguities of General Relativity

#### Sebastian Zell

Based on work<sup>1</sup> with

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École Polytechnique Fédérale de Lausanne

#### 2<sup>nd</sup> June 2022

<sup>1</sup> M. S., A. S., I. T. and S. Z., *Higgs inflation in Einstein-Cartan gravity*, JCAP **02** (2021) 008 arXiv:2007.14978.

G. K., M. S., A. S., and S. Z., *Matter matters in Einstein-Cartan gravity*, Phys. Rev. D **104** (2021) 064036, arXiv:2106.13811.

C. R. and S. Z., *Coupling Metric-Affine Gravity to a Higgs-Like Scalar Field*, arXiv:2204.03003.

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Which formulation should we use?

- From Einstein-Cartan to Metric-Affine Gravity
- 2 Higgs Inflation

Higgs Inflation

Summary O

#### The Geometry of Gravity

Einstein: curvature

 $R^{lpha}_{\ eta\gamma\delta}$ 



Higgs Inflation

Summary 0

#### The Geometry of Gravity

Einstein: curvature

 $R^{lpha}_{\ eta\gamma\delta}$ 

Weyl:<sup>2</sup> non-metricity

$$Q_{lphaeta\gamma} = 
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<sup>2</sup> H. Weyl, *Gravitation und Elektrizität*, Sitzungsber. Preuss. Akad. Wiss. (1918) 465.

Higgs Inflation

Summary 0

#### The Geometry of Gravity



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 <sup>3</sup> É. Cartan, Sur une généralisation de la notion de courbure de Riemann et les espaces à torsion, Comptes Rendus, Ac. Sc. Paris 174 (1922) 593.

**Higgs Inflation** 

#### The Geometry of Gravity



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$$\triangleright R^{\alpha}_{\beta\gamma\delta}$$
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- $\triangleright R^{\alpha}_{\beta\gamma\delta}$  and  $T^{\alpha}_{\beta\gamma}$ : Einstein-Cartan
- ▷ ...

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#### Pure Gravity

▶ Independent fundamental fields:  $g_{\mu\nu}$  and  $\Gamma^{\alpha}_{\beta\gamma}$ 

From Einstein-Cartan to Metric-Affine Gravity  $_{OO} \bullet _{O}$ 

Higgs Inflation

Summary O

#### Pure Gravity

- ▶ Independent fundamental fields:  $g_{\mu\nu}$  and  $\Gamma^{\alpha}_{\beta\gamma}$
- ▶ Minimal action: "Palatini"

$$\mathcal{L}=rac{1}{2}M_P^2R$$

Higgs Inflation

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General Einstein-Cartan

$$\mathcal{L} = \frac{1}{2} M_P^2 R + c_1 M_P^2 T_\alpha^{\ \beta\gamma} T^\alpha_{\ \beta\gamma} + \dots$$

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Equations of motion

$$\frac{\delta \mathcal{L}}{\delta \Gamma^{\alpha}_{\beta \gamma}} \sim T_{\alpha}^{\ \beta \gamma} \qquad \Rightarrow \qquad T_{\alpha}^{\ \beta \gamma} = 0$$

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▶ Torsion vanishes dynamically: equivalent to metric GR

Palatini model

$$\mathcal{L}=rac{1}{2}M_P^2R-rac{1}{2}(\partial_\mu h)^2-V(h)$$

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- Many more non-minimal coupling constants
- ▶ No assumptions about geometry: metric-affine gravity



#### 2 Higgs Inflation

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- ▶ First scenarios: metric<sup>4</sup> and Palatini<sup>5</sup>

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• Match amplitude of perturbations in CMB: metric:  $\xi \sim 10^3$  Palatini:  $\xi \sim 10^7$ 

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#### Review of Higgs Inflation



Planck Collaboration, *Planck 2018 results. I.*, arXiv:1807.06205. BICEP and Keck Collaborations, *Improved Constraints on Primordial Gravitational Waves using Planck, WMAP, and BICEP/Keck Observations*, arXiv:2110.00483.

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u
ho\sigma}T_{
u
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- ▶ Only parts of parameter space studied so far<sup>7</sup>

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- Predictions no longer unique
- Only parts of parameter space studied so far<sup>7</sup>
- Consistent with observations for many choices of couplings
- ► Higgs inflation as probe of GR
- Quantum effects can distinguish: perturbative study of preheating not always possible

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▶ Pure gravity: equivalent formulations of GR

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#### Summary

- ▶ Pure gravity: equivalent formulations of GR
- ► Include matter: distinct observable predictions
- Important implications
  - > Higgs inflation
  - ▷ Fermionic dark matter production<sup>8</sup>
  - ▷ ...

#### Additional Material



3 Seven Formulations of General Relativity

4 Full Action

5 Fermionic Dark Matter

Seven Formulations of General Relativity  $_{\odot O}$ 

Full Actio

Fermionic Dark Matter

#### Four Formulations with Curvature



Fermionic Dark Matter

#### Three formulations without curvature<sup>9</sup>



Equivalence to metric GR in pure gravity

# Generically no free parameters

<sup>9</sup>See J. Jiménez, L. Heisenberg, D. Iosifidis, A. Jiménez-Cano, T. Koivisto, General Teleparallel Quadratic Gravity, arXiv:1909.09045.

Full Action

Fermionic Dark Matter

#### Decomposition

► Torsion  

$$T^{\alpha} = g_{\mu\nu} T^{\mu\alpha\nu}$$

$$\hat{T}^{\alpha} = \epsilon^{\alpha\beta\mu\nu} T_{\beta\mu\nu} ,$$

$$t^{\alpha\beta\gamma} \text{ with } g_{\mu\nu} t^{\mu\alpha\nu} = 0 = \epsilon^{\alpha\beta\mu\nu} t_{\beta\mu\nu}$$

Decomposition

$$T_{lphaeta\gamma}=-rac{2}{3}g_{lpha[eta}T_{\gamma]}+rac{1}{6}\epsilon_{lphaeta\gamma
u}\,\hat{T}^
u+t_{lphaeta\gamma}$$

► Non-metricity

$$egin{aligned} &Q^\gamma =& g_{lphaeta} Q^{\gammalphaeta} \ \hat{Q}^\gamma =& g_{lphaeta} Q^{lpha\gammaeta} \ q^{lphaeta\gamma} & ext{with } g_{lphaeta} q^{\gammalphaeta} = 0 = g_{lphaeta} q^{lpha\gammaeta} \end{aligned}$$

Decomposition

$$Q_{lphaeta\gamma}=rac{1}{18}[g_{eta\gamma}(5Q_lpha-2\hat{Q}_lpha)+2g_{lpha(eta}(4\hat{Q}_{\gamma)}-Q_{\gamma)})]+q_{lphaeta\gamma}$$

#### Full action

$$\begin{split} \mathcal{L} = & \frac{1}{2} \Omega^{2}(h) \mathring{R} - \frac{1}{2} \widetilde{K}(h) g^{\alpha\beta} \partial_{\alpha} h \partial_{\beta} h - V(h) \\ &+ A_{1}(h) \mathring{\nabla}_{\alpha} \widehat{T}^{\alpha} + A_{2}(h) \mathring{\nabla}_{\alpha} T^{\alpha} + A_{3}(h) \mathring{\nabla}_{\alpha} \hat{Q}^{\alpha} + A_{4}(h) \mathring{\nabla}_{\alpha} Q^{\alpha} \\ &+ B_{1}(h) Q_{\alpha} Q^{\alpha} + B_{2}(h) \hat{Q}_{\alpha} \hat{Q}^{\alpha} + B_{3}(h) Q_{\alpha} \hat{Q}^{\alpha} \\ &+ C_{1}(h) T_{\alpha} T^{\alpha} + C_{2}(h) \widehat{T}_{\alpha} \widehat{T}^{\alpha} + C_{3}(h) T_{\alpha} \widehat{T}^{\alpha} \\ &+ D_{1}(h) \epsilon_{\alpha\beta\gamma\delta} t^{\alpha\beta\lambda} t^{\gamma\delta}_{\ \lambda} + D_{2}(h) \epsilon_{\alpha\beta\gamma\delta} q^{\alpha\beta\lambda} q^{\gamma\delta}_{\ \lambda} + D_{3}(h) \epsilon_{\alpha\beta\gamma\delta} q^{\alpha\beta\lambda} t^{\gamma\delta}_{\ \lambda} \\ &+ E_{1}(h) T_{\alpha} Q^{\alpha} + E_{2}(h) \widehat{T}_{\alpha} Q^{\alpha} + E_{3}(h) T_{\alpha} \hat{Q}^{\alpha} + E_{4}(h) \widehat{T}_{\alpha} \hat{Q}^{\alpha} \\ &+ B_{4}(h) q_{\alpha\beta\gamma} q^{\alpha\beta\gamma} + B_{5}(h) q_{\alpha\beta\gamma} q^{\beta\alpha\gamma} \\ &+ C_{4}(h) t_{\alpha\beta\gamma} t^{\alpha\beta\gamma} + E_{5}(h) t^{\alpha\beta\gamma} q_{\beta\alpha\gamma} \end{split}$$

Full Action

Fermionic Dark Matter

#### Equivalent Metric Theory

$$\mathcal{L} = rac{1}{2} \mathring{R} - rac{1}{2} \mathcal{K}(h) g^{lphaeta} \partial_{lpha} h \partial_{eta} h - rac{V(h)}{\Omega^4}$$

► Full metric-affine theory

$$K(h) = \frac{1}{(1+\xi h^2)} \left[ 1 + \frac{h^2 \sum_{n=0}^7 P_n h^{2n}}{M_P^2 (\sum_{m=0}^4 O_m h^{2m})^2} + \frac{6\xi^2 h^2}{(1+\xi h^2)} \right]$$

Einstein-Cartan formulation

$$K(h) = \frac{1}{(1+\xi h^2)} \left[ 1 + \frac{8h^2 \sum_{n=0}^{1} H_n h^{2n}}{M_P^2 \sum_{m=0}^{2} O_m h^{2m}} + \frac{6\xi^2 h^2}{(1+\xi h^2)} \right]$$

• Coupling of fermion  $\Psi$  to torsion (or non-metricity)

$$\left(\xi_1\partial^{\mu}h^2\right)T^{\nu}_{\ \mu\nu}$$

• Coupling of fermion  $\Psi$  to torsion (or non-metricity)

$$\left(\xi_1\partial^{\mu}h^2+\zeta_1\bar{\Psi}\gamma^{\mu}\Psi+\zeta_2\bar{\Psi}\gamma_5\gamma^{\mu}\Psi\right)T^{\nu}_{\ \mu\nu}$$

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Equivalent metric theory

$$\mathcal{L} \supset rac{1}{M_P^2} \left( \zeta_1 \, ar \Psi \gamma^\mu \Psi + \zeta_2 \, ar \Psi \gamma_5 \gamma^\mu \Psi 
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Full Actio

Fermionic Dark Matter ○●○

#### Dark Matter Production<sup>10</sup>

► Singlet fermion *N* in early Universe



Full Actio

Fermionic Dark Matter ○●○

#### Dark Matter Production<sup>10</sup>

• Singlet fermion N in early Universe



Portal to dark matter

Full Actio

Fermionic Dark Matter ○●○

#### Dark Matter Production<sup>10</sup>

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- Portal to dark matter
- Relative abundance

$$\frac{\Omega_N}{\Omega_{DM}} = 10^{-2} \frac{m_N}{10 \text{ keV}} (\zeta_1 + \zeta_2)^4 \frac{T^3}{M_P^3}$$

Full Action

Fermionic Dark Matter ○●○

#### Dark Matter Production<sup>10</sup>

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All of dark matter for

$$m_N = 10 \text{ keV}, \ldots, 10^8 \text{ GeV}$$

#### Example: Warm Dark Matter

Cutoff in Palatini Higgs inflation<sup>11</sup>

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Fermionic Dark Matter 00●

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Characteristic momentum distribution