Summary

Results

Method

Intro

Coleman-Weinberg Abrikosov-Nielsen-Olesen strings

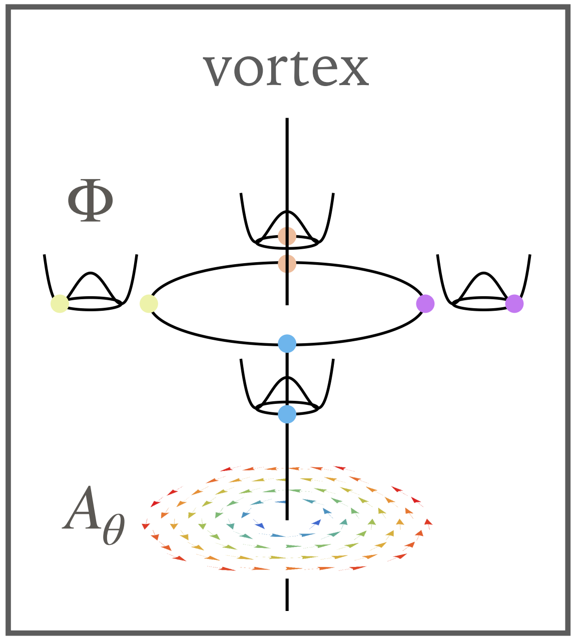
Ryusuke Jinno (IFT)

Planck@Ecole Polytechnique, June 1st, 2022

[2205.04394] with Minoru Eto, Yu Hamada, Muneto Nitta, Masatoshi Yamada

Instituto de Física Teórica UAM-CSIC

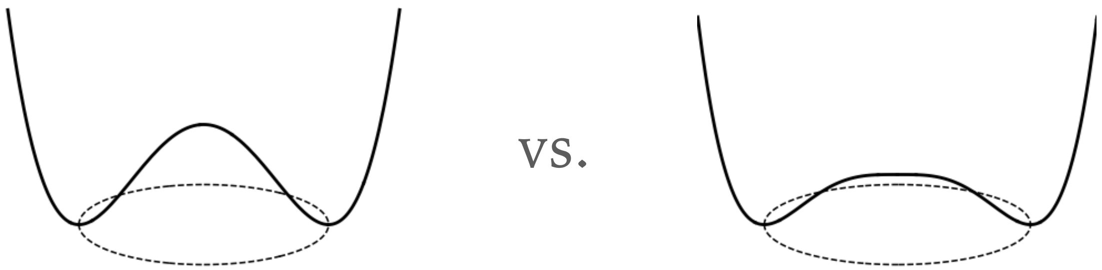
SUMMARY



- We study microphysical properties of U(1) gauged vortices (strings)

- Main interest

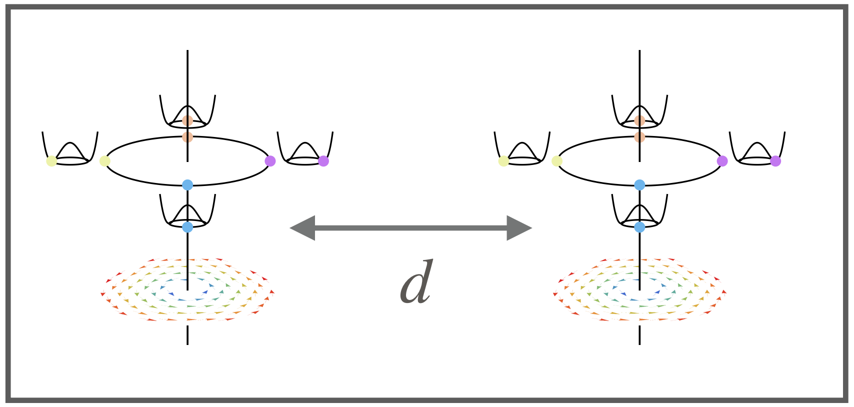
  - Difference btwn. the standard $m|\Phi|^2 + \lambda|\Phi|^4$ and other hep-motivated potentials



- Main result: The force between two strings is...

  - Quadratic-Quartic: only attractive or repulsive

  - Coleman-Weinberg: develops an energy barrier(!)



TOPOLOGICAL DEFECTS

- Topological defects: Ubiquitous objects in quantum field theory

  - Domain walls, Vortices (strings), Monopoles, ...

- Vortices (strings) are interesting in e.g.

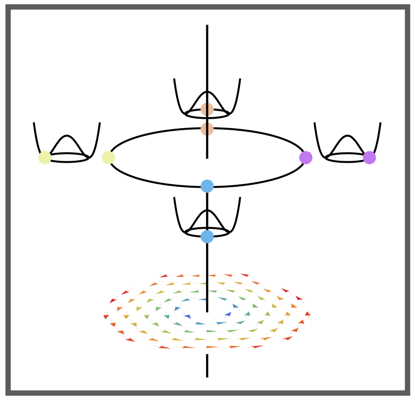
  - Condensed matter systems: { vorticies / magnetic flux tubes } in superconductors

  - Cosmology: cosmic strings and gravitational-wave emission

- Best-known example: Abrikosov-Nielsen-Olesen vortex in superconductors

$V(\Phi) = m^2 |\Phi|^2 + \lambda |\Phi|^4$ with Φ : Cooper pair of electrons

$m^2 < 0 \ \rightarrow \ \langle |\Phi| \rangle = v_\Phi \neq 0$ with $\langle \Phi \rangle$ and $\langle A_\mu \rangle$ being nontrivial $\rightarrow$



ANO vortex

THE STANDARD LORE

➤ Standard ANO strings [Abrikosov '57] [Nielsen, Olesen '73]

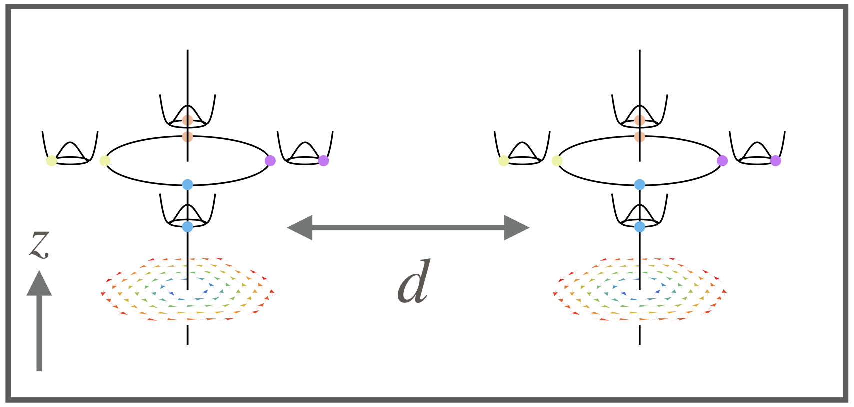
- Described by the Quadratic-Quartic (QQ) potential

$$S = \int d^4x \left(|D_\mu \Phi|^2 - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} - V(\Phi) \right) \qquad V(\Phi) = m^2 |\Phi|^2 + \lambda |\Phi|^4$$

- Only one model parameter determines string properties

$$\beta \equiv \frac{m_\Phi^2}{m_A^2} = \frac{4\lambda v_\Phi^2}{2g^2 v_\Phi^2}$$

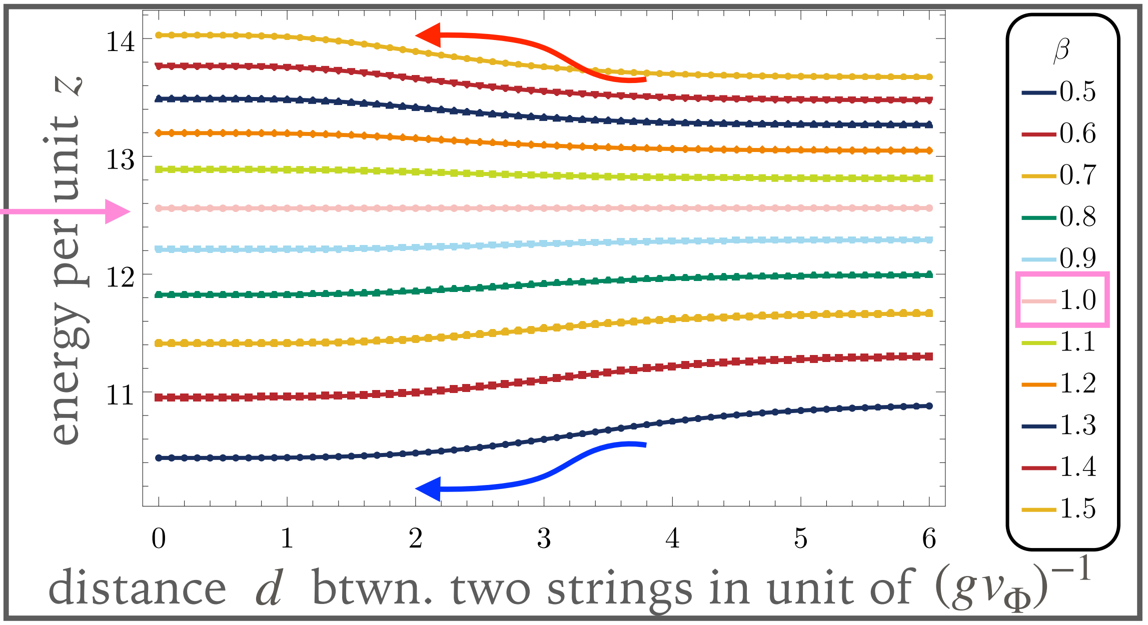
- Force between two strings (both with winding number 1) is attractive or repulsive



$\beta > 1$ (type-II)

$\beta = 1$ (BPS state)

$\beta < 1$ (type-I)



SPONTANEOUS SYMMETRY BREAKING IN HIGH ENERGY PHYSICS

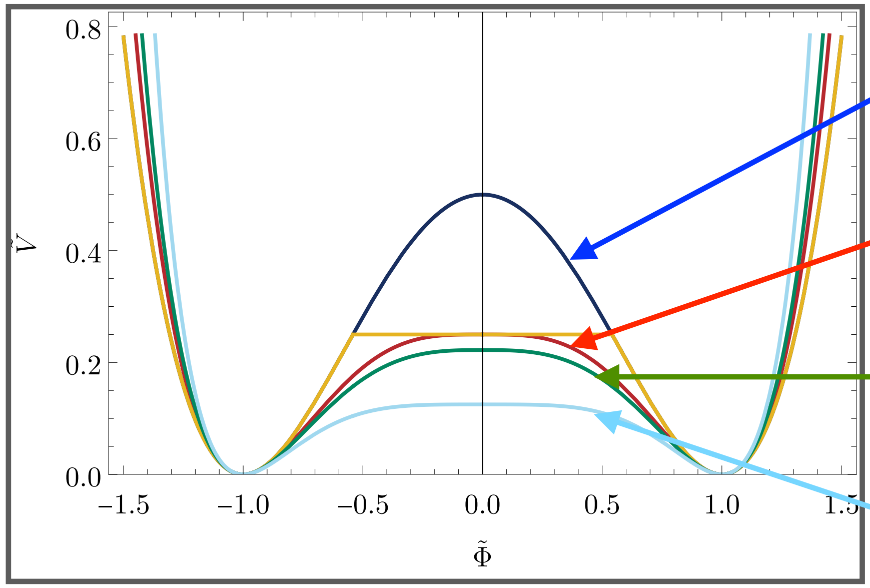
- In high-energy physics, we have a broad class of potentials realizing SSB

  - Higher dimensional operators $V = \lambda |\Phi|^4 + \frac{c}{\Lambda^2} |\Phi|^6$

  - Coleman-Weinberg potentials $V = \lambda(\Phi) |\Phi|^4$ with $\lambda(\Phi) \sim \lambda \ln(|\Phi|^2 / v_\Phi^2)$

    [Coleman, Weinberg '73]

Rescaled potential



$\frac{\beta}{2}(|\Phi|^2 - 1)^2$

$\frac{\beta}{2}\left(\ln|\Phi|^2 - \frac{1}{2}\right)|\Phi|^4$

$\frac{2\beta}{9}(|\Phi|^3 - 1)^2$

$\frac{\beta}{8}(|\Phi|^4 - 1)^2$

Note Both have a flat structure around the origin (→ important later)

- Central question: How differently the resulting vortices behave?


Summary
Results
Method
Intro

RESCALING

- We consider the action

$$S = \int d^4x \left(|D_\mu \Phi|^2 - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} - V(\Phi) \right) \qquad D_\mu = \partial_\mu - igA_\mu$$

with two types of potentials:

$V = \lambda(\lvert\Phi\rvert^2 - v_\Phi^2)^2$ Quadratic-Quartic (QQ)	$V = \lambda \left(\ln \frac{\lvert\Phi\rvert^2}{v_\Phi^2} - \frac{1}{2} \right) \lvert\Phi\rvert^4$ Coleman-Weinberg (CW)

- After rescaling $\Phi \to \Phi/g$ & $A_\mu \to A_\mu/g$ and adopting $gv_\Phi = 1$ unit

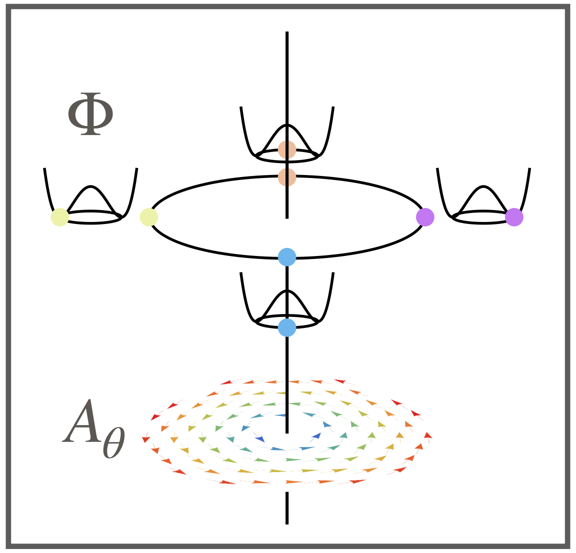
$$S = \frac{1}{g^2} \int d^4x \left(|D_\mu \Phi|^2 - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} - V_\beta(\Phi) \right) \qquad D_\mu = \partial_\mu - iA_\mu$$

$V_\beta = \frac{\beta}{2}(\lvert\Phi\rvert^2 - 1)^2$ (QQ)	$V_\beta = \frac{\beta}{2} \left(\ln \lvert\Phi\rvert^2 - \frac{1}{2} \right) \lvert\Phi\rvert^4$ (CW)

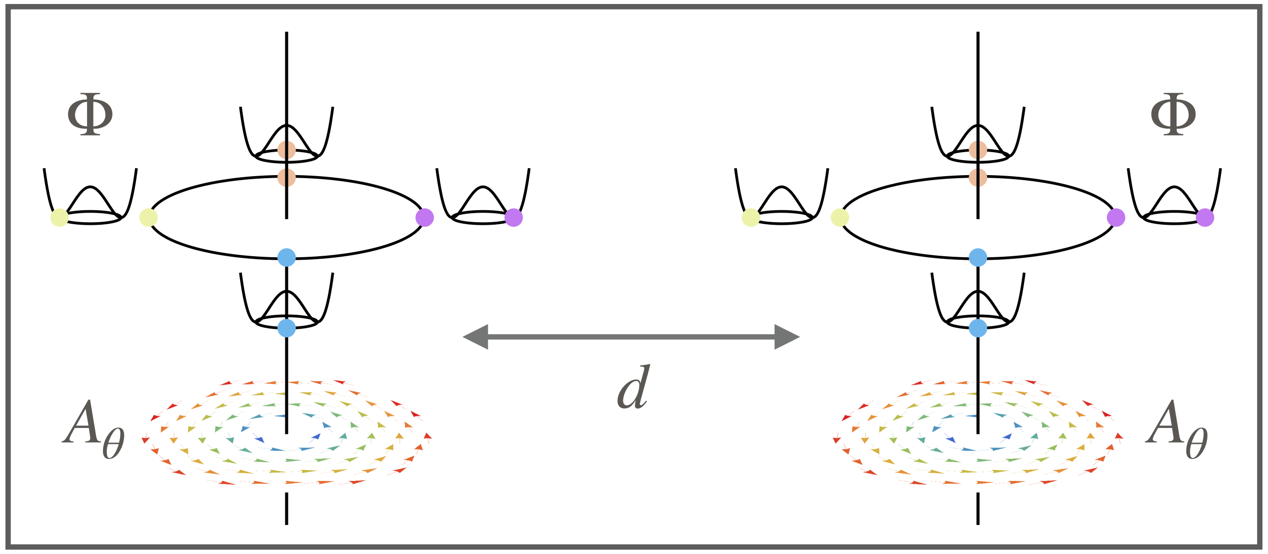
STATIC POTENTIAL

➤ We are interested in the minimum tension (= energy/length) in

- Axisymmetric (≒ one-string) system



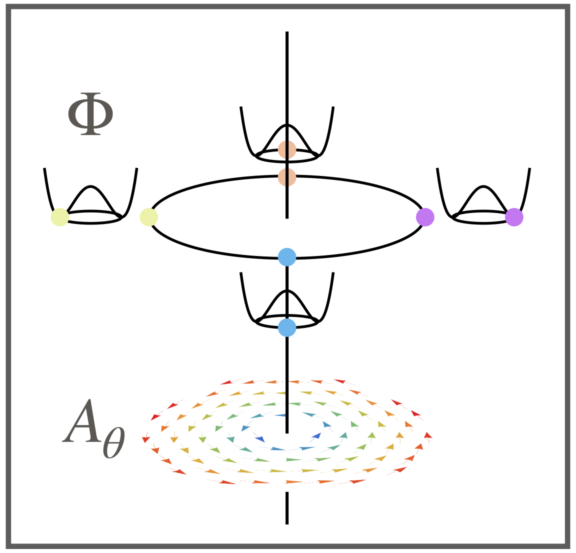
- Two-string system



STATIC POTENTIAL

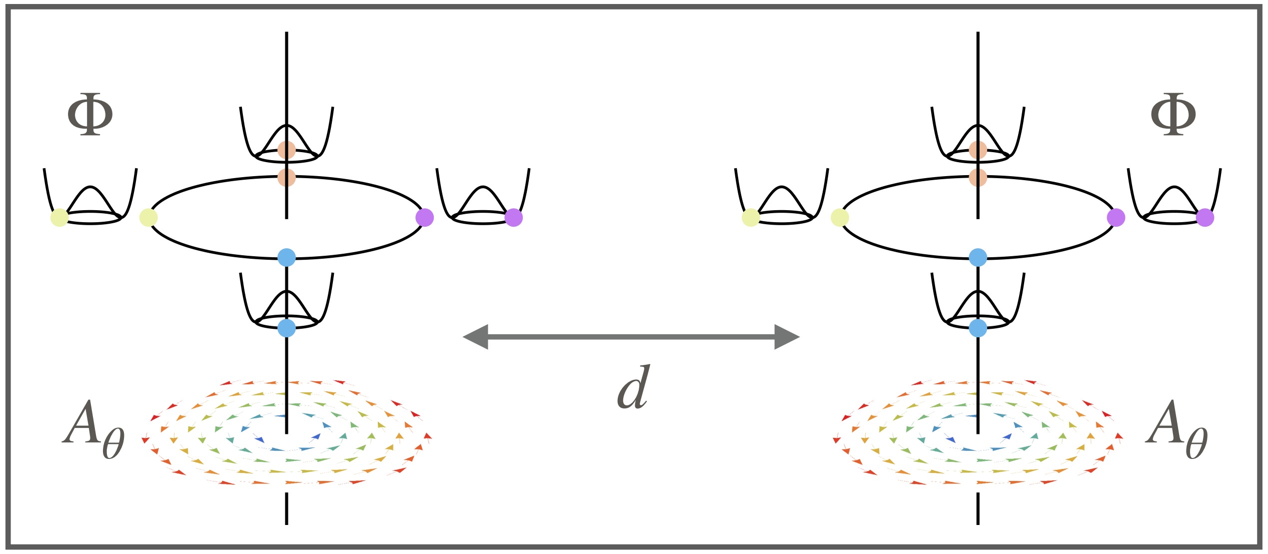
- We are interested in the minimum tension (= energy/length) in

  - Axisymmetric (≒ one-string) system



winding $n = 1$

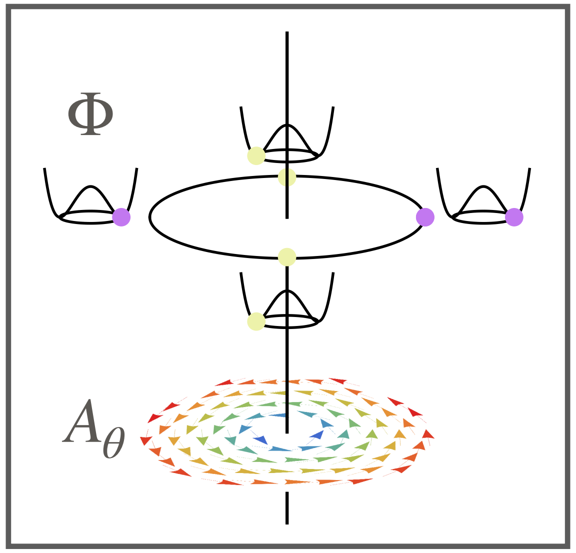
  - Two-string system



STATIC POTENTIAL

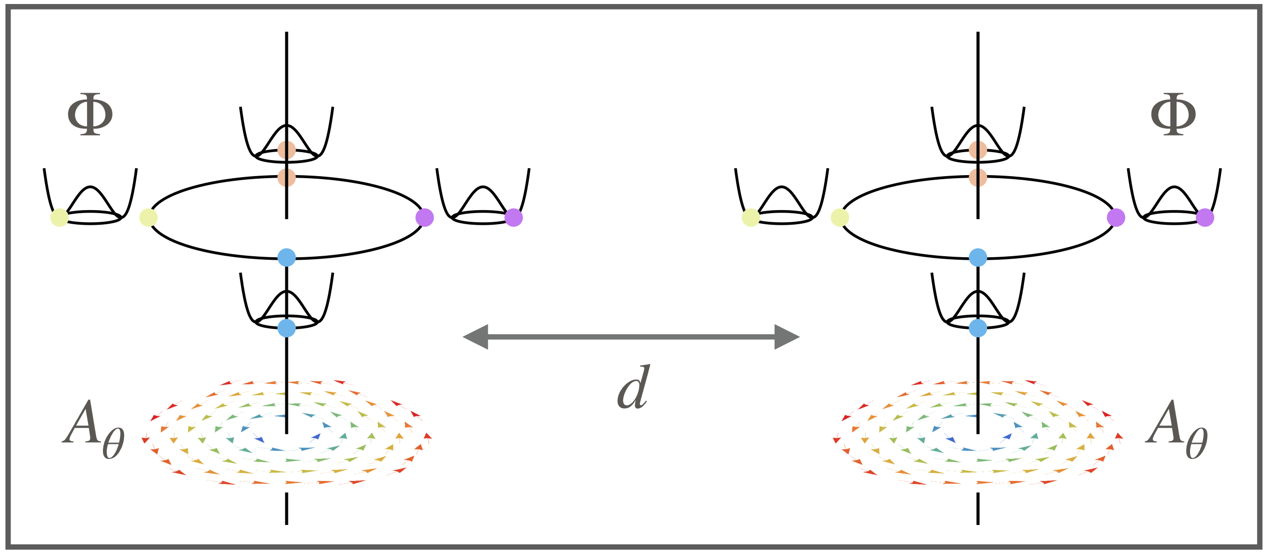
- We are interested in the minimum tension (= energy/length) in

  - Axisymmetric (≒ one-string) system



winding $n = 2$

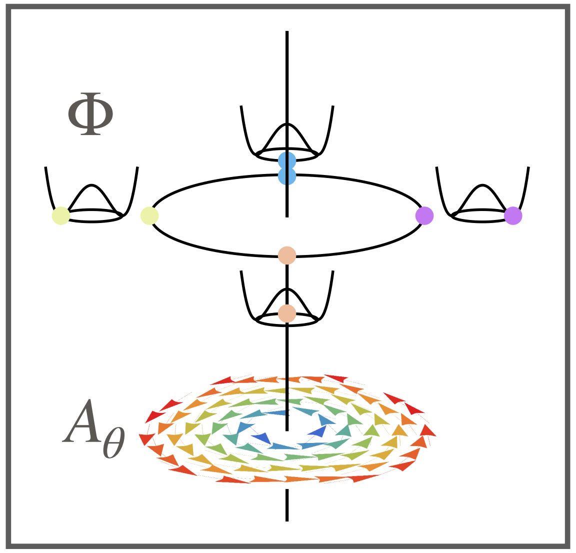
  - Two-string system



STATIC POTENTIAL

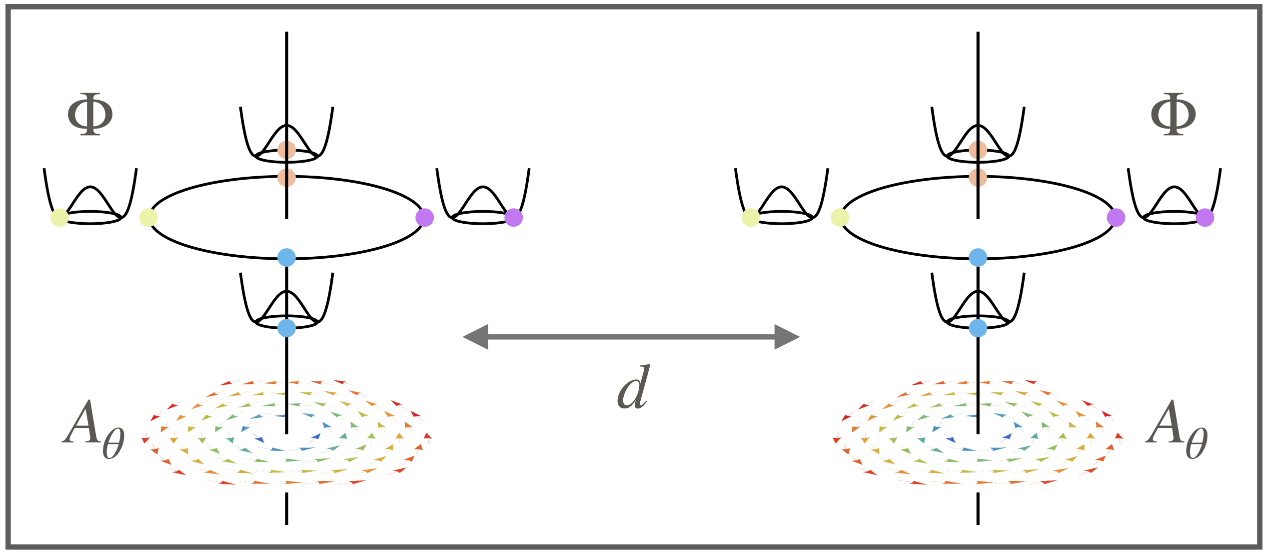
- We are interested in the minimum tension (= energy/length) in

  - Axisymmetric (≒ one-string) system



winding $n = 3$

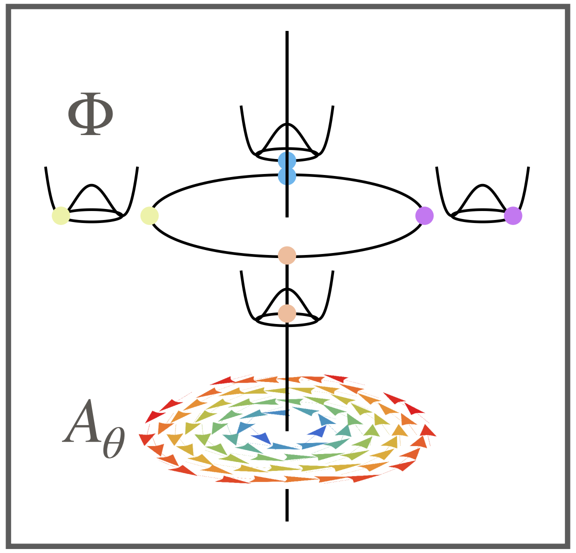
  - Two-string system



STATIC POTENTIAL

- We are interested in the minimum tension (= energy/length) in

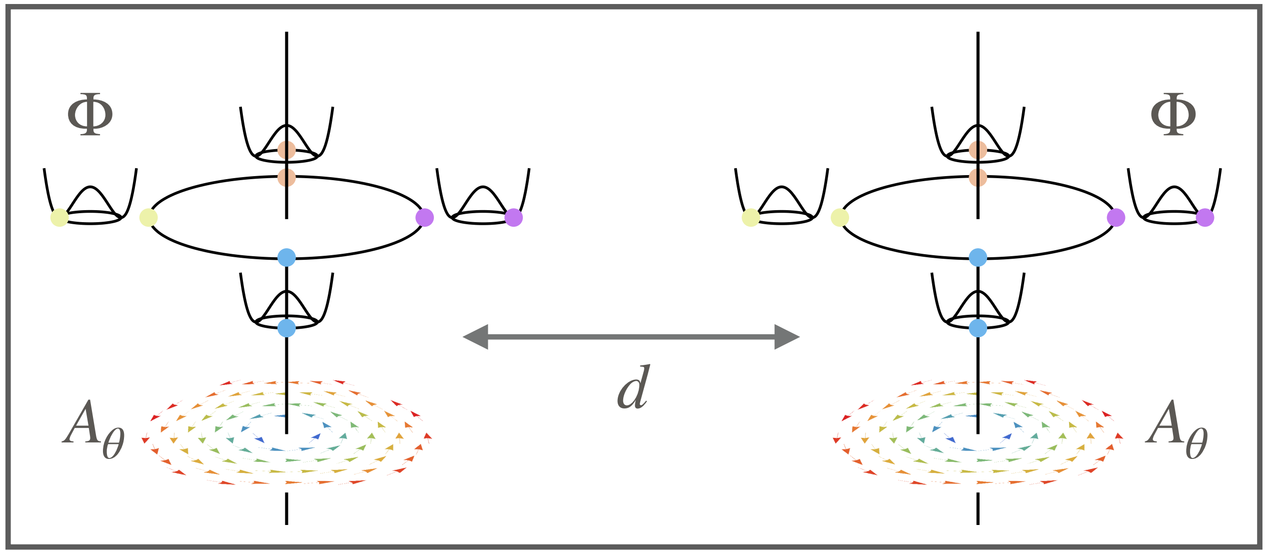
  - Axisymmetric (≒ one-string) system



winding $n = 3$

  - Two-string system

winding n_L



winding n_R

GRADIENT FLOW (FOR TWO-STRING SYSTEM)

- For two-string systems, we minimize the following tension

  - Starting action

$$S = \int d^4x \left(|D_\mu \Phi|^2 - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} - V(\Phi) - \frac{1}{2}(\partial_i A^i)^2 \right)$$

gauge fixing (Coulomb gauge)

  - Tension becomes

$$T = \int dxdy \left[|\partial_i \Phi|^2 + A_i^2 |\Phi|^2 + iA_i(\Phi^* \partial_i \Phi - \Phi \partial_i \Phi^*) + \frac{1}{4}(\partial_i A_j - \partial_j A_i)^2 + \frac{1}{2}(\partial_i A_i)^2 + V(\Phi) \right]$$

Note - Minimum-energy configuration is stationary → any ∂_0 can be dropped

- No electric chage in the system → $A_0 = 0$ from Gauss's law

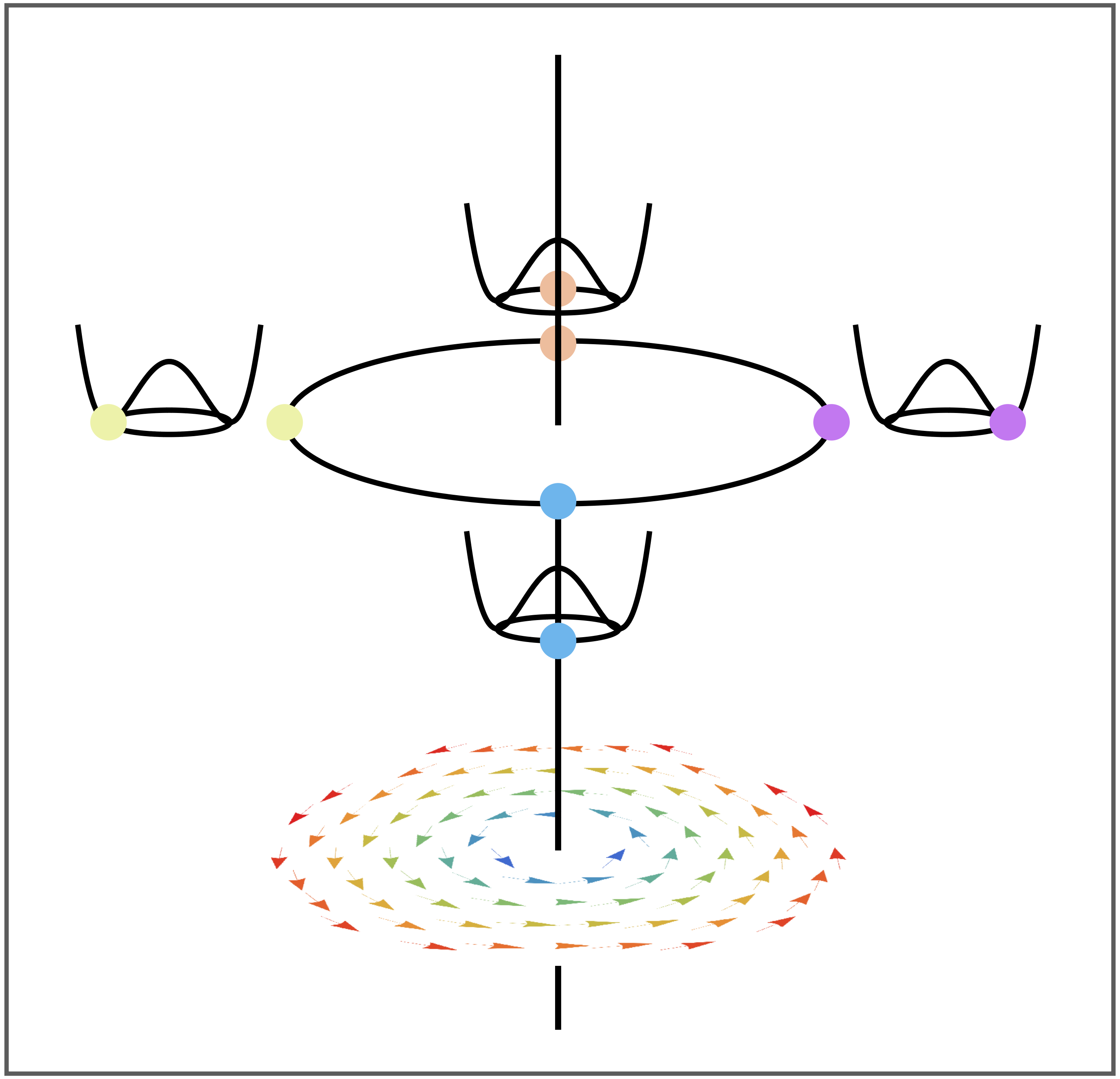
- To minimize the tension, we use gradient flow method (relaxation method)

  - Introduce fictitious time τ s.t. the system becomes diffusive

$$-\frac{\delta T}{\delta X} = \partial_\tau X \qquad X = \Phi \text{ or } A_i$$


Summary
Results
Method
Intro

AXISYMMETRIC STRING (≒ ONE STRING SYSTEM)



AXISYMMETRIC STRING (≒ ONE STRING SYSTEM)

➤ Field configurations

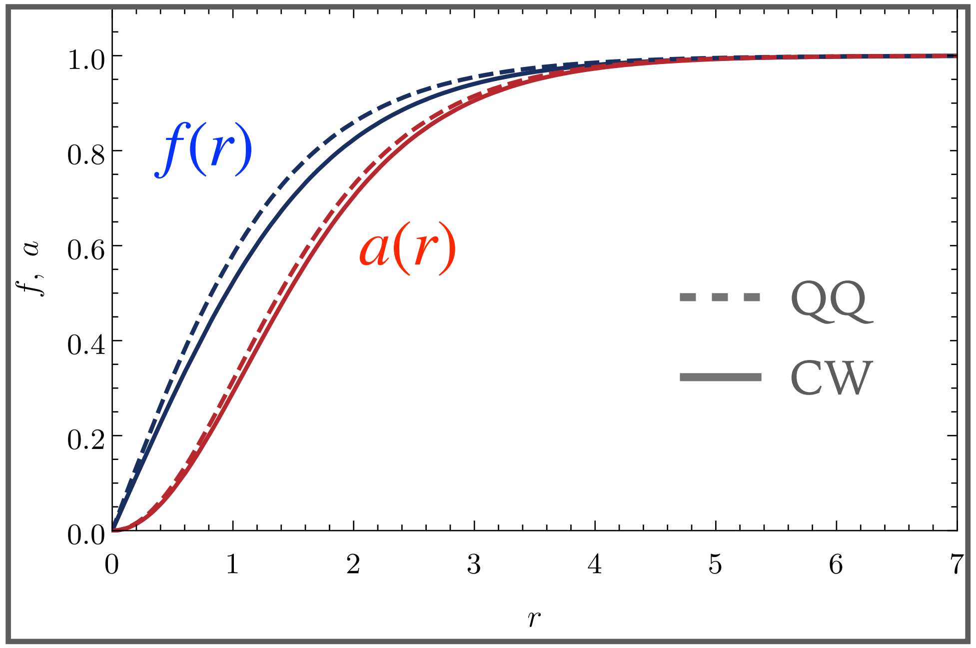
$$\beta \equiv \frac{m_\Phi^2}{m_A^2}$$

- Relevant components are only

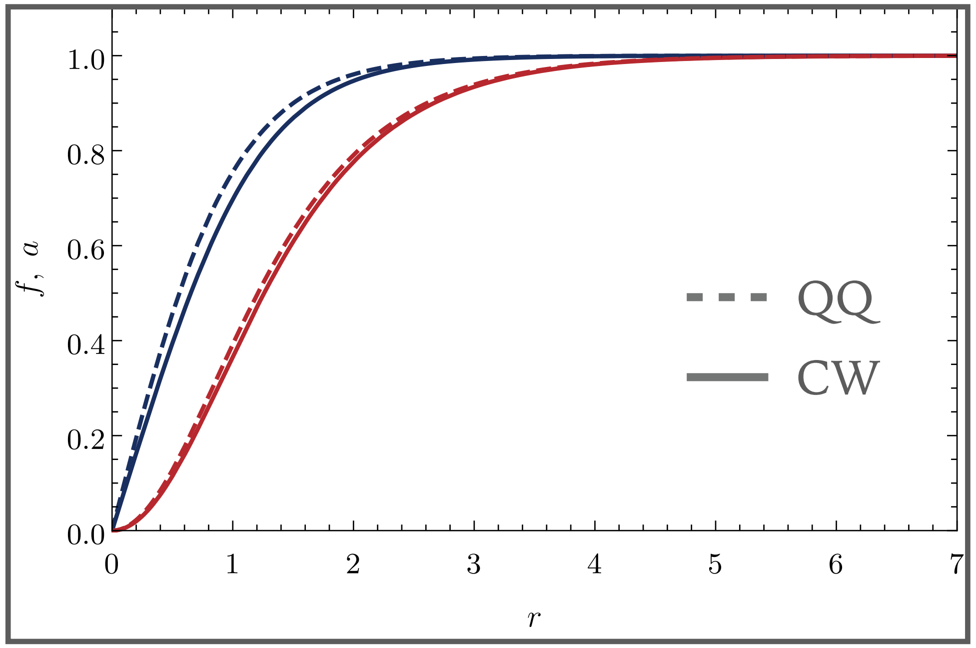
$$\Phi = f(r)e^{in\theta} \qquad A_\theta = na(r) \qquad n \ : \text{winding number}$$

- No significant difference in field configurations (that minimize the tension)

$\beta = 0.5, \quad n = 1$



$\beta = 1.5, \quad n = 1$



AXISYMMETRIC STRING (≒ ONE STRING SYSTEM)

➤ Energy composition

- Definition of the kinetic & potential contributions

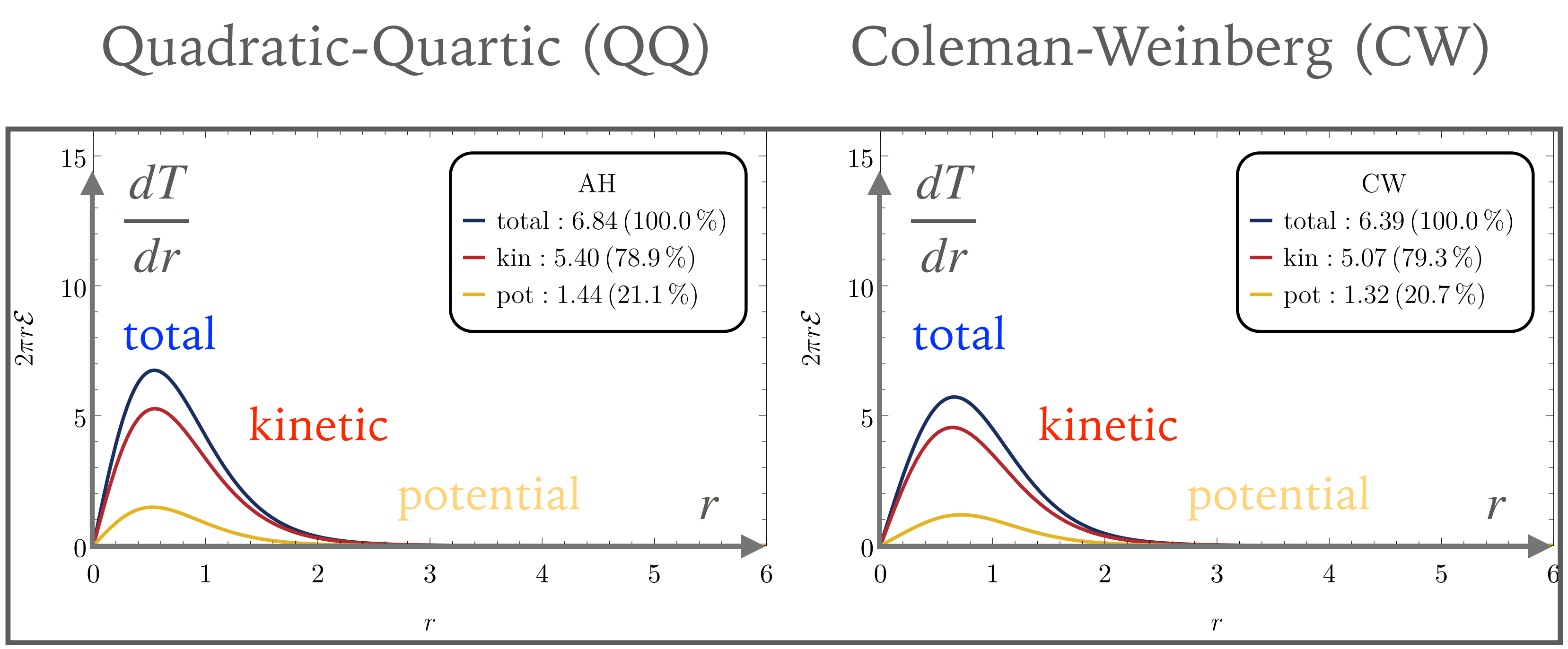
$$T = T_K + T_V \quad \begin{cases} T_V : \text{tension from the potential} \\ T_K : \text{the rest } (\equiv T - T_V) \end{cases}$$

- CW has peaks at slightly larger r (reflecting the flat structure of the potential)

- Overall, no significant difference

mass ratio $\beta = 1.5$

winding number $n = 1$



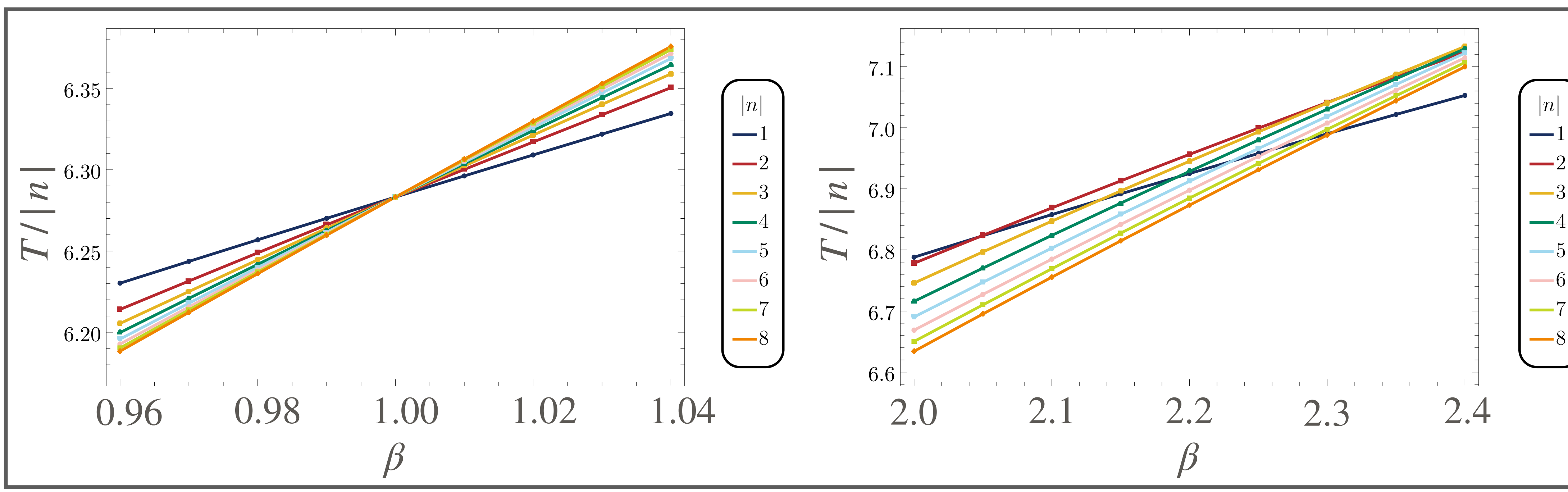
AXISYMMETRIC STRING (≒ ONE STRING SYSTEM)

➤ How tension T behaves for different $\beta \equiv \dfrac{m_\Phi^2}{m_A^2}$ and winding number n

- Quadratic-Quartic crosses at one single point (⭐) → BPS state (next slide)

- No such behavior for Coleman-Weinberg

Quadratic-Quartic (QQ) Coleman-Weinberg (CW)



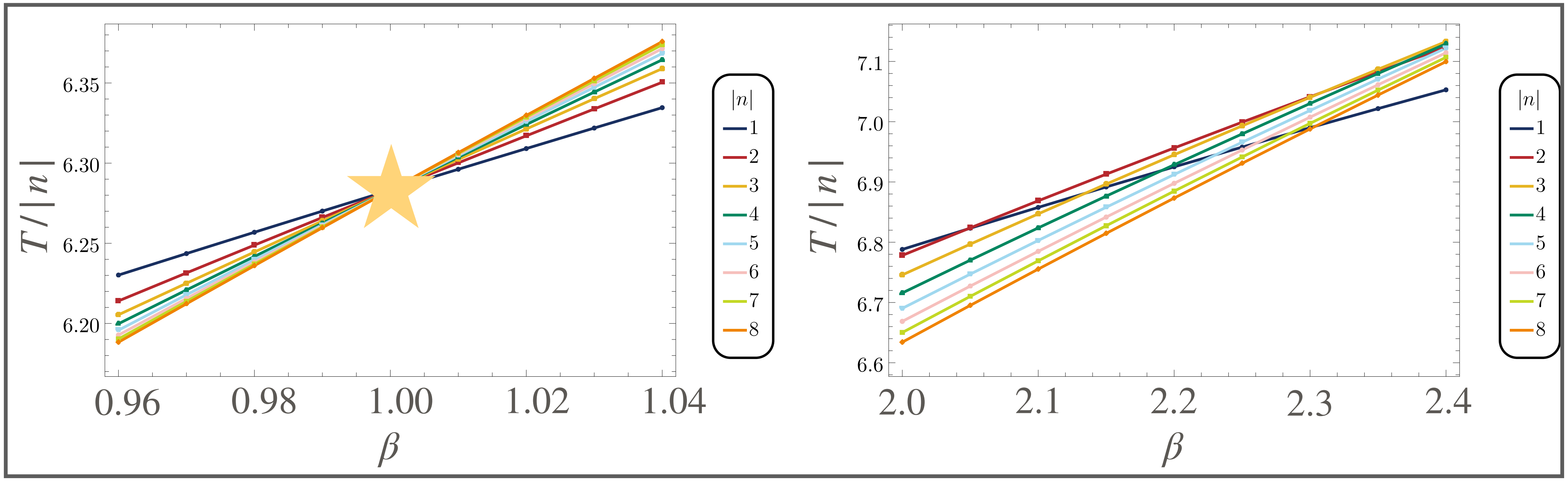
AXISYMMETRIC STRING (≒ ONE STRING SYSTEM)

➤ How tension T behaves for different $\beta \equiv \dfrac{m_\Phi^2}{m_A^2}$ and winding number n

- Quadratic-Quartic crosses at one single point (⭐) → BPS state (next slide)

- No such behavior for Coleman-Weinberg

Quadratic-Quartic (QQ) Coleman-Weinberg (CW)



BPS STATE

➤ BPS (Bogomol'nyi-Prasad-Sommerfield) state

- For Quadratic-Quartic, we can complete the square

$$T = 2\pi|n| + 2\pi\int_0^\infty dr\ r\left[\left(f' + |n|\frac{a-1}{r}f\right)^2 + \frac{n^2}{2r^2}\left(a' + \frac{r}{|n|}(f^2-1)\right)^2 + \frac{1}{2}(\beta-1)(f^2-1)^2\right]$$

- If $\beta = 1$, the last term drops, and the minimization procedure reduces to

$$f' + |n|\frac{a-1}{r}f = 0 \qquad a' + \frac{r}{|n|}(f^2-1) = 0 \qquad \text{BPS equations}$$

- Then tension per unit winding number becomes

$$\frac{T}{|n|} = 2\pi$$

BPS STATE

- BPS (Bogomol'nyi-Prasad-Sommerfield) state

  - For Quadratic-Quartic, we can complete the square

$$T = 2\pi |n| + 2\pi \int_0^\infty dr\ r \left[\left(f' + |n| \frac{a-1}{r} f \right)^2 + \frac{n^2}{r^2} \left(a' + \frac{r}{|n|} (f^2 - 1) \right)^2 + \frac{1}{2} (\beta - 1)(f^2 - 1)^2 \right]$$

Special property only for Quadratic-Quartic

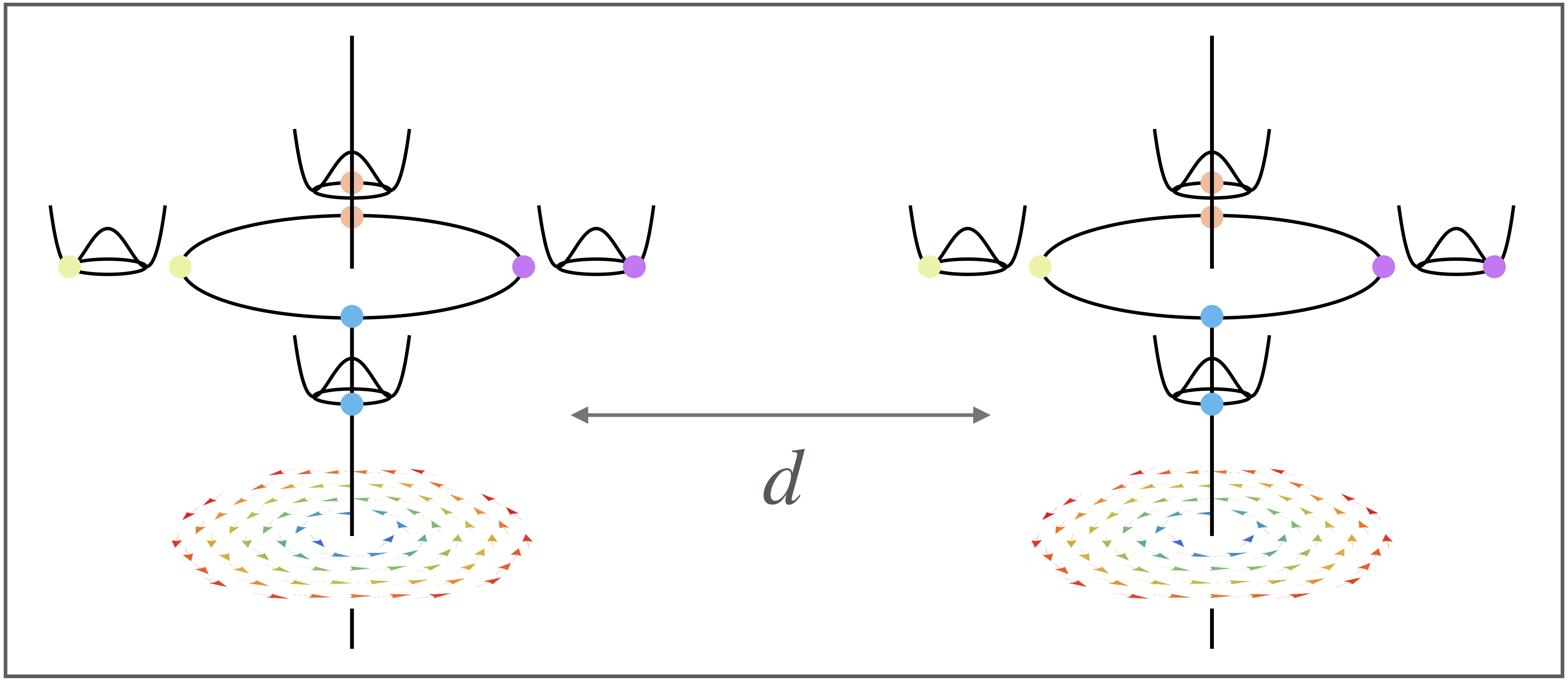
  - If $\beta = 1$, the last term dr[illegible]edure reduces to

$$f' + |n| \frac{a-1}{r} f = 0 \qquad a' + \frac{r}{|n|}(f^2 - 1) = 0 \qquad \text{BPS equations}$$

  - Then tension per unit winding number becomes

$$\frac{T}{|n|} = 2\pi$$

TWO-STRING SYSTEM



TWO-STRING SYSTEM

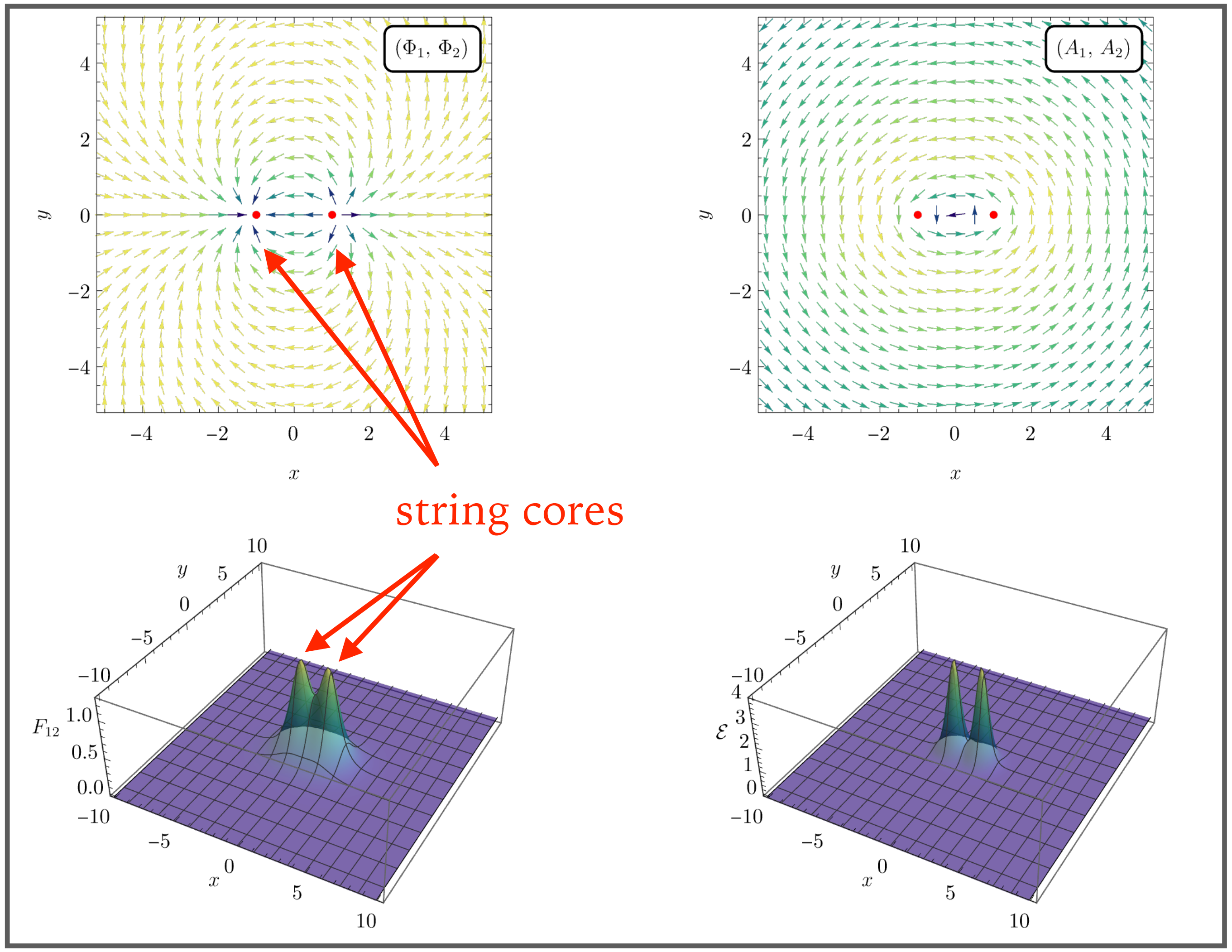
- Field configurations

- Mass ratio $\beta \equiv \frac{m_\Phi^2}{m_A^2} = 2$, winding number $(n_L, n_R) = (1,1)$, string distance $d = 2$

QQ

Scalar field $(\mathrm{Re}\,\Phi, \mathrm{Im}\,\Phi)$

Gauge field (A_x, A_y)



Magnetic flux F_{xy}

Tension density $\frac{d^2T}{dxdy}$

TWO-STRING SYSTEM

➤ Field configurations

- Mass ratio $\beta \equiv \frac{m_\Phi^2}{m_A^2} = 2$, winding number $(n_L, n_R) = (1,1)$, string distance $d = 2$

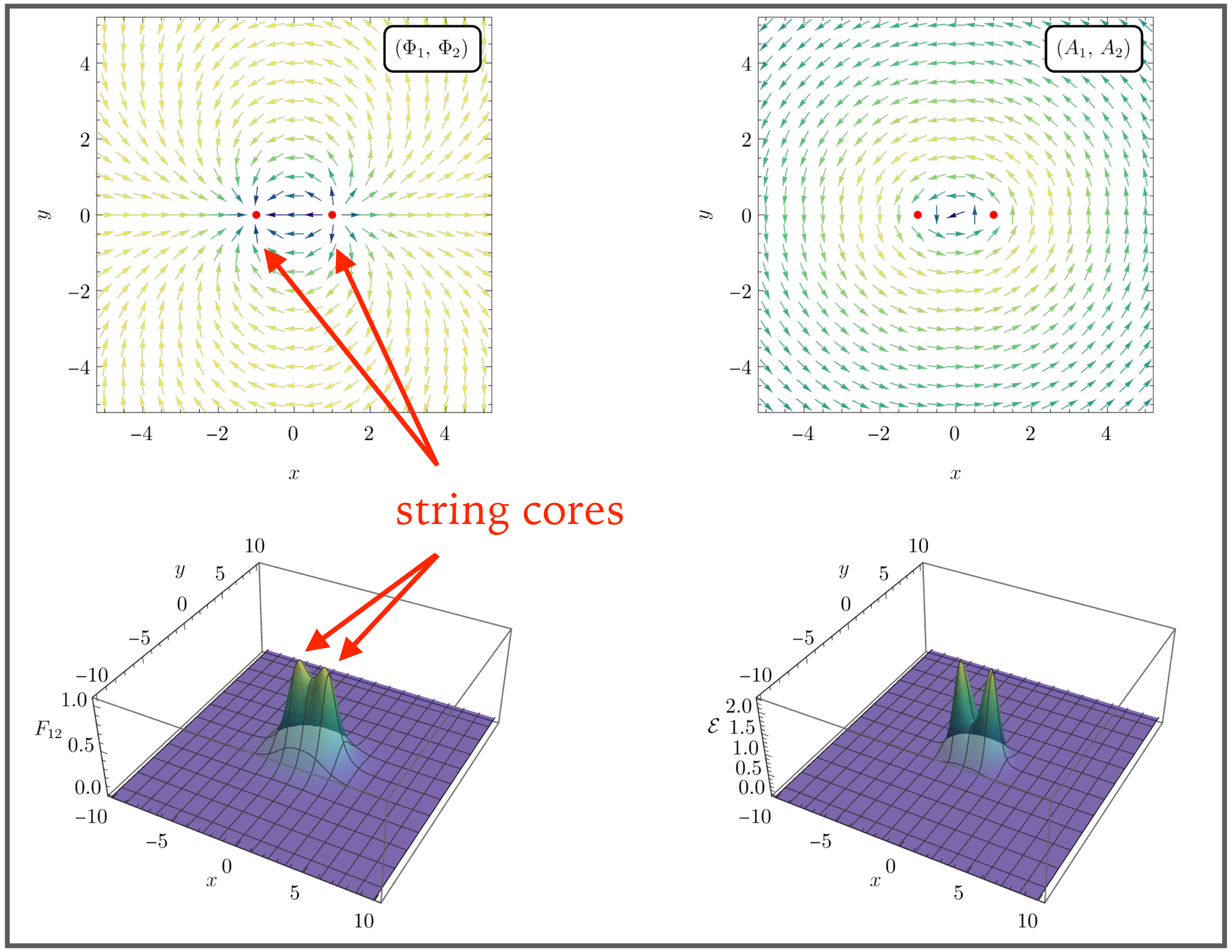
CW

Scalar field

$(\mathrm{Re}\,\Phi, \mathrm{Im}\,\Phi)$

Gauge field

(A_x, A_y)



Magnetic flux

F_{xy}

Tension density

$\frac{d^2T}{dxdy}$

TWO-STRING SYSTEM

➤ Field configurations

- Mass ratio $\beta \equiv \dfrac{m_\Phi^2}{m_A^2} = 2$, winding number $(n_L, n_R) = (1,1)$, string distance $d = 2$

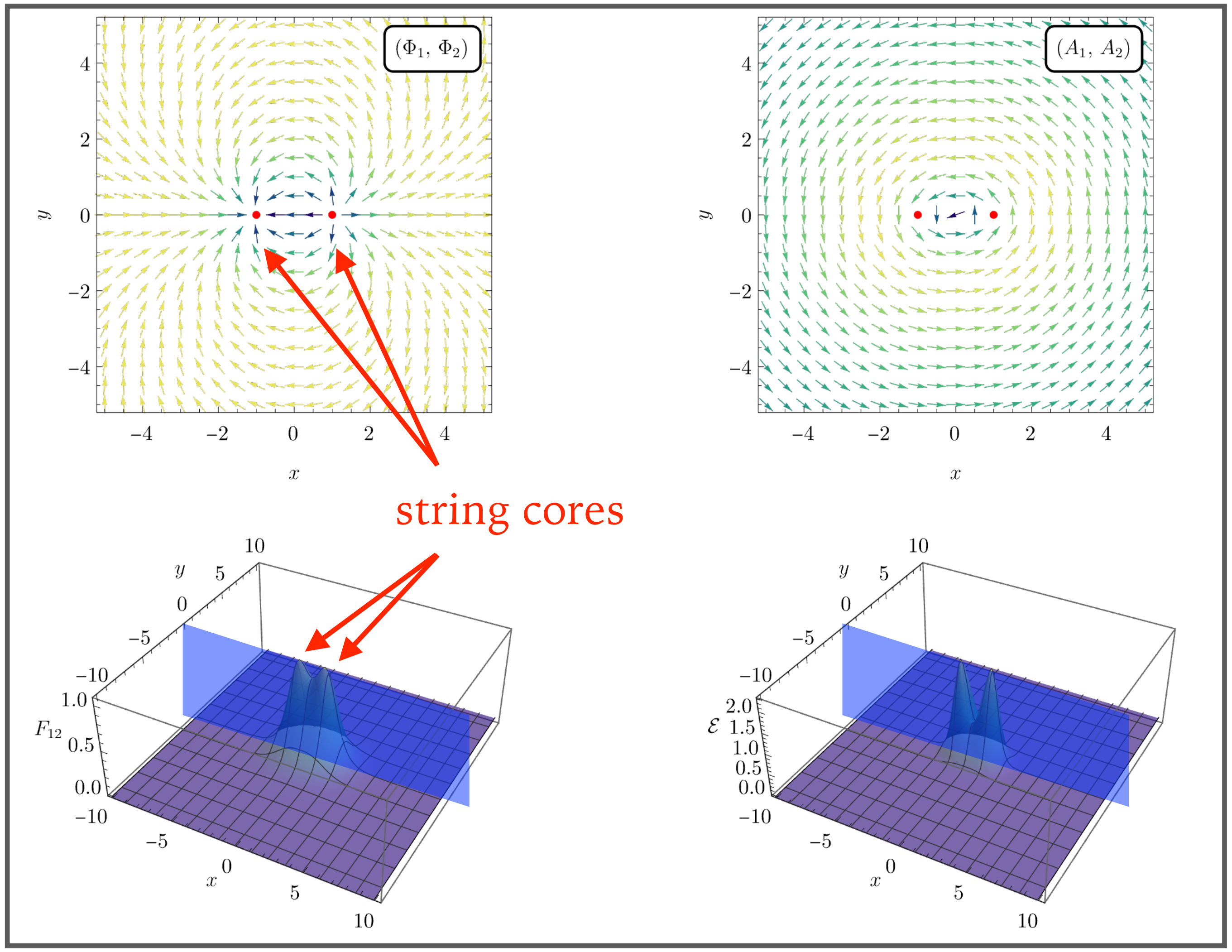
CW

Scalar field

$(\mathrm{Re}\,\Phi, \mathrm{Im}\,\Phi)$

Gauge field

(A_x, A_y)



Magnetic flux

F_{xy}

Tension density

$\dfrac{d^2T}{dxdy}$

TWO-STRING SYSTEM

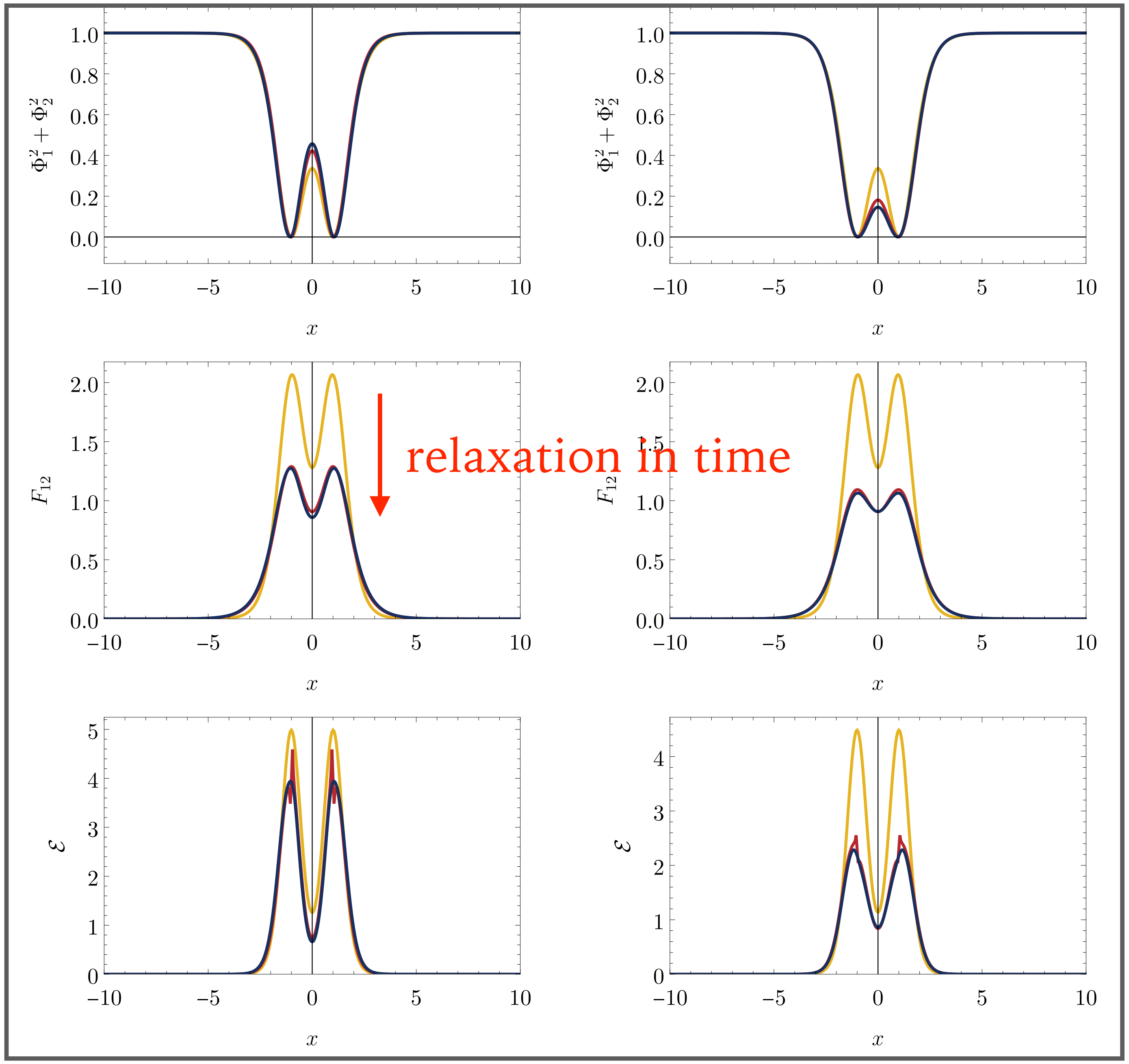
➤ How gradient flow works

QQ CW

$|\Phi|^2$

Magnetic flux F_{xy}

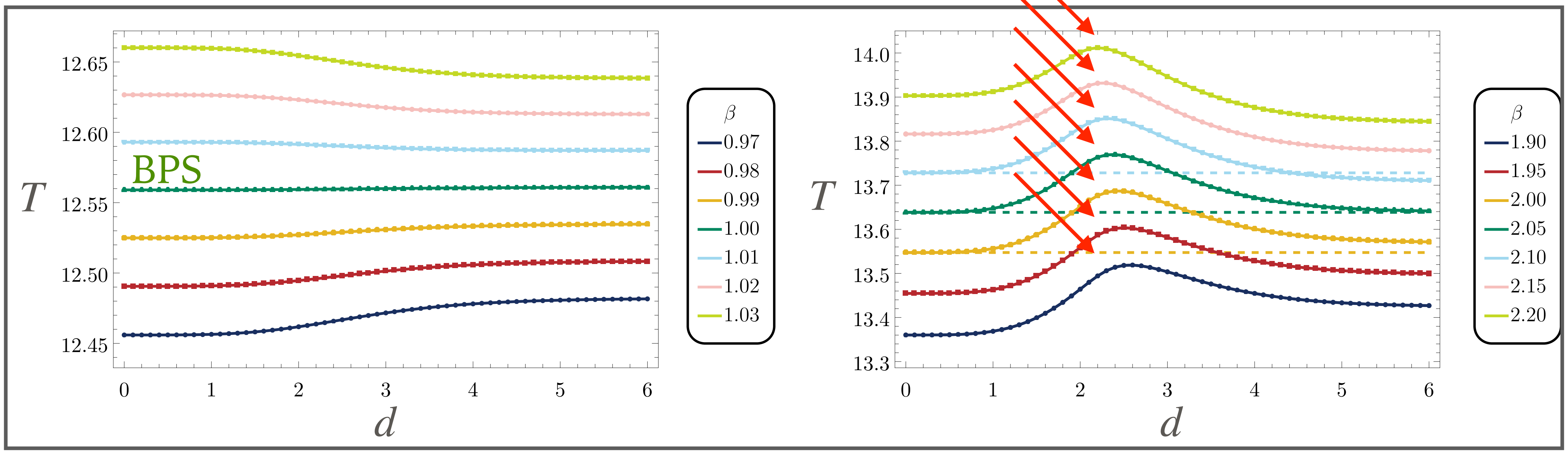
Tension density $\dfrac{d^2T}{dxdy}$



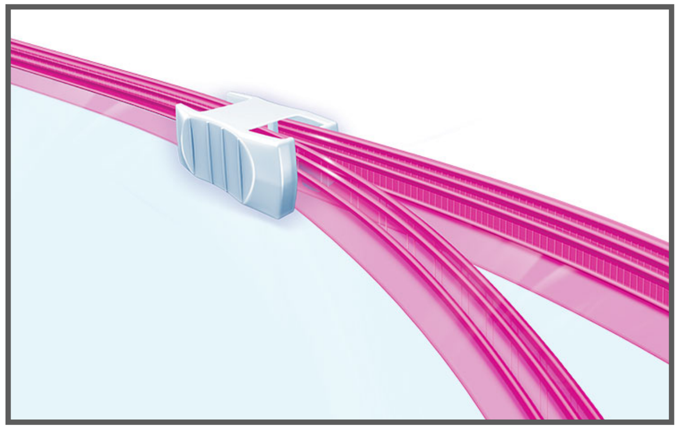
EMERGENCE OF ENERGY BARRIER

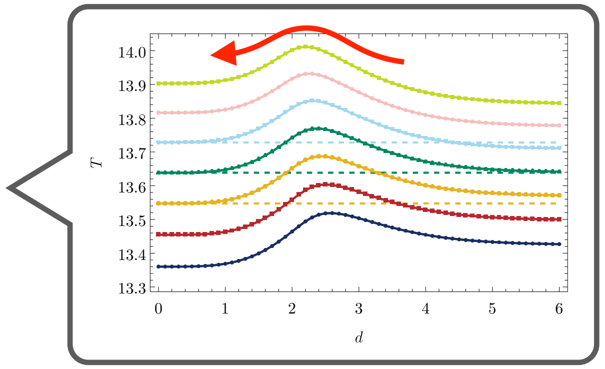
- How tension T behaves for different β for winding number $(n_L, n_R) = (1,1)$

QQ energy barriers! CW



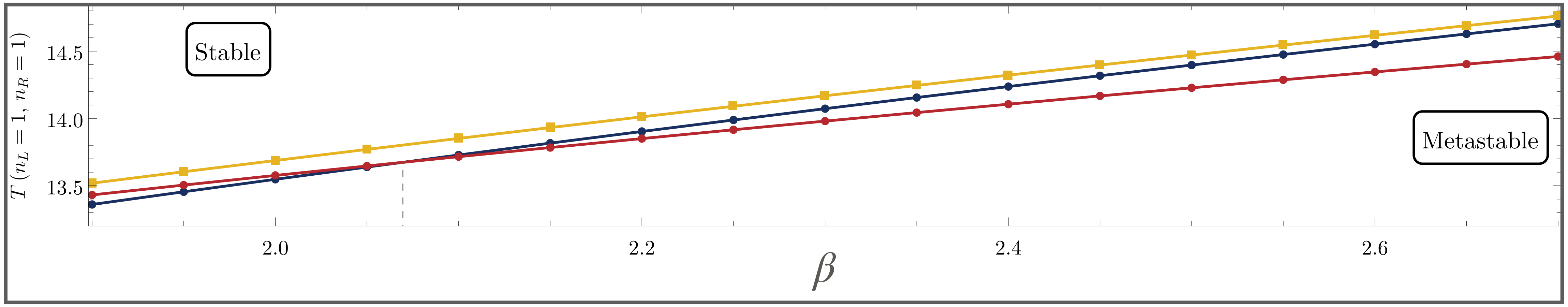
- What does it mean??? ...ziploc!



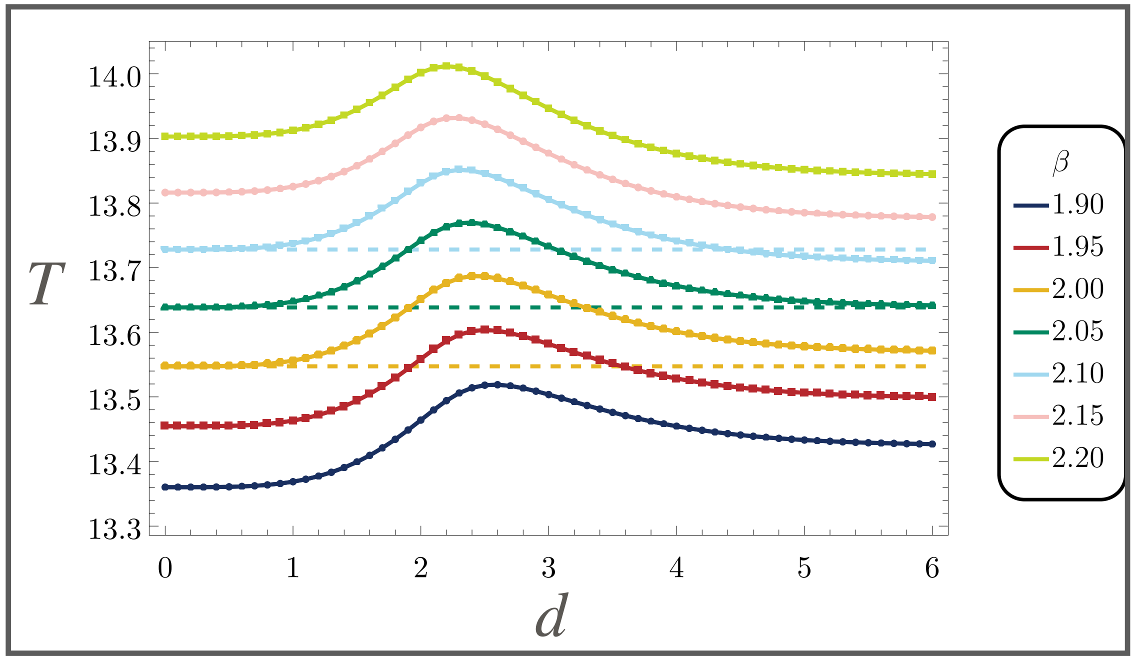


STABILITY & METASTABILITY

- Stability-metastability plot

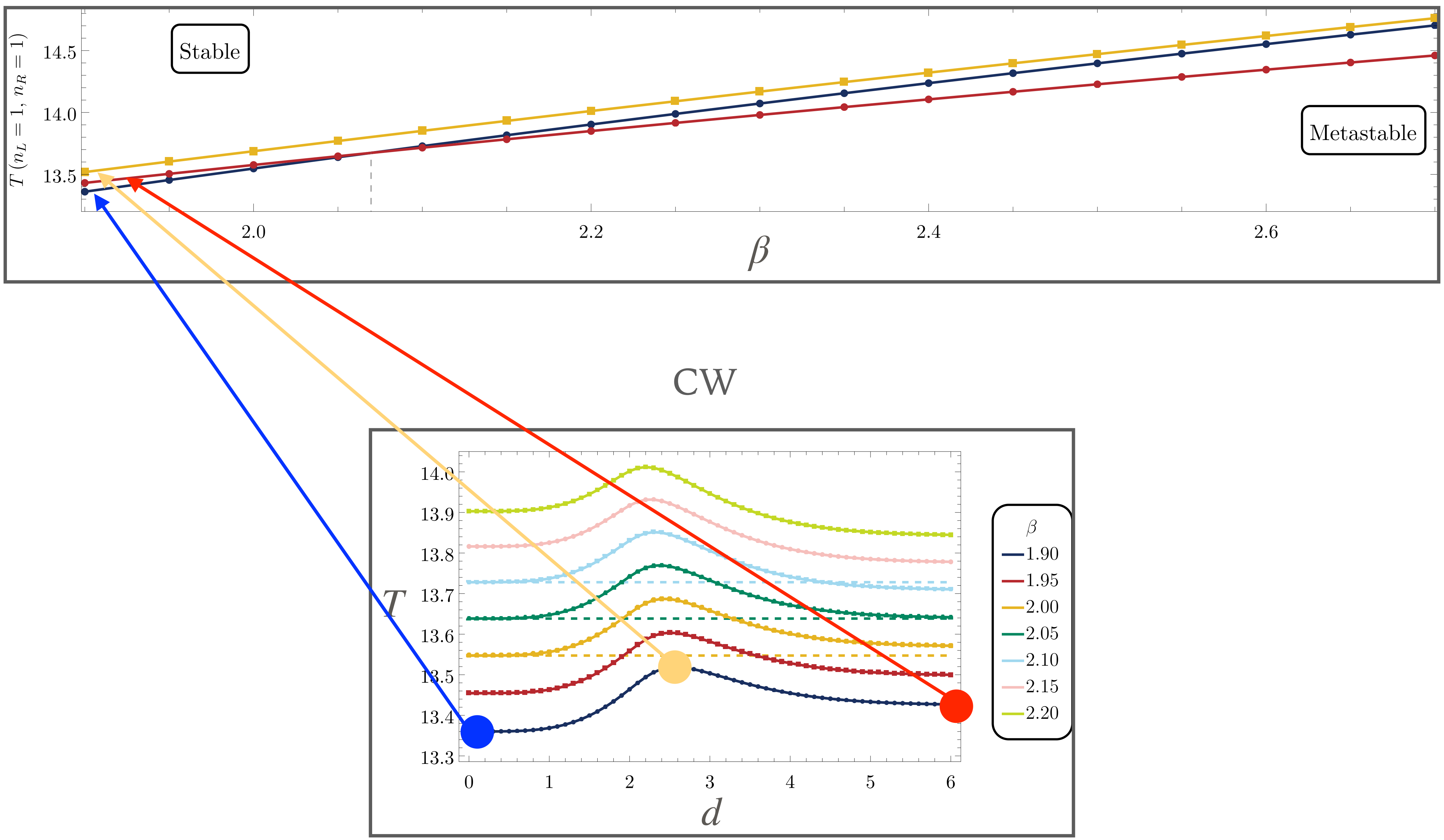


CW



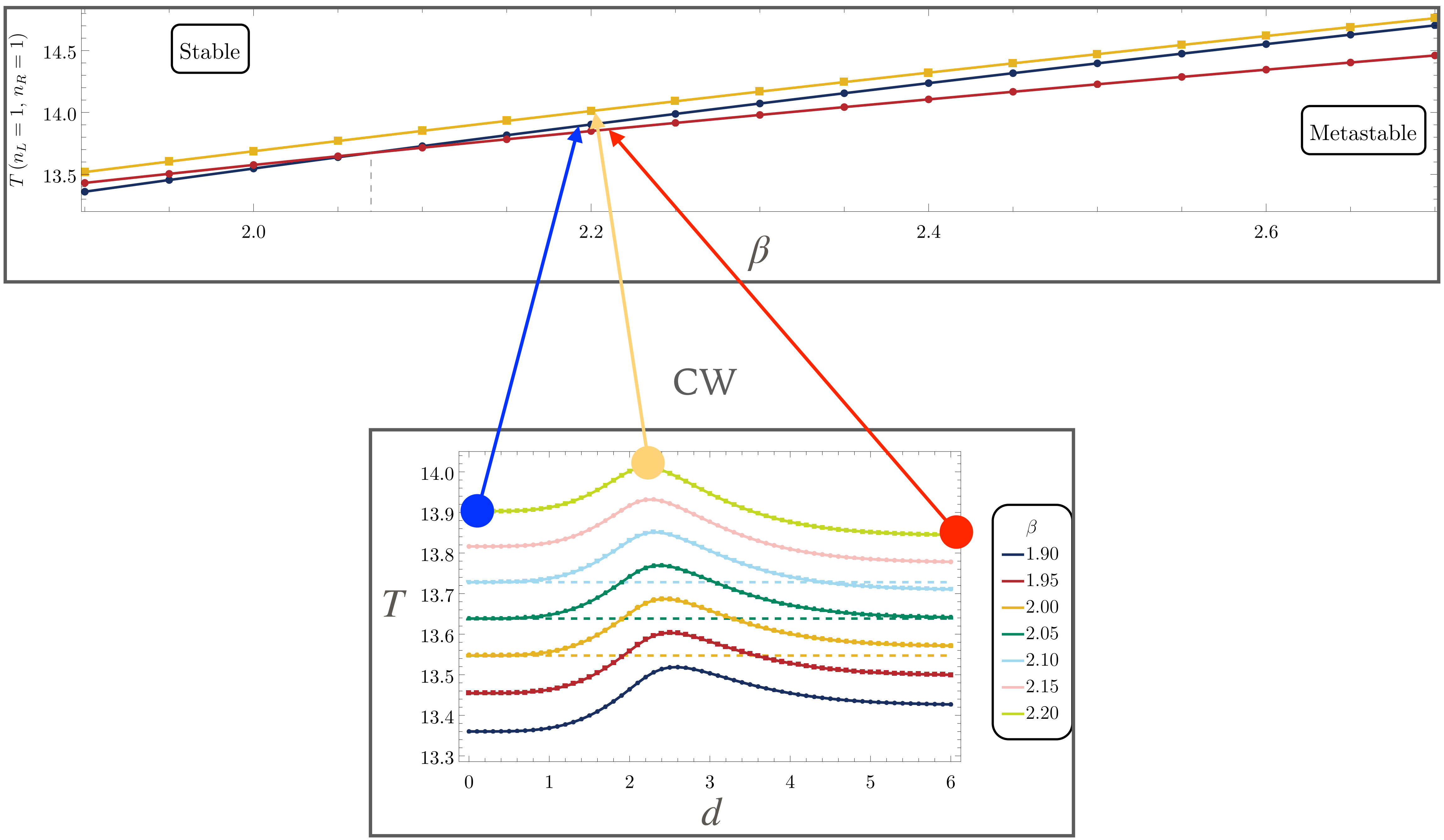
STABILITY & METASTABILITY

➤ Stability-metastability plot



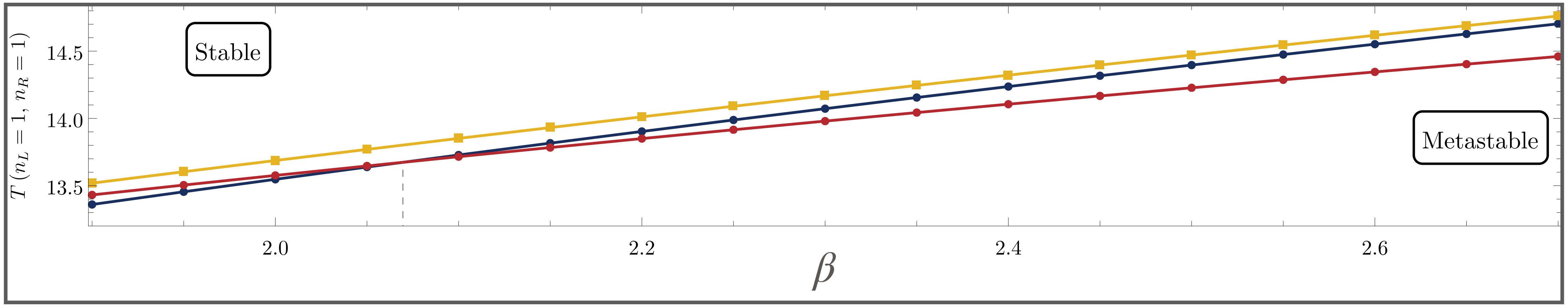
STABILITY & METASTABILITY

➤ Stability-metastability plot

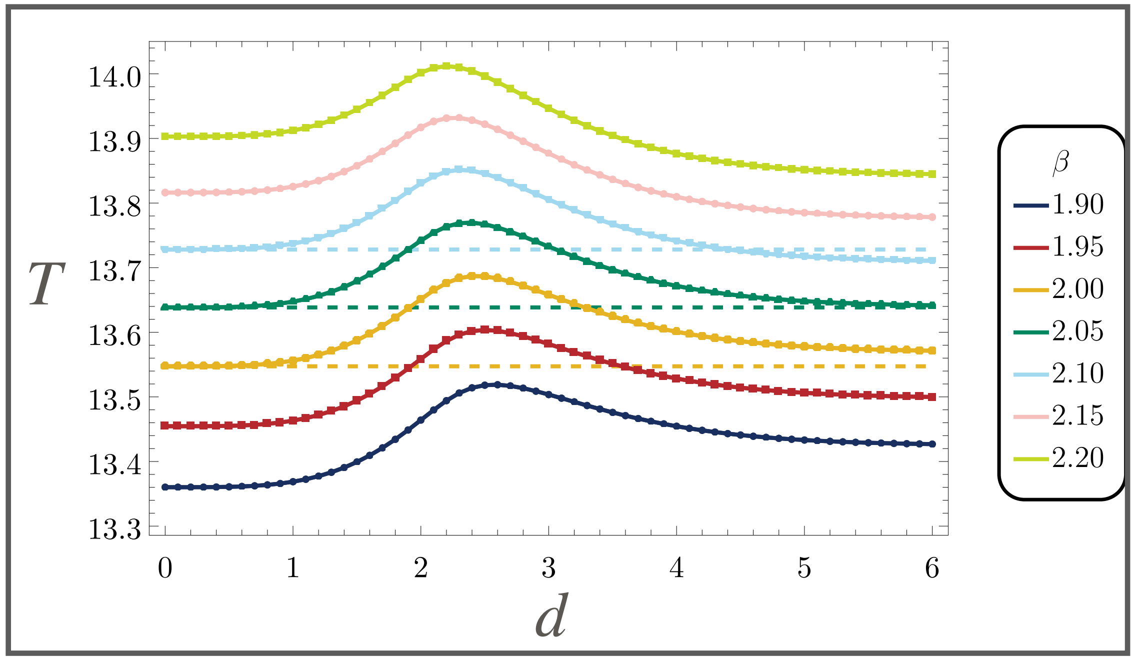


STABILITY & METASTABILITY

➤ Stability-metastability plot

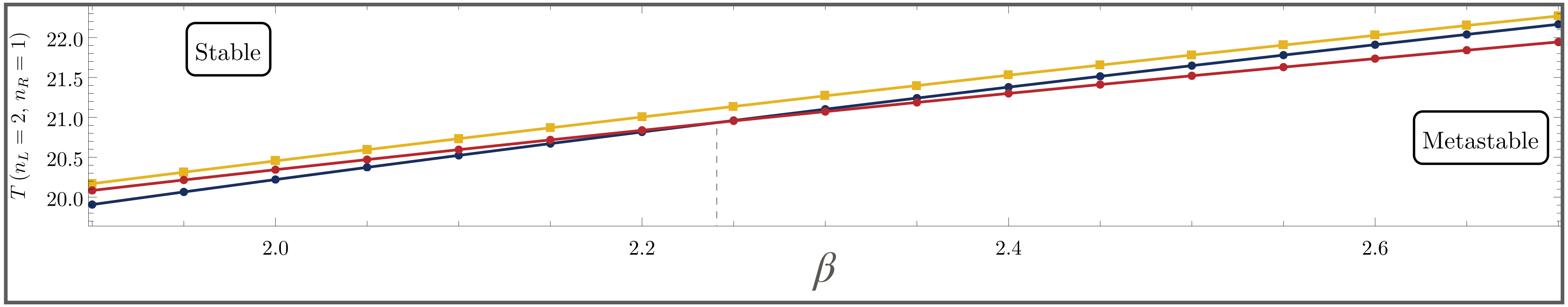


CW



STABILITY & METASTABILITY

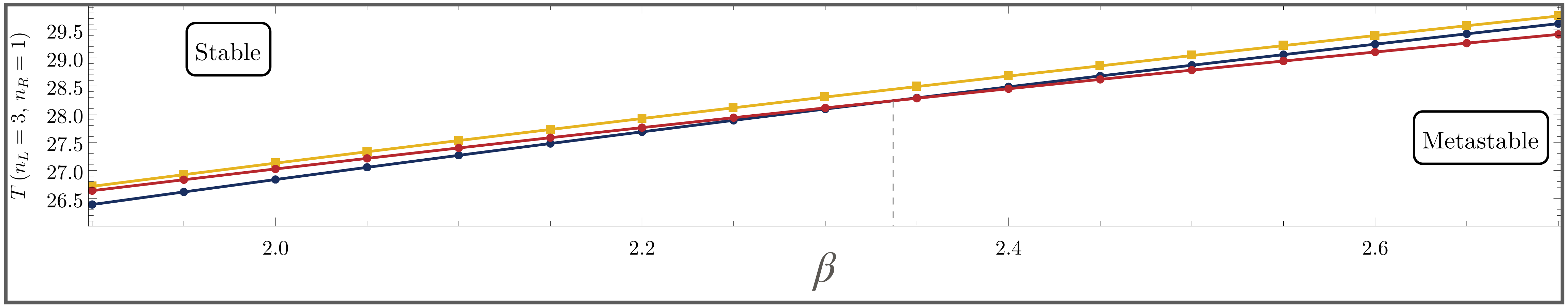
- Stability-metastability plot



$$(n_L, n_R) = (2,1)$$

STABILITY & METASTABILITY

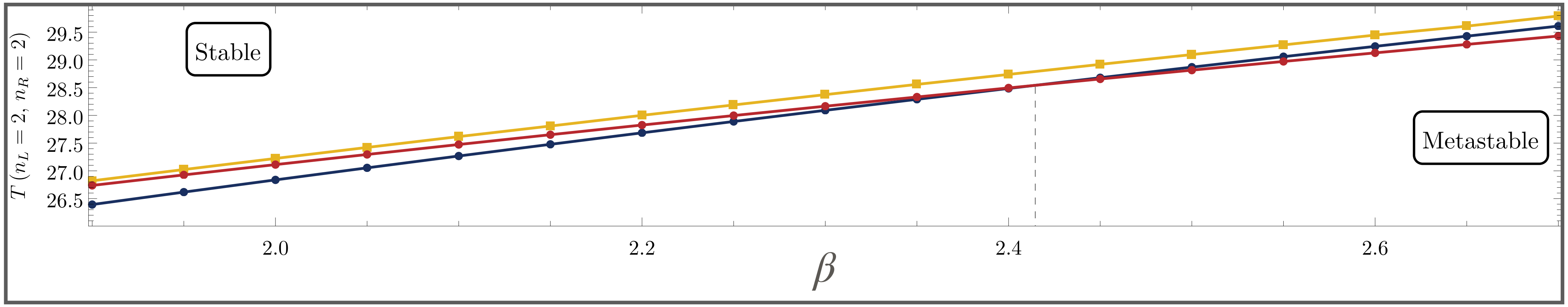
- Stability-metastability plot



$$(n_L, n_R) = (3,1)$$

STABILITY & METASTABILITY

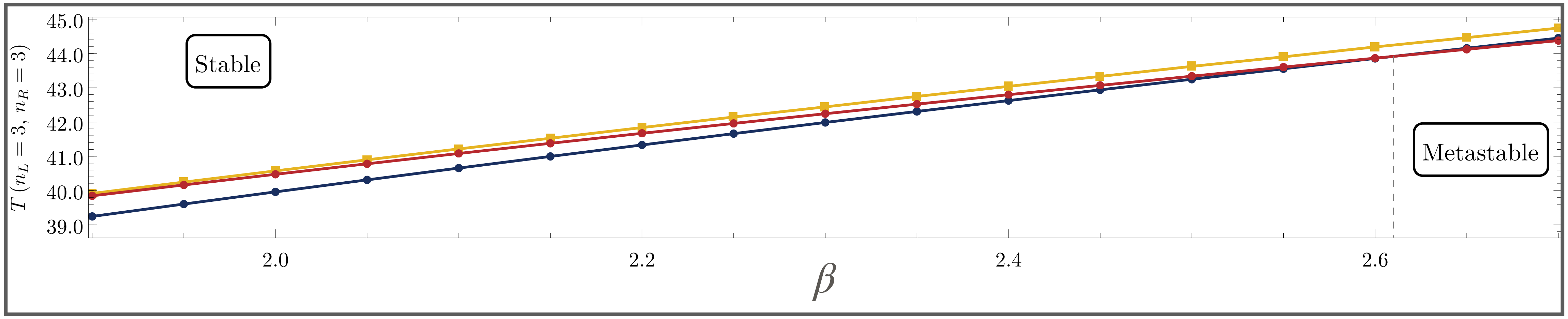
- Stability-metastability plot



$$(n_L, n_R) = (2,2)$$

STABILITY & METASTABILITY

- Stability-metastability plot



$$(n_L, n_R) = (3,3)$$


Summary
Results
Method
Intro

SUMMARY

- We studied microphysical properties of U(1) gauged strings (ANO strings)
- In contrast to Quadratic-Quartic, strings develop an energy barrier in many hep-motivated potentials like the Coleman-Weinberg potential
- Implications
  - Tunneling?
  - Difference in the scaling law? Difference in GW emission?
  - Other possible potentials? Other types of topological defects?

Backup

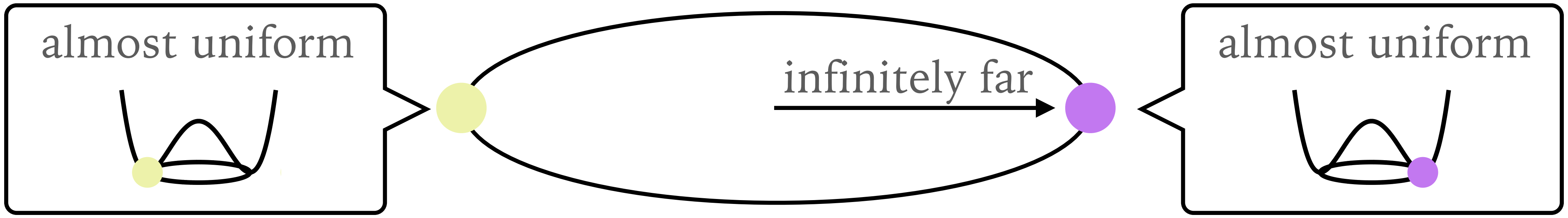
ON THE USE OF THE COLEMAN-WEINBERG POTENTIAL

- The effective potential is just the leading term in the derivative expansion of the effective action, so higher derivatives can in principle be important

$$\Gamma = \int d^4x \left[\tilde{V}(\Phi_B) + Z_\Phi^{(0)} |\partial_\mu \Phi_B|^2 + \cdots \right] = \int d^4x \left[V(\Phi) + |\partial_\mu \Phi|^2 + \cdots \right]$$

- To be conservative, we may at least say the following:

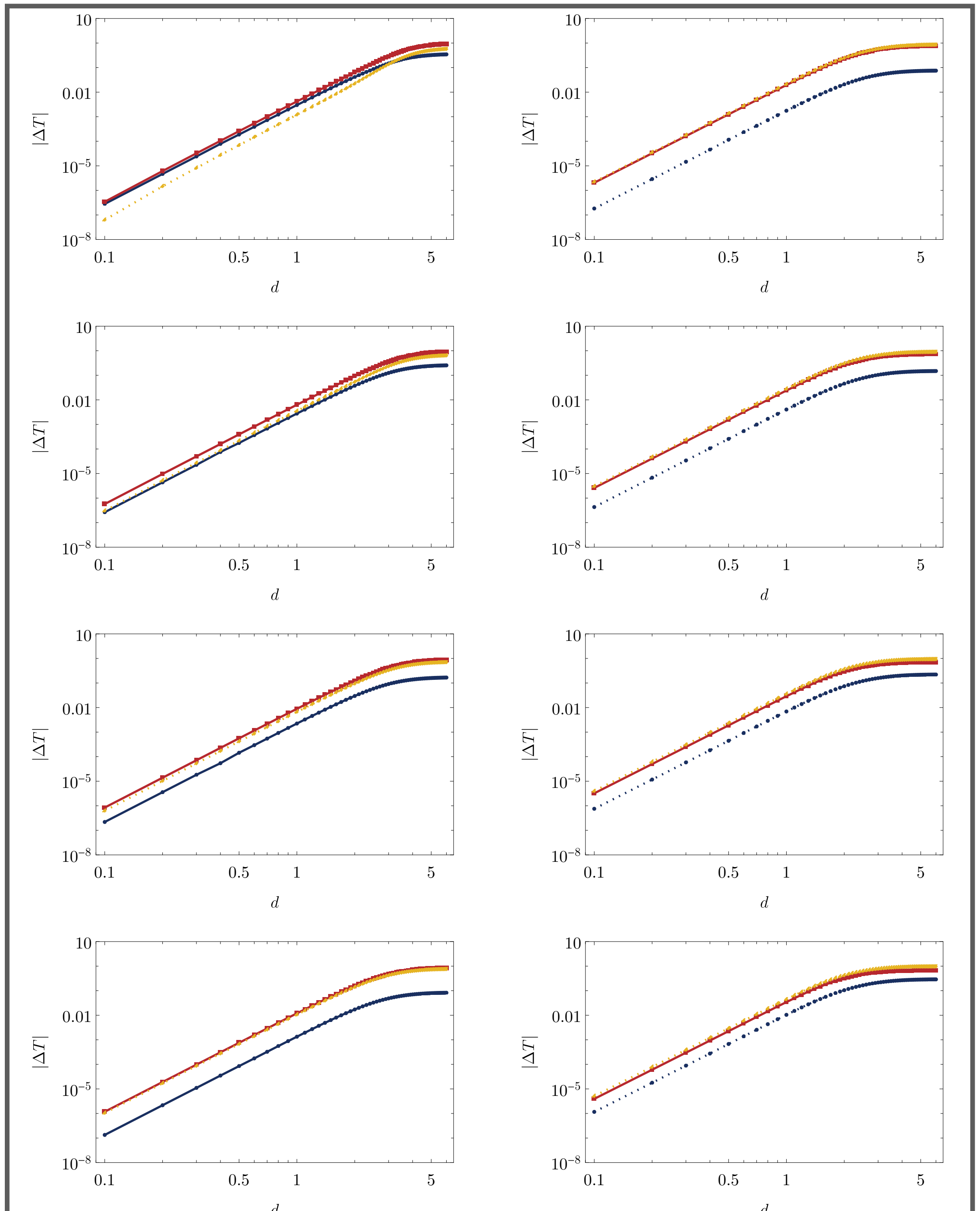
  - Strings do form:



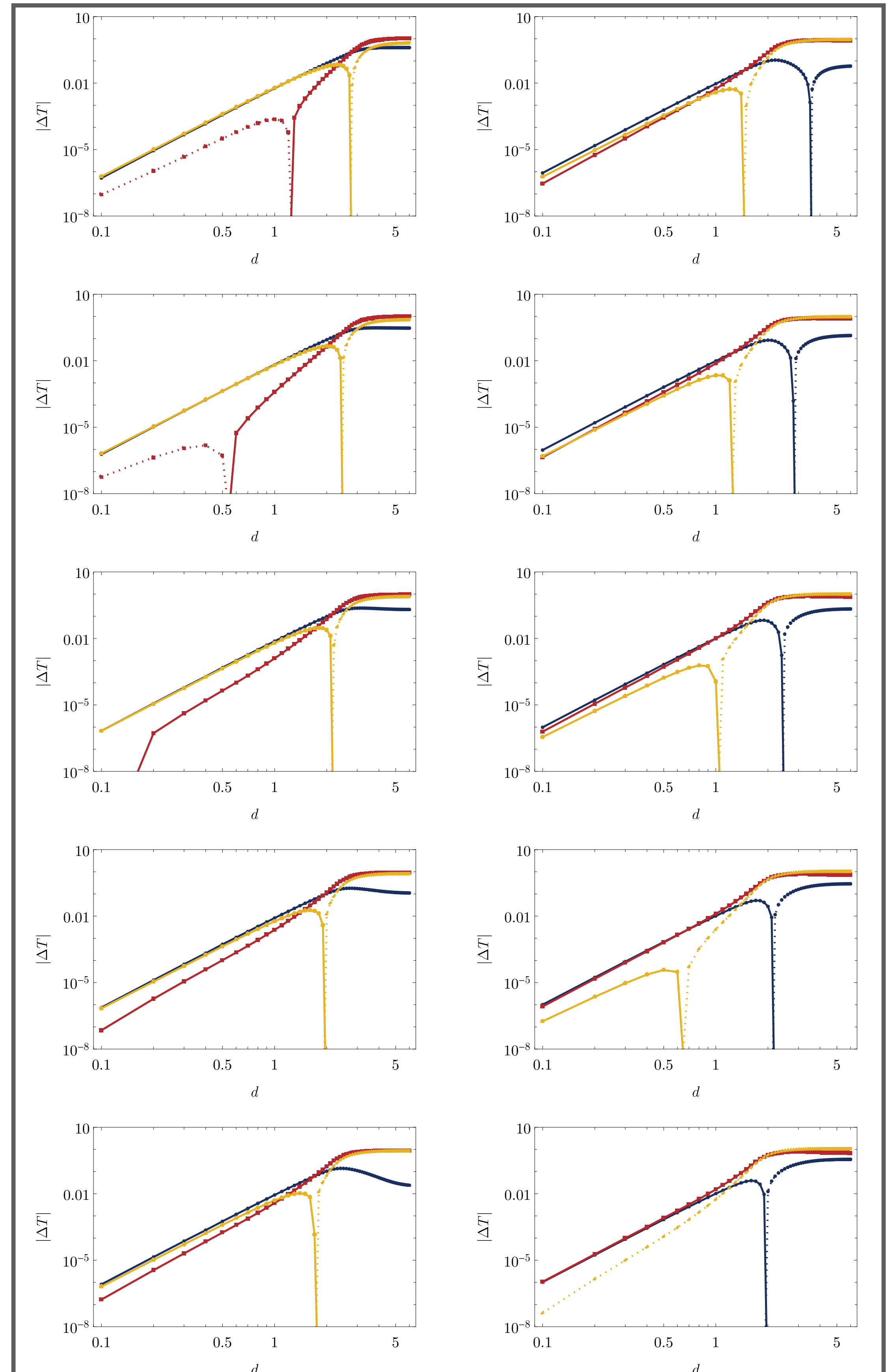
  - The energy barrier seems a universal feature of the two-string system if the effective action is well approximated by a potential flatter than quadratic-quartic (→ next slide)

KINETIC VS. POTENTIAL

QQ



CW

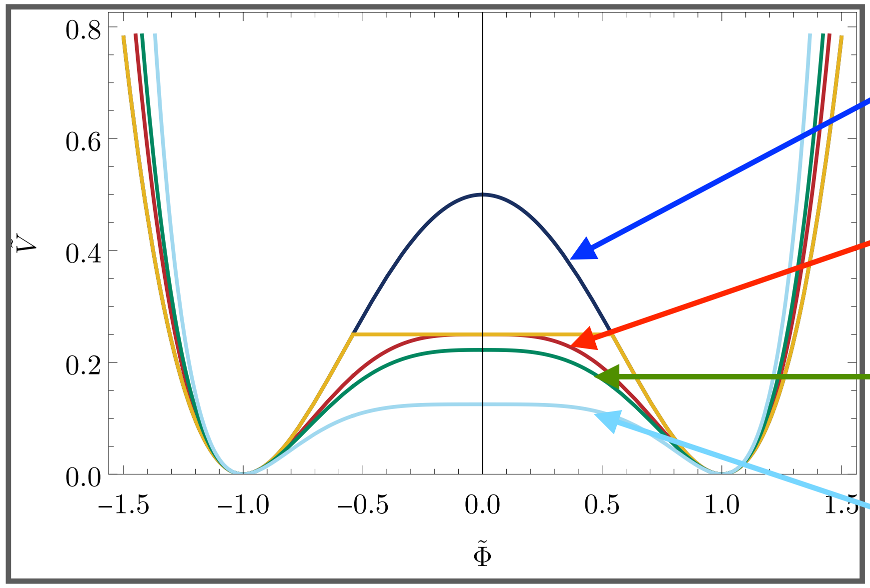


OTHER POTENTIALS

- Energy barrier is a uniervsal feature for flatter potentials

  (in other words, the flat line for BPS in Quadratic-Quartic is very special)

Rescaled potential



$\frac{\beta}{2}(|\Phi|^2-1)^2$

$\frac{\beta}{2}\left(\ln|\Phi|^2-\frac{1}{2}\right)|\Phi|^4$

$\frac{2\beta}{9}(|\Phi|^3-1)^2$

$\frac{\beta}{8}(|\Phi|^4-1)^2$

OTHER POTENTIALS

- Energy barrier is a uniervsal feature for flatter potentials

  (in other words, the flat line for BPS in Quadratic-Quartic is very special)