Fat Zombies from **ALPs** in the stellar graveyard

KONSTANTIN SPRINGMANN

Based on
Reuven Balkin, Javi Serra, Stefan Stelzl, KS and Andreas Weiler 22xx.xxxx (coming soon!)
Reuven Balkin, Javi Serra, KS, Andreas Weiler *JHEP* 07 (2020) 221

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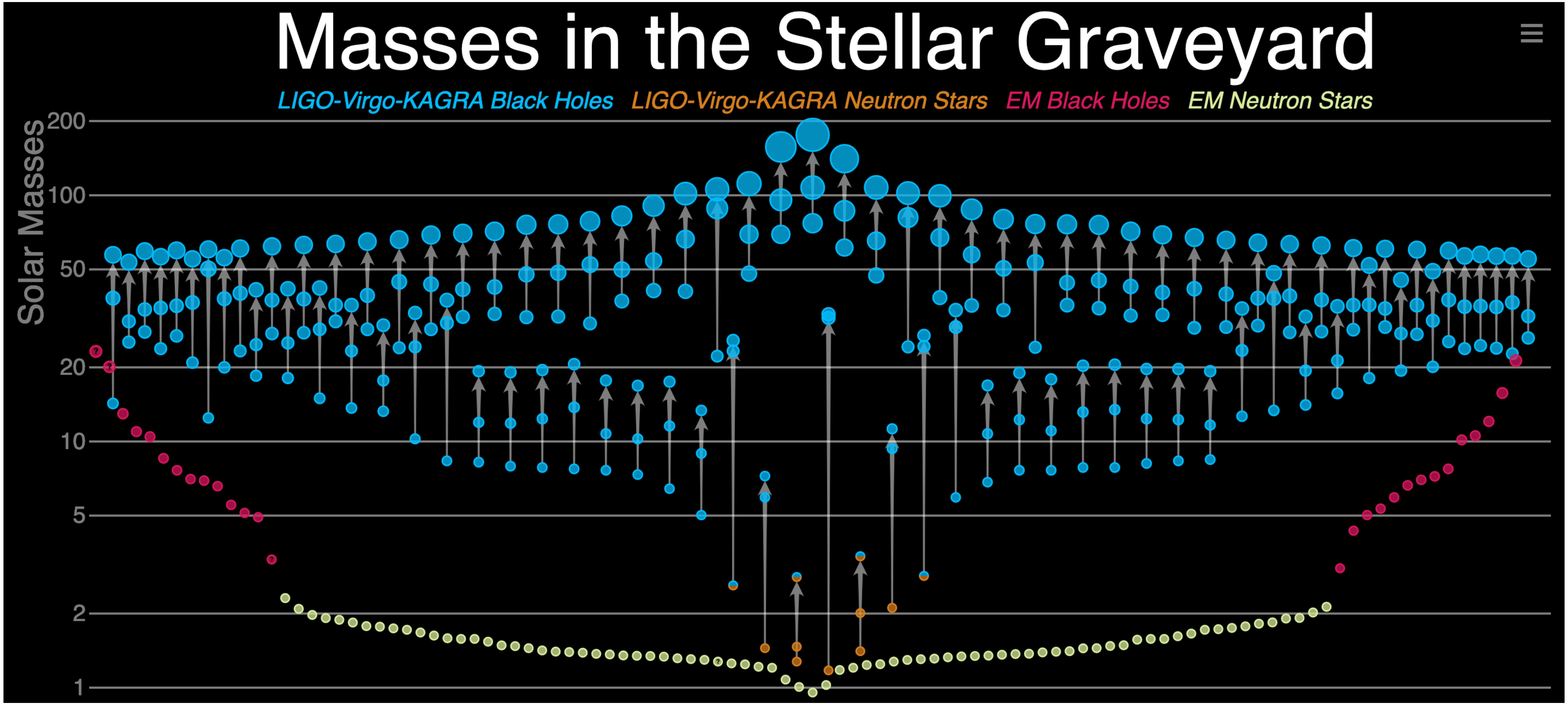
Technische Universität München

June 1st, 2022

MOTIVATION

Gravitational wave astronomy



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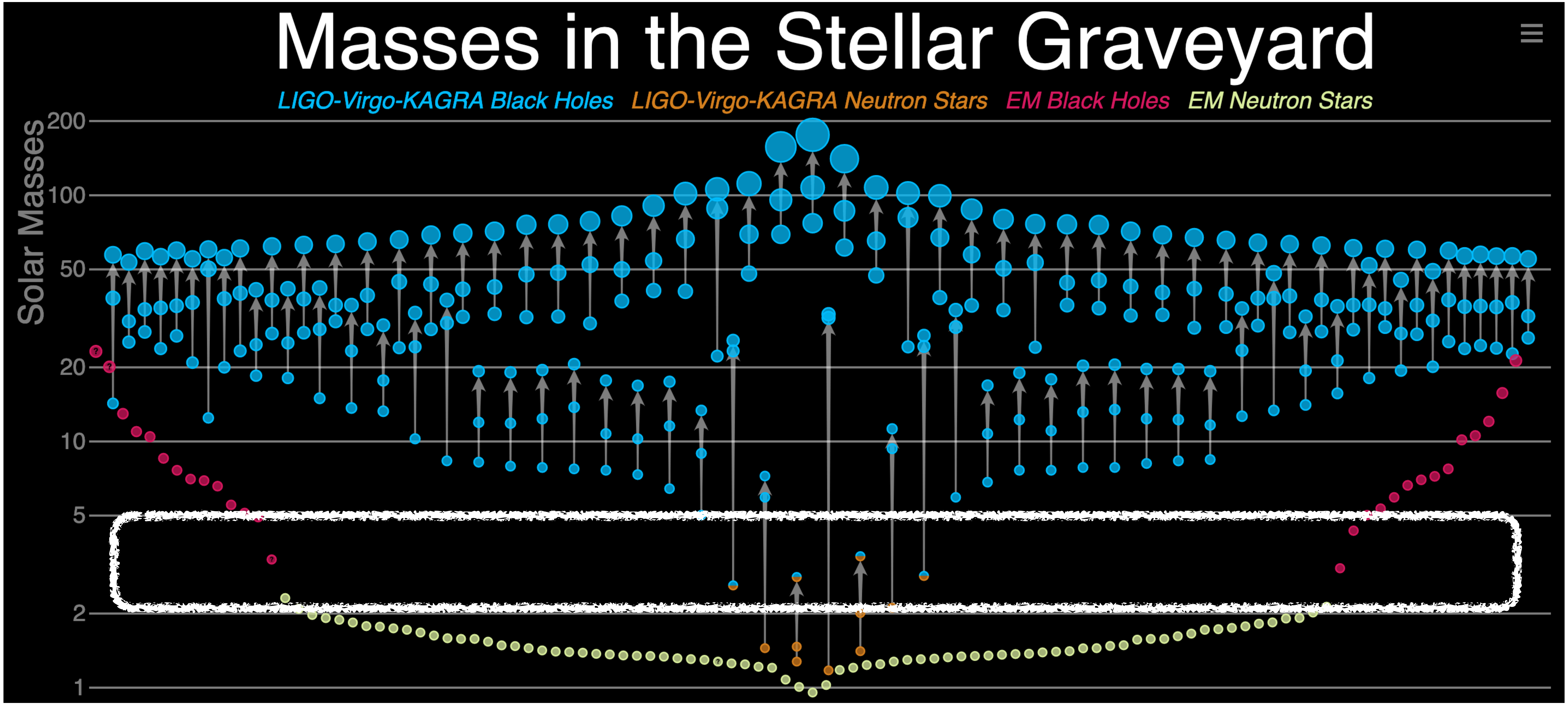

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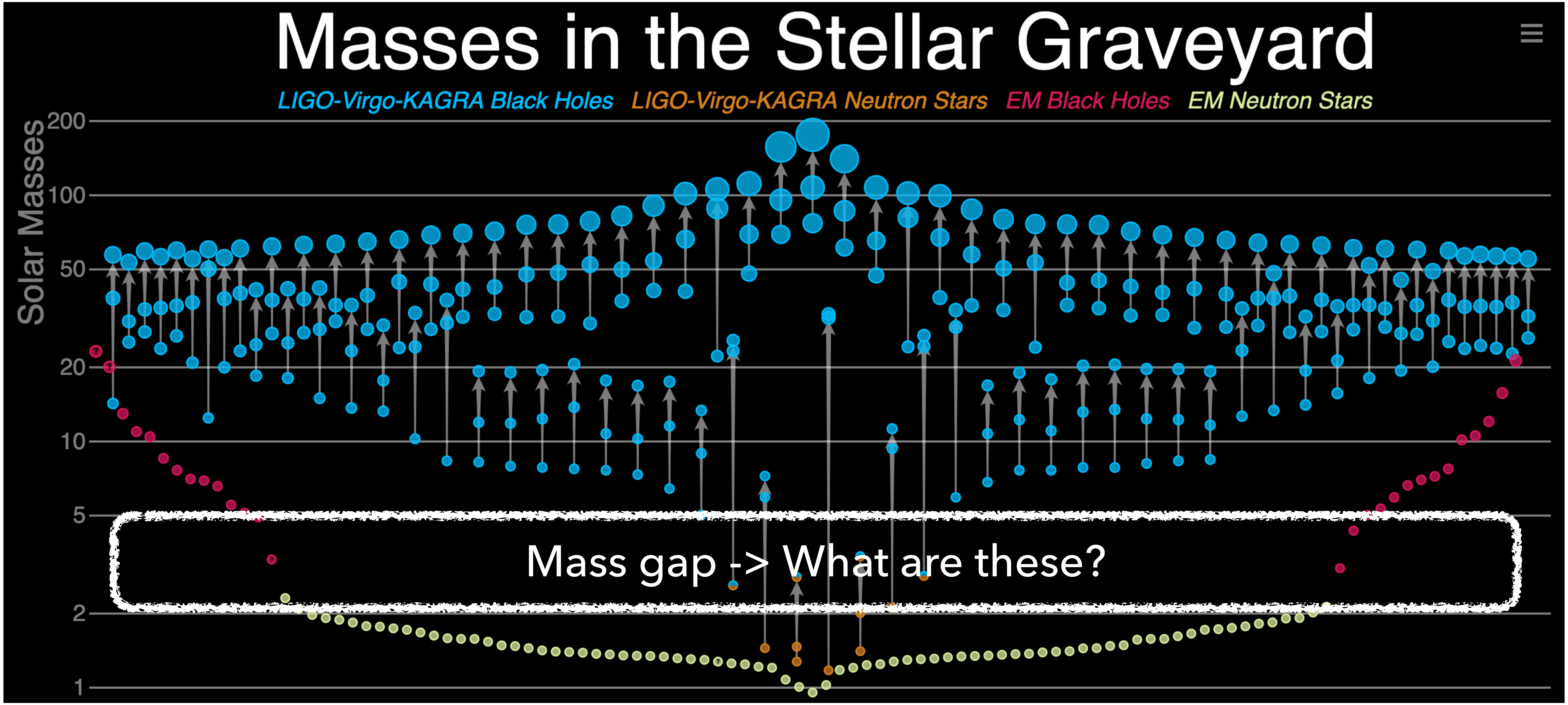
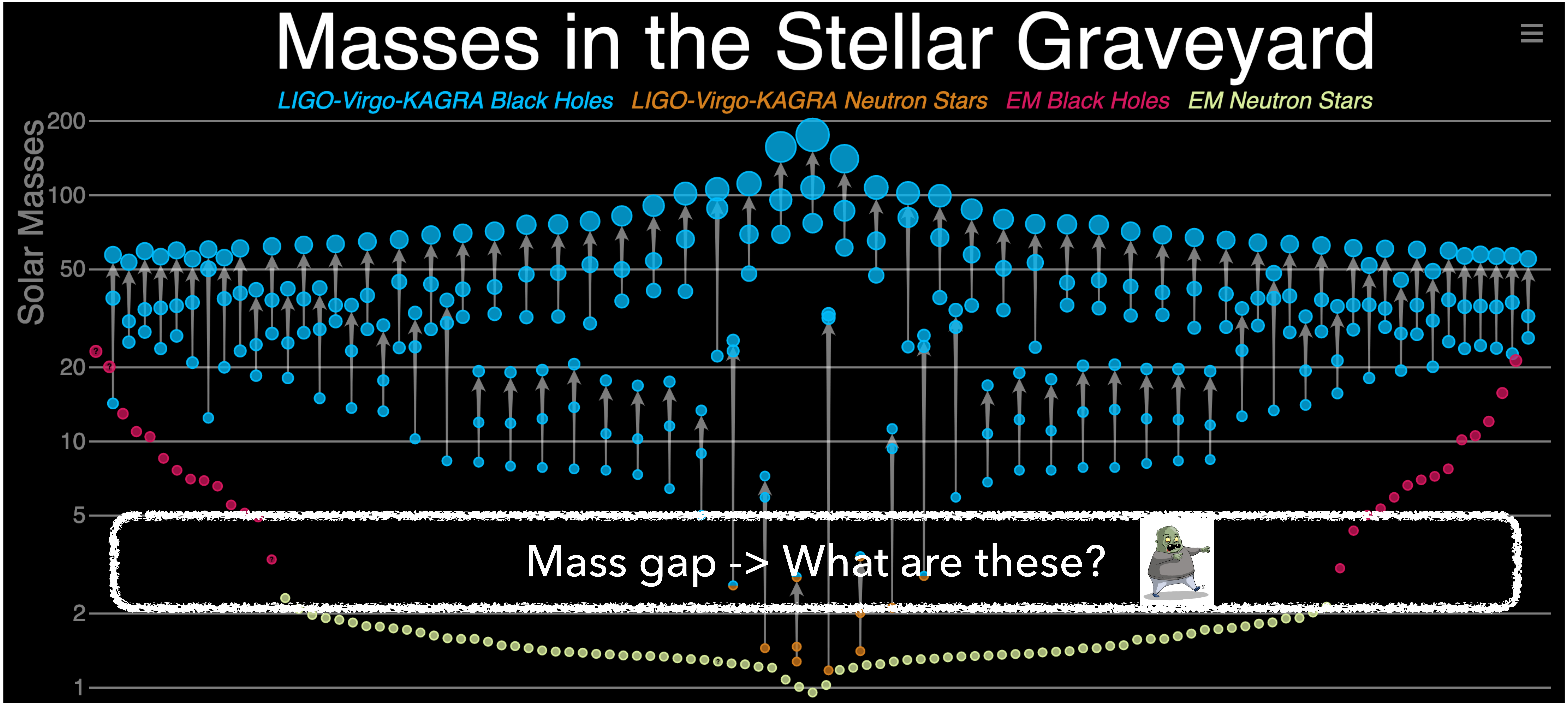


Mass gap -> What are these?

MOTIVATION

Gravitational wave astronomy



Mass gap -> What are these?

Could new physics make neutron stars heavier?

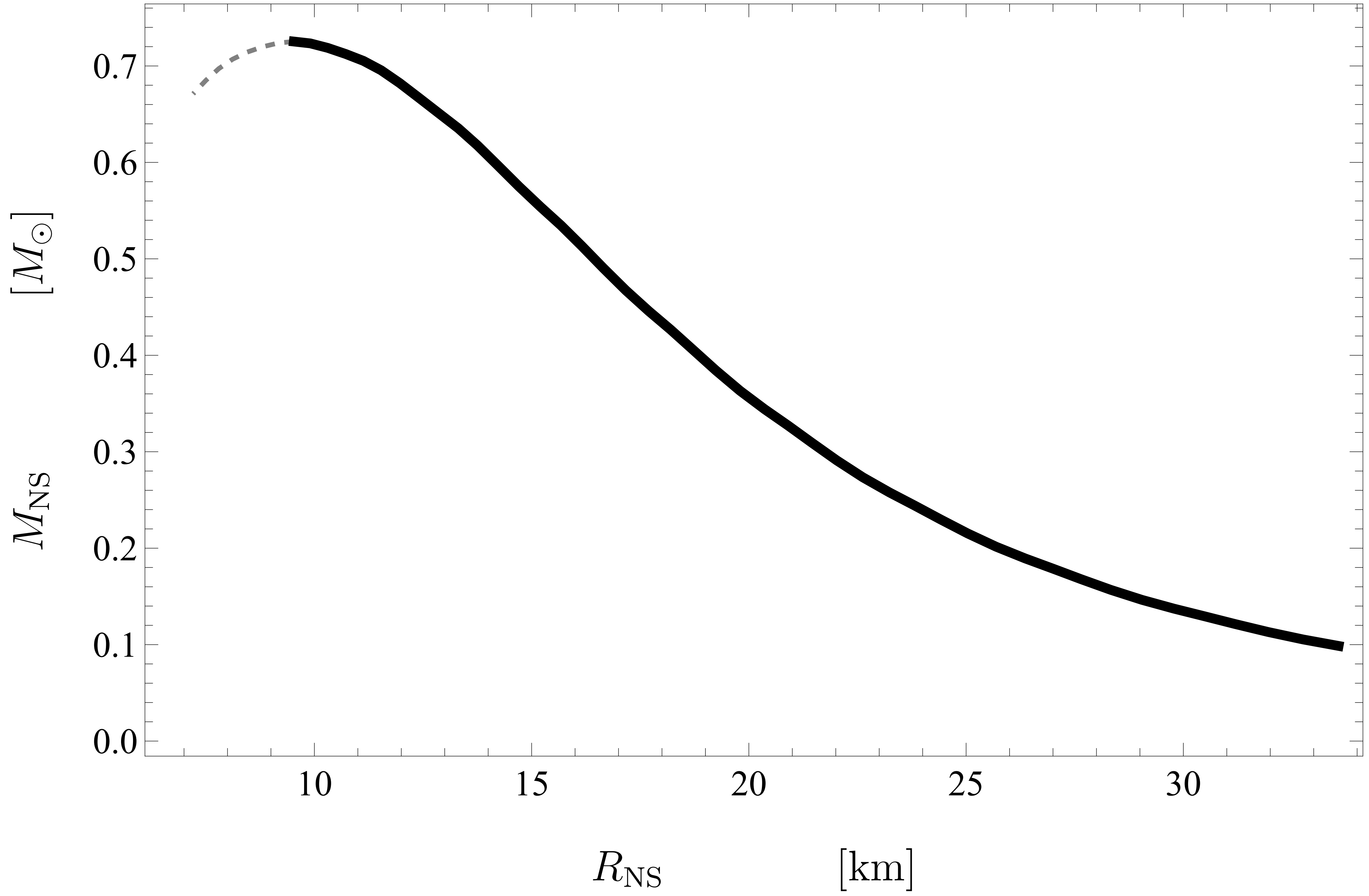
Could these be fat zombies in the stellar graveyard?

A SIMPLE EXAMPLE

Consider non-interacting Fermi gas of neutrons

$$\Rightarrow M_{\max} \sim 0.7 M_{\odot}$$

$$\Rightarrow R_{\max} \simeq 10\,\mathrm{km}$$



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Consider non-interacting Fermi gas of neutrons

$$\Rightarrow M_{\rm max} \sim 0.7 \left(\frac{m_N}{m}\right)^2 M_\odot$$

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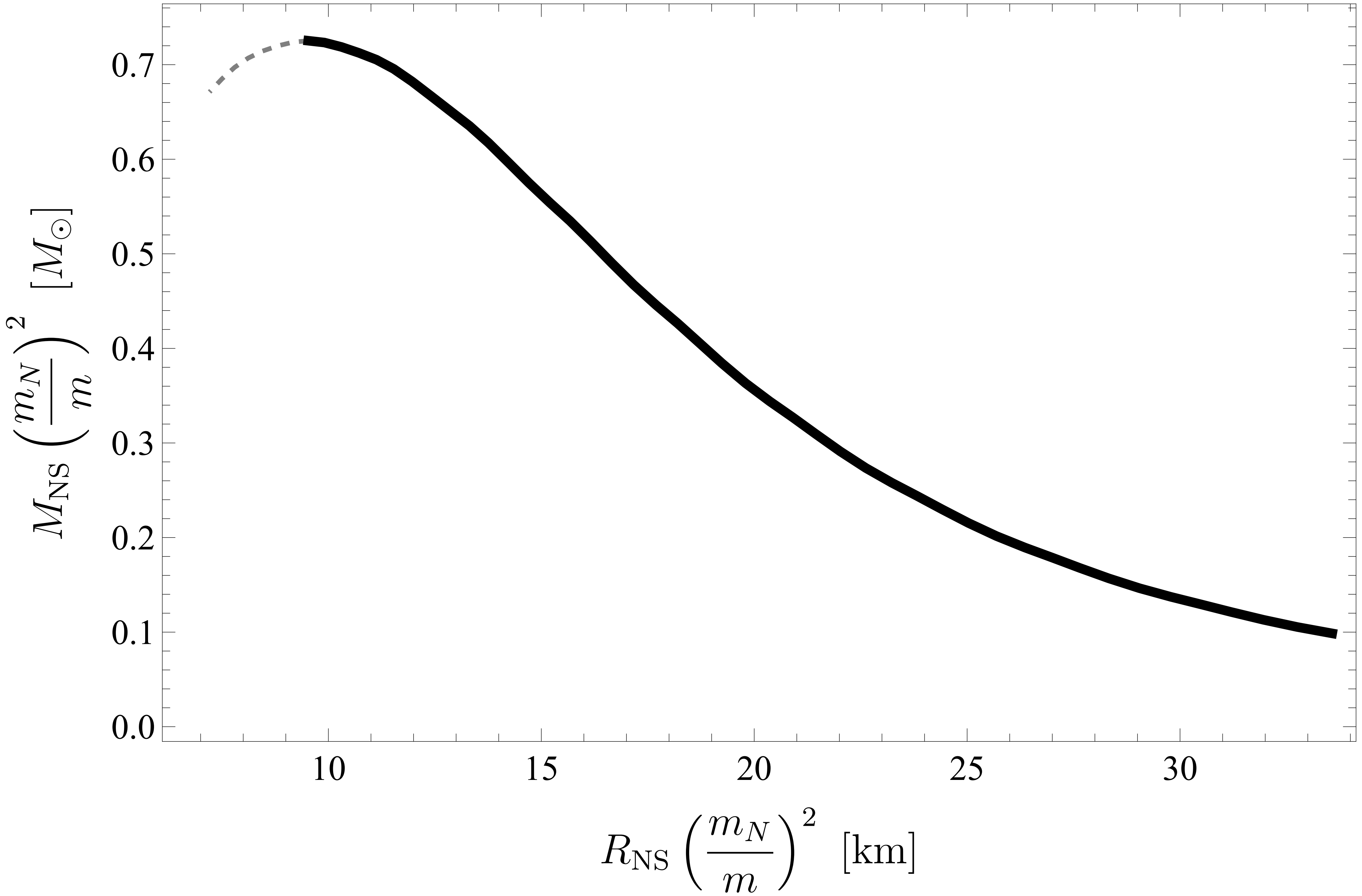

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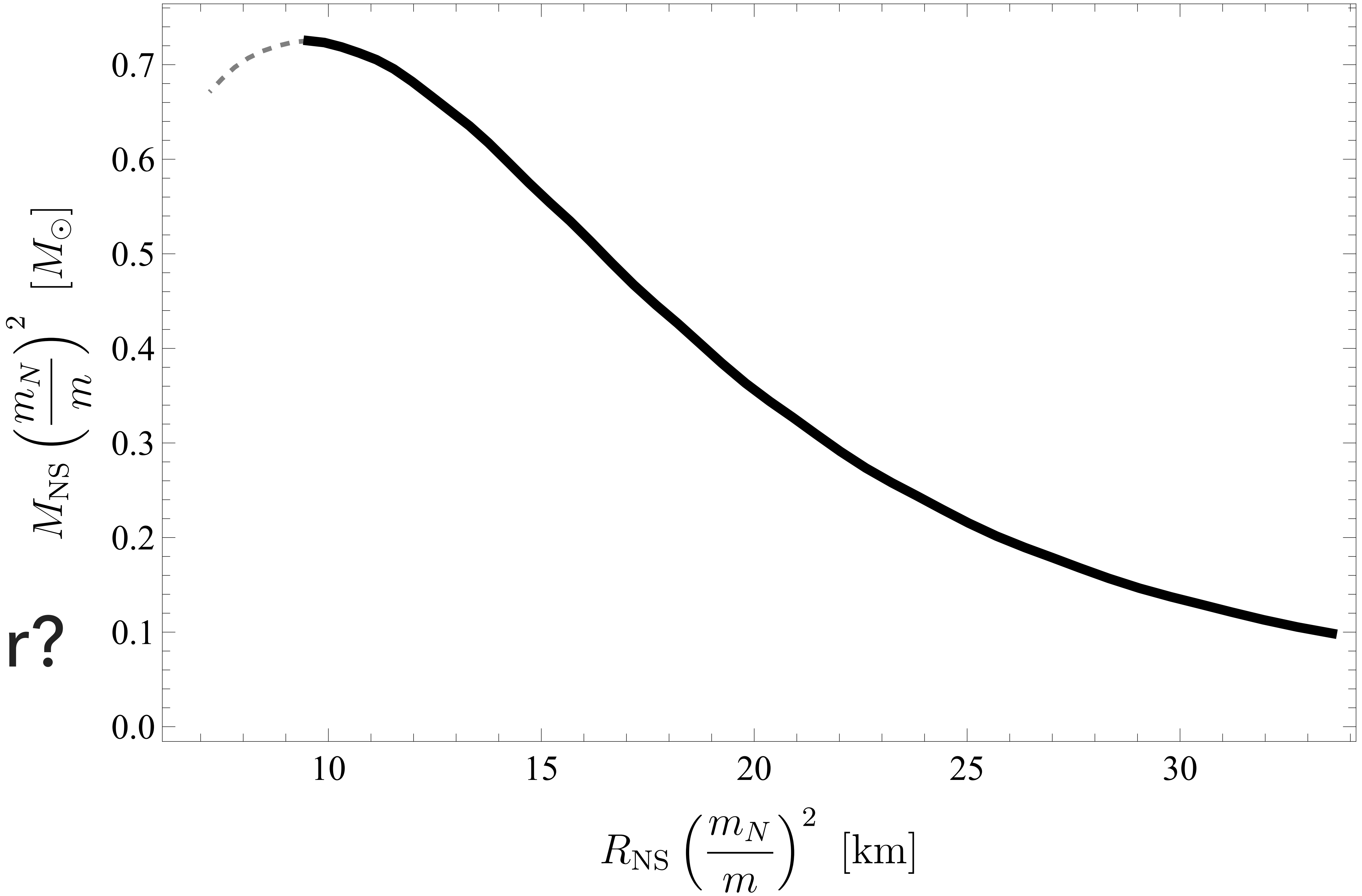
… but what if neutrons where lighter?



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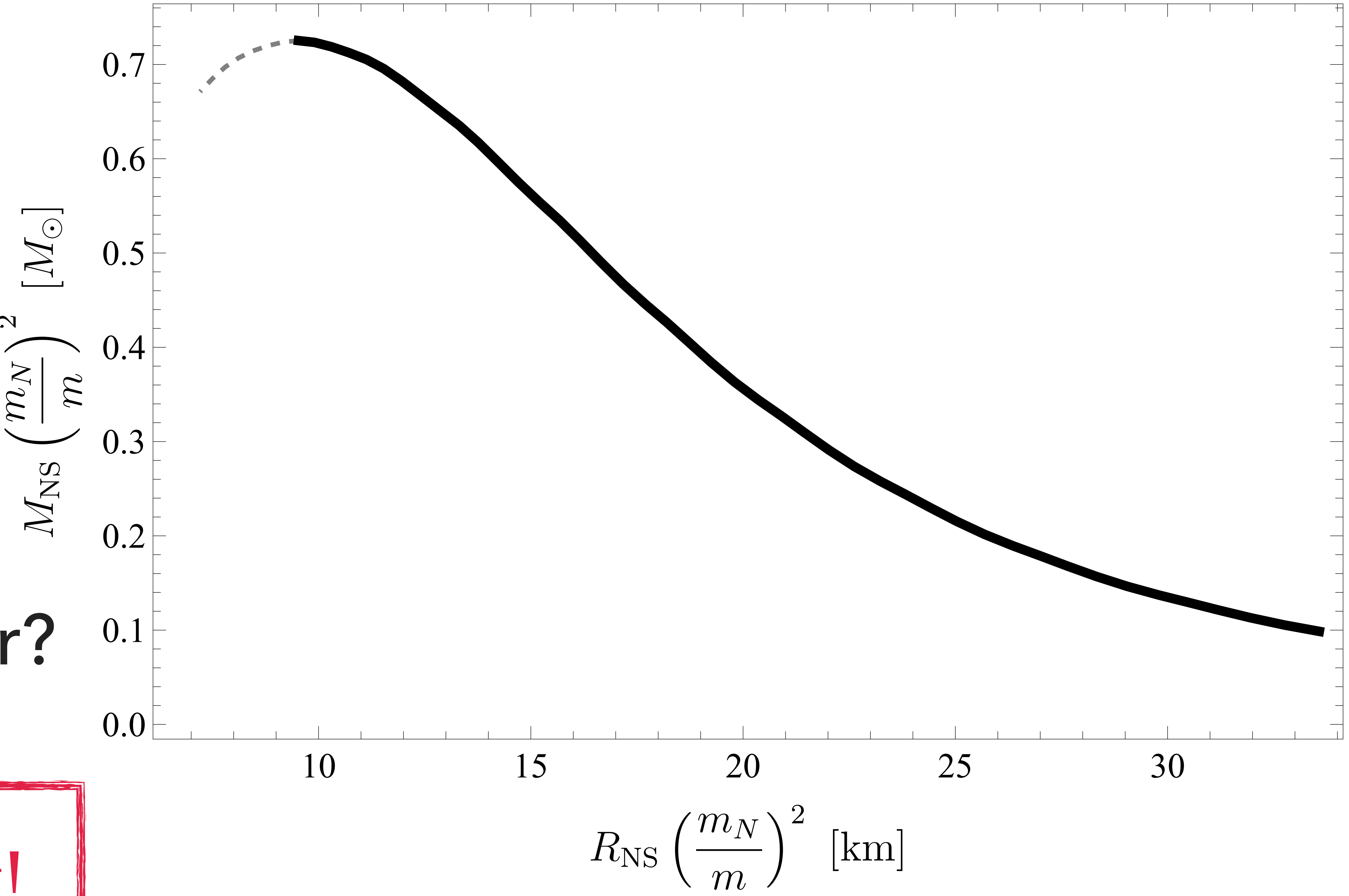

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$$\boxed{m \lesssim m_N/3 \quad \rightarrow \quad \mathcal{O}(10) \;\; \text{effect!}}$$

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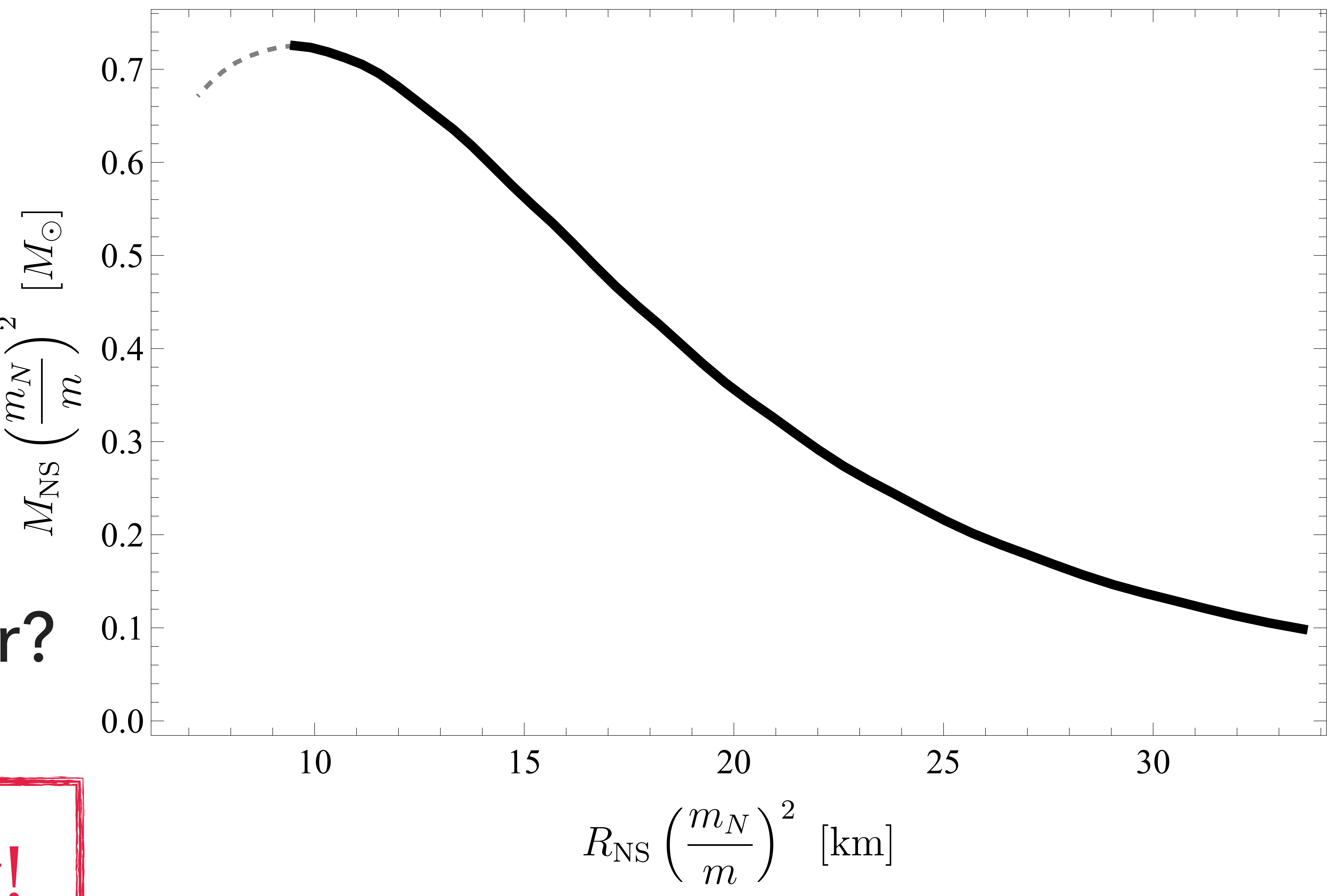

… but what if neutrons where lighter?

$$\boxed{m \lesssim m_N/3 \quad \rightarrow \quad \mathcal{O}(10) \;\; \text{effect!}}$$

So why is that? At fixed energy density need more neutrons $\varepsilon_0 = m\,\rho$

BASIC IDEA

Take axion like particle $V(\phi) = -\Lambda^4 \left(\cos\left(\phi/f\right) - 1\right)$

… with neutron interaction

$$\mathcal{O}_{\phi N} = \frac{g\, m_N}{2} \bar{N} N \cos\left(\frac{\phi}{f}\right)$$

with $1 > g > 0$

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with $1 > g > 0$

Note, this is not the derivative coupling to neutrons

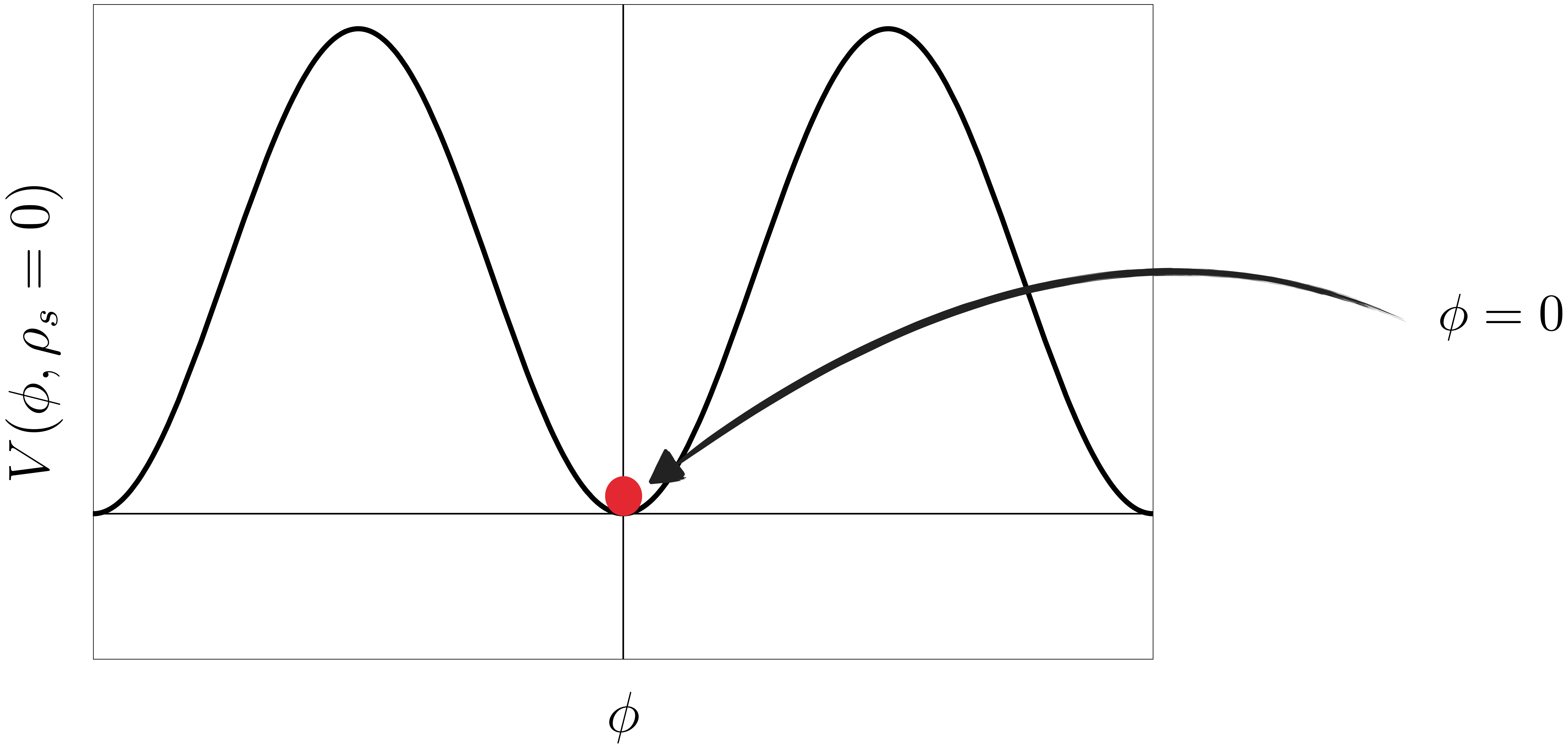
$$\partial_\mu a \bar{N} \gamma^\mu \gamma^5 N$$

But present in vanilla QCD Axion, which we can map to

$$g = \frac{1}{2} \frac{\sigma_N}{m_N} \simeq 0.025 \qquad \Lambda^4 \simeq \frac{m_\pi^2 f_\pi^2}{4}$$

BASIC IDEA

At zero density $\rho_N^s = 0$



BASIC IDEA

This has 2 effects

1) At finite densities the potential is

$$V(\phi, \rho_N^s) = -\left(\Lambda^4 - \frac{g m_N}{2}\rho_N^s\right)\left(\cos\left(\phi/f\right) - 1\right)$$

$$\langle \bar{N}N \rangle \equiv \rho_N^s \simeq \rho$$

This also happens for the QCD axion with

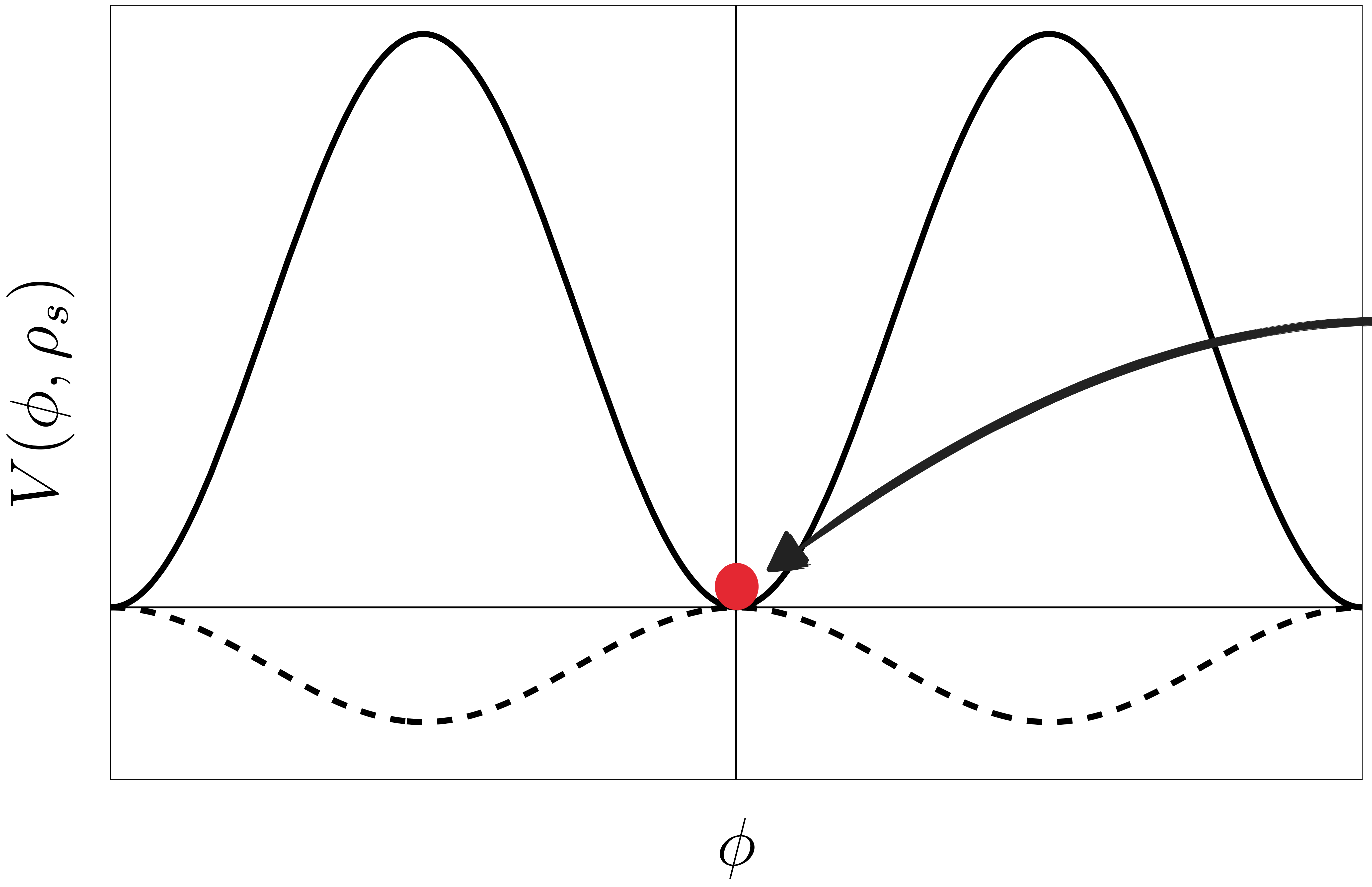
$$g = \frac{1}{2}\frac{\sigma_N}{m_N} \simeq 0.025$$

$$\Lambda^4 \simeq \frac{m_\pi^2 f_\pi^2}{4}$$

BASIC IDEA

This has 2 effects

1) At high densities $\langle \bar{N} N \rangle > \frac{2\Lambda^4}{g\, m_N}$



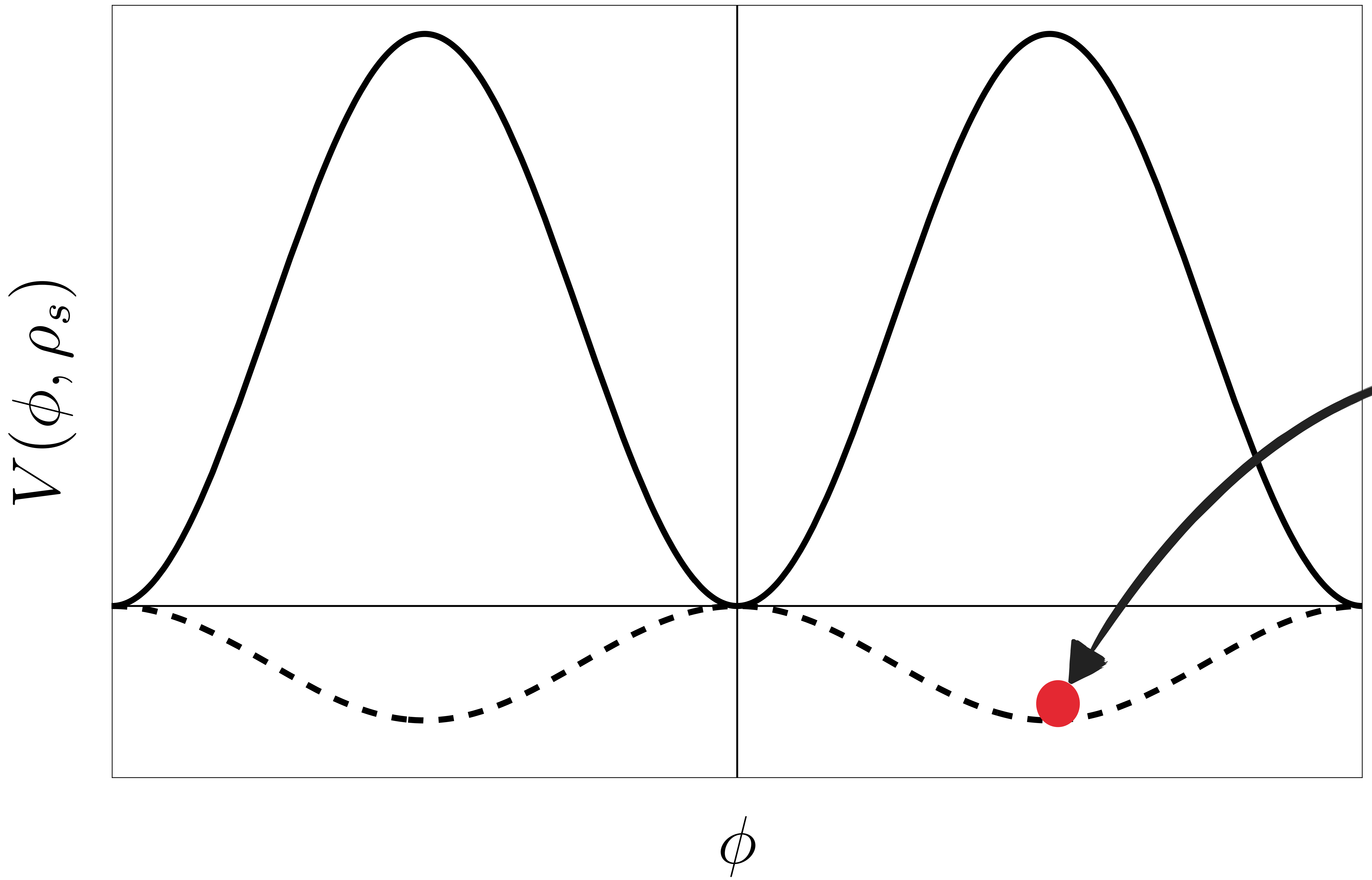
$\phi = 0$

not optimal...

BASIC IDEA

This has 2 effects

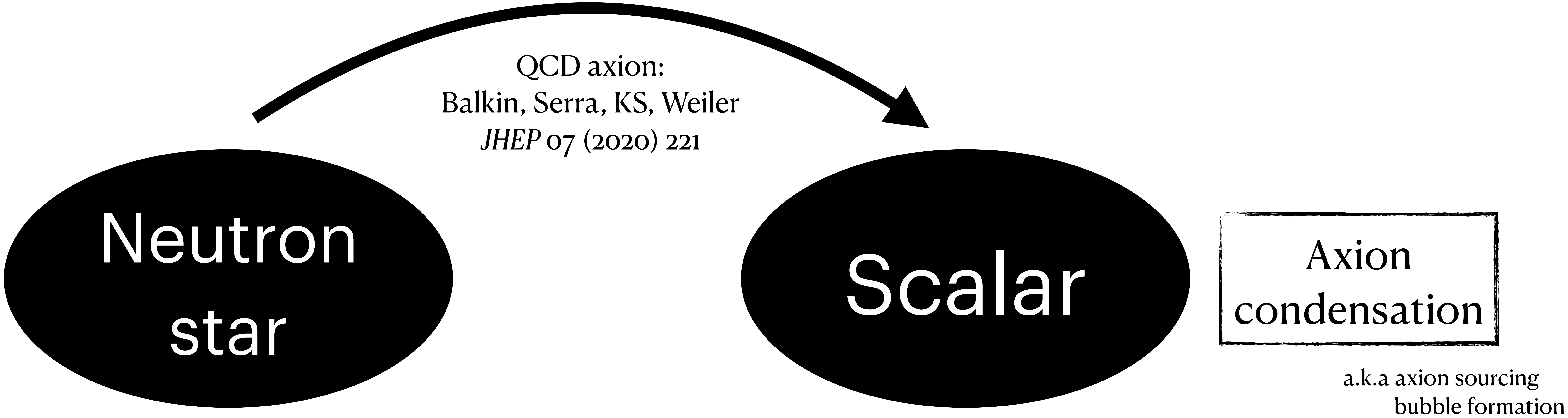
1) At high densities $\langle \bar{N} N \rangle > \dfrac{2\Lambda^4}{g\, m_N}$ the field ϕ is destabilized!



$\phi = \pm\pi$

want to be here!

BASIC IDEA



Neutron star

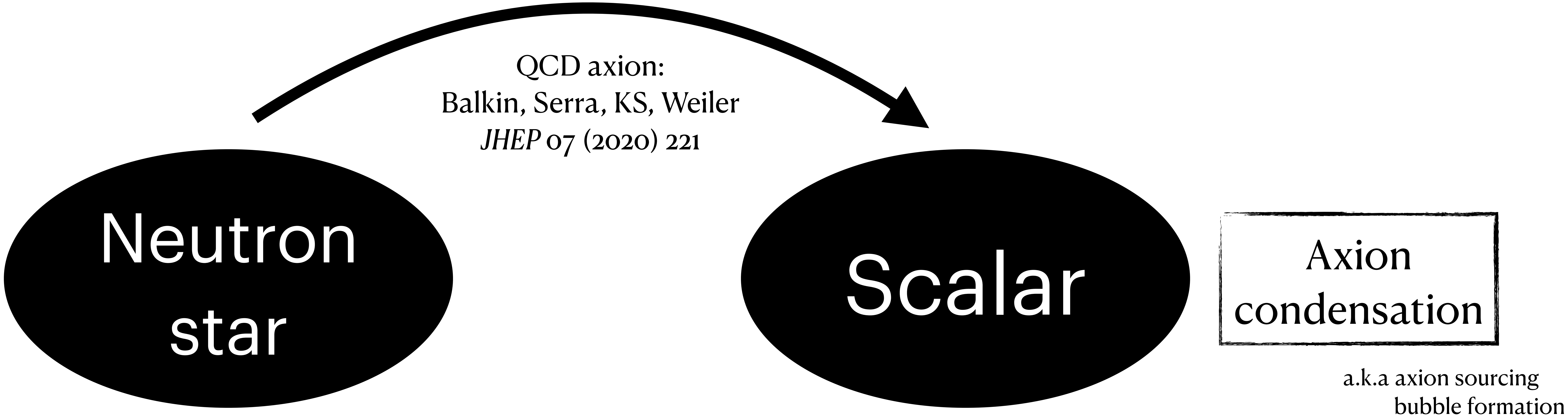
QCD axion:
Balkin, Serra, KS, Weiler
JHEP 07 (2020) 221

Scalar

Axion condensation

a.k.a axion sourcing
bubble formation

BASIC IDEA



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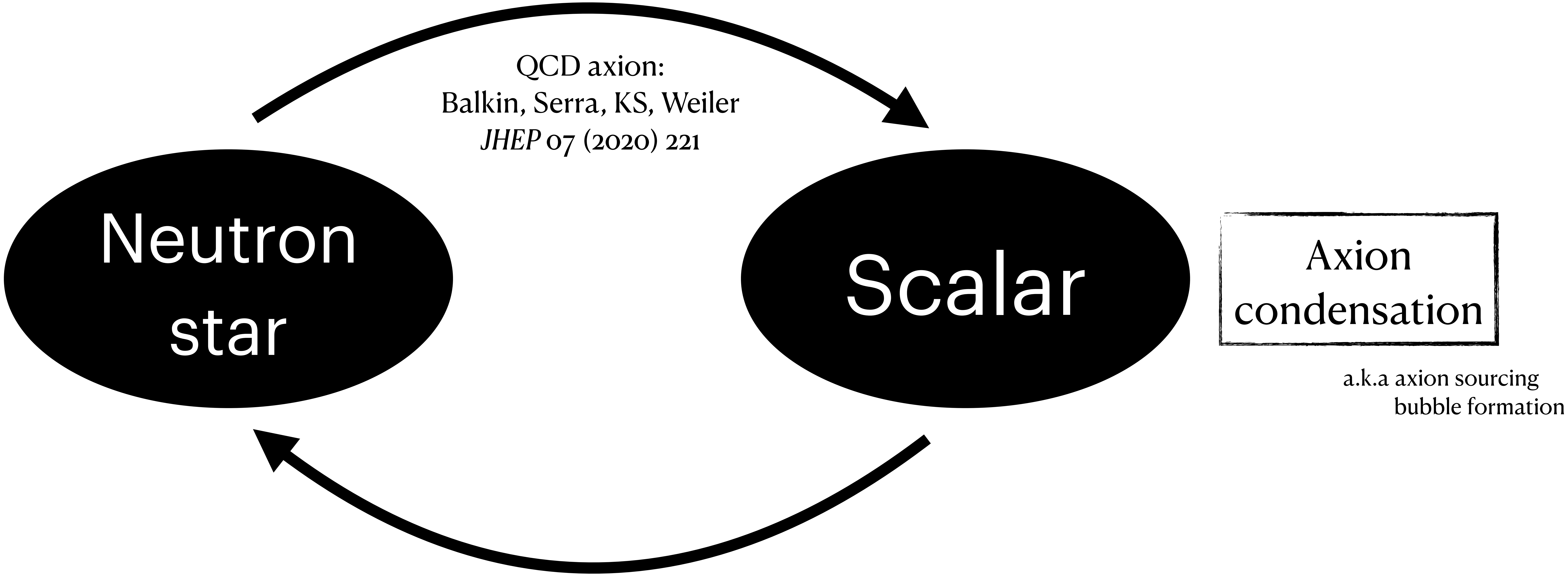
But this is only one way!

BASIC IDEA



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The ALP comes with a lot of energy and couples to neutrons

This talk!
Balkin, Serra, Stelzl, KS, Weiler 22xx.xxxx

BASIC IDEA

This has 2 effects

2) We can write our operator $\mathcal{O}_{\phi N} = \frac{g\, m_N}{2}\bar{N}N\cos\left(\frac{\phi}{f}\right)$

as an effective neutron mass $m_N^* = m_N\left[1 + \frac{g}{2}\left(\cos(\phi/f) - 1\right)\right]$

$$m_N^* = \begin{cases} m_N & \phi = 0 \\ m_N(1-g) & \phi = \pi \end{cases}$$

What happens to the neutron star in this phase?

ALP-FERMION-GRAVITY SYSTEM

Consider one Fermion N, gravity and the ALP

$$\mathcal{L}_{N\phi} = \sqrt{-g}\left[\bar{N}\left(ig^{\mu\nu}\gamma_\mu D_\nu - m_N^*(\phi)\right)N + \frac{1}{2}g^{\mu\nu}(\partial_\mu\phi)(\partial_\nu\phi) - V(\phi)\right],$$

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ALP neutron interaction

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ALP self-interaction

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Outside the dense object

$$\left.\frac{\partial V(\phi)}{\partial\phi}\right|_{\phi_0} = 0 \qquad V(\phi_0) = 0 \qquad m_N^*(\phi_0) = m_N$$

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Effectively decoupled

ALP-FERMION-GRAVITY SYSTEM

Minimising the action $\frac{\delta S}{\delta g_{\mu\nu}} = \frac{\delta S}{\delta \phi} = 0$

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can be solved numerically, very technical

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coupled system
fermion, gravity, scalar

can be solved numerically, very technical

Luckily, there is a simplifying limit!

ALP-FERMION-GRAVITY SYSTEM: ZERO GRADIENT LIMIT

Assume scale hierarchy

Scale of the system

Scale of ϕ

$$R \gg \Delta R = \frac{f}{\sqrt{\varepsilon_{\rm pot}}}$$

$$\varepsilon_{\rm pot} \simeq g\, m_N\, \rho - 2\Lambda^4$$

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Can forget about the scalar gradient: $\phi'(r) = 0$

This is very nice because now the system is effectively decoupled!

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The opposite limit: prevents sourcing in nuclei

EQUATION OF STATE

$\frac{\partial \varepsilon}{\partial \phi} = 0$ minimising the potential energy

$$\varepsilon(\rho, \phi) = \varepsilon_N(\rho, \phi) + V(\phi)$$

$$p(\rho, \phi) = p_N(\rho, \phi) - V(\phi)$$

$$\frac{\partial V}{\partial \phi} + \rho_s(\rho, \phi) \frac{\partial m_N^*(\phi)}{\partial \phi} = 0$$

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$$\begin{tikzcd}
\frac{\partial V}{\partial \phi} + \rho_s(\rho, \phi) \frac{\partial m_N^*(\phi)}{\partial \phi} = 0 \arrow[r] \arrow[d, bend right] & \phi(\rho) & \\
\varepsilon(\rho), p(\rho) \arrow[r, Rightarrow] & p(\varepsilon) & \text{equation of state}
\end{tikzcd}$$

EQUATION OF STATE

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$$\frac{\partial V}{\partial \phi} + \rho_s(\rho,\phi)\frac{\partial m_N^*(\phi)}{\partial \phi} = 0 \longrightarrow \phi(\rho)$$

$$\longrightarrow \varepsilon(\rho), p(\rho) \quad \Rightarrow \quad p(\varepsilon)$$ equation of state

plug into textbook TOV equations

$$p' = -\frac{(p+\varepsilon)}{8\pi r^2 M_{\rm pl}^2}\left(1 - \frac{M}{4\pi r M_{\rm pl}^2}\right)^{-1}\left(4\pi r^3 p + M\right) \,,$$

$$M' = 4\pi r^2 \varepsilon \,,$$

EQUATION OF STATE

What kind of EOS do we get? There are 2 competing effects

1) Mass reduction $m_N^* < m_N$ stiffens the EOS

$$\varepsilon = \text{const.} = m_N^* \rho$$

$$\Rightarrow M_{\text{max}} \sim 0.7 \left(\frac{m_N}{m}\right)^2 M_\odot$$

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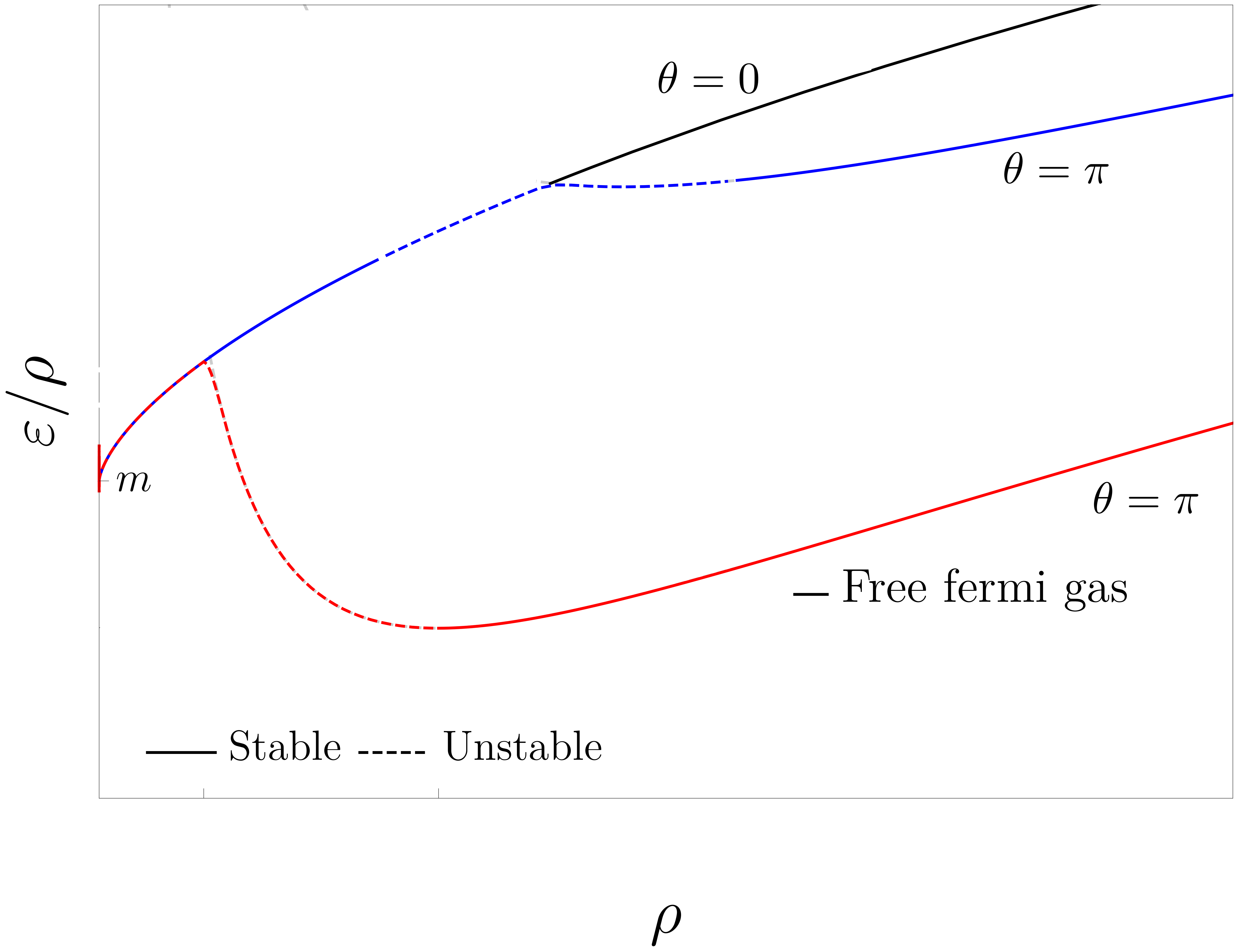
2) Vacuum energy $V(\pi) = 2\Lambda^4$ softens the EOS   additional energy density gravitates

ENERGY PER PARTICLE

Difference between NGS and CE region

$$p = \rho^2 \frac{\partial(\varepsilon/\rho)}{\partial\rho}$$



New ground state: $\{\Lambda_1, g\}$

$\varepsilon/\rho < m_N$ for some ρ

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- Can even reach less energy per particle as well separated ordinary neutrons!
- Nucleons want to be at finite density!

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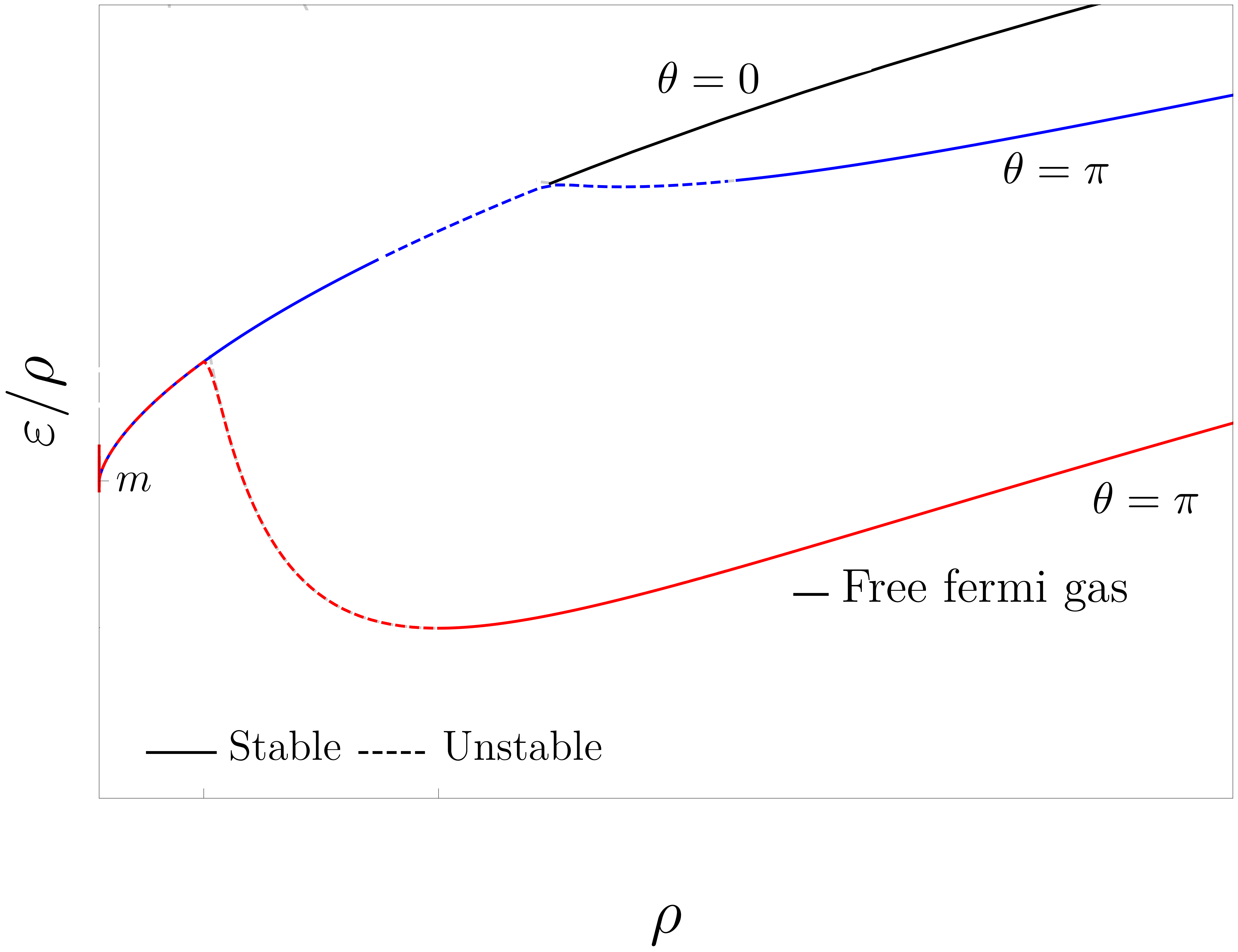

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Coexistence: $\{\Lambda_2, g\}$

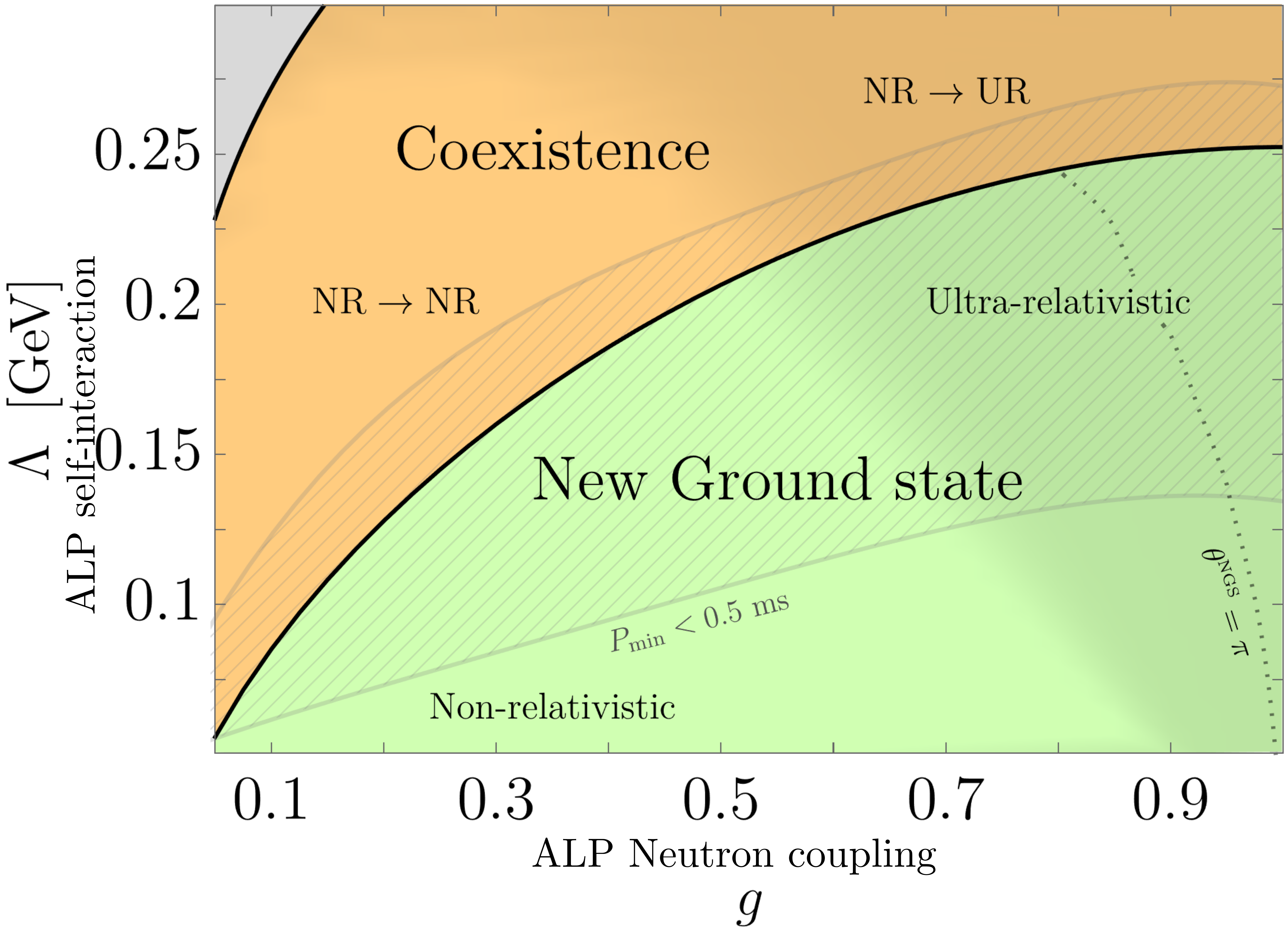
$\varepsilon/\rho > m_N$ for all ρ

- At higher densities, mass contributes less to the total energy density

ALP PARAMETER SPACE

$$V(\phi) = -\Lambda^4 \left(\cos\left(\phi/f\right) - 1\right)$$

$$m_N^*(\phi) = m_N \left[1 + \frac{g}{2}\left(\cos(\phi/f) - 1\right)\right]$$



ALP PARAMETER SPACE: COEXISTENCE

$$\phi = \theta f$$

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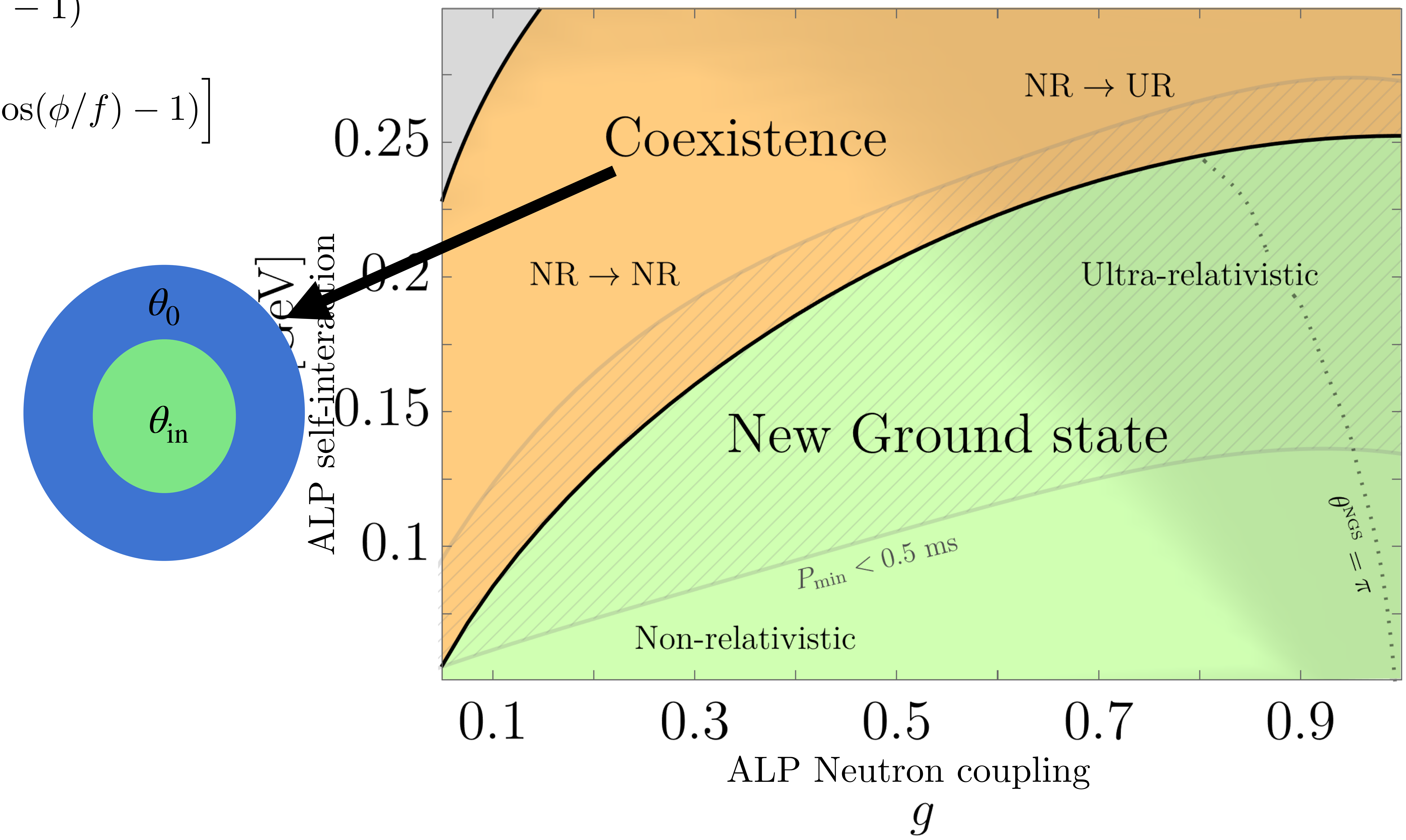
$$m_N^*(\phi) = m_N \left[1 + \frac{g}{2}\left(\cos(\phi/f) - 1\right)\right]$$

Hybrid stars

$M_{\rm max} \lesssim 1\ M_\odot$

Softens EOSs!



ALP PARAMETER SPACE: FAT ZOMBIES

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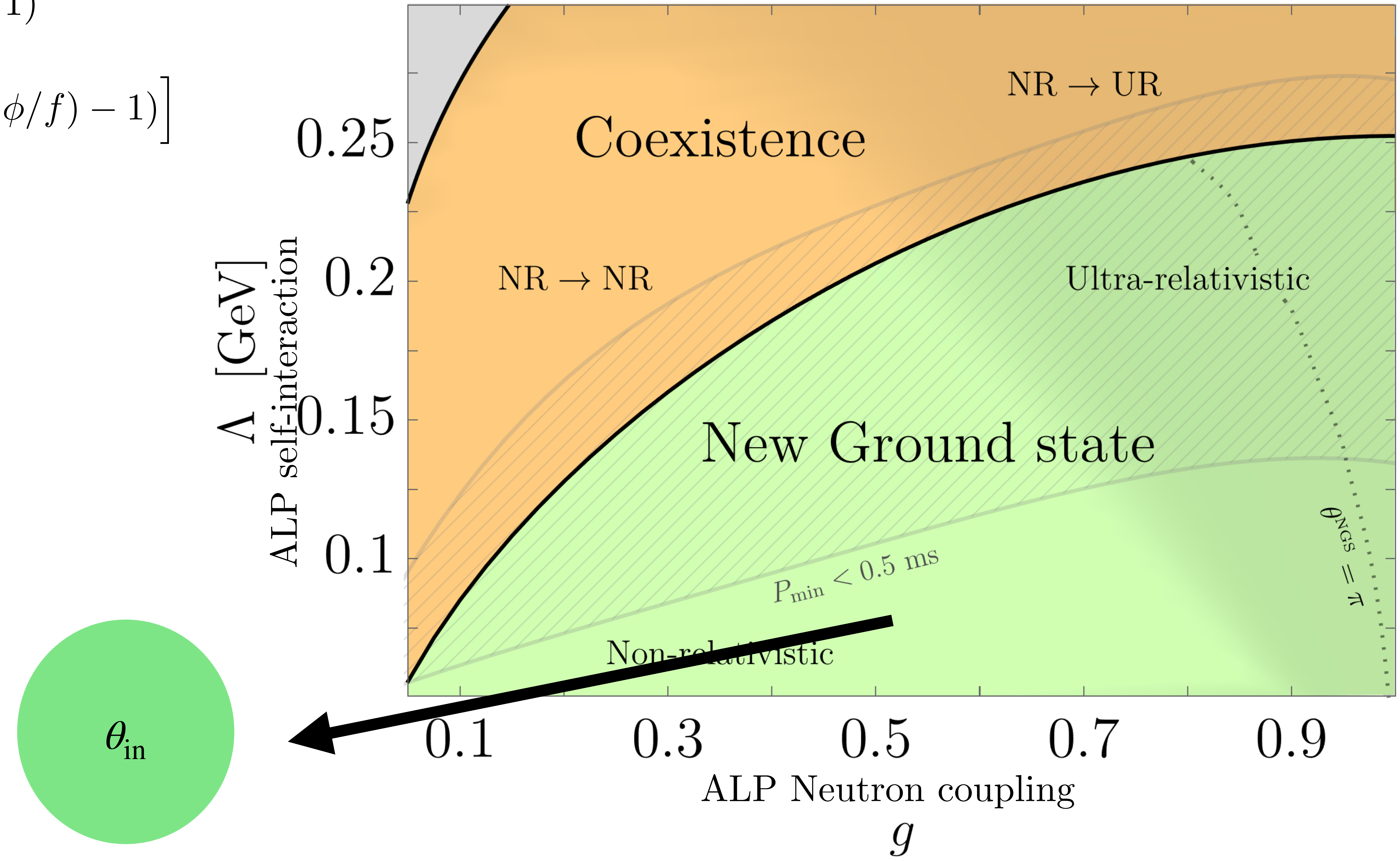

New ground state

$\max[M_{\rm max}] \gg M_\odot$

EOS is stiffer!

NGS PHASE: FAT ZOMBIES

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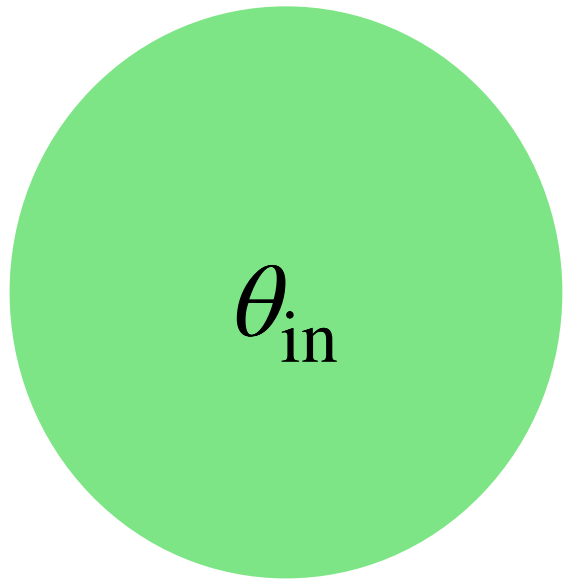
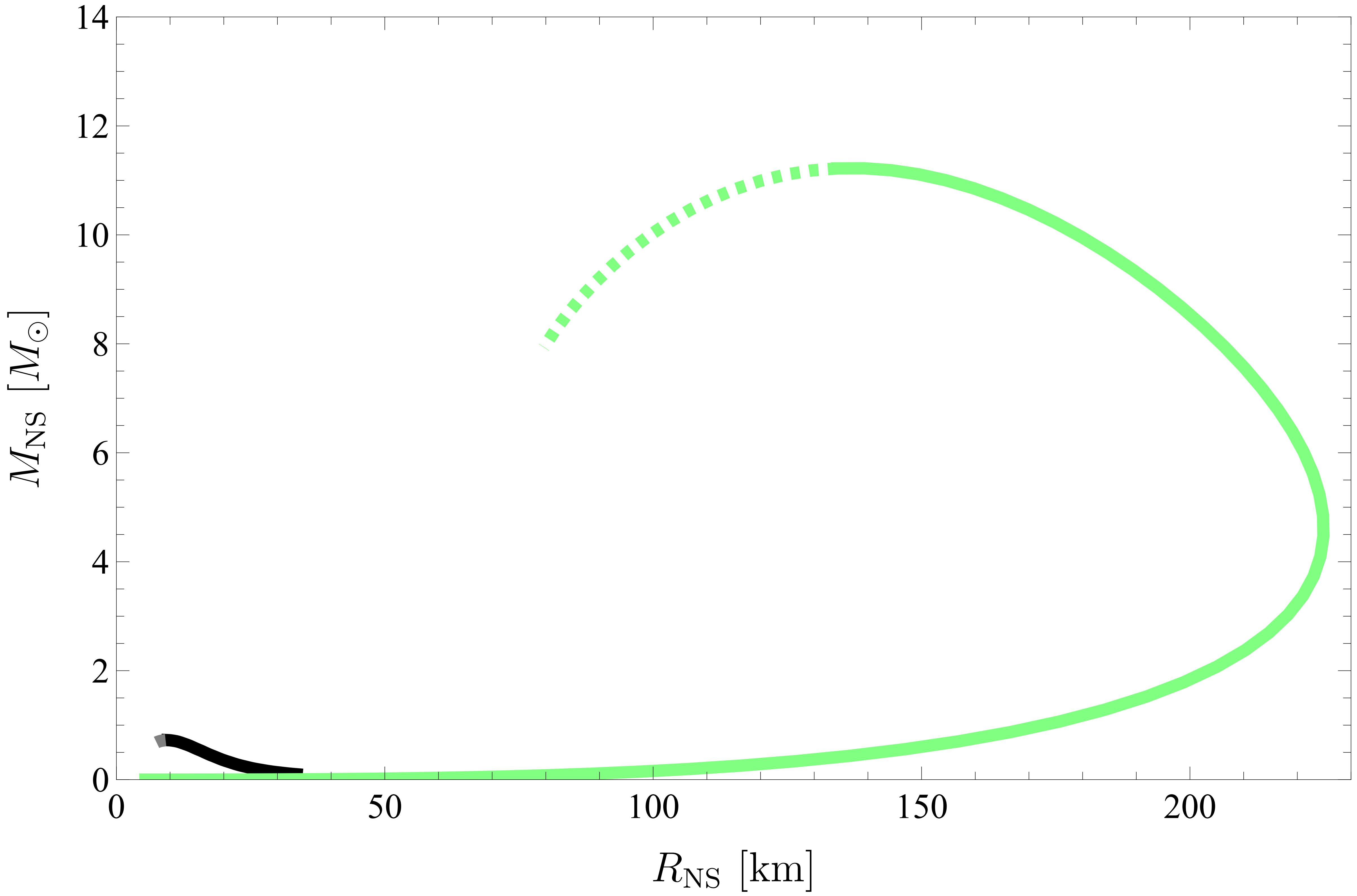


NGS for $\Lambda = 5\,\mathrm{MeV},\;\; g = 0.75$

This is a huge effect

$$M_{\max} \simeq 11.2 M_\odot$$
$$R_{\max} \simeq 160\,\mathrm{km}$$



NGS PHASE: FAT ZOMBIES

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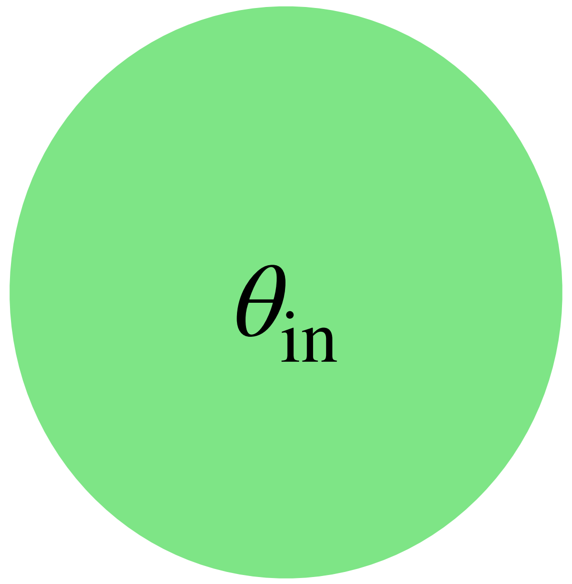
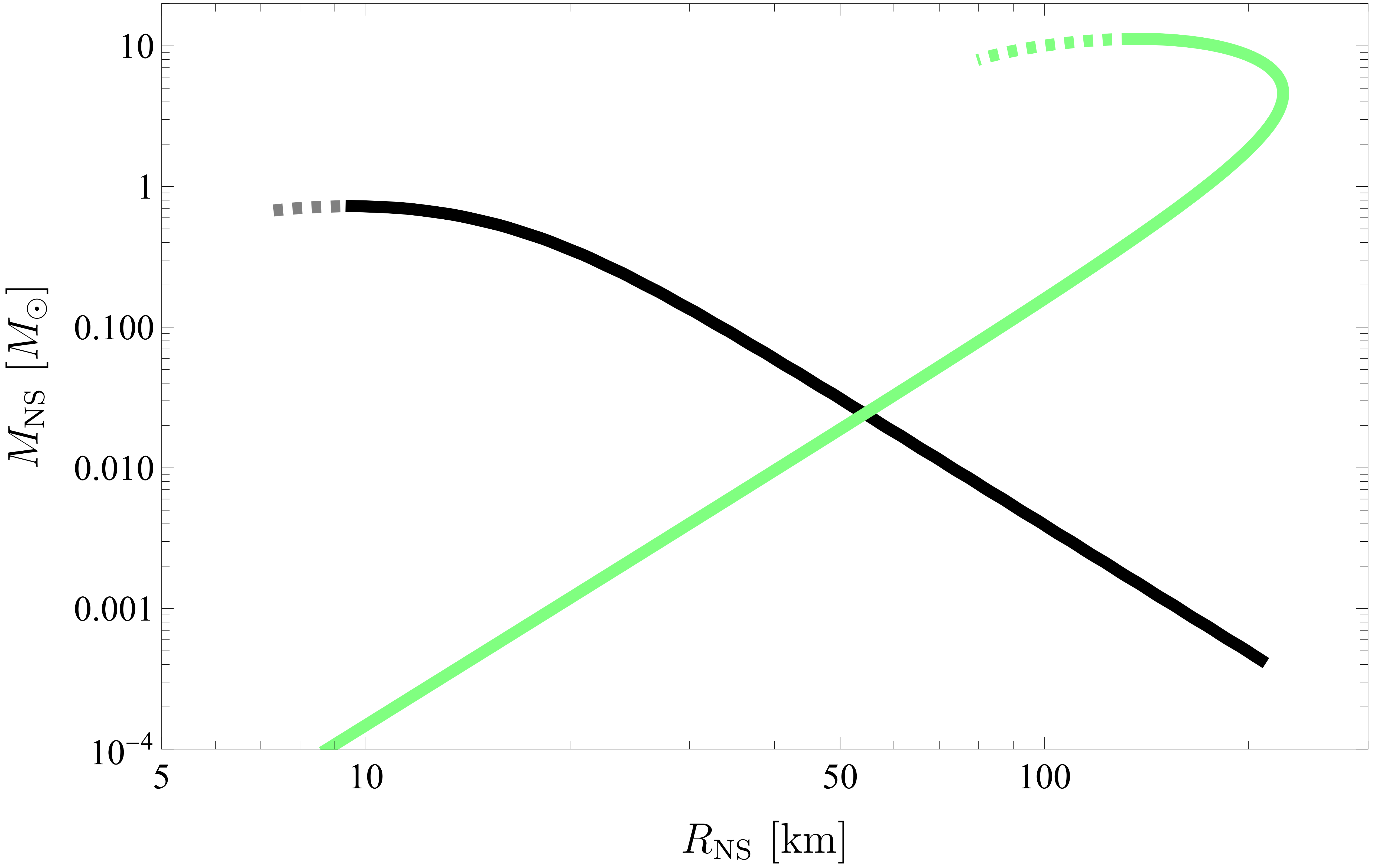


Also interesting on a log plot



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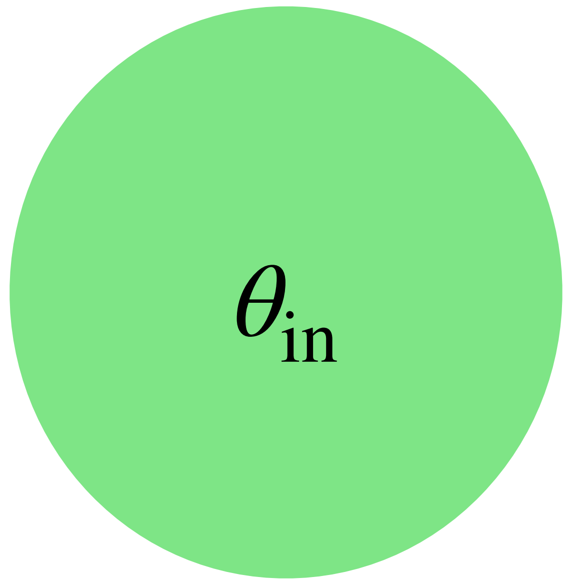


Also interesting on a log plot

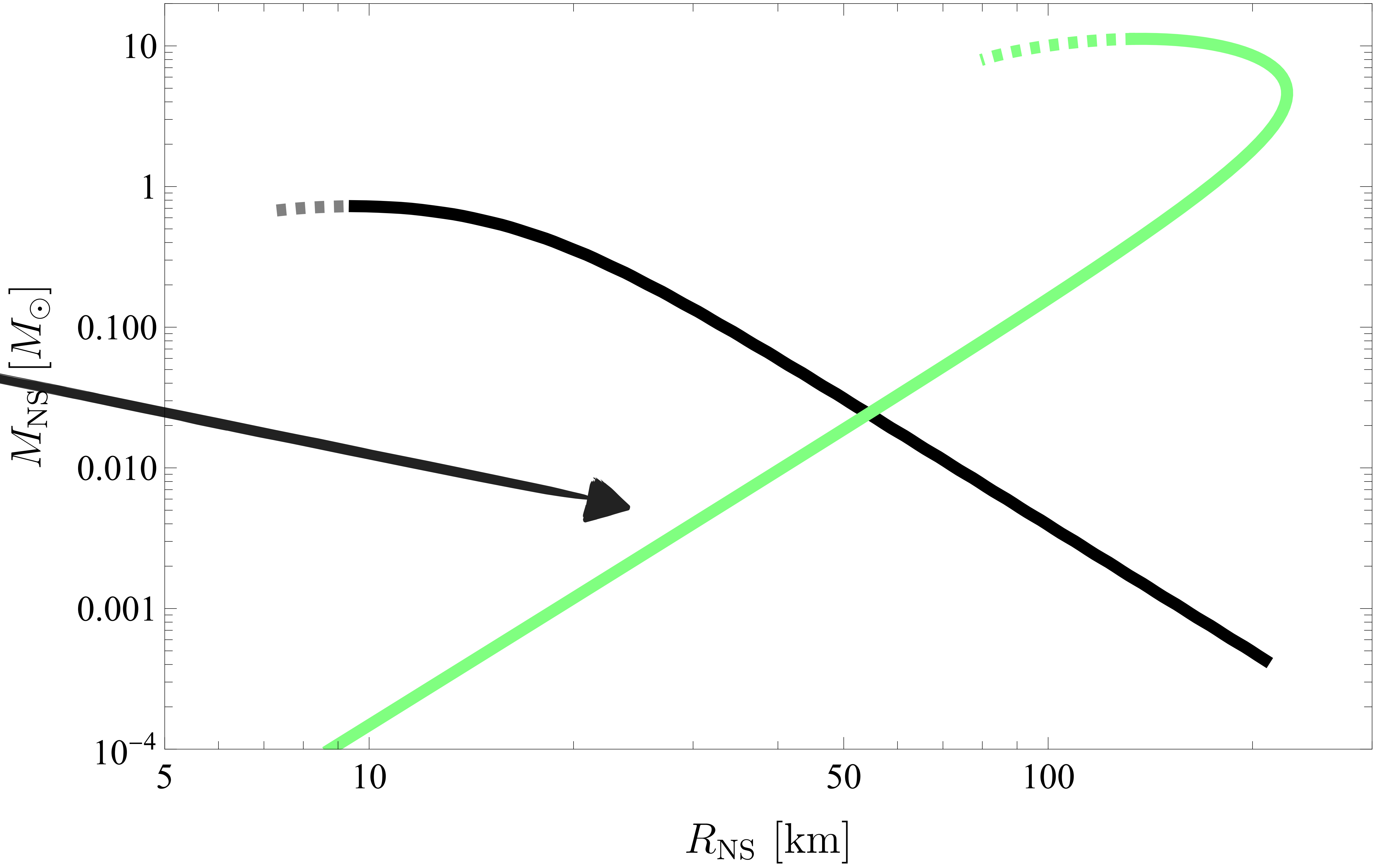
Self bound objects

$$M \simeq \varepsilon^{\text{NGS}} R^3$$

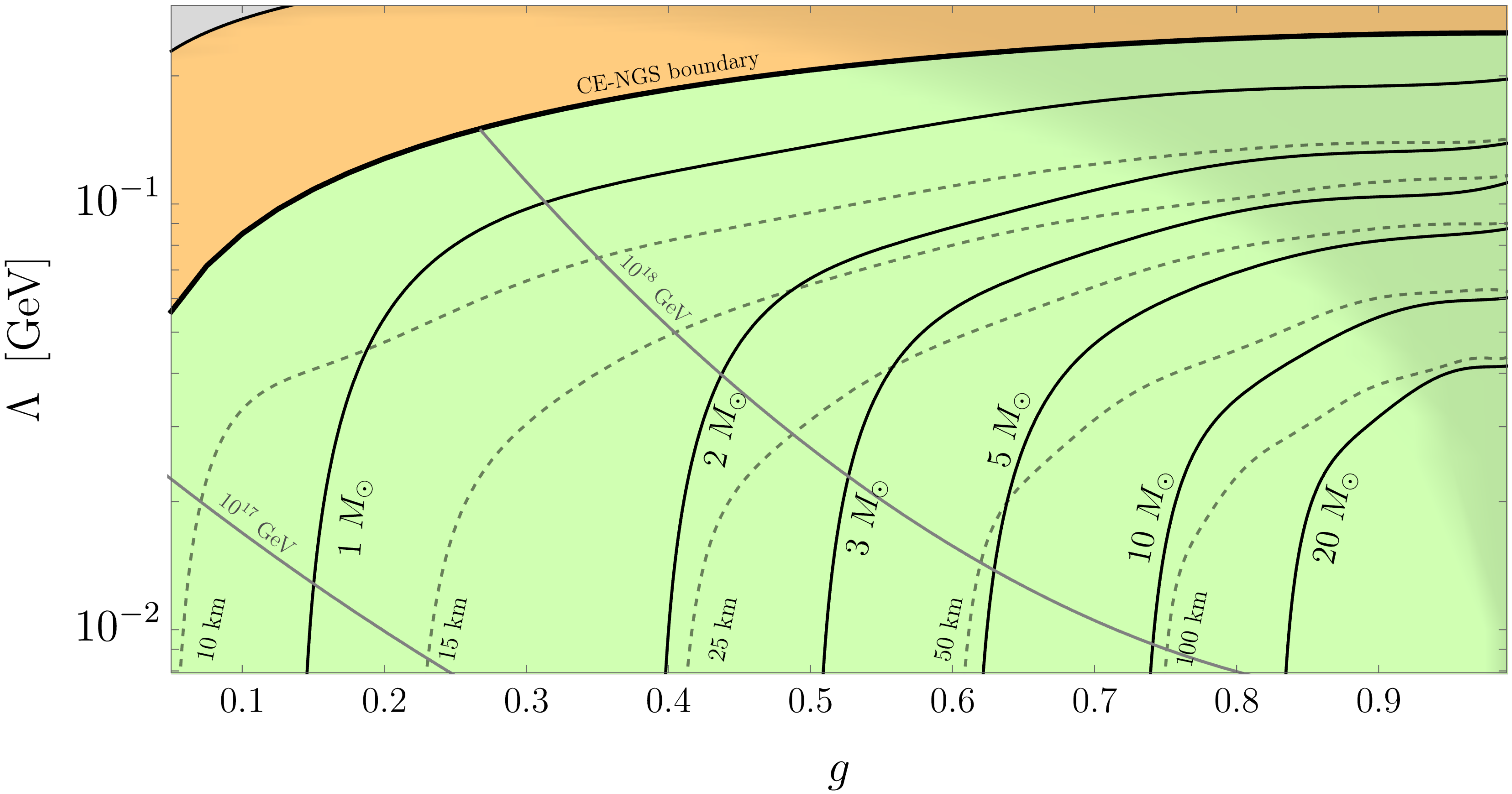
Minimal size given by gradient

$$R_{\min} \simeq \frac{f}{\Lambda^2}$$

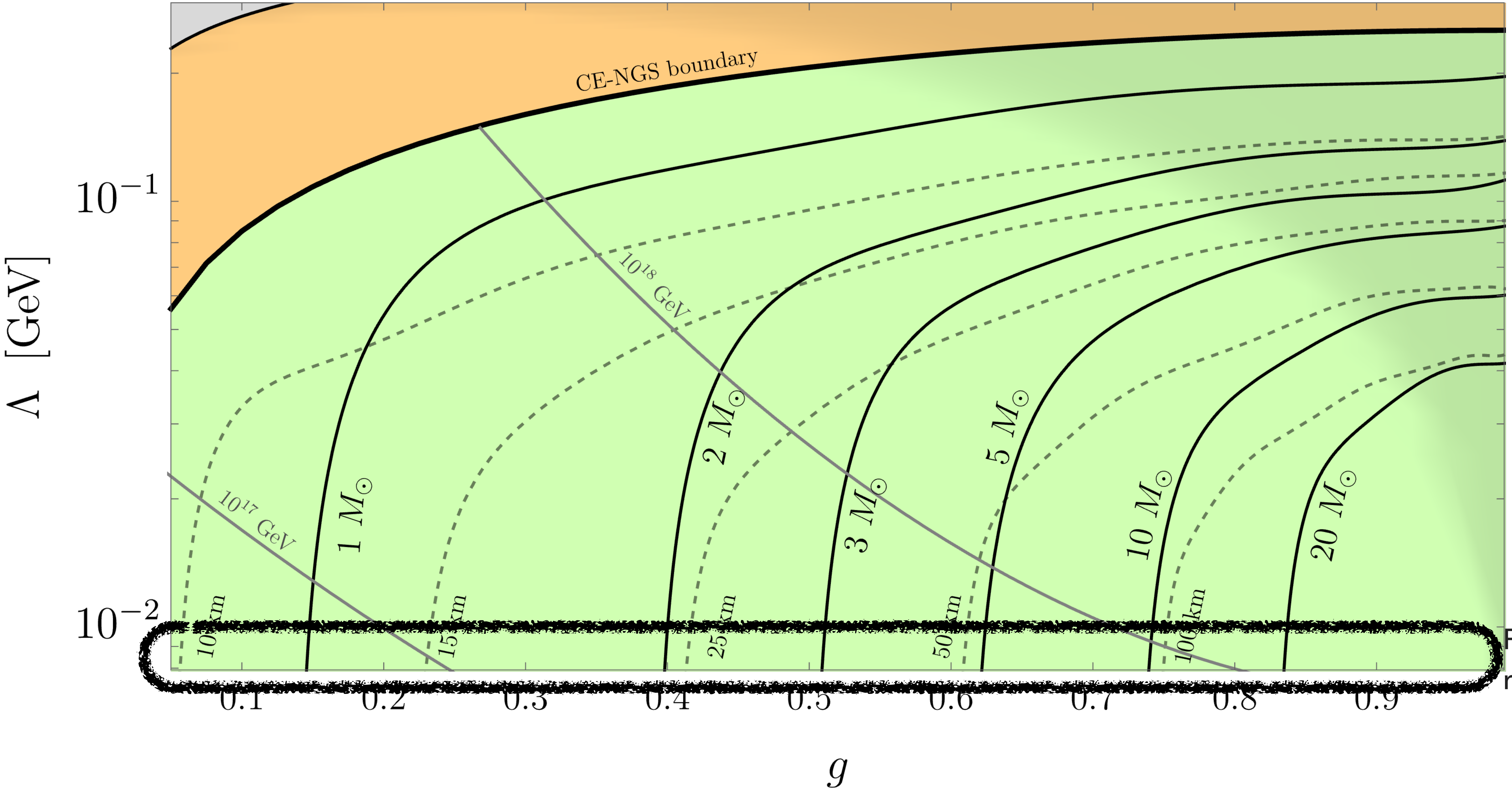
Field has to fit inside R^3



GLOBAL VIEW

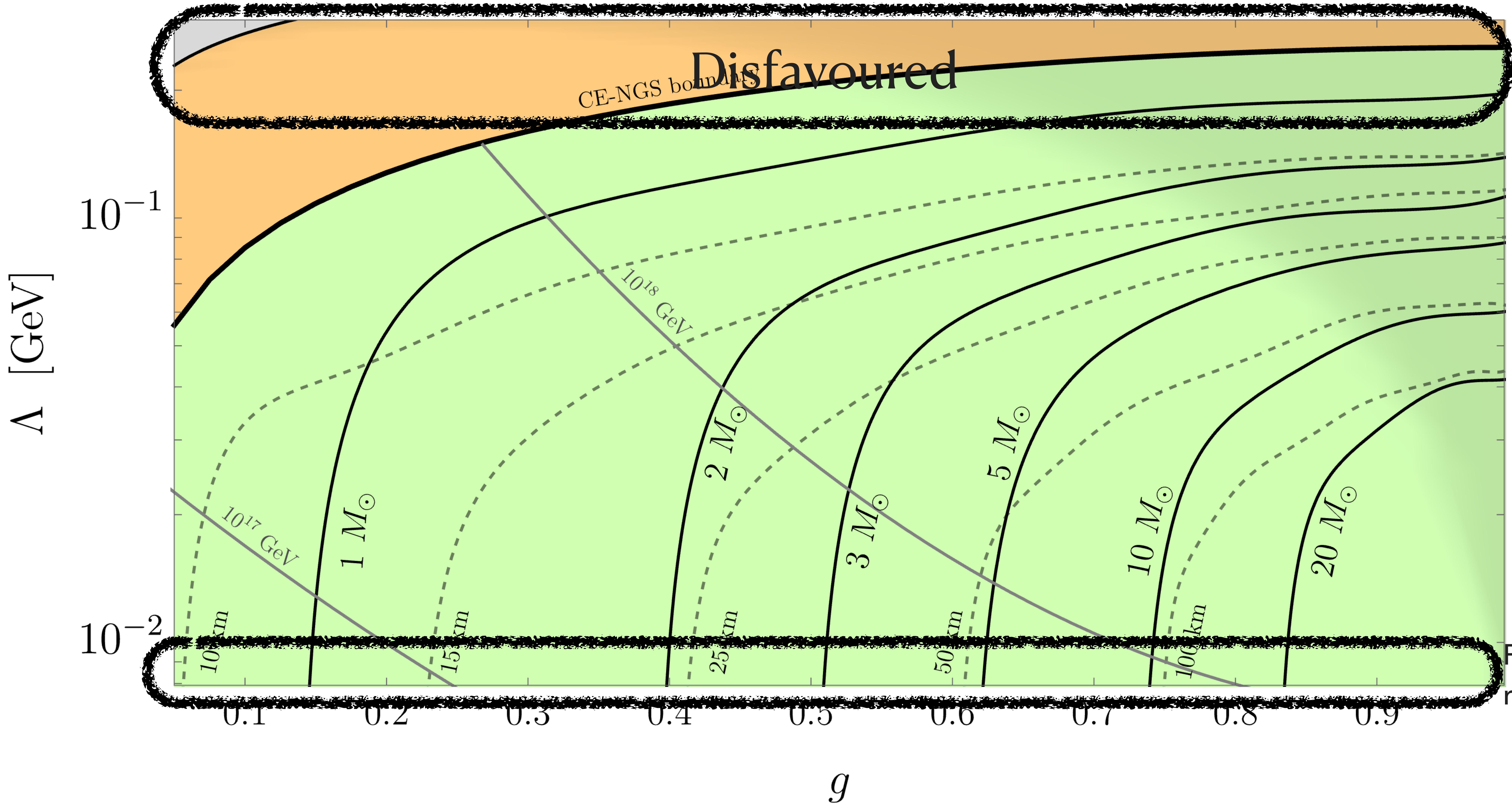

CE-NGS boundary
10^{18} GeV
10^{17} GeV
$1\ M_\odot$
$2\ M_\odot$
$3\ M_\odot$
$5\ M_\odot$
$10\ M_\odot$
$20\ M_\odot$
10 km
15 km
25 km
50 km
100 km
10^{-1}
10^{-2}
Λ [GeV]
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
g

GLOBAL VIEW



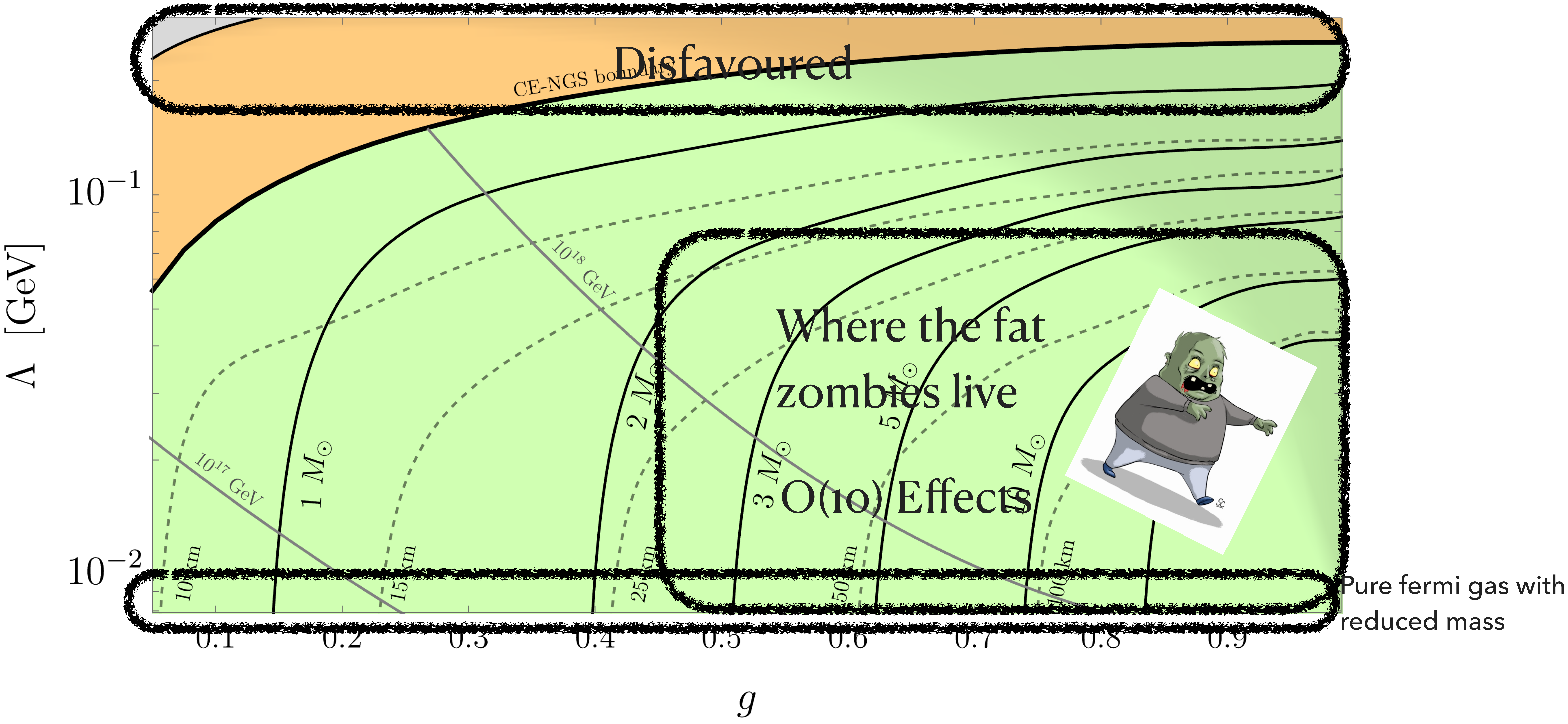
Pure fermi gas with reduced mass

GLOBAL VIEW



Pure fermi gas with reduced mass

GLOBAL VIEW


Disfavoured
CE-NGS boundary
Where the fat zombies live
O(10) Effects
Pure fermi gas with reduced mass
10^{18} GeV
10^{17} GeV
1 $M_\odot$
2 $M_\odot$
3 $M_\odot$
5 $M_\odot$
10 $M_\odot$
10 km
15 km
25 km
50 km
100 km
10^{-1}
10^{-2}
Λ [GeV]
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
g


Masses in the Stellar Graveyard
LIGO-Virgo-KAGRA Black Holes
LIGO-Virgo-KAGRA Neutron Stars
EM Black Holes
EM Neutron Stars
Solar Masses
200
100
50
20
10
5
2
1

SUMMARY AND OUTLOOK

- Coupling to a light scalar
  - ‣ Hybrid stars: disfavored by massive NSs .
  - ‣ A new ground state: $\mathcal{O}(10)$ effects on star properties.

- Finite gradient energy: new phase not accessible in small systems
  → does not mess with nuclear physics
  → evade potential constraints, e.g 5th force, Rhoades and Raffini bound

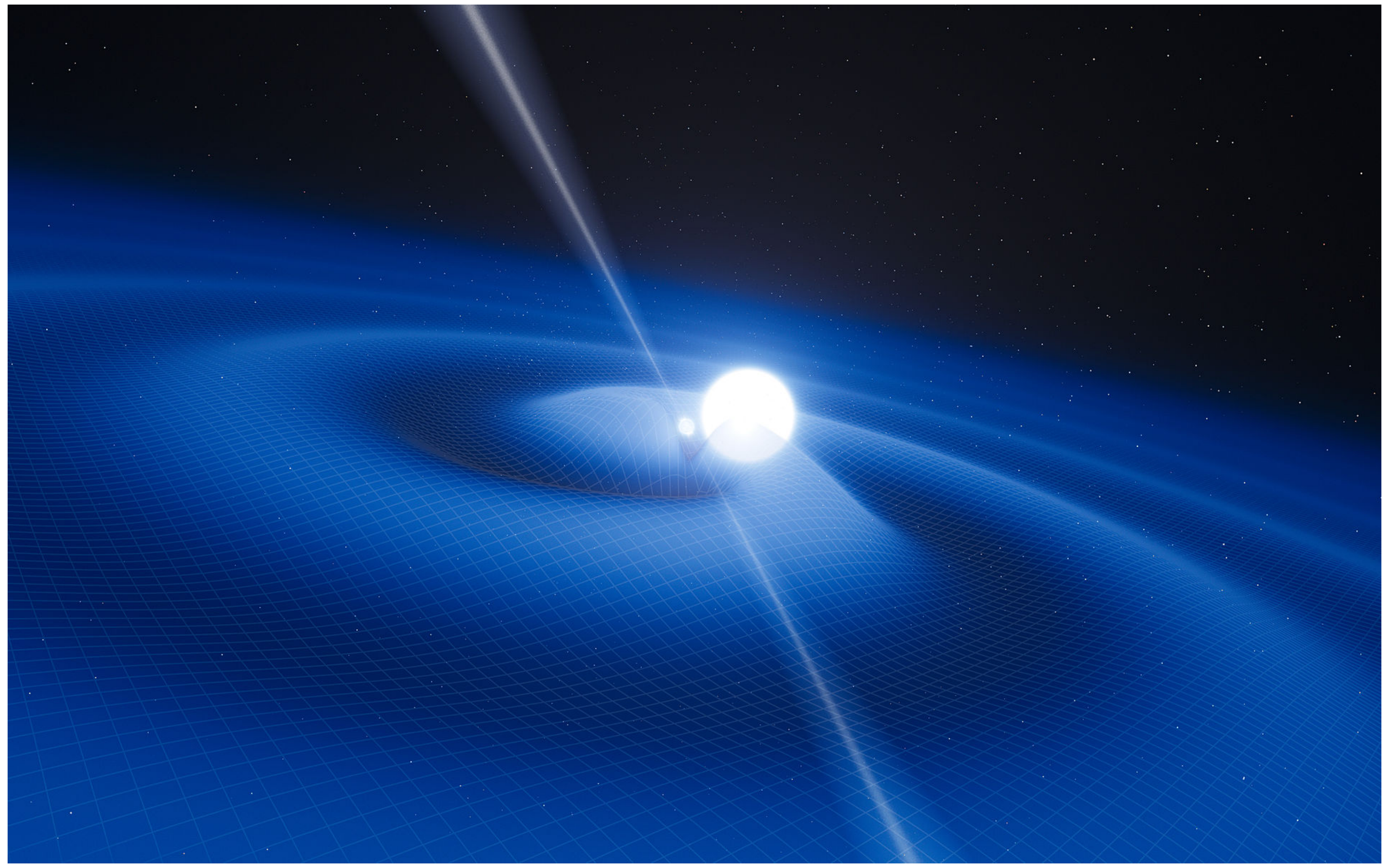
- More to do:
  - Phenomenology of macroscopic-sized self-bound objects
  - Formation?

BACKUP

MOTIVATION

Observation of very massive neutron stars: $M_{\mathrm{NS}} > 2M_{\odot}$

e.g. binary system PSR J0348+0432: Pulsar + Red Giant



MOTIVATION

Observation of very massive neutron stars: $M_{\mathrm{NS}} > 2M_{\odot}$

Hard to explain with Standard Model physics

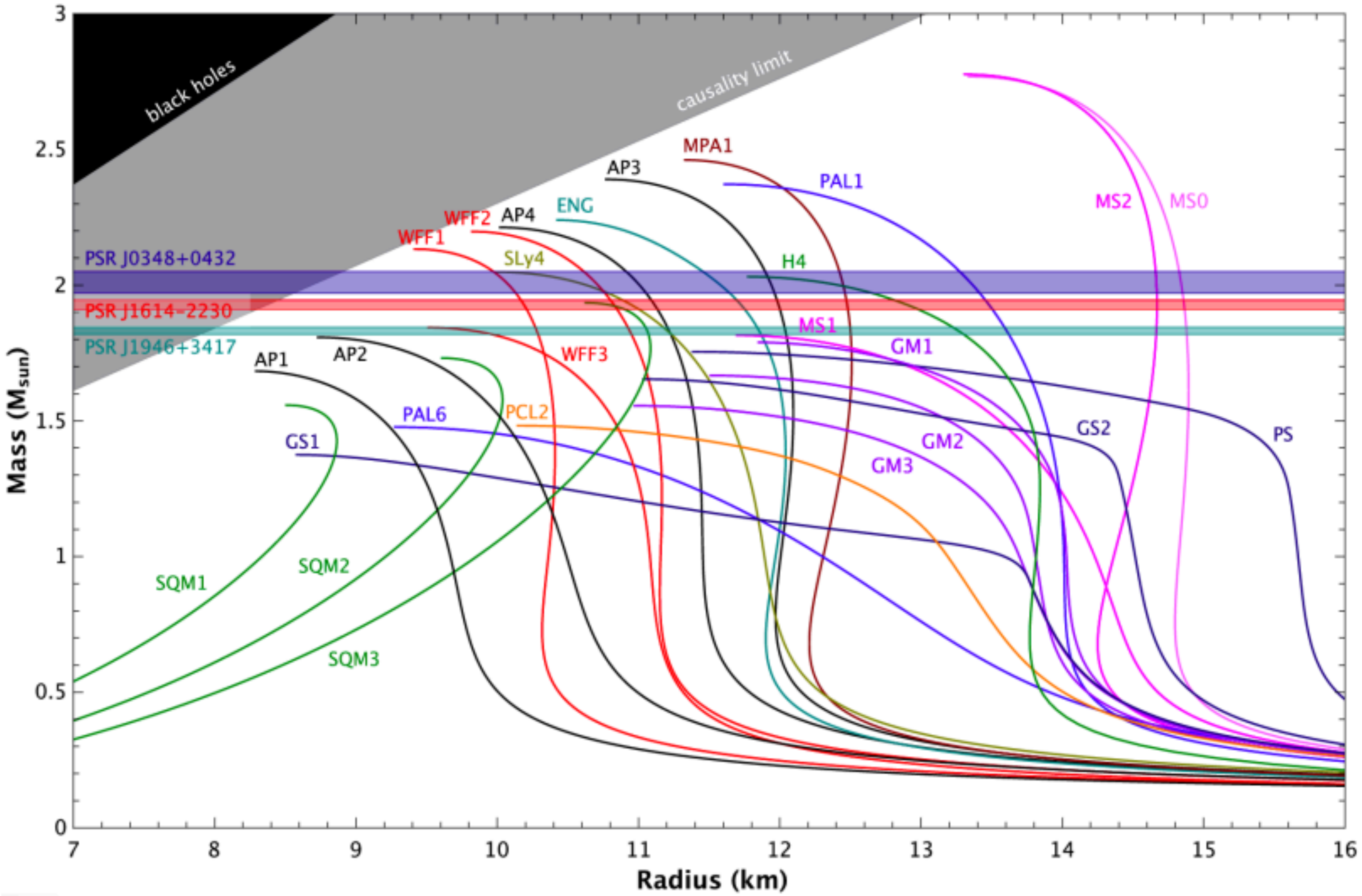
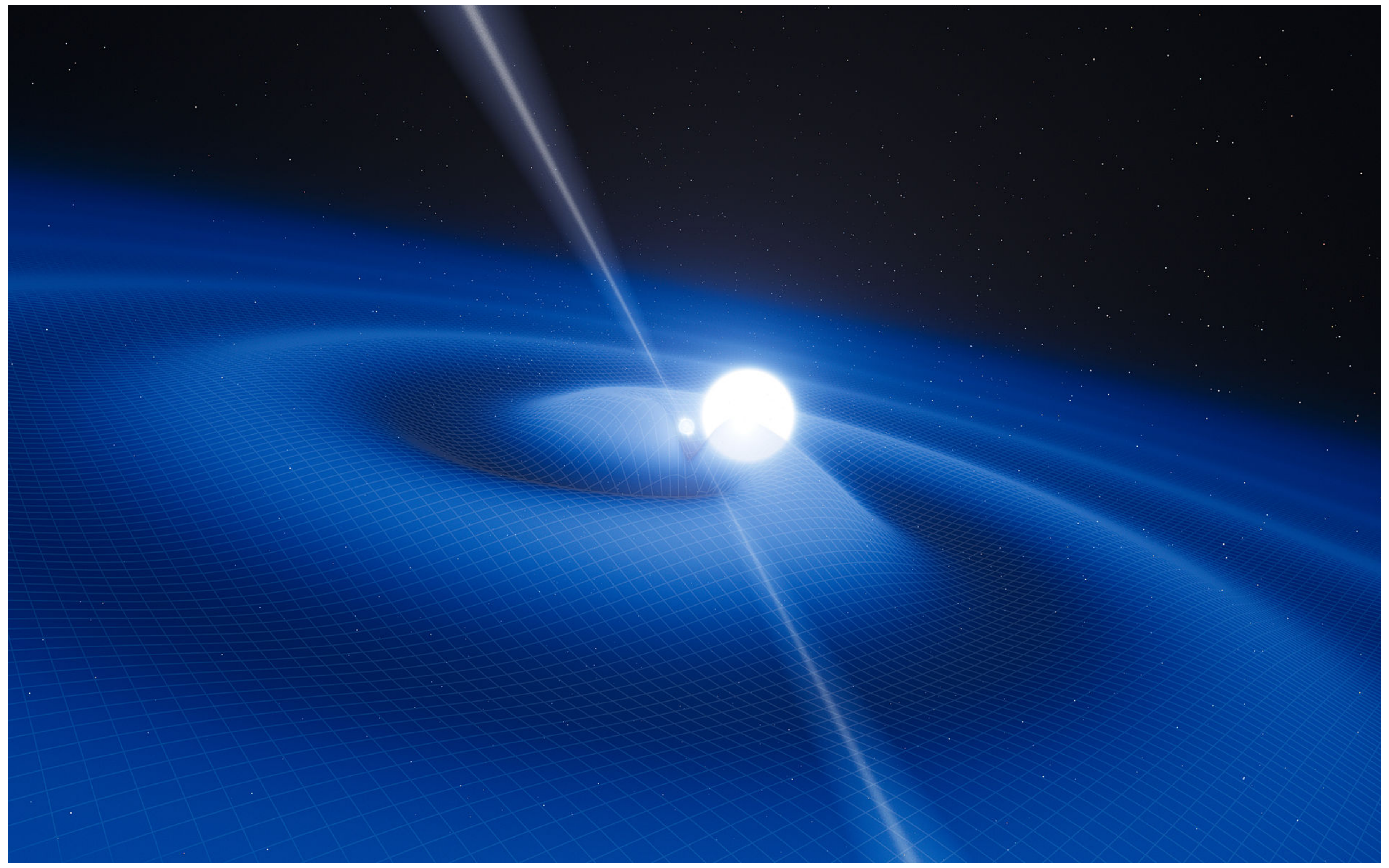


Figure created by Norbert Wex



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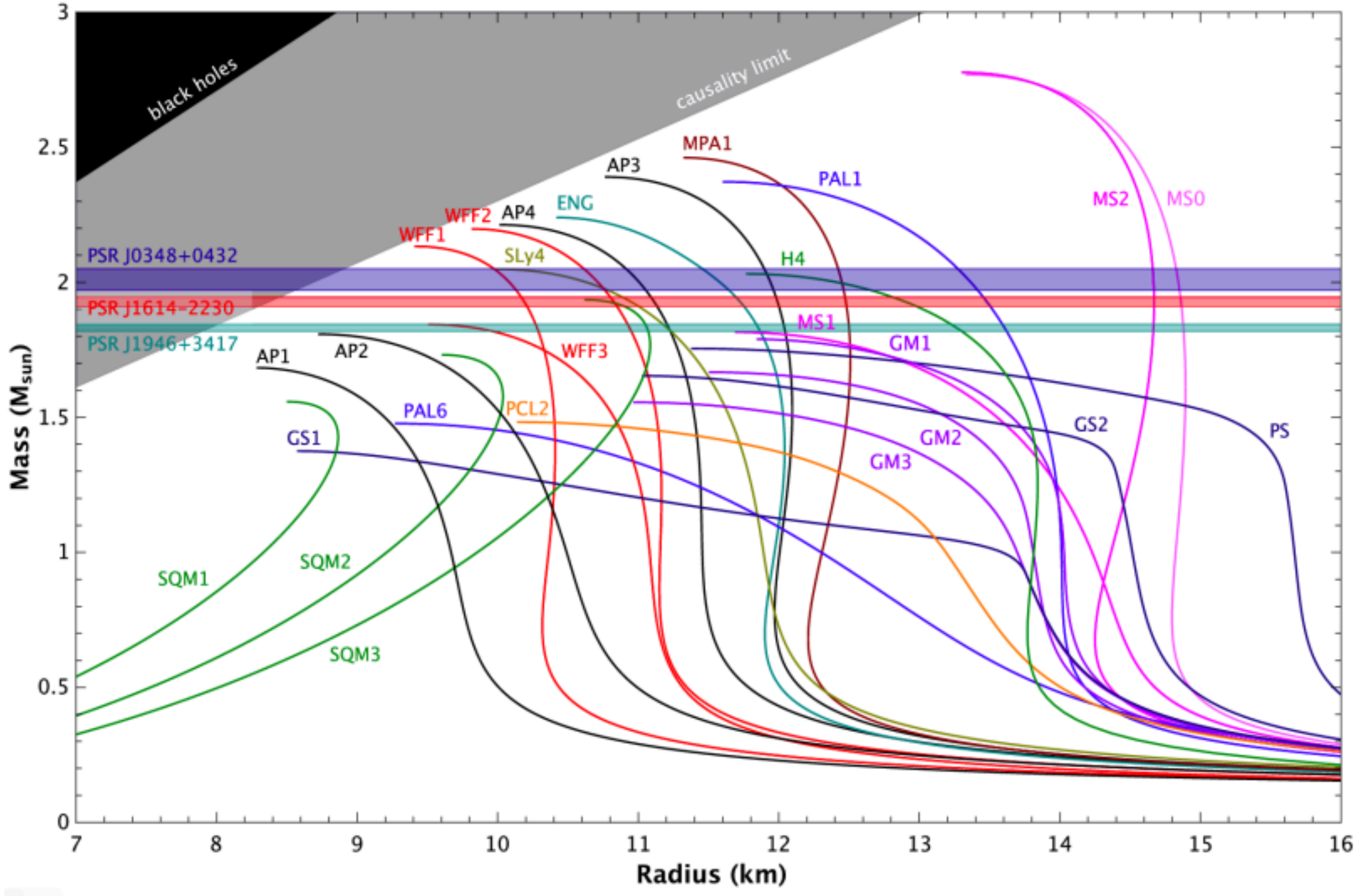


Figure created by Norbert Wex

- Neutron degeneracy pressure vs gravity

$$\Rightarrow M_{\max} \sim O(1)\, M_\odot$$

$$\Rightarrow R_{\max} \sim 10\,\mathrm{km}$$

- SM not well understood at high densities

$$\gtrsim 2\rho_0, \qquad \rho_0 = 0.16\,\mathrm{fm}^{-3}$$

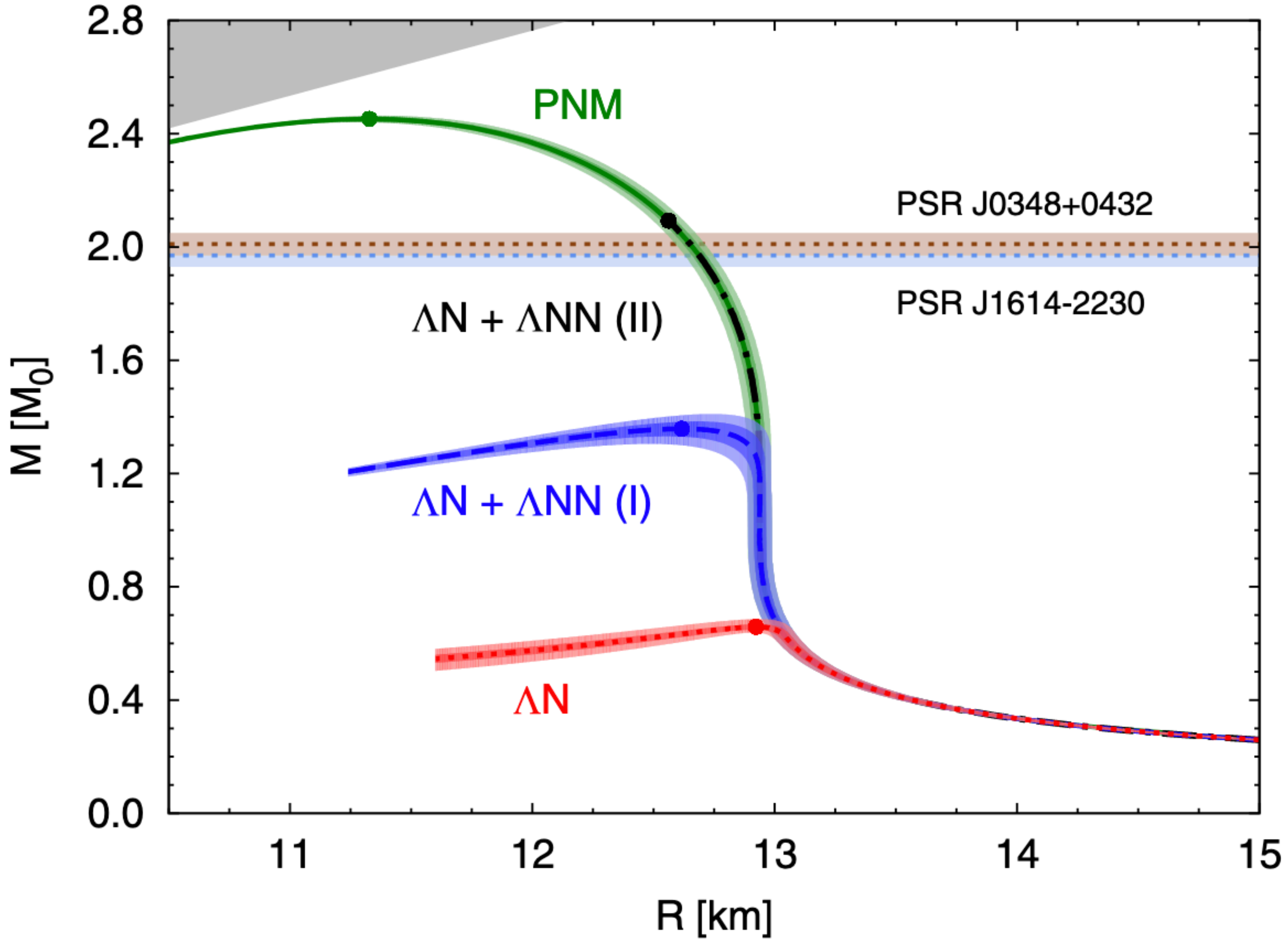
- Complicated: non perurbative nature of QCD, meson condensation, hyperons,..

MOTIVATION

Observation of very massive neutron stars: $M_{\mathrm{NS}} > 2M_{\odot}$

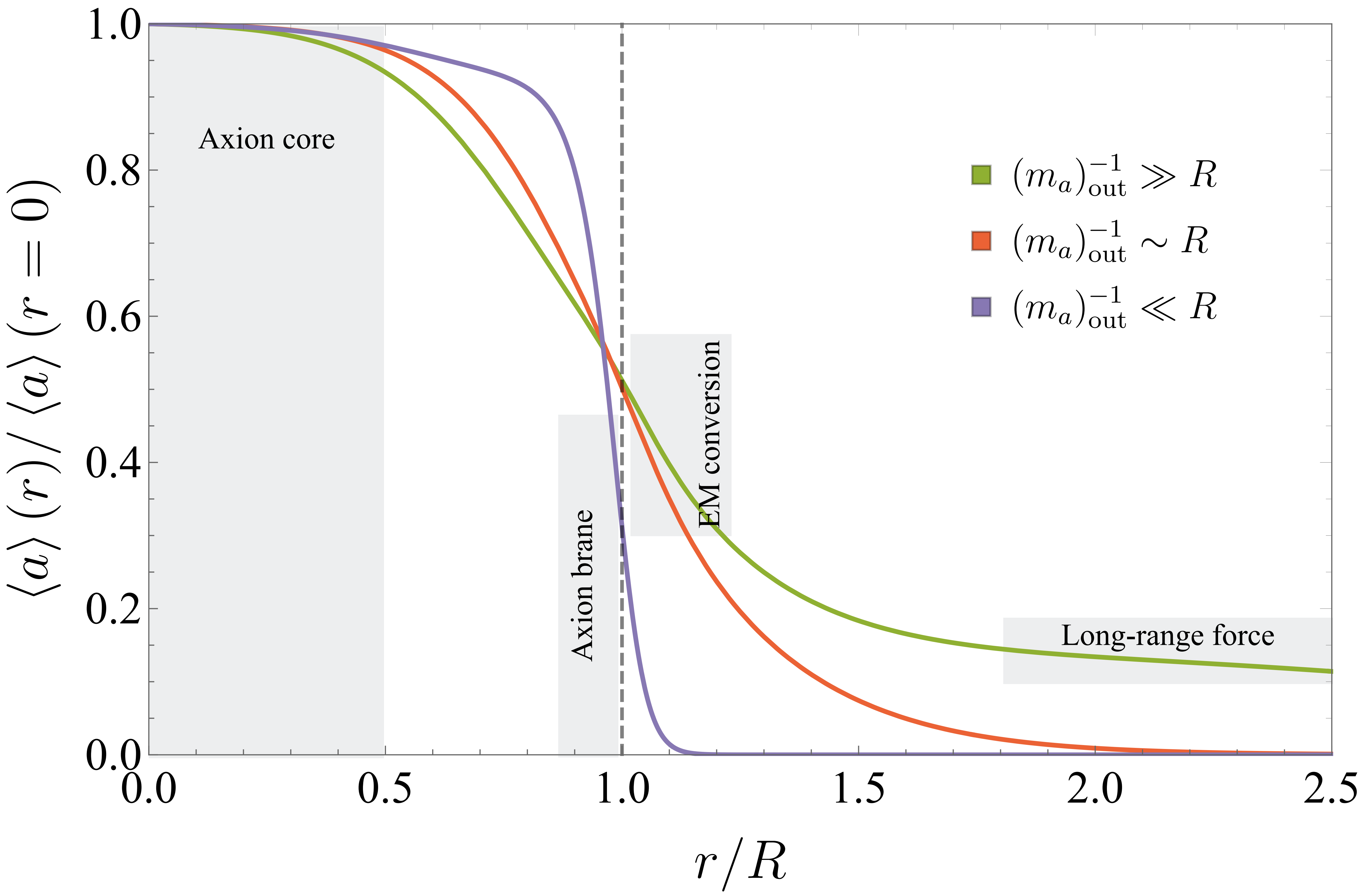
Adding SM degrees of freedom softens equation of state: Leads to lighter neutron stars , e.g. hyperon puzzle, quark stars…

PRL 114, 092301



BASIC IDEA

Scalar field condensation: the axion get’s a vev



New features of NSs

QCD axion:
Balkin, Serra, KS, Weiler
JHEP 07 (2020) 221

ALP-FERMION-GRAVITY SYSTEM: ZERO GRADIENT LIMIT

But why don’t we measure much lighter neutron masses in the lab?

Nuclei are very dense but tiny!

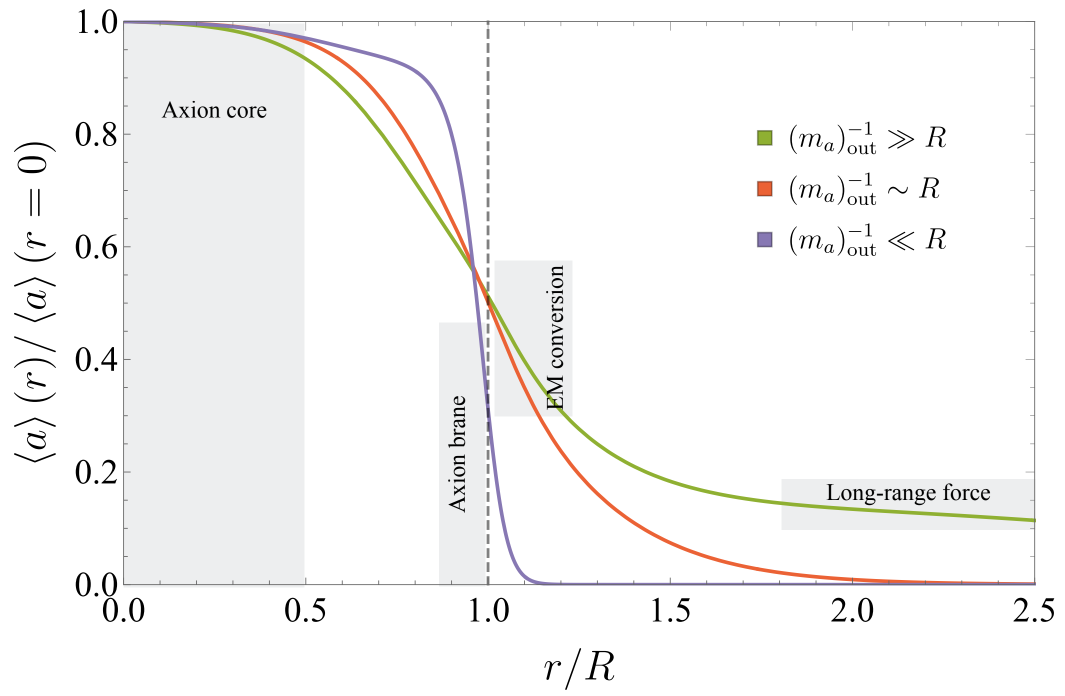
$$\Delta R = \frac{f}{\sqrt{\varepsilon_{\rm pot}}} \ll R$$

ϕ has to change within $R \sim 1\,\mathrm{fm}$ which requires small values f

We do not mess with nuclear physics on earth

ALP-FERMION-GRAVITY SYSTEM: ZERO GRADIENT LIMIT

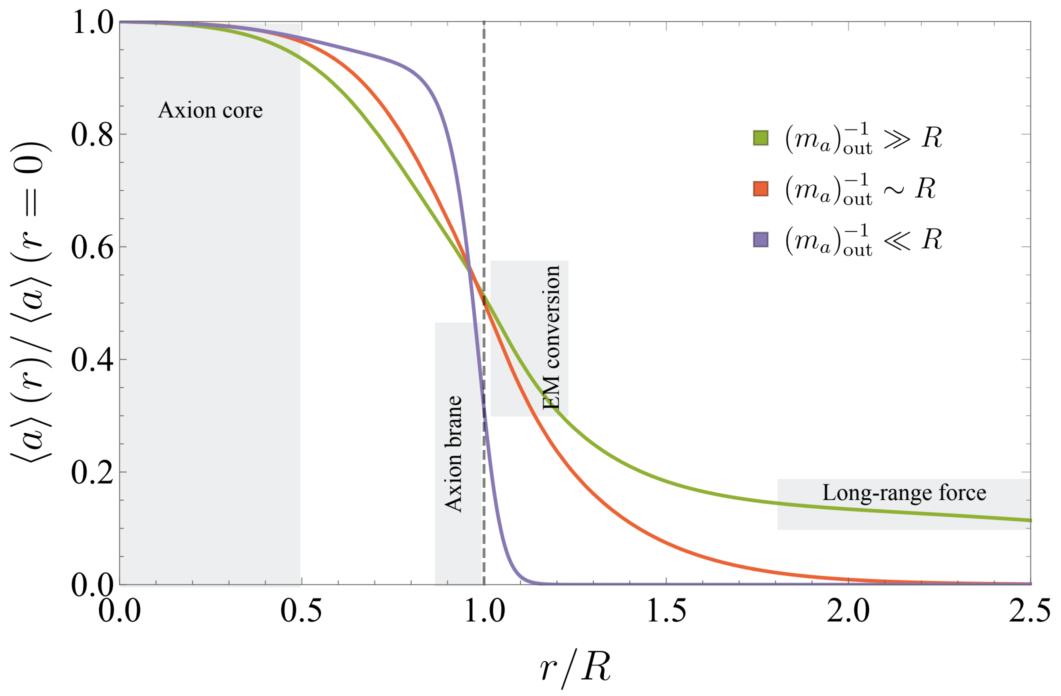
At finite density: ϕ is displaced from its vacuum value



ALP-FERMION-GRAVITY SYSTEM: ZERO GRADIENT LIMIT

At finite density: ϕ is displaced from its vacuum value

This costs energy! $$\varepsilon_{\text{grad}} \simeq \frac{f^2}{\Delta R^2}$$ “Kinetic energy”



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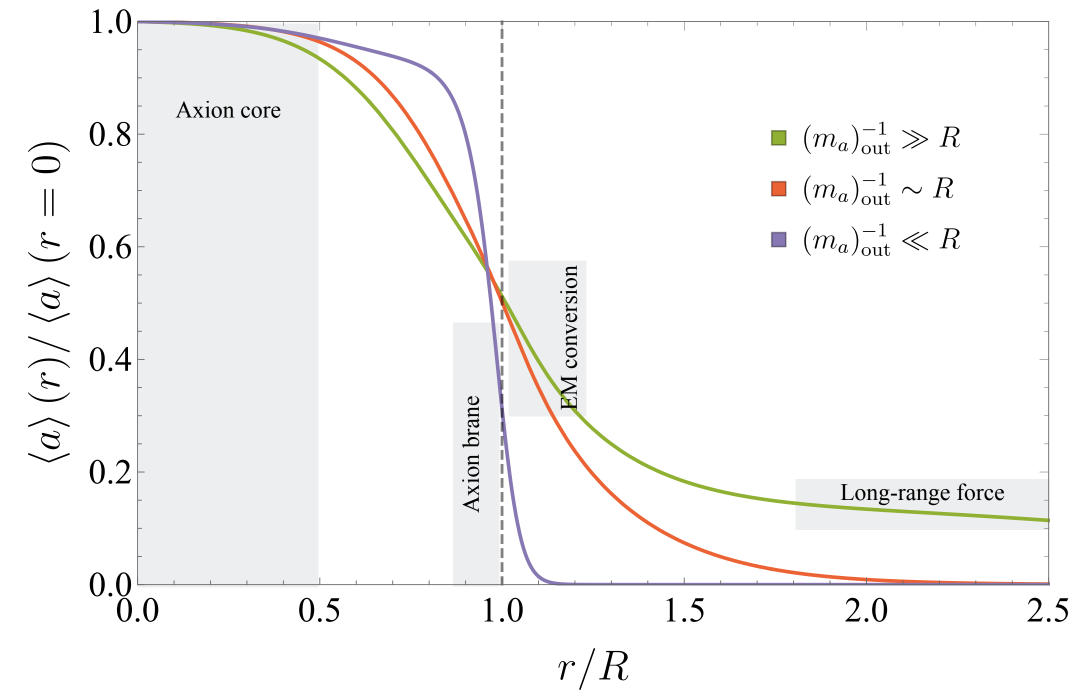

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… which has to be balanced by the gain in potential energy

$$\varepsilon_{\text{pot}} \simeq g\, m_N\, \rho - 2\Lambda^4$$

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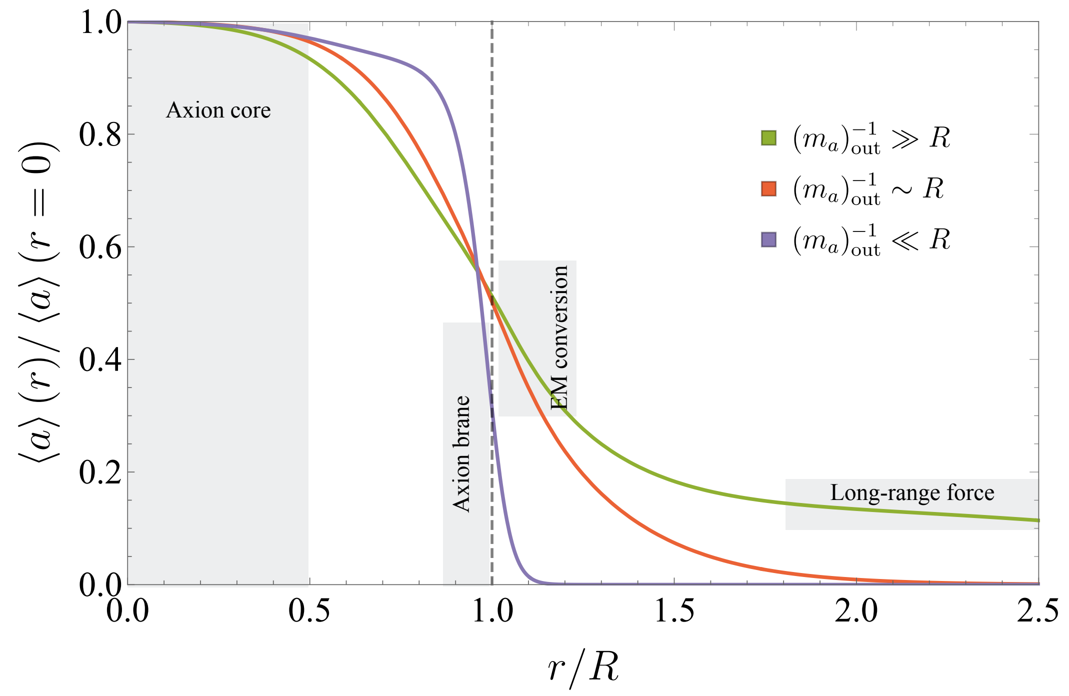

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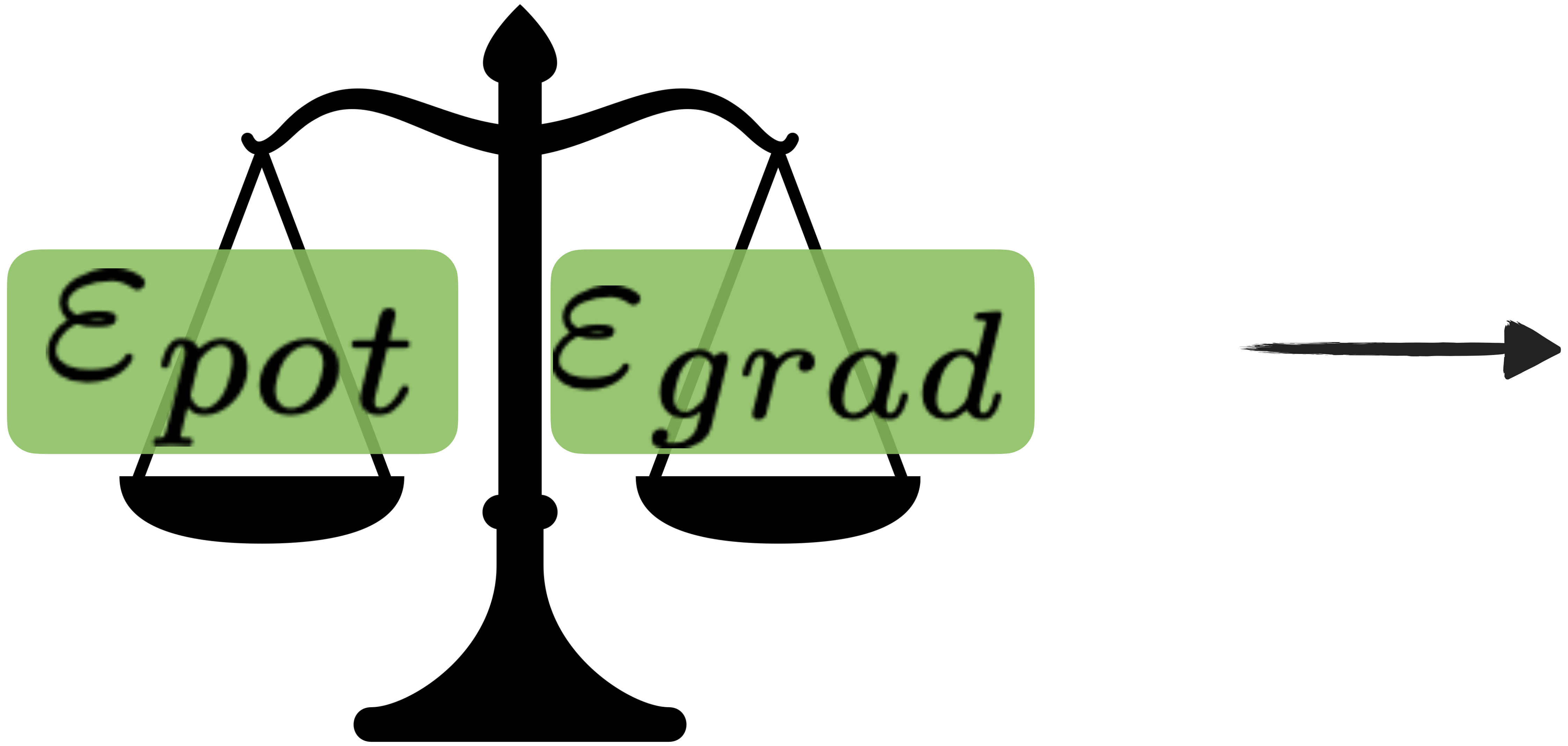

Typical scale of ϕ

$$\Delta R = \frac{f}{\sqrt{\varepsilon_{\rm pot}}}$$

ALP-FERMION-GRAVITY SYSTEM: ZERO GRADIENT LIMIT

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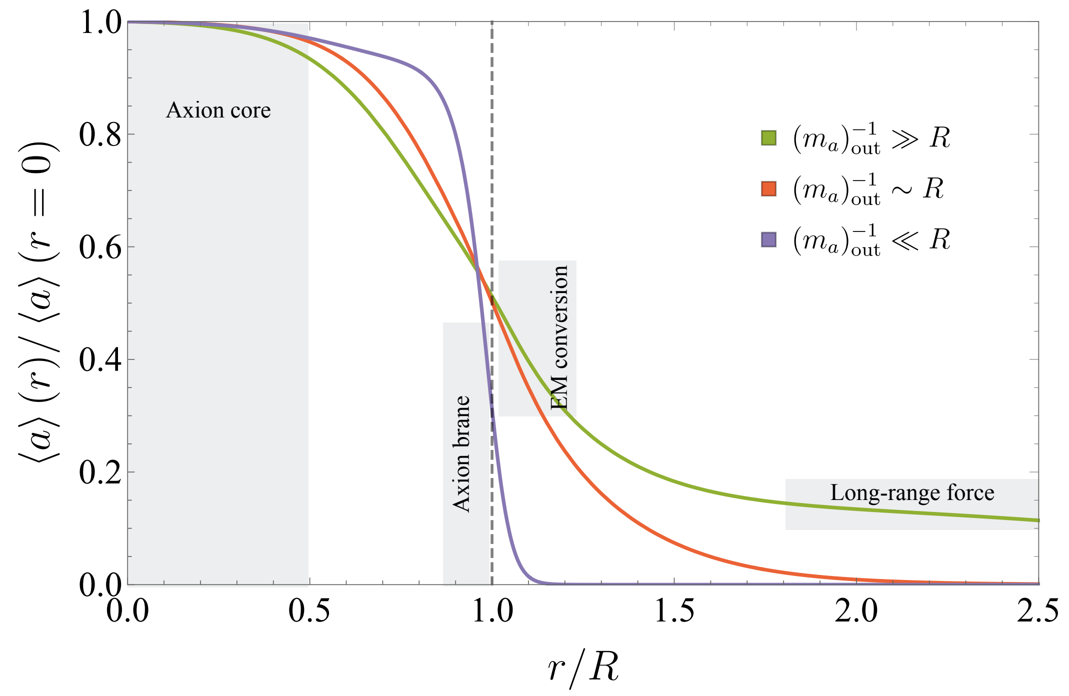

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Consider system large system $R \gg \Delta R$

ALP-FERMION-GRAVITY SYSTEM: ZERO GRADIENT LIMIT

Corresponds to systems much larger than the typical scale of ϕ

$$E(R) \simeq R^2 \Delta R \left(\frac{f}{\Delta R} \right)^2 + R^3 \varepsilon_{\mathrm{pot}} \simeq R^3 \varepsilon_{\mathrm{pot}}$$

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Can forget about the scalar gradient $\partial_\mu \phi = 0$

This is very nice because now the system is effectively decoupled!

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→ Can forget about the scalar gradient $\partial_\mu \phi = 0$

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$\frac{\partial \varepsilon}{\partial \phi} = 0$ + Neutron Fermi gas → Equation of state

$\frac{\delta S}{\delta g_{\mu\nu}} = 0$ → Pressure - Gravity balance equations

(Also known as TOV equations)

BACKUP

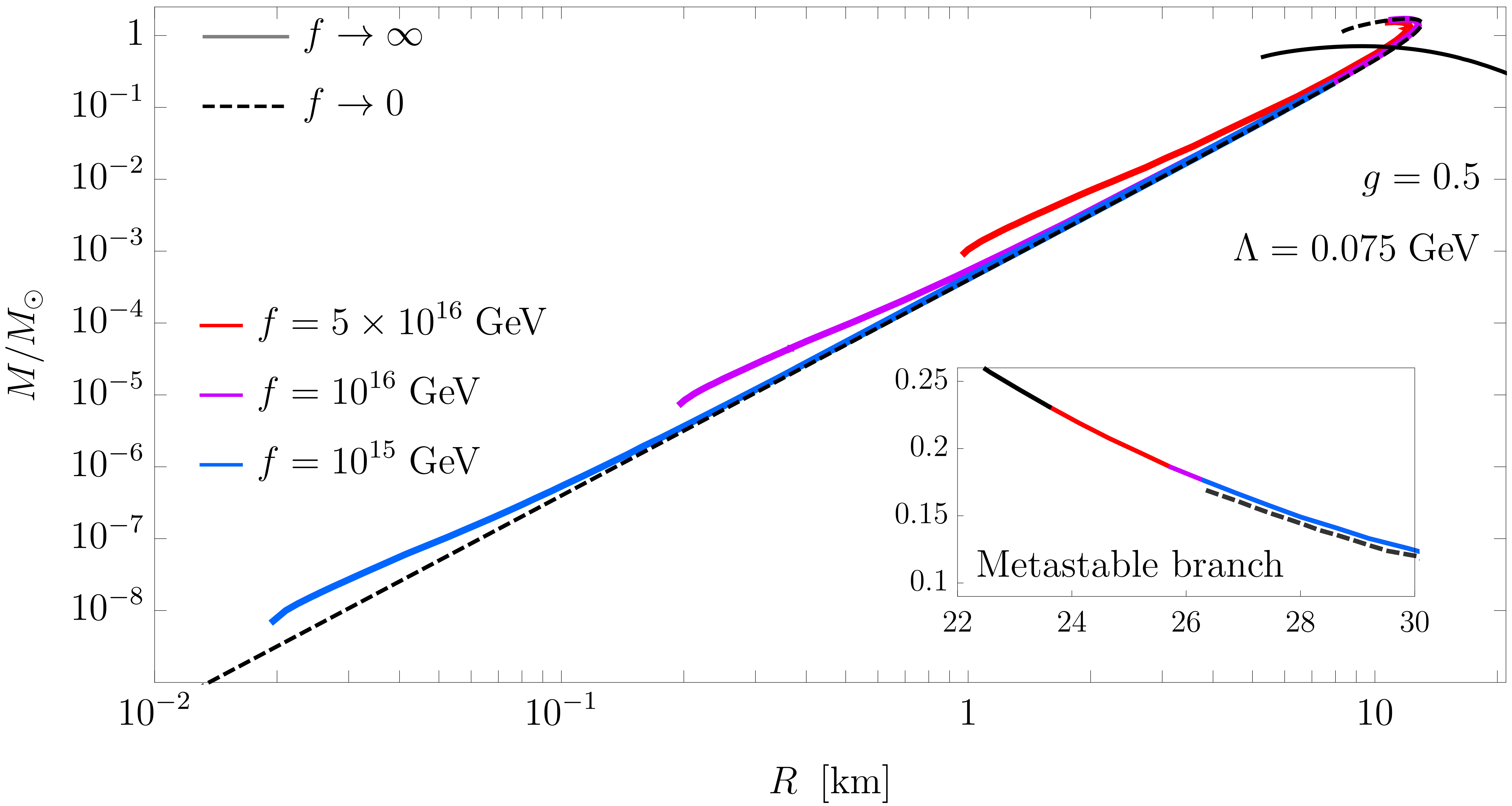
The full coupled system

$$p' + \phi' \left(\frac{\mathrm{d}V}{\mathrm{d}\phi} \right) = - \frac{(\epsilon + p)\, e^{\sigma}}{2r} \left[1 - e^{-\sigma} + \kappa r^2 \left(p + \frac{e^{-\sigma}}{2} (\phi')^2 \right) \right],$$

$$\sigma' = \kappa r e^{\sigma} \left[\epsilon + \frac{e^{-\sigma}}{2} (\phi')^2 \right] - \frac{e^{\sigma} - 1}{r},$$

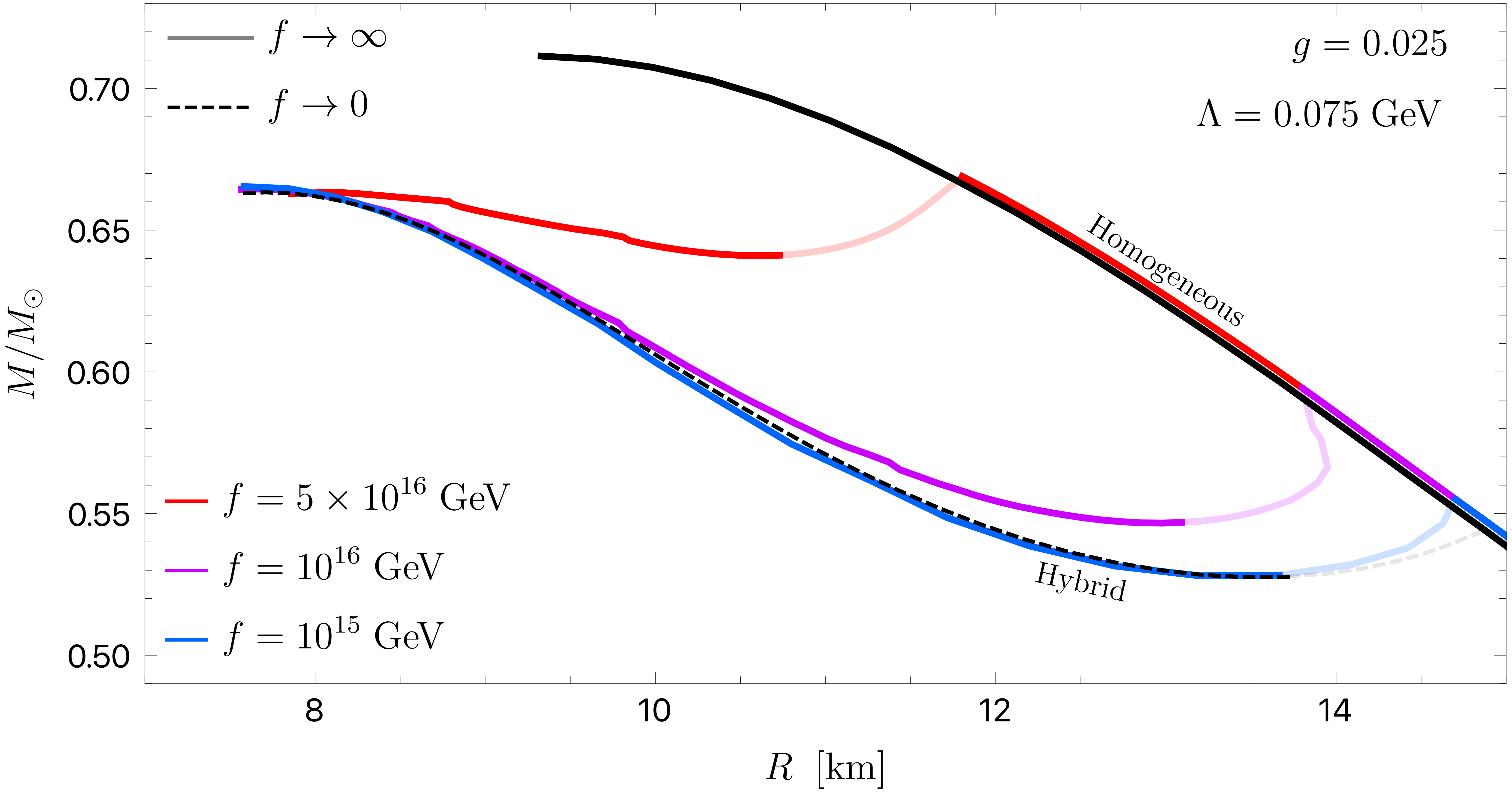
$$\phi'' + \frac{2}{r} \left[\frac{1 + e^{\sigma}}{2} + \frac{\kappa r^2 e^{\sigma}}{4} (p - \epsilon) \right] \phi' = e^{\sigma} \frac{\mathrm{d}V}{\mathrm{d}\phi}.$$

BACKUP



$f \to \infty$
$f \to 0$
$g = 0.5$
$\Lambda = 0.075$ GeV
$f = 5 \times 10^{16}$ GeV
$f = 10^{16}$ GeV
$f = 10^{15}$ GeV
$M/M_\odot$
1
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
10^{-6}
10^{-7}
10^{-8}
10^{-2}
10^{-1}
1
10
R [km]
Metastable branch
0.25
0.2
0.15
0.1
22
24
26
28
30

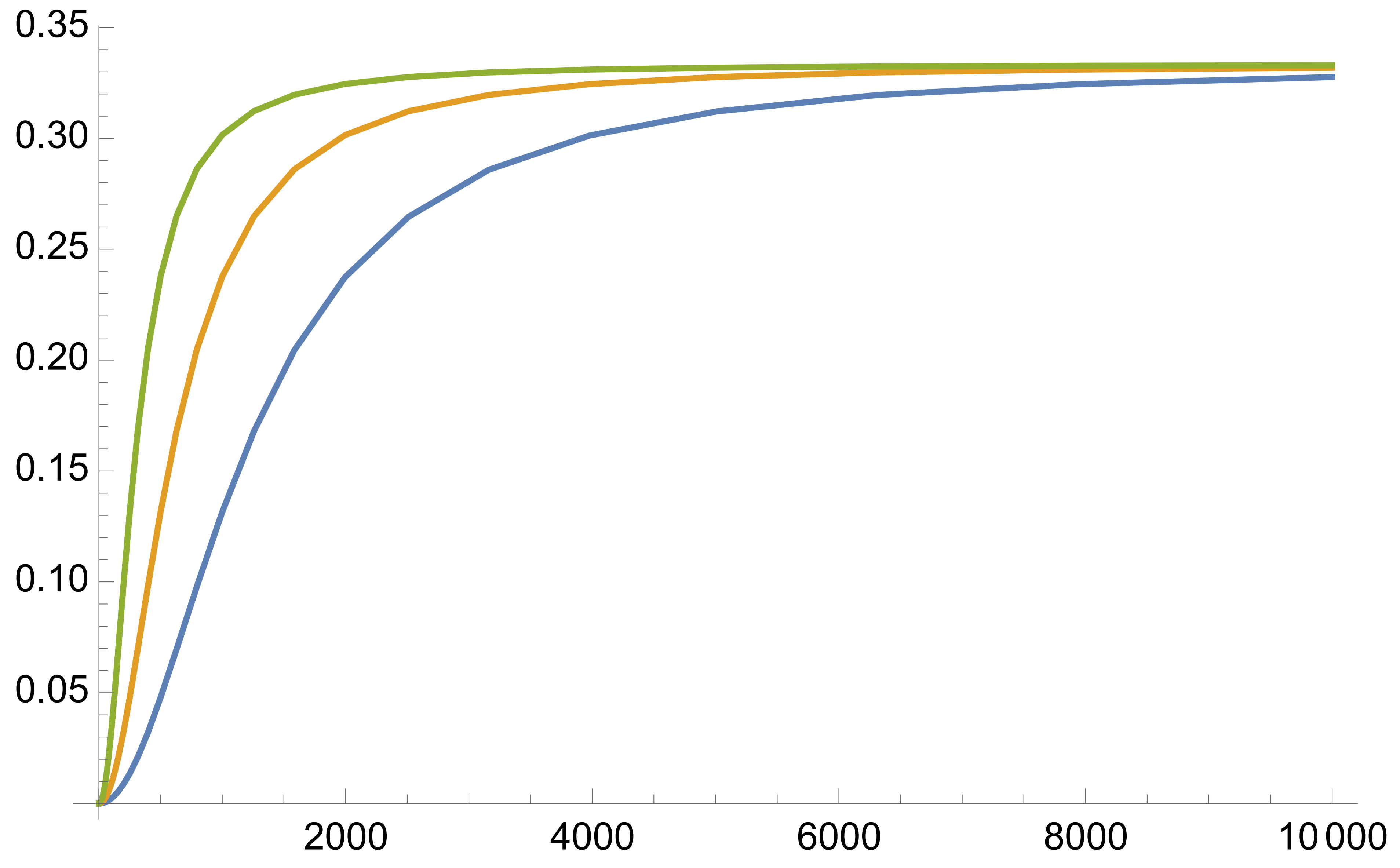
BACKUP



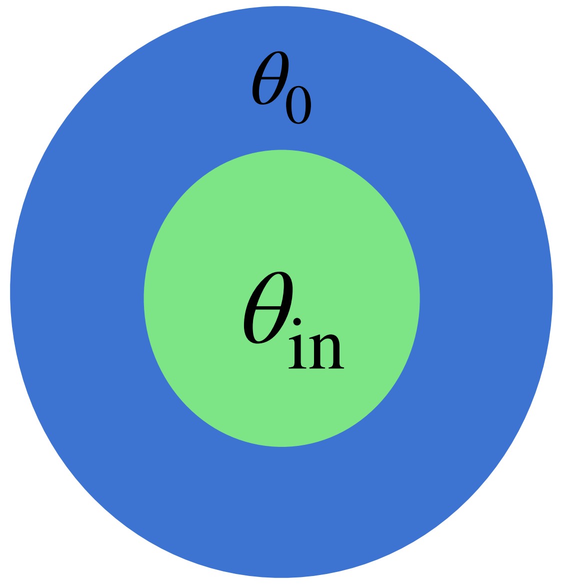
$f \to \infty$
$f \to 0$
$g = 0.025$
$\Lambda = 0.075$ GeV
Homogeneous
Hybrid
$f = 5 \times 10^{16}$ GeV
$f = 10^{16}$ GeV
$f = 10^{15}$ GeV
$M/M_\odot$
0.70
0.65
0.60
0.55
0.50
8
10
12
14
R [km]

BACKUP

Sound speed squared for neutron star EOS with lighter masses



COEXISTENCE PHASE: COEXISTENCE

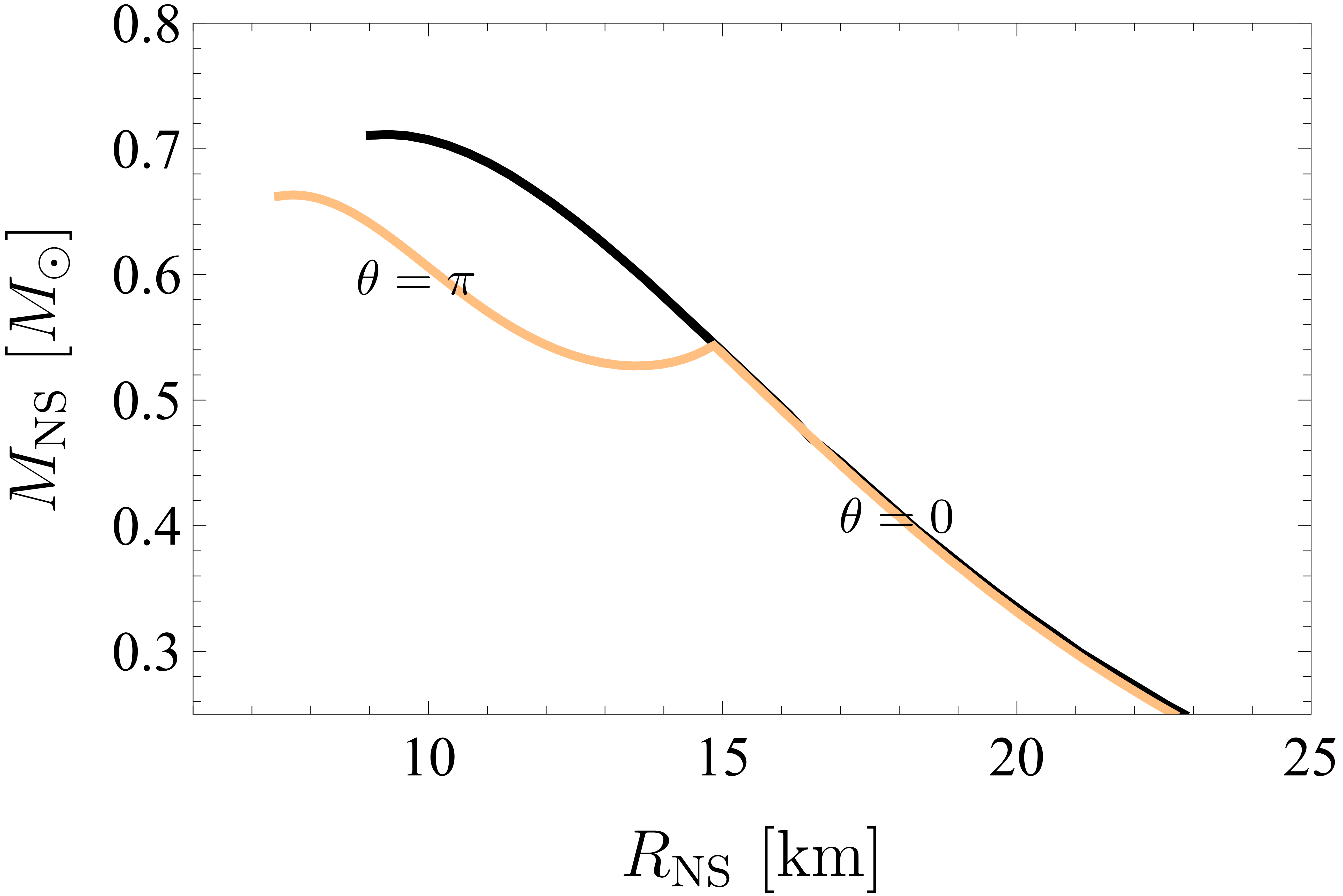


1st order phase transition:

- Generically softens EOSs

  e.g. Kaon condensation

- Clearly disfavoured

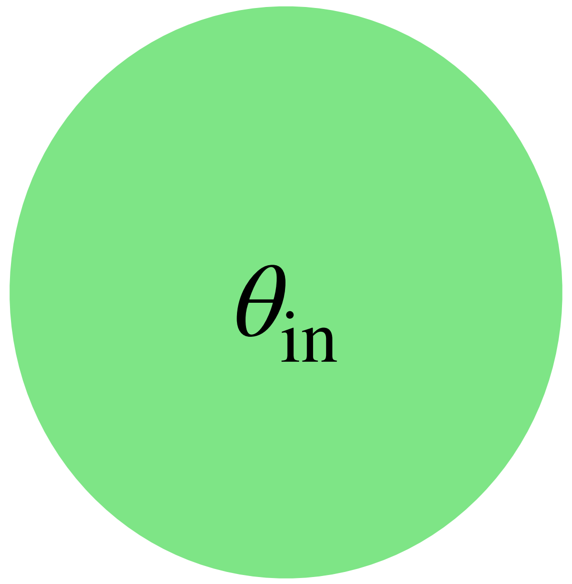


NGS PHASE: FAT ZOMBIES

New ground state

$$\max[M_{\text{max}}] \gg \; M_\odot$$

EOS (can be) stiffer!



Also interesting on a log plot

Gravity becomes important